Chiral and deconfinement aspects of QCD transition at finite temperature and the equation of state

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How does chiral symmetry restoration happens at high *T* ?

What is the interplay between chiral symmetry restoration and axial ($U_A(1)$) anomaly?

How does EoS changes with increasing temperatures

How do we understand deconfinement in QCD with light quarks (absence of order parameter) ?

Is there is an interplay between chiral transition and deconfinement transition ?

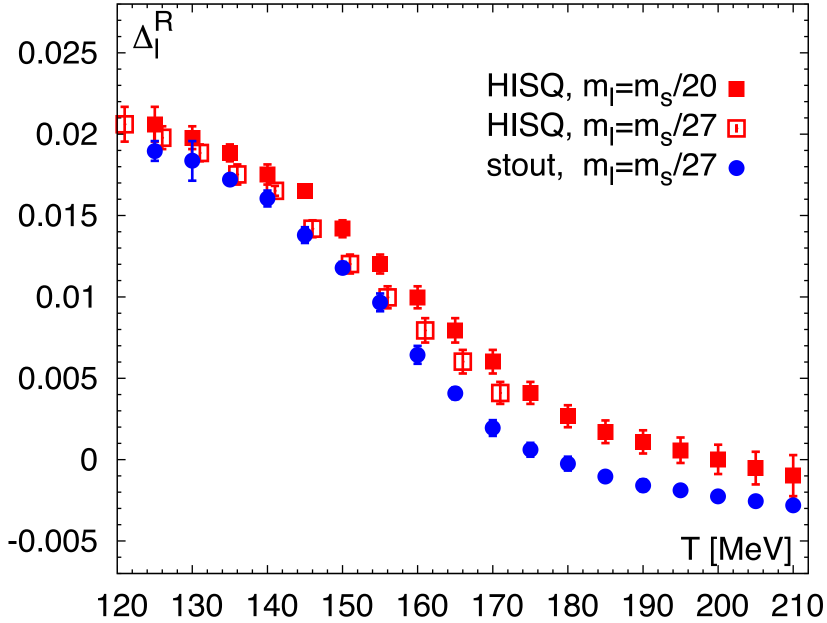
The temperature dependence of chiral condensate

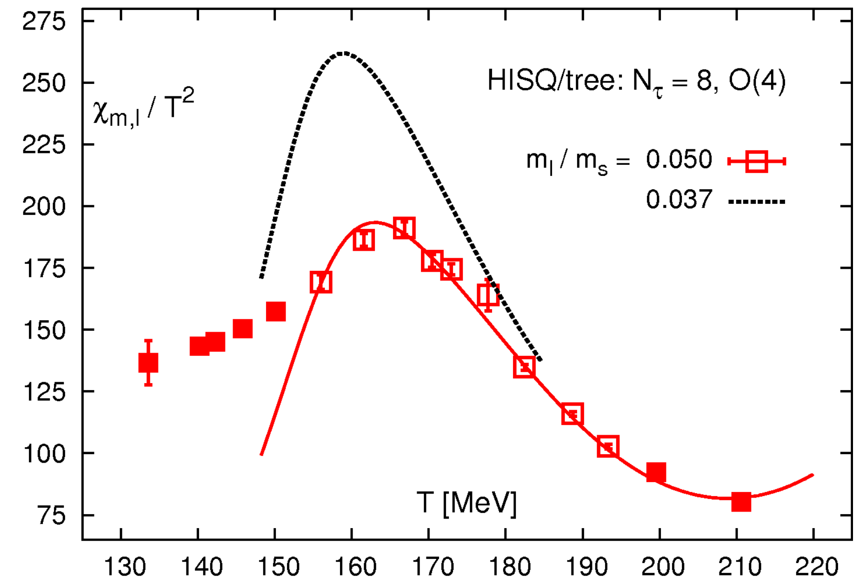
Renormalized chiral condensate introduced by Budapest-Wuppertal collaboration

$$\langle\bar\psi\psi\rangle_q \Rightarrow \Delta_q^R(T) = m_s r_1^4 \left(\langle\bar\psi\psi\rangle_{q,T} - \langle\bar\psi\psi\rangle_{q,T=0}\right) + d, \quad q = l, s$$

with our choice : $d = \langle\bar\psi\psi\rangle_{m_q=0}^{T=0}$

HotQCD : Phys. Rev. D85 (2012) 054503;
Bazavov, PP, RRD 87(2013) 094505





The chiral transition temperature in the continuum limit at physical point:

$$T_c = (154 \pm 8 \pm 1(scale))\text{MeV}$$

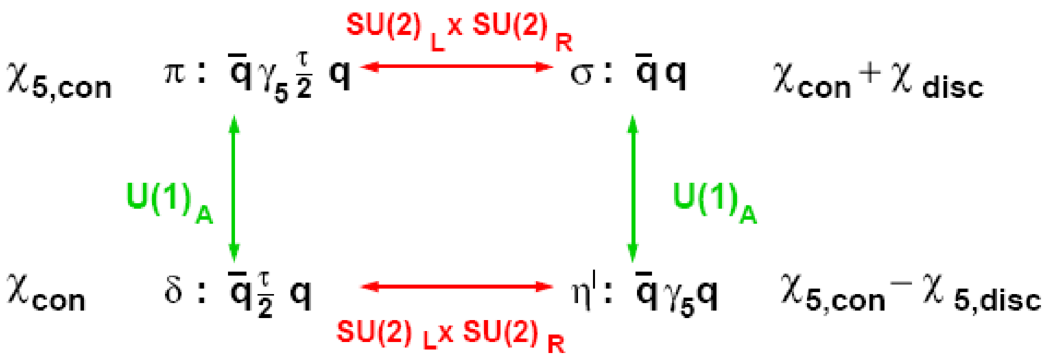
The chiral transition is compatible with O(N) scaling and thus with 2nd order transition for $m_l=0$

