



# QCD and EW corrections for precision Montecarlo generators

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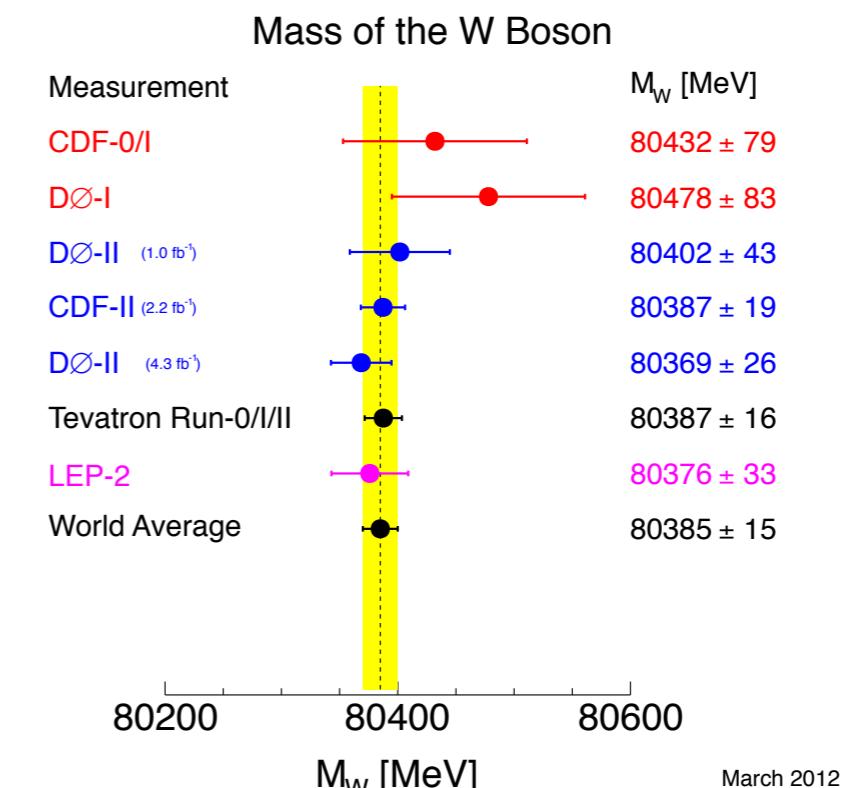
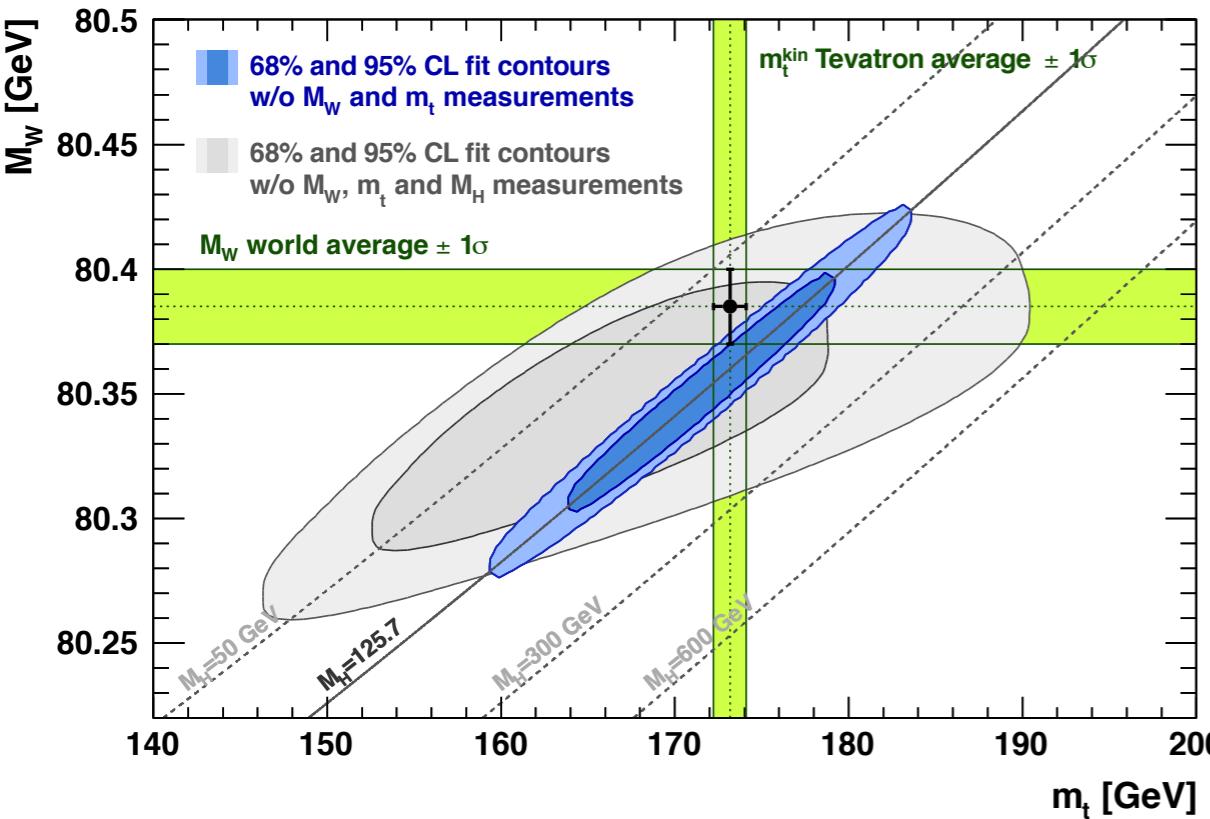
TIFR, January 6th 2014

# Plan of the talk

- observables relevant for the MW measurement
- QCD effects in fixed order and relevance of all-orders resummation of multiple parton emissions
- ambiguities affecting the W transverse momentum distribution, which in turn affect the observables relevant for MW
- QED/EW effects in fixed order (NLO and beyond)
- relevance of all-orders resummation of multiple photon emissions
- non-universality of ISR corrections
- approximations to include the bulk of mixed QCDxEW effects
- the POWHEG solution
- remaining ambiguities related to different equivalent prescriptions
- classification of the impact of different classes of radiative corrections / sources of uncertainties as a shift in the extracted MW value

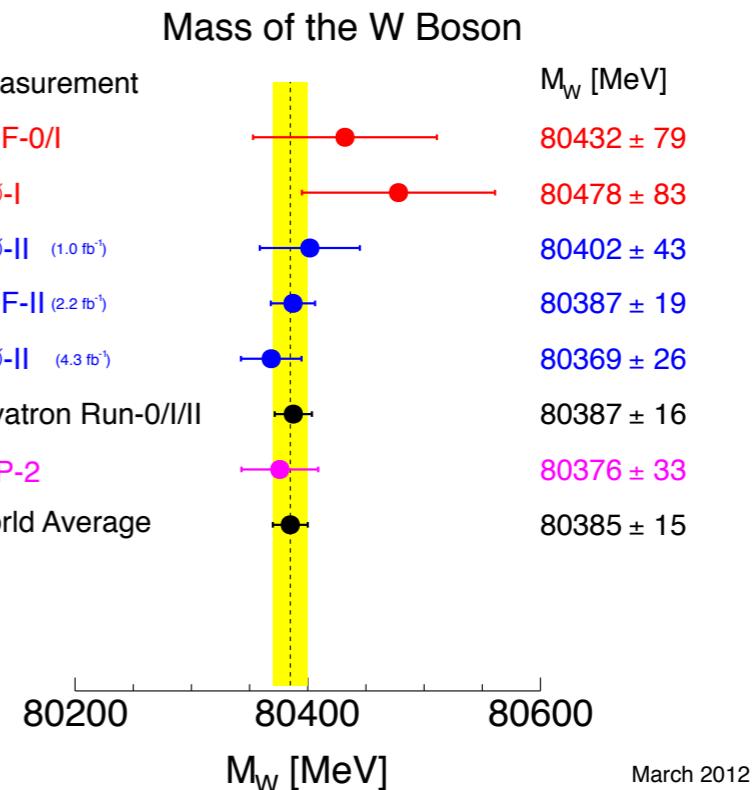
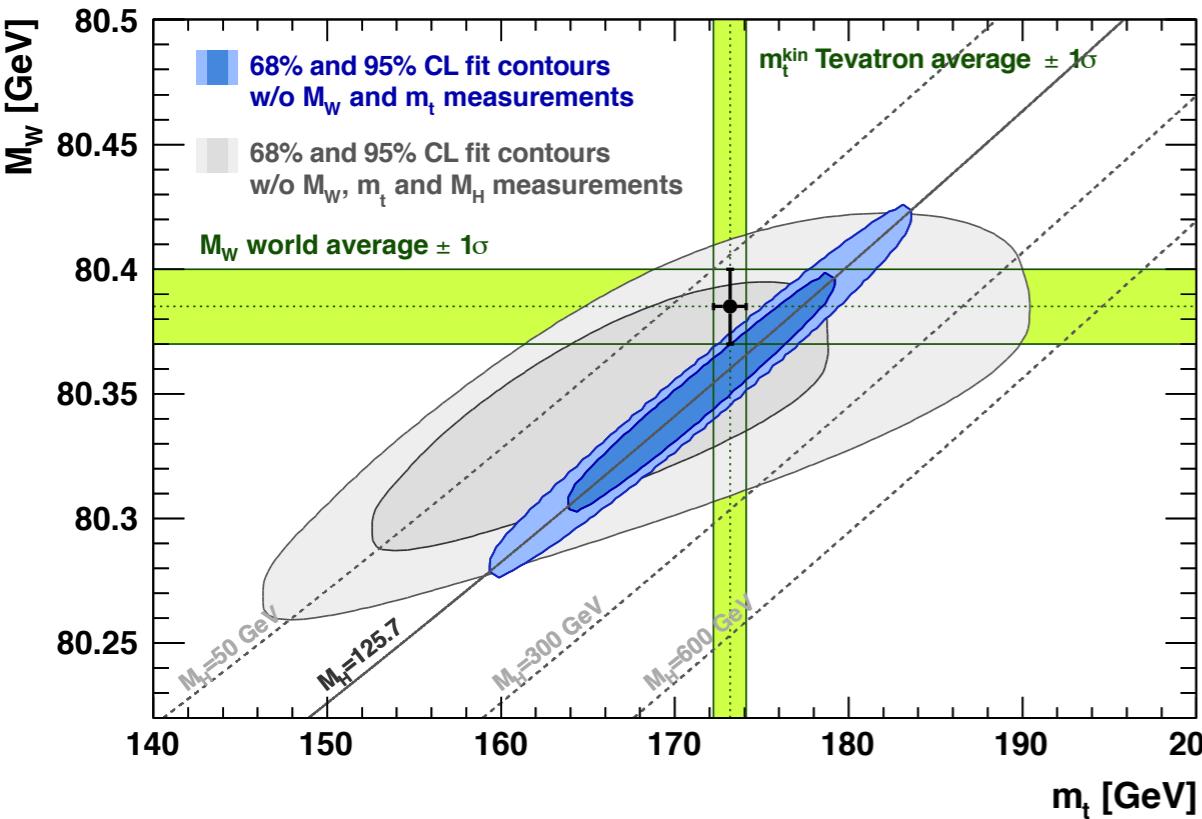
# Motivations

A precise measurement of  $M_W$  provides a crucial test of the SM

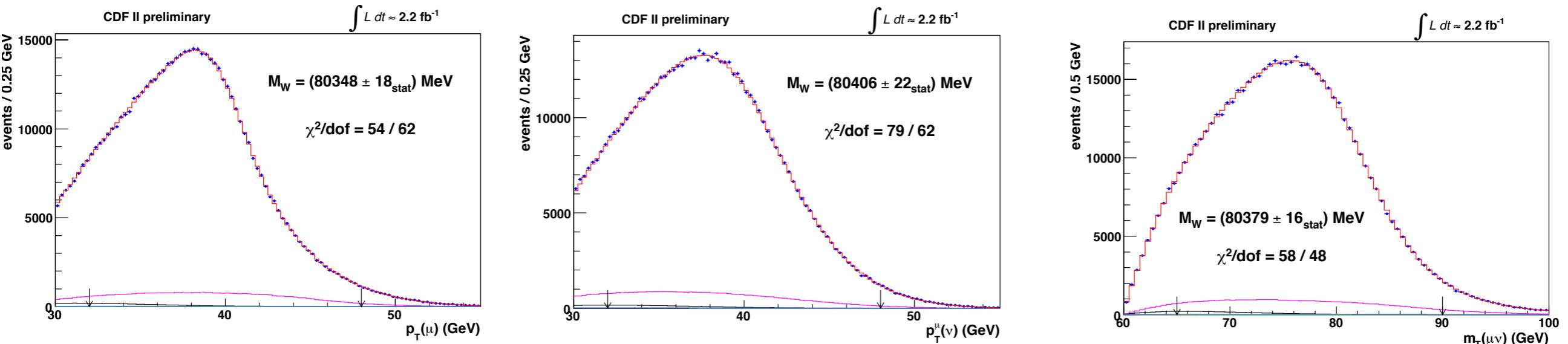


# Motivations

A precise measurement of MW provides a crucial test of the SM



MW is extracted with a template fit technique of various distributions of CC-DY  
 An event generator that includes the best available results in terms of radiative corrections  
 is necessary to minimize the theoretical systematic error in the fit

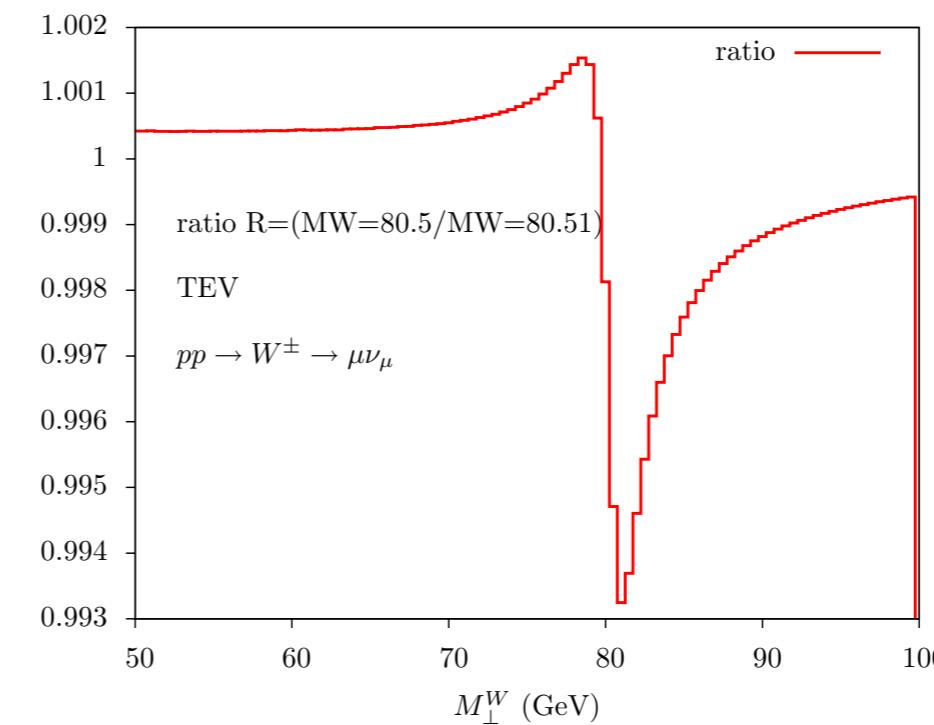
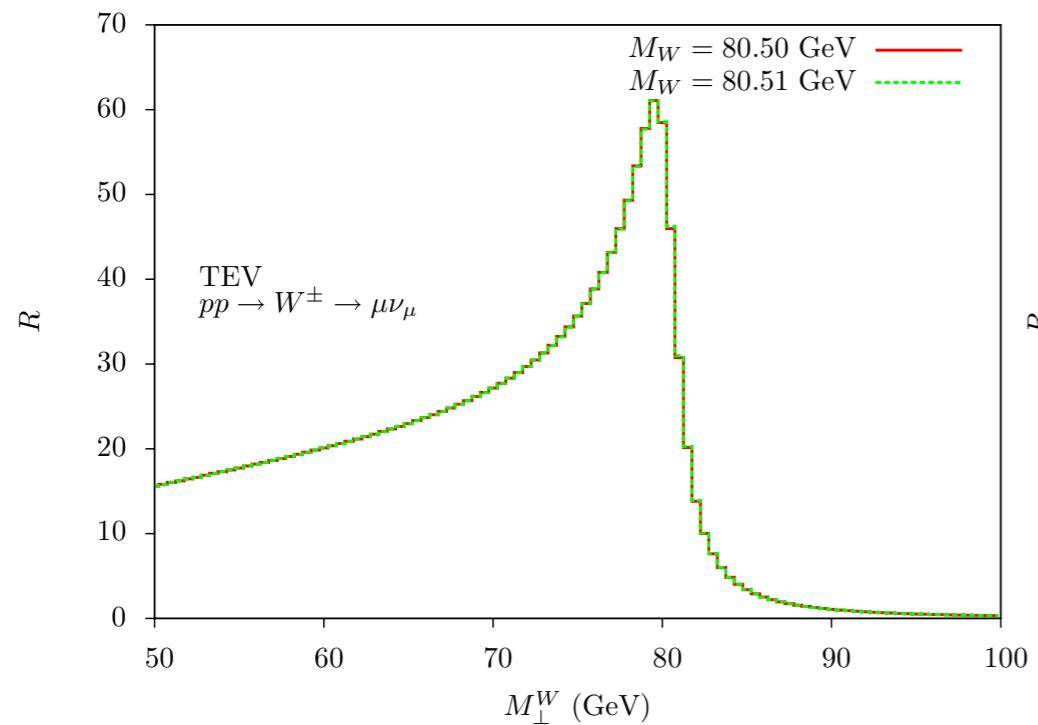


# Template fit and theoretical accuracy

In a template fit approach

- the best theoretical prediction for a distribution is computed several times, with different values of MW
- each template is compared to the data
- the measured MW is the one of the template that maximizes the agreement with the data

Which level of accuracy do we need?

