Remnants of O(4) criticality on the freeze-out line

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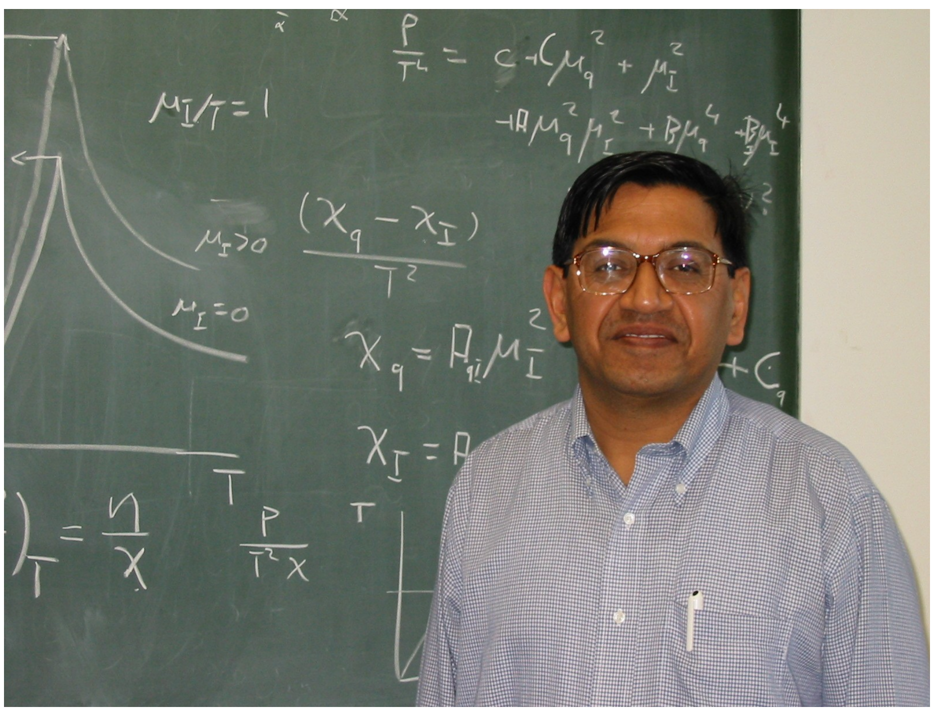


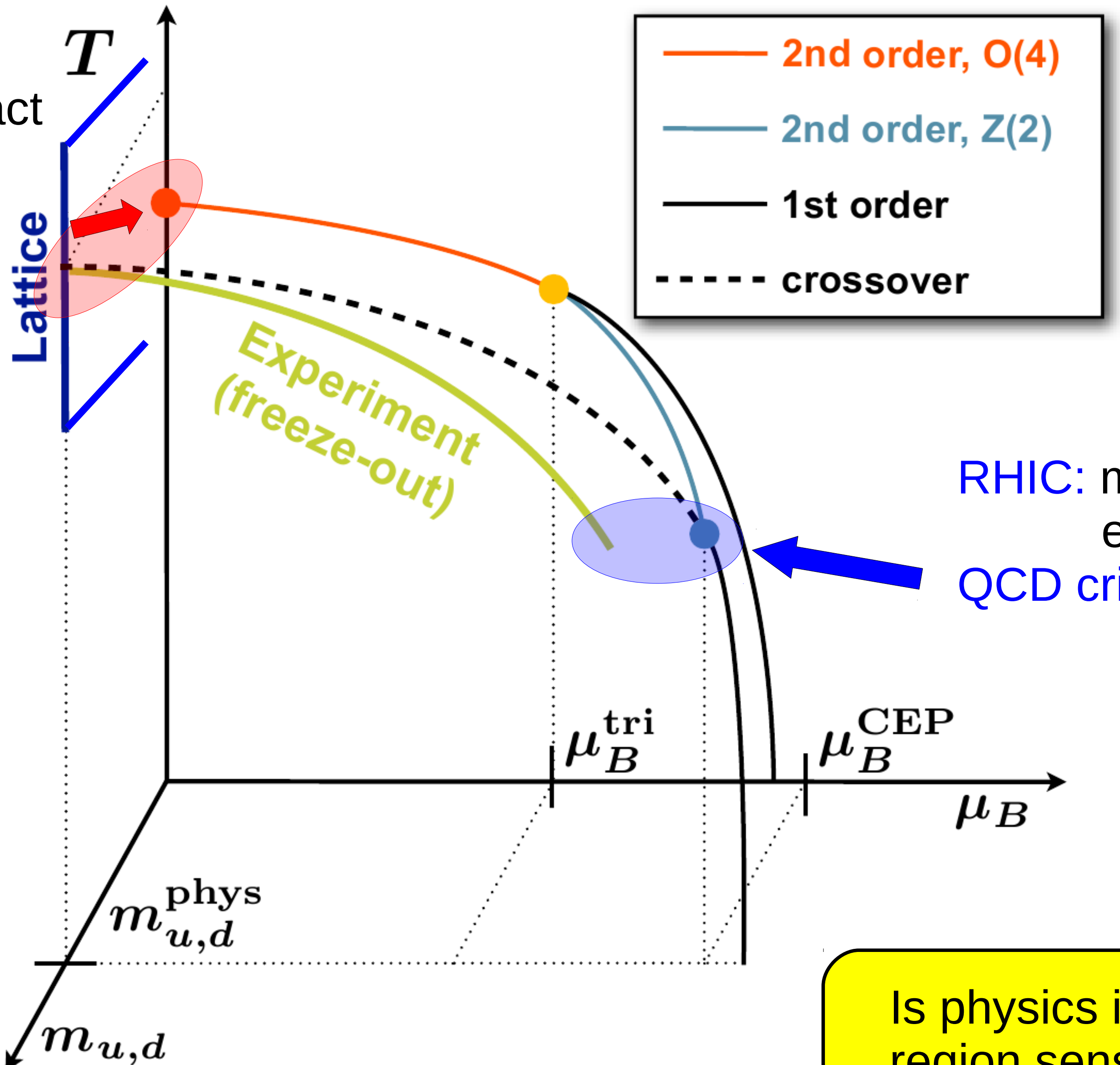
October 2004

OUTLINE

- conserved charge fluctuations in QCD and HIC
- at $\mu_B = 0$
- at $\mu_B > 0$

Chiral critical point and QCD critical endpoint

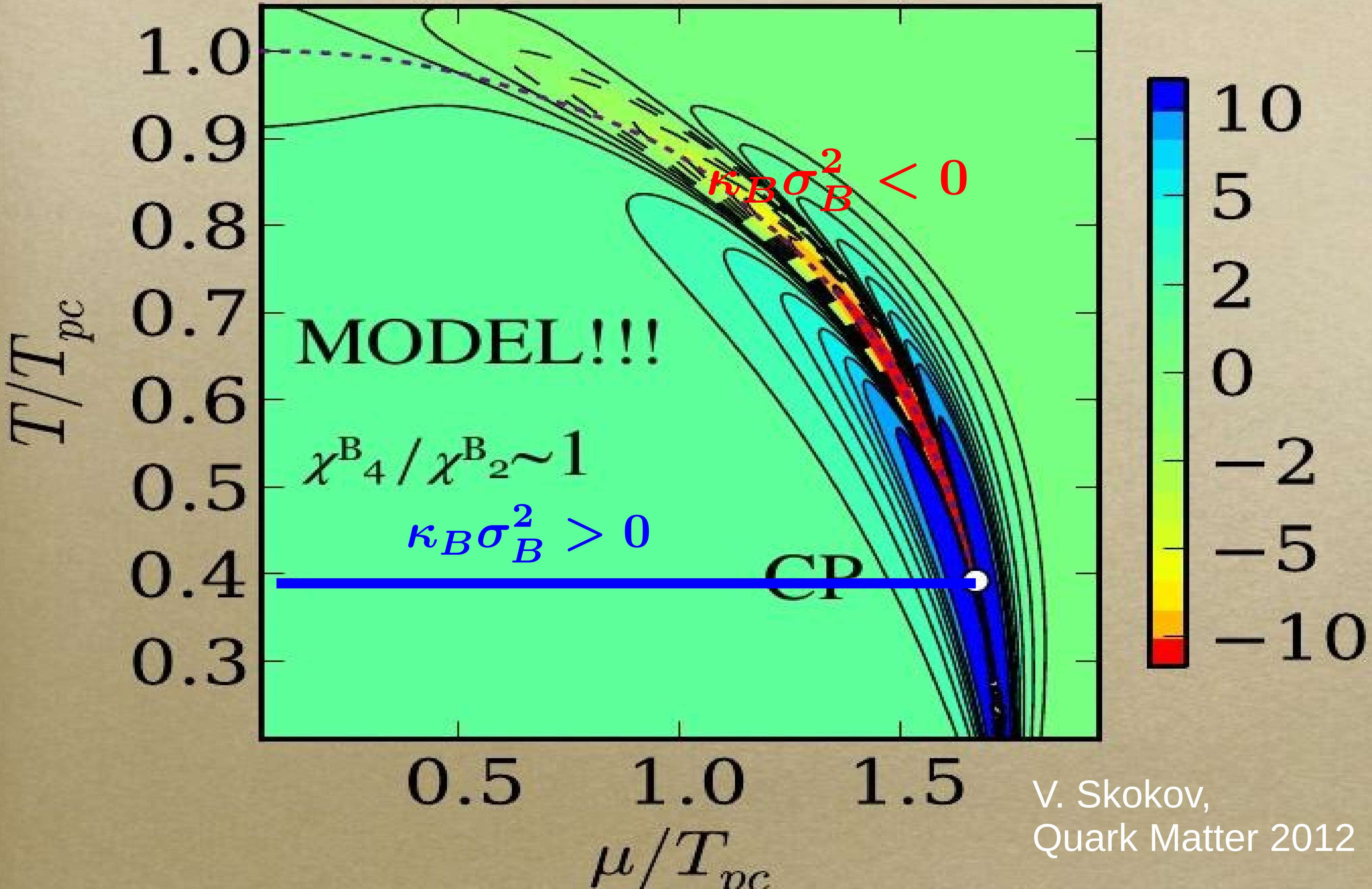
LHC: may establish contact with the QCD chiral PHASE transition



RHIC: may establish evidence for the QCD critical end point

Is physics in the freeze-out region sensitive to critical behavior?

