Principal Component Analysis of Event-by-Event Fluctuations — Eigenmode Analysis of Anisotropic Flow

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Outline

- Introduction
- Principal Component Analysis (PCA)
- Flow picture
- Application of PCA to two-particle correlation matrix
- Results – AMPT $(\eta,\ p_T)$, ALICE (p_T)
- Discussion

Analysis of anisotropic flow v_n

- Methods currently in use (event-plane, cumulants, ..): devised before the importance of flow fluctuations was recognized
- New method: extraction of flow fluct. directly from data on 2-particle correls; uses all the information
- Based on Principal Component Analysis (PCA) — applied to the 2-particle correlation matrix, $\langle \cos n\Delta\phi \rangle$
- Leading eigenmode $\longleftrightarrow$ usual v_2, v_3
  Subleading modes of v_2, v_3 revealed for the 1st time

Principal Component Analysis (PCA)

- Statistical procedure to elucidate the underlying covariance structure in the multi-dimensional data
- To identify the directions (PC) where there is the most variance, and possibly reduce the dimension of data
- Diagonalize the covariance matrix: Eigenvector with the largest eigenvalue is the direction of greatest variance; that with the 2nd largest eigenvalue ...
- PCA can be thought of as fitting a hyper-ellipsoid to the cloud of data points — each of its axes representing a PC

Flow Picture

- Single-particle distribution in an event:
  $\frac{dN}{d\mathbf{p}} = \sum_{n=-\infty}^{\infty} V_n(p) \exp(in\phi)$
- Pair distribution averaged over events:
  $\left\langle \frac{dN_{pairs}}{d\mathbf{p_1} d\mathbf{p_2}} \right\rangle = \left\langle \frac{dN}{d\mathbf{p_1}} \frac{dN}{d\mathbf{p_2}} \right\rangle + \mathcal{O}(N) \leftarrow$ nonflow correl.
  $= \sum_{n=-\infty}^{\infty} V_{n\Delta}(p_1, p_2) \exp(in(\phi_1 - \phi_2))$
- Two-particle correlation matrix:
  $V_{n\Delta}(p_1, p_2) = \langle V_n(p_1) V_n^*(p_2) \rangle$,
  neglecting non-flow correlations

Covariance Matrix

- $V_{n\Delta}(p_1, p_2) = \langle V_n(p_1) V_n^*(p_2) \rangle$ is a covariance matrix.
- Covariance matrix is positive semidefinite, and its eigenvalues are non-negative.
- PCA yields $V_{n\Delta}(p_1, p_2) \simeq \sum_{\alpha=1}^{k} V_n^{(\alpha)}(p_1) V_n^{(\alpha)*}(p_2)$ $=$ sum over modes of flow fluct. Here $(k \leq N_{dim})$.
- If no flow fluctuations, then factorization occurs: $V_{n\Delta}(p_1, p_2) \simeq V_n^{(1)}(p_1) V_n^{(1)*}(p_2)$

Method

- Divide the detector acceptance into several bins in p_T and/or η. Let p: bin index.
- Flow vector in an event: $Q_n(p) \equiv \sum_{j=1}^{M(p)} \exp(in\phi_j)$
- Covariance matrix $V_{n\Delta}(p_1, p_2)$
  $\equiv \langle Q_n(p_1) Q_n^*(p_2) \rangle - \langle M(p_1) \rangle \delta_{p_1 p_2} - \langle Q_n(p_1) \rangle \langle Q_n^*(p_2) \rangle$
  RHS_2: subtracts self-correl. RHS_3: singles out fluct.
- PC are obtained by diagonalizing $V_{n\Delta}(p_1, p_2)$:
  Eigenvalues: $\lambda^{(1)} > \lambda^{(2)} > \lambda^{(3)} \ldots$
  Eigenvectors: $\psi^{(\alpha)}(p) = V_n^{(\alpha)}(p)/\sqrt{\lambda^{(\alpha)}}$

Method (contd.)

Flow in a given event is a sum over eigenmodes:

$V_n(p) = \sum_{\alpha=1}^{k} \xi^{(\alpha)} V_n^{(\alpha)}(p),$ where

$\xi^{(\alpha)}$: complex, random variables with zero mean and unit variance, i.e.,

$\left\langle \xi^{(\alpha)} \right\rangle = 0, \quad \left\langle \xi^{(\alpha)} \xi^{(\beta)*} \right\rangle = \delta^{\alpha,\beta}$

We define

$$v_n^{(\alpha)}(p) \equiv \frac{V_n^{(\alpha)}(p)}{\langle M(p) \rangle}$$

$n = 0$: rel. multiplicity; $n \neq 0$: anisotropic flow

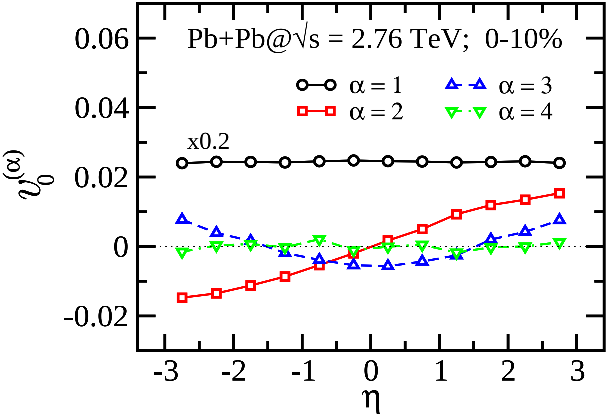
Thus $v_0^{(\alpha)}(p)$: relative multiplicity fluctuations,