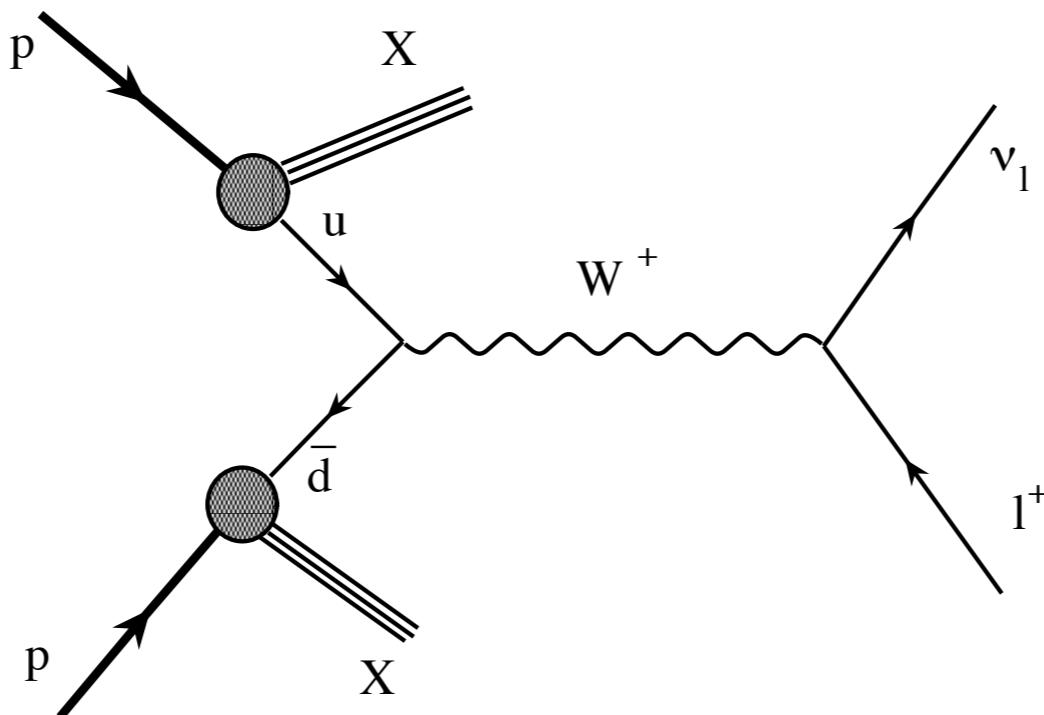


If we aim at measuring MW with **10 MeV** of error, are we able to control the **shape** of the distributions and the theoretical uncertainties at the **few per mille level?**

Not all the radiative corrections have the same impact on the MW measurement  
not all the uncertainties are equally bad on the final error

# Perturbative expansion of the Drell-Yan cross section

$$\sigma_{tot} = \sigma_0 + \boxed{\alpha_s \sigma_{\alpha_s} + \alpha_s^2 \sigma_{\alpha_s^2} + \dots} \quad \text{QCD}$$
$$+ \boxed{\alpha \sigma_\alpha + \alpha^2 \sigma_{\alpha^2} + \dots} \quad \text{EW}$$
$$+ \boxed{\alpha \alpha_s \sigma_{\alpha \alpha_s} + \alpha \alpha_s^2 \sigma_{\alpha \alpha_s^2} + \dots} \quad \text{mixed QCDxEW}$$



Which corrections modify the shape of the distributions?  
affect the extraction of MW?

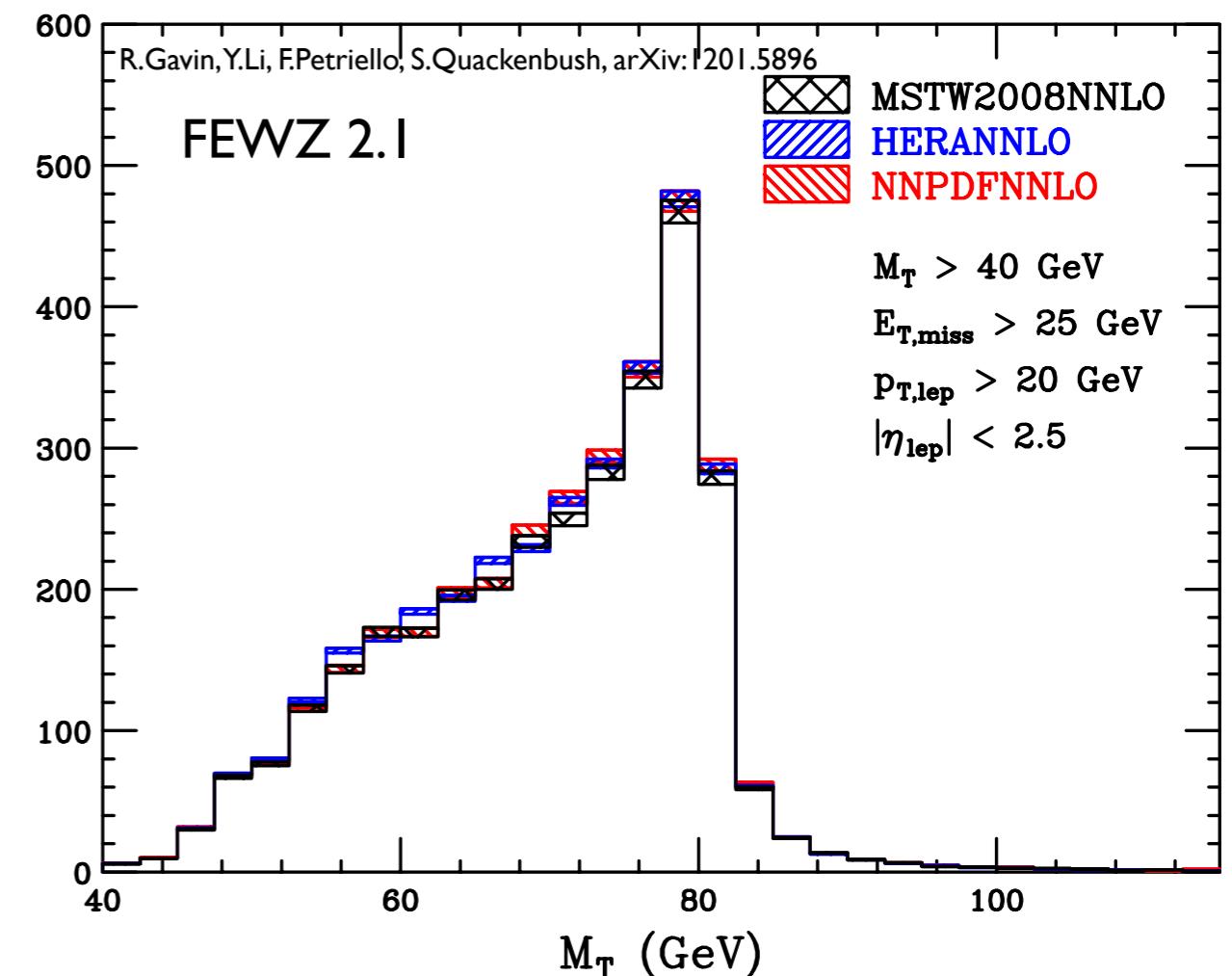
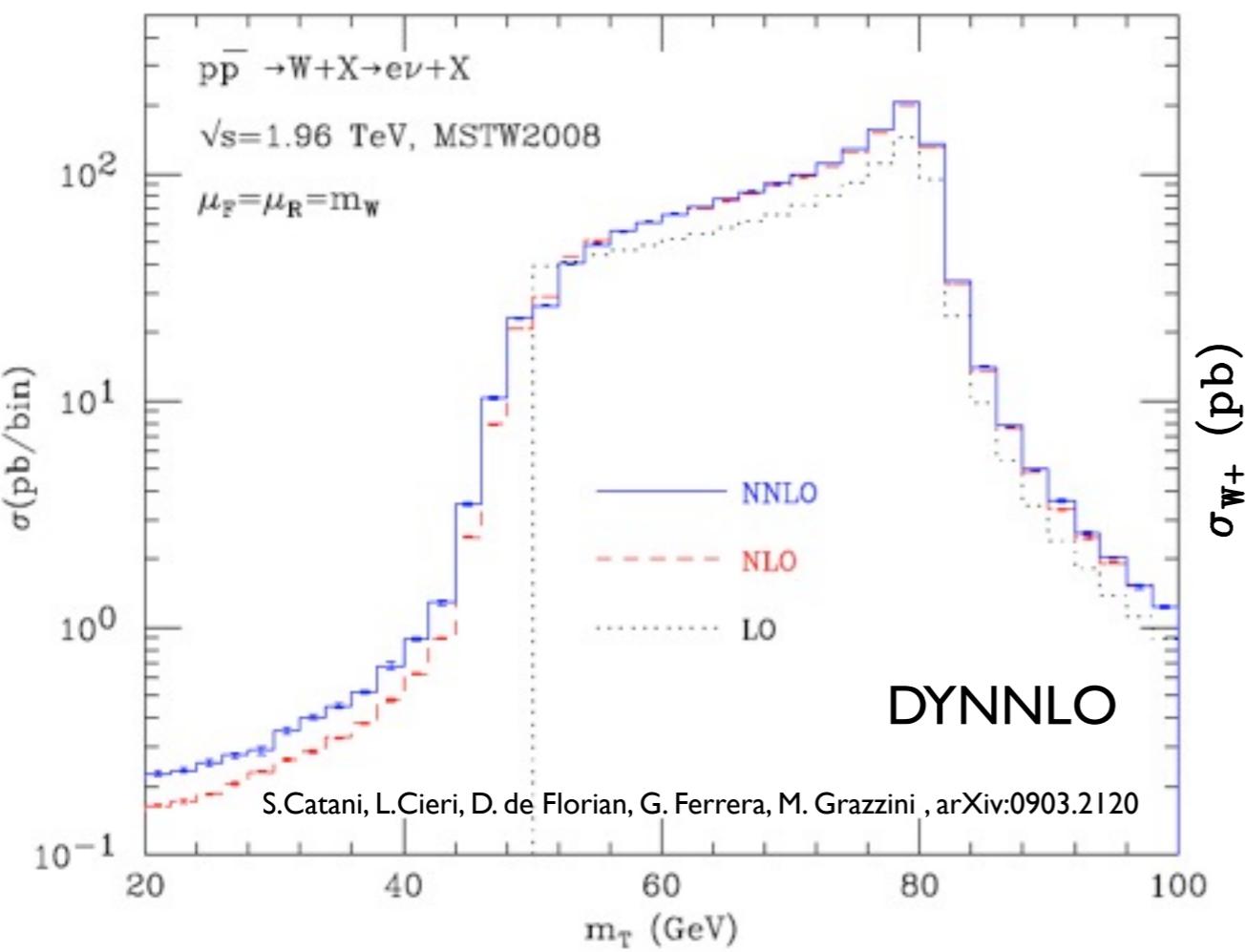
# Perturbative expansion of the Drell-Yan cross section

$$\sigma_{tot} = \sigma_0 + \boxed{\alpha_s \sigma_{\alpha_s} + \alpha_s^2 \sigma_{\alpha_s^2}} + \dots \quad \text{MCFM,FEWZ,DYNNLO}$$

$$+ \boxed{\alpha \sigma_\alpha} + \alpha^2 \sigma_{\alpha^2} + \dots \quad \text{WGRAD, RADY, HORACE, SANC}$$

$$+ \alpha \alpha_s \sigma_{\alpha \alpha_s} + \alpha \alpha_s^2 \sigma_{\alpha \alpha_s^2} + \dots$$

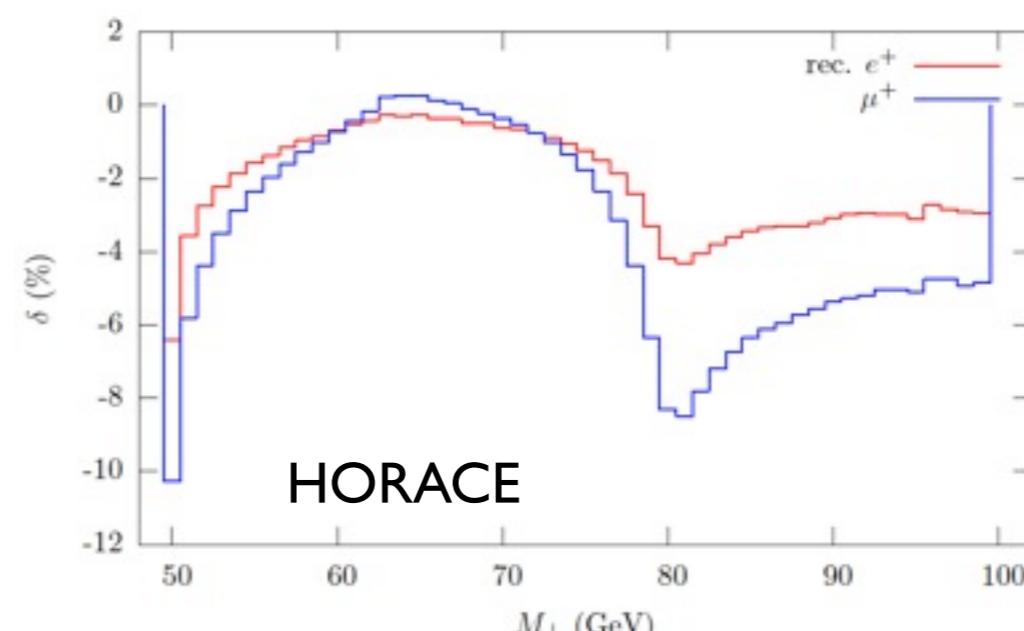
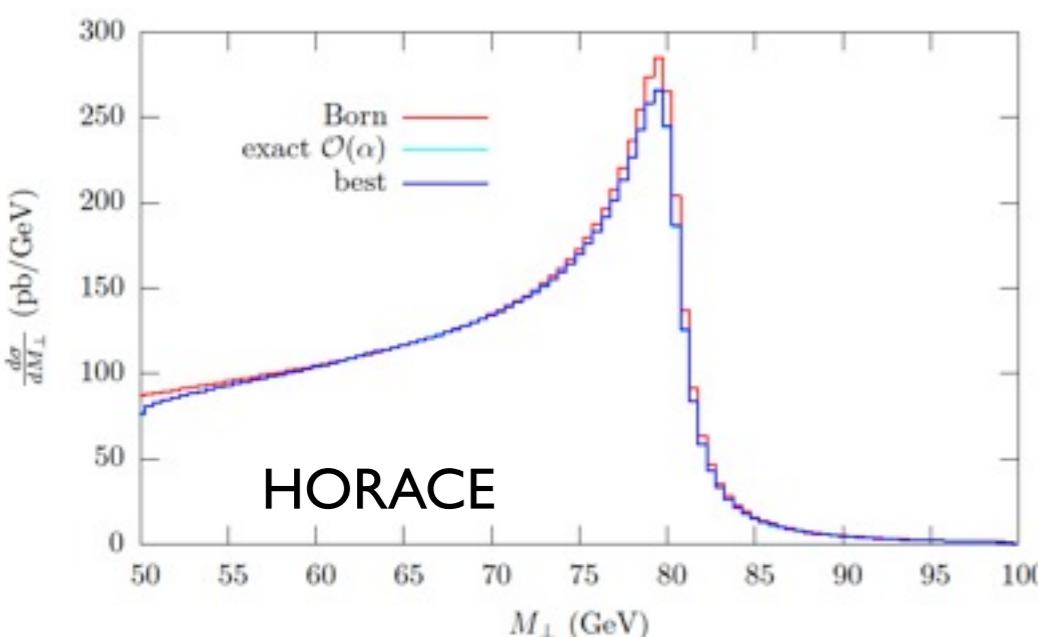
Fixed order corrections exactly evaluated and available in simulation codes



# Perturbative expansion of the Drell-Yan cross section

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$$+ \alpha \alpha_s \sigma_{\alpha \alpha_s} + \alpha \alpha_s^2 \sigma_{\alpha \alpha_s^2} + \dots$$

Fixed order corrections exactly evaluated and available in simulation codes



The change of the final state lepton distribution yields a huge shift in the extracted MW value

$$\Delta M_W^\alpha = 110 \text{ MeV}$$

# Perturbative expansion of the Drell-Yan cross section

$$\sigma_{tot} = \sigma_0 + \boxed{\alpha_s \sigma_{\alpha_s} + \alpha_s^2 \sigma_{\alpha_s^2}} + \dots$$
$$+ \boxed{\alpha \sigma_\alpha} + \boxed{\alpha^2 \sigma_{\alpha^2}} + \dots$$
$$+ \boxed{\alpha \alpha_s \sigma_{\alpha \alpha_s}} + \alpha \alpha_s^2 \sigma_{\alpha \alpha_s^2} + \dots$$

Fixed order corrections exactly evaluated and available in simulation codes

Subsets of corrections partially evaluated or approximated

$\mathcal{O}(\alpha^2)$

EW Sudakov logs J.Kühn,A.Kulesza, S.Pozzorini, M.Schulze, Nucl.Phys.B797:27-77,2008, Phys.Lett.B651:160-165,2007, Nucl.Phys.B727:368-394,2005.

