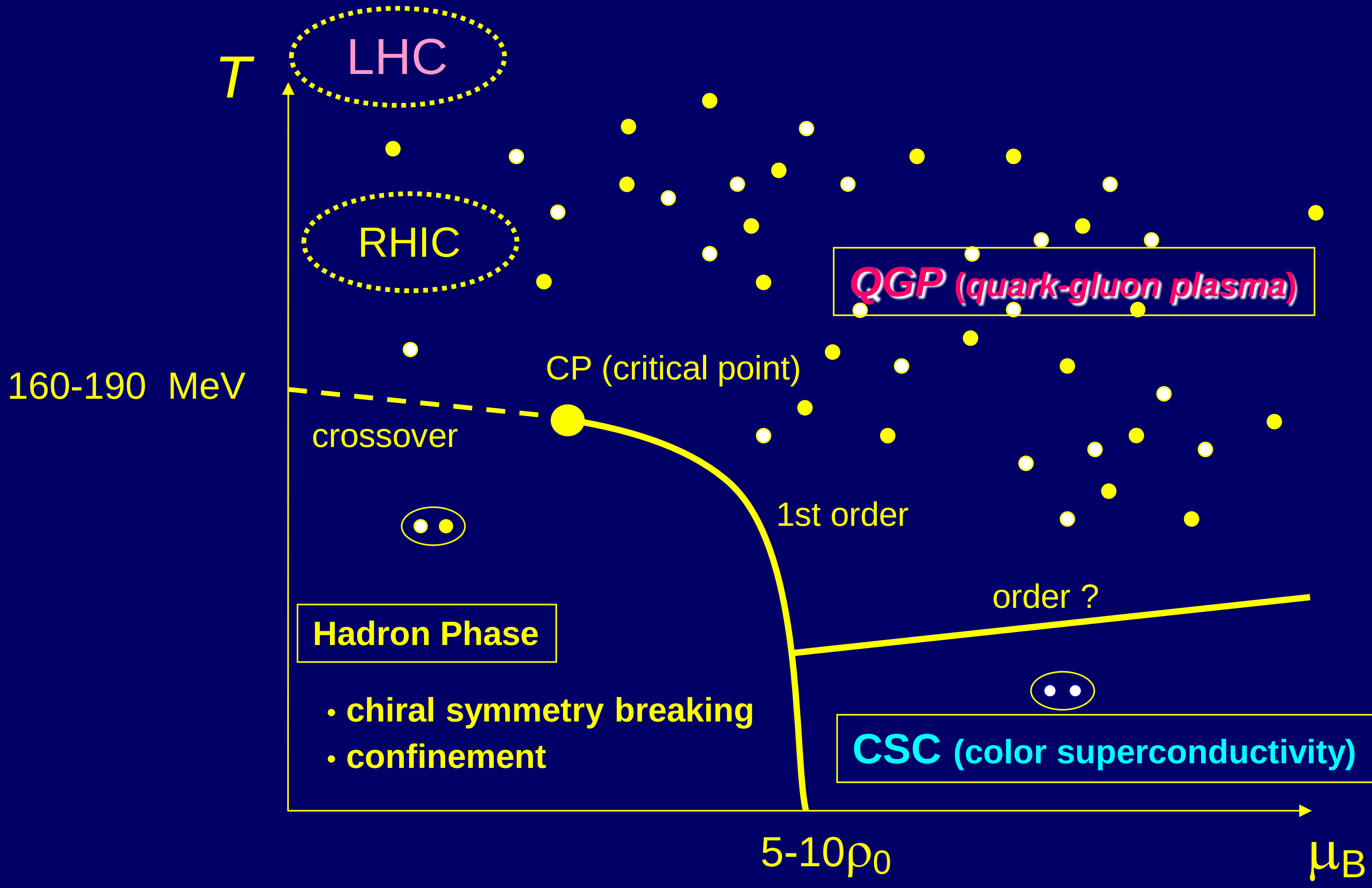
Conserved Charge Fluctuations: Myths and Facts

Masayuki Asakawa

Department of Physics, Osaka University

With M. Kitazawa, H. Ono, and M. Sakaida

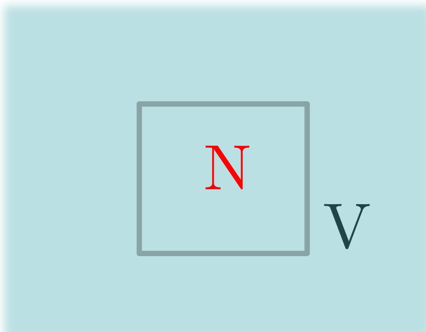
QCD Phase Diagram

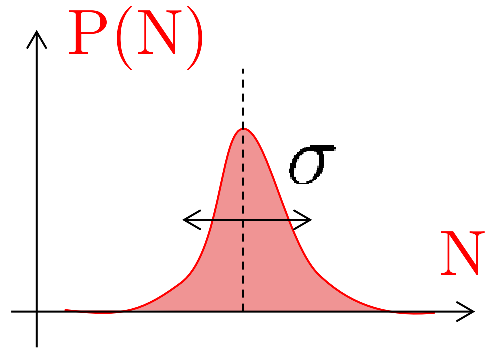


M. Asakawa (Osaka University)

Fluctuations, or Cumulants $\langle \delta N^n \rangle_c$

Observables in equilibrium are fluctuating.



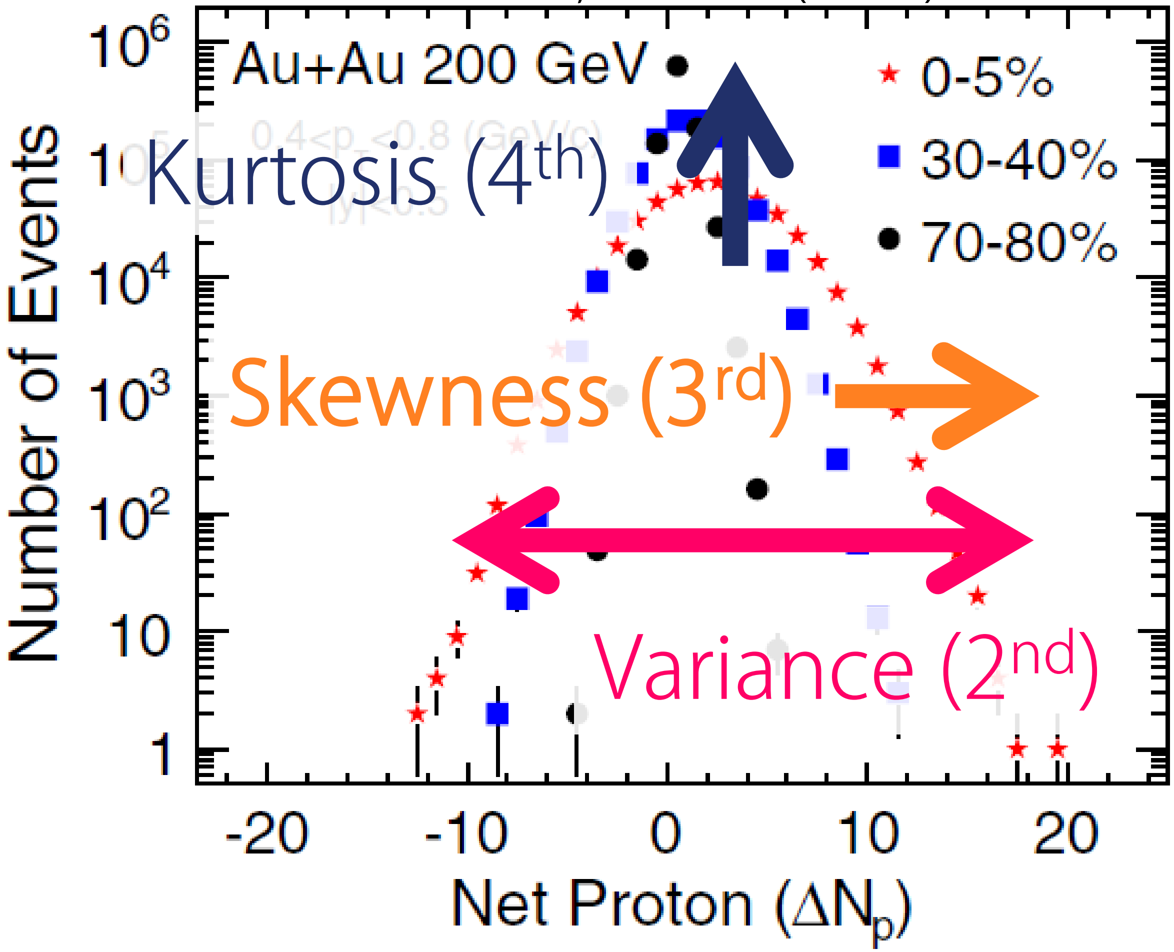


- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$ $\delta N = N - \langle N \rangle$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

Non-Gaussianity

Variance, Skewness, and Kurtosis

STAR, PRL**105** (2010)



Why Conserved Charge Fluctuations ?

- Their values do not change during the phase transition
- Their values in QGP and Hadron Phase are different
- They change in Hadron Phase only by diffusion

D measure for electromagnetic charge fluctuation

Heinz, Müller, M.A., Jeon, Koch, 2000

- Charge Fluctuation and Baryon Number Fluctuations are well-defined quantities, and can be measured on the lattice

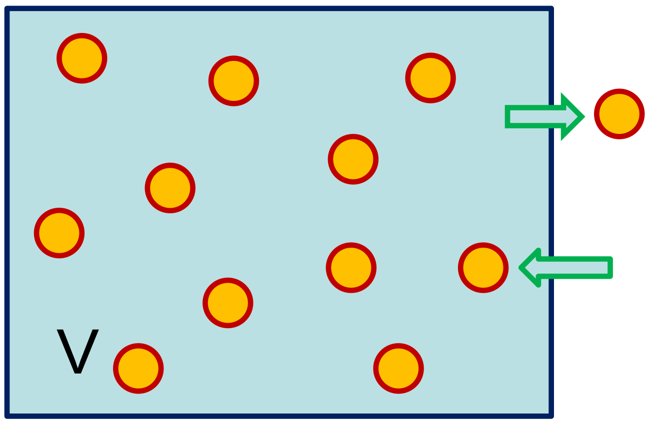
- Lattice results and Effective Model results (equilibrium thermodynamics) are often compared with experimental results

  Does this make sense?

Conserved and Non-Conserved Charge Fluc.

Necessary to consider dynamical evolution of fluctuation!

Conserved Charge

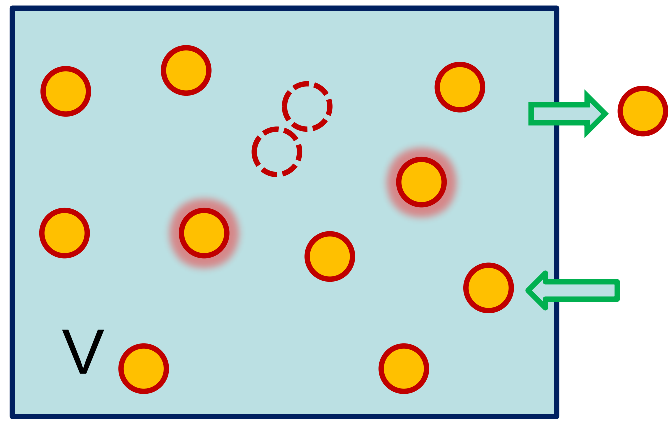


Only diffusion changes
the number of charge

relaxation time $\tau \to \infty$

for $V \to \infty$

Non-Conserved Charge



Charge can change
anywhere in the volume

$\tau \to \text{const.}$

for $V \to \infty$

Three Myths for Fluctuations

- *Proton number is a proxy of baryon number*

- *Freeze-out parameters: lattice meets experiment*

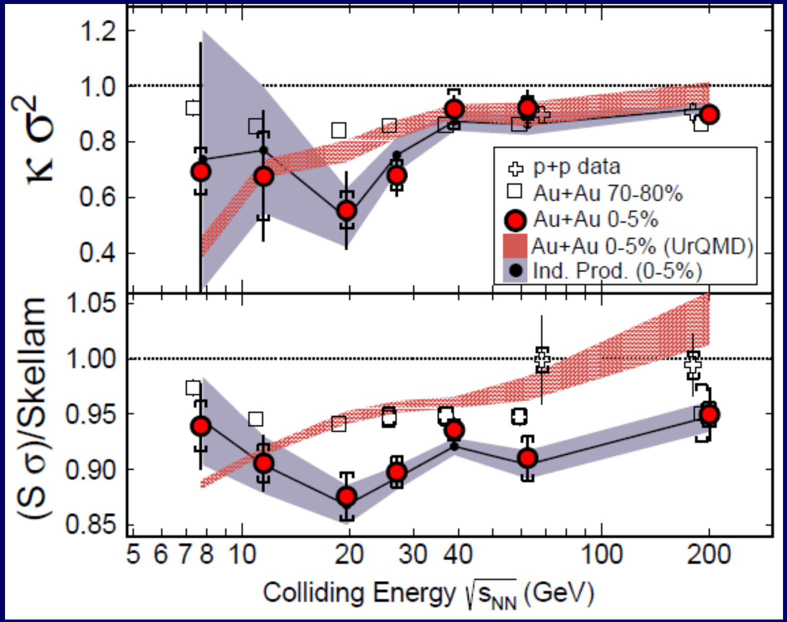
- *Global Charge Conservation is important even at LHC*

Proton Number Cumulants

- Proton Number Fluctuation has been attracting a lot of interest because it can be observed experimentally
- Proton Number Fluctuation diverges at CP

Hatta and Stephanov, 2003

- Comparisons of experimental results and lattice predictions have been made （e.g. Gupta et al., Science 2011)



$$\chi_B^{(n)}\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)=\frac{1}{T^n}\frac{\partial^n}{\partial(\mu_B/T)^n}P\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)\Bigg|_{T/T_c}$$

$$S\sigma=\frac{T\chi_B^{(3)}}{\chi_B^{(2)}}$$

$$\kappa\sigma^2=\frac{T^2\chi_B^{(4)}}{\chi_B^{(2)}}$$

STAR, PRL 2014

Experiment: Net Proton
Theory: Net Baryon

Is this harmless?

Protons and Baryons

The question here is how these are related to each other:

$$\left\langle \left(\delta N_p\right)^n \right\rangle_c \Longleftrightarrow \left\langle \left(\delta N_\mathrm{B}\right)^n \right\rangle_c$$

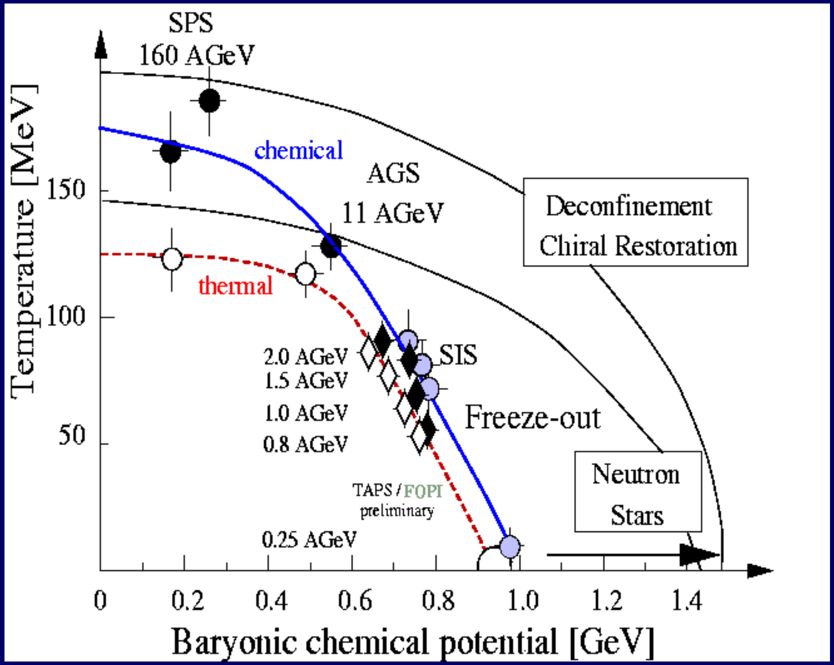
In free nucleon gas in equilibrium,

$$\left\langle \left(\delta N_\mathrm{B}\right)^n \right\rangle_c = 2\left\langle \left(\delta N_p\right)^n \right\rangle_c$$

Otherwise, in general,

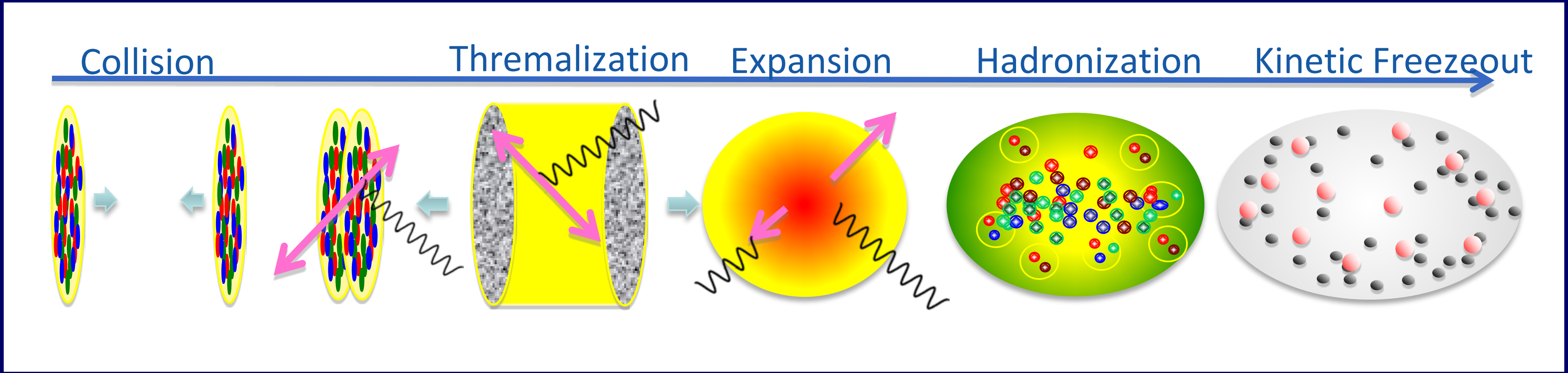
$$\left\langle \left(\delta N_\mathrm{B}\right)^n \right\rangle_c \neq 2\left\langle \left(\delta N_p\right)^n \right\rangle_c$$

Freezeouts

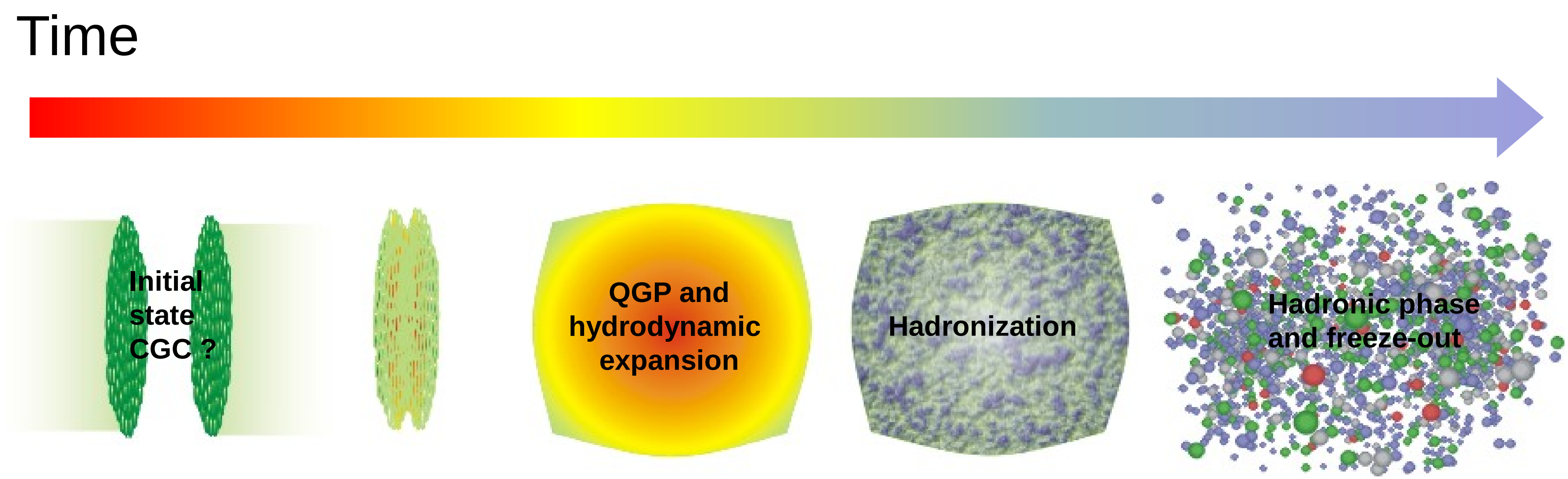


- Net proton number may be considered as a proxy of net baryon number
- Chemical freezeout is close to the crossover, and (anti)proton number is expected to be fixed early (?)