Problem: straggered fermions

Chiral and transitions with Domain Wall Fermions

1st order chiral in QCD restoration if $U_A(1)$ is restored at T_c

$$\chi_i = \int d^4x G_i(x)$$

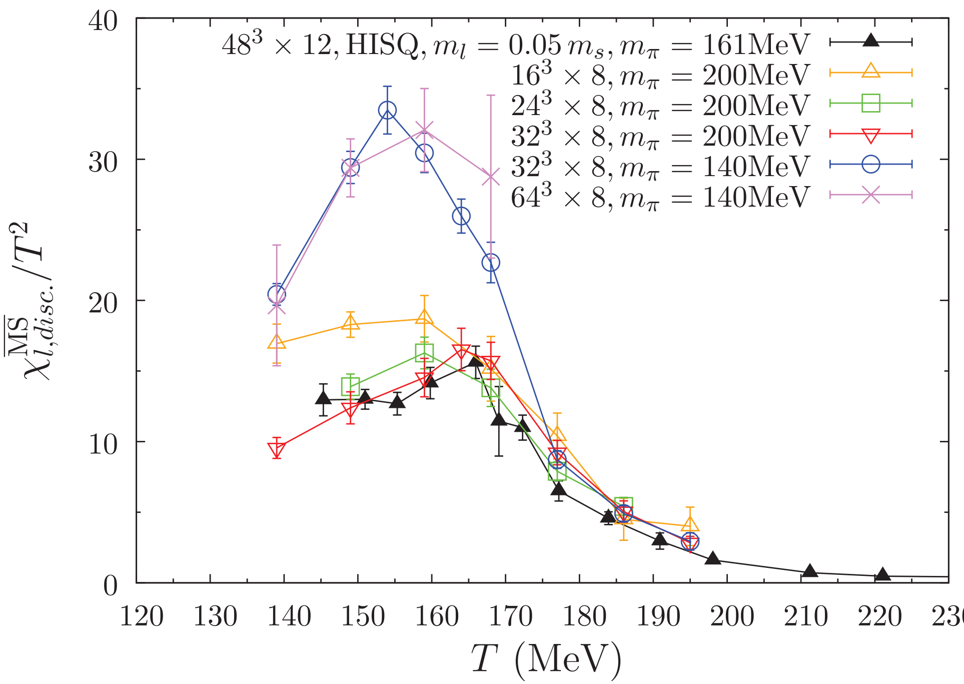


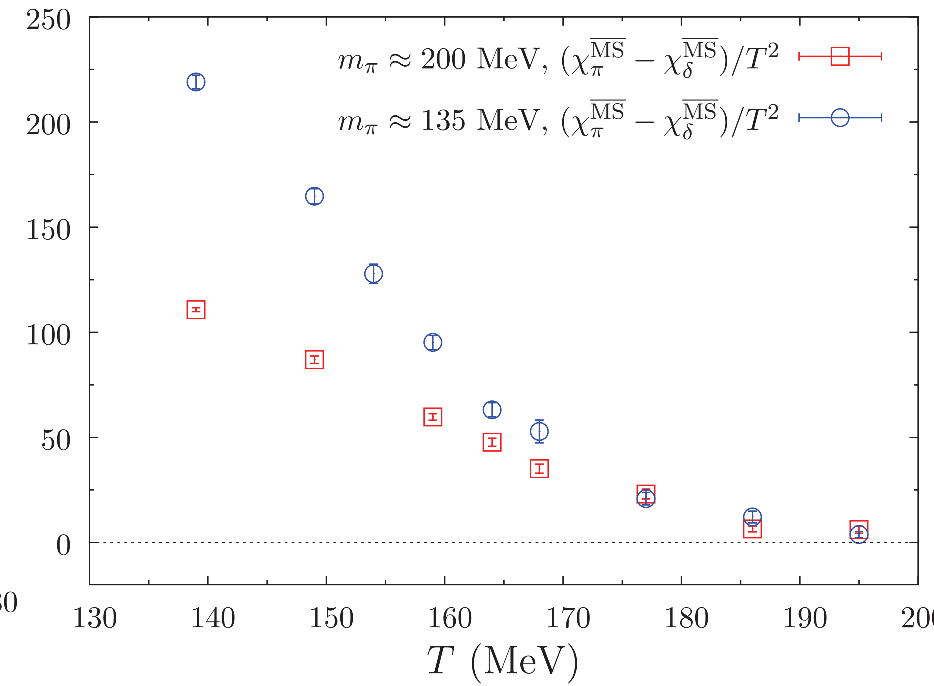
chiral: $\chi_{disc} = \chi_{5,disc}$

axial: $\chi_{disc} = -\chi_{5,disc}$

$$\chi_{disc} = 0$$

Bhattacharya et al (HotQCD) PRL113 (2014) 082001

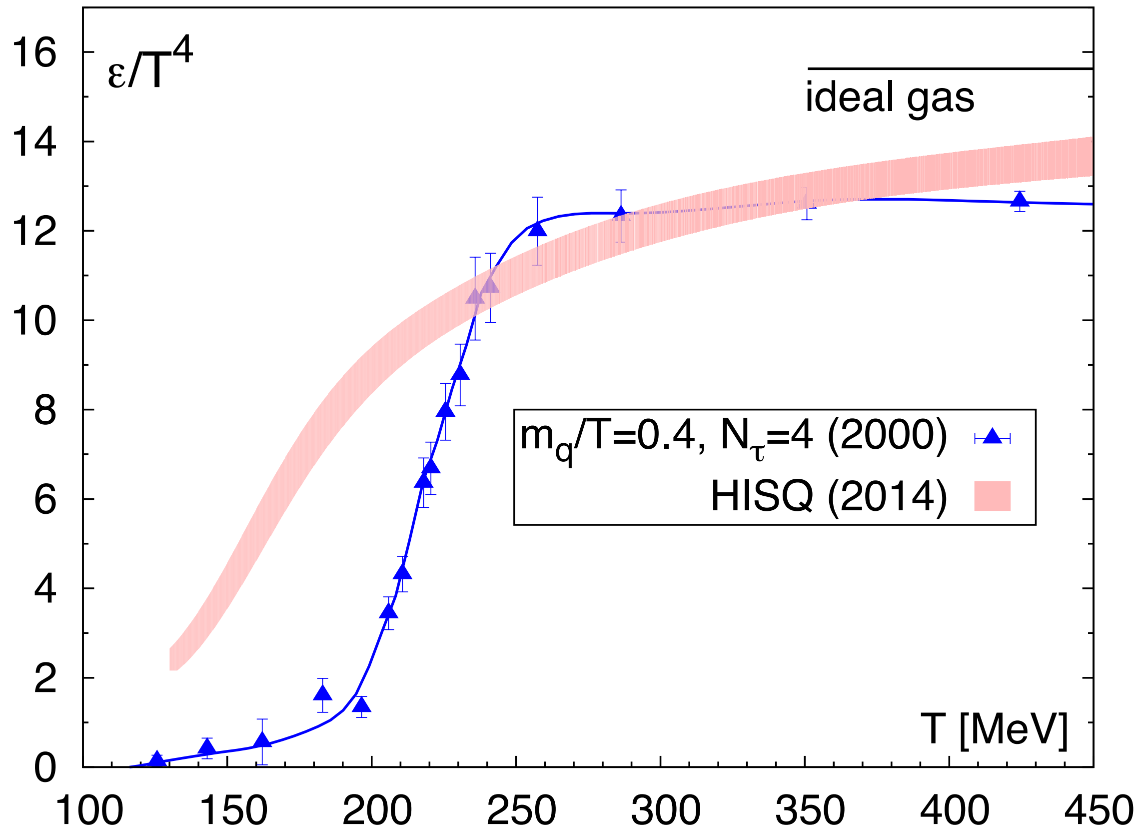




Peak position roughly agrees with staggered results, T_c=*155(1)(8)* MeV

axial symmetry is effectively restored *T>200* MeV !

How Equation of state changed since 2000



- Much smoother transition to QGP
- The energy density keeps increasing up to 450 MeV instead of flattening

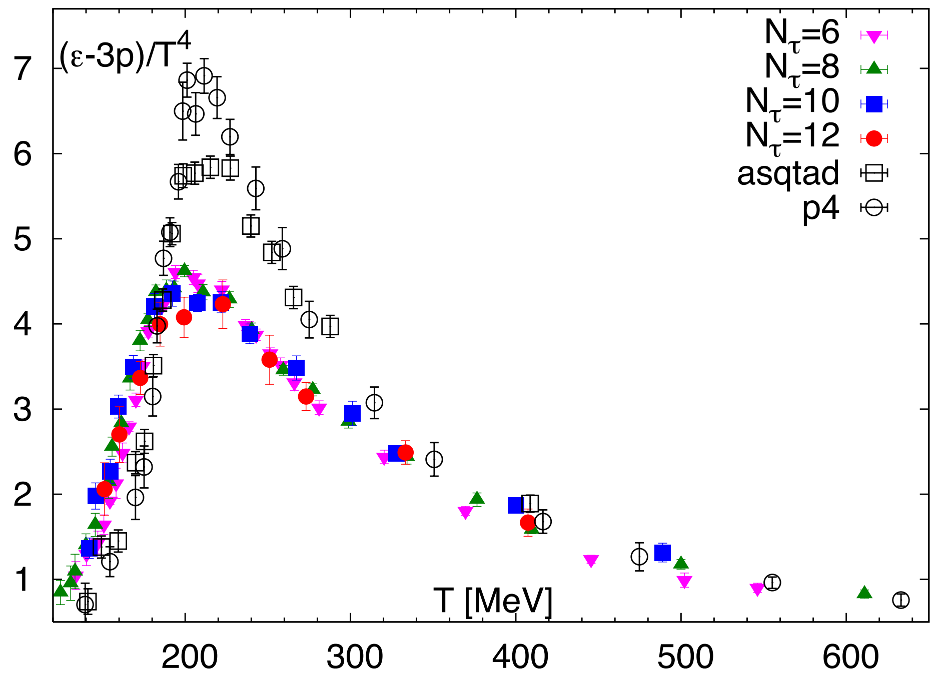
Trace anomaly and the integral method

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\varepsilon - 3p}{T^4} = T\frac{d}{dT}\left(\frac{p}{T^4}\right) \Rightarrow \frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Theta^{\mu\mu}(T')}{T'^5},$$

$$\frac{\Theta^{\mu\mu}(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = R_\beta \{\langle S_G\rangle_0 - \langle S_G\rangle_T\} - R_\beta R_m \{2m_l(\langle\bar{q}q\rangle_0 - \langle\bar{q}q\rangle) + m_s(\langle\bar{s}s\rangle_0 - \langle\bar{s}s\rangle_T)\}$$

$\Theta_G^{\mu\mu}$ $\Theta_F^{\mu\mu}$

$$R_\beta(\beta) = -a\frac{d\beta}{da},\ R_m = \frac{1}{m_q(\beta)}\frac{dm_q(\beta)}{d\beta},\ \beta = 10/g^2$$



Bazavov et al, PRD90 (2014) 094503

The peak height is much reduced compared to the asqtad and p4 N_τ=8 calculations

Agreement with p4 and asqtad calculations for T>350 MeV

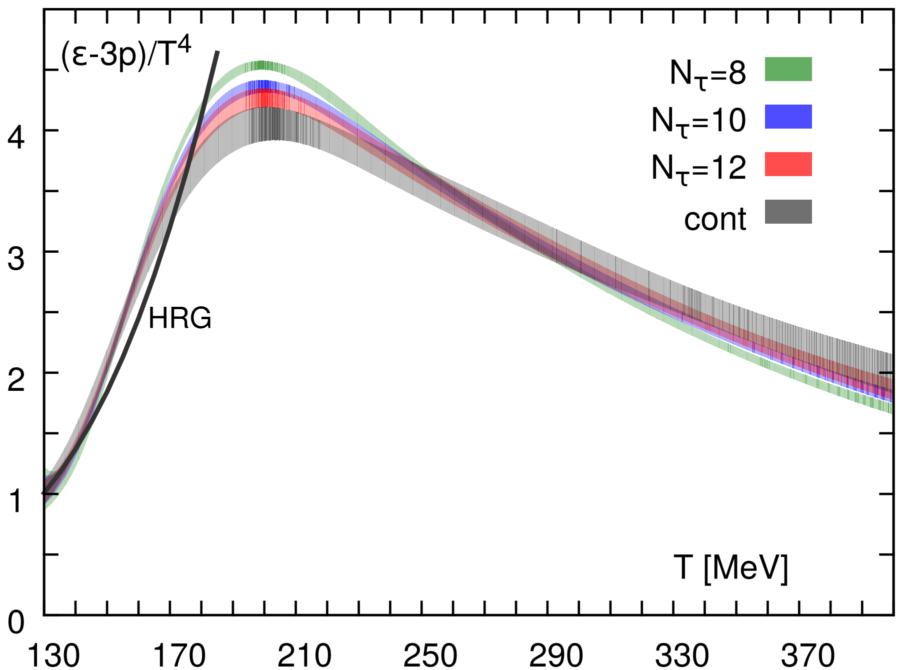
Small cutoff effects for HISQ except for N_τ=6

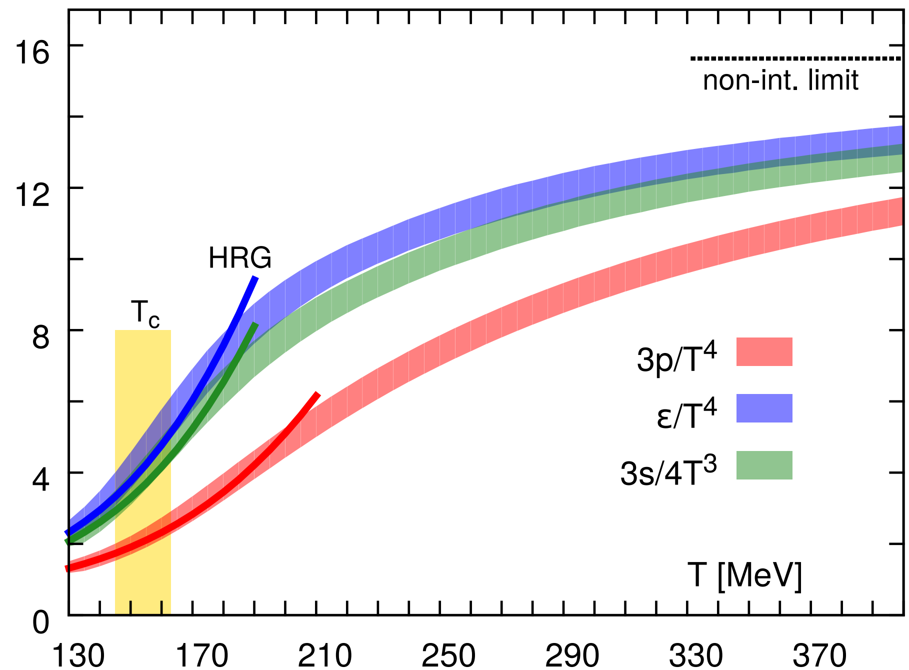
Equation of state in the continuum limit