QED LL

QED NLL (approximated)

additional light pairs (approximated)

$\mathcal{O}(\alpha \alpha_s)$

EW corrections to  $\bar{f}f + \text{jet}$  production

QCD corrections to  $\bar{f}f + \gamma$  production

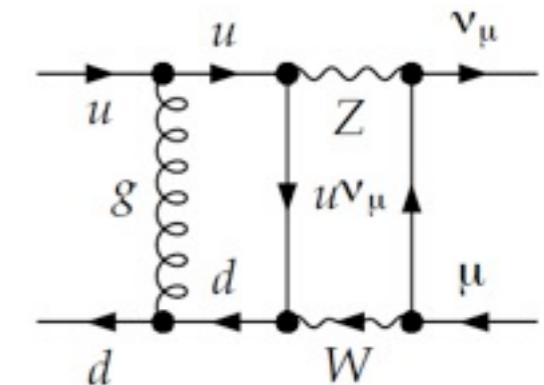
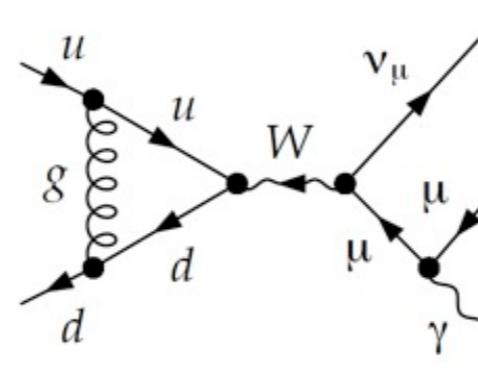
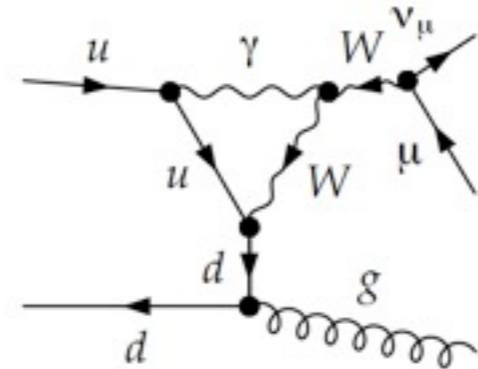
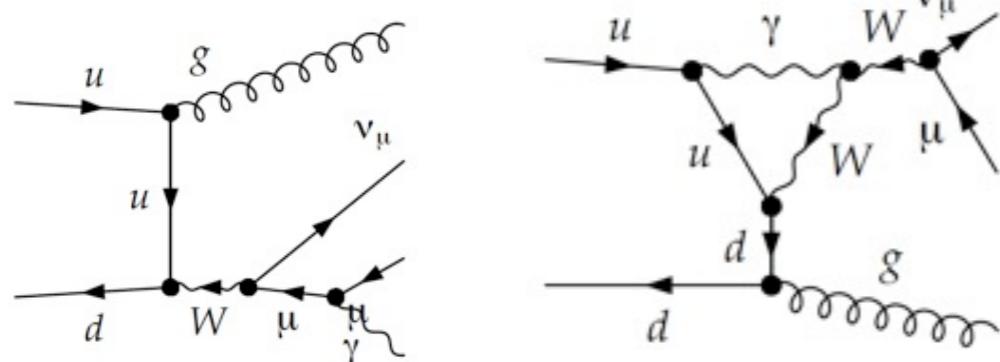
A.Denner, S.Dittmaier, T.Kasprzik, A.Mueck, arXiv:0909.3943, arXiv:1103.0914

# Mixed QCDxEW corrections the Drell-Yan cross section

$$\begin{aligned}\sigma_{tot} = & \sigma_0 + \alpha_s \sigma_{\alpha_s} + \alpha_s^2 \sigma_{\alpha_s^2} + \dots \\ & + \alpha \sigma_\alpha + \alpha^2 \sigma_{\alpha^2} + \dots \\ & + \boxed{\alpha \alpha_s \sigma_{\alpha \alpha_s}} + \alpha \alpha_s^2 \sigma_{\alpha \alpha_s^2} + \dots\end{aligned}$$

- The first mixed QCDxEW corrections include different contributions:

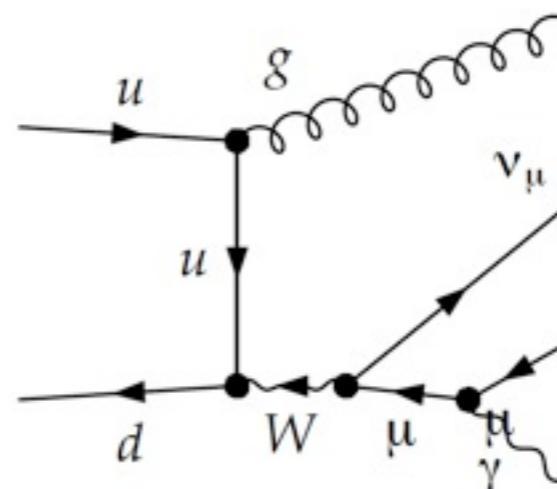
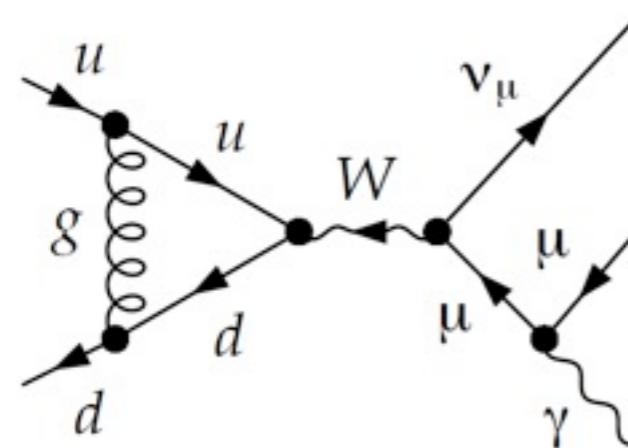
- emission of two real additional partons (one photon + one gluon/quark)
- emission of one real additional parton (one photon with QCD virtual corrections, one gluon/quark with EW virtual corrections)
- two-loop virtual corrections



- an exact complete calculation is not yet available, neither for DY nor for single gauge boson production

W.B. Kilgore, C. Sturm, arXiv:1107.4798

- The bulk of the mixed QCDxEW corrections, relevant for a precision MW measurement, is factorized in QCD and EW contributions:  
 $(\text{leading-log part of final state QED radiation}) \times (\text{leading-log part of initial state QCD radiation} \parallel \text{NLO-QCD contribution to the K-factor})$



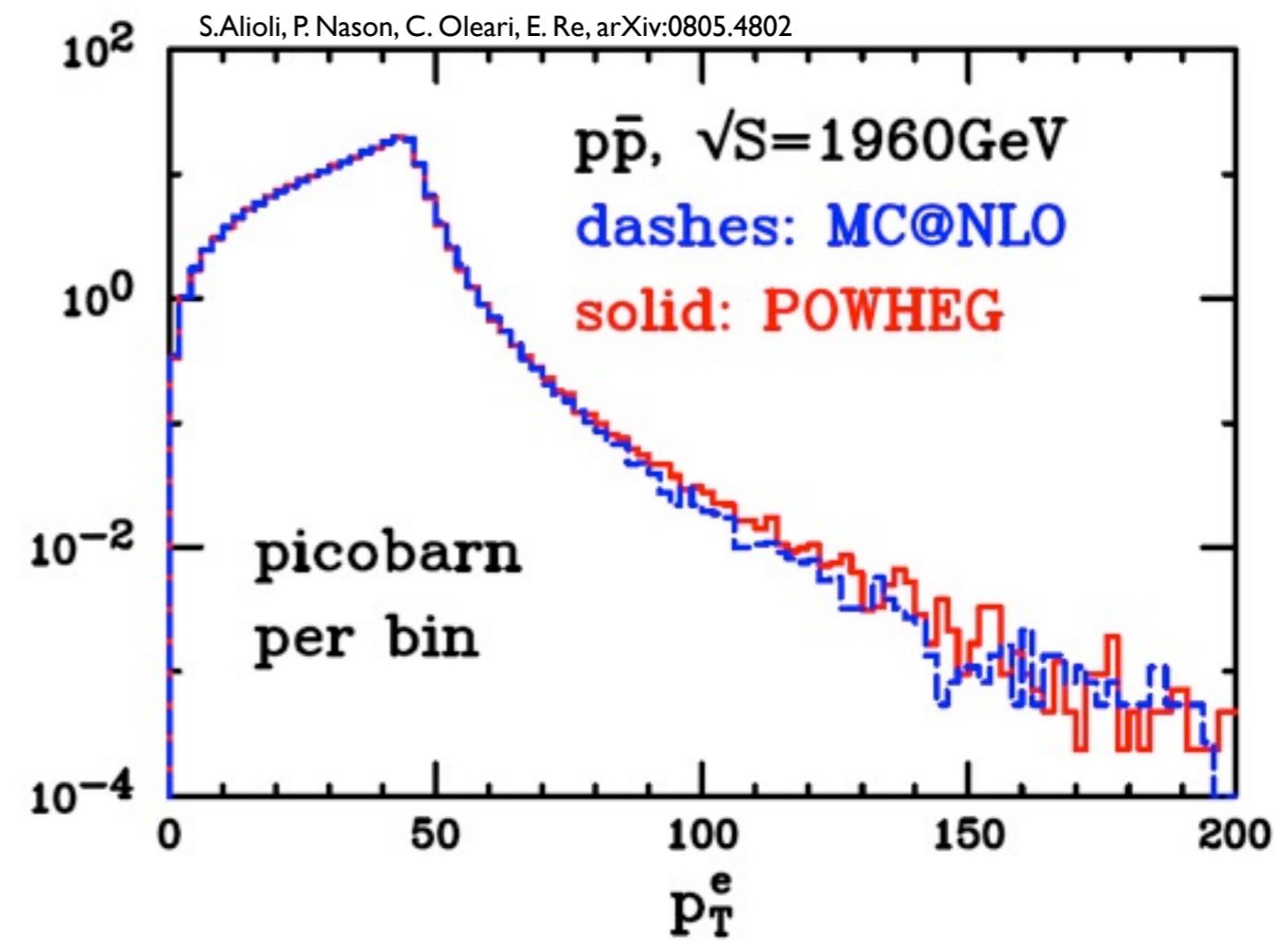
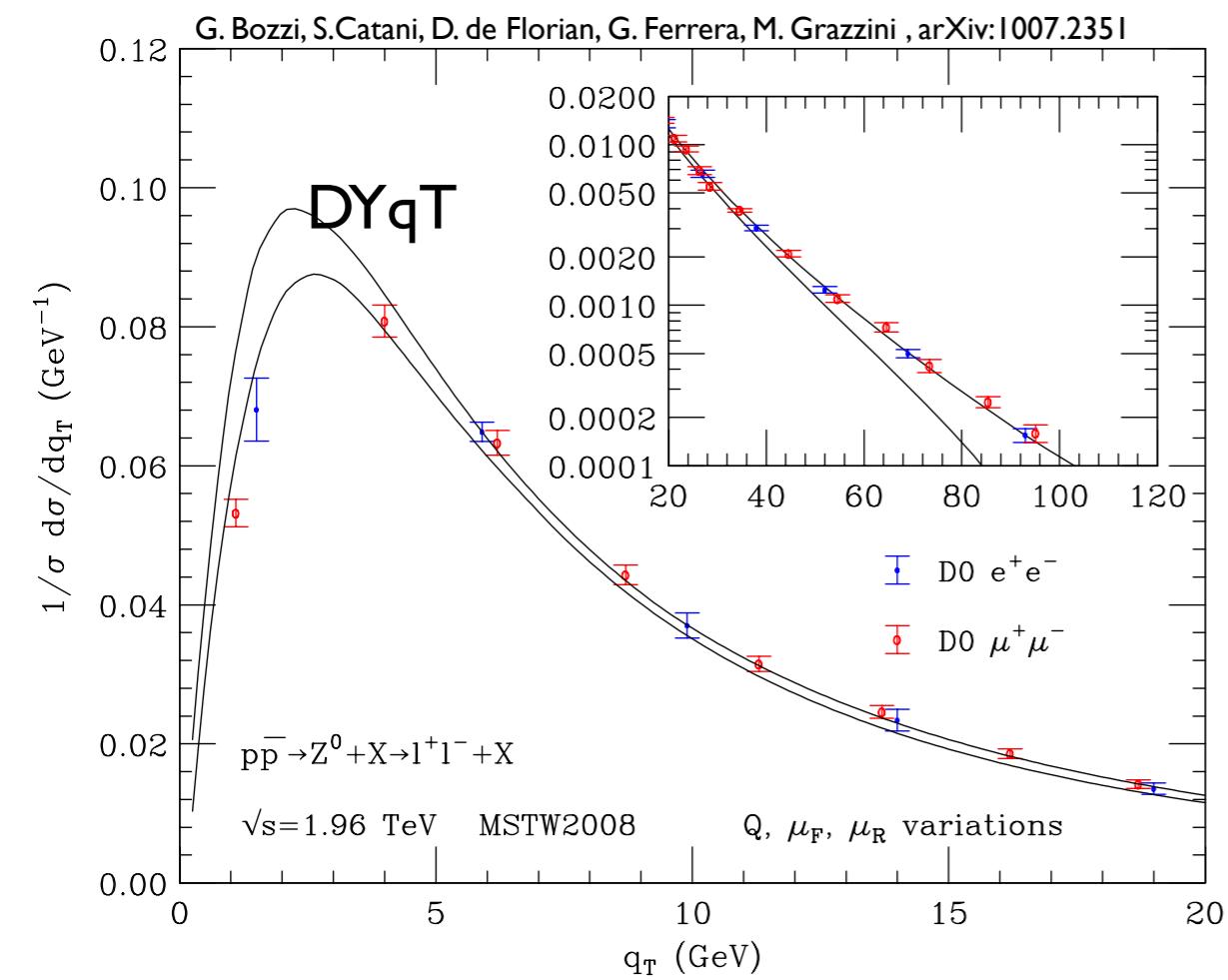
In any case, a fixed order description of the process is not sufficient...

# The relevance of multiple gluon emission

numerical simulation of IS QCD multiple gluon emission via Parton Shower (Herwig, Pythia, Sherpa)

matching of NLO-QCD results with QCD Parton Shower (MC@NLO, POWHEG)

analytical resummation of initial state QCD multiple gluon emission (Resbos, DYqT)



## Classification of radiative corrections: QCD

the QCD expansion can be organized with respect to  $L_V \equiv \log \left( \frac{p_\perp^V}{M_V} \right)$

$$\sigma = \sigma_0 +$$

$$\begin{aligned} & A_1 \alpha_s L_V + B_1 \alpha_s + \xleftarrow{\hspace{10em}} \\ & A_2 \alpha_s^2 L_V^2 + B_2 \alpha_s^2 L_V + C_2 \alpha_s^2 + \xleftarrow{\hspace{10em}} \\ & A_3 \alpha_s^3 L_V^3 + B_3 \alpha_s^3 L_V^2 + C_3 \alpha_s^3 L_V + D_3 \alpha_s^3 + \dots \xleftarrow{\hspace{10em}} \\ & \uparrow \quad \uparrow \quad \uparrow \\ \text{LL-QCD} & \quad \text{NLL-QCD} \quad \text{NNLL-QCD} \quad \dots \end{aligned}$$

NLO-QCD

NNLO-QCD

NNNLO-QCD

DYRes description of ptV spectrum with NNLL- + NNLO-QCD accuracy  
(beta-version with fully differential leptonic decay exists)

Resbos description of ptV spectrum with NNLL-QCD accuracy  
(partial inclusion of NNLO contributions) analytical resummation