➢ But Not all particle numbers and fluctuations are fixed at chemical freezeout



Heavy Ion Physics 101



- Electromagnetic probes (γ's, dileptons) leave QGP without interaction

  no exceptions

- On the other hand, hadrons keep on interacting with each other until freezeouts
- There are two freeze-outs, chemical and thermal

These are (phenomenological) results of dynamics

Some exceptions may exist ➡ need to understand physics

Exception

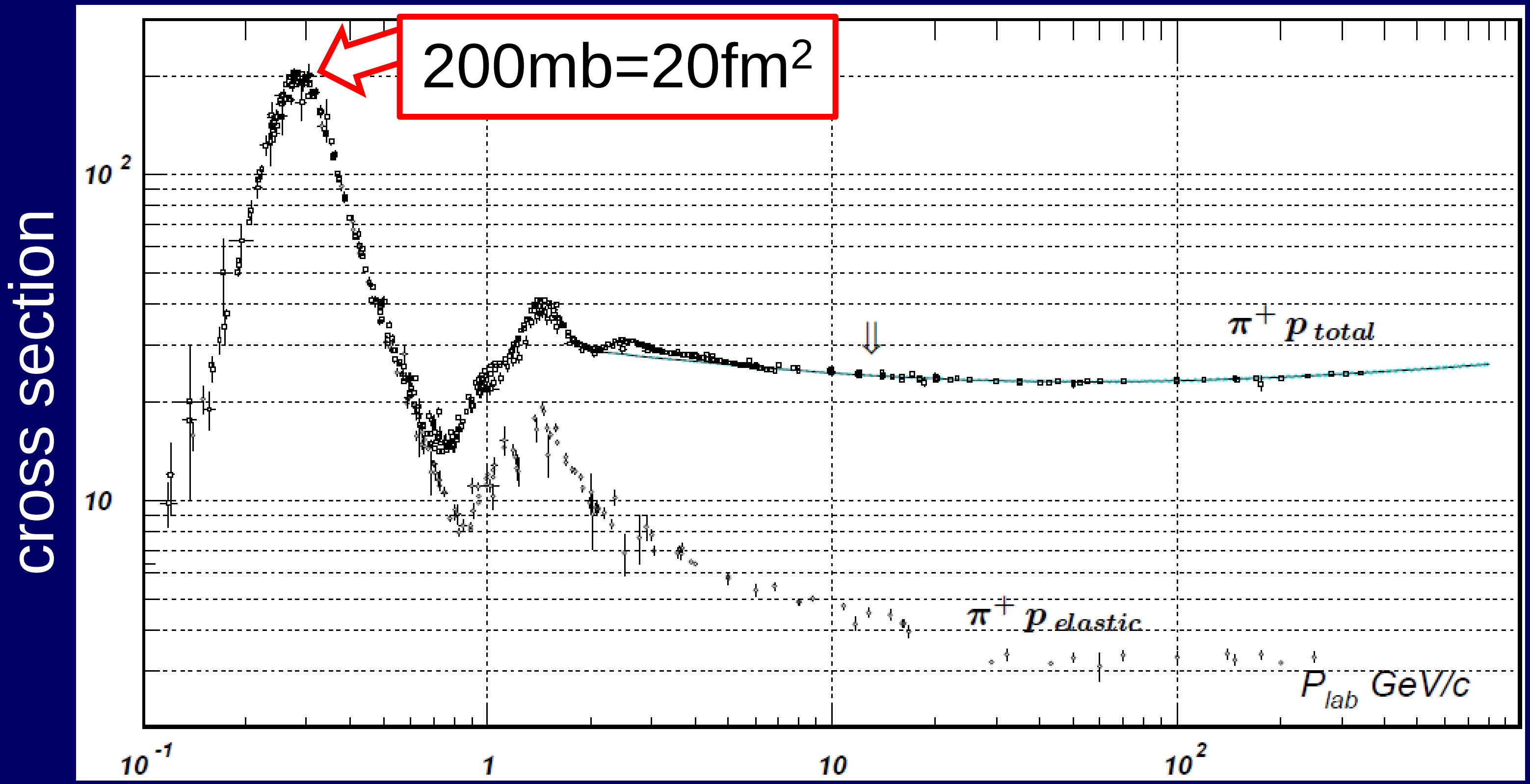
If there are low mass resonances with large cross sections, this exception happens

In our case at hand, Δ resonances

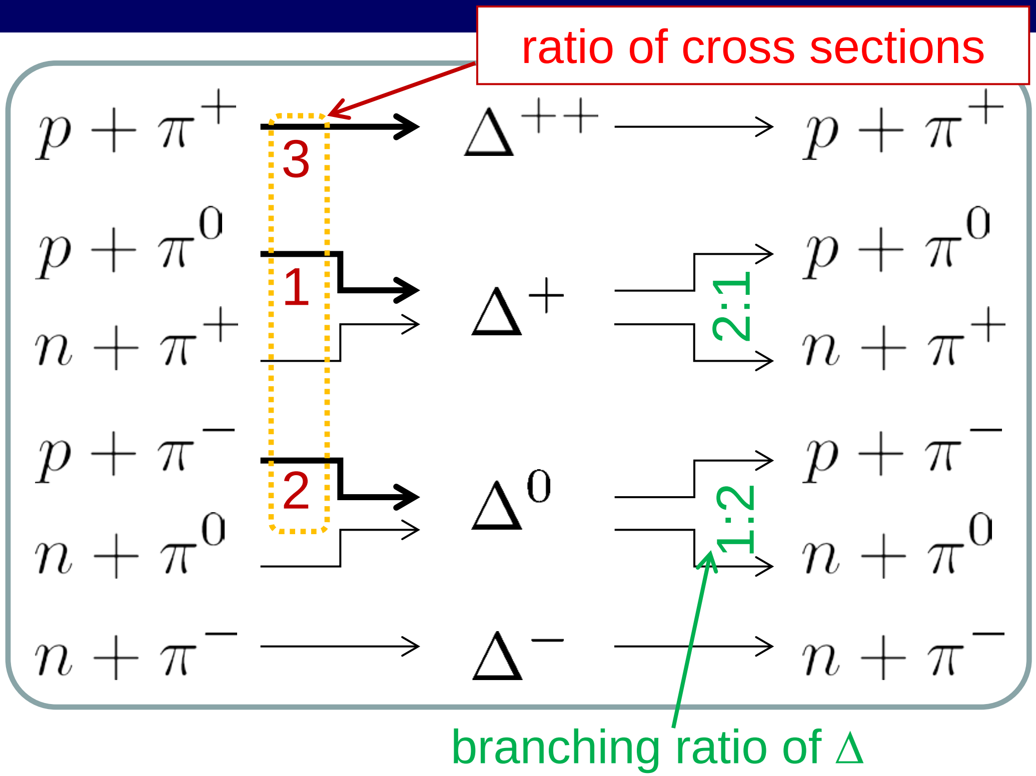
$$p, n \searrow \quad \Delta(1232) \nearrow p, n$$
$$\pi \nearrow \quad\quad\quad\quad \searrow \pi$$

$$I = 3/2$$

$$\Gamma \simeq 1.8 \text{ [fm]}$$

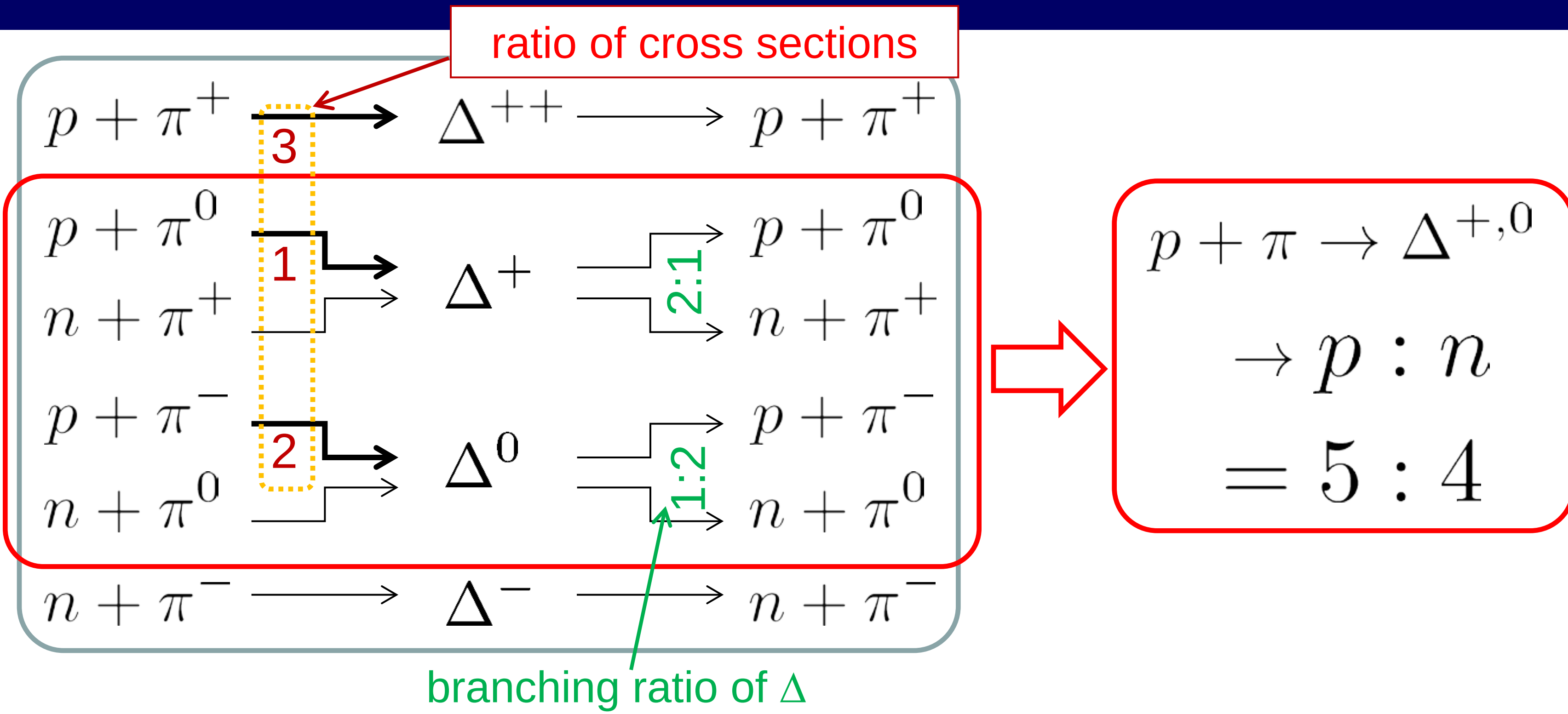


Effect of Δ

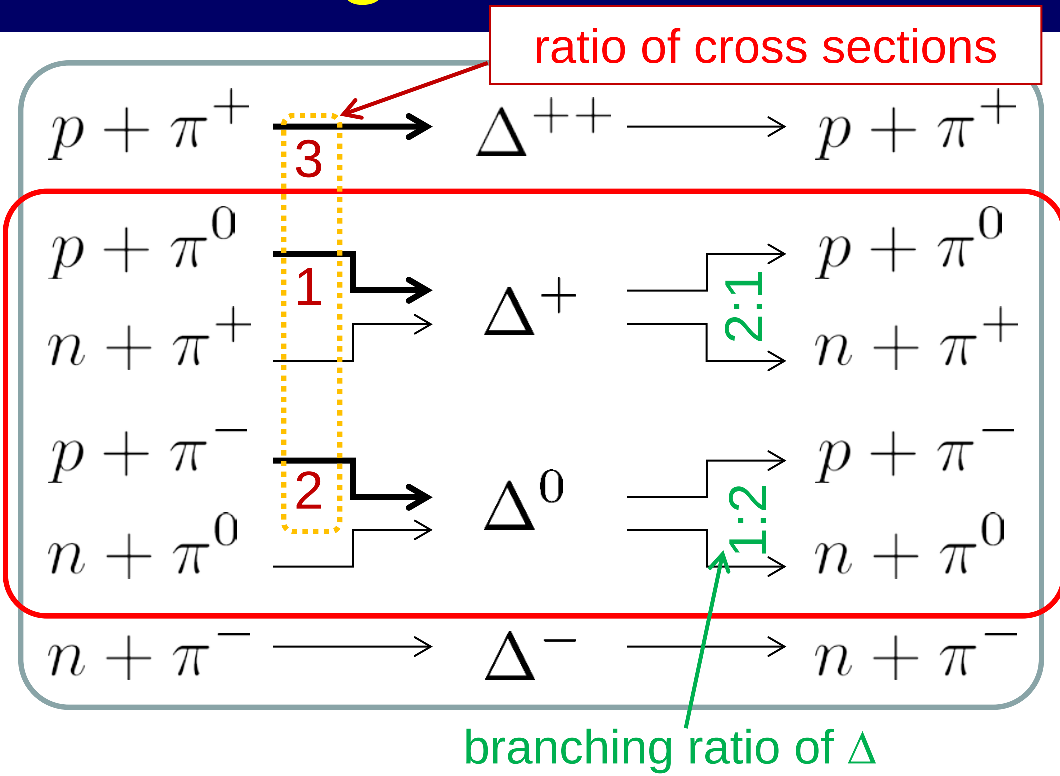


ratio of cross sections
$p + \pi^+$
3
Δ^{++}
$p + \pi^+$
$p + \pi^0$
1
$n + \pi^+$
Δ^+
2:1
$p + \pi^0$
$n + \pi^+$
$p + \pi^-$
2
$n + \pi^0$
Δ^0
1:2
$p + \pi^-$
$n + \pi^0$
$n + \pi^-$
Δ^-
$n + \pi^-$
branching ratio of Δ

Effect of Δ


ratio of cross sections
$p+\pi^+$ 3 Δ^{++} $p+\pi^+$
$p+\pi^0$ 1 Δ^+ 2:1 $p+\pi^0$
$n+\pi^+$ $n+\pi^+$
$p+\pi^-$ 2 Δ^0 $p+\pi^-$
$n+\pi^0$ 1:2 $n+\pi^0$
$n+\pi^-$ Δ^- $n+\pi^-$
branching ratio of Δ
$p+\pi\rightarrow\Delta^{+,0}$
$\rightarrow p:n$
$=5:4$

How long is the mean free time?

