The Equation of State from Lattice QCD

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[Bi-BNL-CCNU collaboration]

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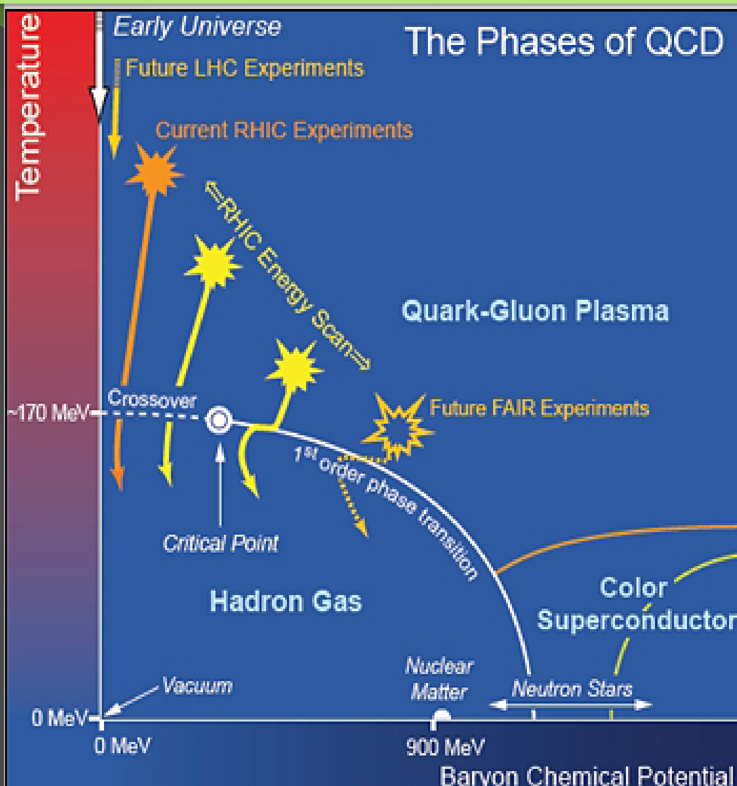
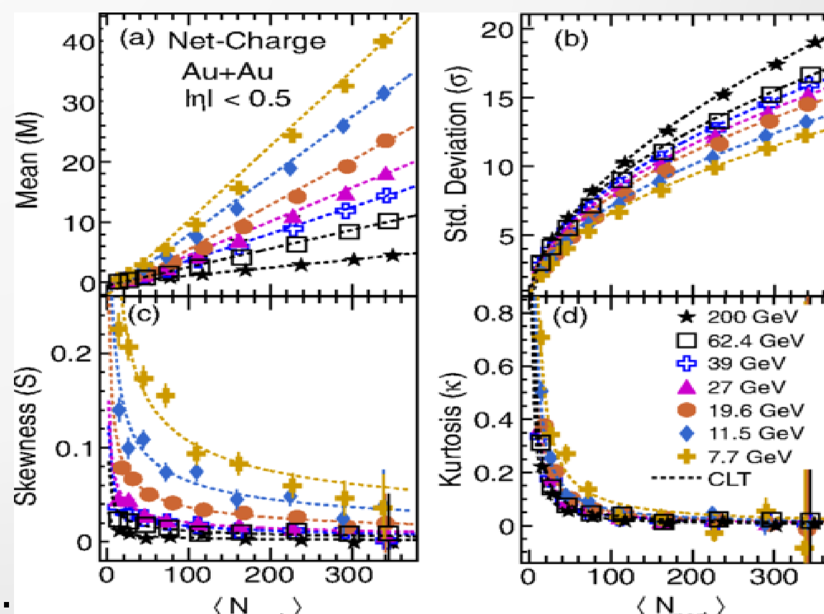
January 27, 2015.

Workshop on QCD at High Density,
TIFR, Mumbai.