Perform spline interpolation of all the $N_\tau > 6$ data with spline coefficients of the form $a + b/N_\tau^2$, stabilize the spline demanding that $\varepsilon\text{-}3p$ is given by HRG at $T=130$ MeV

Set the lower integration limit to $T_0=130$ MeV and take $p_0=p^{HRG}(T=130 \text{ MeV})$ → $p(T)$

Bazavov et al, PRD90 (2014) 094503





Hadron resonance gas (HRG):
Interacting gas of hadrons = non-interacting gas of hadrons and hadron resonances
(virial expansion, Prakash & Venugopalan)

HRG agrees with the lattice for $T < 145$ MeV

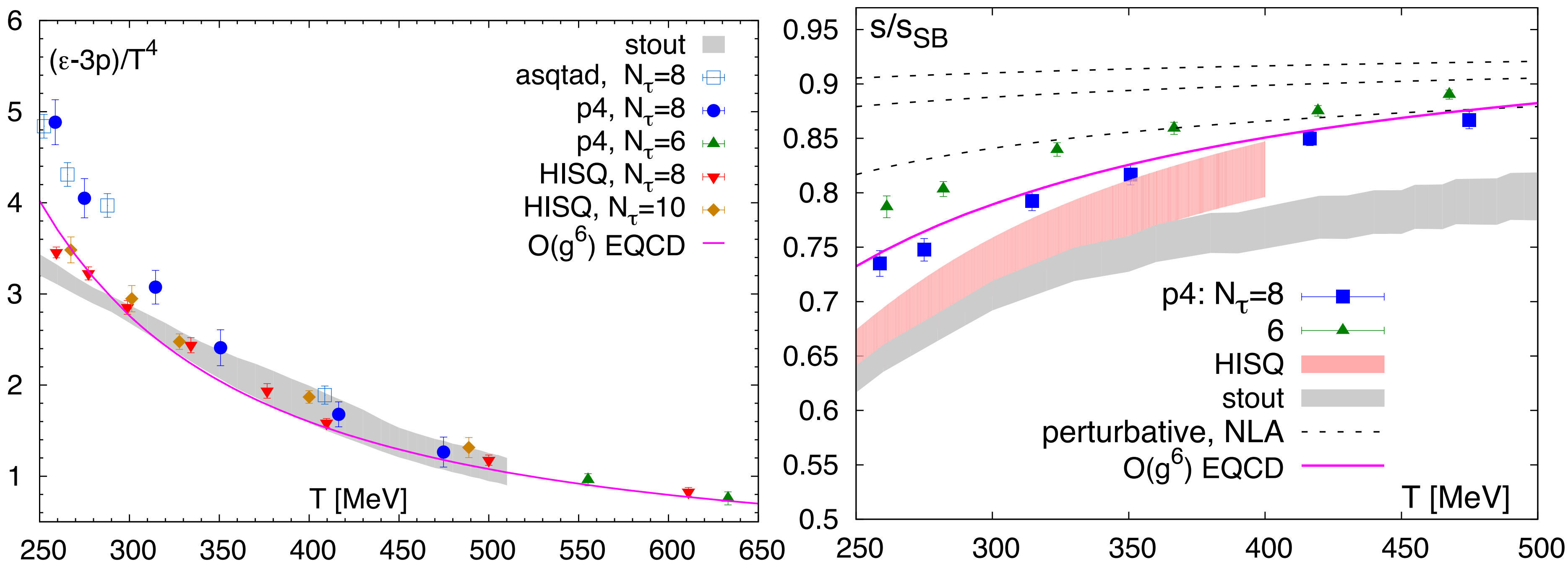
$$T_c = (154 \pm 9)\text{MeV}$$

$$\epsilon_c \simeq 300\text{MeV}/\text{fm}^3$$

$$\epsilon_{low} \simeq 180\text{MeV}/\text{fm}^3 \leftrightarrow \epsilon_{nucl} \simeq 150\text{MeV}/\text{fm}^3$$

$$\epsilon_{high} \simeq 500\text{MeV}/\text{fm}^3 \leftrightarrow \epsilon_{proton} \simeq 450\text{MeV}/\text{fm}^3$$

Equation of State on the lattice and in the weak coupling



The high temperature behavior of the trace anomaly is not inconsistent with weak coupling calculations (EQCD) for $T>300$ MeV

For the entropy density the continuum lattice results are below the weak coupling calculations For $T< 500$ MeV

At what temperature can one see good agreement between the lattice and the weak coupling results ?

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$ quark

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c)|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}(\langle X^2 \rangle - \langle X \rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$

↓

information about carriers of the conserved charges (hadrons or quarks)

↓

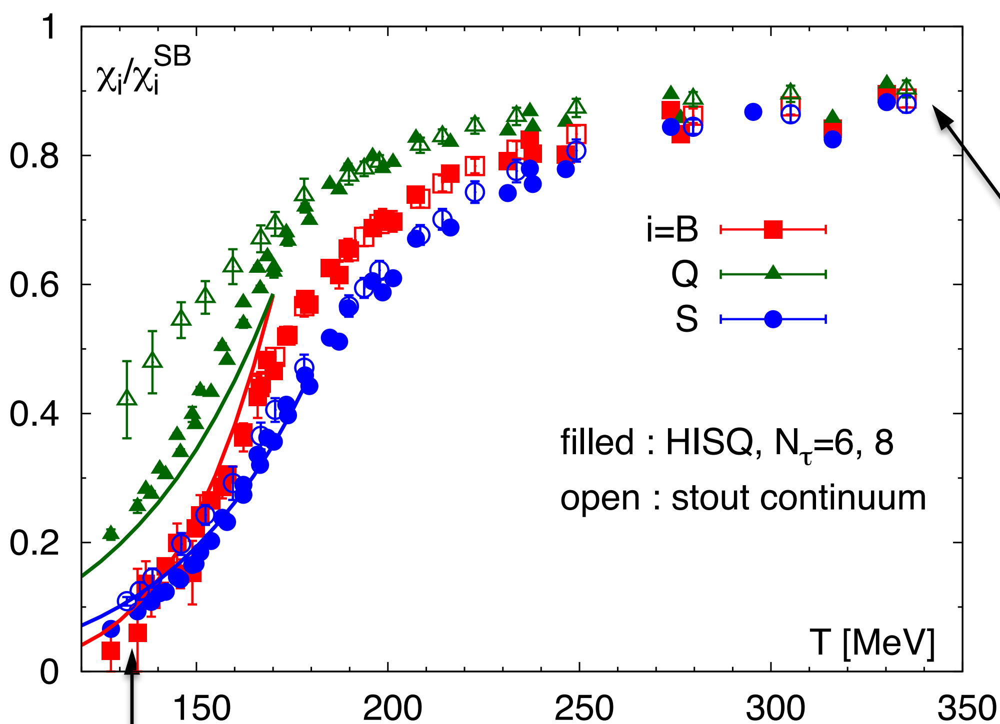
probes of deconfinement

Deconfinement : fluctuations of conserved charges

$$\chi_{\rm B}^{\rm SB} = \frac{1}{VT^3}(\langle B^2\rangle - \langle B\rangle^2)$$ baryon number

$$\chi_{\rm Q}^{\rm SB} = \frac{1}{VT^3}(\langle Q^2\rangle - \langle Q\rangle^2)$$ electric charge

$$\chi_{\rm S}^{\rm SB} = \frac{1}{VT^3}(\langle S^2\rangle - \langle S\rangle^2)$$ strangeness



conserved charges are carried by massive hadrons

Ideal gas of massless quarks :

$$\chi_B^{\rm SB} = \frac{1}{3} \qquad \chi_Q^{\rm SB} = \frac{2}{3}$$

$$\chi_B^{\rm SB} = 1$$

conserved charges carried by light quarks

HotQCD: PRD86 (2012) 034509

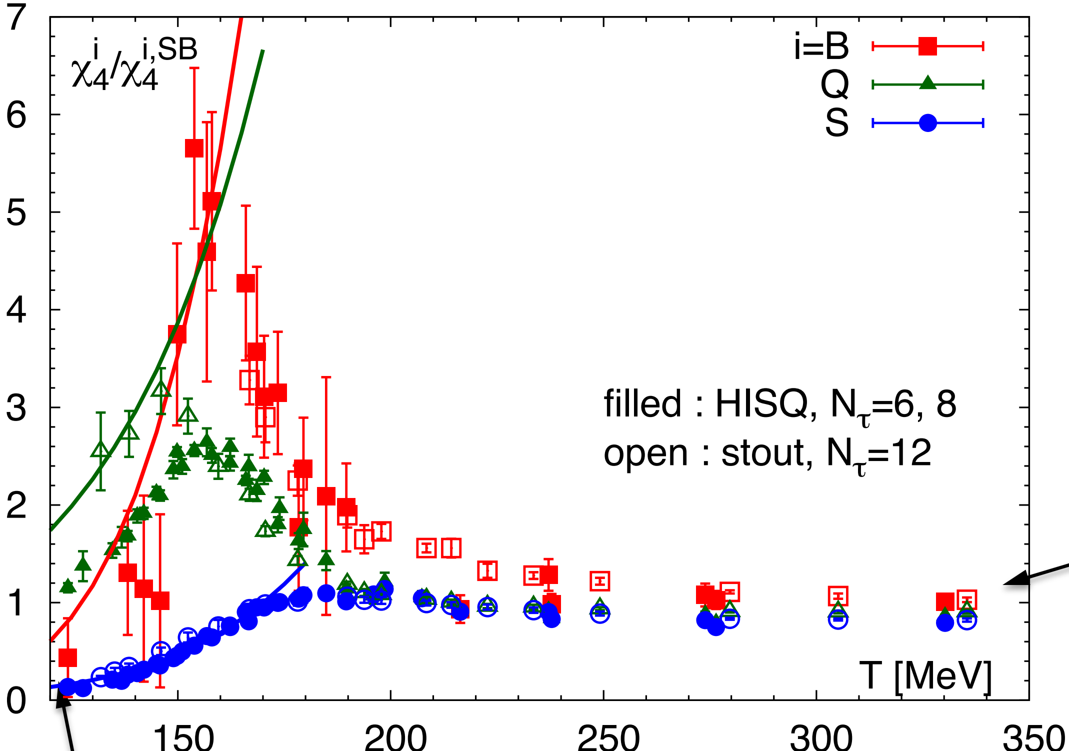
BW: JHEP 1201 (2012) 138,

Deconfinement : fluctuations of conserved charges

$$\chi_4^B = \frac{1}{VT^3}(\langle B^4\rangle - 3\langle B^2\rangle^2)$$ baryon number

$$\chi_4^Q = \frac{1}{VT^3}(\langle Q^4\rangle - 3\langle Q^2\rangle^2)$$ electric charge

$$\chi_4^S = \frac{1}{VT^3}(\langle S^4\rangle - 3\langle S^2\rangle^2)$$ strangeness



