Instabilities in Anisotropic Chiral Plasmas

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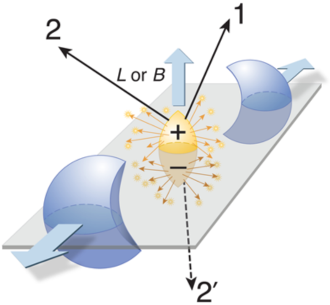
Motivation

- Recently it has been proposed that P and CP violation should manifest in heavy ion collisions through the electric charge separation with respect to reaction plane in non-central collisions.



- Magnetic field created in off central Au-Au collisions (at 100GeV per nucleon) at RHIC ($eB(\tau = 0.2fm) = 10^3 - 10^4\ MeV^2 \sim 10^{17}$ Gauss).

D. Kharzeev, Phys. Lett. B **633**, 260 (2006),
D. Kharzeev, L.D. Mclerran and H.J. Warringa, Nucl. Phys. A **803**, 227 (2008),
D. Kharzeev, A. Zhitnitsky, Nucl. Phys. A **797**, 67 (2007),

Motivation

- QCD contains field configuration which are characterized by a topological invariant, the winding number $Q_w = \frac{g^2}{32\pi^2}\int d^4x F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a = 0, \pm1, \pm2, ...$
- Configuration with nonzero Q_w leads to the non-conservation of axial currents even in chiral limit.

$$\partial_\mu j^{\mu 5} = -\frac{N_f g^2}{16\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a .$$

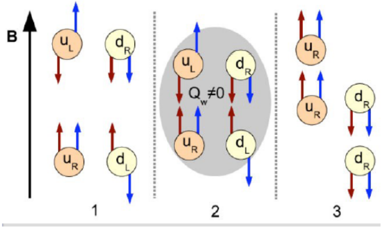
- Let at $t = -\infty$ we have $N_L = N_R$ then it follows from above equation,

$$.(N_L - N_R)|_{t=\infty} - (N_L - N_R)|_{t=-\infty} = 2N_f Q_w$$

- Thus non-zero Q_w leads to chiral imbalance or finite μ_5.

D. Kharzeev, L.D. Mclerran and H.J. Warringa, Nucl. Phys. A **803**, 227 (2008),

Motivation



(a)

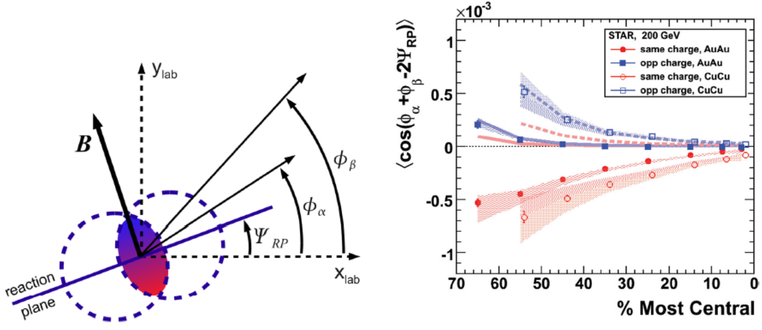
Figure: Illustrate a mechanism by which configuration with non zero Q_w can separate charge in presence of a background magnetic field leading to CME (Chiral Magnetic effect).

- In presence of magnetic field spins of quarks align parallel or anti-parallel to magnetic field (depending upon the sign of electric charge).
- $Q_w = -1$ will convert left handed up/down quark into right handed up/down quark by reversing direction of momentum.
- Right handed up quarks will move upward and right down quarks will move downward. A charge difference $Q = 2e$ will be created.

D. Kharzeev, L.D. Mclerran and H.J. Warringa, Nucl. Phys. A **803**, 227 (2008),

Motivation

- First preliminary result of such a study has been presented recently by STAR Collaboration at RHIC by measuring three particle azimuthal correlator $\langle\cos(\phi_\alpha + \phi_\beta - 2\psi_{RP})\rangle$ with respect to collision centrality.



(a) (b)

- From the fig (c) it is clear that correlations of same charge and opposite charge particles separates out on opposite sides.
- Correlations increases in more peripheral case.

I. V. Selyuzhenkov[STAR Collaboration], Rom. Rep. Phys. B **58**, 49 (2006),
B. I. Abelev et. al.[STAR Collaboration], Phys. Rev. C. **81**, 054908 (2010),
B. I. Abelev et. al.[STAR Collaboration], Phys. Rev. Lett. **103**, 251601 (2009),

Motivation

- Recently, based on Berry curvature corrections, a modified kinetic theory has been developed which allows one to study the CP violating or chiral effects in non-equilibrium conditions.
- It has been found that for modified kinetic theory the presence of CP-violating effects can lead to the instabilities in the transverse branch of dispersion relation in the quasi-stationary limit.
- Typical time scale of such instability is $\tau = 1/(\alpha^2\mu)$.
- However in many realistic situations in plasma physics it is important to consider initial distribution function to be anisotropic in momentum space. It is well known that momentum anisotropy can lead to so called Weibel instability. Therefore it is important to consider effect of anisotropy in modified kinetic theory and to see how the two instabilities compete with each other.