Beam energy scan at RHIC



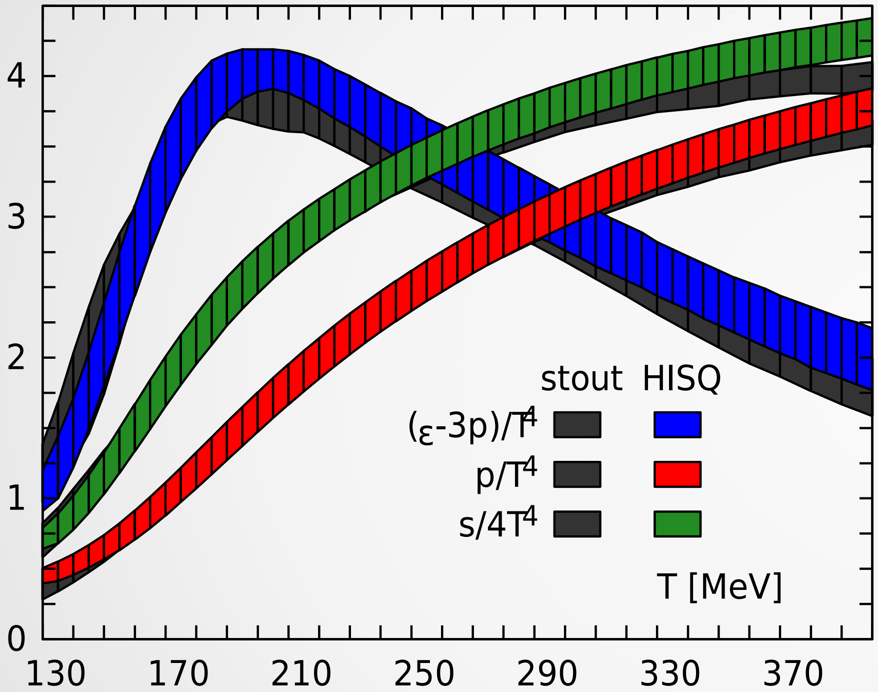
- Possible existence of a critical point at large mu_B.
  - Talk by F.Karsch this morning.
- Search up to mu_B ~ 400-450 MeV.
- Look at higher moments of net charge (electric, baryon number,...) distributions.
  - Talks by N.Xu, B.Mohanty, R.Lacey,...



STAR collaboration, PRL 113, 0902301 (2014).