Chiral model and negative $\chi^{B}_{4}/\chi^{B}_{2}$:

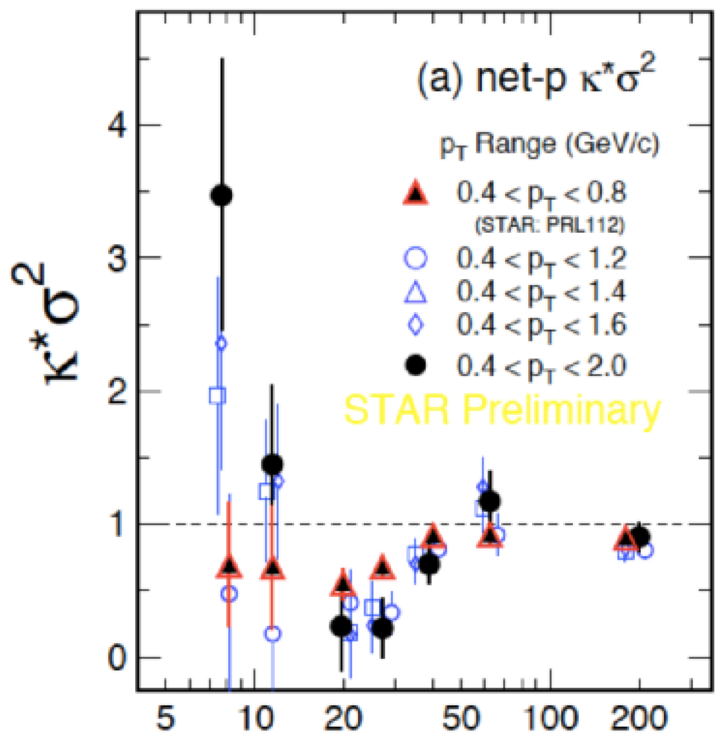


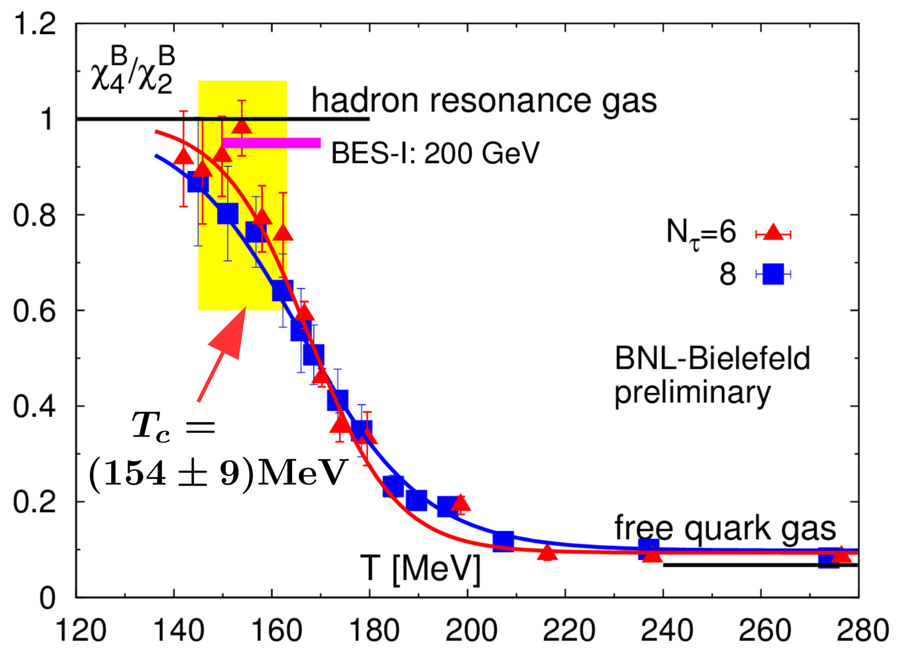
V. Skokov,
Quark Matter 2012

Cumulant ratios of conserved net-charge fluctuations

kurtosis*variance: $$(\kappa\sigma^2)_X = \frac{\chi_4^X}{\chi_2^X} \quad , \quad X = B,\ Q,\ S$$

$$\chi_n^X = \frac{\partial^n P/T^4}{\partial(\mu_X/T)^n}$$





Xiaofeng Luo (for the STAR Collaboration), CPOD 2014

Chiral Transition at small μ_B/T

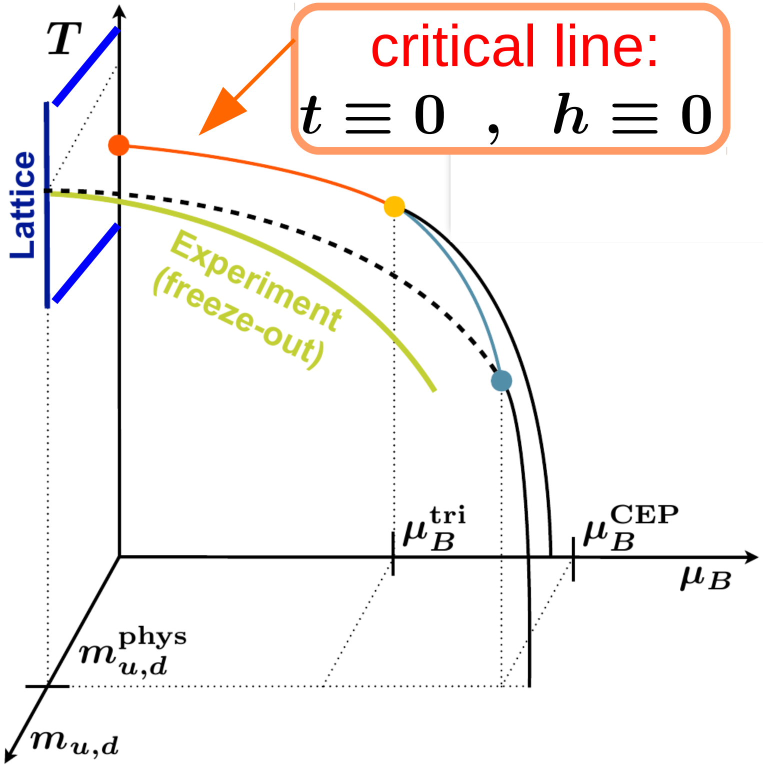
- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal O(4) scaling function

singular | regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -f_s(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

critical line: $t \equiv 0 \;,\; h \equiv 0$

$$t \sim \frac{T-T_c}{T_c} + \kappa_q \left(\frac{\mu_q}{T}\right)^2 \;,\; h \sim \frac{m_q}{T_c}$$



suppressing dependence on strange quark chemical potential

higher order derivatives==cumulants become sensitive to singular structure

$$\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k} p/T^4}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}$$

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	O(4)
α	-0.213
β	0.380
δ	4.824

$(2-\alpha)/\beta\delta = 1 + 1/\delta$

critical line:
$t \equiv 0 \quad , \quad h \equiv 0$

$$t \sim \frac{T-T_c}{T_c} + \kappa_q \left(\frac{\mu_q}{T}\right)^2$$

- details are controlled by four non-universal, QCD specific parameters (for $\mu_s = \mu_I = 0,\ \hat{\mu}_q = \mu_q/T \lesssim\ 1$)

$$z = z_0 \left(\frac{T-T_c}{T_c} + \kappa_q\,\hat{\mu}_q^2\right)\left(\frac{m_s}{m_q}\right)^{1/\beta\delta}$$

$$\frac{p}{T^4} = -h_0^{-1/\delta}\left(\frac{m_q}{m_s}\right)^{1+1/\delta} f_f(z) - f_r(V,T,\vec{\mu})$$

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- details are controlled by four non-universal, QCD specific parameters (for μ_s = ... = ...)

width

location

curvature

$$z = z_0 \left(\frac{T - T_c}{T_c} + \kappa_q \, \hat{\mu}_q^2\right) \left(\frac{m_s}{m_q}\right)^{1/\beta\delta}$$

$$\frac{p}{T^4} = -h_0^{-1/\delta} \left(\frac{m_q}{m_s}\right)^{1+1/\delta} f_f(z) - f_r(V,T,\vec{\mu})$$

strength

Critical and pseudo-critical temperature

How close is the pseudo-critical (crossover) temperature to the true chiral PHASE transition temperature?

In the scaling regime this is controlled by a single non-universal parameter

$$\frac{P}{T^4} = -h_0^{-1/\delta}\, H^{1+1/\delta} f_f(z) - f_r(V,T,\vec{\mu}) \quad , \quad H = m_q/m_s$$

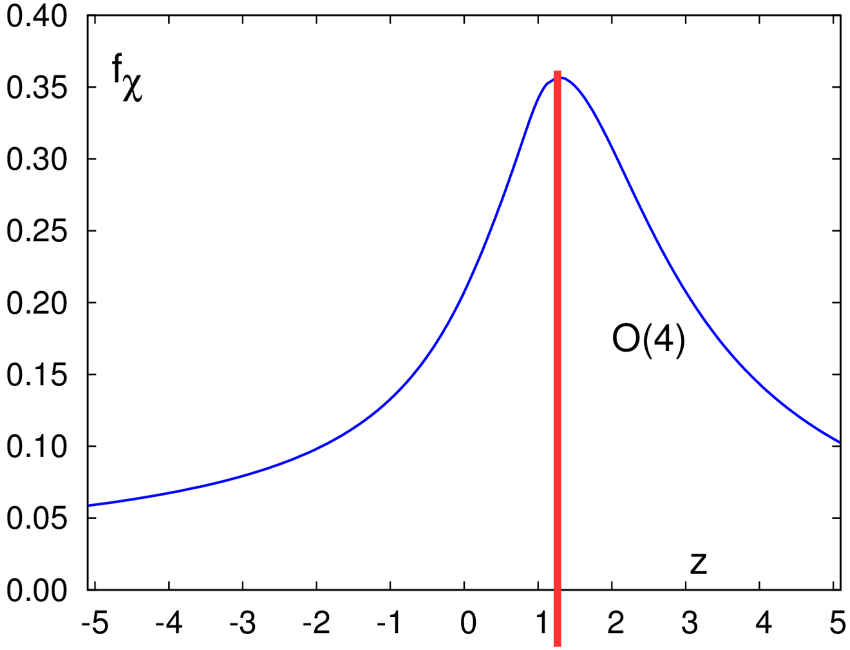
H derivatives
chiral condensate,
chiral susceptibility

$$\frac{\chi_{m,q}}{T^2} = \frac{\partial^2 P/T^4}{\partial (m_q/T)^2}$$

$$= \left(\frac{T}{m_s}\right)^2 \frac{\partial^2 P/T^4}{\partial H^2}$$

$$\sim A_q H^{1/\delta - 1} f_\chi(z)$$

with $A_q = \left(\frac{T_c}{m_s}\right)^2 h_0^{-1/\delta}$



$z_p = 1.33(5)$

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chiral susceptibility

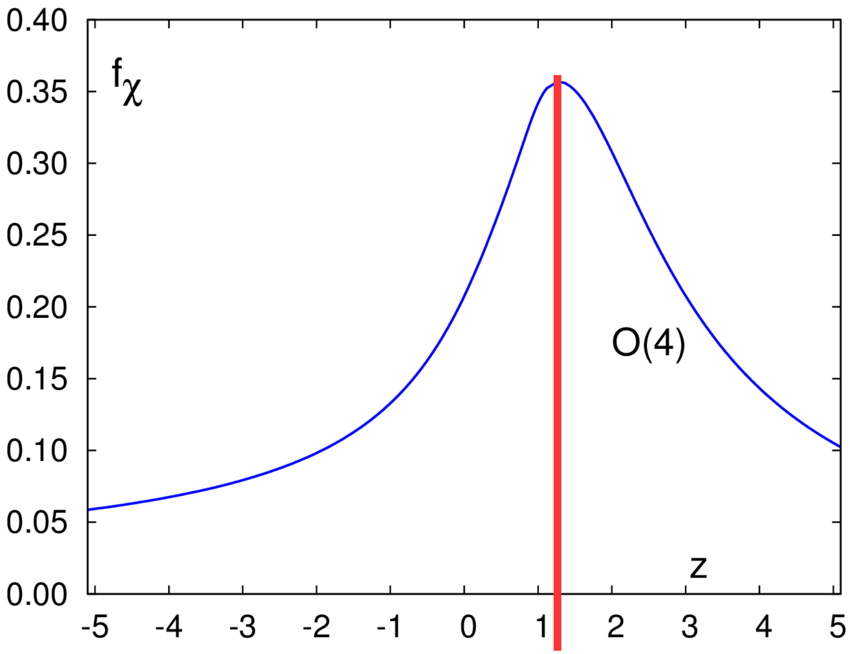
$$\frac{\chi_{m,q}}{T^2} = \frac{\partial^2 P/T^4}{\partial(m_q/T)^2} \sim A_q H^{1/\delta-1} f_\chi(z)$$