D. T. Son and N. Yamamoto, Phys. Rev. D **87**, 085016 (2013) [arxiv:1210.815],
Y. Akamatsu and N. Yamamoto, Phys. Rev. Lett. **111**, 052002 (2013),
E.S. Weibel, Phys. Rev. Lett. **2**, 83, (1959).
Berry Curvature: When Hamiltonian depends on some external time dependent parameter which changes slowly then by adiabatic theorem wave function picks up an additional dynamical phase factor apart from the usual one. That additional dynamical phase is called Berry phase and can expressed as the surface integral of a curl of a vector (called Berry connection), if parameter describe a closed loop. The curl of Berry connection is called Berry curvature. Which can be easily calculated for the case of chiral fermion.

Intuitive understanding of these two instabilities

Chiral Instability:

- The number and energy densities of the particles with chiral chemical potentials μ respectively given by μT^2 and $\mu^2 T^2$.
- Number density $n \sim \alpha k A^2$, A is the gauge field.
- When number density associated with the gauge field and particles are same we have $k \sim \frac{\mu T^2}{\alpha A^2}$.
- The typical energy for the gauge field $\epsilon_A \sim k^2 A^2 = \mu^2 T^2 \frac{T^2}{\alpha^2 A^2}$.
- Thus for $\frac{T^2}{\alpha^2} < A^2$ there exists a state where the gauge field for this particular k can lower its energy density (in comparison with the energy density of the matter) by increasing A. This leads to the chiral-imbalance instability.

Y. Akamatsu and N. Yamamoto, Phys. Rev. Lett. **111**, 052002 (2013), M. Joyce and M. Shaposhnikov, Phys. Rev. Lett. **79**, 1193 (1997).

Intuitive understanding of these two instabilities

Weibel Instability:

- Let us consider the distribution function $n^0_p = \frac{1}{1+e^{-(\sqrt{p_x^2+(1+\xi)p_y^2+p_z^2})/T}}$.
- If in this situation a disturbance with a magnetic field spontaneously arises from noise, one can write the Lorentz force term in the kinetic equation as,
  $$e(v \times B) \cdot \partial_p n^0_p = e[\xi(v_z B_x - v_x B_z)\frac{p_y}{T}] \left(\frac{-e^{-(\sqrt{p_x^2+(1+\xi)p_y^2+p_z^2})/T}}{1+e^{-(\sqrt{p_x^2+(1+\xi)p_y^2+p_z^2})/T}} \right).$$
- This would be zero for isotropic plasma i.e. for $\xi = 0$.
- This Lorentz force will produce current-sheets which will generate magnetic field that enhances the original magnetic field thus perturbation grows.

E. S. Weibel, Phys. Rev. Lett. **2**, 83 (1959),
B. D. Fried, Phys. Fluids **2**, 337 (1959).

Chiral kinetic theory

- In this description we have considered the weak gauge field limit, where there is no essential difference between Abelian and non-Abelian gauge fields up to color and flavor degrees of freedom.

$$\dot{n}_{\mathbf{p}} + \frac{1}{1+e\mathbf{B}\cdot\boldsymbol{\Omega}_{\mathbf{p}}}\Big[\Big(e\tilde{\mathbf{E}} + e\tilde{\mathbf{v}}\times\mathbf{B} + e^2(\tilde{\mathbf{E}}\cdot\mathbf{B})\boldsymbol{\Omega}_{\mathbf{p}}\Big)\cdot\frac{\partial n_{\mathbf{p}}}{\partial\mathbf{p}} + \Big(\tilde{\mathbf{v}} + e\tilde{\mathbf{E}}\times\boldsymbol{\Omega}_{\mathbf{p}} + e(\tilde{\mathbf{v}}\cdot\boldsymbol{\Omega}_{\mathbf{p}})\mathbf{B}\Big)\cdot\frac{\partial n_{\mathbf{p}}}{\partial\mathbf{x}}\Big] = 0,$$

- where $\tilde{\mathbf{v}} = \frac{\partial\epsilon_{\mathbf{p}}}{\partial\mathbf{p}}$, $e\tilde{\mathbf{E}} = e\mathbf{E} - \frac{\partial\epsilon_{\mathbf{p}}}{\partial\mathbf{x}}$, $\epsilon_{\mathbf{p}} = p(1 - e\mathbf{B}\cdot\boldsymbol{\Omega}_{\mathbf{p}})$ and $\boldsymbol{\Omega}_{\mathbf{p}} = \pm\mathbf{p}/2p^3$. Here $\pm$ sign corresponds to right and lefted handed fermions respectively.
- If $\boldsymbol{\Omega}_{\mathbf{p}} = 0$, above equation reduces to Vlasov equation.
- From above equation it is easy to get,

$$\partial_t n + \nabla\cdot\mathbf{j} = \mathbf{e}^2\int\frac{\mathbf{d^3p}}{(2\pi)^3}\left(\boldsymbol{\Omega}_{\mathbf{p}}\cdot\frac{\partial\mathbf{n_p}}{\partial\mathbf{p}}\right)\mathbf{E}\cdot\mathbf{B},$$

where,

$$n = \int\frac{d^3p}{(2\pi)^3}(1 + e\mathbf{B}\cdot\boldsymbol{\Omega}_{\mathbf{p}})n_{\mathbf{p}},$$