POWHEG matching of NLO-QCD results

MC@NLO with LL- and (possibly) NLL-QCD

resummation via Parton Shower

# Matching NLO calculations with resummation: DYqT

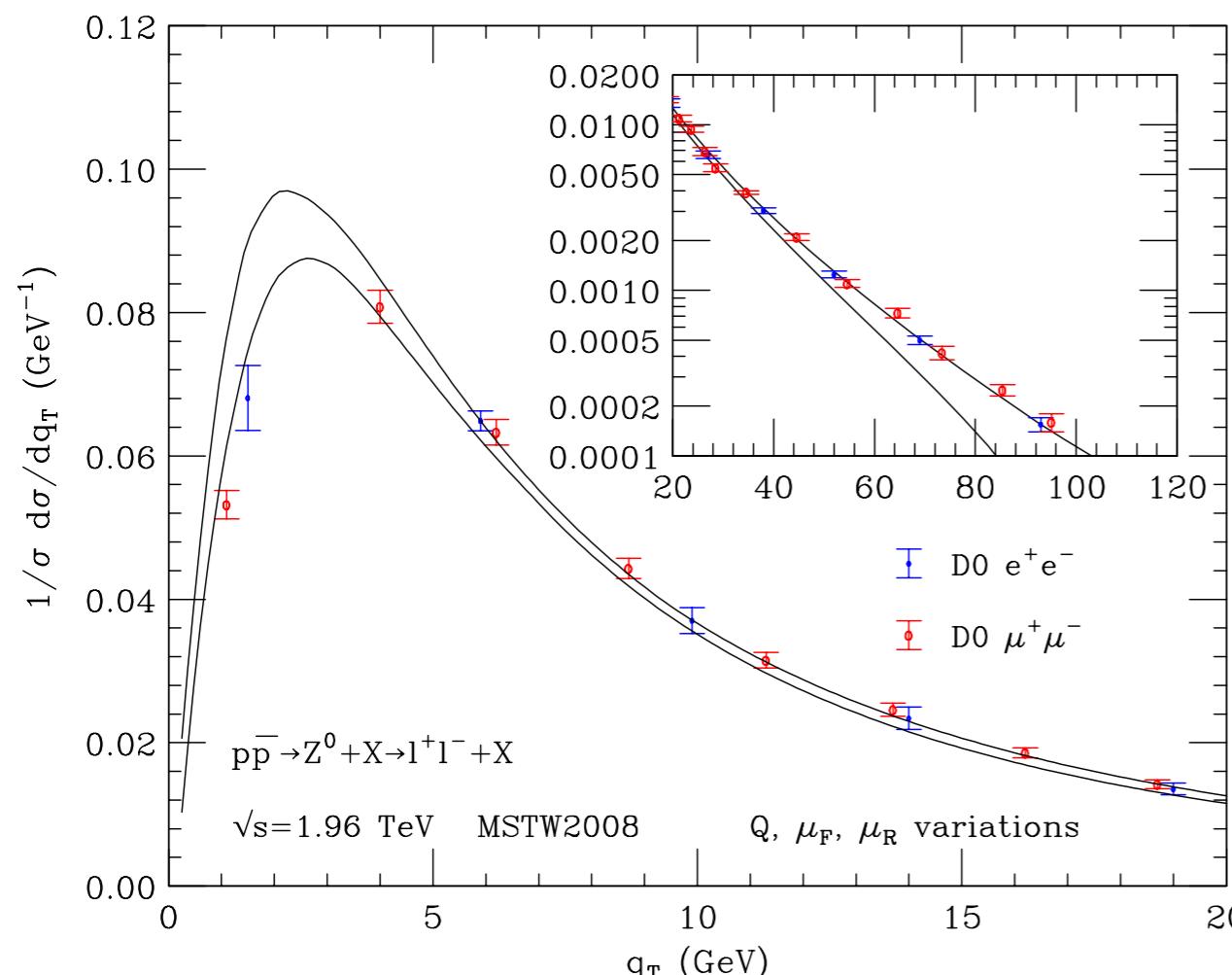
Bozzi, Catani, De Florian, Ferrera, Grazzini

$$\frac{d\hat{\sigma}_{Vab}^{(\text{res.})}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2) = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(bq_T) \mathcal{W}_{ab}^V(b, M, \hat{s}; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2) ,$$

$$\mathcal{W}_N^V(b, M; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2) = \mathcal{H}_N^V(M, \alpha_S(\mu_R^2); M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \times \exp\{\mathcal{G}_N(\alpha_S(\mu_R^2), L; M^2/\mu_R^2, M^2/Q^2)\} ,$$

process dependent  
universal

G. Bozzi, S. Catani, D. de Florian, G. Ferrera, M. Grazzini, arXiv:1007.2351



$Q$  is the resummation scale

the fixed order total cross section  
is by construction reproduced

a non-perturbative smearing factor  
can be applied on top of the pQCD result

## Classification of radiative corrections: EW

Fermion masses regulate collinear emission  $\rightarrow$  mass logarithm  $L \equiv \log \left( \frac{s}{m_l^2} \right)$   
Final state lepton masses are physical parameters

Perturbative expansion in  $\alpha$   
at each order, classification w.r.t. powers of  $L$

$$\sigma = a_0 + a_1 \alpha L + b_1 \alpha + a_2 \alpha^2 L^2 + b_2 \alpha^2 L + c_2 \alpha^2 + a_3 \alpha^3 L^3 + b_3 \alpha^3 L^2 + c_3 \alpha^3 L + d_3 \alpha^3 + \dots$$

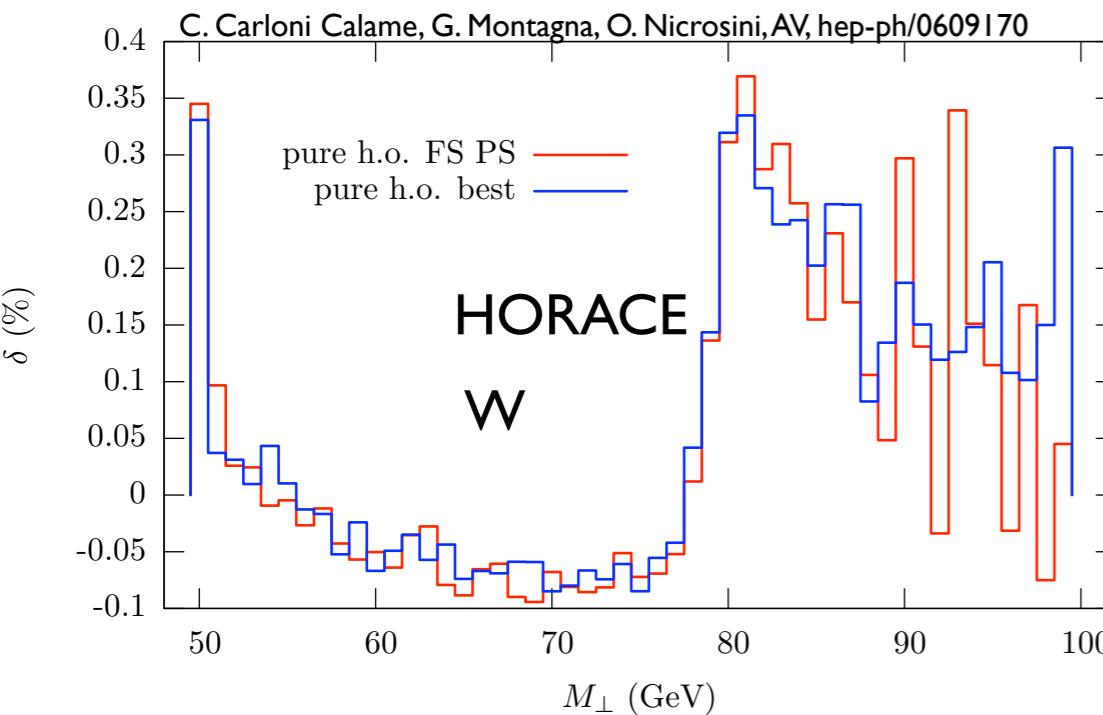
Photos  
HORACE-matched  
DKM  
Winhac

WGRAD  
DK  
HORACE-matched  
SANC

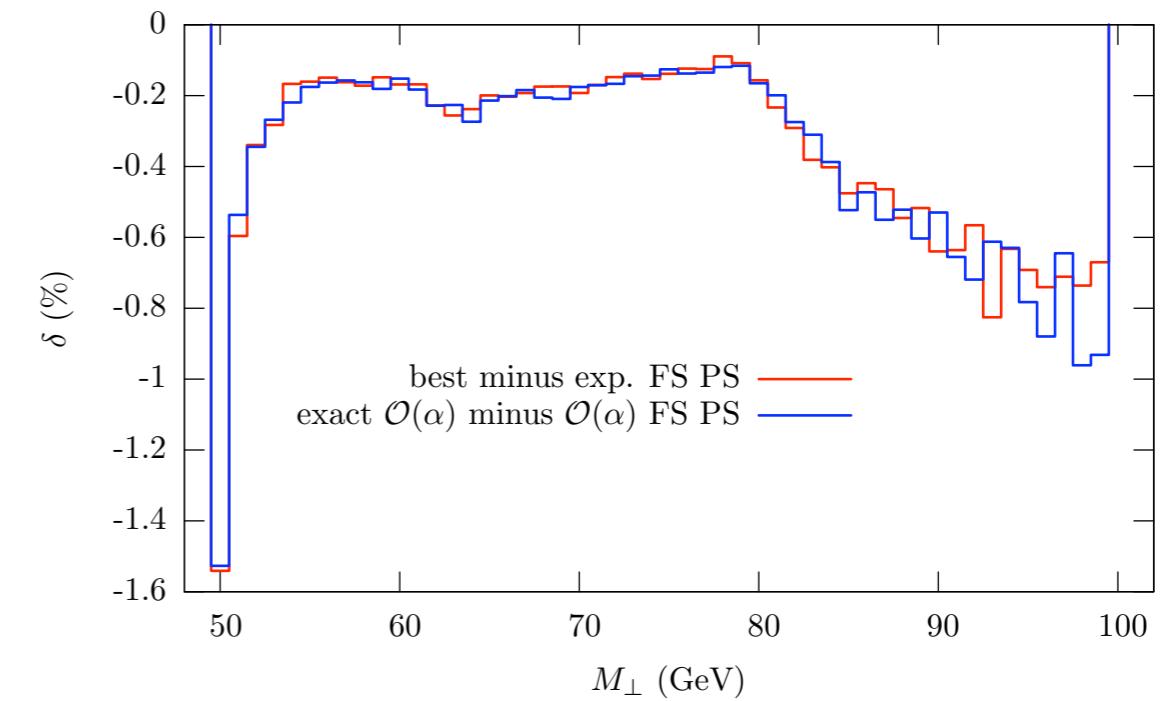
various codes include  
different subsets

numerical simulation of final state QED multiple photon emission via Parton Shower (Photos, HORACE)  
matching of NLO-EW results with complete QED Parton Shower (HORACE)

# Impact of EW higher-order radiative corrections



higher orders beyond  $\mathcal{O}(\alpha)$  coming from the Parton Shower



pure Parton Shower compared with the full calculation

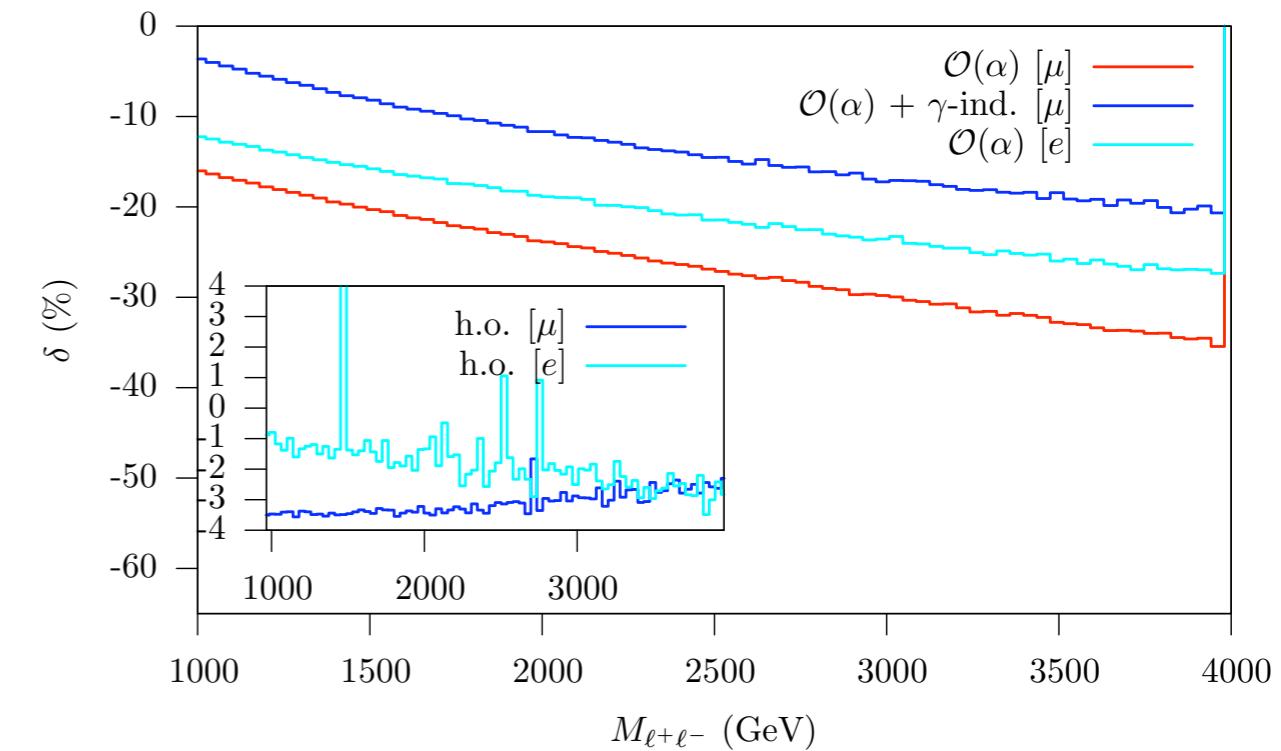
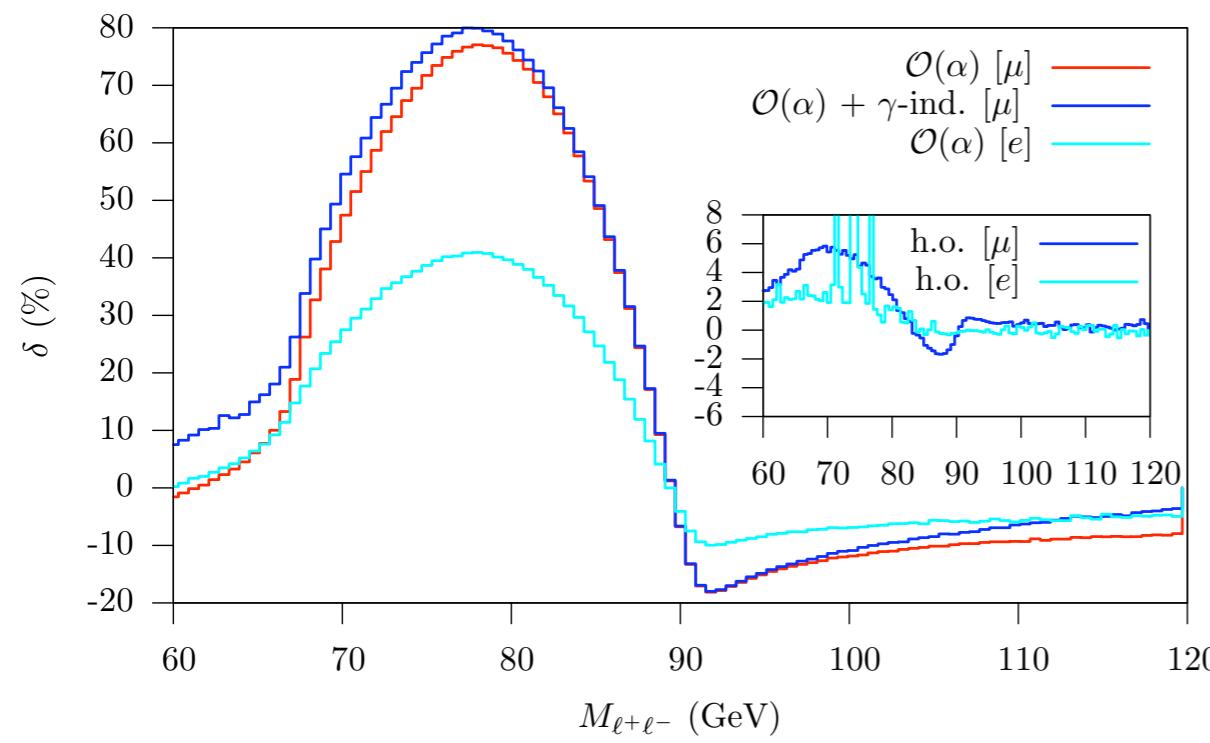
- Shift induced in the extraction of MW from higher order QED effects

$$\Delta M_W^\alpha = 110 \text{ MeV}$$

$$\Delta M_W^{exp} = -10 \text{ MeV}$$

- In HORACE each approximation is gauge invariant and offers a sensible definition of the observables
- The HORACE matching differs from a simple additive prescription, for it includes in the factorized formulation also higher order subleading contributions