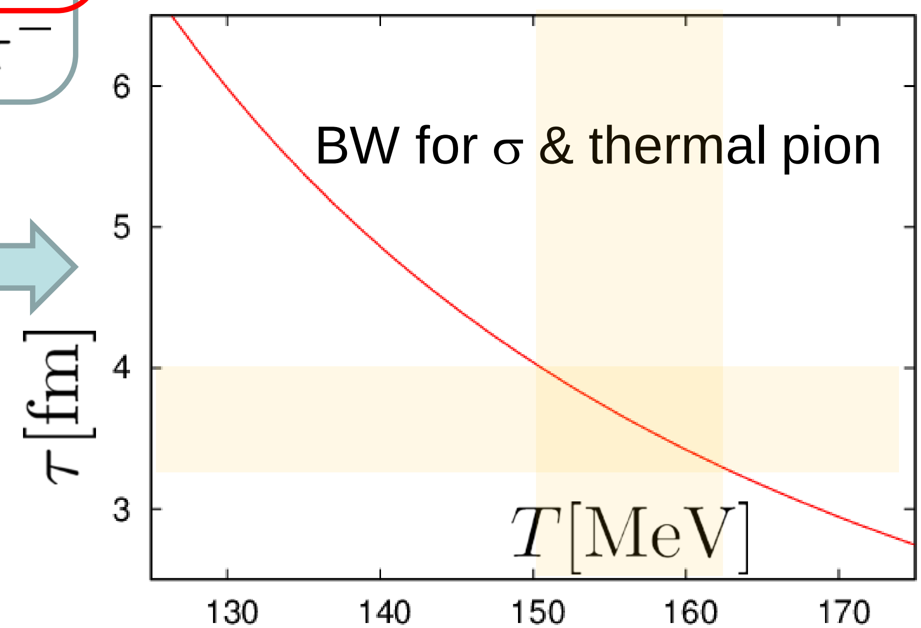


$$p+\pi \to \Delta^{+,0}$$
$$\to p : n$$
$$= 5 : 4$$

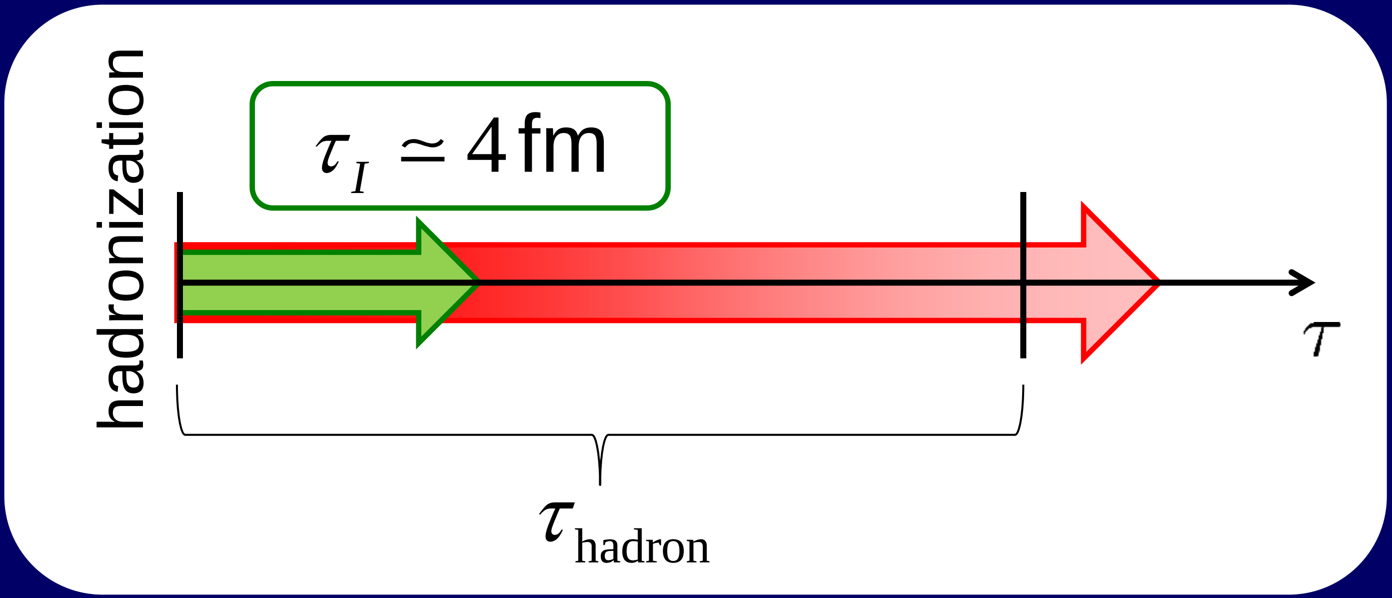
Meantime to create Δ^+ or Δ^0

$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3}\sigma(E_{\rm cm})v_\pi n(E_\pi)$$

$\tau \leq$ a few fm



Time Scales



- τ_I : time scale to realize isospin randomization
- τ_{hadron} : time scale of hadron phase duration

τ_{hadron} ⇐ result of state-of-art hydro + cascade calculation

Result of Hydro+Cascade Calculation

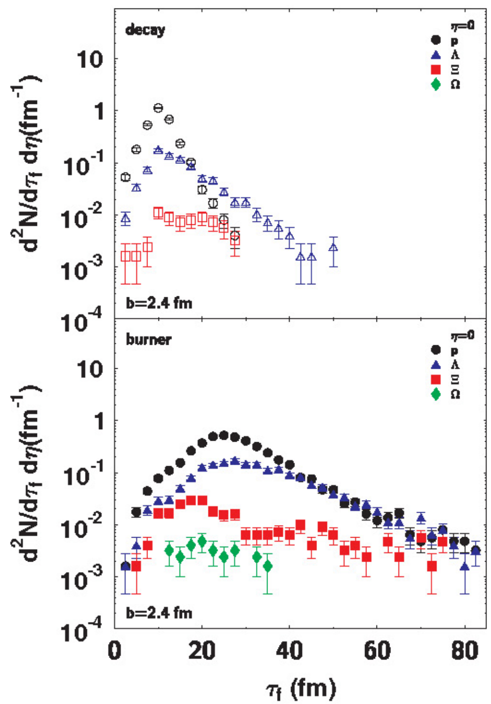


FIG. 22. (Color online) Freeze-out time distribution of baryons for hydro+decay (open symbols, above) and hydro+UrQMD (solid symbols, below) at midrapidity.

providing us with an estimate on the lifetime of the hadronic phase around 10–20 fm/c. Note that this estimate is subject to the same systematic uncertainties discussed previously in the context of the overall lifetime of the system.

Freezeout time distribution

without after-burner

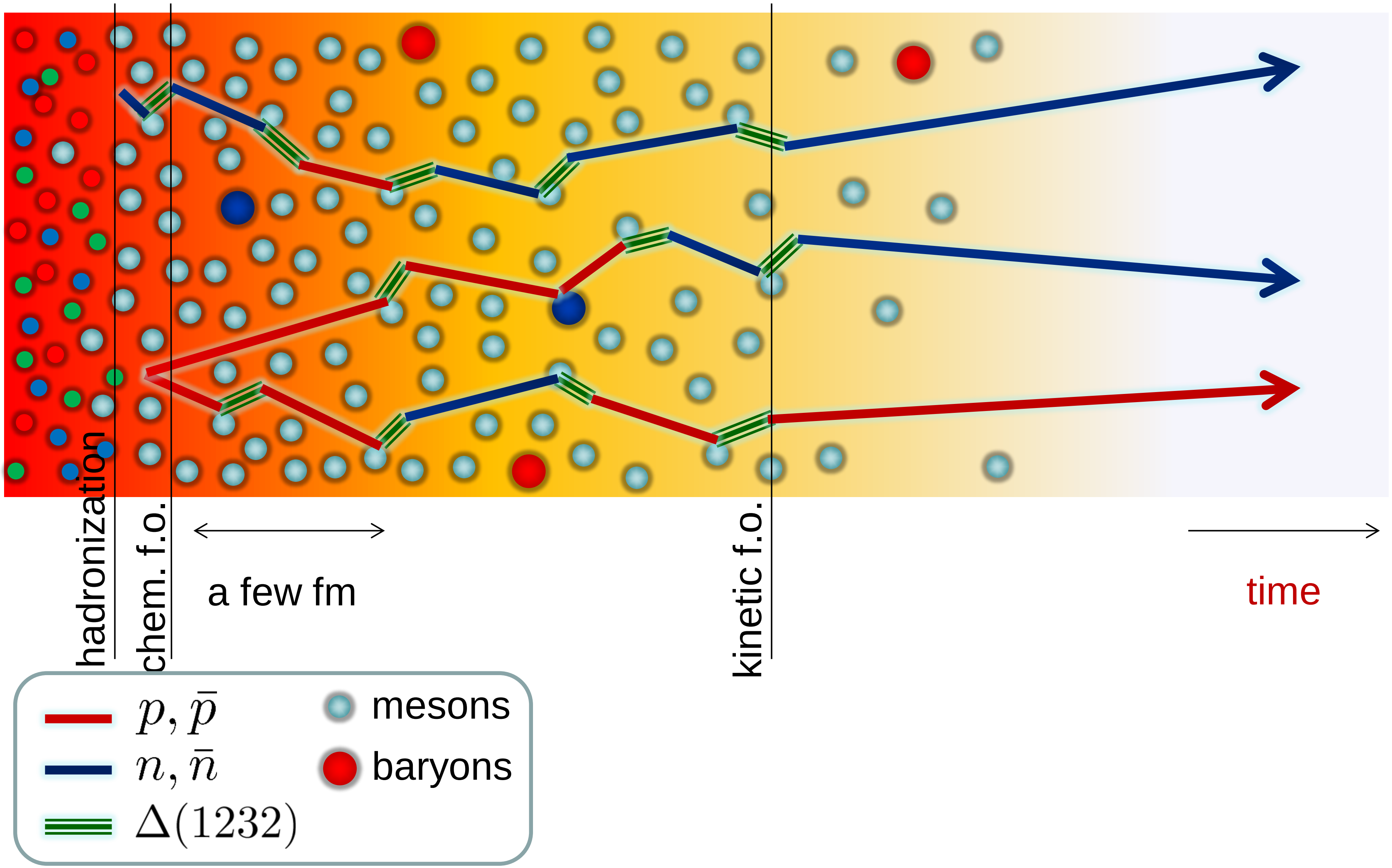
with after-burner

τ_{hadron} : 10 ~ 20 fm

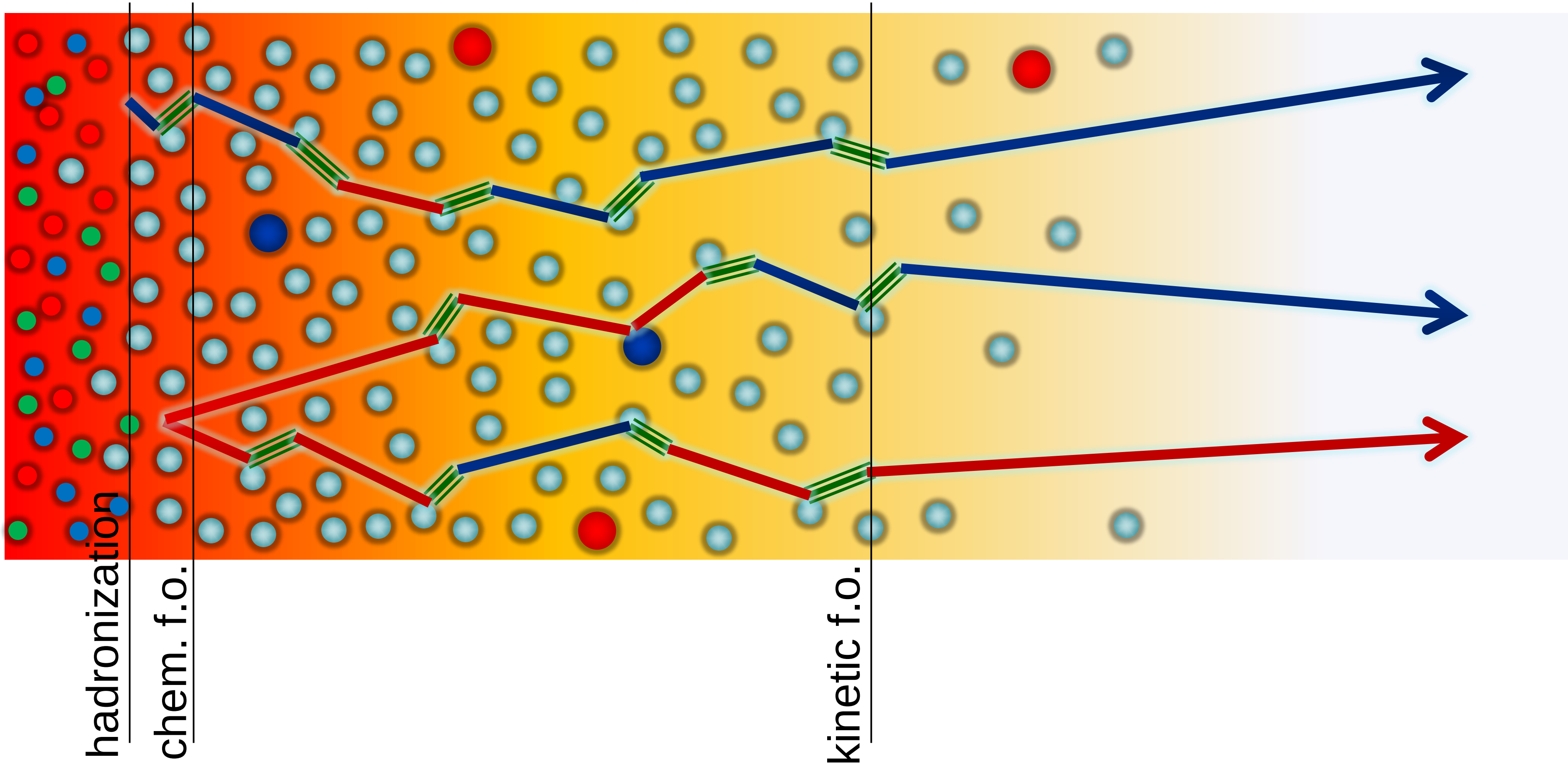
Nonaka and Bass, PRC 2007

$\tau_I \ll \tau_{\text{hadron}}$ isospin: randomized

Nucleon Isospin Randomization in Pion Gas

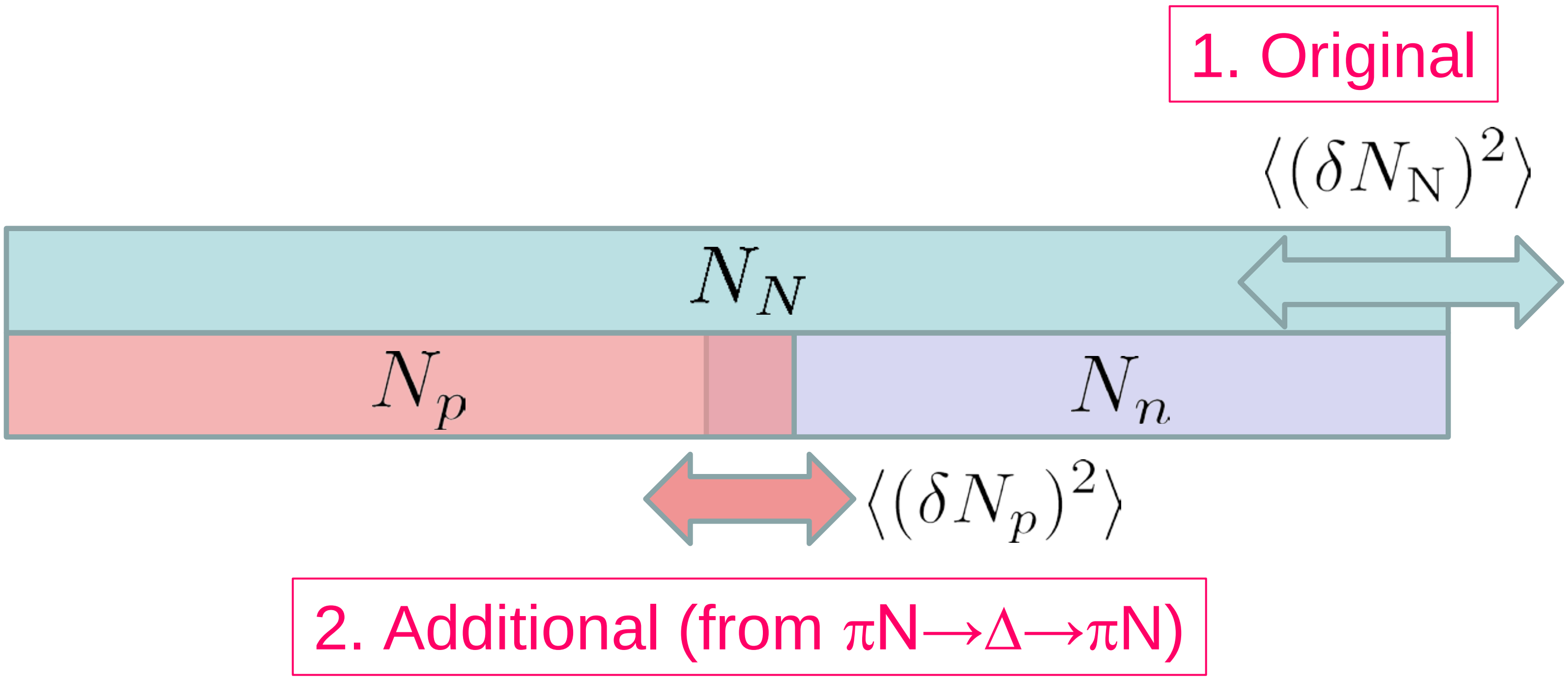


Probability Distribution



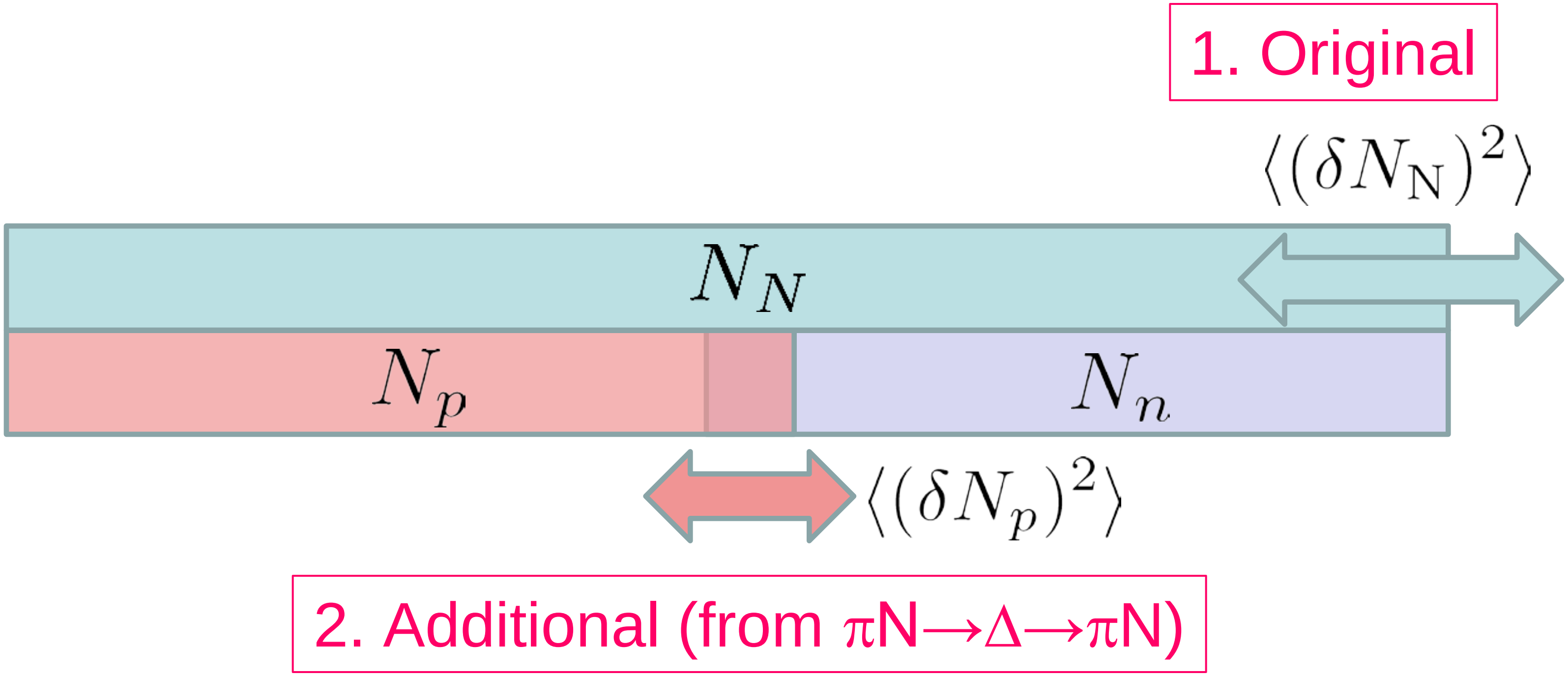
$$P_i(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) \Rightarrow P_f(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$