D. T. Son and N. Yamamoto, Phys. Rev. D **87**, 085016 (2013) [arxiv:1210.815].,

Chiral kinetic theory

$$\mathbf{j} = -e\int \frac{d^3p}{(2\pi)^3}\left[\epsilon_{\mathbf{p}}\frac{\partial n_{\mathbf{p}}}{\partial p} + e\left(\mathbf{\Omega_p}\cdot\frac{\partial \mathbf{n_p}}{\partial \mathbf{p}}\right)\epsilon_{\mathbf{p}}\mathbf{B} + \epsilon_{\mathbf{p}}\mathbf{\Omega_p}\times\frac{\partial n_{\mathbf{p}}}{\partial x}\right] + \mathbf{E}\times\boldsymbol{\sigma}.$$

$$\boldsymbol{\sigma} = \int \frac{d^3p}{(2\pi)^3}\mathbf{\Omega_p} n_{\mathbf{p}}.$$

- We follow the power counting scheme $A_\mu = O(\epsilon)$ and $\partial_x = O(\delta)$ where ϵ and δ are small parameters.

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}}\right)n_{\mathbf{p}} + \left(e\mathbf{E} + e\mathbf{v}\times\mathbf{B} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{x}}\right)\cdot\frac{\partial n_{\mathbf{p}}}{\partial p} = 0$$

- Where $v = \frac{\mathbf{p}}{p}$.

D. T. Son and N. Yamamoto, Phys. Rev. D **87**, 085016 (2013) [arxiv:1210.815].,

$$\dot{n}_{\mathbf{p}} + \frac{1}{1+e\mathbf{B}\cdot\mathbf{\Omega_p}}\left[\left(e\tilde{\mathbf{E}} + e\tilde{\mathbf{v}}\times\mathbf{B} + e^2(\tilde{\mathbf{E}}\cdot\mathbf{B})\mathbf{\Omega_p}\right)\cdot\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} + \left(\tilde{\mathbf{v}} + e\tilde{\mathbf{E}}\times\mathbf{\Omega_p} + e(\tilde{\mathbf{v}}\cdot\mathbf{\Omega_p})\mathbf{B}\right)\cdot\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{x}}\right] = 0,$$

,

Linear response analysis of anisotropic chiral plasma

- We consider the distribution function of the form

$$n^0_{\mathbf{p}} = 1/[e^{(\epsilon_{\mathbf{p}}-\mu)/T} + 1]$$

$$n^0_{\mathbf{p}} = n^{0(0)}_{\mathbf{p}} + e n^{0(\epsilon\delta)}_{\mathbf{p}},$$

- where, $n^{0(0)}_{\mathbf{p}} = \frac{1}{[e^{(\tilde{p}-\mu)/T}+1]}$ and $n^{0(\epsilon\delta)}_{\mathbf{p}} = \left(\frac{\mathbf{B}\cdot\mathbf{v}}{2\tilde{p}T}\right)\frac{e^{(\tilde{p}-\mu)/T}}{[e^{(\tilde{p}-\mu)/T}+1]^2}$.
- $\tilde{p} = \sqrt{p^2 + \xi(\mathbf{p}\cdot\hat{\mathbf{n}})^2}$.
- Anomalous Hall current depends on electric field, it can be of order $O(\epsilon\delta)$ or higher. We are interested in finding deviations in current and distribution function up to order $O(\epsilon\delta)$, only $n^{0(0)}_{\mathbf{p}}$ will contribute to the Hall current term.

$$\boldsymbol{\sigma} = \frac{1}{2}\int d\Omega d\tilde{p}\frac{\mathbf{v}}{[1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})]^{1/2}}\frac{1}{(1+e^{(\tilde{p}-\mu)/T})} = 0.$$

- Anomalous Hall current vanishes.

The distribution function can be decomposed into separate scales as follows,

$$n_{\mathbf{p}} = n^0_{\mathbf{p}} + e(n^{(\epsilon)}_{\mathbf{p}} + n^{(\epsilon\delta)}_{\mathbf{p}}).$$

P. Romatschke and M. Strickland, Phys. Rev. **D 68**, 036004 (2003).

C. Manuel and J. M. Torres-Rincon, arXiv:1312.1158[hep-ph] (2014).

Linear response analysis of anisotropic chiral plasma

$$\Pi_{+}^{ij}(K) = m_D^2 \int \frac{d\Omega}{4\pi} \frac{v^i(v^l + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})\hat{n}^l)}{(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^2} \left(\delta^{jl} + \frac{v^j k^l}{v.k + i\epsilon}\right),$$

$$\Pi_{-}^{im}(K) = C_E \int \frac{d\Omega}{4\pi} \left[\frac{i\epsilon^{jlm} k^l v^j v^i (\omega + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})(\mathbf{k}.\hat{\mathbf{n}}))}{(v.k+i\epsilon)(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^{3/2}} + \left(\frac{v^i + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})\hat{n}^i}{(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^{3/2}}\right) i\epsilon^{iml} k^l v^j \right.$$
$$\left. - i\epsilon^{ijl} k^l v^j \left(\delta^{mn} + \frac{v^m k^n}{v.k+i\epsilon}\right)\left(\frac{v^n + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})\hat{n}^n}{(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^{3/2}}\right)\right]$$