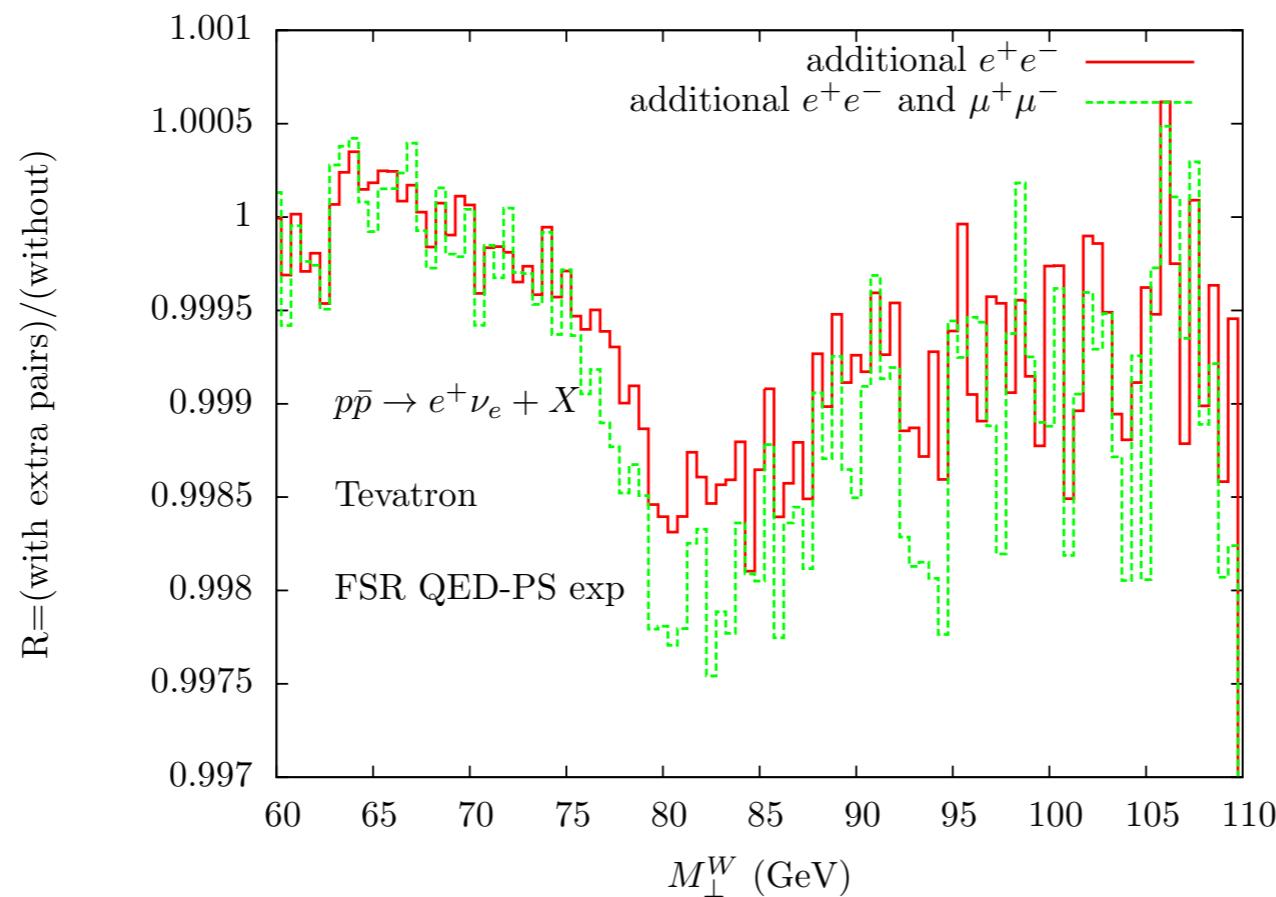
# Impact of EW higher-order radiative corrections



- FSR modifies the momenta of the final state leptons and induces a migration of events from the Z resonance to lower invariant masses; multiple photon effects are at the several percent level
- The HORACE matching differs from a simple additive prescription, for it includes in the factorized formulation also higher order subleading contributions
- The effect of higher order terms can be at the few per cent level in the large invariant mass tail (interplay between photonic and EW Sudakov corrections)

# Additional lepton-pair emission at $\mathcal{O}(\alpha^2)$

- The HORACE shower has been improved beyond the simulation of purely photonic radiation and approximately includes non-singlet contributions due to the conversion of a photon into a light-fermion pair
- These contributions are of  $\mathcal{O}(\alpha^2)$ , like e.g. the second radiated real photon; they are logarithmically enhanced by the parent fermion mass in the collinear limit of the pair, but also from the integration over the phase space of the two fermions of the pair.
- Observables inclusive over this additional radiation do not suffer of the  $\log^3(s/m^2)$  enhancement of these corrections, that cancels between real and virtual contributions



# Impact of EW radiative corrections on MW determination

Barzè, Bizjak, Montagna, Nicrosini, Piccinini, Vicini, in preparation

- each set of radiative corrections induces a distortion of the shape of the observables
- with a template-fit approach, the distortion of the shape is translated into a MW shift
- study performed in the Tevatron setup (energy and acceptance cuts)
- the available subsets of corrections **MUST** be included in the analysis

			$m_T$		$p_T^l$		$\cancel{E}_T$	
line	approximation 1	approximation 2	$e$	$\mu$	$e$	$\mu$	$e$	$\mu$
1	BORN	LL1 $\gamma$	$-143 \pm 3.1$	$-148 \pm 2.1$	$-167 \pm 3.7$	$-198 \pm 3.1$	$-104 \pm 4.0$	$-89 \pm 2.5$
2	BORN	LL $n\gamma$	$-138 \pm 3.1$	$-138 \pm 2.1$	$-162 \pm 3.7$	$-184 \pm 3.1$	$-104 \pm 4.0$	$-85 \pm 2.5$
3	LL1 $\gamma$	LL $n\gamma$	$5 \pm 3.5$	$10 \pm 2.3$	$5 \pm 4.4$	$15 \pm 3.3$	$1 \pm 4.5$	$5 \pm 2.5$
4	BORN	$\mathcal{O}(\alpha)$	$-147 \pm 2.8$	$-153 \pm 2.5$	$-174 \pm 3.5$	$-208 \pm 3.5$	$-105 \pm 3.7$	$-91 \pm 2.8$
5	BORN	MATCH	$-137 \pm 3.0$	$-138 \pm 3.4$	$-163 \pm 3.7$	$-190 \pm 3.4$	$-96 \pm 4.0$	$-78 \pm 2.7$
6	$\mathcal{O}(\alpha)$	MATCH	$11 \pm 3.0$	$12 \pm 3.0$	$11 \pm 3.5$	$16 \pm 3.3$	$12 \pm 4.0$	$13 \pm 3.8$
7	LL1 $\gamma$	$\mathcal{O}(\alpha)$	$-1 \pm 3.4$	$-3 \pm 2.5$	$-3 \pm 4.1$	$-5 \pm 3.7$	$-1 \pm 4.4$	$-1 \pm 3.0$
8	LL $n\gamma$	MATCH	$4 \pm 3.5$	$5 \pm 2.4$	$4 \pm 4.2$	$2 \pm 3.5$	$10 \pm 4.5$	$10 \pm 2.8$

			$m_T$
line	approximation 1	approximation 2	$e \ \mu$
1	exp-LL	exp-LL + $e^+e^-$	-2 -3
2	exp-LL	exp-LL + $e^+e^- + \mu^+\mu^-$	-3 -3

- an estimate of remaining sources of uncertainty can enter in the theoretical systematic error

line	approx.1	approx.2	$m_T$		$p_T^l$		$\cancel{E}_T$	
			$e$	$\mu$	$e$	$\mu$	$e$	$\mu$
1	exp-LL $\kappa = 1.5$	exp-LL $\kappa = 1$	4.0	5.9	4.0	7.7	2.4	3.8
2	$\mathcal{O}(\alpha)$ LL $\kappa = 1.5$	$\mathcal{O}(\alpha)$ LL $\kappa = 1$	1.9	4.8	1.8	5.9	1.5	2.3
	$\Delta M_W^{\alpha^2}$ according to Eq. (25)		2.1	1.1	2.2	1.8	0.9	1.5

line	approx. 1	approx. 2	$m_T$		$p_T^l$		$\cancel{E}_T$	
			$e$	$\mu$	$e$	$\mu$	$e$	$\mu$
1	$\mathcal{O}(\alpha) \alpha_0$	$\mathcal{O}(\alpha) G_\mu - I$	-9.0	-11.6	-10.8	-11.8	-2.8	-7.4
2	$\mathcal{O}(\alpha) \alpha_0$	$\mathcal{O}(\alpha) G_\mu - II$	1.2	-0.3	-0.2	0.2	1.7	-0.7
3	$\mathcal{O}(\alpha) G_\mu - I$	$\mathcal{O}(\alpha) G_\mu - II$	10.1	11.2	10.6	12.0	4.4	6.6
4	matched $\alpha_0$	matched $G_\mu - I$	-0.1	-0.1	0.0	-1.1	2.0	1.8
5	matched $\alpha_0$	matched $G_\mu - II$	1.7	1.1	1.3	-0.3	4.0	2.6
6	matched $G_\mu - I$	matched $G_\mu - II$	1.8	1.2	1.0	0.8	2.0	0.9

Uncertainties are of  $\mathcal{O}(\alpha^2)$  and are due to:

- 1) different renormalization schemes
- 2) missing  $\mathcal{O}(\alpha^2)$  matrix elements (QED subleading terms, 2-loop EW corrections,...)
- 3) precise definition of the  $W$  mass in the complex plane

Estimate of the uncertainties

- 1) fit of  $MW$  using codes in different renormalization schemes  
the spread of the results is reduced when using HORACE matched  
w.r.t. fixed  $\mathcal{O}(\alpha)$  calculation

$$\mathcal{O}(\alpha) : MW(\alpha_0) - MW(G_\mu) = 9 \text{ MeV}$$

$$\text{matched} : MW(\alpha_0) - MW(G_\mu) = 1 \text{ MeV}$$

- 2) missing  $\mathcal{O}(\alpha^2)$  QED subleading terms can be probed in the pure QED-PS approach  
varying the QED-PS scale by a factor  $\kappa$ , and then comparing with the reference  $\kappa=1$

$$\Delta m_W^{\alpha^2}{}_{NLL}(\kappa) = (m_W^{\text{exp-LL}}(\kappa) - m_W^{\text{exp-LL}}(\kappa = 1)) - (m_W^{\alpha-LL}(\kappa) - m_W^{\alpha-LL}(\kappa = 1))$$

line	approx.1	approx.2	$m_T$	$p_T^l$	$E_T$			
			$e$	$\mu$	$e$	$\mu$		
1	exp-LL $\kappa = 1.5$	exp-LL $\kappa = 1$	4.0	5.9	4.0	7.7	2.4	3.8
2	$\mathcal{O}(\alpha)$ LL $\kappa = 1.5$	$\mathcal{O}(\alpha)$ LL $\kappa = 1$	1.9	4.8	1.8	5.9	1.5	2.3
	$\Delta M_W^{\alpha^2}$ according to Eq. (25)		2.1	1.1	2.2	1.8	0.9	1.5

Uncertainties estimated with HORACE do not span ALL possible combinations of higher orders  
the best predictions obtained with other codes might differ at  $\mathcal{O}(\alpha^2)$   
→ provide a more conservative envelope

# Previous combinations of QCD and EW corrections to Drell-Yan

LL approximation in Shower MC

no tuned comparisons on these tools

combined use of MC@NLO + HORACE + HERWIG

G. Balossini, C.M.Carlone Calame, G.Montagna, M.Moretti, O.Nicrosini, F.Piccinini, M.Treccani, A.Vicini, JHEP 1001:013, 2010

the combination of MC@NLO+PHOTOS in N.Adam, V.Halyo, S.Yost, W.Zhu, JHEP 0809:133,2008  
the (QCD+EW) combination in S.Jadach, M.Skrzypek, P.Stephens, Z.Was, W.Placzek, Acta.Phys.Polon.B38:2305 (2007)

# Previous combinations of QCD and EW corrections to Drell-Yan

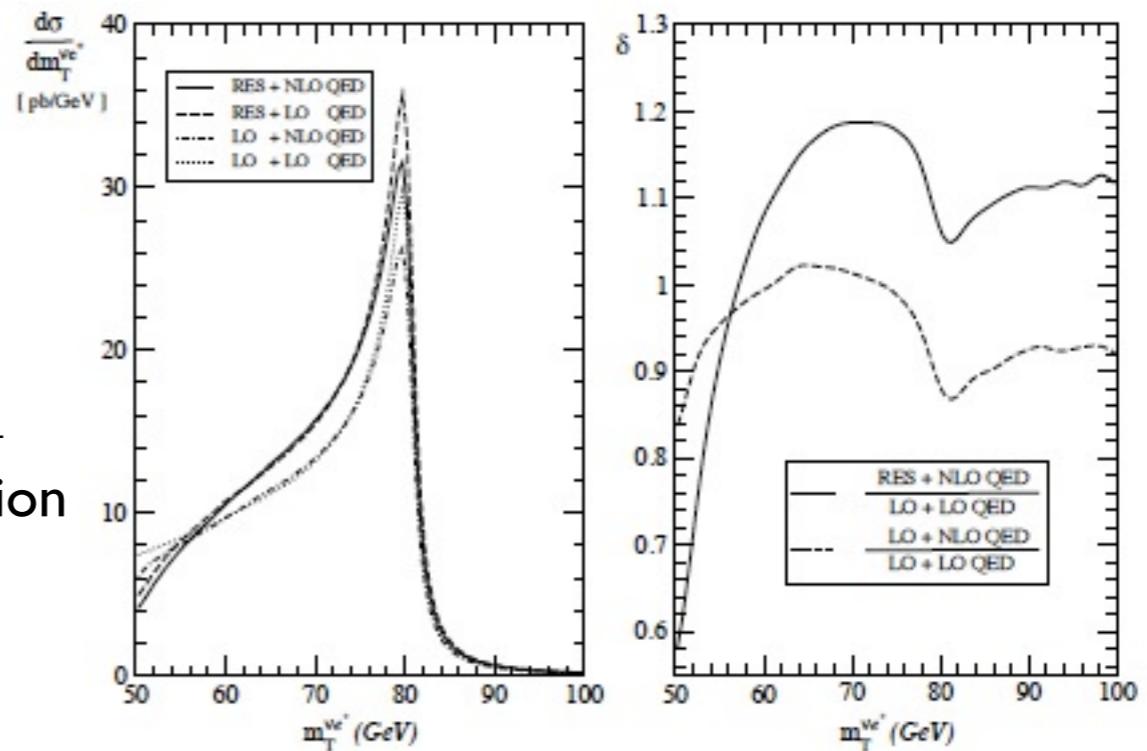
LL approximation in Shower MC

no tuned comparisons on these tools

**Resbos-A**

Q.-H. Cao and C.-P. Yuan, Phys. Rev. Lett. **93** (2004) 042001

soft gluon resummation + NLO final state QED radiation



combined use of MC@NLO + HORACE + HERWIG

G. Balossini, C.M. Carloni Calame, G. Montagna, M. Moretti, O. Nicrosini, F. Piccinini, M. Treccani, A. Vicini, JHEP 1001:013, 2010

the combination of MC@NLO+PHOTOS in  
the (QCD+EW) combination in

N.Adam, V.Halyo, S.Yost, W.Zhu, JHEP 0809:133,2008

S.Jadach, M.Skrzypek, P.Stephens, Z.Was, W.Placzek, Acta Phys. Polon. B38:2305 (2007)