Production of Additional Fluctuation



- In, general, fluctuations of N_N and N_p are different
- Additional N_p fluctuations are created by (thermal) pions

Proton and Nucleon Moments



$$\left\langle\left(\delta N_p^{(\mathrm{net})}\right)^2\right\rangle = \frac{1}{4}\left\langle\left(\delta N_{\rm N}^{(\mathrm{net})}\right)^2\right\rangle + \frac{1}{4}\left\langle N_{\rm N}^{(\mathrm{tot})}\right\rangle$$

$$\left\langle\left(\delta N_{\rm N}^{(\mathrm{net})}\right)^2\right\rangle = 4\left\langle\left(\delta N_p^{(\mathrm{net})}\right)^2\right\rangle - 2\left\langle N_p^{(\mathrm{tot})}\right\rangle$$

for isospin symmetric matter

- For free nucleon gas

$$\left\langle\left(\delta N_p^{(\mathrm{net})}\right)^2\right\rangle = \frac{1}{2}\left\langle\left(\delta N_{\rm N}^{(\mathrm{net})}\right)^2\right\rangle$$

Proton and Nucleon Moments

Similarly,

$$N_{\mathrm{N}} \longrightarrow N_p$$

$$\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^3 \right\rangle = \frac{1}{8}\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^3 \right\rangle + \frac{3}{8}\left\langle \delta N_{\mathrm{N}}^{(\mathrm{net})} \delta N_{\mathrm{N}}^{(\mathrm{tot})} \right\rangle$$

$$\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^4 \right\rangle_c = \frac{1}{16}\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^4 \right\rangle_c + \frac{3}{8}\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^2 \delta N_{\mathrm{N}}^{(\mathrm{tot})} \right\rangle + \frac{3}{16}\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^2 \right\rangle - \frac{1}{8}\left\langle N_{\mathrm{N}}^{(\mathrm{tot})} \right\rangle$$

$$N_p \longrightarrow N_{\mathrm{N}}$$

$$\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^3 \right\rangle = 8\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^3 \right\rangle - 12\left\langle \delta N_p^{(\mathrm{net})} \delta N_p^{(\mathrm{tot})} \right\rangle + 6\left\langle N_p^{(\mathrm{net})} \right\rangle$$

$$\left\langle \left(\delta N_{\mathrm{N}}^{(\mathrm{net})}\right)^4 \right\rangle_c = 16\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^4 \right\rangle_c - 48\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^2 \delta N_p^{(\mathrm{tot})} \right\rangle + 48\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^2 \right\rangle + 12\left\langle \left(\delta N_p^{(\mathrm{tot})}\right)^2 \right\rangle - 26\left\langle N_p^{(\mathrm{tot})} \right\rangle$$

$$\left\langle \delta N^4 \right\rangle_c = \left\langle (\delta N)^4 \right\rangle - 3\left\langle (\delta N)^2 \right\rangle^2$$

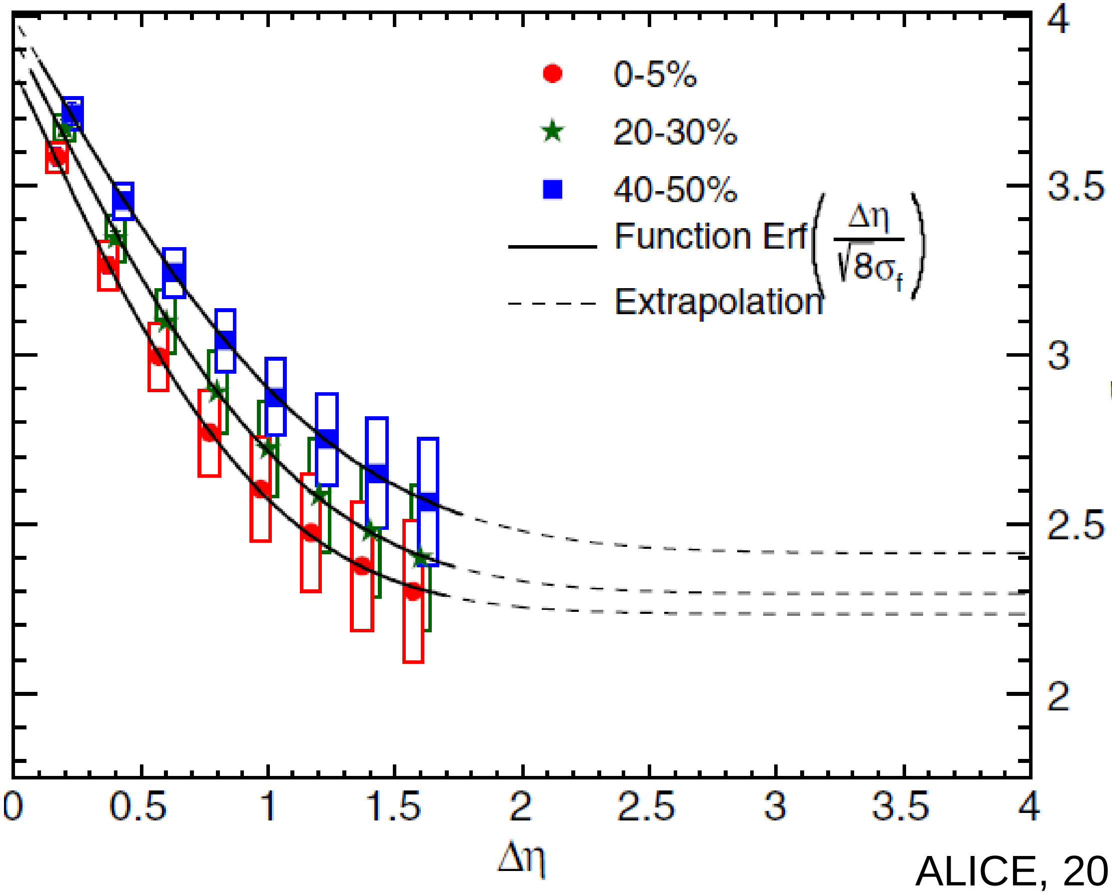
Conclusion 1

- *Proton number is NOT a proxy of baryon number*

  - This statement is true at least at RHIC and LHC

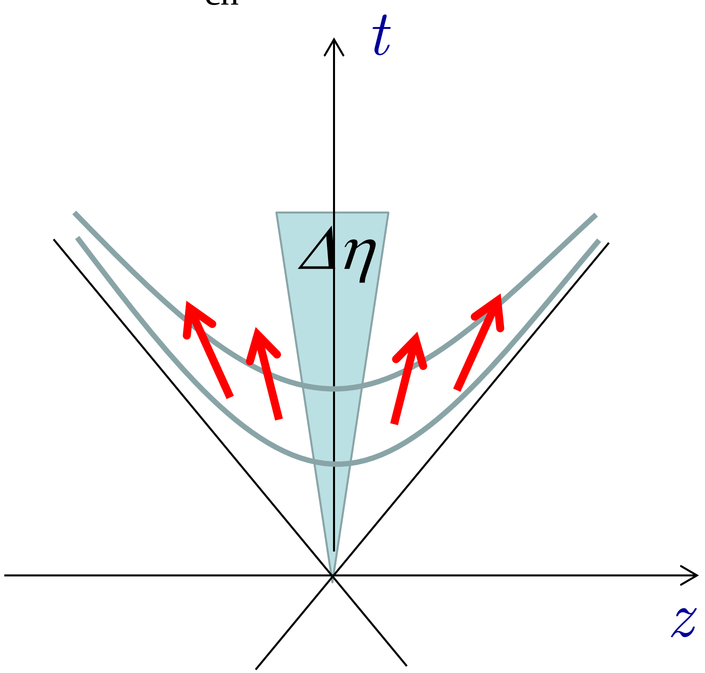
  - At BES energies,
    where pion density is small and τ_{hadron} is not large,
    proton number could be a proxy of baryon number approximately

$\Delta\eta$ Dependence @ ALICE



ALICE, 2013

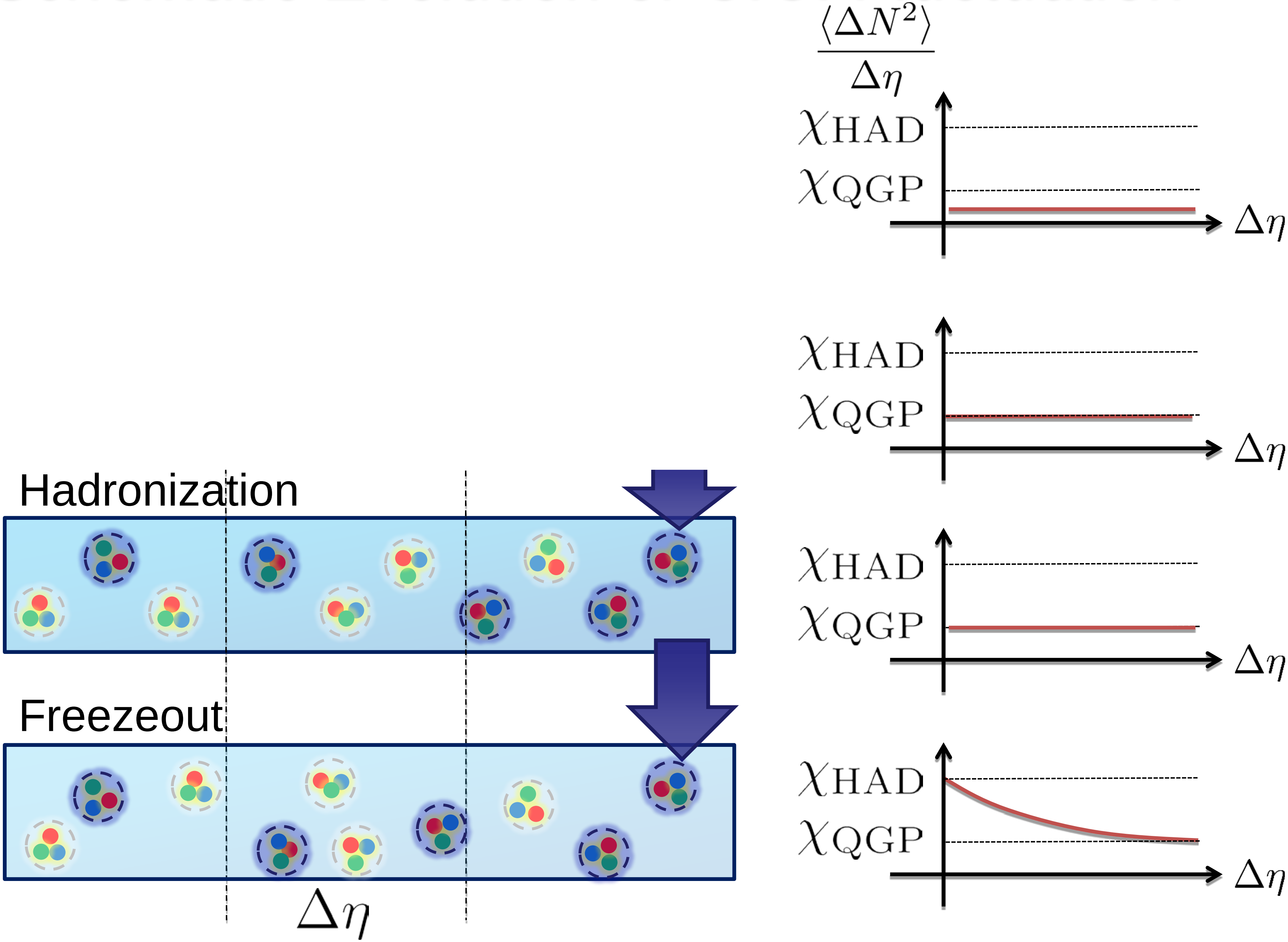
$$D = 4\frac{\langle \delta N_Q^2 \rangle}{N_{ch}}$$



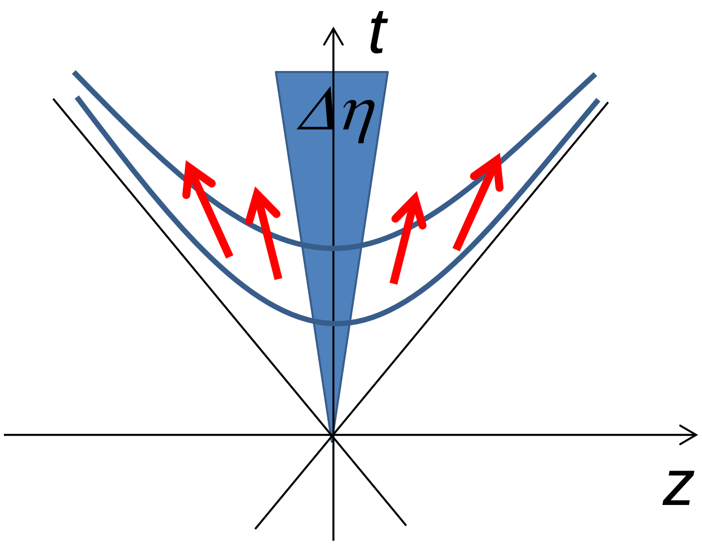
- *Freeze-out parameters: lattice meets experiment*

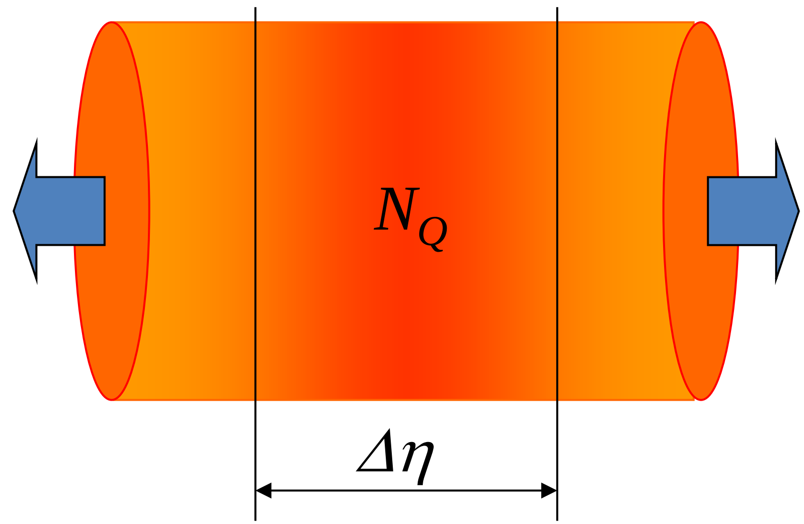
In this argument, no rapidity window dependence is taken into account