where,

$$m_D^2 = -\frac{1}{2\pi^2}\int_0^\infty d\tilde{p}\tilde{p}^2 \left[\frac{\partial n_{\tilde{\mathbf{p}}}^{0(0)}(\tilde{p}-\mu)}{\partial\tilde{p}} + \frac{\partial n_{\tilde{\mathbf{p}}}^{0(0)}(\tilde{p}+\mu)}{\partial\tilde{p}}\right]$$

$$C_E = -\frac{1}{4\pi^2}\int_0^\infty d\tilde{p}\tilde{p} \left[\frac{\partial n_{\tilde{\mathbf{p}}}^{0(0)}(\tilde{p}-\mu)}{\partial\tilde{p}} - \frac{\partial n_{\tilde{\mathbf{p}}}^{0(0)}(\tilde{p}+\mu)}{\partial\tilde{p}}\right]$$

- After performing above integrations one can get $m_D^2 = \frac{\mu^2}{2\pi^2} + \frac{T^2}{6}$ and $C_E = \frac{\mu}{4\pi^2}$. It can be noticed that the terms with anisotropy parameter ξ are contributing in both parity-even and odd part of the self-energy or polarization tensor.

$$j^{\mu}_{ind} = \Pi^{\mu\nu}(K) A_{\nu}(K),$$

.

Linear response analysis of anisotropic chiral plasma

- Maxwell equation,

$$\partial_\nu F^{\nu\mu} = j^\mu_{ind} + j^\mu_{ext}.$$
$$j^\mu_{ind} = \Pi^{\mu\nu}(K)A_\nu(K),$$

- $\Pi^{\mu\nu}(K)$ is the retarded self energy in Fourier space. Here we denote the Fourier transform as $F(K) = \int d^4x e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x})}F(x,t)$.

- Choosing temporal gauge $A_0 = 0$

$$[(k^2 - \omega^2)\delta^{ij} - k^i k^j + \Pi^{ij}(K)]E^j = i\omega j^i_{ext}(k).$$

- From this one can define,

$$[\Delta^{-1}(K)]^{ij} = (k^2 - \omega^2)\delta^{ij} - k^i k^j + \Pi^{ij}(K).$$

- The poles of $[\Delta(K)]^{ij}$ will give us the dispersion relation.

Finding the Poles of $[\Delta(K)]^{ij}$ or Dispersion relation

- We decompose first $\Pi^{ij}(K)$ in following six tensorial basis,

$$\Pi^{ij} = \alpha P_T^{ij} + \beta P_L^{ij} + \gamma P_n^{ij} + \delta P_{kn}^{ij} + \lambda P_A^{ij} + \chi P_{An}^{ij}.$$

- Where,

$$\begin{aligned} P_T^{ij} &= \delta^{ij} - k^i k^j / k^2 \\ P_L^{ij} &= k^i k^j / k^2 \\ P_n^{ij} &= \tilde{n}^i \tilde{n}^j / \tilde{n}^2 \\ P_{kn}^{ij} &= k^i \tilde{n}^j + k^j \tilde{n}^i \\ P_A^{ij} &= i\epsilon^{ijk} \hat{k}^k \\ P_{An}^{ij} &= i\epsilon^{ijk} \tilde{n}^k. \end{aligned}$$

- α,β, γ, δ λ and χ are some scalar functions of k and ω which can be determined by $\alpha = (P_T^{ij} - P_n^{ij})\Pi^{ij}$, $\beta = P_L^{ij}\Pi^{ij}$, $\gamma = (2P_n^{ij} - P_T^{ij})\Pi^{ij}$, $\delta = \frac{1}{2k^2\tilde{n}^2} P_{kn}^{ij}\Pi^{ij}$ $\lambda = -\frac{1}{2} P_A^{ij}\Pi^{ij}$ and $\chi = -\frac{1}{2\tilde{n}^2} P_{An}^{ij}\Pi^{ij}$.

Finding the Poles of $[\Delta(K)]^{ij}$ or Dispersion relation

- We shall do the analysis in the small ξ limit (Very weak anisotropy),

$$\begin{aligned}
\alpha &= \Pi_T + \xi\Big[\frac{z^2}{12}(3+5\cos 2\theta_n)m_D^2 - \frac{1}{6}(1+\cos 2\theta_n)m_D^2 + \frac{1}{4}\Pi_T\,\Big((1+3\cos 2\theta_n) - z^2(3+5\cos 2\theta_n)\Big)\Big];\\
z^{-2}\beta &= \Pi_L + \xi\Big[\frac{1}{6}(1+3\cos 2\theta_n)m_D^2 + \Pi_L\Big(\cos 2\theta_n - \frac{z^2}{2}(1+3\cos 2\theta_n)\Big)\Big];\\
\gamma &= \frac{\xi}{3}(3\Pi_T - m_D^2)(z^2-1)\sin^2\theta_n;\\
\delta &= \frac{\xi}{3k}(4z^2 m_D^2 + 3\Pi_T(1-4z^2))\cos\theta_n;\\
\lambda &= -\frac{\mu k}{4\pi^2}\left[(1-z^2)\frac{\Pi_L}{m_D^2}\right] - \xi\frac{\mu k}{8\pi^2}\left[(1-z^2)\frac{\Pi_L}{m_D^2}((3\cos 2\theta_n - 1)\right.\\
&- \left. 2z^2(1+3\cos 2\theta_n)) + \frac{2z^2}{3}(1-3\cos 2\theta_n) - \frac{43}{15} + \frac{22}{10}(1+\cos 2\theta_n)\right];\\
\chi &= \xi\,[f(\omega,k)]\,,
\end{aligned}$$