Ideal gas of massless quarks :

$$\chi_4^B{}_{SB} = \frac{2}{9\pi^2} \qquad \chi_4^Q{}_{SB} = \frac{4}{3\pi^2}$$

$$\chi_4^S{}_{SB} = \frac{6}{\pi^2}$$

conserved charges carried by light quarks

conserved charges are carried by massive hadrons

BNL-Bielefeld : talk by C. Schmidt
BW: talk by Borsanyi
@ Confinement X conference

Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

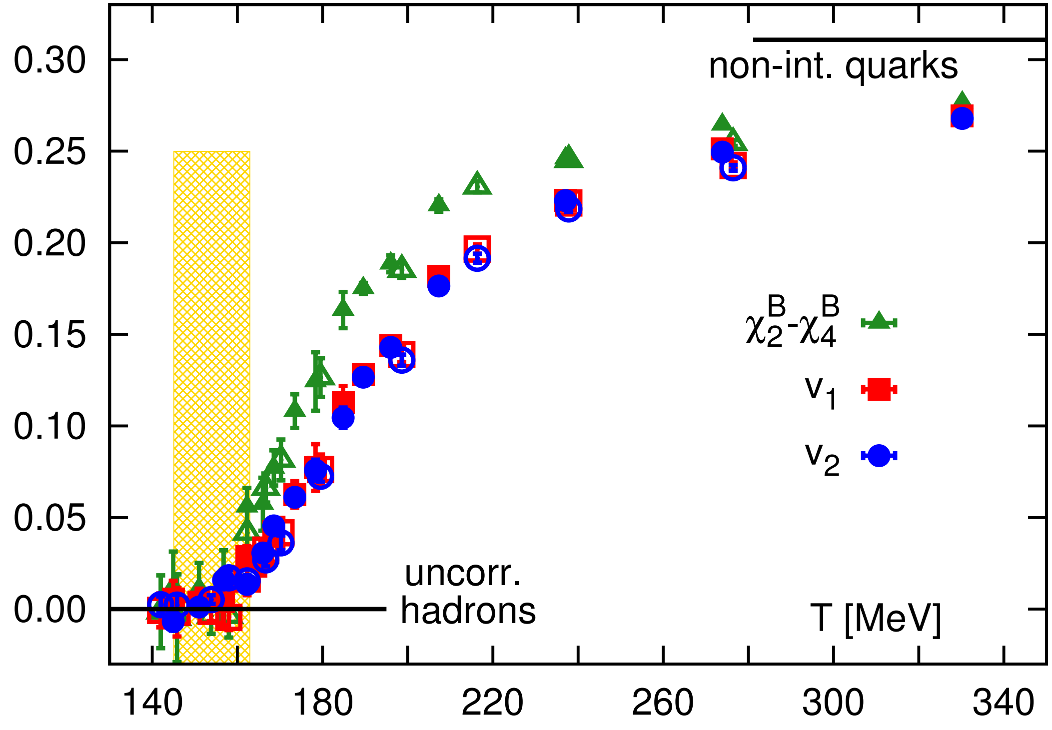
$$P_S = \frac{p(T) - p_{S=0}(T)}{T^4} = M(T)\cosh\left(\frac{\mu_S}{T}\right) +$$
$$B_{S=1}(T)\cosh\left(\frac{\mu_B - \mu_S}{T}\right) + B_{S=2}(T)\cosh\left(\frac{\mu_B - 2\mu_S}{T}\right) + B_{S=3}(T)\cosh\left(\frac{\mu_B - 3\mu_S}{T}\right)$$

$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$
$$v_2 = \frac{1}{3}\left(\chi_4^S - \chi_2^S\right) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

should vanish !

- v_1 and v_2 do vanish within errors at low T
- v_1 and v_2 rapidly increase above the transition region, eventually reaching non-interacting quark gas values

Bazavov et al, PRL 111 (2013) 082301

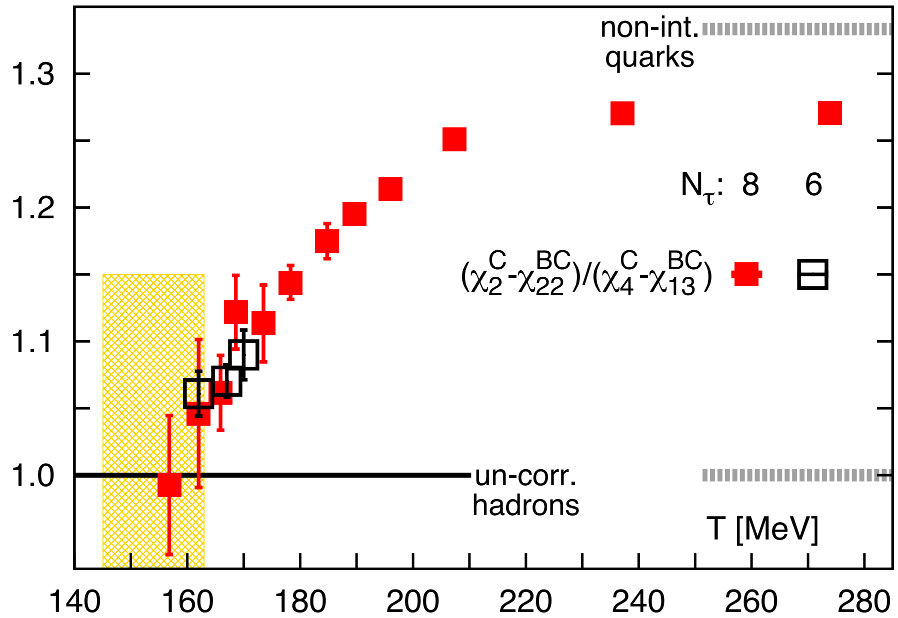


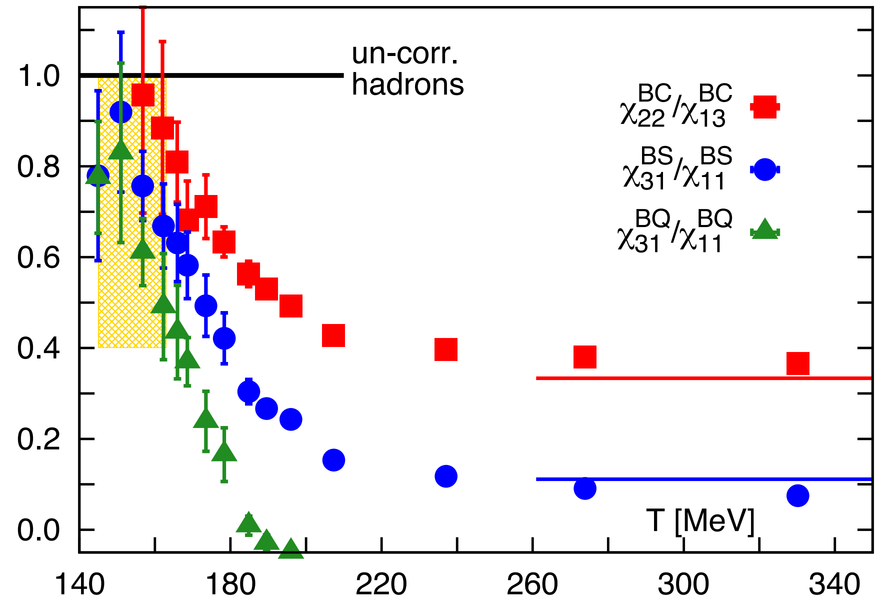
What about charm hadrons ?

We could introduce chemical potential for charm quarks and study the derivatives of the pressure with respect to the charm chemical potential
Bazavov et al, PLB737 (2014) 210

$m_c \gg T$ ⇨ only |*C*|=*1* sector contributes

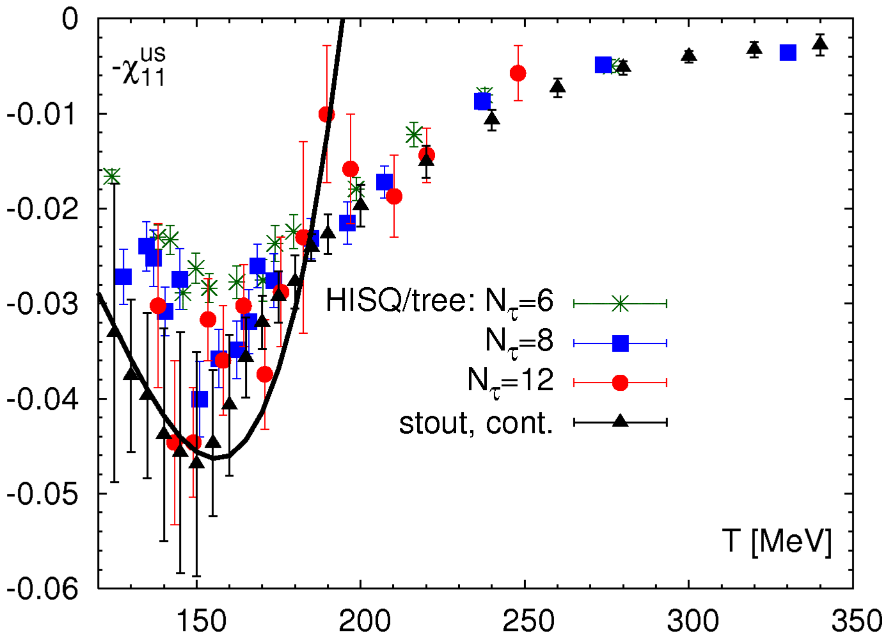
In the hadronic phase all *BC*-correlations are the same !