Schematic Evolution of C.C. Fluctuation

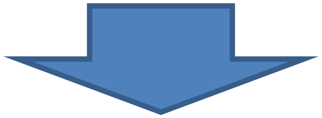


Time Evolution of Conserved Charge



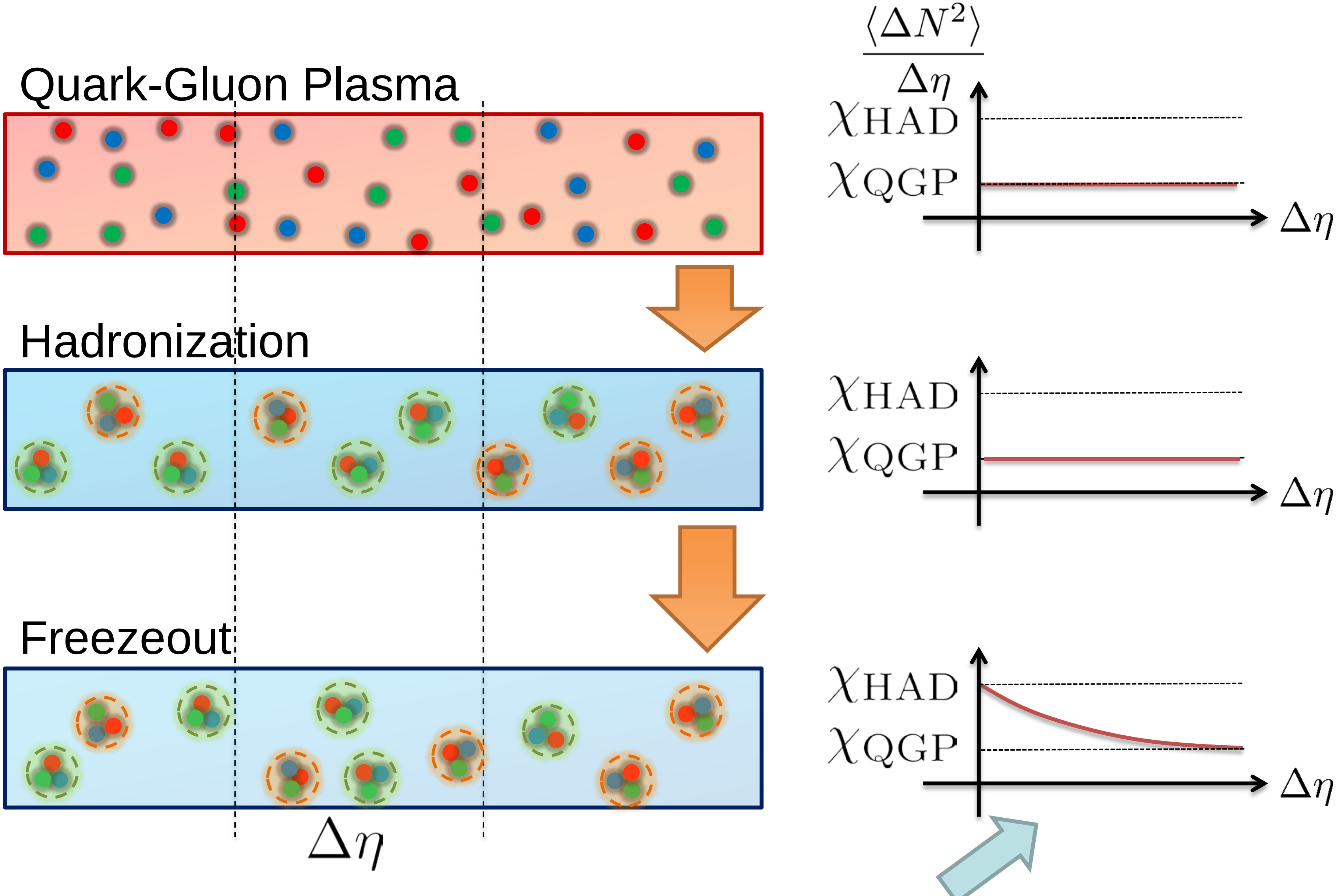


Variation of a conserved charge in Δη is achieved only through diffusion



The larger Δη, the slower diffusion

Time Evolution of C.C. fluctuation



In the Δη dependence of C.C. Fluctuation, history of system is encoded

Conservation Charge Transport in Hadron Phase

Naively,

Diffusion Equation,

$$\partial_\tau n = D\partial_\eta^2 n$$

Plus Fluctuation

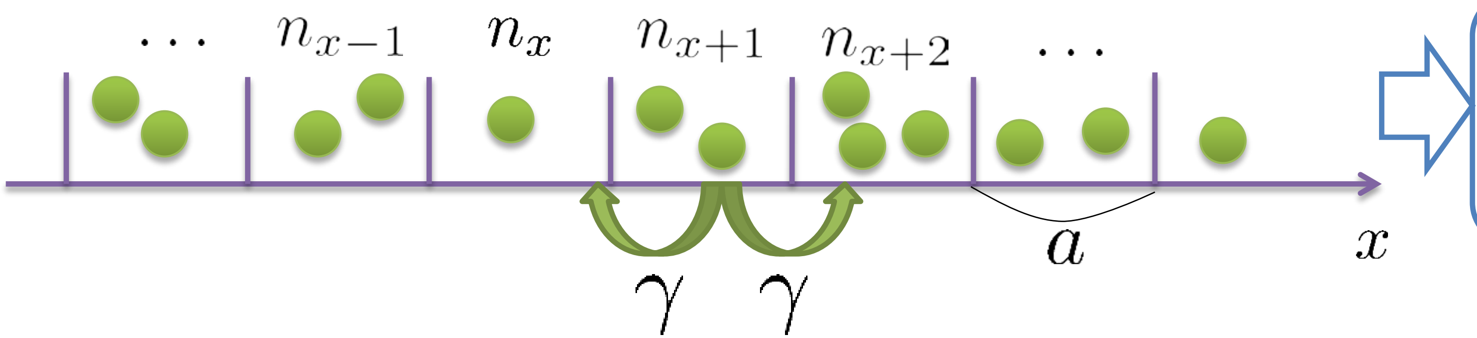
$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

But it is known *stochastic forces for*
"Markov process for continuum variable(s)" are Gaussian

⇒ We will use a discrete formulation

Diffusion Master Equation (DME)

Divide spatial coordinate into discrete cells



probability

$P(\mathbf{n})$

Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x + 1)\{P(\mathbf{n}+\mathbf{e}_x - \mathbf{e}_{x+1}) + P(\mathbf{n}+\mathbf{e}_x - \mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

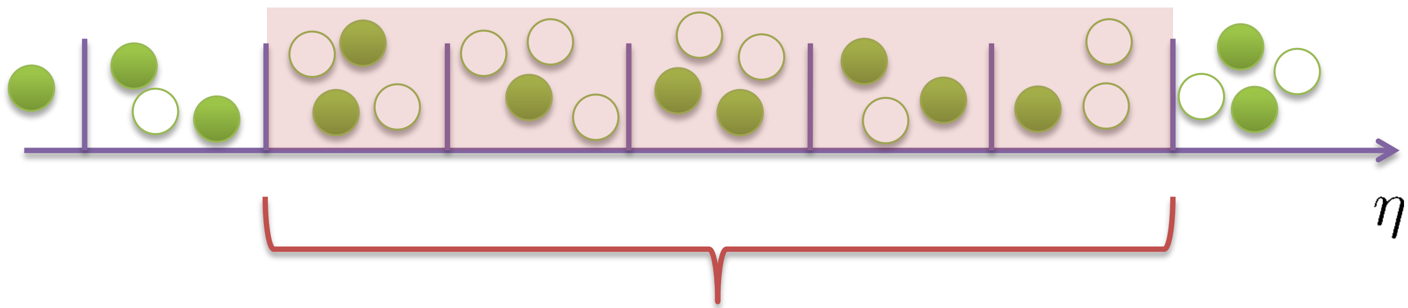
Solve the DME **exactly**, and take *a*→0 limit

No approximation is needed

Ono, Kitazawa, M.A., PLB 2014

Net Charge Number

Prepare 2 species of (non-interacting) particles



$$\bar{Q}(\tau,\Delta\eta)=\int_0^{\Delta\eta}\big(n_1(\tau,\eta)-n_2(\tau,\eta)\big)\,d\eta$$

Let us investigate

$\langle\bar{Q}^2\rangle_c \quad \langle\bar{Q}^4\rangle_c$ at freezeout time t

Net and Total Charge Numbers

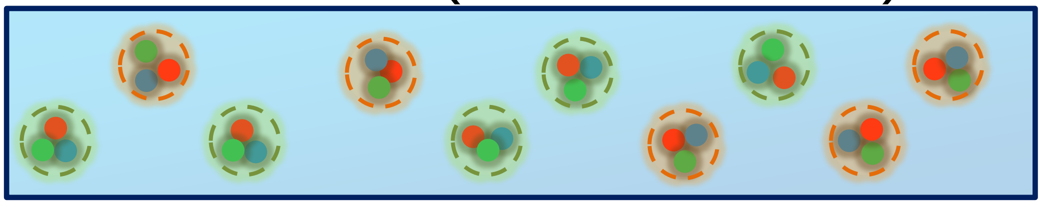
In the following, it is important to distinguish "net" and "total" charges

$$Q_{(\mathrm{net})}(\tau,\Delta\eta)=\bar{Q}(\tau,\Delta\eta)=\int_{-\Delta\eta/2}^{\Delta\eta/2}\big(n(\tau,\eta)-\overline{n}(\tau,\eta)\big)\,d\eta$$ conserved

$$Q_{(\mathrm{tot})}(\tau,\Delta\eta)=\int_{-\Delta\eta/2}^{\Delta\eta/2}\big(n(\tau,\eta)+\overline{n}(\tau,\eta)\big)\,d\eta$$ non-conserved

Evolution of C.C. Fluctuation in Hadron Phase

Hadronization (initial condition)



Time evolution via DME

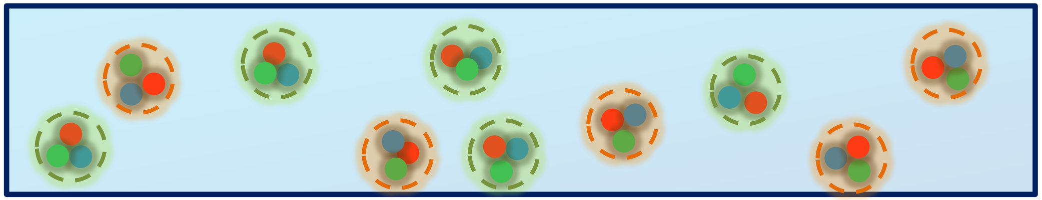
- ☐ Boost invariance / infinitely long system
- ☐ Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ — suppression owing to local charge conservation

$\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$

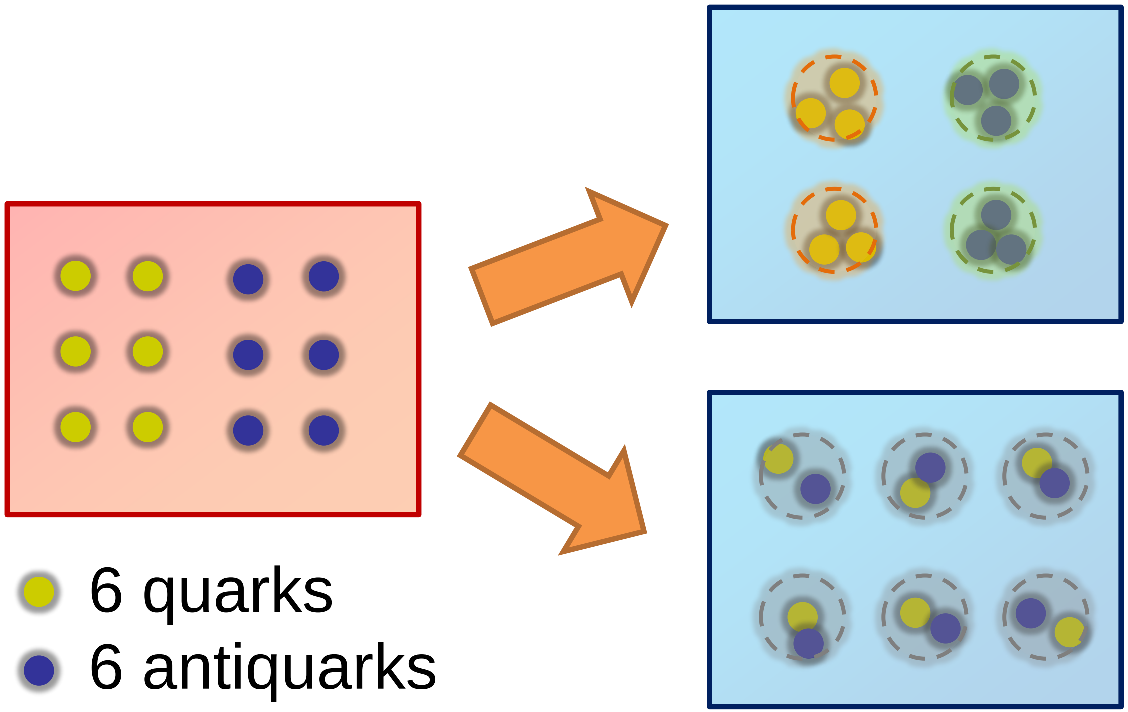
$\langle Q_{(\mathrm{tot})}^2 \rangle_c$ — sensitive to hadronization mechanism

Freezeout



Why Hadronization Mechanism Matters

For example, *even within* the recombination model,



- 6 quarks
- 6 antiquarks

$$N_B^{(\mathrm{net})} = 0$$
$$N_B^{(\mathrm{tot})} = 4$$

$$N_B^{(\mathrm{net})} = 0$$
$$N_B^{(\mathrm{tot})} = 0$$

In other scenarios, $N_B^{(\mathrm{tot})}$ may differ, but $N_B^{(\mathrm{net})}$ does not

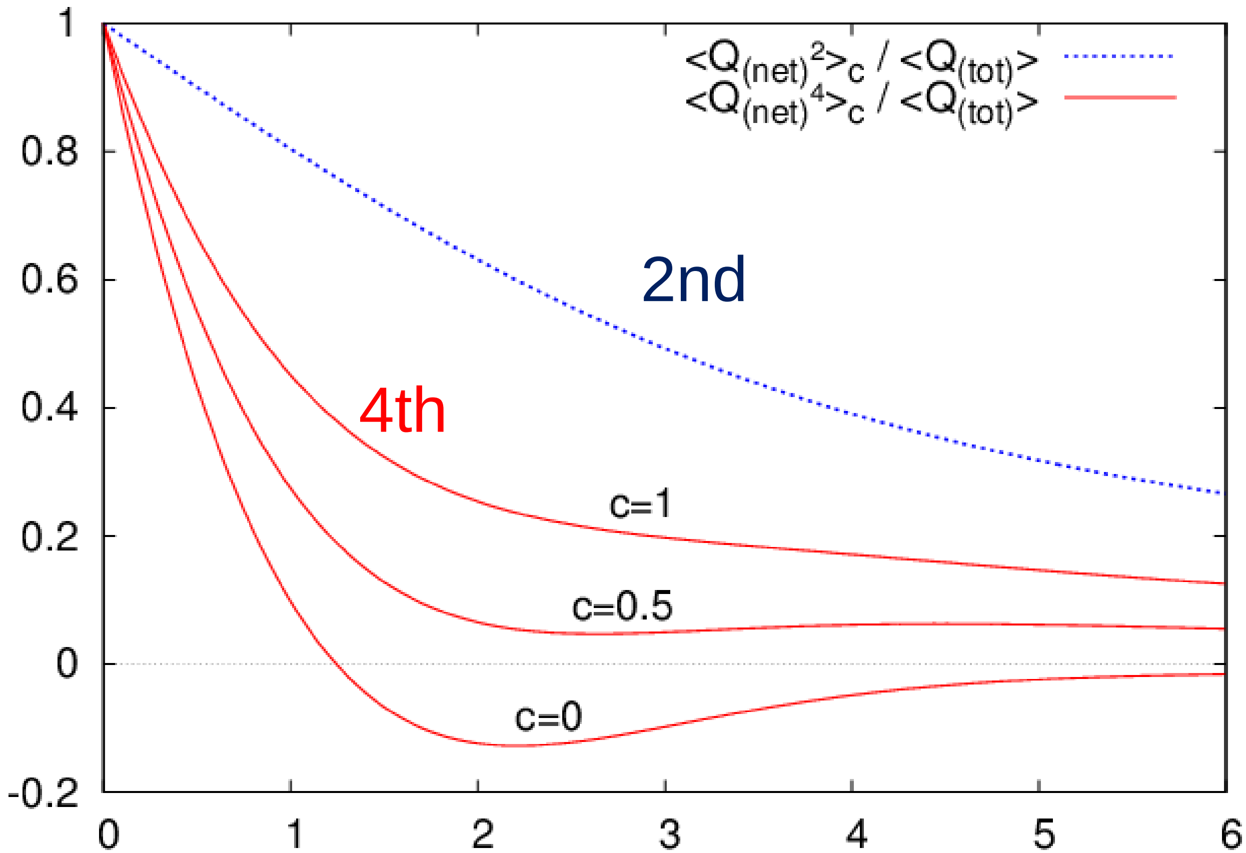
$\Delta\eta$ Dependence at Kinetic Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$

$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

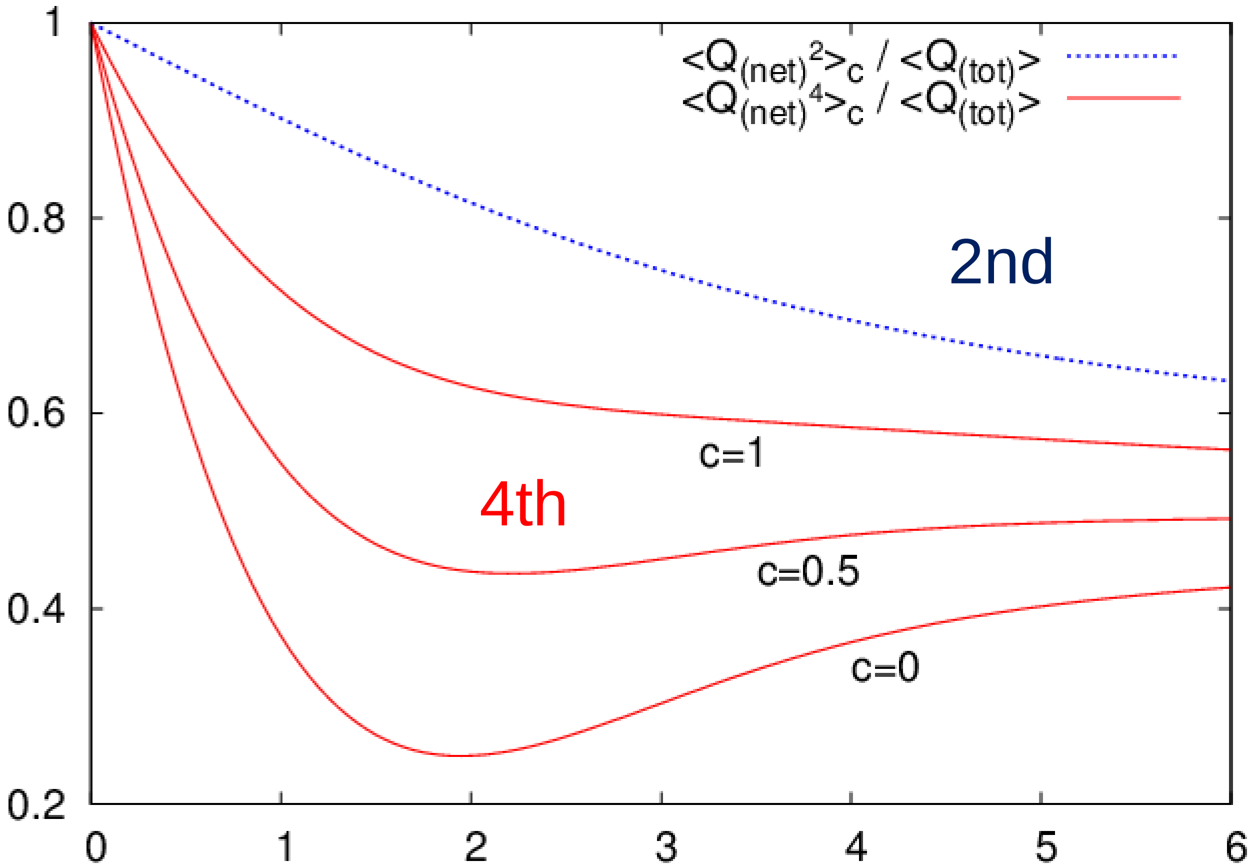


$\Delta\eta / 2\sqrt{D\tau}$ $\quad (\gamma a^2 \rightarrow D \ \ (a \rightarrow 0))$

$\Delta\eta$ Dependence at Kinetic Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0.5 \langle Q_{(\mathrm{tot})} \rangle$$