- Expressions for Π_T, Π_L are given as,

$$\Pi_T = m_D^2\frac{\omega^2}{2k^2}\left[1 + \frac{k^2-\omega^2}{2\omega k}\ln\frac{\omega+k}{\omega-k}\right],$$

$$\Pi_L = m_D^2\left[\frac{\omega}{2k}\ln\frac{\omega+k}{\omega-k} - 1\right],$$

Finding the Poles of $[\Delta(K)]^{ij}$ or Dispersion relation

- Similarly we can write $[\Delta^{-1}(k)]^{ij}$ as

$$[\Delta^{-1}(K)]^{ij} = C_T P_T^{ij} + C_L P_L^{ij} + C_n P_n^{ij} + C_{kn} P_{kn}^{ij} + C_A P_A^{ij} + C_{An} P_{An}^{ij}.$$

- Coefficients C's and α's have the following relationship.

$$\begin{aligned} C_T &= k^2 - \omega^2 + \alpha \\ C_L &= -\omega^2 + \beta \\ C_n &= \gamma \\ C_{kn} &= \delta \\ C_A &= \lambda \\ C_{An} &= \chi. \end{aligned}$$

- So once we know α,β, γ, δ λ and χ we can determine coefficient C's.
- But In order to get dispersion relation we have to find poles of $[\Delta(K)]^{ij}$ not of $[\Delta^{-1}(K)]^{ij}$. We can use the following formula,

$$[\Delta^{-1}(K)]^{ij}[\Delta(K)]^{jl} = \delta^{il}$$

Finding the Poles of $[\Delta(K)]^{ij}$ or Dispersion relation

- we can obtain following formula for $[\Delta(K)]^{ij}$.

$$[\Delta(K)]^{ij} = aP_L^{ij} + bP_T^{ij} + cP_n^{ij} + dP_{kn}^{ij} + eP_A^{ij} + fP_{An}^{ij}$$

- where,

$$a = \frac{C_A^2 - C_T(C_n + C_T)}{2k\tilde{n}^2 C_A C_{An} C_{kn} + C_A^2 C_L + \tilde{n}^2 C_{An}^2 (C_n + C_T) - C_T(-k^2\tilde{n}^2 C_{kn}^2 + C_L(C_n + C_T))}$$

$$b = \frac{k^2\tilde{n}^2 C_{kn}^2 - C_L(C_n + C_T)}{2k\tilde{n}^2 C_A C_{An} C_{kn} + C_A^2 C_L + \tilde{n}^2 C_{An}^2 (C_n + C_T) - C_T(-k^2\tilde{n}^2 C_{kn}^2 + C_L(C_n + C_T))}$$

$$c = \frac{(C_A C_{An} + kC_{kn} C_T)/k}{2k\tilde{n}^2 C_A C_{An} C_{kn} + C_A^2 C_L + \tilde{n}^2 C_{An}^2 (C_n + C_T) - C_T(-k^2\tilde{n}^2 C_{kn}^2 + C_L(C_n + C_T))}$$

$$d = \frac{k\tilde{n}^2 C_{An} C_{kn} + C_A C_L}{2k\tilde{n}^2 C_A C_{An} C_{kn} + C_A^2 C_L + \tilde{n}^2 C_{An}^2 (C_n + C_T) - C_T(-k^2\tilde{n}^2 C_{kn}^2 + C_L(C_n + C_T))}$$

$$e = \frac{\tilde{n}^2(C_{An}^2 - k^2 C_{kn}^2) + C_L C_n}{2k\tilde{n}^2 C_A C_{An} C_{kn} + C_A^2 C_L + \tilde{n}^2 C_{An}^2 (C_n + C_T) - C_T(-k^2\tilde{n}^2 C_{kn}^2 + C_L(C_n + C_T))}$$

$$f = \frac{kC_A C_{kn} + C_{An}(C_n + C_T)}{2k\tilde{n}^2 C_A C_{An} C_{kn} + C_A^2 C_L + \tilde{n}^2 C_{An}^2 (C_n + C_T) - C_T(-k^2\tilde{n}^2 C_{kn}^2 + C_L(C_n + C_T))}$$

Finding the Poles of $[\Delta(K)]^{ij}$ or Dispersion relation

- Therefore the dispersion relation is,

$$2k\tilde{n}^2 C_A C_{An} C_{kn} + C_A^2 C_L + \tilde{n}^2 C_{An}^2 (C_n + C_T) - C_T(-k^2\tilde{n}^2 C_{kn}^2 + C_L(C_n + C_T)) = 0.$$

- In the weak anisotropy limit, one can write the dispersion relation as,

$$C_A^2 C_L - C_T C_L(C_n + C_T)) = 0,$$

- Which give following two branches of Dispersion relation,

$$C_A^2 - C_T^2 - C_n C_T = 0.$$
$$C_L = 0.$$

- When $C_A = 0$, above equations reduces to exactly the same dispersion relation discussed in Ref. given below for an anisotropic plasma where there is no parity violating effect.