$$z_p = z_0 H^{-1/\beta\delta} \frac{T_{pc} - T_c}{T_c}$$

$$T_{pc} = T_c \left(1 + \frac{z_p}{z_0} H^{1/\beta\delta}\right)$$

$$\simeq T_c(1 + 0.22/z_0)$$

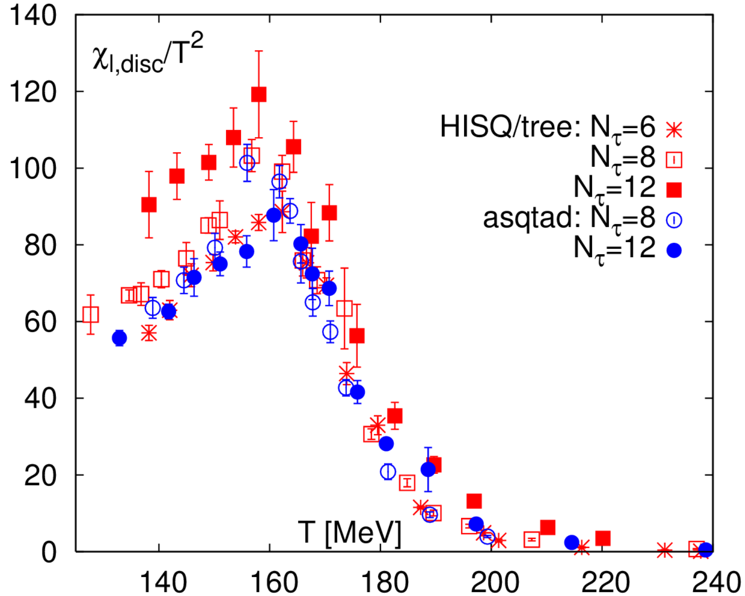
for $H = 1/27$



$z_p = 1.33(5)$

z_0 controls deviations from criticality ⇔ width of critical region

Chiral Transition Temperature

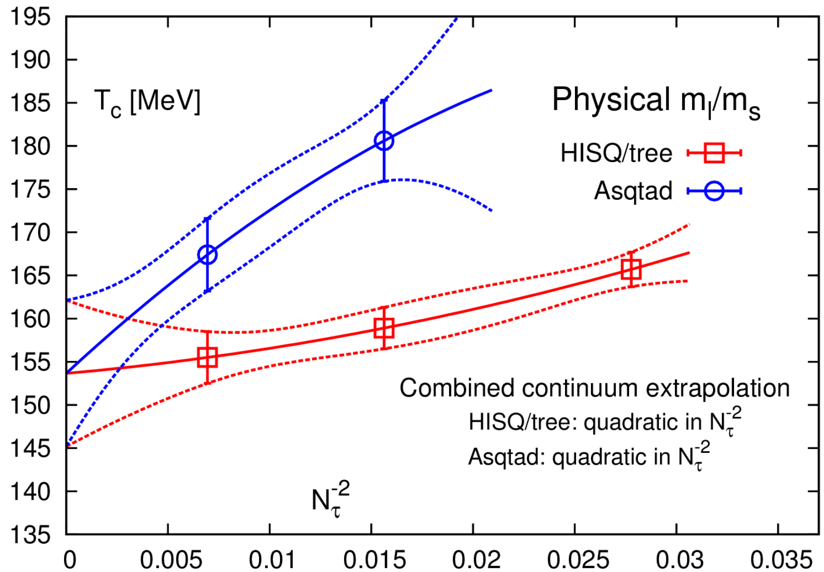


– locate pseudo-critical temperature from chiral susceptibility

$$\frac{\chi_{m,l}}{T^2} = \frac{\partial^2 p/T^4}{\partial(m_l/T)^2} = \frac{\partial\langle\bar{\psi}\psi\rangle_l}{\partial m_l}$$

$$= \frac{\chi_{l,disc}}{T^2} + \frac{\chi_{l,con}}{T^2}$$

– peak location defines pseudo-critical temperature on $N_\sigma^3 N_\tau$ lattice, $T \equiv 1/N_\tau a$



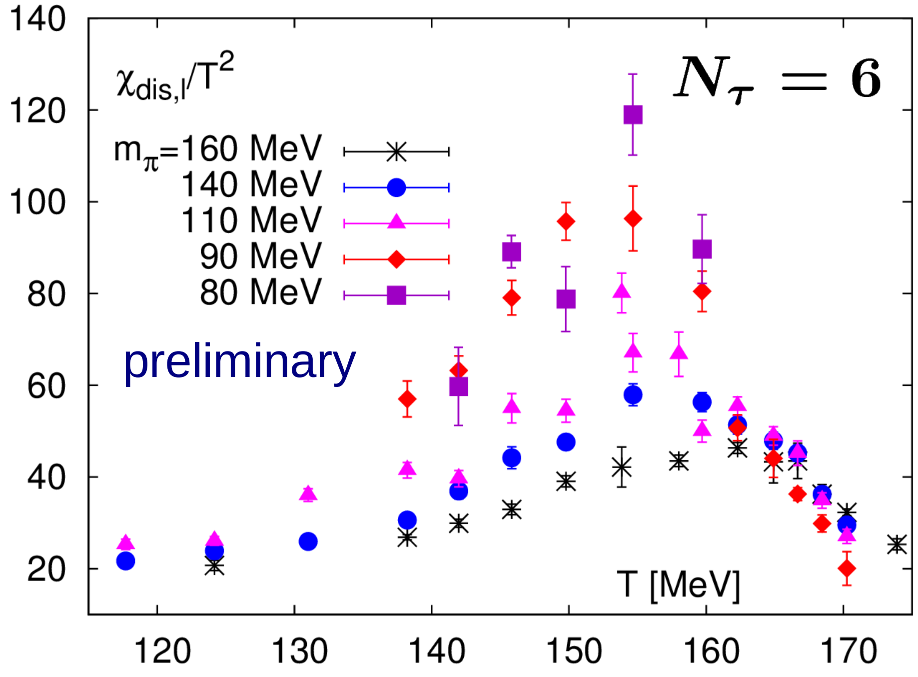
continuum extrapolation of pseudo-critical temperatures at physical light and strange quark masses for two different lattice discretization schemes

$$\mathrm{T}_{pc} = (154 \pm 9)\ \mathrm{MeV}$$

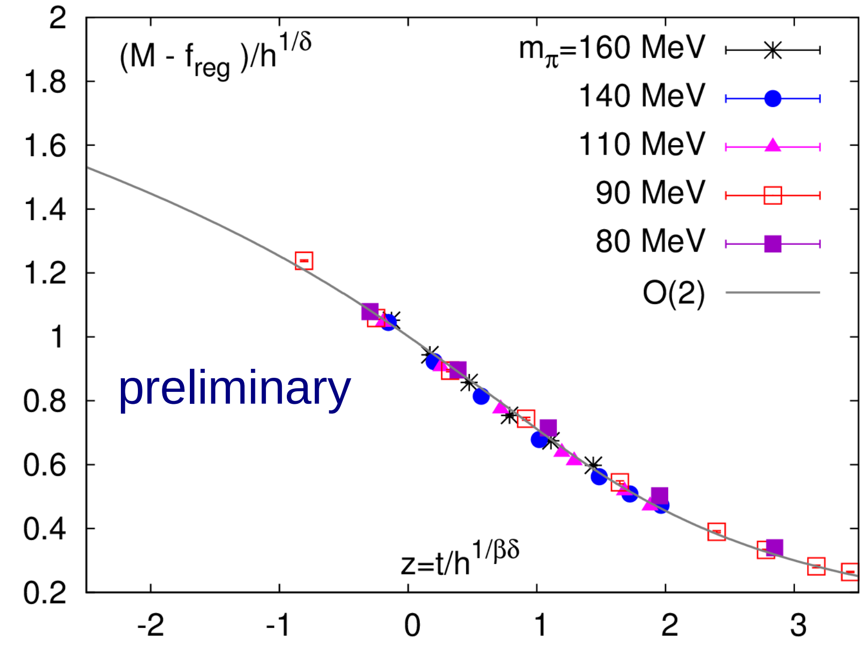
A. Bazavov et al. [hotQCD Collaboration] Phys. Rev. D 85, 054503 (2012)

consistent with: Y. Aoki et al., JHEP 0906, 088 (2009)