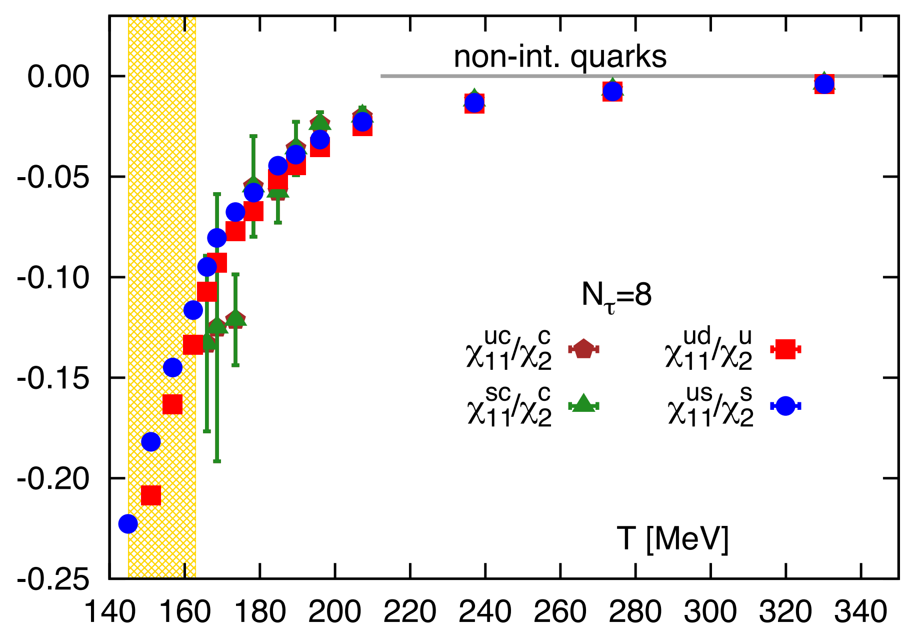


Hadronic description breaks down just above T_c
⇒ open charm deconfines above T_c

Quark number correlations



P.P. J.Phys. G39 (2012) 093002



Courtesy of S. Mukherjee

- Correlations between strange and light quarks at low *T* are due to the fact that strange hadrons contain both strange and light quarks but very small at high *T (>250* MeV*)* weakly interacting quark gas $\chi_{11}^{us} \sim \alpha_s^6$?

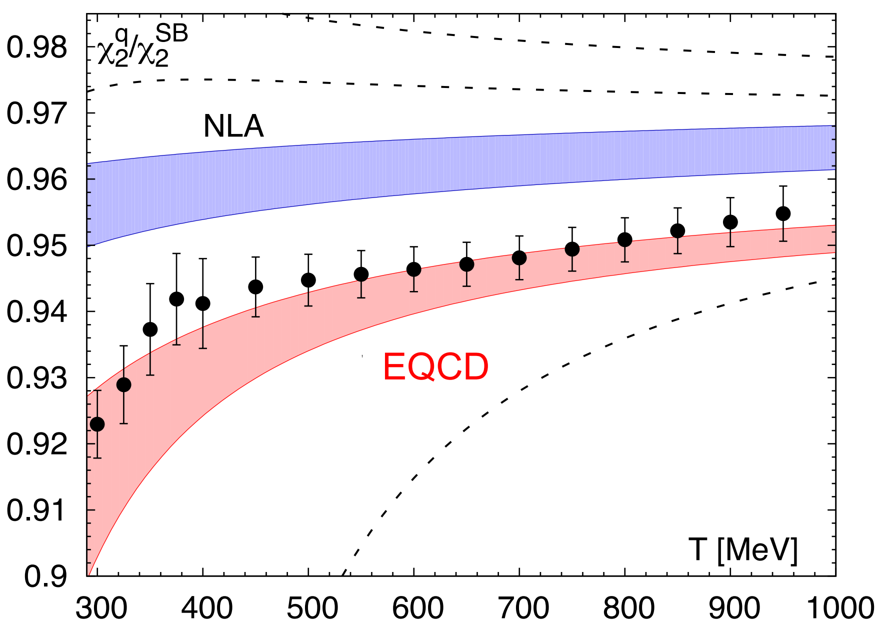
- Quark number correlations are flavor independent, correlations are unlikely due to bound states

- The transition region where degrees of freedom change from hadronic to quark-like is broad ~ (100-150) MeV

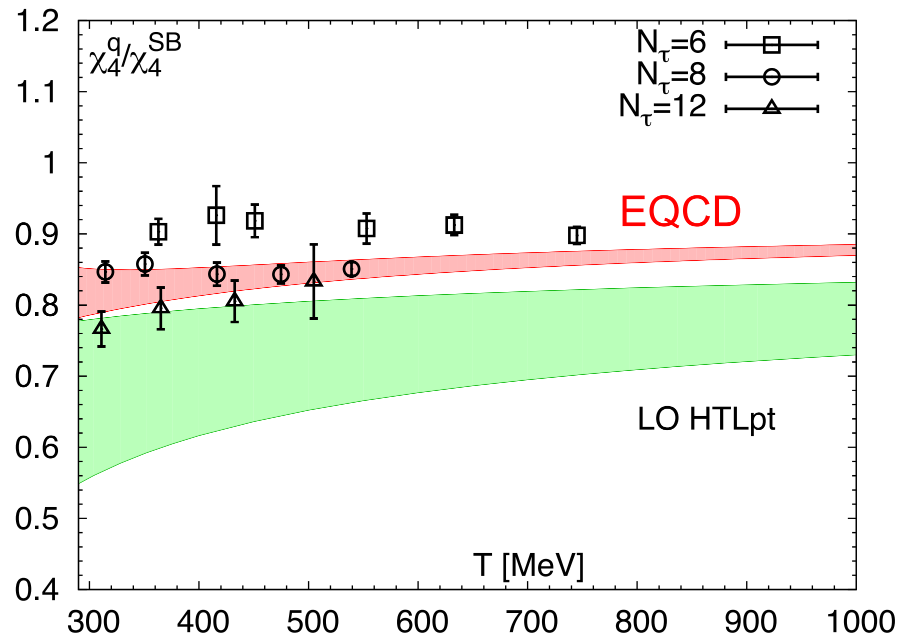
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

2nd order quark number fluctuations



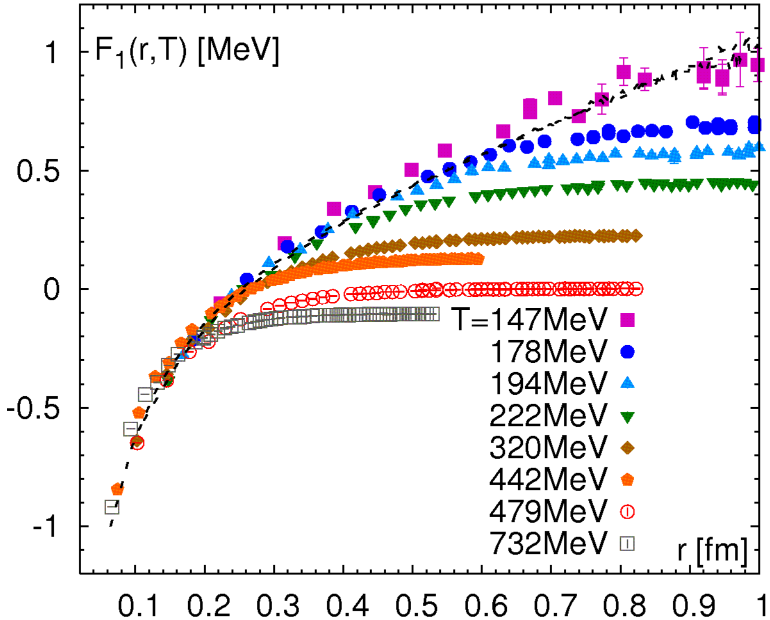
4th order quark number fluctuations

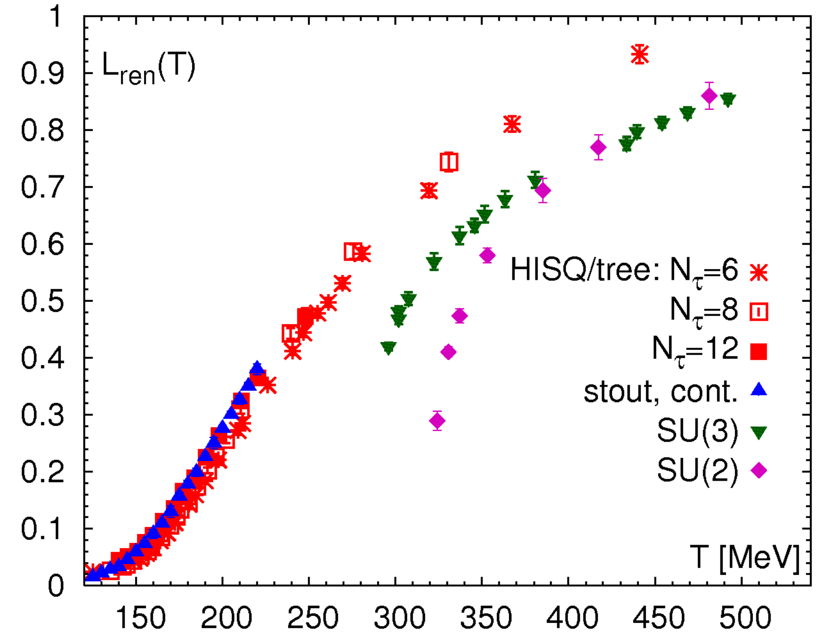


Bazavov et al, PRD88 (2013) 094021

- Lattice results converge as the continuum limit is approached
- Good agreement between lattice and the weak coupling approach for 2nd order quark number fluctuations
- For 4th order the weak coupling results are in reasonable agreement with lattice