Equation of state: What we know

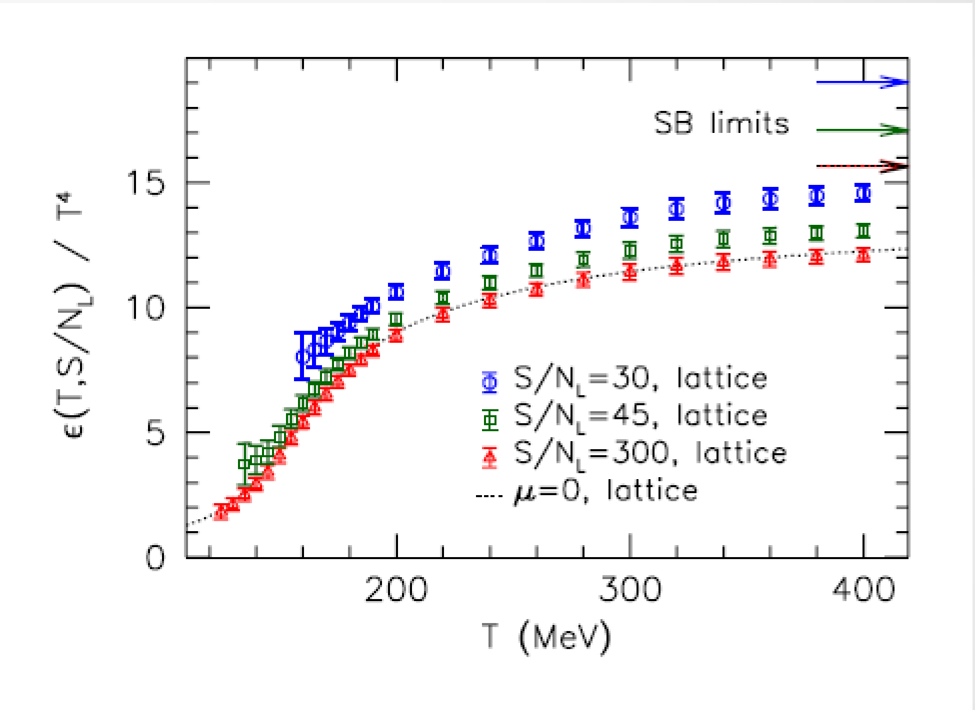
0^{th}-order equation of state



A. Bazavov et al. [HotQCD collaboration] Phys. Rev. D90, 054903 (2014); see talk by P.Petreczky at this workshop.

S. Borsanyi et al. [BMW collaboration], JHEP 1011, 077 (2010).

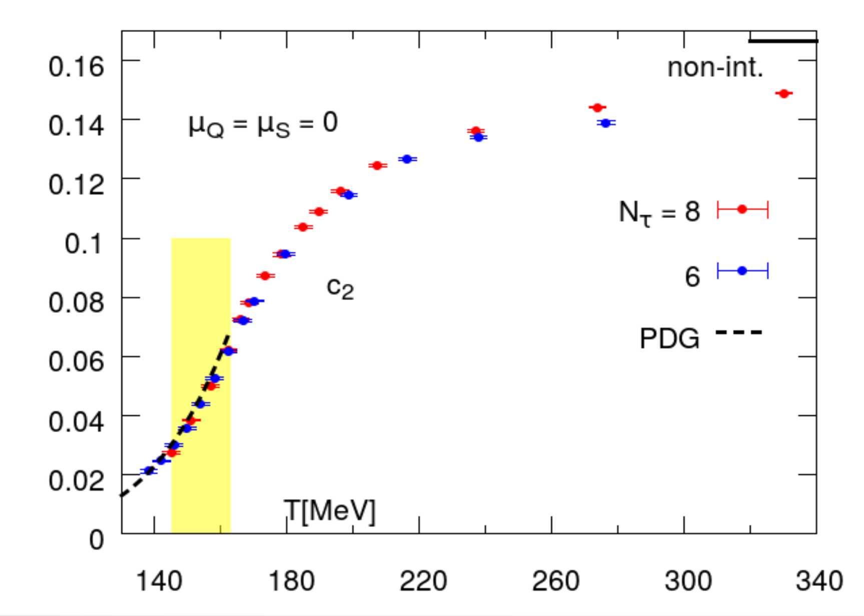
2^{nd}-order equation of state

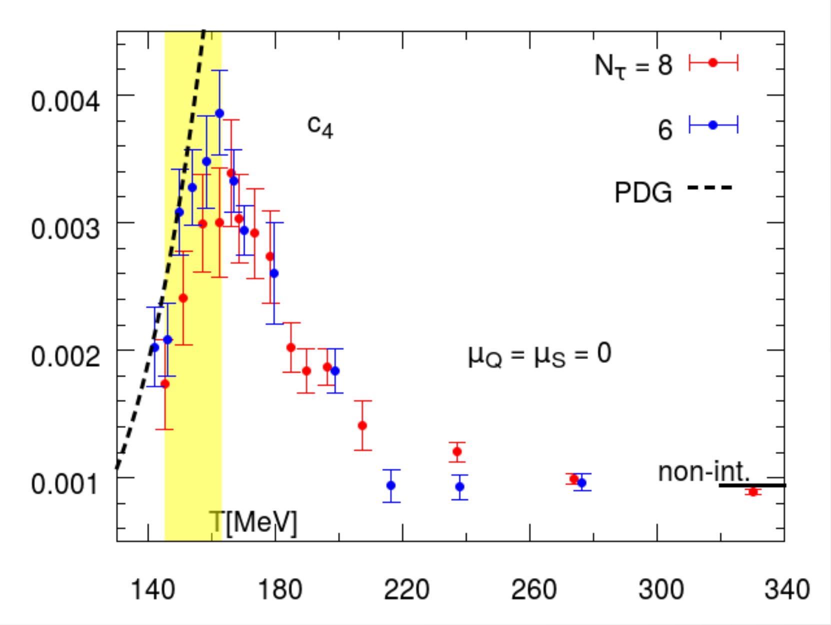


S. Borsanyi et al. [BMW collaboration], JHEP 1208, 053 (2012).

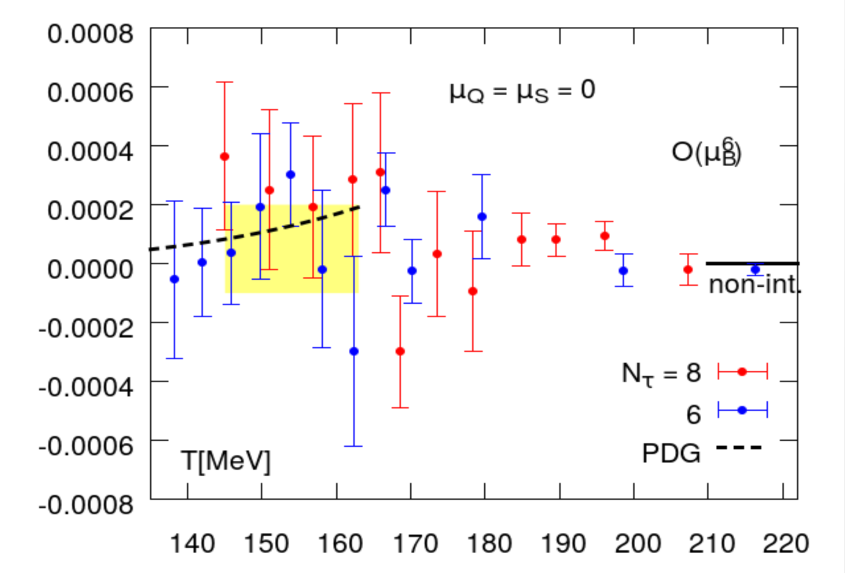
Here we will present results for a 4^{th}-order equation of state.