# Recent developments of QCD and EW corrections to Drell-Yan

FEWZ, NC-DY : NNLO-QCD + NLO-EW additive combination

Li, Petriello, arXiv:1208.5967

POWHEG, CC-DY: NLO-(QCD+EW) matched with QCD/QED Parton Shower

Bernaciak, Wackerth, arXiv:1201.4804

Barzè, Montagna, Nason, Nicrosini, Piccinini, arXiv:1202.0465

POWHEG, NC-DY: NLO-(QCD+EW) matched with QCD/QED Parton Shower

Barzè, Montagna, Nason, Nicrosini, Piccinini, Vicini, arXiv:1302.4606

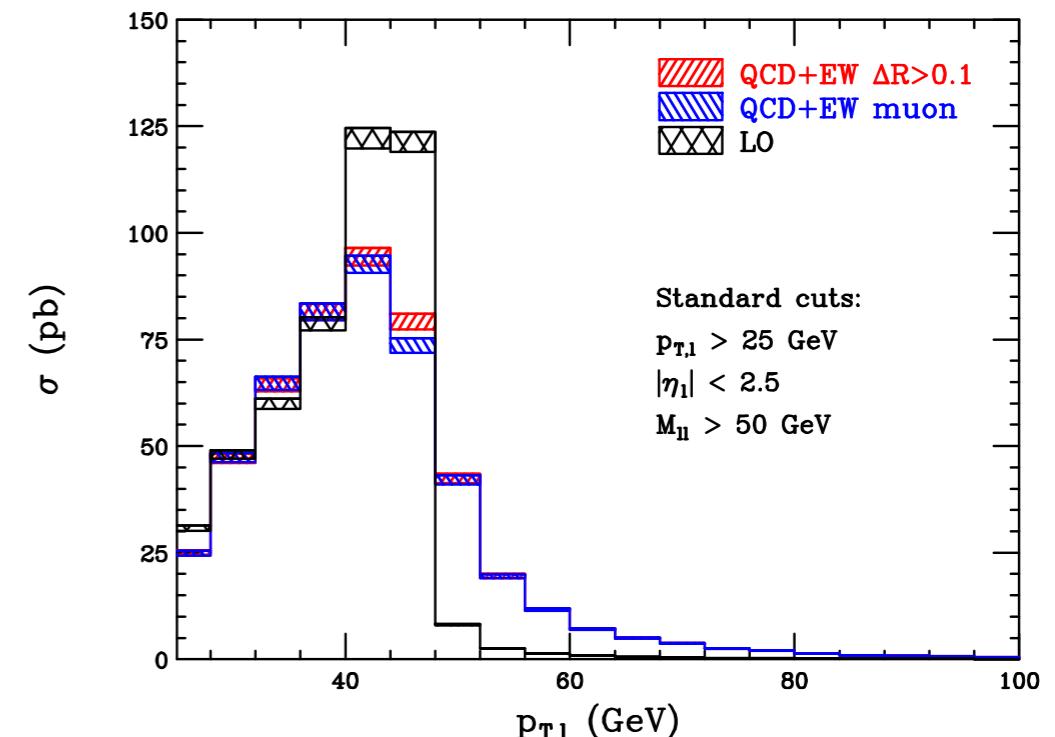
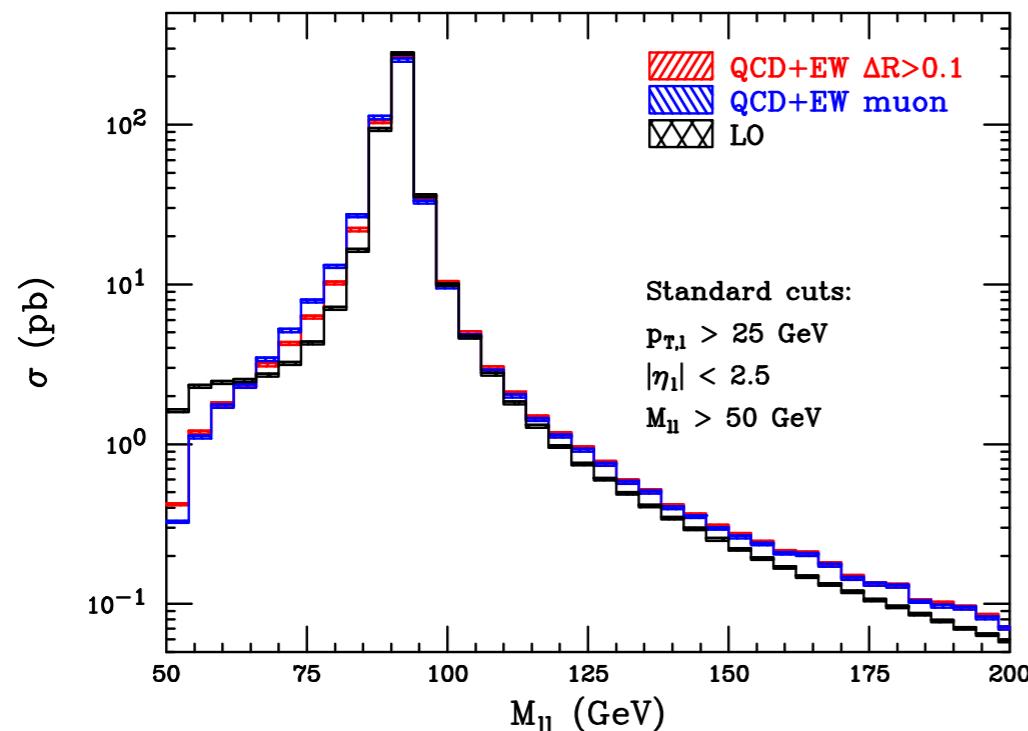
# Inclusion in FEWZ of exact $O(\alpha)$ EW corrections to NC-DY

FEWZ, NC-DY : NNLO-QCD + NLO-EW additive combination

Li, Petriello, arXiv:1208.5967,

Boughezal, Li, Petriello, arXiv:1312.3972

$$\mathcal{O} = \mathcal{O}_{LO} \left( 1 + \delta_{QCD}^{NLO+NNLO} + \delta_{EW}^{NLO} \right)$$



- accurate prediction of the invariant mass distribution
- missing effects of multiple photon radiation and of mixed higher orders (few % in the tails)

- the large bins avoid the appearance of the double peak structure typical of fixed order results

# Inclusion in POWHEG of the exact $\mathcal{O}(\alpha)$ EW corrections

POWHEG, CC-DY: NLO-(QCD+EW) matched with QCD/QED Parton Shower

Bernaciak, Wackerlo, arXiv:1201.4804

Barzè, Montagna, Nason, Nicrosini, Piccinini, arXiv:1202.0465

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<http://powhegbox.mib.infn.it/>

$$d\sigma = \sum_{f_b} \bar{B}^{f_b}(\Phi_n) d\Phi_n \left\{ \Delta^{f_b}(\Phi_n, p_T^{\min}) + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \frac{[d\Phi_{rad} \theta(k_T - p_T^{\min}) \Delta^{f_b}(\Phi_n, k_T) R(\Phi_{n+1})]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \right\}$$

Upward arrow: (differential) overall normalization factor exact NLO QCD+EW accuracy (Born+virtual+integrated real)

Leftward arrow: no emission probability (Sudakov form factor)

Rightward arrow: exact emission probability of one parton (either one photon or gluon or quark) requested to be the hardest emission (Sudakov form factor)

- the events generated in this way are then passed to PYTHIA/HERWIG for **QCD and QED** showering
- the effect of radiative corrections on the distributions is ruled by the (modified) Sudakov form factor and is factorized w.r.t. the lowest order kinematics  $\underline{B}$

# The POWHEG method

(Nason 2004, Frixione Nason Oleari 2007, Alioli Nason Oleari Re 2009)

matching NLO-QCD matrix elements with QCD Parton Shower

- avoiding double counting between the first emission (hard matrix element) and the PS radiation
- generating positive weight events
- independent of the details of the (vetoed) shower adopted

$$d\sigma = \sum_{f_b} \bar{B}^{f_b}(\Phi_n) d\Phi_n \left\{ \Delta^{f_b}(\Phi_n, p_T^{min}) + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \frac{[d\Phi_{rad} \theta(k_T - p_T^{min}) \Delta^{f_b}(\Phi_n, k_T) R(\Phi_{n+1})]_{\alpha_r}^{\Phi_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \right\}$$

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- **NLO-(QCD+EW) accuracy of the total cross section:** inclusion of virtual corrections, integral over the whole phase space of (subtracted) real matrix element

$$\begin{aligned} \bar{B}^{f_b}(\Phi_n) = & [B(\Phi_n) + V(\Phi_n)]_{f_b} + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \int [\theta(k_T(\Phi_{n+1}) - p_T) R(\Phi_{n+1})]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n} d\Phi_{rad} \\ & + \sum_{\alpha_{\oplus} \in \{\alpha_{\oplus} | f_b\}} \int \frac{dz}{z} G_{\oplus}^{\alpha_{\oplus}}(\Phi_{n,\oplus}) + \sum_{\alpha_{\ominus} \in \{\alpha_{\ominus} | f_b\}} \int \frac{dz}{z} G_{\ominus}^{\alpha_{\ominus}}(\Phi_{n,\ominus}) \end{aligned}$$

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- (N)LO-(QCD+QED) accuracy of the real emission probability: exact real matrix elements, are used also in the Sudakov form factor (instead of the collinear splitting function)

$$\Delta^{f_b}(\Phi_n, p_T) = \exp \left\{ - \sum_{\alpha_r \in \{\alpha_r | f_b\}} \int \frac{[\theta(k_T(\Phi_{n+1}) - p_T) R(\Phi_{n+1})]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} d\Phi_{rad} \right\}$$

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- The **curly bracket**, integrated over the whole phase space, is equal to 1 : the NLO accuracy of the total cross section is preserved

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- The curly bracket, integrated over the whole phase space, is equal to 1 : the NLO accuracy of the total cross section is preserved
- The POWHEG (first) emission is by construction the hardest: HERWIG/PYTHIA are bound to radiate partons with lower virtuality (transverse momentum)

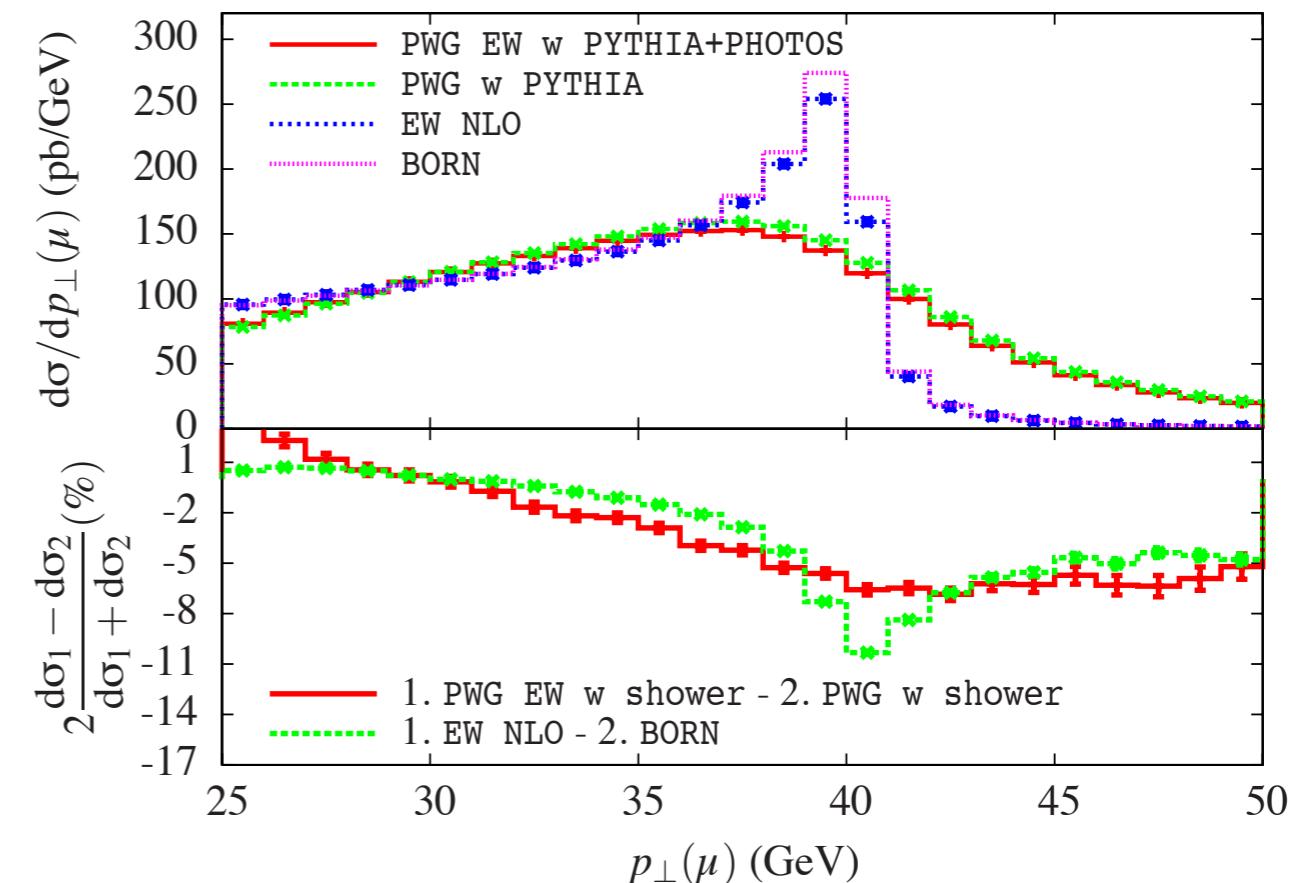
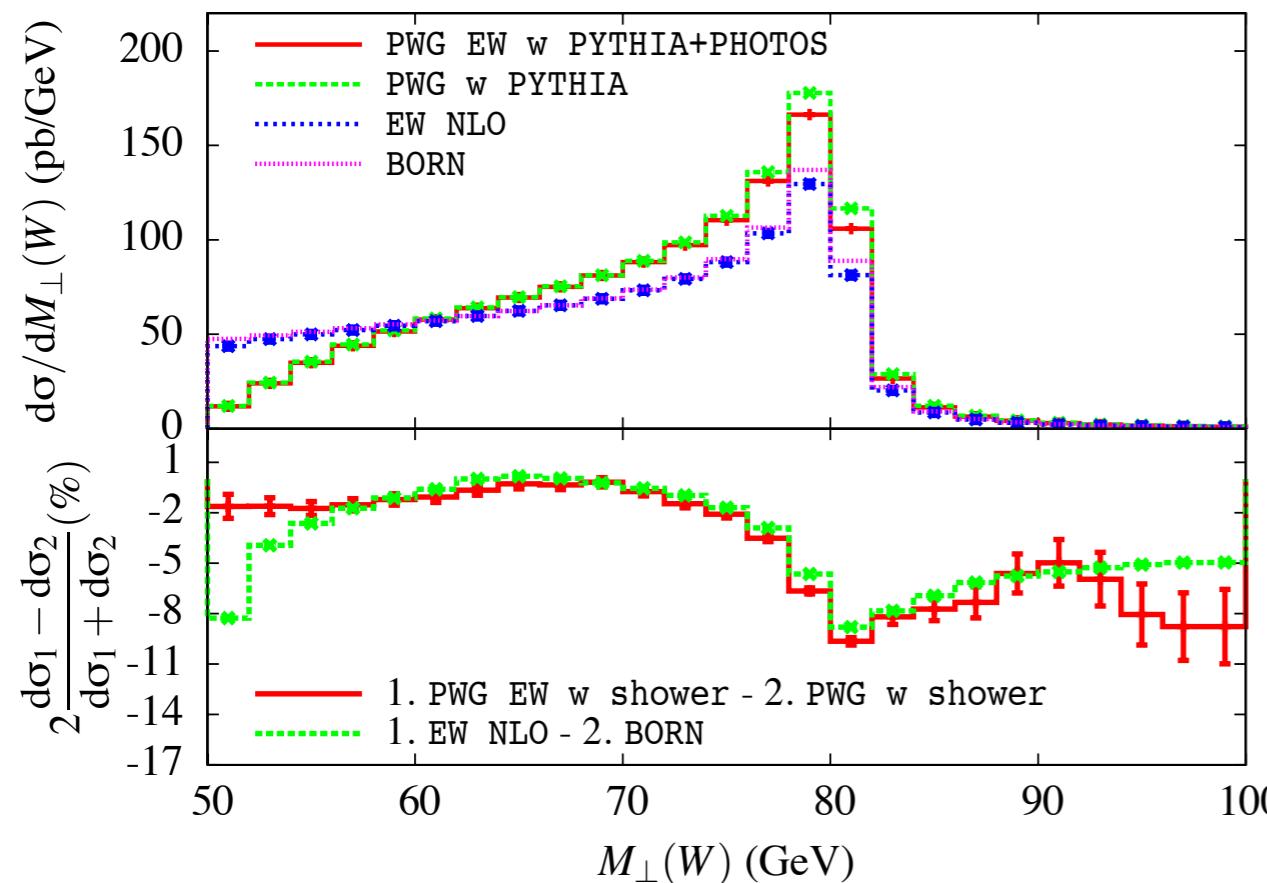
# Inclusion in POWHEG of the exact $O(\alpha)$ corrections (NLO-EW)