Deconfinement and color screening





free energy of static quark anti-quark pair shows Debye screening at high temperatures

Pure glue ≠ QCD !

$$L = \mathrm{tr}\mathcal{P} e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})} \Rightarrow L_{ren} = \exp(-F_Q(T)/T)$$

$$F_1(r) = -\frac{4}{3}\frac{\alpha_s}{r}\exp(-m_D r) + 2F_Q(T),\ m_D \sim T$$

$$F_Q(T) \simeq \Lambda_{QCD} - C_F \alpha_s m_D$$

infinite in the pure glue theory or large in the “hadronic” phase ~600MeV

Decreases in the deconfined phase

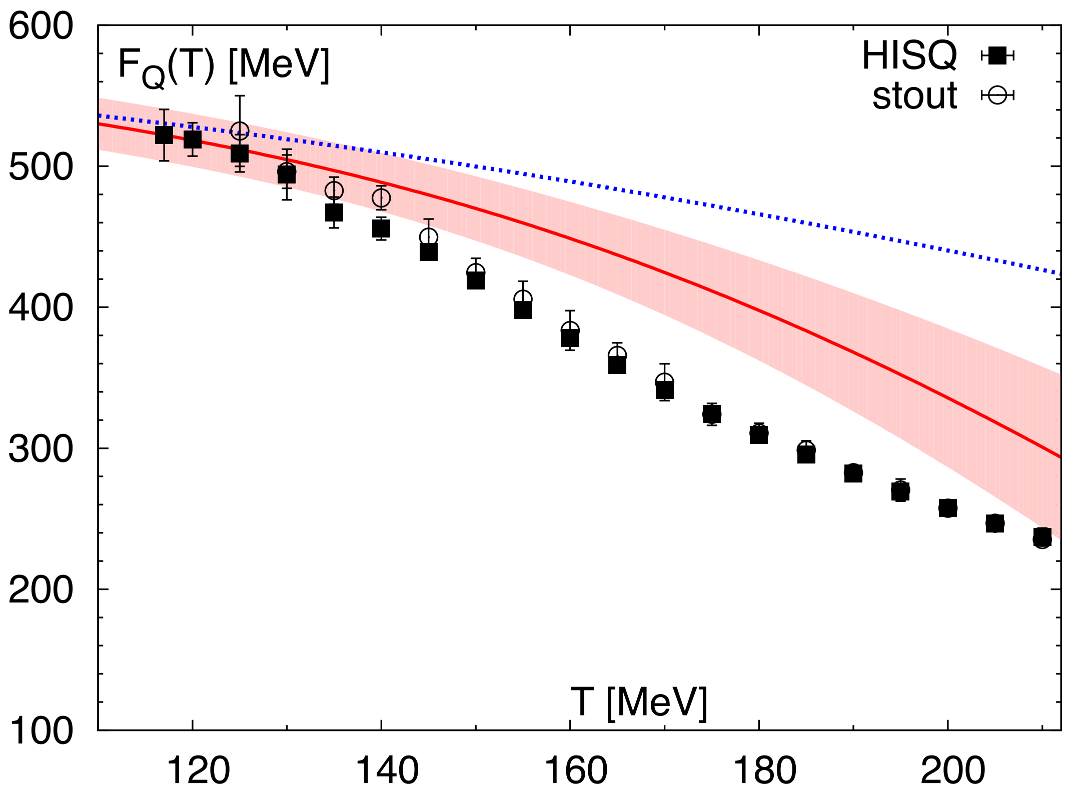
melting of bound states of heavy quarks => quarkonium suppression at RHIC: $r_{bound} > 1/m_D$

Polyakov and gas of static-light hadrons

$$Z_{Q\bar{Q}}(T)/Z(T) = \sum_n \exp(-E_n^{Q\bar{Q}}(r \to \infty)/T)$$

Energies of static-light mesons: $E_n^{Q\bar{Q}}(r \to \infty) = M_n - m_Q$

Free energy of an isolated static quark: $F_Q(T) = -\frac{1}{2}(T \ln Z_{Q\bar{Q}}(T) - T \ln Z(T))$



Megias, Arriola, Salcedo, PRL 109 (12) 151601

Bazavov, PP, PRD 87 (2013) 094505

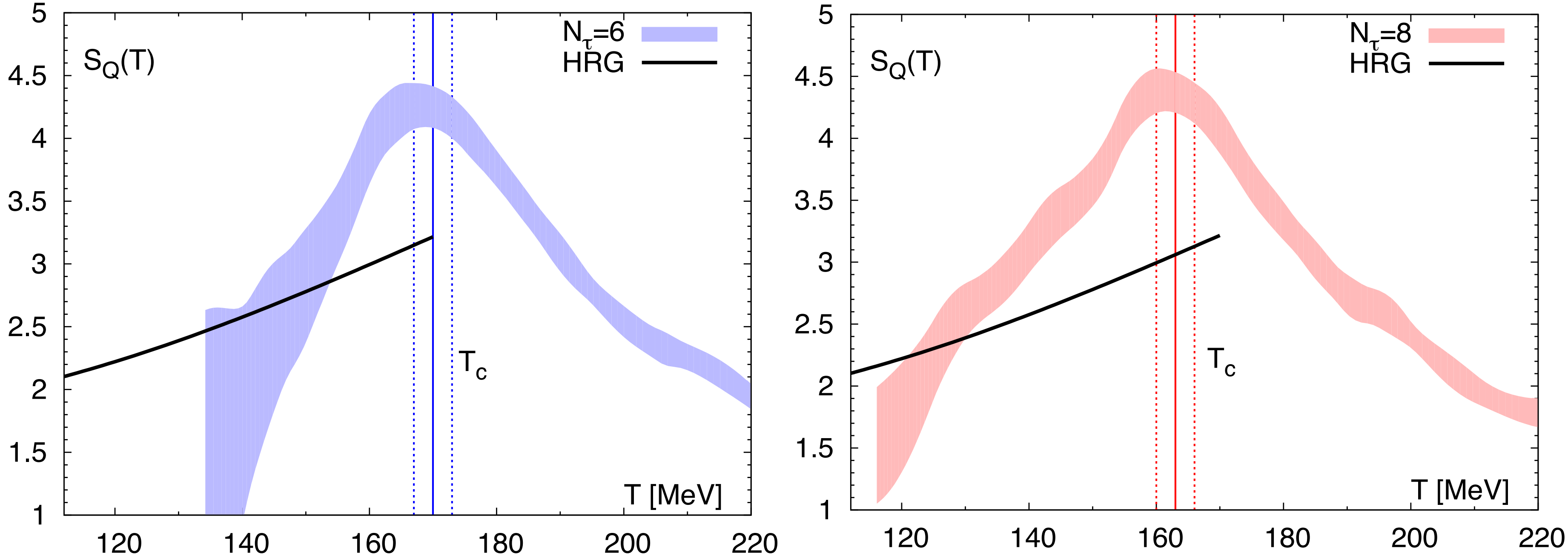
Ground state and first excited states are from lattice QCD
Michael, Shindler, Wagner, arXiv1004.4235
Wagner, Wiese, JHEP 1107 016,2011

Higher excited state energies are estimated from potential model

Gas of static-light mesons only works for *T < 145* MeV

The entropy of static quark

$$S_Q = -\frac{\partial F_Q}{\partial T}$$



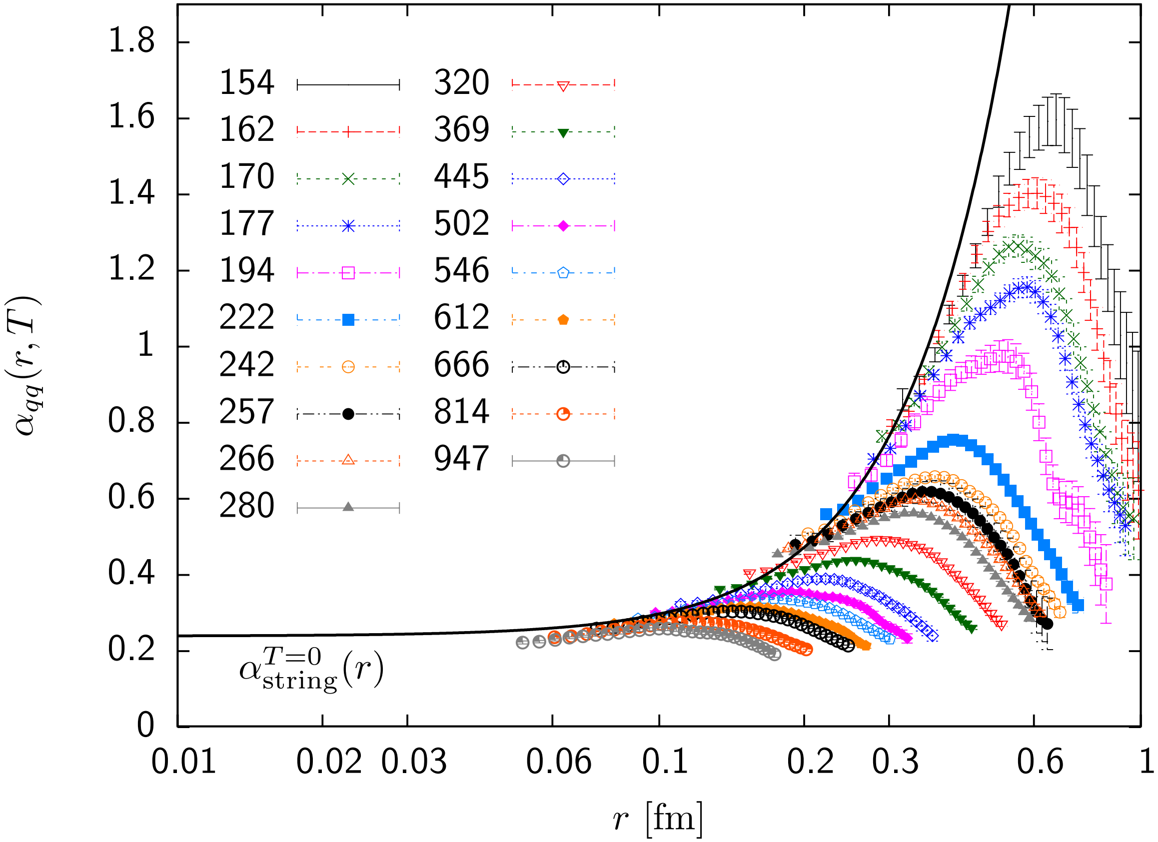
At low T the entropy S_Q increases reflecting the increase of states the heavy quark can be coupled to