$\Delta\eta / 2\sqrt{D\tau}$ $\quad (\gamma a^2 \to D \ (a \to 0))$

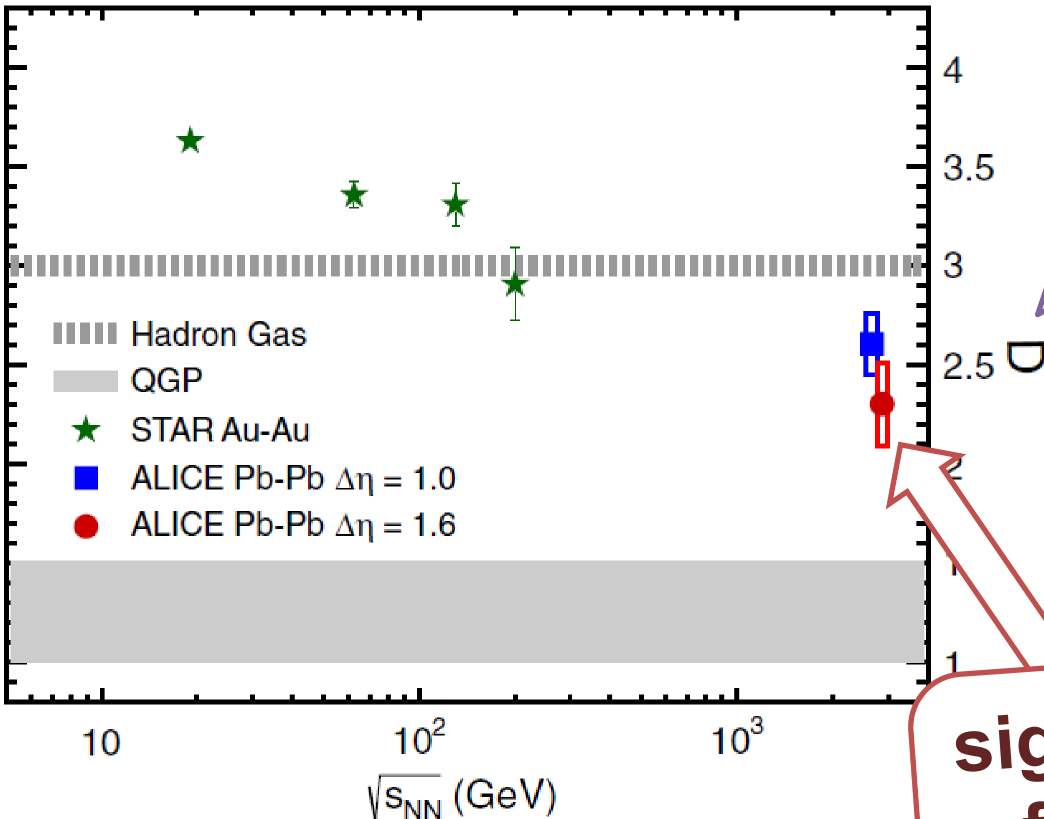
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

Conclusion 2

- *Freeze-out parameters: lattice meets experiment*
  - Diffusion effect in the hadron phase is important (observed fluctuations do not reflect their values at chemical freezeout)
  - Necessary to measure $\Delta\eta$ dependence of cumulants
  - 4th order cumulant includes information of hadronization mechanism

Charge Fluctuation @LHC



ALICE, PRL110,152301(2013)

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

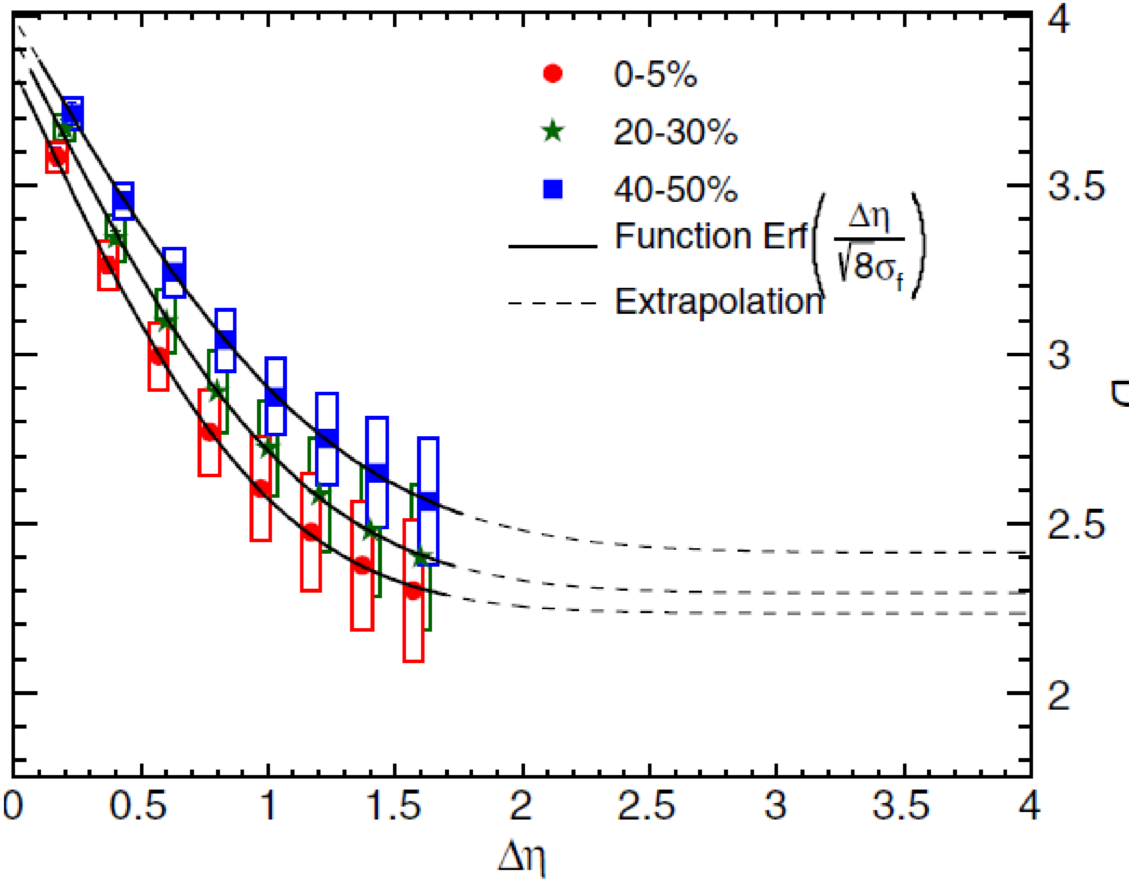
- $D \sim$ 3-4 Hadronic
- $D \sim 1$ Quark

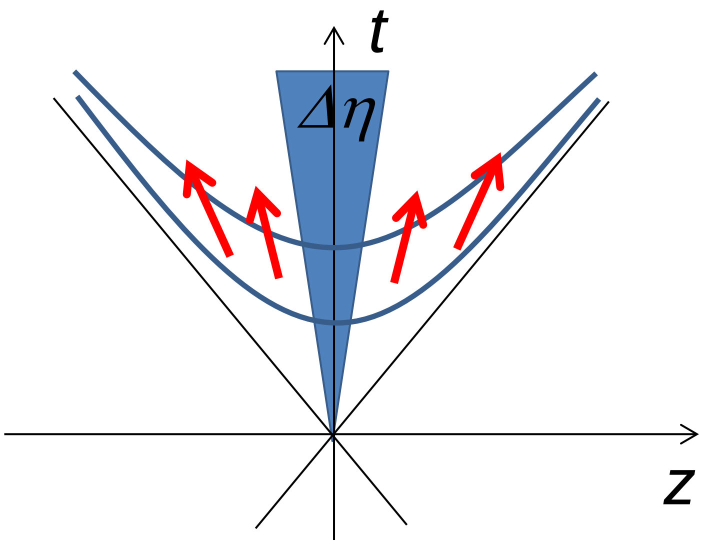
significant suppression from hadronic value at LHC energy!

$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Closer Look: $\Delta\eta$ dependence

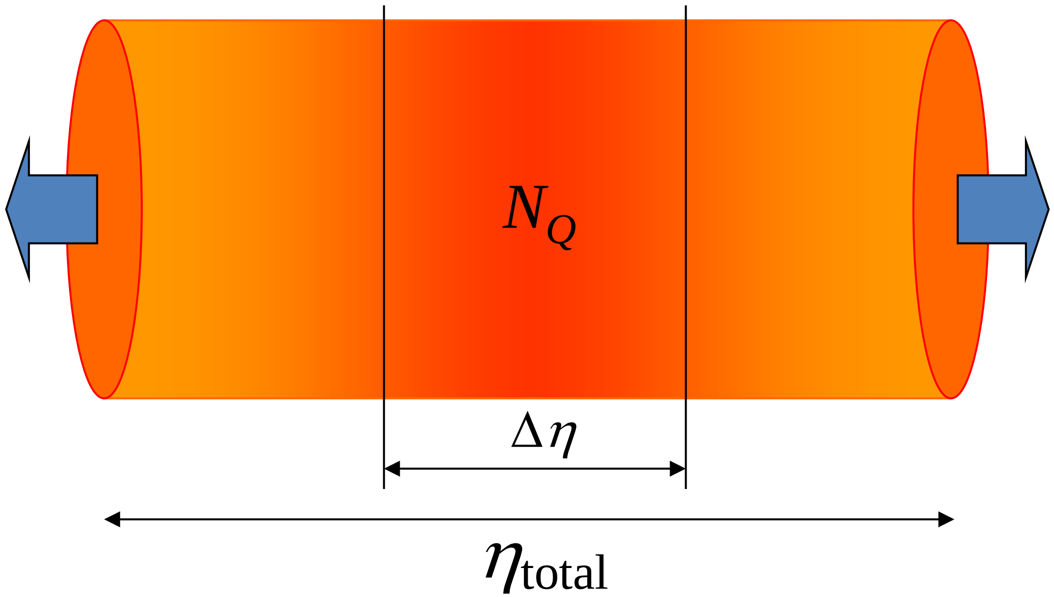
ALICE
PRL 2013





rapidity window

Finite Size Effect (Global Charge Conservation)?

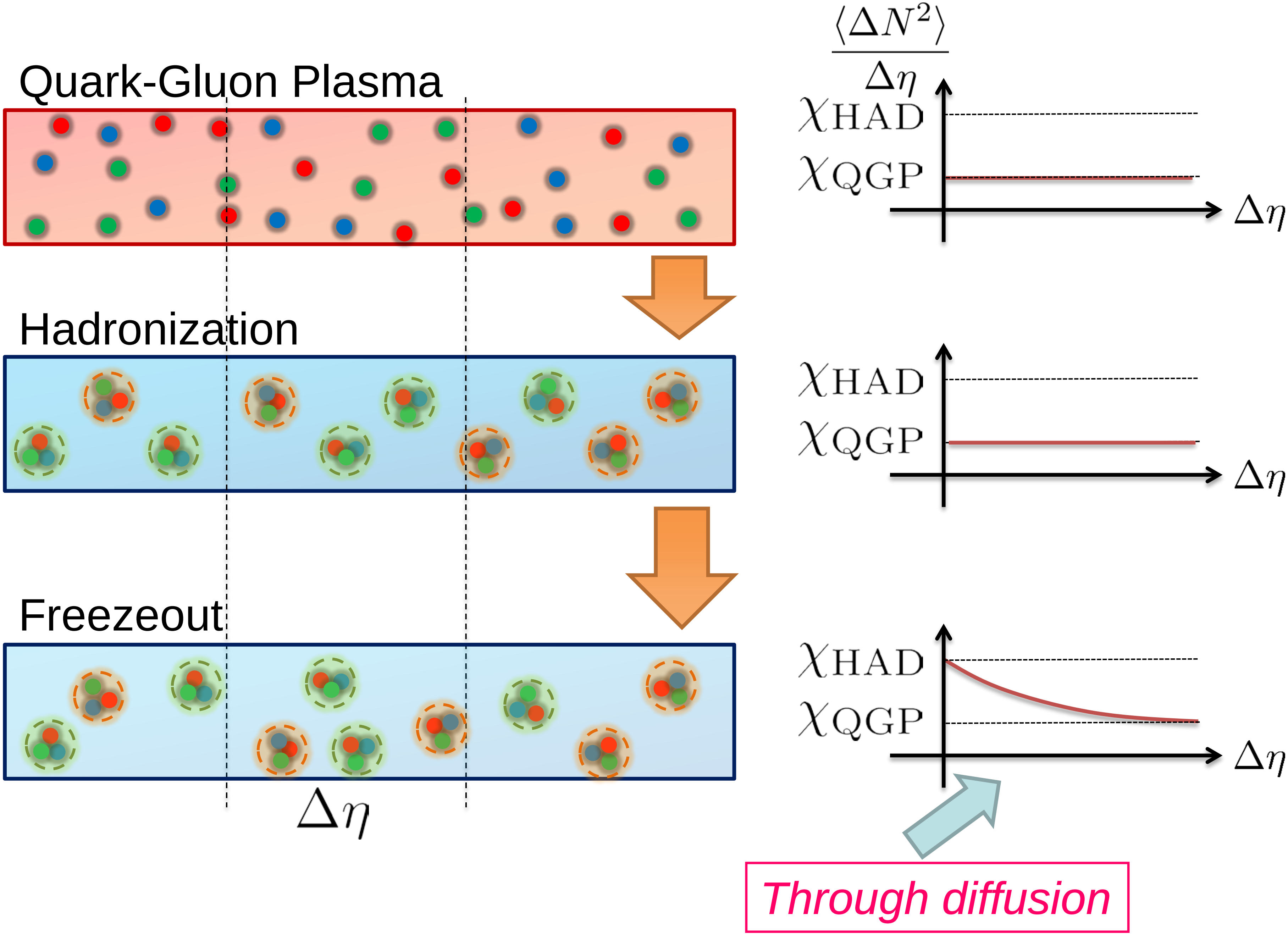


C. C. Fluctuation: 0 if the whole system is observed

$$\Rightarrow \quad \left\langle \delta Q^2 \right\rangle_{\text{obs}} = \left\langle \delta Q^2 \right\rangle_{\text{equil}} \times \left(1 - \frac{\Delta\eta}{\eta_{\text{total}}} \right) \quad ?$$

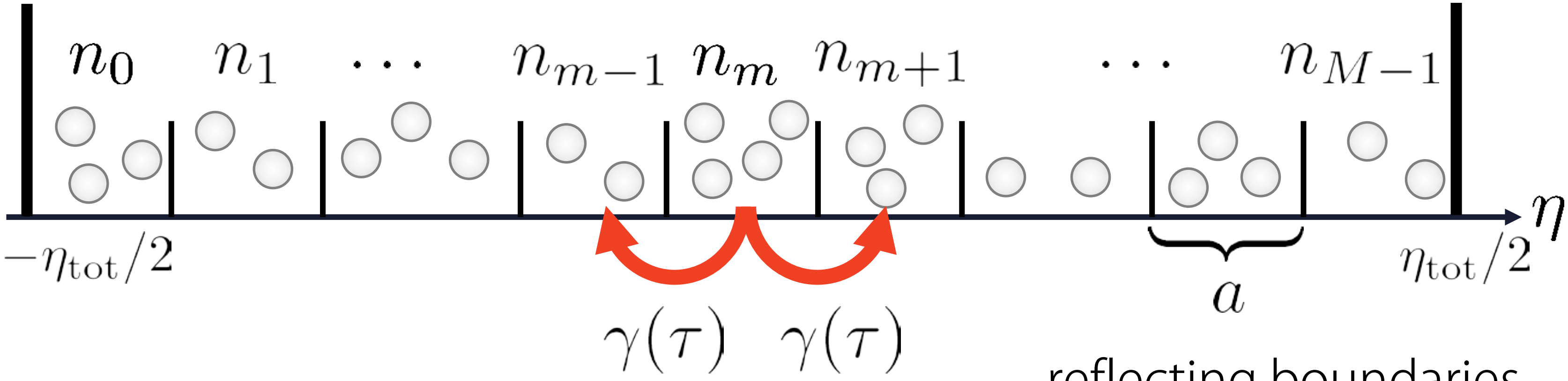
if *the whole system is thermalized* (Bleicher, Jeon, Koch)

Time Evolution of C.C. fluctuation



DME with Reflecting Boundaries

Sakaida, Kitazawa, M.A. , PRC 2014



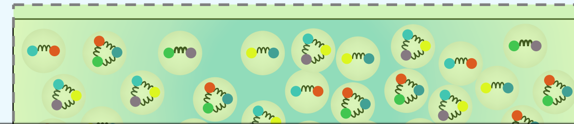
Diffusion Master Equation + Boundary Condition(GCC Effect) Particles do NOT flow in/out.