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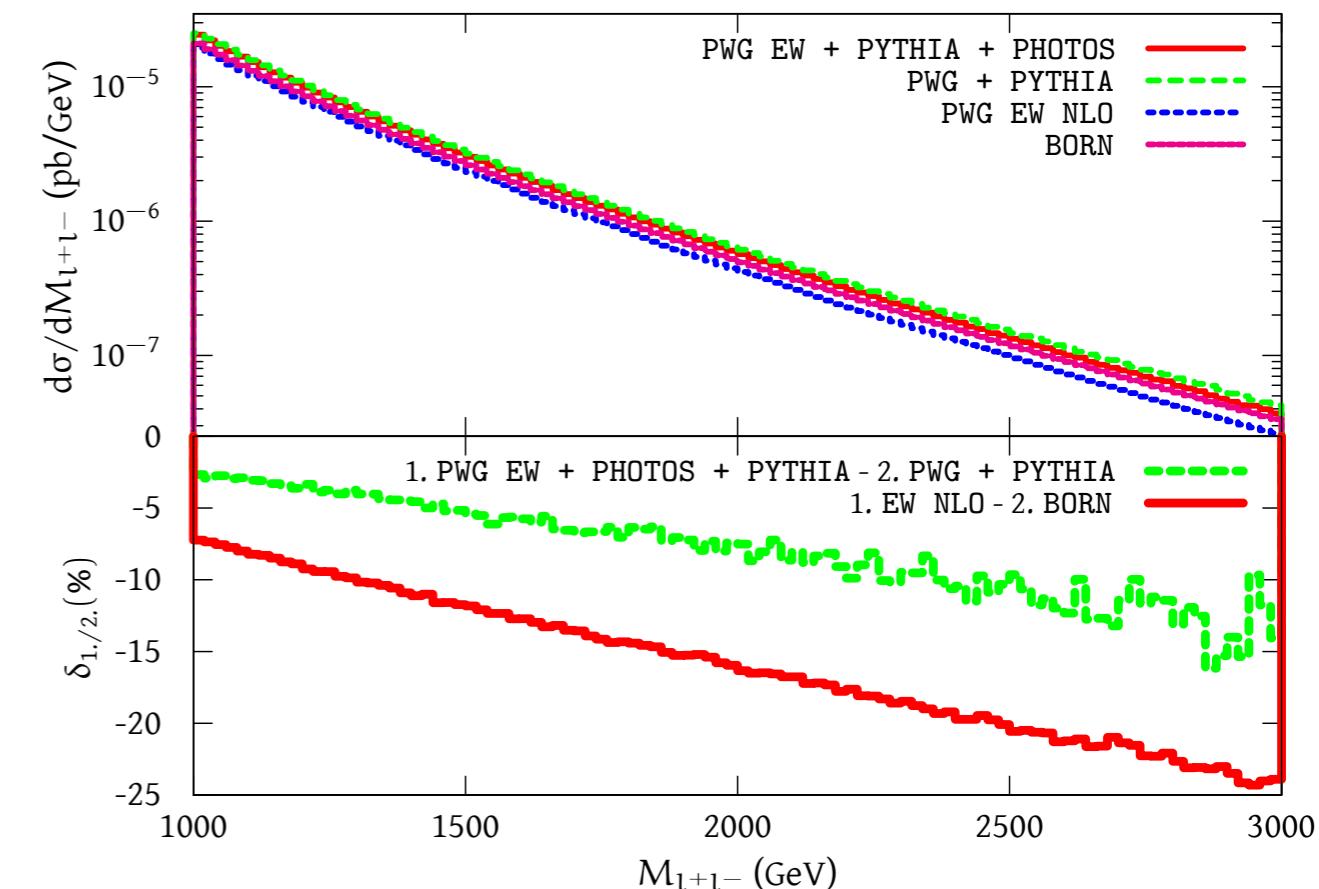
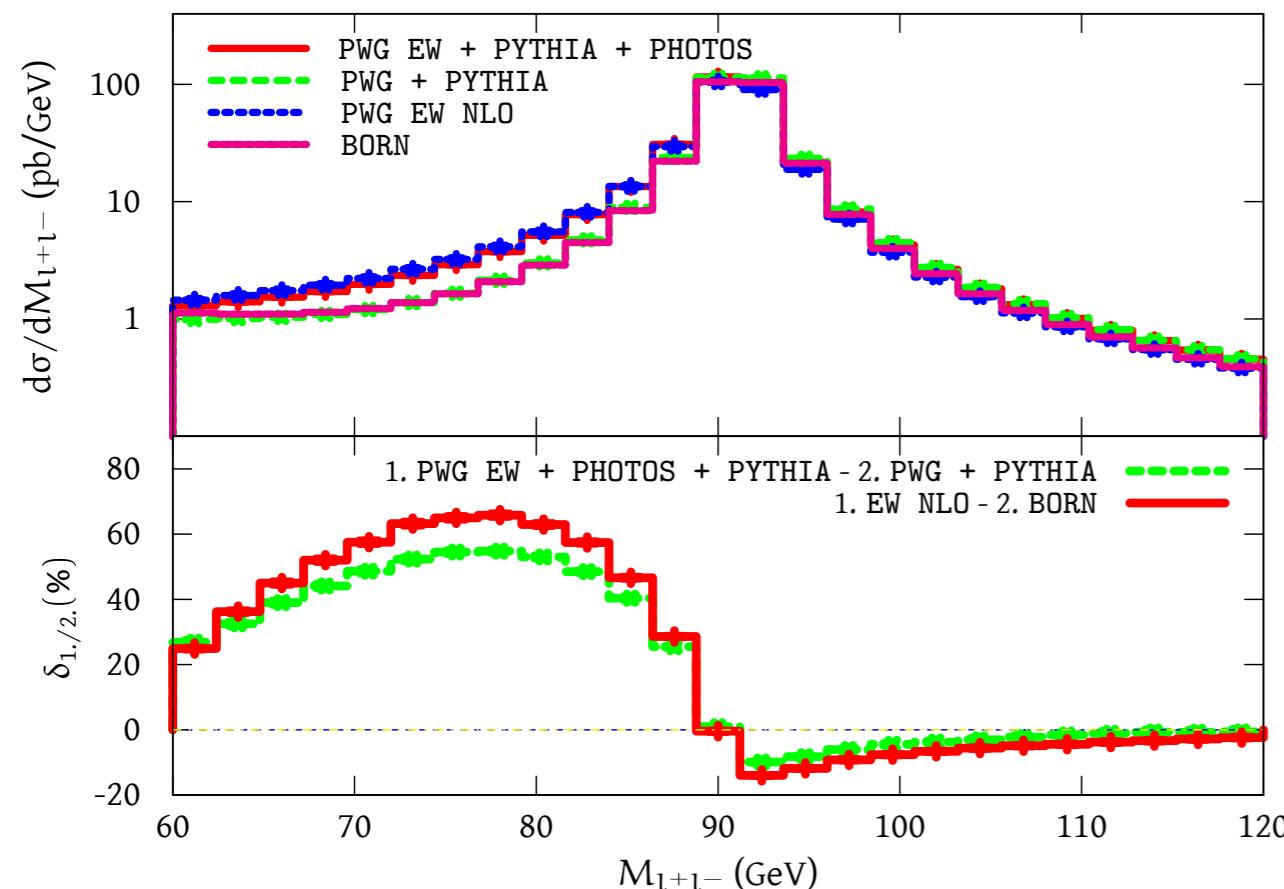
- the final state may contain 0 or 1 additional partons  
the parton can be 1 gluon or 1 photon (qqbar subprocess) or 1 quark (qg subprocess)
- the virtuality (transverse momentum) of the emitted parton sets the largest virtuality that the Parton Shower can reach
- the Parton Shower can be a pure QCD shower (BW) or a mixed QCD/QED shower (BMNNP)
- the process has three regions of collinear singularity, associated to the emission of one final state photon, one initial state photon, one initial state gluon/quark  
the Sudakov form factor is given by the product of the three individual form factors, for the three regions of collinearity
- the soft/collinear divergences have been regularized by phase-space slicing and final state lepton masses (BW) or in a mixed scheme using dimensional regularization to treat the quark and photon singularities and the lepton mass as natural cut-off of the final state mass singularities (BMNNP)
- the virtual corrections have been implemented according to the WGRAD results (BW) or reproducing independently the HORACE results (BMNNP) with the option of working in the complex mass scheme

# CC-DY: QCD+EW effects

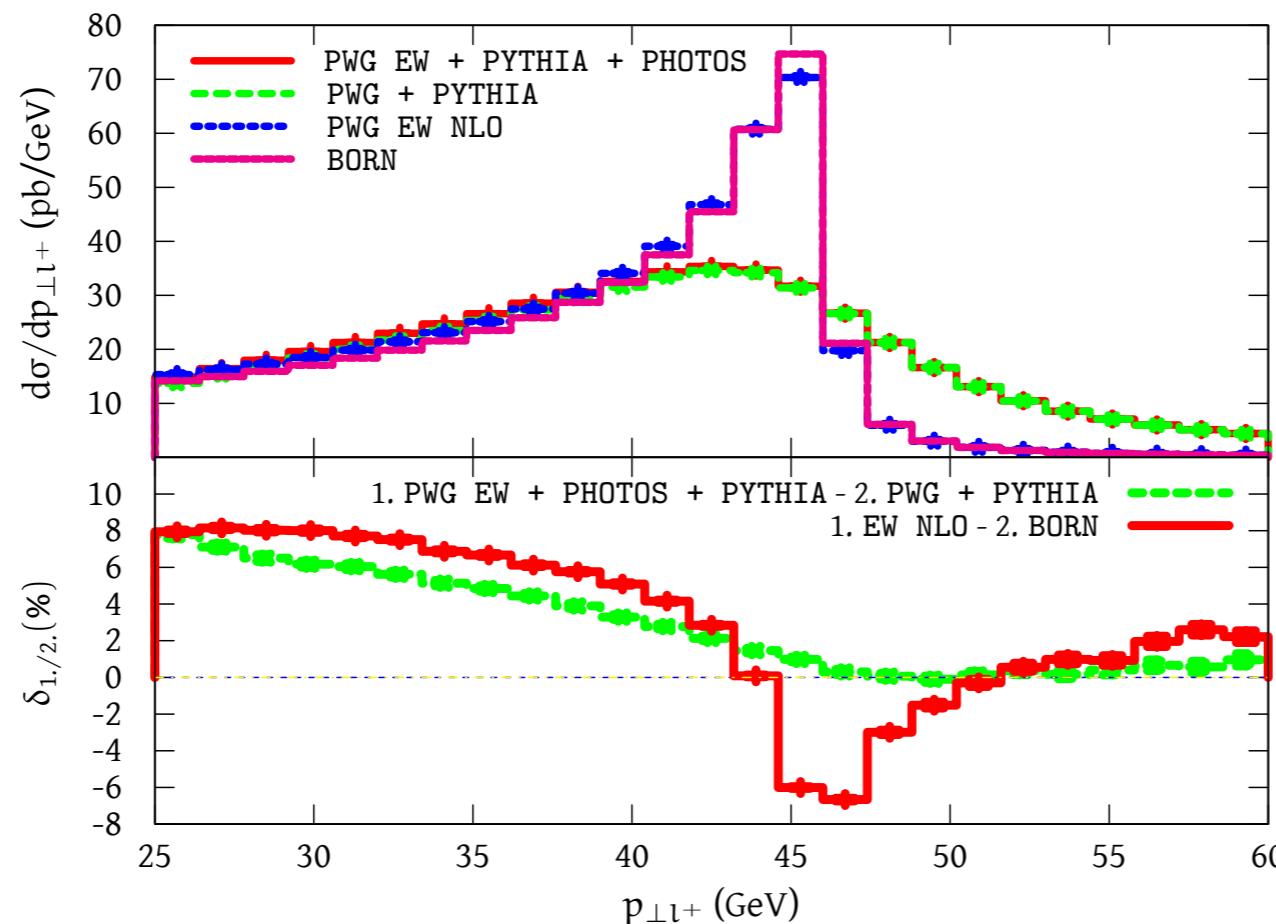
Barzè, Montagna, Nason, Nicrosini, Piccinini, arXiv:1202.0465



- all the results in the  $G\mu$  input scheme; multiple photon radiation included with PHOTOS
- transverse mass stable against QCD corrections  $\rightarrow$  NLO-EW effects are preserved after showering
- the lepton transverse momentum is more sensitive to multiple gluon radiation  
the sharp peak due to EW corrections is reduced by the QCD-Parton Shower
- the interplay between QCD and EW corrections yields effects at the per cent level



- all the results in the  $\alpha_0$  input scheme; first photon emission is described exactly with matrix elements  
FSR multiple photon radiation included with PHOTOS, ISR with PYTHIA
- the invariant mass is stable against QCD corrections  $\rightarrow$  the bulk of the NLO-EW effects are preserved after showering
- the interplay between QCD and EW corrections of  $\mathcal{O}(\alpha\alpha_s)$  yields effects at the per cent level in the peak region  
at the 10% level in the tails

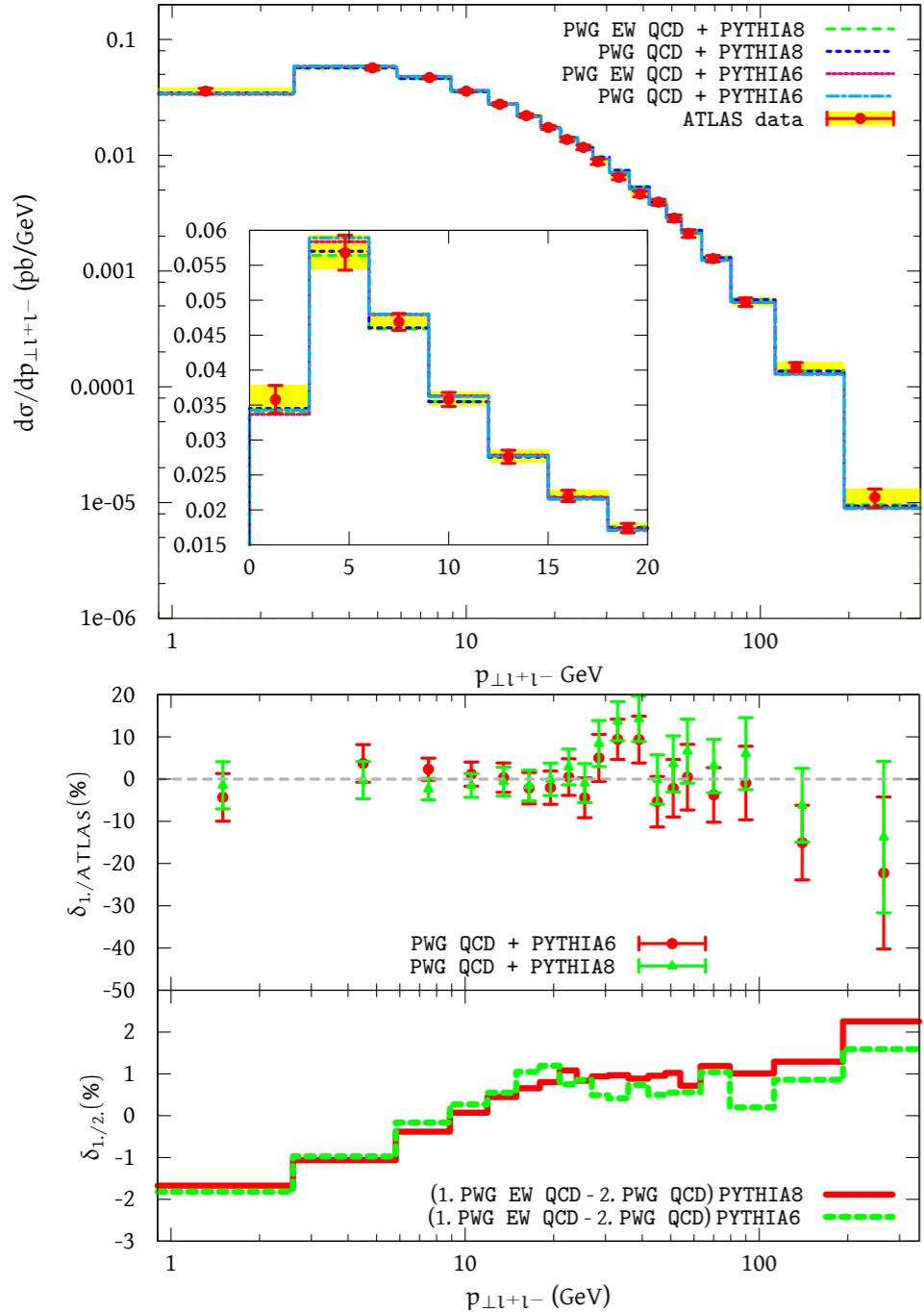


- the lepton transverse momentum is very sensitive to multiple gluon radiation
- the sharp peak due to EW corrections is reduced by the interplay with the QCD-Parton Shower; factorizable  $\mathcal{O}(\alpha\alpha_s)$  corrections are at the level of 7%
- an additive prescription to combine QCD+EW effects instead preserves the peak

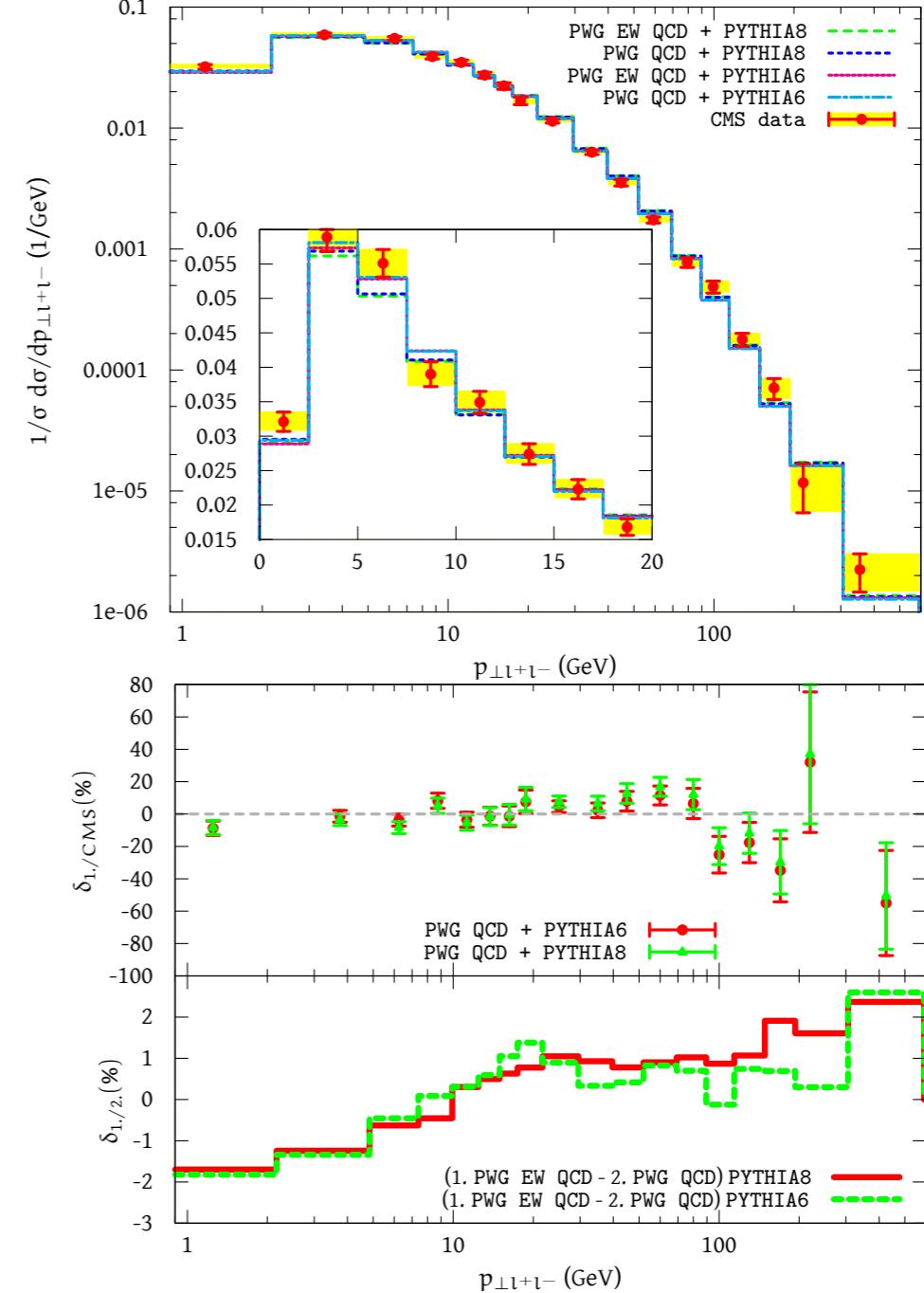
the fixed-order QCD description of the lepton transverse momentum distribution is poor, a resummation is needed

the combination of NLO-EW effects with multiple gluon emission strongly smears both the NLO-QCD fixed order spectrum and the peaked NLO-EW correction