Chiral limit: O(4) scaling



$$\chi_{\rm dis}/T^2 \sim m_\pi^{2(1/\delta-1)}$$



$$M = m_s \langle \bar\psi \psi \rangle / T^4$$

magnetic equation of state: $M = h^{1/\delta} f_G(z)$

– scaling analysis in (2+1)-flavor QCD with HISQ fermions

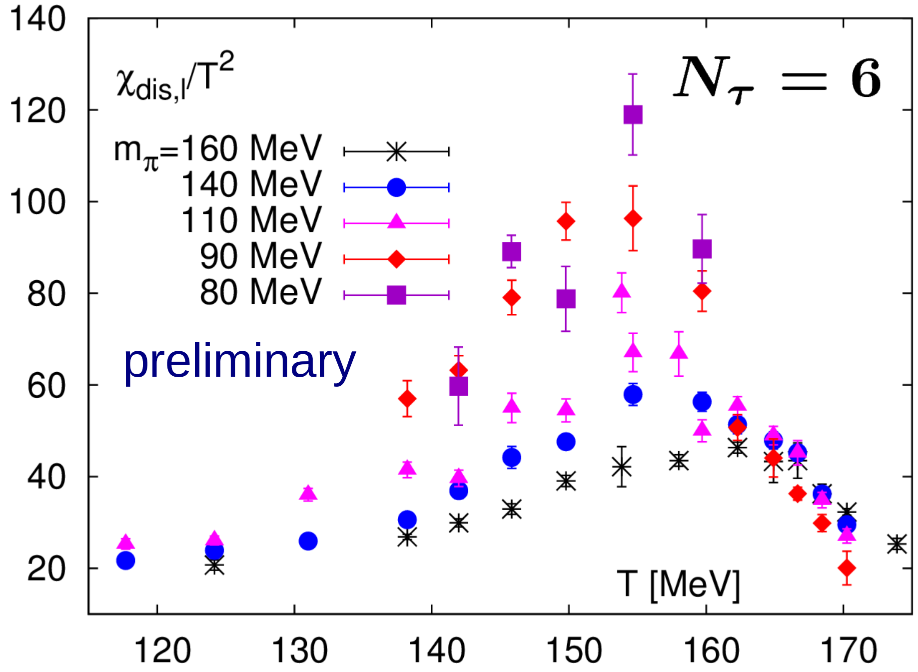
→ $m_\pi = 140\mathrm{MeV}$

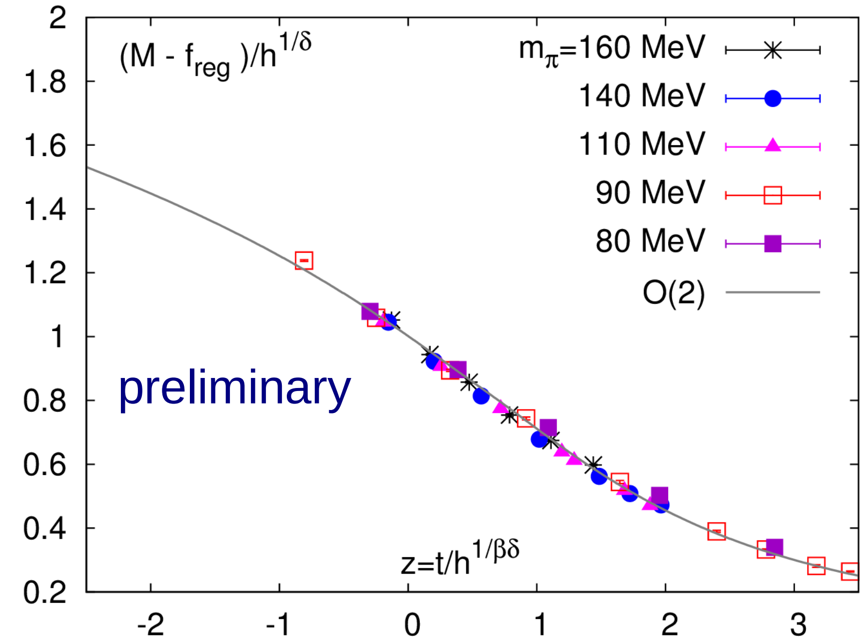
small enough to be sensitive to O(4) scaling behavior in the chiral limit

H.-T. Ding et al., CPOD 2014

staggered fermions: O(2) instead of O(4) for non-zero cut-off

Chiral limit: O(4) scaling





$\chi_{\rm dis}/T^2 \sim m_\pi^{2(1/\delta-1)}$

$M = m_s \langle \bar\psi\psi \rangle / T^4$

magnetic equation of state: $M = h^{1/\delta} f_G(z)$

– scaling analysis in (2+1)-flavor QCD with HISQ fermions

eventually fixes the non-universal parameters: $z_0,\ h_0,\ T_c$

$T_c \simeq 145$ MeV

$h_0^{1/\delta} \simeq 0.057$

$z_0 \simeq 2$

O(4) Scaling in QCD: Curvature of the critical line

Bielefeld-BNL, Phys. Rev. D83, 014504 (2011)

p4-action: $N_\tau = 4$

- "thermal" fluctuations of the order parameter

$$t \equiv \frac{1}{t_0}\left(\left(\frac{T}{T_c} - 1\right) + \kappa_q \left(\frac{\mu_q}{T}\right)^2\right) \;,\; z = t/h^{1/\beta\delta}$$

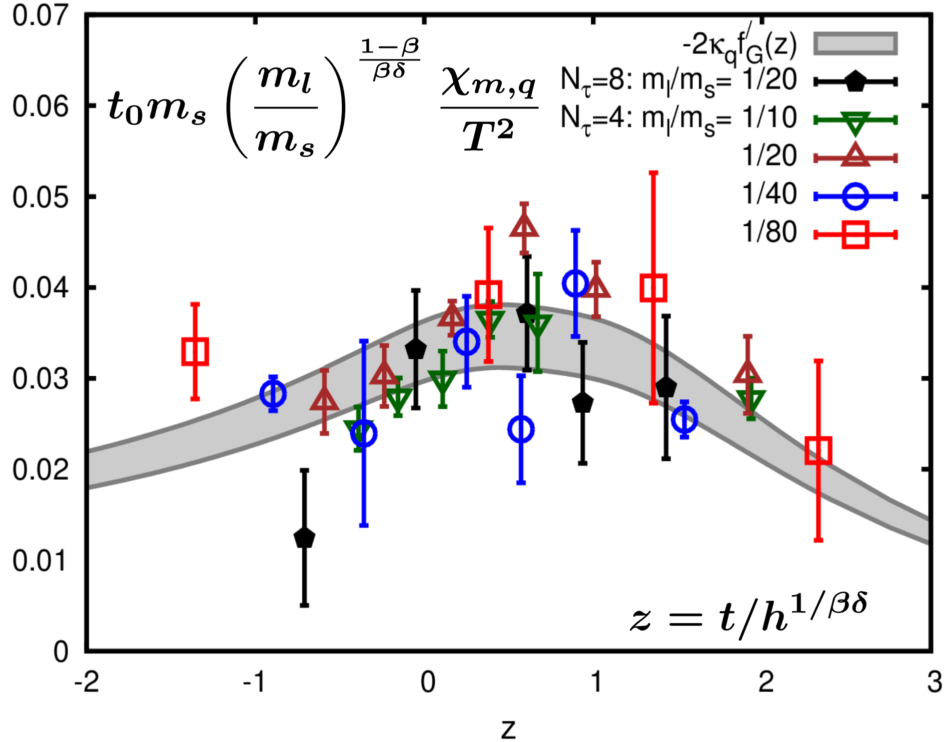
$$M_b \equiv \frac{m_s \langle\bar{\psi}\psi\rangle}{T^4} = h^{1/\delta} f_G(z)$$

$$\frac{\chi_{m,q}}{T} = \frac{\partial^2 \langle\bar{\psi}\psi\rangle / T^3}{\partial(\mu_q/T)^2}$$

$$= \frac{2\kappa_q T}{t_0 m_s} h^{(\beta-1)/\delta\beta} f_G'(z)$$

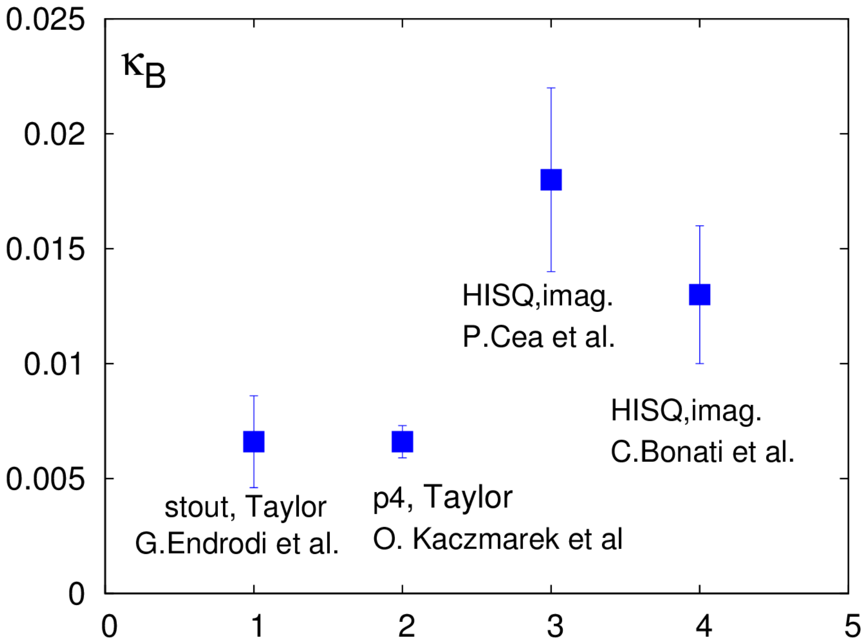
$$\kappa_B = \kappa_q/9 = 0.0066(7)$$

scaling function of order parameter



O(4) Scaling in QCD

summary of current values for the curvature term:



$$0.006 \lesssim \kappa_B \lesssim 0.02$$

Current knowledge of the non-universal parameters controlling singular behavior in QCD at small values of the chemical potential:

$$T_c \simeq 145 \text{ MeV}$$
$$h_0^{1/\delta} \simeq 0.057$$
$$z_0 \simeq 2$$
$$\kappa_q \simeq 0.1$$

Critical behavior of conserved charge fluctuations

$$\frac{P}{T^4} = -2h_0^{-1/\delta}\, H^{(2-\alpha)/\beta\delta} f_f(z) - f_r(V,T,\vec{\mu})$$

H derivatives
chiral condensate,
chiral susceptibility

$$\frac{\chi_{m,q}}{T^2} = \frac{\partial^2 P/T^4}{\partial (m_q/T)^2}$$

$$\sim A_q H^{1/\delta-1} f_\chi(z)$$

$$A_q = 2\left(\frac{T_c}{m_s}\right)^2 h_0^{-1/\delta}$$

μ_q/T derivatives
energy density, specific heat
quark number susceptibilities

$$\text{e.g. } \chi_6^q = \left.\frac{\partial^6 P/T^4}{\partial(\mu_q/T)^6}\right|_{\mu_{q,c}=0}$$

$$\sim -A_6^q H^{-(1+\alpha)/\beta\delta} f_f'''(z)$$

at $\hat{\mu}_{q,c} = 0$

$$A_6^q = 30(2\kappa_q z_0)^3 h_0^{-1/\delta}$$

$$\hat{\mu} \equiv \mu/T$$

$$f_\chi(z) = a_0 f_f(z) + a_1 f_f'(z) + a_2 f_f''(z)$$

$$A_6^q \simeq 5 - 50$$

Critical behavior of conserved charge fluctuations

$$\frac{P}{T^4} = -2h_0^{-1/\delta}\, H^{(2-\alpha)/\beta\delta} f_f(z) - f_r(V,T,\vec{\mu})$$

μ_q/T derivatives
energy density, specific heat
quark number susceptibilities

a factor 2 uncertainty
in $\kappa_q z_0$ generates
a factor 8 uncertainty in
χ_6^q

$$\text{e.g. } \chi_6^q = \left.\frac{\partial^6 P/T^4}{\partial(\mu_q/T)^6}\right|_{\mu_{q,c}=0}$$

$$\sim -A_6^q H^{-(1+\alpha)/\beta\delta} f_f^{(3)}(z)$$

at $\hat{\mu}_{q,c} = 0$

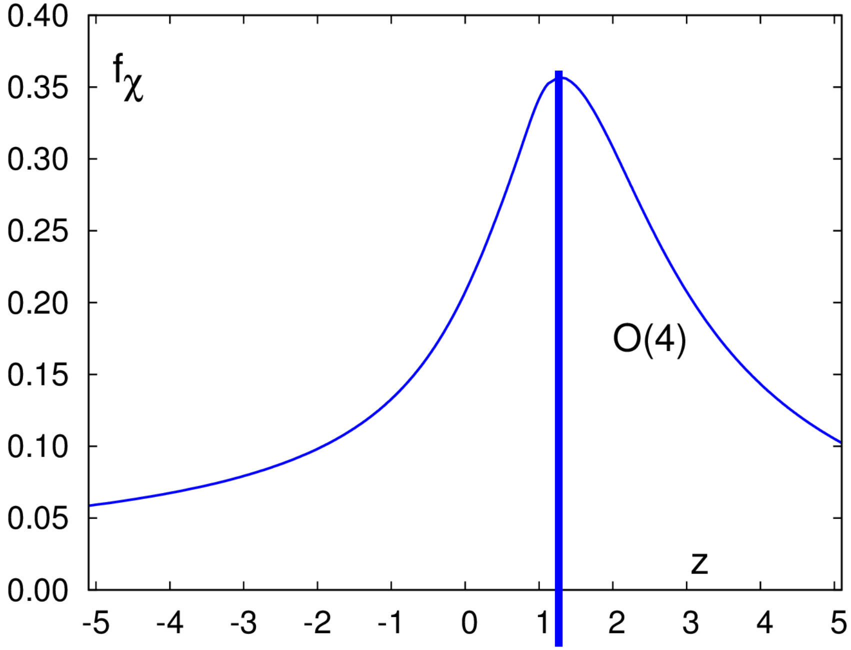
$$A_6^q = 30(2\kappa_q z_0)^3 h_0^{-1/\delta}$$

prefactor is known
with about a factor 10
uncertainty?