- Equation for transverse modes give the following solution,

$$(k^2 - \omega^2) = \frac{-(2\alpha + \gamma) \pm 2\lambda}{2}.$$

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[14] P. Romatschke, M. Strickland, Phys. Rev. D **68** 036004 (2003)

Poles of $[\Delta(K)]^{ij}$ or Dispersion relation

- In the quasi stationary limit $|\omega| << k$ one can get the final form of dispersion relation as $\omega = i\rho(k)$, where $\rho(k)$ is given by.

$$\rho(k) = \left(\frac{4\alpha^3\mu^3}{\pi^4 m_D^2}\right) k_N^2 \left[1 - k_N + \frac{\xi}{12}(1 + 5\cos 2\theta_n) + \frac{\xi}{12}(1 + 3\cos 2\theta_n)\frac{\pi^2 m_D^2}{\mu^2\alpha^2 k_N}\right].$$

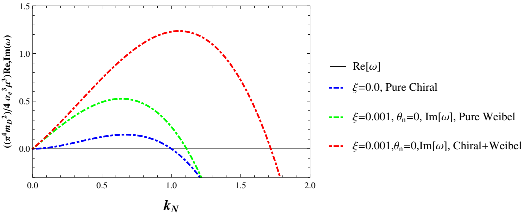
- Where $k_N = \frac{\pi k}{\mu\alpha}$, and $\alpha = \frac{e^2}{4\pi}$ is the electromagnetic coupling.
- In the limit $\xi \to 0$ we will get,

$$\rho(k) = \left(\frac{4\alpha^3\mu^3}{\pi^4 m_D^2}\right) k_N^2 \left[1 - k_N\right]$$

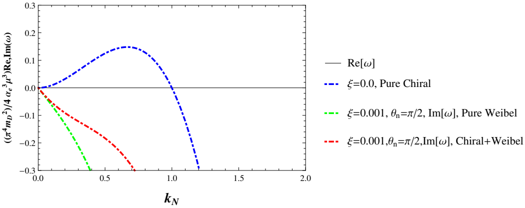
- From here it is easy to determine that the two instabilities have comparable growth at a critical angle $\theta_c = \frac{1}{2}\cos^{-1}\left[\left(\frac{2}{27}\right)^{2/3}\frac{12\mu^2\alpha^2}{\xi\pi^2 m_D^2} - \frac{1}{3}\right]$.

Y. Akamatsu and N. Yamamoto, Phys. Rev. Lett. **111**, 052002 (2013).

Results and discussion



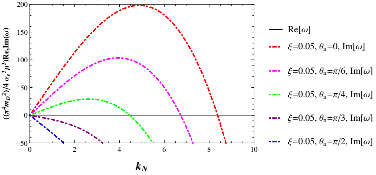
(c)



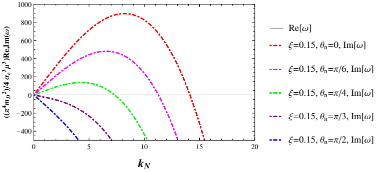
(d)

Figure: Shows plots of real and imaginary part of the dispersion relation. Here θ_n is the angle between the wave vector k and the anisotropy vector. Real part of dispersion relation is zero. Fig. (1a-1b) show plots for three cases: (i) Pure chiral (no anisotropy), (ii) Pure Weibel (chiral chemical potential=0) and (iii) When both chiral and Weibel instabilities are present.

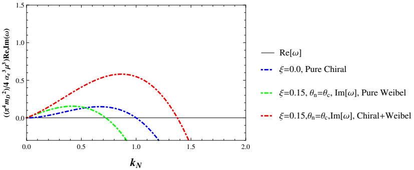
Results and discussion



(a)



(b)



(c)

Figure: Shows plots of real and imaginary part of the dispersion relation. Here θ_n is the angle between the wave vector k and the anisotropy vector. Real part of dispersion relation is zero. Fig. (2a-2b) represent the case when both the instabilities are present but the anisotropy parameter varies at different values of θ_n. Fig. (2c) represents the case when for a particular value of $\theta_n \sim \theta_c$ two instabilities have equal growth at different ξ values. Here frequency is normalized in unit of $\omega / \left(\frac{4\alpha^3\mu^3}{\pi^4 m_D^2}\right)$ and wave-number k by $k_N = \frac{\pi}{\mu\alpha}k$.

Summary

- We have studied collective modes in anisotropic chiral plasmas. We have considered two cases of the instabilities together namely chiral imbalance instability and Weibel instability.
- We found that even for small value of anisotropy parameter ($\xi << 1$), Weibel instability dominates over chiral imbalance instability.
- For $\xi > 0$, the growth rate and range increases significantly when the wave vector k is in the direction parallel to anisotropy vector $\mathbf{n}$.
- Instability gets suppressed when k is in the direction perpendicular to $\mathbf{n}$.
- Growth for the two instabilities become comparable at a critical angle $\theta_c = \frac{1}{2}\cos^{-1}\left[\left(\frac{2}{27}\right)^{2/3}\frac{12\mu^2\alpha^2}{\xi\pi^2 m_D^2} - \frac{1}{3}\right]$.