- ∗ Diffusion from Hadronization to Thermal Freeze-out
- ∗ Initial Condition : No Fluctuations or Fluctuations in Thermal QGP

Rapidity Window Dependence of Charge Fluctuations

Diffusion + Global Charge Conservation

If one looks at the Total System,



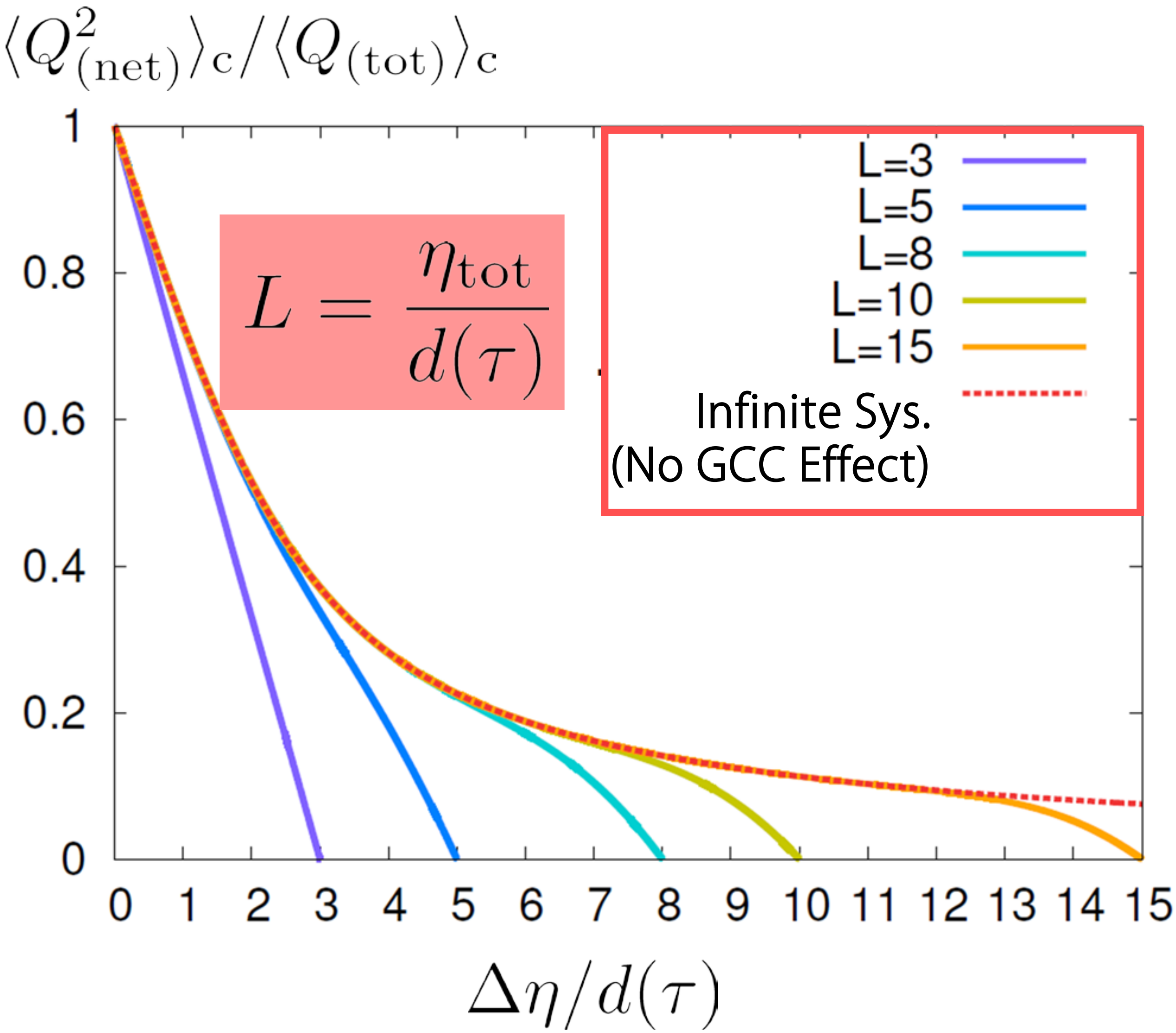
Previous Study	Global Charge Conservation	Time Evolution	Higher Fluctuations
Bleicher, Jeon, Koch (2000)	○	×	×
Shuryak, Stephanov (2001)	×	○	×
Kitazawa, Asakawa, Ono (2013)	×	○	○
Our Study	○	○	○

η_{tot}

$\Delta\eta$: Rap. Win.

Bleicher, Jeon, Koch (2000)

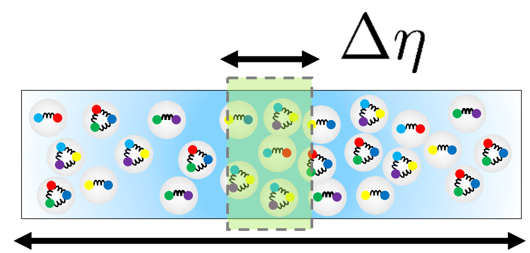
Boundary Effect



$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

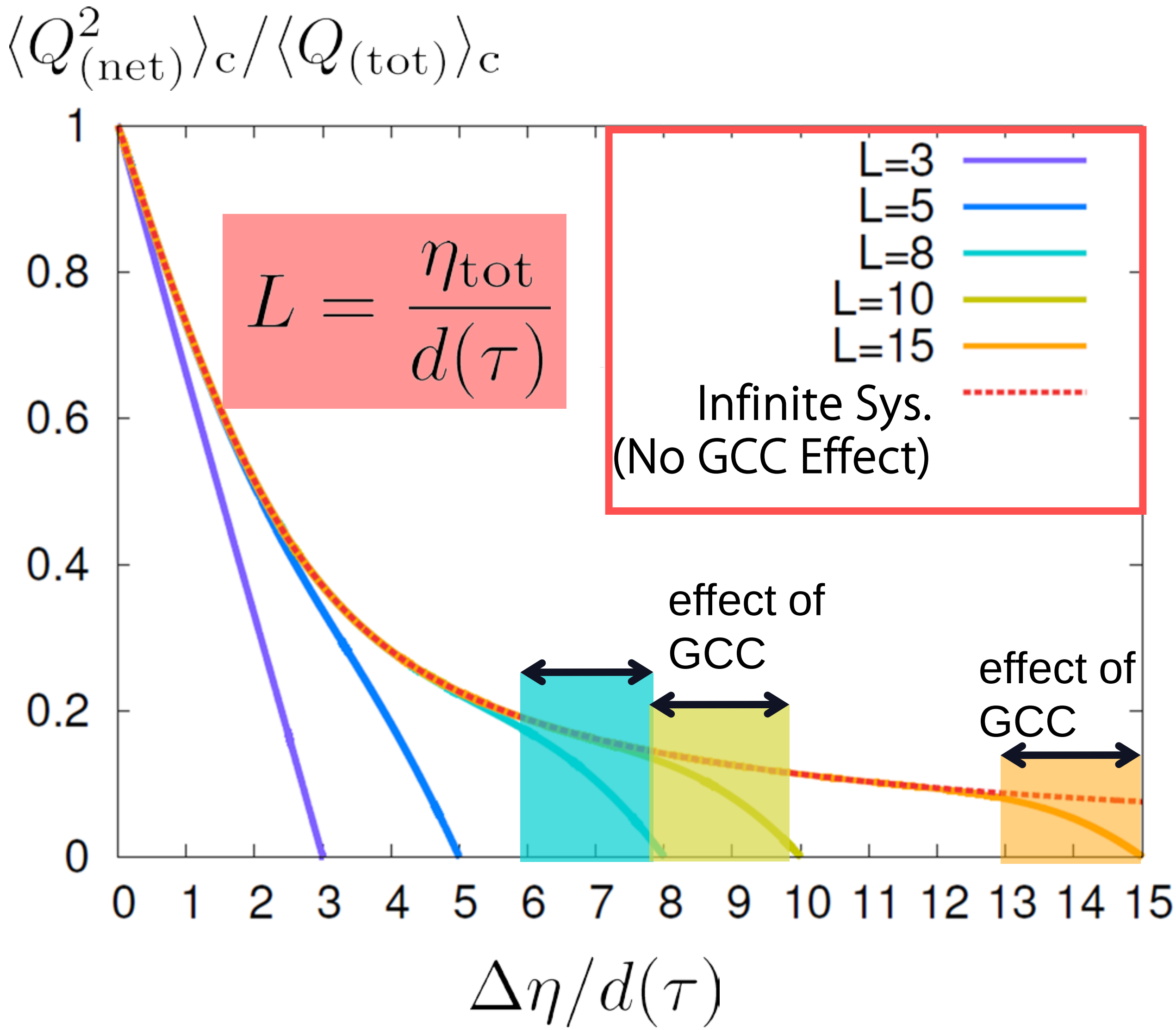
: Average Diffusion Length

$D(\tau)$: Diffusion Coefficient



η_{tot} : Total Rapidity Length

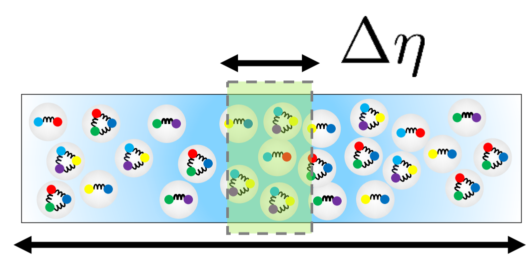
Boundary Effect



$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

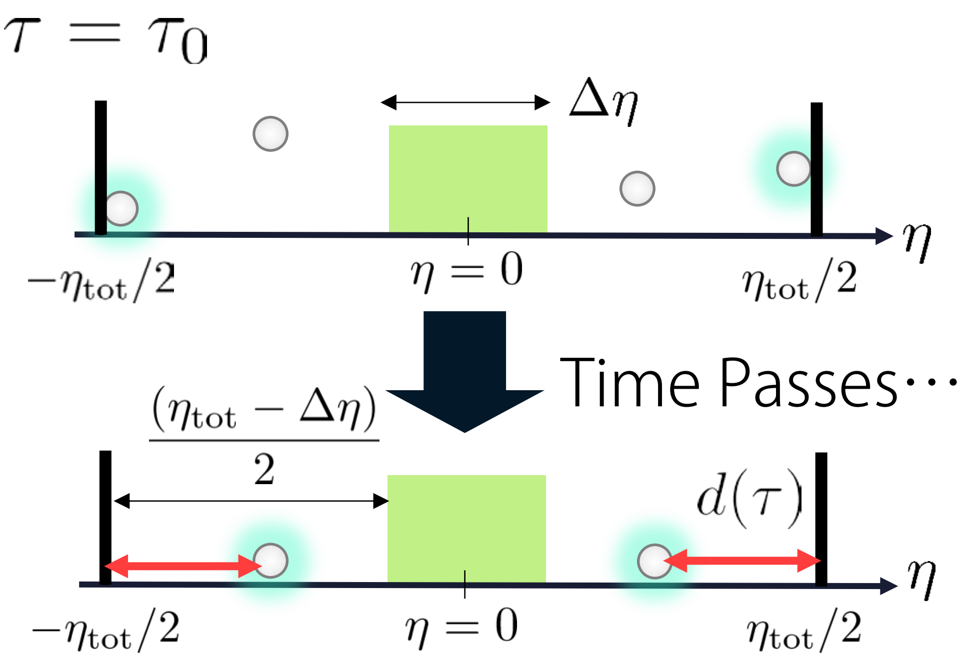
: Average Diffusion Length

$D(\tau)$: Diffusion Coefficient



η_{tot} : Total Rapidity Length

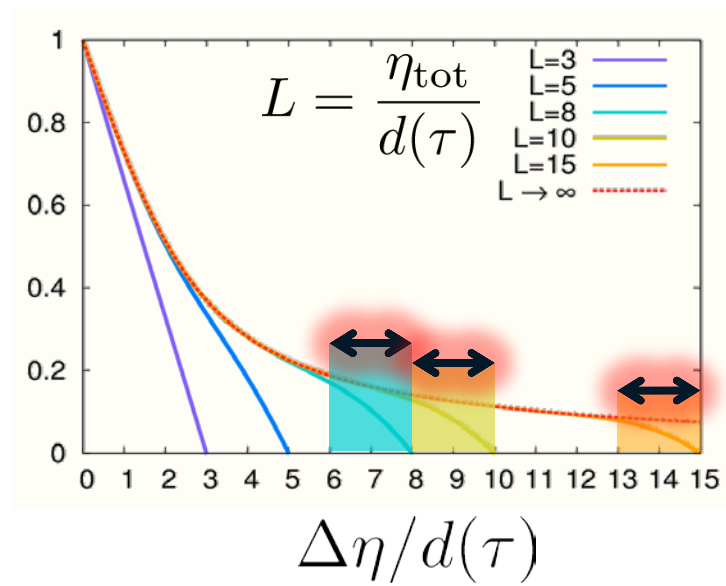
Physical Interpretation



$d(\tau)$: Average Diffusion Distance

$D(\tau)$: Diffusion Coefficient

η_{tot} : Total Length of Matter

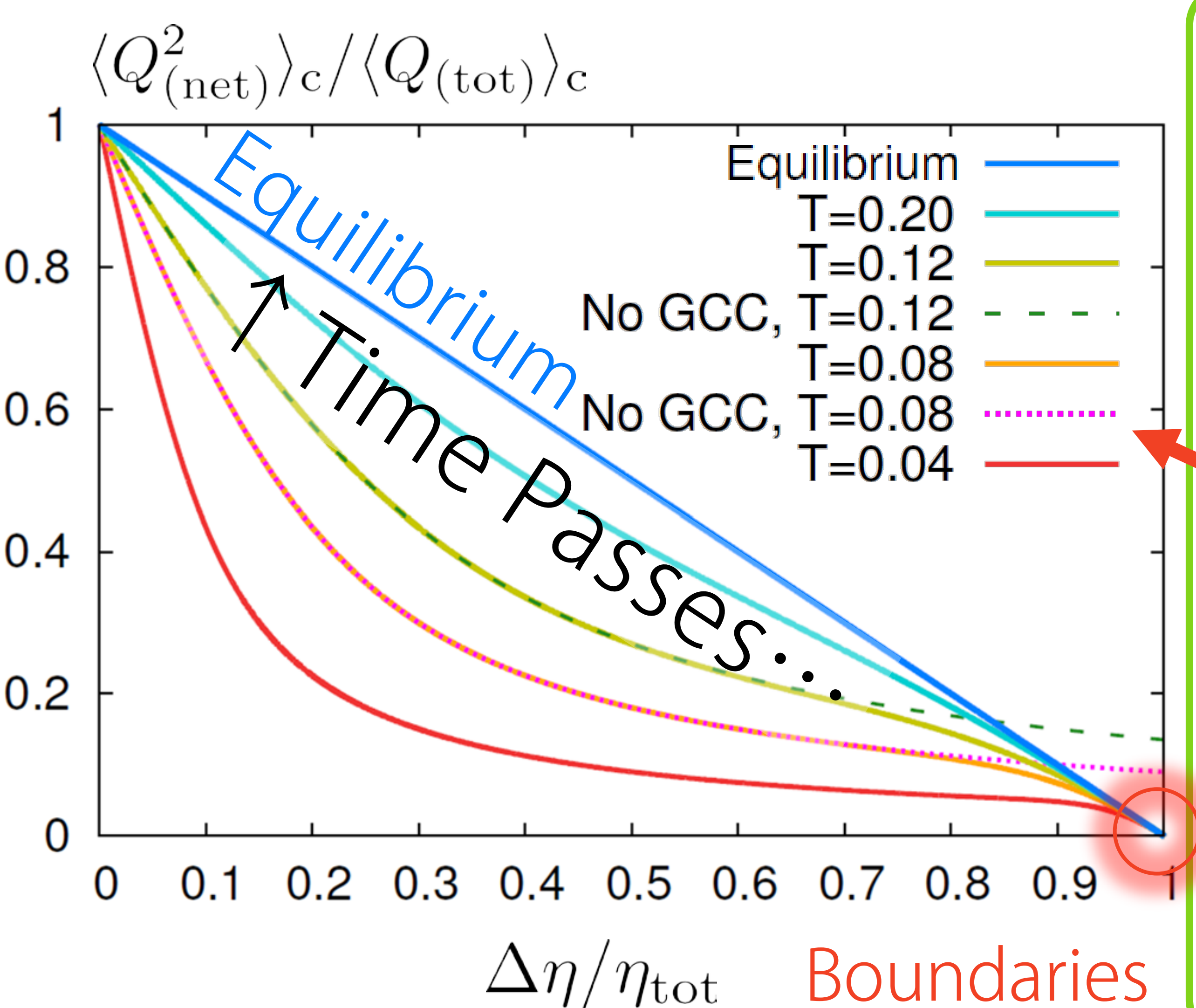


Condition for effects of the GCC

$$\eta_{tot} - \Delta\eta \le 2d(\tau) \Leftrightarrow L - 2 \le \frac{\Delta\eta}{d(\tau)}, L = \frac{\eta_{tot}}{d(\tau)}$$

Effects of the GCC appear only near the boundaries.