At high temperature the static quark only "sees" the medium within a Debye radius, as T increases the Debye radius decreases and S_Q also decreases

The onset of screening corresponds to peak is S_Q and its position coincides with T_c

Color screening and strong coupling constant in QGP

Define a *T*-dependent effective running coupling constant



$$\alpha_{eff}(r,T) = \frac{3}{4} r^2 \frac{dF_1(r,T)}{dr}$$

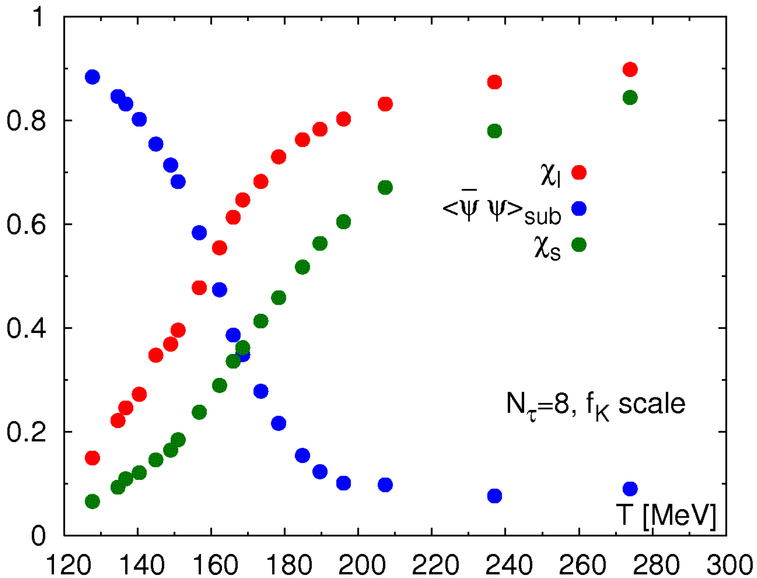
In collaboration with J. Webber

- For $r<0.5/T$ the coupling constant runs with the distance as at $T=0$
- It reaches a maximum at $r=r_{max} \approx 0.5/T$
- The maximal value of the α_{eff} is about *1.6* at T_c and *0.5* at $T=300$ *MeV*

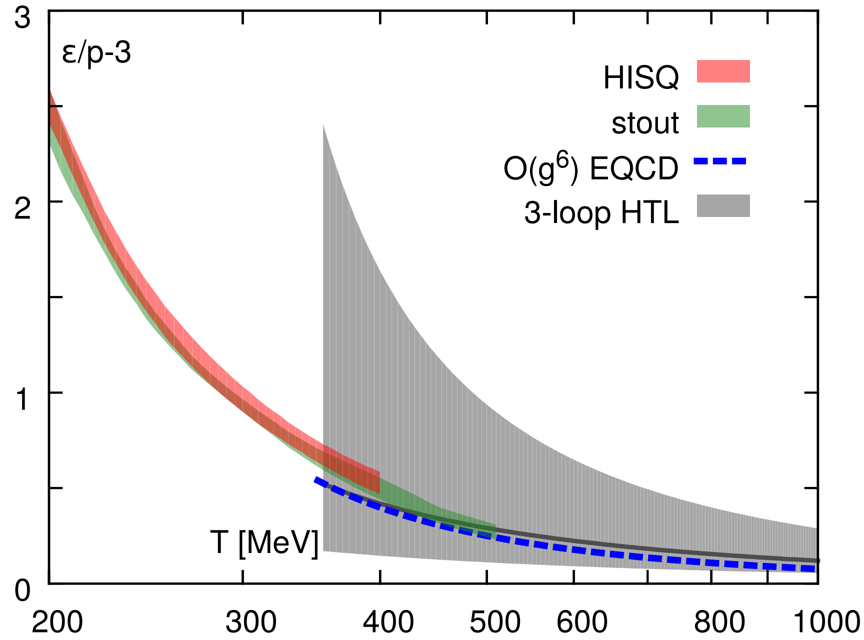
Summary

- The value chiral transition temperature is now well established in the continuum
  $T_c=154(9)$ MeV
- $U_A(1)$ restoration happens above T_c and does not effect the chiral transition
- Equation of state is known in the continuum limit up to $T=400$ MeV
- Deconfinement transition can be studied in terms of fluctuations and correlations of conserved charges, it manifest itself as a abrupt breakdown of hadronic description that occurs around the chiral transition temperature
- Deconfinement in terms of color screening occurs at the chiral transition
- For $T > (300\text{-}400)$ MeV weak coupling expansion works well for certain quantities (e.g. quark number susceptibilities)

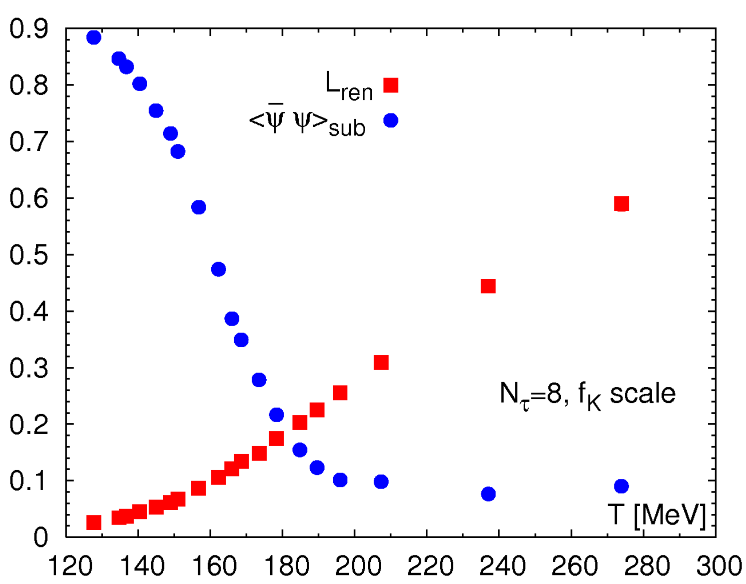
Back-up I:



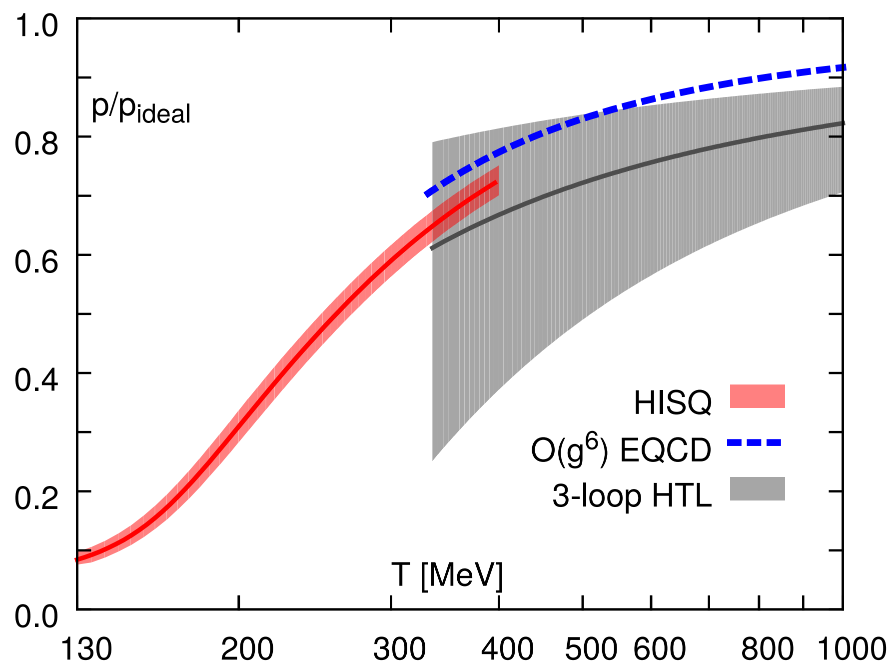
χ_l
$<\bar{\psi}\,\psi>_{sub}$
χ_s
N_τ=8, f_K scale
T [MeV]



ε/p-3
HISQ
stout
O(g^6) EQCD
3-loop HTL
T [MeV]