The method of Taylor expansions





$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}}{i!\,j!\,k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$



Estimate the pressure for $mu_B > 0$ using n^{th}-order expansion.

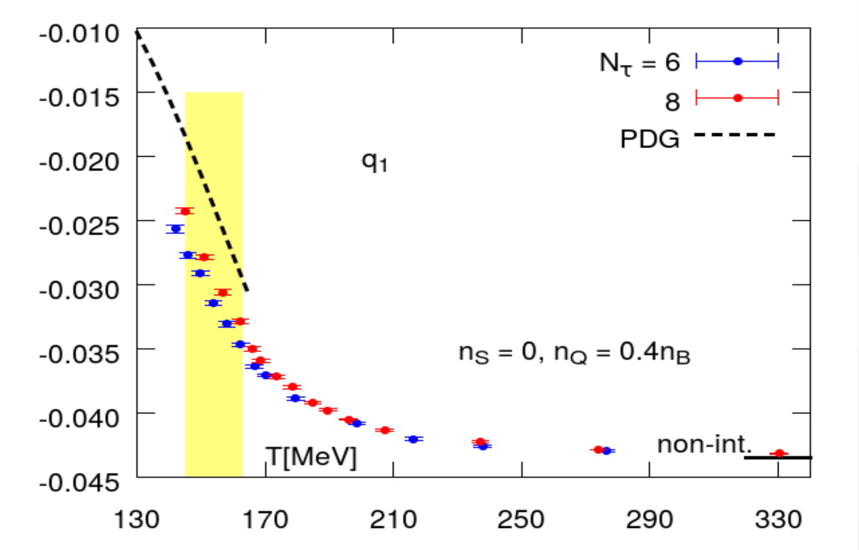
Initial conditions in heavy-ion collisions

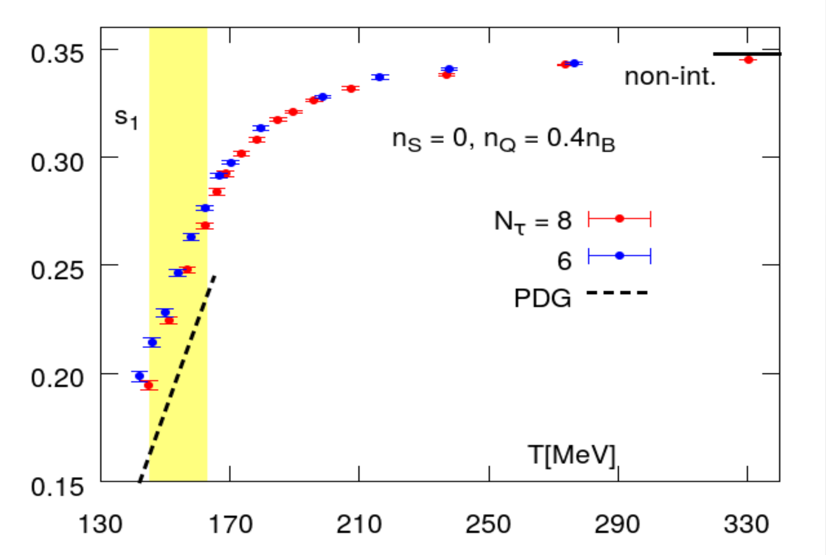
Fix mu$_Q$ and mu$_S$ from initial conditions:

- N_S = 0 (no valence strange quarks),
- r = $N_p/(N_p+N_n)$ = const. (fixed Z-to-A ratio).