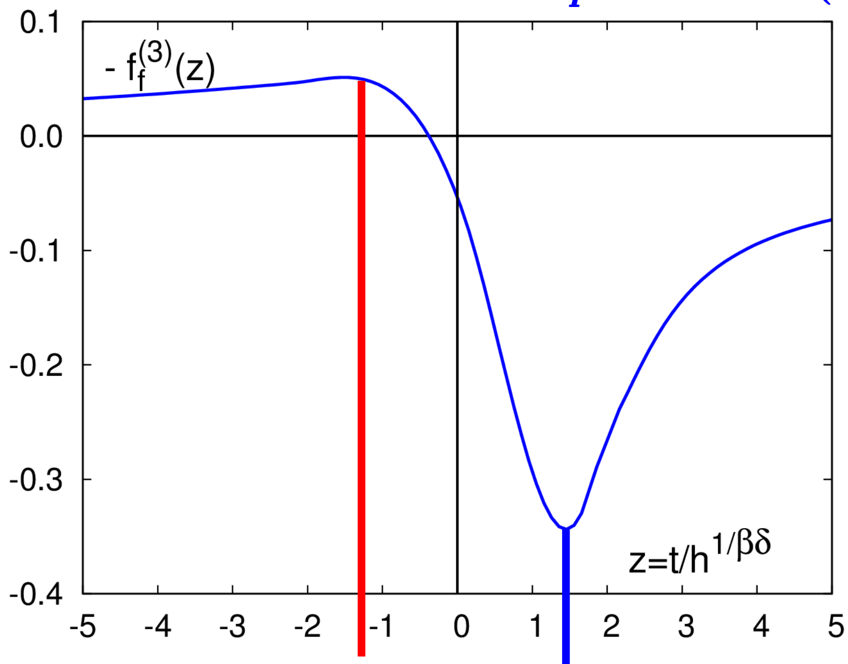
$$\hat{\mu} \equiv \mu/T$$

$$\chi_6^q \simeq (20-200)\ f_f^{(3)}(z) - f_r^{(6)}(V,T,\vec{\mu}=0)$$

$$A_6^q \simeq 5-50$$



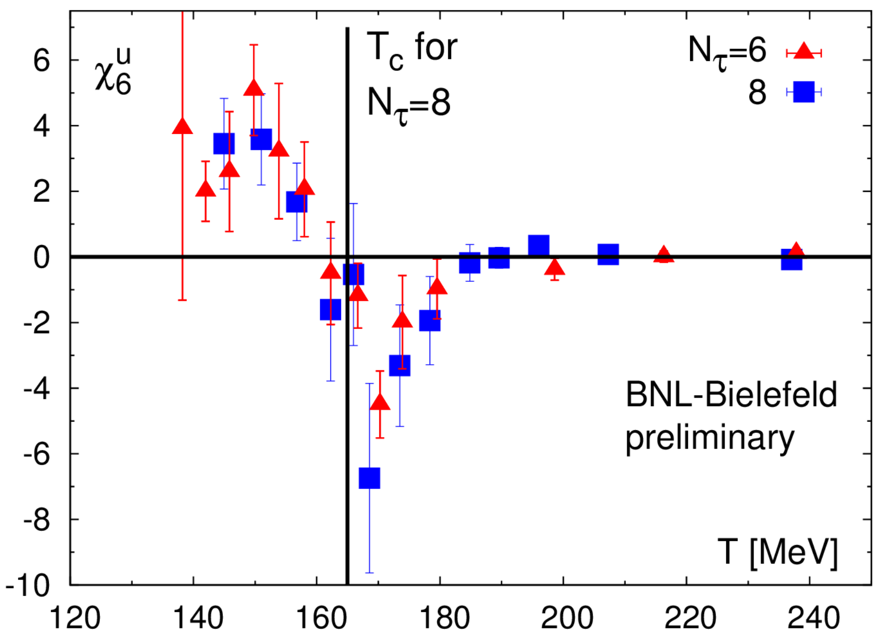
$z_p = 1.33(5)$



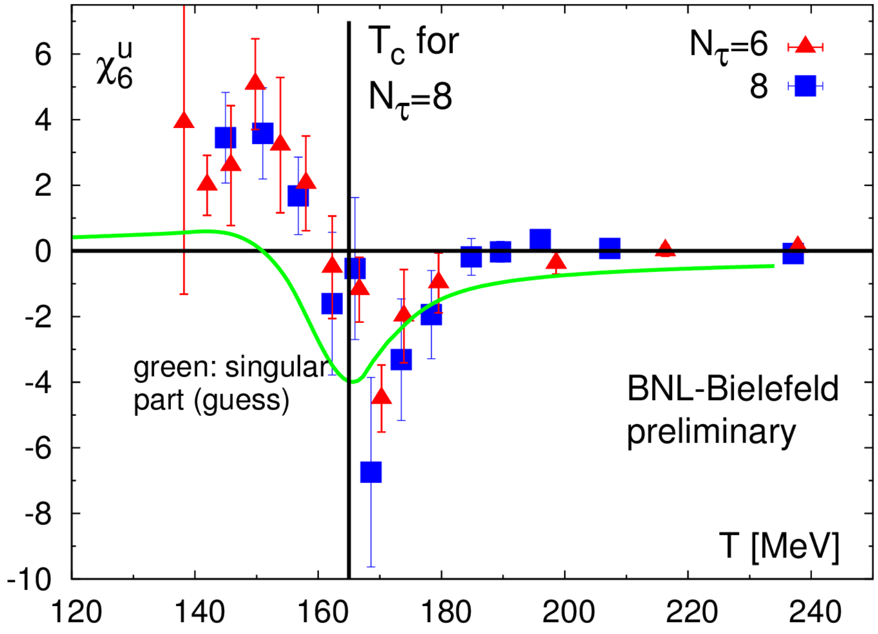
$z_- \simeq -1.50$ $\quad z_+ \simeq 1.48$

The peak in the scaling function that determines the location of the chiral crossover transition as seen by the chiral susceptibility is at (almost) the same temperature, at which the 6th order quark number susceptibility has its minimum – if contributions from regular terms are small!!

6-th order net "up-ness" fluctuations



qualitatively as expected

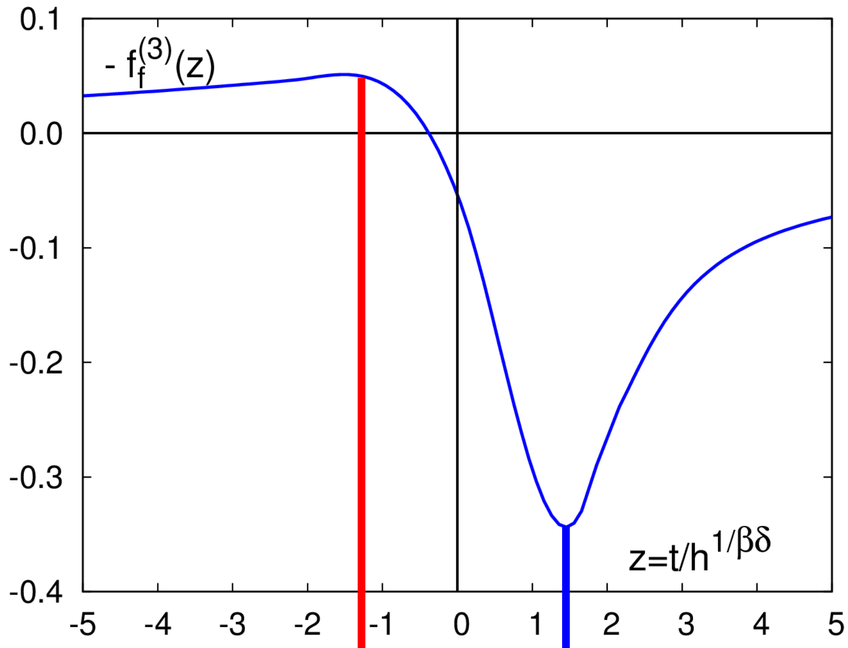


– the dip reflects behavior of $-f_f^{(3)}(z)$

– the width of the transition region (as seen by χ_6^q)

$$\Delta z = z_+ - z_- = (t_+ - t_-)/h^{1/\beta\delta}$$

universal numbers $\Delta z = z_+ - z_- \simeq 3$



$z_- \simeq -1.50$ $z_+ \simeq 1.48$

$$T_+ - T_- = \frac{\Delta z}{z_0} \left(\frac{m_l}{m_s}\right)^{1/\beta\delta} T_c$$

$T_+ - T_- \simeq 0.25 T_c$

for $m_l/m_s = 1/27$

qualitatively as expected

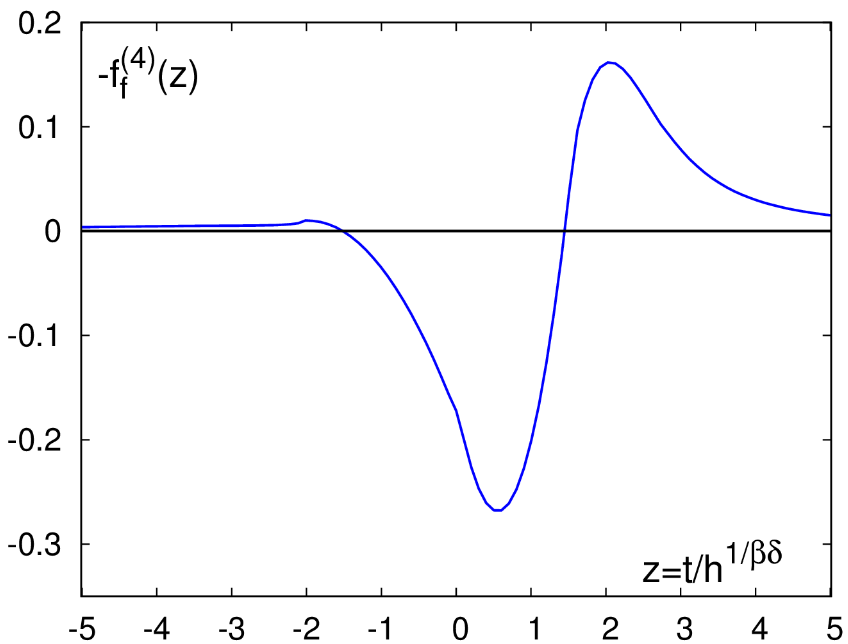
Conserved charge fluctuations in QCD and HIC $\mu_B > 0$