$v_n^{(\alpha)}(p)$ for $n \neq 0$: anisotropic flow fluctuations

AMPT simulations (Lin, Ko, Li, Zhang, Pal 2005)

- Initial conditions from HIJING 2.0 (Deng, Wang, Xu 2011)
- These contain nontrivial e-by-e fluctuations
- Flow generated mainly as a result of partonic cascade
- Resonance formations & decays $\longrightarrow$ nonflow effects
- In agreement with v_2 to v_6 versus p_T for all centralities at 2.76 TeV. (Except perhaps, ultra-central)
- 0-10% centrality, $\sim 10^4$ events

Relative Multiplicity Fluctuations vs Pseudorapidity

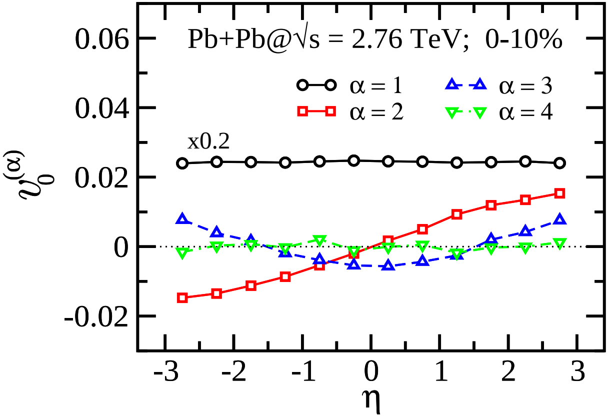


- PCA results for the leading and 3 subleading modes
- PC: Alternating parities; Mutually orthogonal

Orthogonality of Principal Components

$$\sum_p V_n^{(\alpha)}(p) V_n^{(\beta)*}(p) = 0, \quad \text{if} \quad \alpha \neq \beta$$

Relative Multiplicity Fluctuations vs Pseudorapidity



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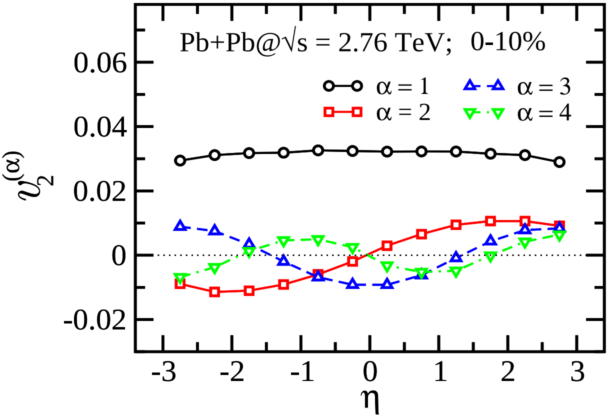
- Subleading modes $\ll$ Leading modes; Eigenvalues $\lambda^{(3)} \ll \lambda^{(2)} \ll \lambda^{(1)}$

Elliptic flow fluctuations vs Pseudorapidity



- PCA results for the leading and 3 subleading modes
- PC: Alternating parities; Mutually orthogonal
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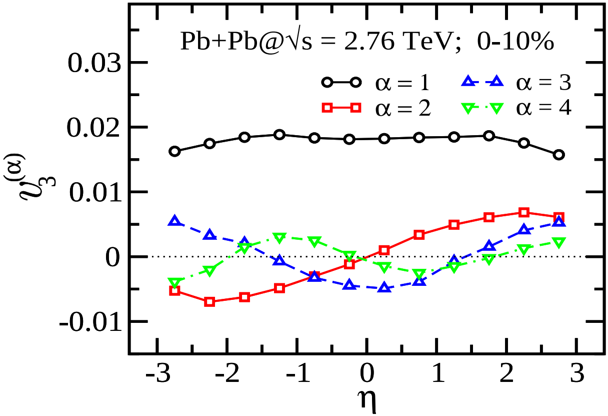
Triangular flow fluctuations vs Pseudorapidity