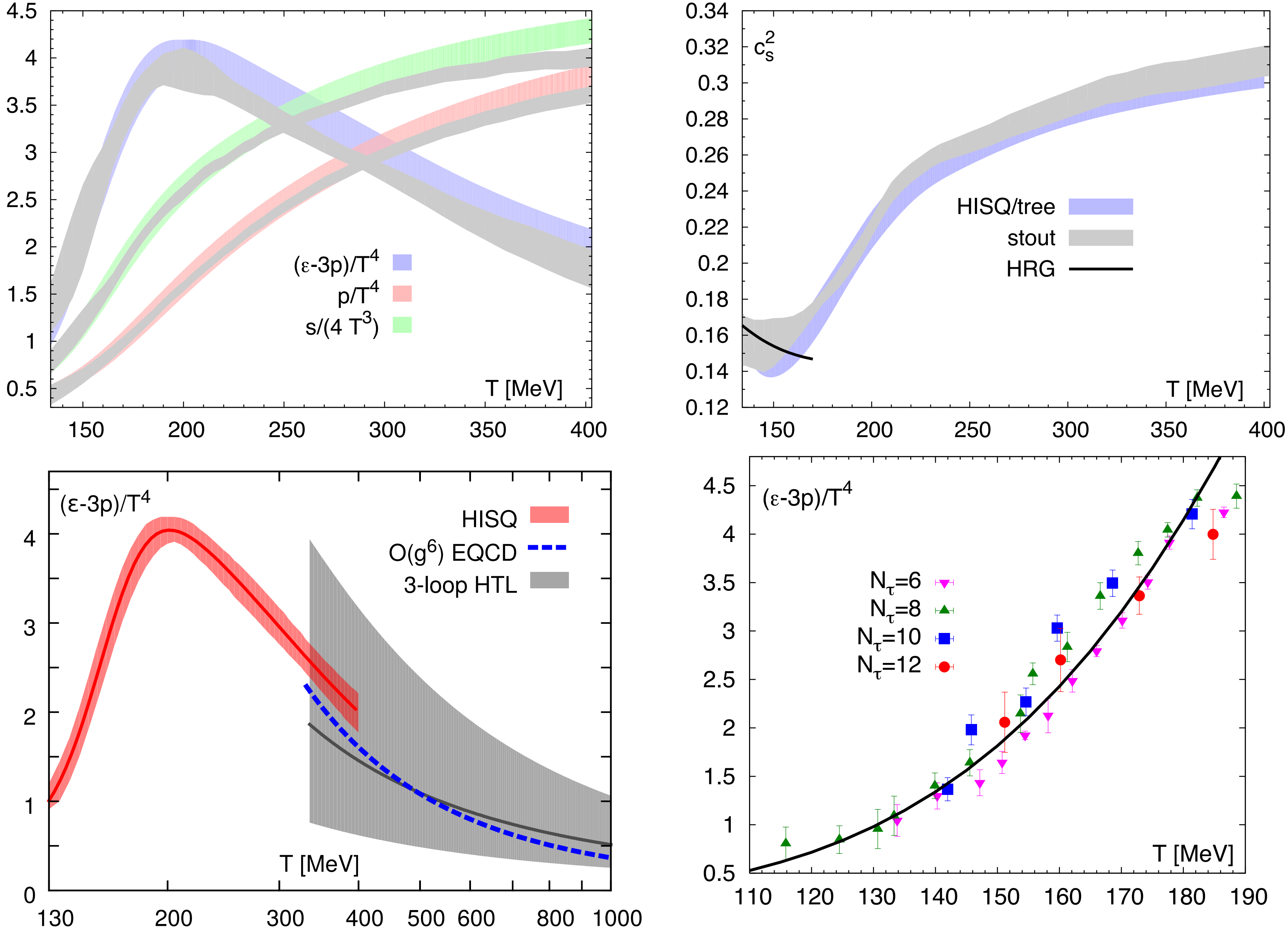
L_{ren}
$<\bar{\psi}\,\psi>_{sub}$
N_τ=8, f_K scale
T [MeV]



p/p_{ideal}
HISQ
O(g^6) EQCD
3-loop HTL
T [MeV]

HISQ action vs. stout action

Continuum results obtained with stout and HISQ action agree reasonably well given their errors



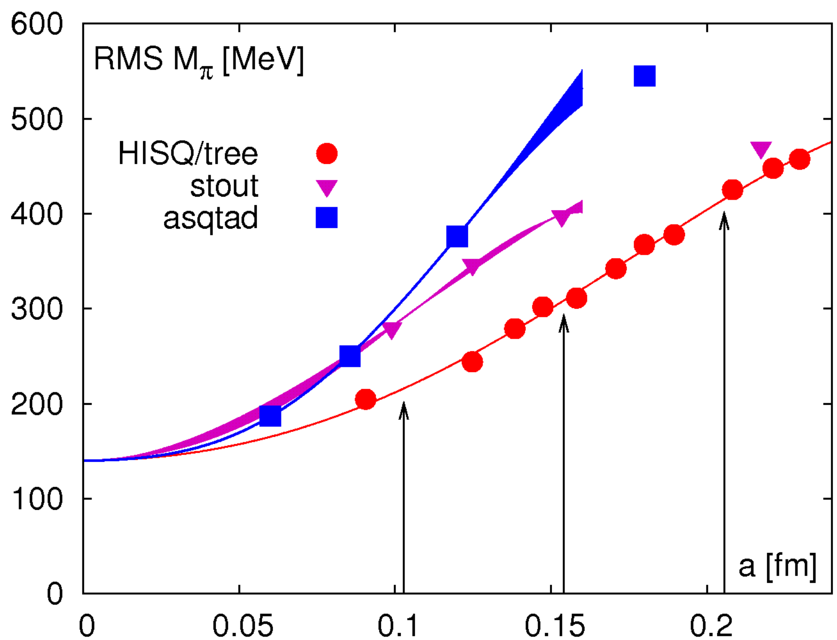
Improved staggered calculations at finite temperature

low T region
$T<200$ MeV
$\mathcal{O}(\alpha_s^n(a\Lambda_{QCD})^2)$ errors
$a>0.125fm$
hadronic degrees of freedom

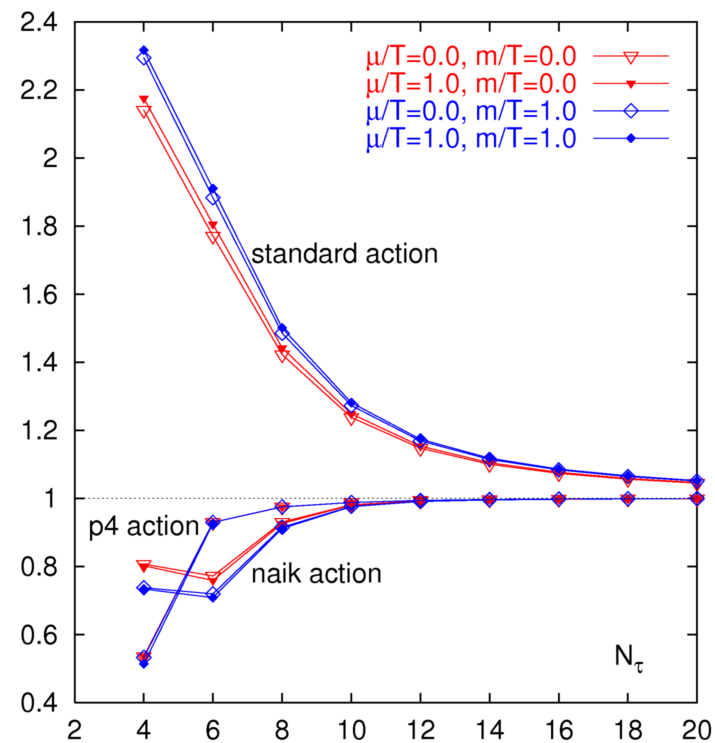
improvement of the flavor symmetry is important $\rightarrow$ fat links

cutoff effects are different in :
$a = 1/(TN_\tau)$
$N_\tau = 8$
for #flavors < 4
rooting trick
$detD \rightarrow (detD)^{\frac{n_f}{4}}$

high-T region
$T>200$MeV
$\mathcal{O}((aT)^2)$ errors
$a<0.125fm$
quark degrees of freedom
quark dispersion relation



p4, asqtad, HISQ, stout



The Highly Improved Staggered Quark (HISQ) Action

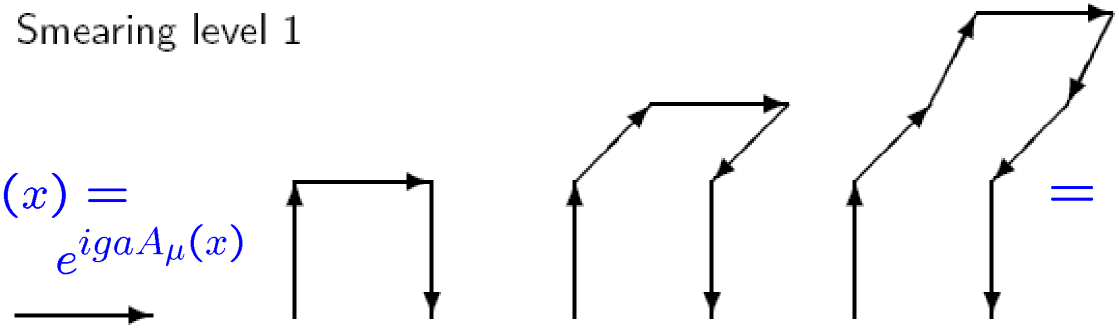
HISQ action

two levels of gauge field smearing with re-unitarization

Follana et al, PRD75 (07) 054502

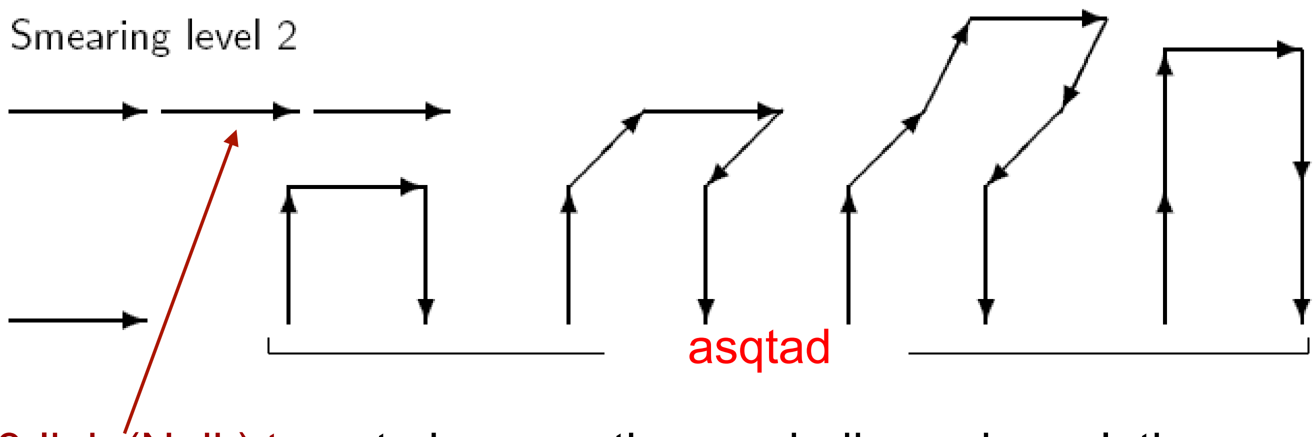
Smearing level 1

$$U_\mu(x) = e^{igaA_\mu(x)}$$



$$= U_\mu^{fat7} \rightarrow \tilde{U}_\mu = \frac{U_\mu^{fat7}}{\sqrt{U_\mu^{fat7} U_\mu^{fat7\dagger}}}$$

Smearing level 2



projection onto U(3) improves flavor symmetry
Hasenfratz, arXiv:hep-lat/0211007

3-link (Naik) term to improve the quark dispersion relation + asqtad smearing