THANK YOU

Berry's Phase

- Consider the Hamiltonian of the system with an external time dependent parameter $R(t)$ denoting it as H(R(t)).

- The ket $|n(R(t))\rangle$ of the n^{th} energy eigenstate corresponding to R(t) satisfies the eigenvalue equation at time t.

$$H(R(t))|n(R(t))\rangle = E_n(R(t))|n(R(t))\rangle$$

$$\langle n(R(t))|n'(R(t))\rangle = \delta_{n,n'}$$

- Let R evolve in time from $R(0) = R_0$.
- Let at time t the state ket is $|n(R_0), t_0; t\rangle$

- Time dependent Schrödinger equation that the state ket obeys is.

$$H(R(t))|n(R_0), t_0; t\rangle = i\hbar\frac{\partial}{\partial t}|n(R_0), t_0; t\rangle \tag{1}$$

- where $t_0 = 0$.

J. J. Sakurai, Modern Quantum Mechanics: (Pearson Education, 1994),

Berry's Phase

- When R(t) is slow enough, we expect from the adiabatic theorem that $|n(R_0), t_0; t\rangle$ would be proportional to the n^{th} energy eigenket $|n(R(t))\rangle$ of $H(R(t))$ at time t.

$$|n(R_0), t_0; t\rangle = A_n(t)\exp\left\{ -\frac{i}{\hbar}\int_0^t E_n(R(t'))dt' \right\}|n(R(t))\rangle. \qquad (2)$$

Using Eq.(2) in Eq.(1), one can get;

$$\frac{dA_n(t)}{dt} = -A_n(t)\langle n(R(t))|\frac{d}{dt}|n(R(t))\rangle$$

$$A_n(t) = A_n(0)\exp\left\{ -\underbrace{\int_0^t dt'\langle n(R(t'))|\frac{d}{dt'}|n(R(t'))\rangle}_{i\gamma_n(t)} \right\}$$

- Let us call it phase factor $\gamma_n(t)$

$$\gamma_n(t) = -i\int_0^t dt'\langle n(R(t'))|\frac{d}{dt'}|n(R(t'))\rangle$$

$$|n(R_0), t_0; t\rangle = A_n(0)\exp\left\{ \underbrace{-i\gamma_n(t)}_{\textbf{Berry Phase}} - \frac{i}{\hbar}\int_0^t E_n(R(t'))dt' \right\}|n(R(t))\rangle.$$

Berry's Phase

- $\gamma_n(t)$ can also be represented by a path-integral in parameter $\mathbf{R(t)}$ space as,

$$\gamma_n(t) = -i \int_{\mathbf{R_0}}^{\mathbf{R_f}} d\mathbf{R} \langle n(R(t'))|\nabla_{\mathbf{R}}|n(R(t'))\rangle$$

- If $\mathbf{R}$ describes a closed loop in parameter space i.e. $\mathbf{R_f} = \mathbf{R_0}$

$$\begin{aligned} \gamma_n(t) &= -i \oint_c d\mathbf{R} \langle n(R(t'))|\nabla_{\mathbf{R}}|n(R(t'))\rangle \\ &= \int\int_{S(c)} dS \cdot \nabla_{\mathbf{R}} \times Q(\mathbf{R}) \end{aligned}$$

- Where, $\mathbf{Q}(\mathbf{R}) = -i\langle n(R(t'))|\nabla_{\mathbf{R}}|n(R(t'))\rangle$. $\Rightarrow$ Berry connection
- While $\nabla_{\mathbf{R}} \times \mathbf{Q}(\mathbf{R}) = \boldsymbol{\Omega}(\mathbf{R})$ $\Rightarrow$ Berry Curvature.

Berry Curvature for Chiral Fermions

- Consider a chiral fermion expressed by the two-component spinor $\mathbf{u_p}$ satisfying the Weyl equation.

$$(\sigma \cdot \mathbf{p})\mathbf{u_p} = \pm|p|\mathbf{u_p}$$

- Two component spinor described above has a nonzero Berry connection

$$\mathbf{Q_p} \equiv -i\mathbf{u_p}^{\dagger}\nabla_{\mathbf{p}}\mathbf{u_p}$$

- Nonzero Berry curvature,

$$\mathbf{\Omega(p)} \equiv \nabla_{\mathbf{p}} \times \mathbf{Q_p} = \pm\frac{\hat{\mathbf{p}}}{2|p|^2}$$

- where $\hat{\mathbf{p}} = \frac{\hat{\mathbf{p}}}{|p|}$ is a unit vector.

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[17] D. T. Son and N. Yamamoto, Phys. Rev. D **87**, 085016 (2013) [arxiv:1210.815].,

Chiral Kinetic Theory

- Considering a charged fermion in electromagnetic fields and Berry curvature, the action.

$$S(x,p) = \int dt[(p^i + eA^i(x))\dot{x}^i - \mathbf{Q^i}(\mathbf{p})\dot{p}^i - \epsilon_p(p) - A^0(x)]$$

$$S(\xi) = \int dt[\Sigma_a(\xi)\dot{\xi}^a - H(\xi)]$$

- Where, $\Sigma_a(\xi) = (p^i + eA_i(x), -\mathbf{Q^i}(\mathbf{p}))$ and $\xi^a = (x^i, p^i)$
- Equations of motion of the action read.

$$\Sigma_{ab}\dot{\xi}^b = -\frac{\partial H(\xi)}{\partial \xi^a}$$

- Where $\Sigma_{ab} = \frac{\partial \Sigma_a(\xi)}{\partial \xi^b} - \frac{\partial \Sigma_b(\xi)}{\partial \xi^a}$.