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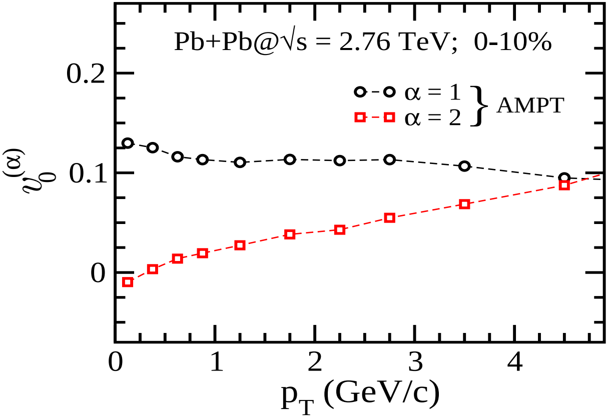
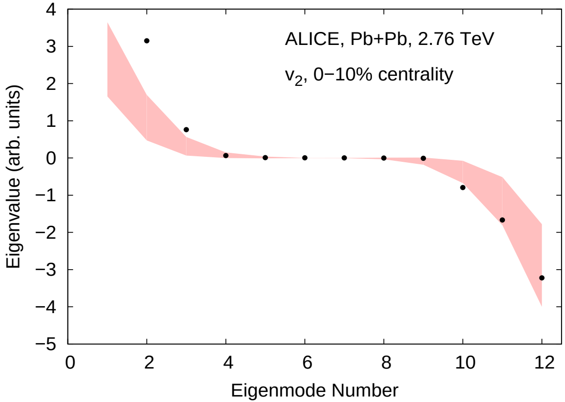
Relative Multiplicity Fluctuations vs Transverse Momentum



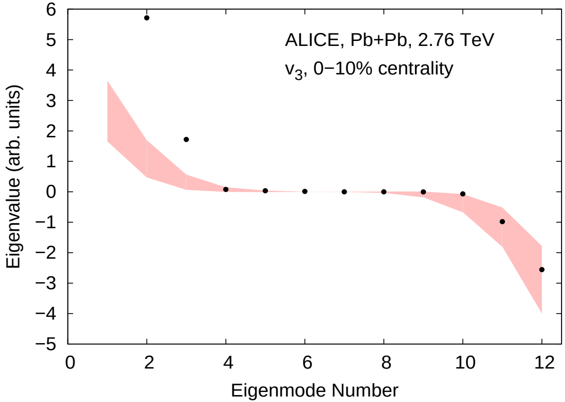
- PCA results for the leading and subleading modes
- Subleading modes $\ll$ Leading modes; Eigenvalue $\lambda^{(2)} \ll \lambda^{(1)}$
- No experimental data available for any result shown so far

PCA – Eigenvalues for $v_2(p_T)$



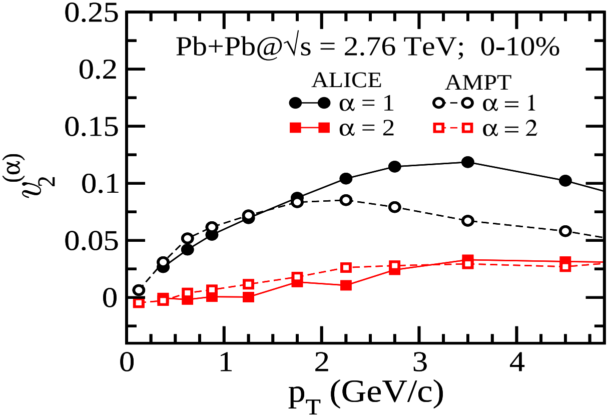
- Band: PCA applied to purely statistical fluctuations
- Negative eigenvalues of $V_{n\Delta}(p_1, p_2)$ are compatible with those of large random matrices. Can be attributed to stat. fluct.
- Note the few leading eigenmodes which clearly stand out

PCA – Eigenvalues for $v_3(p_T)$



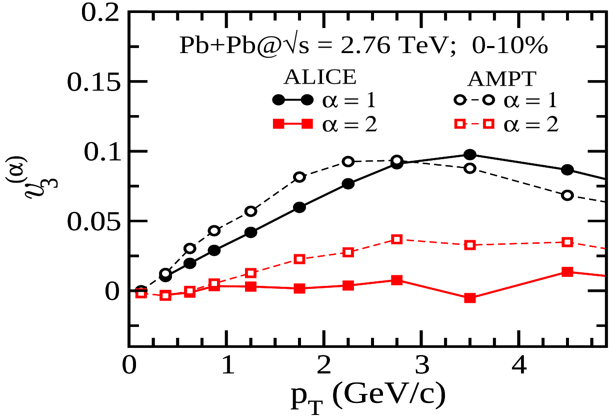
- Band: PCA applied to purely statistical fluctuations
- Negative eigenvalues of $V_{n\Delta}(p_1, p_2)$ are compatible with those of large random matrices. Can be attributed to stat. fluct.
- Note the few leading eigenmodes which clearly stand out

Elliptic Flow Fluctuations vs Transverse Momentum



- PCA results for the leading and subleading modes
- Subleading modes ≪ Leading modes; Eigenvalue $\lambda^{(2)} \ll \lambda^{(1)}$

Triangular flow fluctuations vs Transverse Momentum



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Discussion

- 2-particle azimuthal correlations depend on momenta of both particles. Traditional methods: one of the momenta integrated over. New method: Makes use of both the momenta, i.e., all the info in correl matrix.
- PCA has revealed previously unknown substructure in multiplicity, elliptic flow, and triangular flow fluctuations.
- We anticipate a rich experimental and theoretical program studying the dynamics of subleading flow vectors, which can be used to further constrain the plasma response to the initial geometry.

THANK YOU