# NC-DY: QCD+EW effects

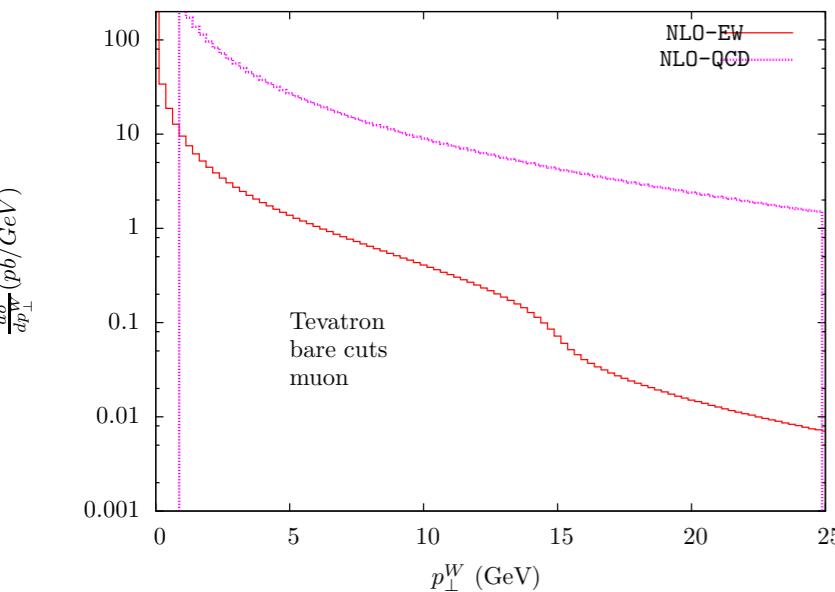


# lepton-pair transverse momentum



- the description of the lepton-pair transverse momentum distribution data is in general good
- default values for the non-perturbative parameters in PYTHIA6 and PYTHIA8 have been used (further tuning possible)
- full NLO-EW matrix element → bulk of the QED effects on  $ptZ$ ; multiple photon radiation has negligible impact
- QED radiation affects differently  $ptW$  and  $ptZ$ , both in its FSR and in its ISR components

# QED induced W(Z) transverse momentum

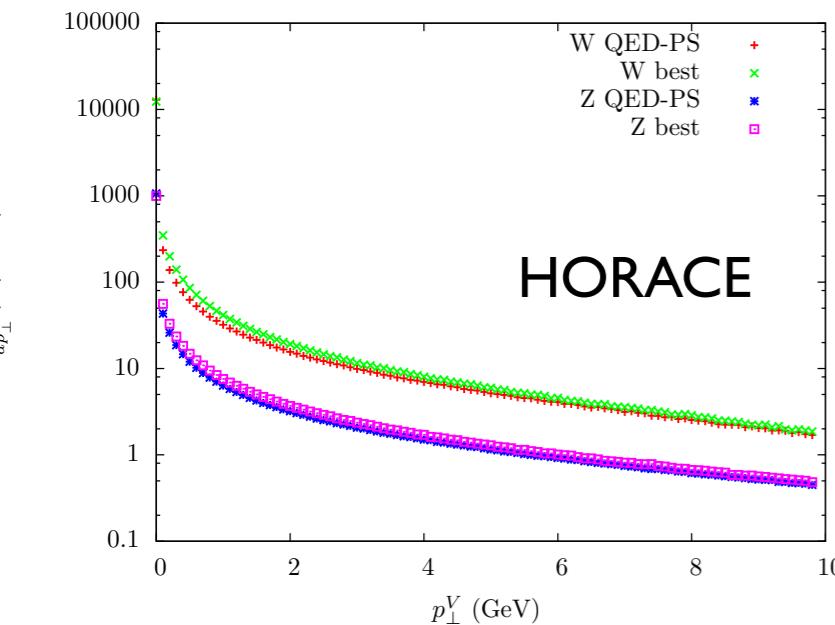


Uncertainty on  $p_T^W$  directly translates into an uncertainty on  $M_W$ .

Photon radiation yields a tiny gauge boson transverse momentum.

The gauge boson transverse momentum is different in the CC and NC channels because of the different flavor structure.

A possible estimate of the “non-final state” component differs in the 2 cases by  $54 (Z) - 33 (W) = 21 \text{ MeV}$



$$\langle p_T^V \rangle = \begin{array}{llll} Z & \text{FSR-PS} & 0.409 & \text{GeV} \\ Z & \text{best} & 0.463 & \text{GeV} \\ W & \text{FSR-PS} & 0.174 & \text{GeV} \\ W & \text{best} & 0.207 & \text{GeV} \end{array}$$

The fit of the non-perturbative PYTHIA parameters from the  $Z$  transverse momentum should be done using POWHEG (QCD+EW) + PYTHIA, in order to remove completely the EW corrections to the NC channel from the tuning  
→ the PYTHIA parameters will encode only non-perturbative QCD information

In the simulation of the CC channel, the use of POWHEG (QCD+EW), with the above PYTHIA parameters, will yield the proper combination of QCD and EW effects

# Combining QCD + EW corrections: old results

G. Balossini, C.M. Carloni Calame, G. Montagna, M. Moretti, O. Nicrosini, F. Piccinini, M. Treccani, A. Vicini, JHEP 1001:013, 2010

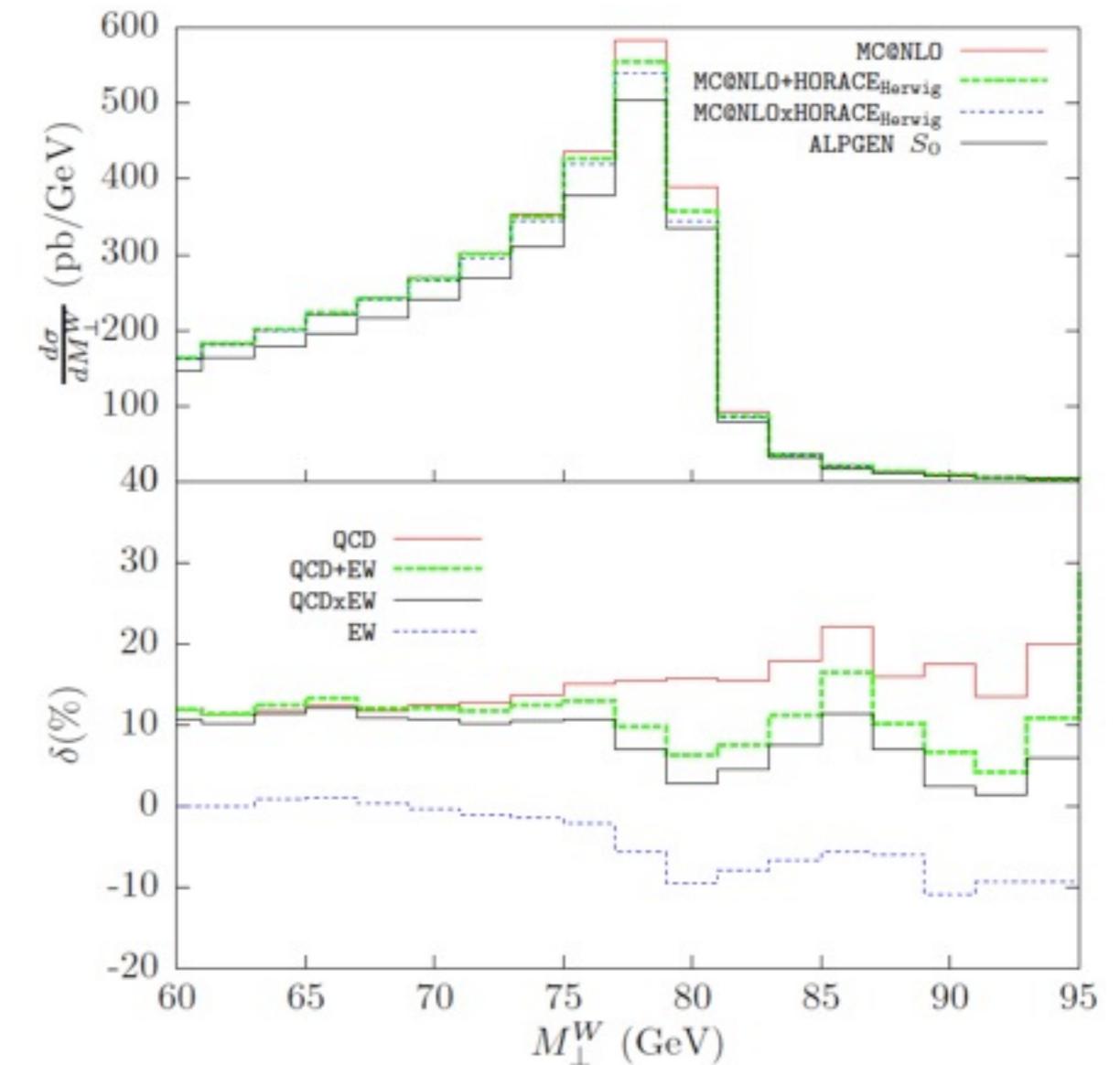
factorized prescription

$$\left[ \frac{d\sigma}{d\mathcal{O}} \right]_{QCD \otimes EW} = \left( 1 + \frac{\left[ \frac{d\sigma}{d\mathcal{O}} \right]_{MC@NLO} - \left[ \frac{d\sigma}{d\mathcal{O}} \right]_{HERWIG\ PS}}{\left[ \frac{d\sigma}{d\mathcal{O}} \right]_{LO/NLO}} \right) \times \left\{ \left[ \frac{d\sigma}{d\mathcal{O}} \right]_{EW} \right\}_{HERWIG\ PS}$$

additive prescription

$$\left[ \frac{d\sigma}{d\mathcal{O}} \right]_{QCD \oplus EW} = \left\{ \frac{d\sigma}{d\mathcal{O}} \right\}_{QCD} + \left\{ \left[ \frac{d\sigma}{d\mathcal{O}} \right]_{EW} - \left[ \frac{d\sigma}{d\mathcal{O}} \right]_{Born} \right\}_{HERWIG\ PS}$$

- the factorized and the additive formulae differ by few per cent
- different inclusion of higher orders  $\mathcal{O}(\alpha_s^2)$  and  $\mathcal{O}(\alpha\alpha_s)$



# Inclusion in POWHEG of the exact $\mathcal{O}(\alpha)$ corrections (NLO-EW)

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- the POWHEG basic formula
  - is additive in the overall normalization,
  - it describes exactly one parton emission (photon/gluon/quark) (but NOT two partons)
  - includes in a factorized form mixed and higher order corrections relevant in the distributions in particular the bulk of the  $\mathcal{O}(\alpha\alpha_s)$  corrections (but it has NOT  $\mathcal{O}(\alpha\alpha_s)$  accuracy)
- difference with respect to
  - $\mathcal{O} = \mathcal{O}_{LO} \left( 1 + \delta_{QCD}^{NLO+NNLO} + \delta_{EW}^{NLO} \right)$  1) purely additive prescription
  - $\mathcal{O} = \mathcal{O}_{LO} \left( 1 + \delta_{QCD}^{NLO+NNLO} \right) \left( 1 + \delta_{EW}^{NLO} \right)$  2) factorized use of (differential) K-factors
- POWHEG accounts for multiple emission effects
- the kinematics of multiple emissions is exact (fully differential)
- the subtraction of IS QED collinear singularities is consistent only with NNPDF2.3QED, where the evolution kernel of the parton densities includes also a QED term

# Ambiguities affecting the shape of the $ptV$ distribution

The prediction of the  $ptV$  distribution depends on the:

- logarithmic accuracy of the resummation uncertainty parametrized by the resummation scale  $Q$  (analytical approach)
- prescription to match fixed-order results and Parton Shower variation of  $h_{\text{fact}}$  in the general formulation of NLO-matched Shower MC
- QED and mixed QCDxQED effects

Any choice of the scale  $Q$  or of the factor  $h_{\text{fact}}$ , for a given PDF set, will then require a corresponding tuning of the model dependent part of the simulation

- non-perturbative “intrinsic” transverse momentum component measured from  $ptZ$ ; validity of the extrapolation to a different phase-space?  
to a different flavor combination?

PDF set choice: partial correlation between  $ptZ$  and  $ptW$ , in particular via the gluon density

# PYTHIA tuning and QCD scale uncertainties

- define a maximum level of disagreement (in terms of a  $\Delta\chi^2$ ) to consider the combination POWHEG+PYTHIA as a good description of  $Z$  data
- for a given choice of
  - PDF set and replica
  - renormalization scale
  - factorization scale
  - $h_{\text{fact}}$  scale
- using POWHEG (QCD+EW) NC  
perform a tuning of PYTHIA non-perturbative QCD parameters to describe  $Z$  observables
- repeat the above procedure for different choices of the scales (e.g.  $3^3$  canonical combinations) exploring the range of combinations that yield agreement with the data, defined above
- take each of these models (i.e. choice of scales and corresponding PYTHIA tune) and use it in POWHEG (QCD+EW) CC to predict MW-observables
- the spread of MW values provides an estimate of QCD uncertainties, for a given PDF set
- **caveat:** by no means we can fit/measure the QCD scales  
(the exact result to all orders does not depend on them) !  
but the tune of PYTHIA parameters depends on their value
- the PDF uncertainty of a given set can be studied after that the PYTHIA tuning with the central replica has been determined

# On-going benchmarking study within the LHC-EWWG

see <http://lpcc.web.cern.ch/lpcc/>

- the authors of the following codes are actively participating to this study
  - HORACE, RADY, SANC, WZGRAD
  - PHOTOS, WINHAC
  - DYNNLO, FEWZ
  - POWHEG (only QCD and QCD+EW)
- in a first phase, technical agreement (same inputs  $\Rightarrow$  same outputs)  
at LO, NLO-QCD, NLO-EW has been reached on differential distributions at better than 0.5% level
- given this common starting point with NLO accuracy,  
we are now exploring the impact of higher order corrections (pure QCD, pure EW, mixed QCDxEW)
  - corrections available only in some codes (e.g. NNLO-QCD vs QCD-PS)
  - ambiguities which can not be fixed without an explicit full next-order calculation (e.g. EW inputs)

# Conclusions

- different behaviour of observables inclusive vs more exclusive w.r.t. QCD radiation  
need for resummed calculations (either analytical or via Parton Shower)
- full NLO-(QCD+EW) matched with QCD+QED showers are available in POWHEG  
both in CC and in NC
- the merging procedure to combine QCD and EW corrections may follow different prescriptions  
yielding different results  
POWHEG QCD+EW provides a motivated *Ansatz* that includes systematically several higher-order  
mixed contributions
- matching resummation/Parton Shower with fixed order results introduces some ambiguities  
which affect the shape of  $pt_V$  distribution (and in turn of  $pt_I$  or  $MT_W$  and in turn of  $M_W$ )
- a detailed tune of PYTHIA parameters can be performed with POWHEG QCD+EW NC  
and the result consistently applied to the CC process

# Back-up

# POWHEG formulation and the role of *hfact*

$$d\sigma^{\text{NLO+PS}} = d\Phi_B \bar{B}^s(\Phi_B) \left[ \Delta^s(p_\perp^{\min}) + d\Phi_{R|B} \frac{R^s(\Phi_R)}{B(\Phi_B)} \Delta^s(p_T(\Phi)) \right] + d\Phi_R R^f(\Phi_R) + d\Phi_R R_{reg}(\Phi_R)$$

$$\bar{B}^s = B(\Phi_B) + \left[ V(\Phi_B) + \int d\Phi_{R|B} R^s(\Phi_{R|B}) \right]$$

$R = R_{reg} + R_{div}$  is the sum of all the real emission squared matrix elements, with a regular (divergent) behaviour in the collinear limit

$R^s$  enters in the Sudakov form factor  $\Delta^s(p_T(\Phi))$

**MC@NLO**

$$R^s = \frac{h^2}{h^2 + p_T^2} R_{div} \quad R^f = \frac{p_T^2}{h^2 + p_T^2} R_{div}$$

$$R^s \propto \frac{\alpha_s}{t} P_{ij}(z) B(\Phi_B)$$

$$R^f = R - R^s$$

at low  $pT$ , the damping factor  $\rightarrow 1$ ,  $R_{div}$  tends to its collinear approximation,  
at large  $pT$ , the damping factor  $\rightarrow 0$  and suppresses  $R_{div}$  in the Sudakov and in the square bracket

the scale  $h$  fixes the upper limit for the Sudakov form factor to play a role,  
**effectively is the upper limit for the inclusion of multiple parton emissions**

the total cross section does NOT depend on the value of  $h$