18 19 20 21

[18] G. Sundaram and Q. Niu, Phys. Rev. B**59**, 14915 (1999)
[19] D. T. Son and N. Yamamoto, Phys. Rev. D **87**, 085016 (2013) [arxiv:1210.815].,

[20] D. Xiao, J. Shi, and Q. Niu. Sundaram and Q. Niu, Phys. Rev. Lett.**95**, 137204 (2005)
[21] C. Duval and Z. Horvath, P. A Horvathy, L. Martina and P. Stichel, Mod. Phys. lett. B**20**, 373 (2006)

Chiral Kinetic Theory

- Further we rewrite above equation as,

$$\dot{\xi}^a = -(\Sigma^{-1})^{ab}\frac{\partial H(\xi)}{\partial \xi^b}$$

- Hamilton's equation of motion is,

$$\dot{\xi}^a = -\{\xi^a, H(\xi)\} = -\{\xi^a, \xi^b\}\frac{\partial H(\xi)}{\partial \xi^b}$$

- $\Rightarrow \{\xi^a, \xi^b\} = (\Sigma^{-1})^{ab}$
- Explicit form of poision brackets with berry curvature,
- $\{\dot{x}^i, \dot{x}^j\} = \frac{\epsilon_{ijk}\Omega_k}{1+e\mathbf{B}\cdot\mathbf{\Omega}}$, $\{\dot{x}^i, \dot{p}^j\} = -\frac{\delta_{ij}+e\Omega_i B_j}{1+e\mathbf{B}\cdot\mathbf{\Omega}}$, $\{\dot{p}^i, \dot{p}^j\} = -\frac{e\epsilon_{ijk}B_k}{1+e\mathbf{B}\cdot\mathbf{\Omega}}$, .
- Where $B^i = \epsilon^{ijk}\frac{\partial A^k}{\partial x^j}$

Kinetic Equation

- Invariant Phase space gets modified, $d\Gamma = \sqrt{det\Sigma_{ab}}d\xi = (1 + e\mathbf{B}\cdot\mathbf{\Omega})\frac{dpdx}{2\pi^3}$.
- Equivalent Liouville's theorem,

$$\dot{n}_{\mathbf{p}} - (\Sigma)^{-1}_{ab}\frac{\partial H(\xi)}{\partial \xi^b}\frac{\partial n_{\mathbf{p}}}{\partial \xi^a} = 0$$

- Taking $H = \epsilon_p + A_0$, One can explicitly write down the kinetic equation as,

Linear responce analysis of anisotrpic chiral plasma

Therefore,

$$\mathbf{j} = -\int \frac{d^3p}{(2\pi)^3}\left[\epsilon_{\mathbf{p}}\frac{\partial n_{\mathbf{p}}}{\partial p} + e\left(\mathbf{\Omega}_{\mathbf{p}}\cdot\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}\right)\epsilon_{\mathbf{p}}\mathbf{B} + \epsilon_{\mathbf{p}}\mathbf{\Omega}_{\mathbf{p}}\times\frac{\partial n_{\mathbf{p}}}{\partial x}\right].$$

The distribution function can be decomposed into separate scales as follows,

$$n_{\mathbf{p}} = n^0_{\mathbf{p}} + e(n^{(\epsilon)}_{\mathbf{p}} + n^{(\epsilon\delta)}_{\mathbf{p}}).$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}}\right) n^{(\epsilon)}_{\mathbf{p}} = -(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\frac{\partial n^{0(0)}_{\mathbf{p}}}{\partial p}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}}\right)(n^{0(\epsilon\delta)}_{\mathbf{p}} + n^{(\epsilon\delta)}_{\mathbf{p}}) = -\frac{1}{e}\frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{x}}\cdot\frac{\partial n^{0(0)}_{\mathbf{p}}}{\partial p}$$

$$\mathbf{j}^{\mu(\epsilon)} = e^2\int\frac{d^3p}{(2\pi)^3}v^{\mu}n^{(\epsilon)}_{\mathbf{p}}$$

$$\mathbf{j}^{\mathbf{i}(\epsilon\delta)} = e^2\int\frac{d^3p}{(2\pi)^3}\left[v^i n^{(\epsilon\delta)}_{\mathbf{p}} - \left(\frac{v^j}{2p}\frac{\partial n^{0(0)}_{\mathbf{p}}}{\partial p^j}\right)B^i - \epsilon^{ijk}\frac{v^j}{2p}\frac{\partial n^{(\epsilon)}_{\mathbf{p}}}{\partial x^k}\right]$$

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$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}}\right)n_{\mathbf{p}} + \left(e\mathbf{E} + e\mathbf{v}\times\mathbf{B} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{x}}\right)\cdot\frac{\partial n_{\mathbf{p}}}{\partial p} = 0$$

,