4th order cumulant: A dip in the kurtosis ?

$$\mu_B > 0: \quad \chi^q_{4,\mu} = -2h_0^{-1/\delta}\Big(3(2\kappa_q z_0)^2 H^{-\alpha/\Delta} f_f^{(2)}(z) - 6(2\kappa_q z_0)^3(\hat{\mu}_q^c)^2 H^{-(1+\alpha)/\Delta} f_f^{(3)}(z) - (2\kappa_q z_0)^4(\hat{\mu}_q^c)^4 H^{-(2+\alpha)/\Delta} f_f^{(4)}(z)\Big) + \text{regular}$$

dominates in the chiral limit

vanishes at $z = z_p$



all singular contributions are negative for $T \lesssim T_{pc}$

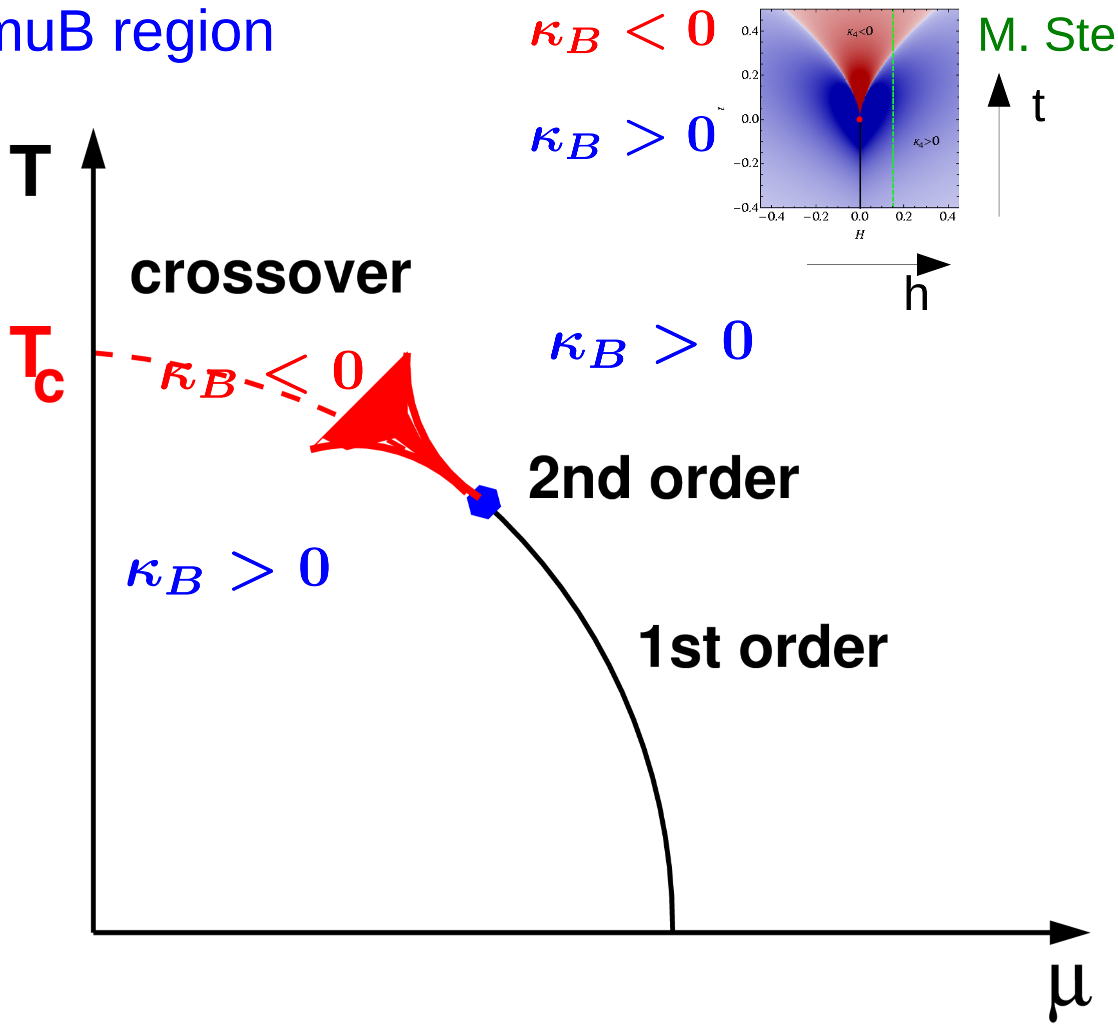
$\Rightarrow \chi_4^B(\mu_B) < 0$

B.Friman, FK, K.Redlich,V.Skokov, Eur. Phys. J. C71, 1694 (2011)

4^{th} order cumulant (kurtosis) and the critical point

in the vicinity of the critical point the kurtosis will be negative in a certain T, muB region

M. Stephanov, PRL 107, 052301 (2011)



mapping of the Ising variables t, h on the T, μ_B plane is non-trivial

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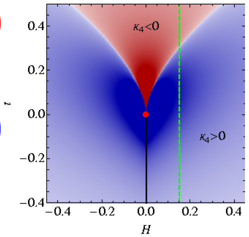
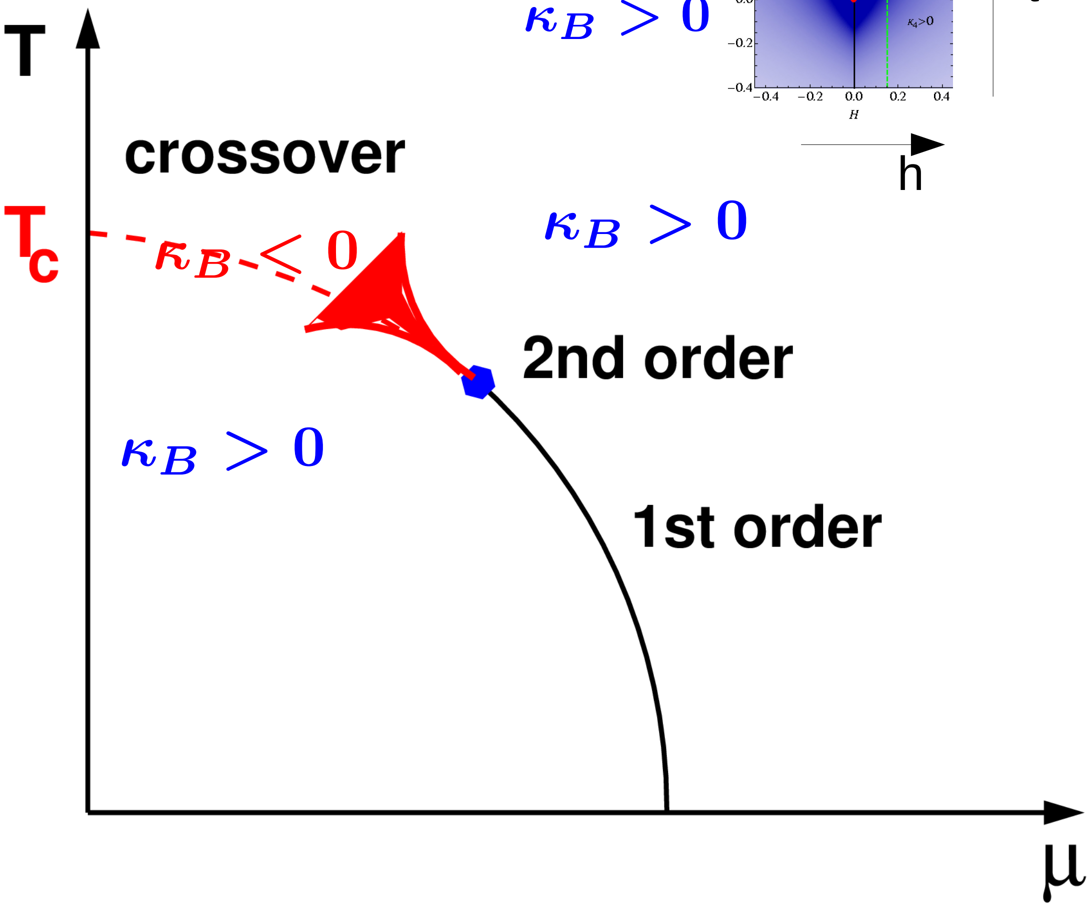
M. Stephanov, PRL 107, 052301 (2011)





mapping of the Ising variables t, h on the T, μ_B plane is non-trivial

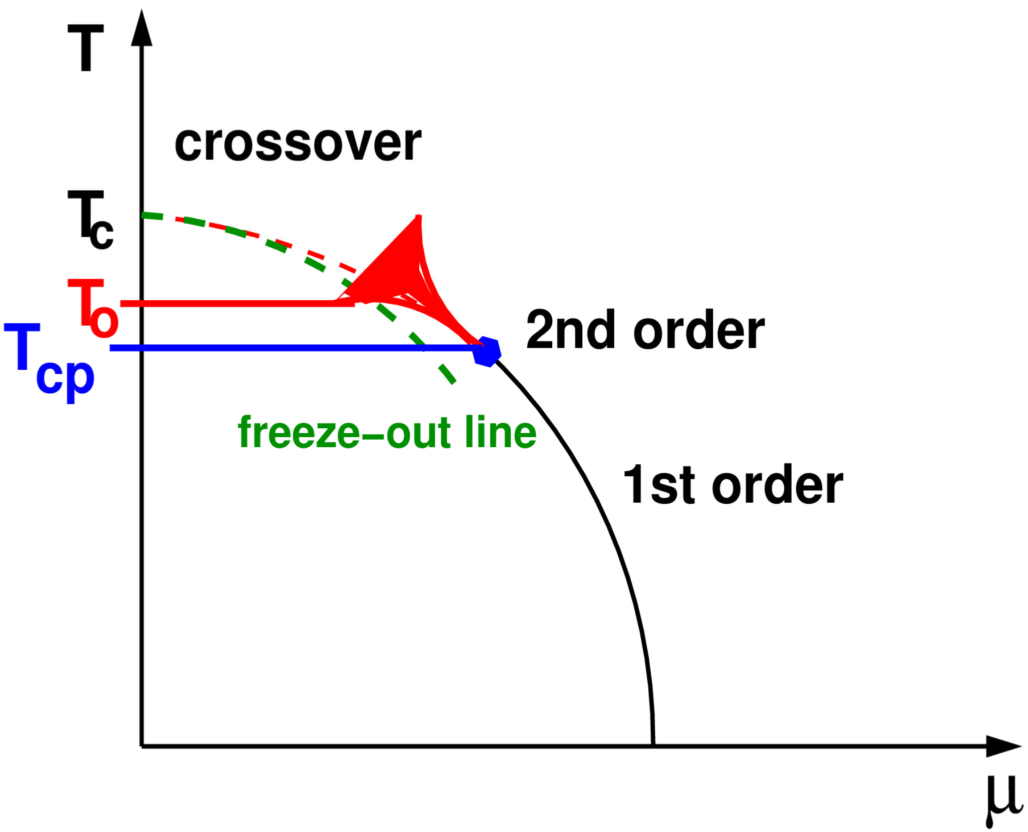
generically, expect:

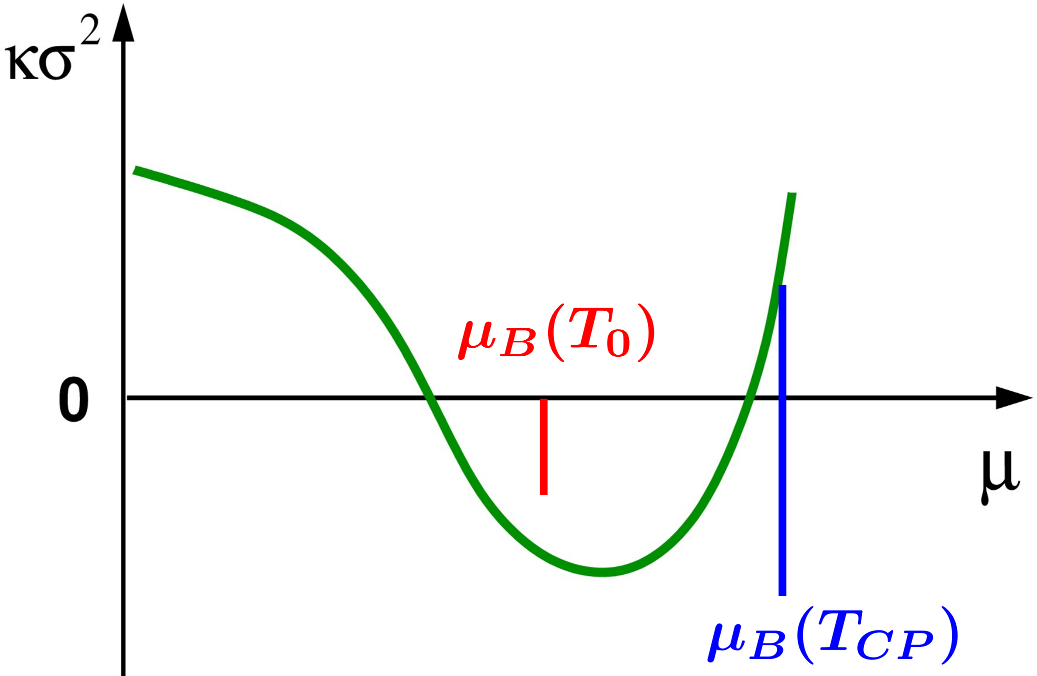
$\kappa_B > 0$ for $T \le T_{CP}$

expect all cumulants to be positive on the line of fixed $T \equiv T_{CP}$

prerequisite for well-behaved estimates of the location of the critical point based on the radius of convergence of the Taylor series for $\chi_{B,\mu}$

Kurtosis on the freeze-out curve





to determine the importance of regular terms and the non-universal scales requires
lattice QCD

a dip in the kurtosis seems to be generic:
whether or not it becomes negative depends on the magnitude of regular terms in the QCD partition function (pressure)

Taylor expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_B,\mu_S,\mu_Q)$$
$$= \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

generalized susceptibilities: $\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k} \right|_{\mu=0}$

– can be evaluated using standard MC simulation algorithms;
- valid up to radius of convergence: μ_c (critical point?)

– radius of convergence corresponds to a critical point only, iff

$$\chi_n > 0 \text{ for all } n \geq n_0$$

forces P/T^4 and χ_B^n
to be monotonically growing with μ_B/T

Chiral Transition at small μ_B/T in the chiral limit

$$\frac{p}{T^4} = -h_0^{-1/\delta} \left(\frac{m_q}{m_s}\right)^{1+1/\delta} f_f(z) - f_r(V,T,\vec{\mu})$$

$$z = z_0 \left(\frac{T-T_c}{T_c} + \kappa_q\, \hat{\mu}_q^2\right) \left(\frac{m_s}{m_q}\right)^{1/\beta\delta}$$

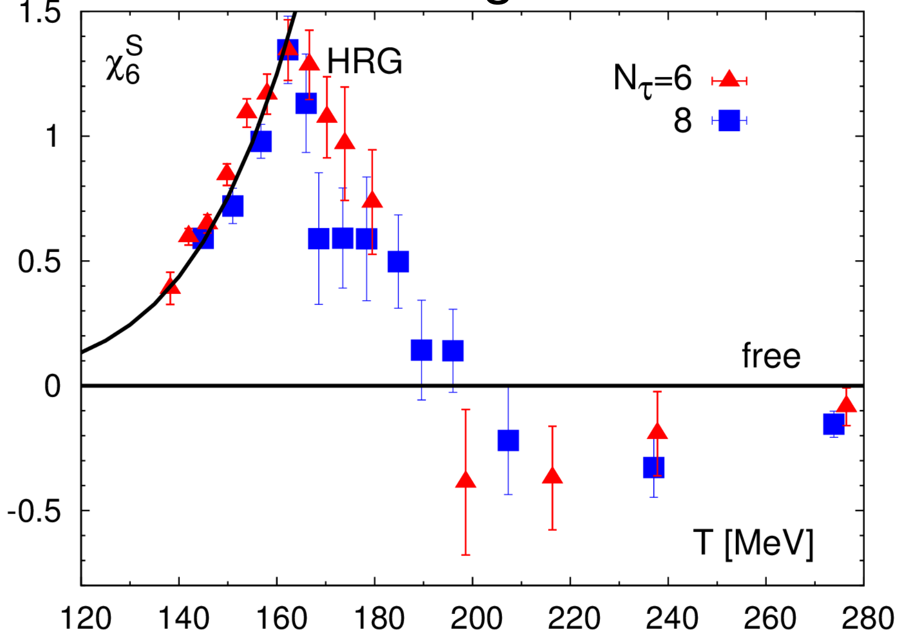
In the chiral limit $m_q \rightarrow \infty$ below T_c the behavior of the scaling function for $z \rightarrow -\infty$ controls the critical behavior for small μ_B/T

$$f_f(z) = (-z)^{2-\alpha} \left(c_0^- + \mathcal{O}((-z)^{-1/(2\beta\delta)})\right)$$

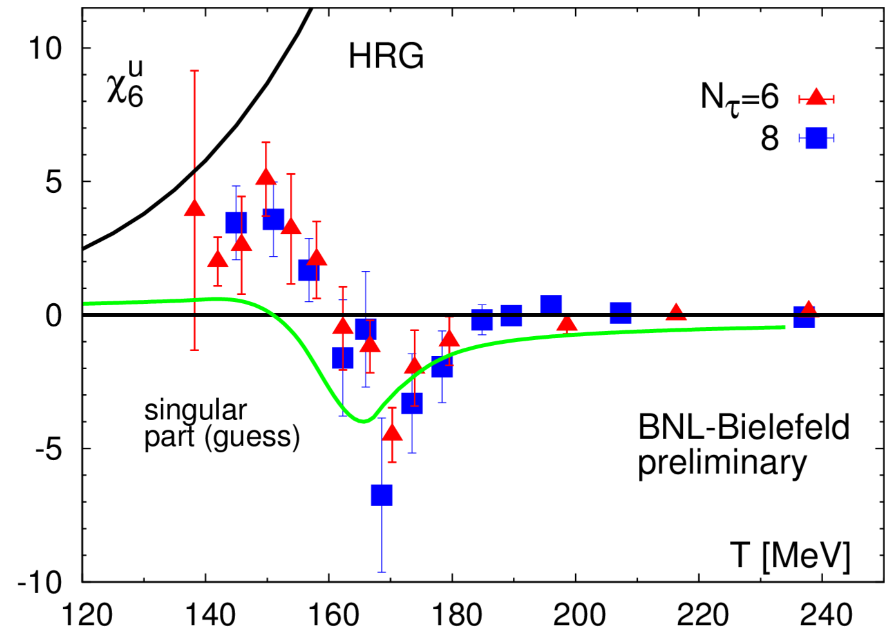
$$\left(\frac{\mu_q}{T}\right)^{\chi}_{crit} \equiv \lim_{n\rightarrow\infty} r_n^{\chi} = \lim_{n\rightarrow\infty} \sqrt{\left|\frac{n(n-1)\chi_n^q}{\chi_{n+2}^q}\right|} = \sqrt{\frac{T-T_c}{\kappa_q T_c}}$$

6th order light and strange quark number cumulants

strange



light



– no evidence for
'typical' O(4) singular structure
– regular contribution dominates

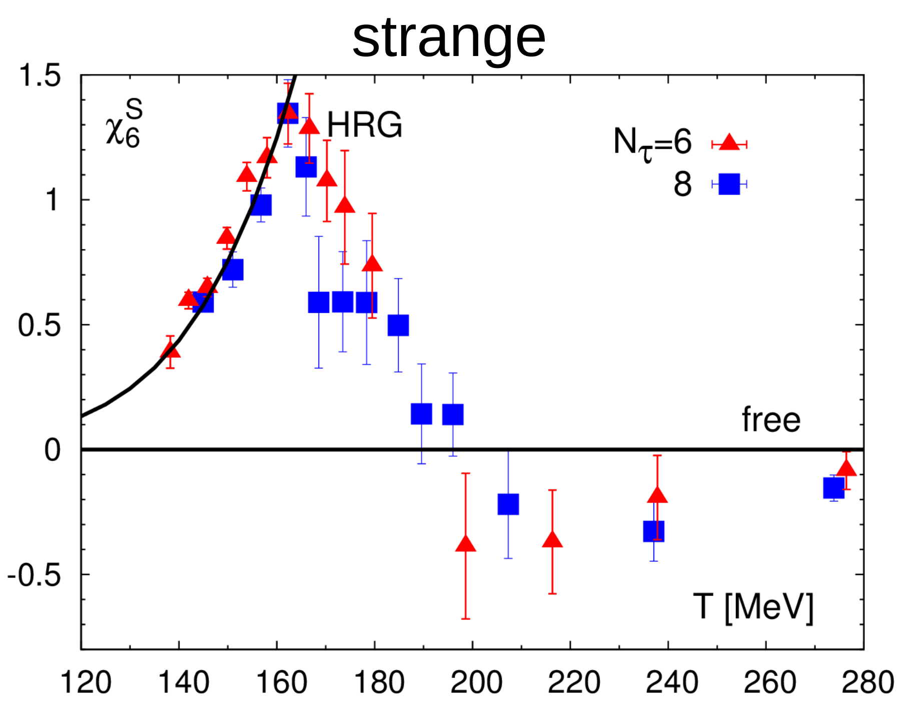
– clear evidence for
'typical O(4) singular structure
– regular and singular contributions