Another View: Time Evolution

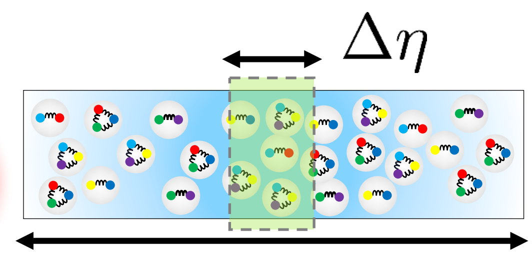


$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Distance

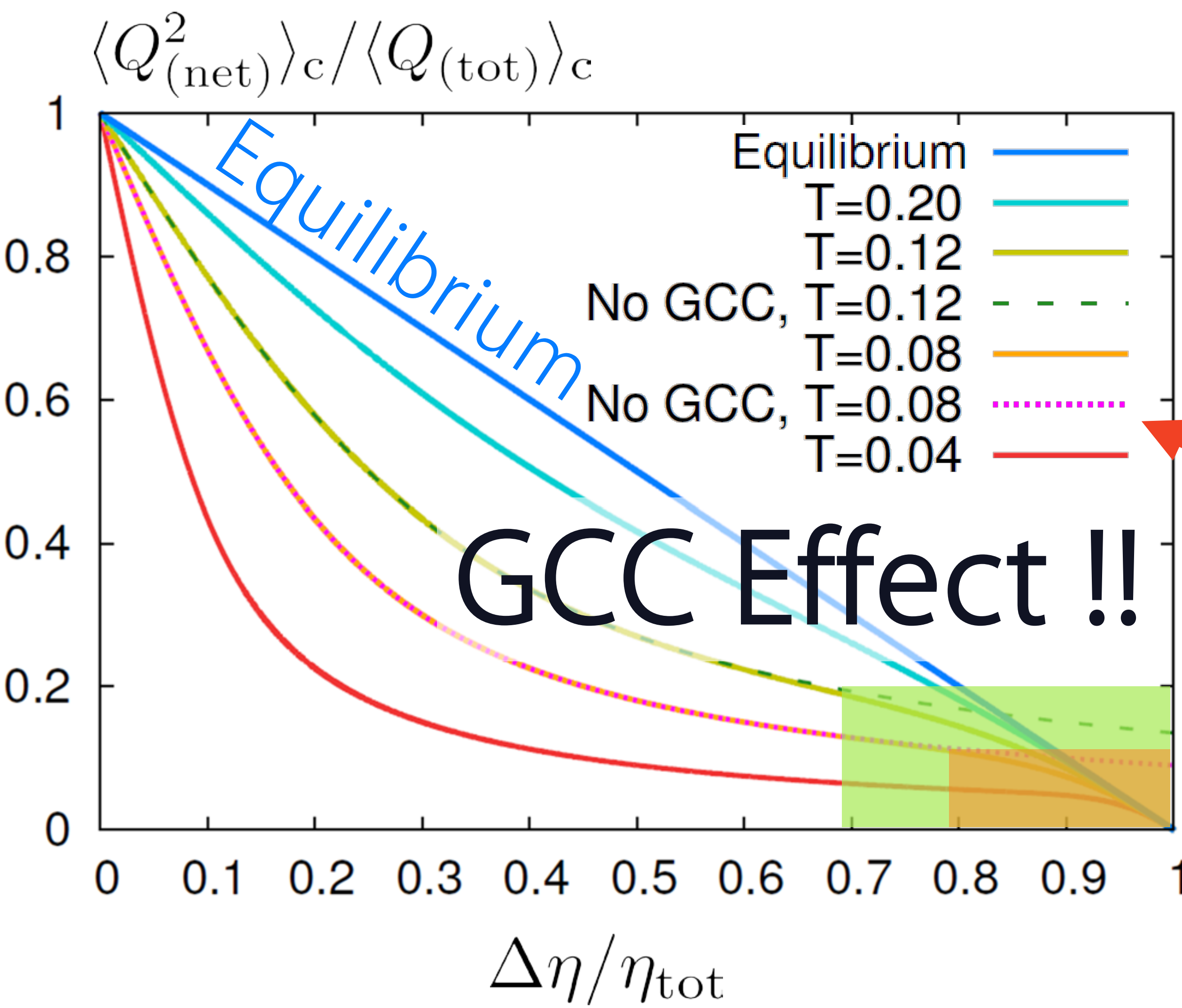
$D(\tau)$: Diffusion Coefficient

$$T = \frac{d(\tau)}{\eta_{\mathrm{tot}}}$$



η_{tot} : Total Rapidity Length

Another View: Time Evolution

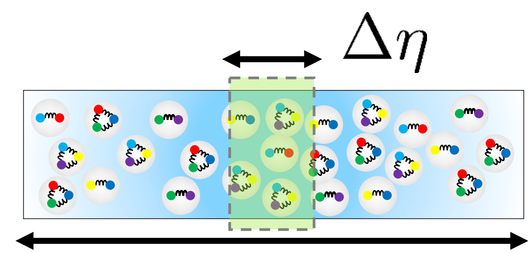


$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Distance

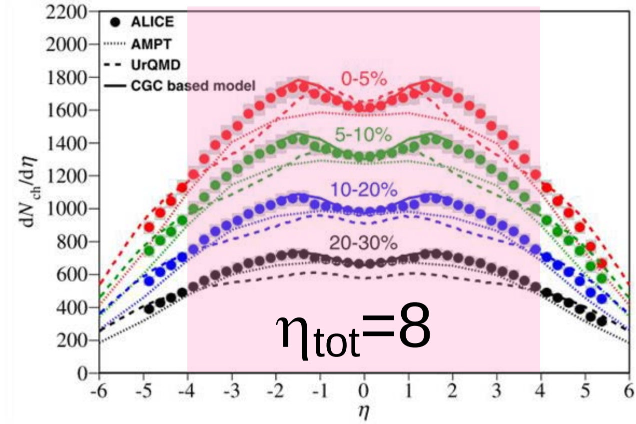
$D(\tau)$: Diffusion Coefficient

$$T = \frac{d(\tau)}{\eta_{\mathrm{tot}}}$$



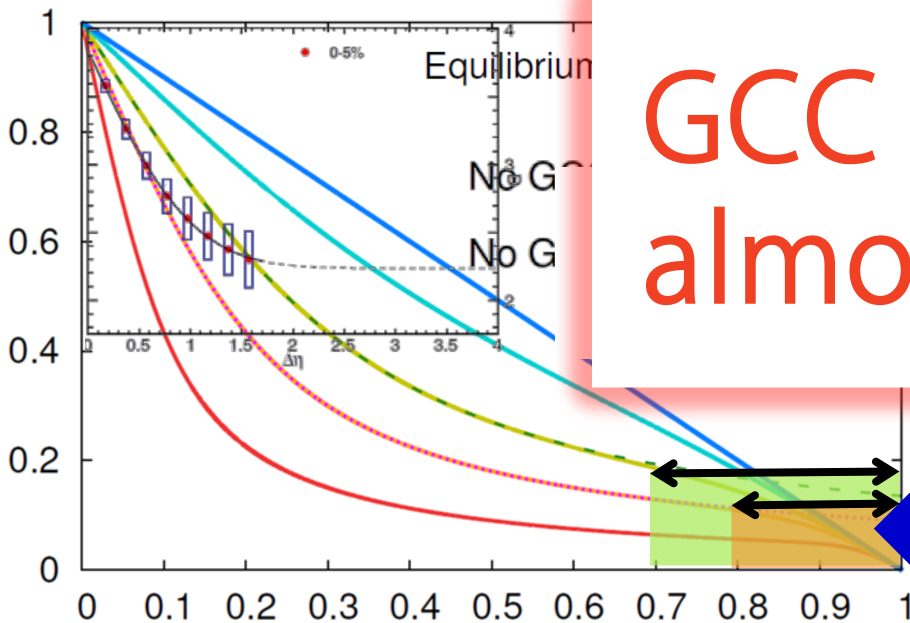
η_{tot} : Total Rapidity Length

Comparison with ALICE Data

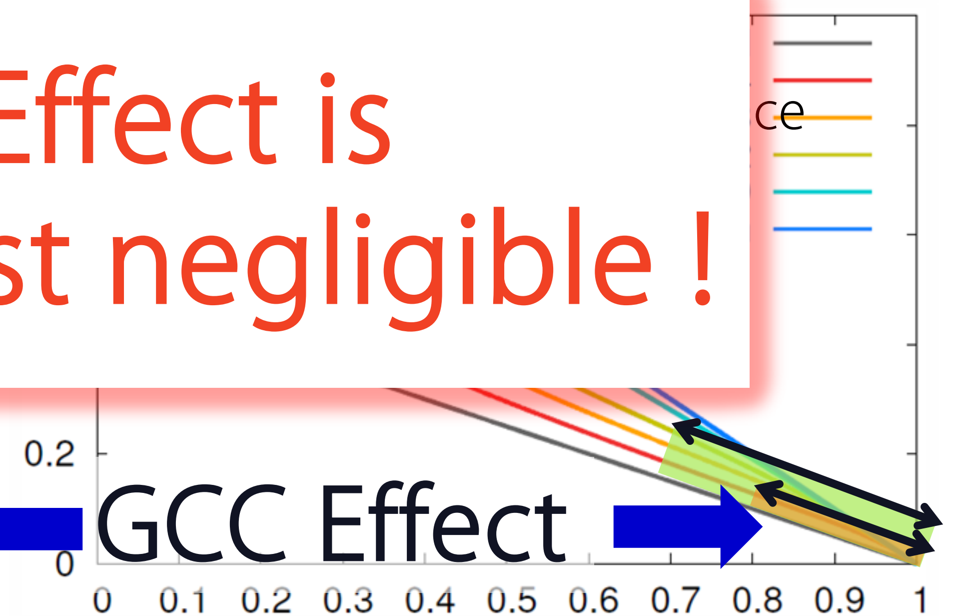


$$\langle Q^2_{(\mathrm{net})}\rangle_{\mathrm{c}} / \langle Q_{(\mathrm{tot})}\rangle_{\mathrm{c}}$$

without initial fluc.



with initial fluc.

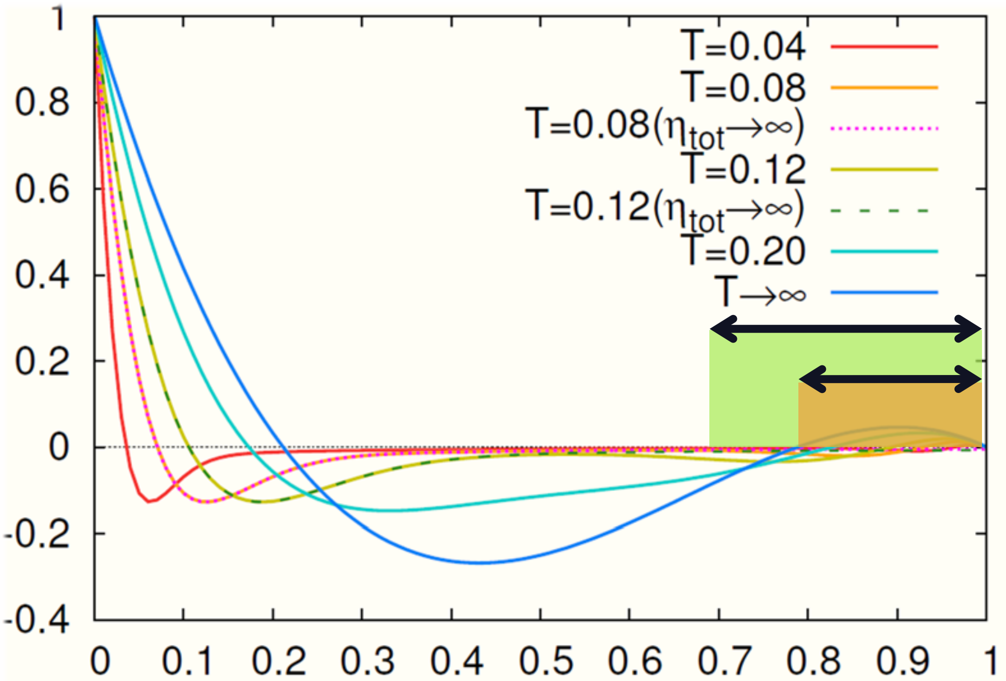


GCC Effect is almost negligible !

GCC Effect

Pb-Pb, 2.76TeV

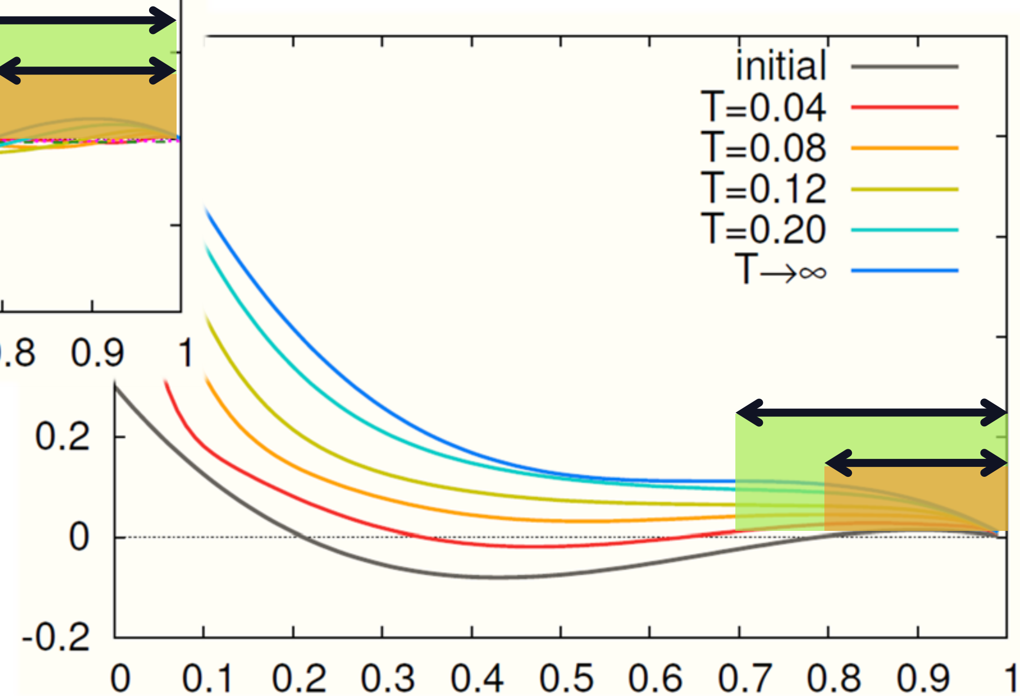
Kurtosis



No Initial Fluctuation

$$\left\langle Q^4_{\rm (net)} \right\rangle_c \Big/ \left\langle Q_{\rm (tot)} \right\rangle_c$$

+ Initial Fluctuation

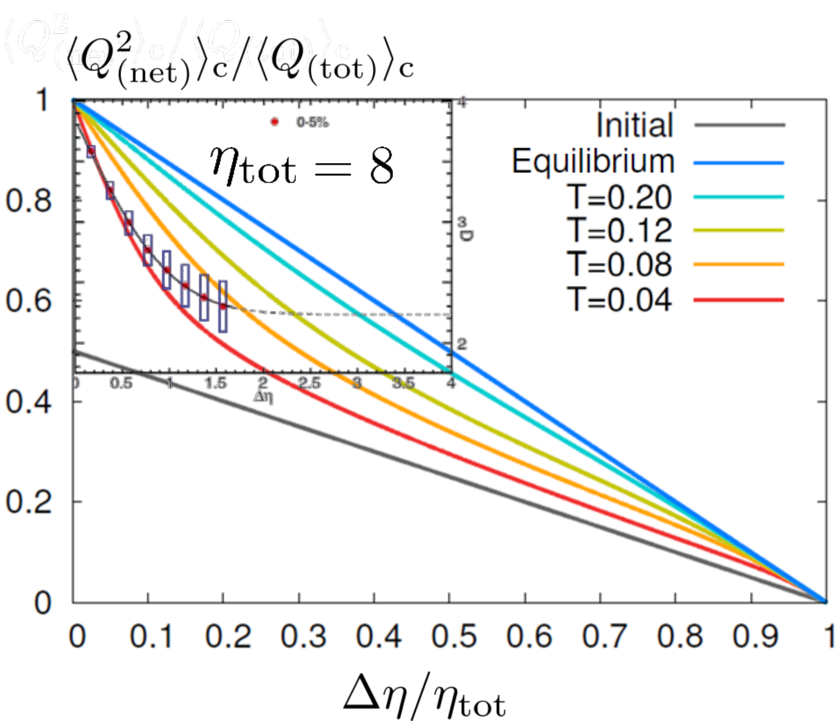


Again, GCC Effect appears ONLY near Boundaries !

Conclusion 3

- ~~*Global Charge Conservation is important even at LHC*~~

Suppression of Charge Fluctuation observed @ALICE
→ ~~Global Charge Conservation~~
Fluctuations are NOT Equilibrated!!



Information on

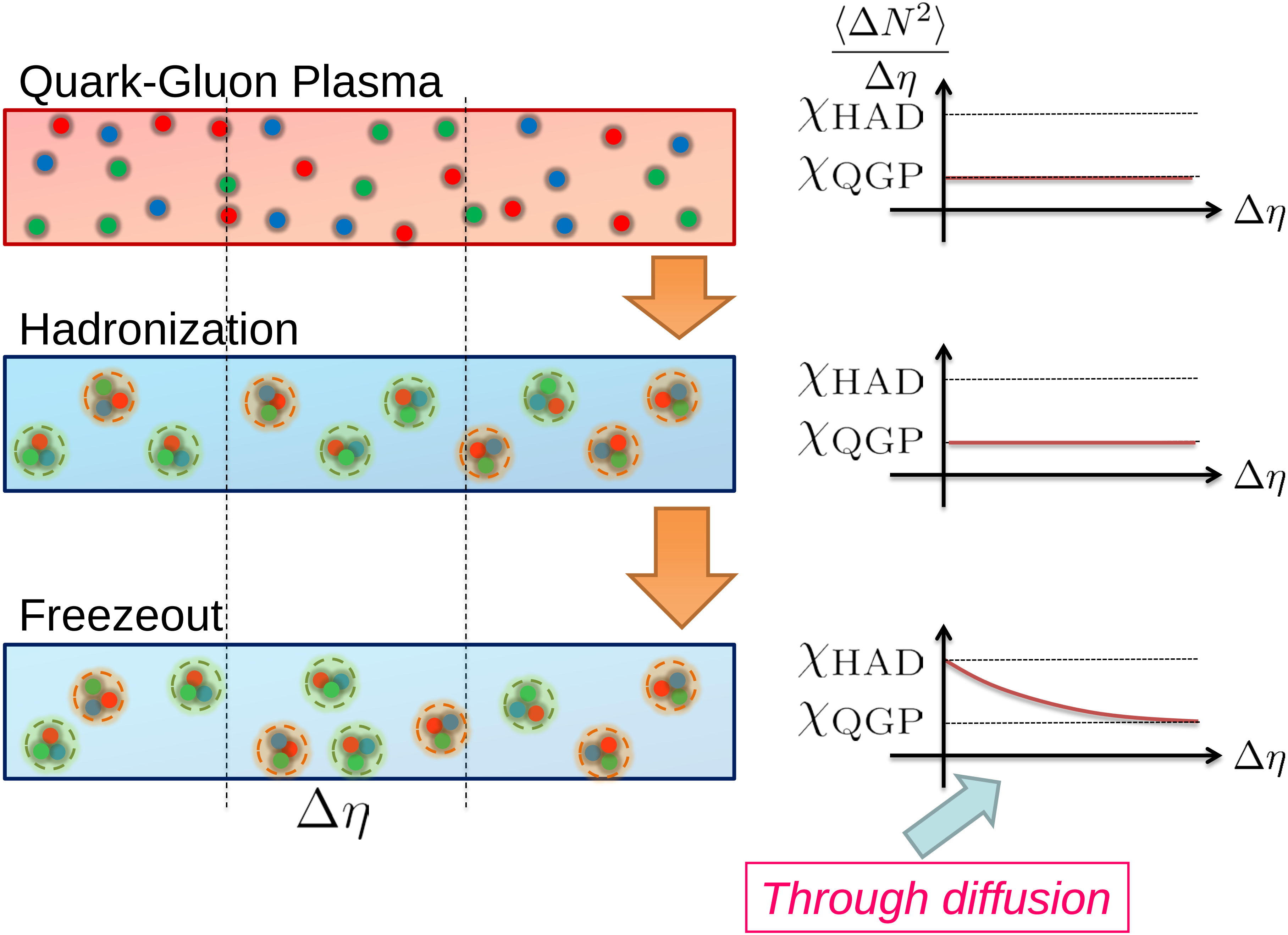
＊Fluctuation in QGP
＊Time Evolution
＊Diffusion Coefficient

…etc.

are encoded

Sakaida, Kitazawa, M.A., PRC 2014

Time Evolution of C.C. fluctuation



In Summary

- *Proton number is a proxy of baryon number*
- *Freeze-out parameters: lattice meets experiment*
- *Global Charge Conservation is important even at LHC*

➡ Each of these needs to be reconsidered again

At low energies (e.g., BES energies)

□ Larger effect of global charge conservation
(smaller system size)

□ Violation of Bjorken scaling
(lost correspondence
between spacetime rapidity and rapidity)