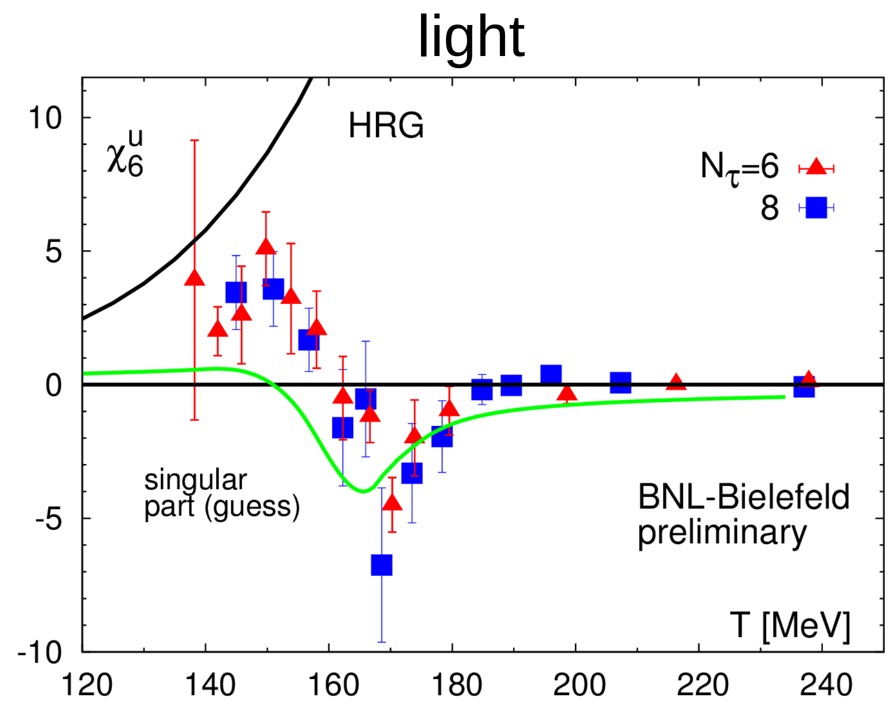
any "significant" overshooting of the HRG values can not arise from O(4) criticality

depth of the minimum at high T fixes maximal singular contribution at low T !!!

$$\left(\frac{\chi_6^{u,min}}{\chi_6^{u,max}}\right)_{\mathbf{sing}} = -6.7$$

6^{th} order light and strange quark number cumulants





– no evidence for
'typical' O(4) singular structure
– regular contribution dominates

any "significant" overshooting of the HRG values can not arise from O(4) criticality

→

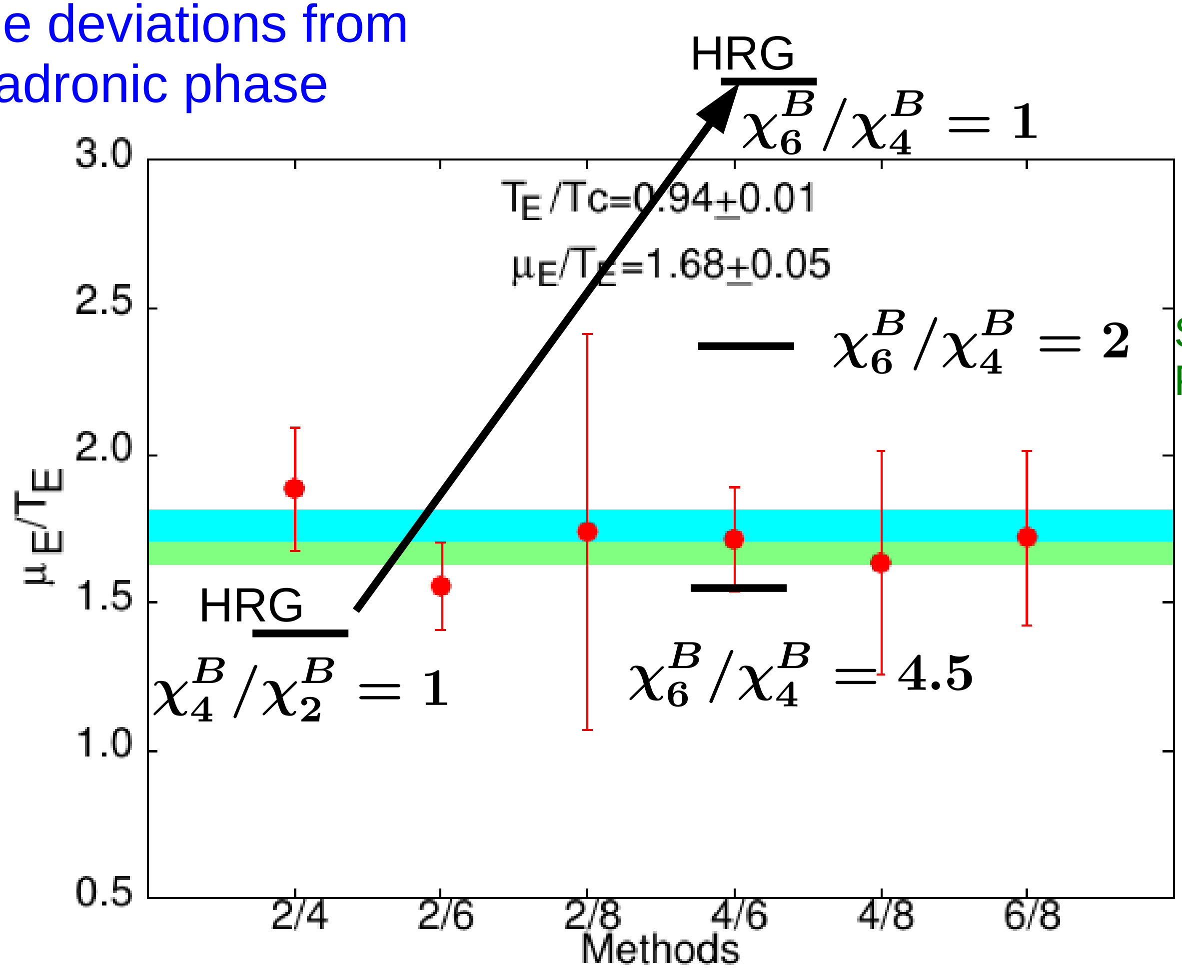
so far none of the cumulants of charge fluctuations that are statistically well under control show a "significant" overshooting of HRG values

Estimates of the radius of convergence

a challenging prediction
from susceptibility series:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

suggests large deviations from
HRG in the hadronic phase



huge deviations
from HRG in
6th order cumulants!

S. Datta et al.,
PoS Lattice2013 (2014) 202

at present, we
cannot rule it out!

BNL-Bielefeld

Conclusions

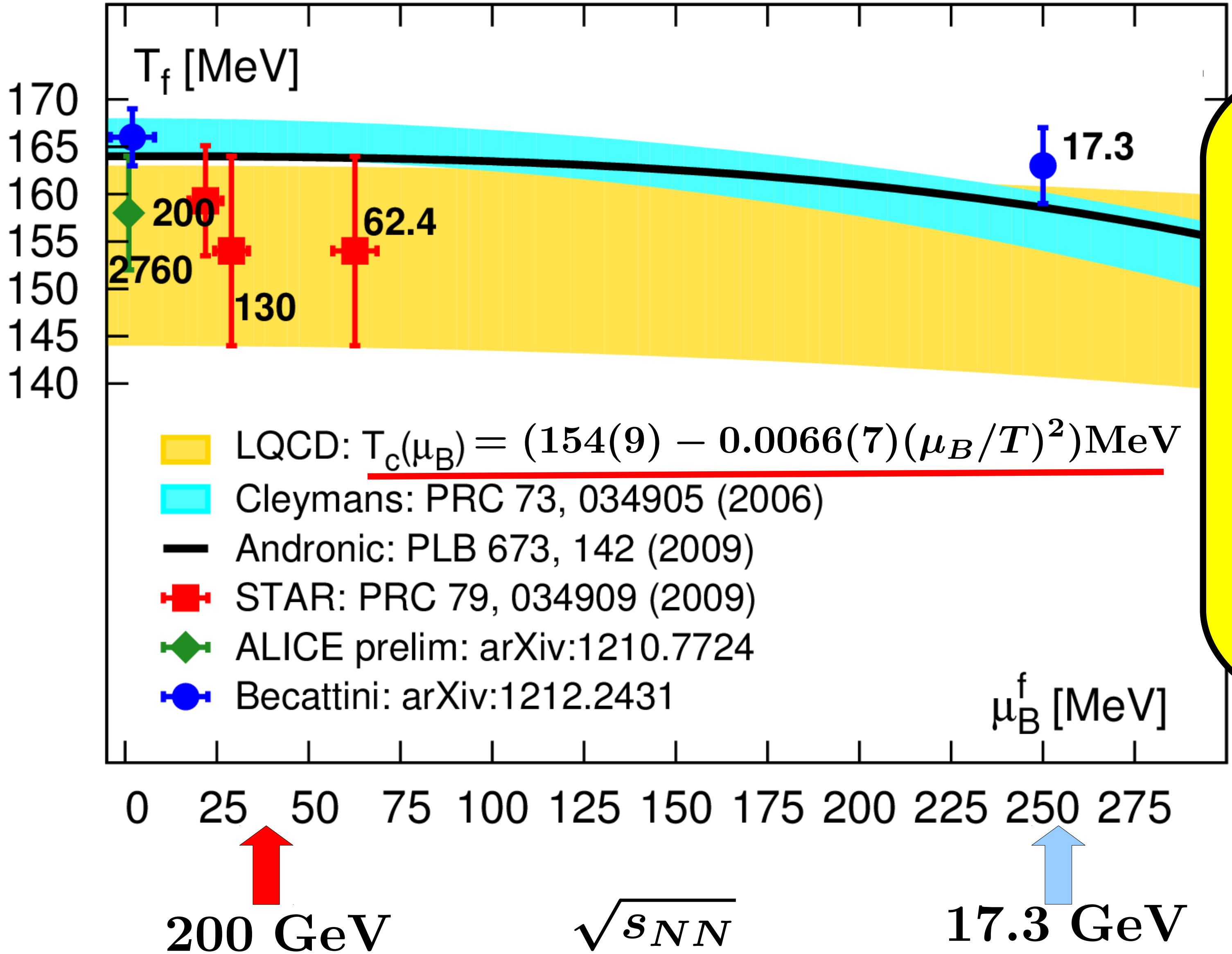
- only small deviations from HRG model calculations at freeze-out and sensitivity to O(4) criticality in the crossover region are not inconsistent with each other

- 6th order cumulants are sensitive to O(4) scaling but will pick up only a small singular contribution below Tc.

- Any large increase of cumulants above HRG values below Tc thus are not due to O(4) critical behavior but may be indicative for a critical end point

- a decreasing kurtosis*variance is likely to show up as a consequence of O(4) criticality. A possible critical end point needs to be in a region with $\kappa_B\sigma_B > 1$

Chiral transition and freeze-out



phenomenological freeze-out / hadronization curve, QCD transition line and experimental data (obtained by assuming the validity of the HRG model) are consistent for

$$\mu_B/T \lesssim 2$$