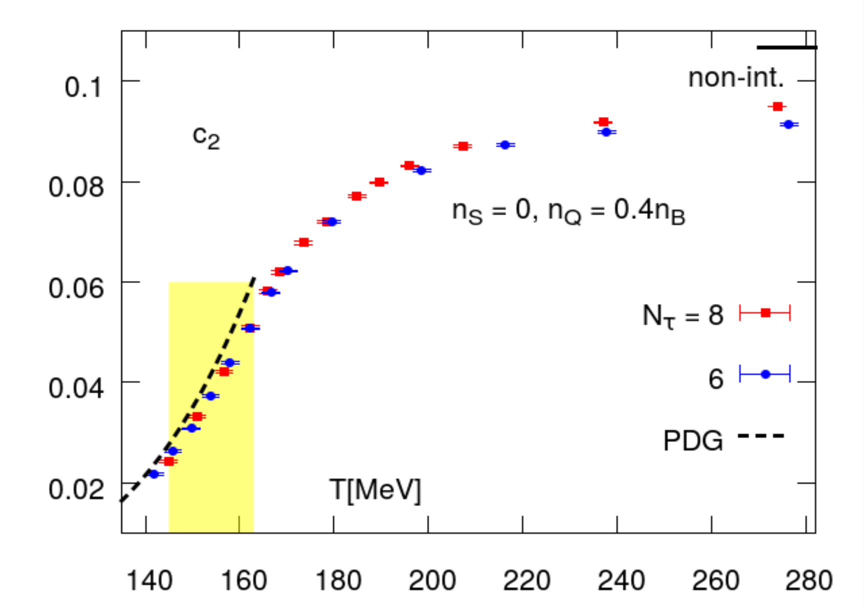
$$\frac{\mu_Q}{T} = q_1 \frac{\mu_B}{T} + q_3 \left(\frac{\mu_B}{T}\right)^3 + \ldots$$

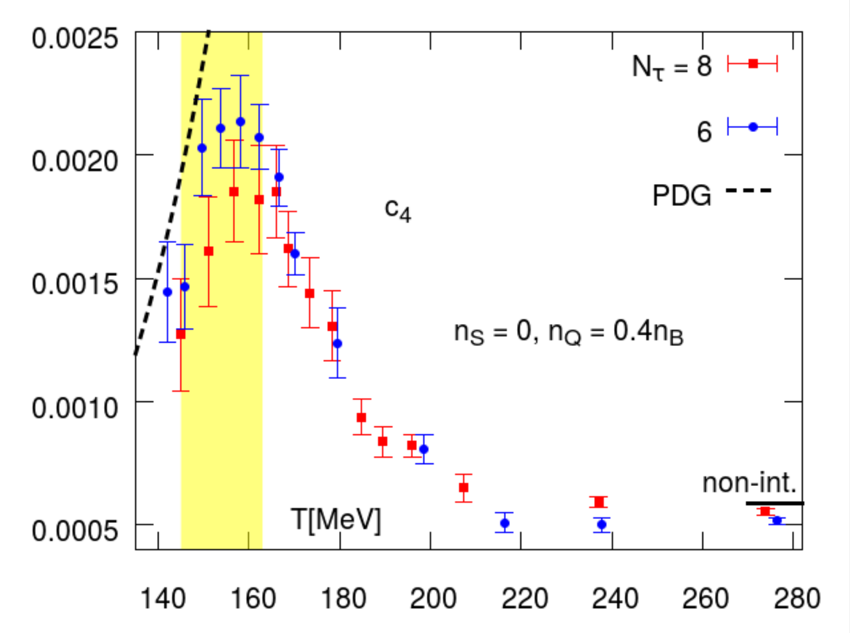
$$\frac{\mu_S}{T} = s_1 \frac{\mu_B}{T} + s_3 \left(\frac{\mu_B}{T}\right)^3 + \ldots$$

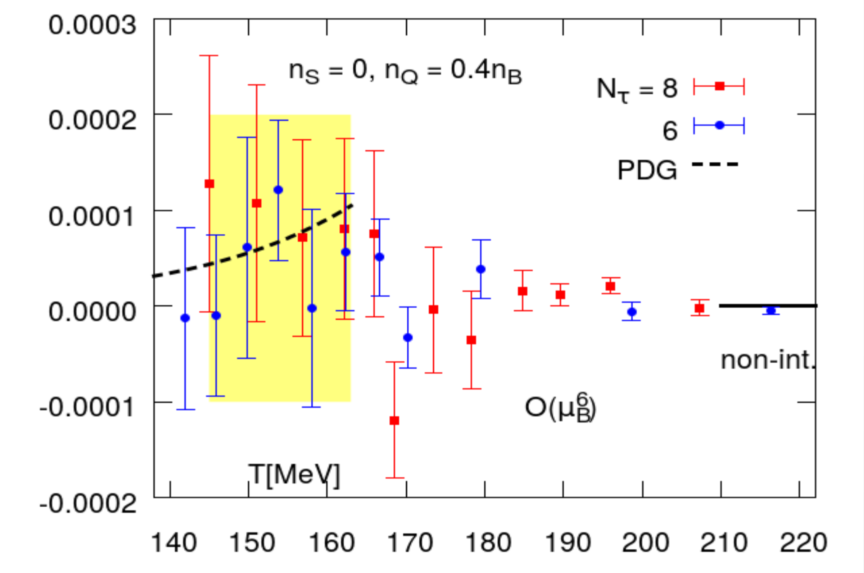




Susceptibilities: Constrained case







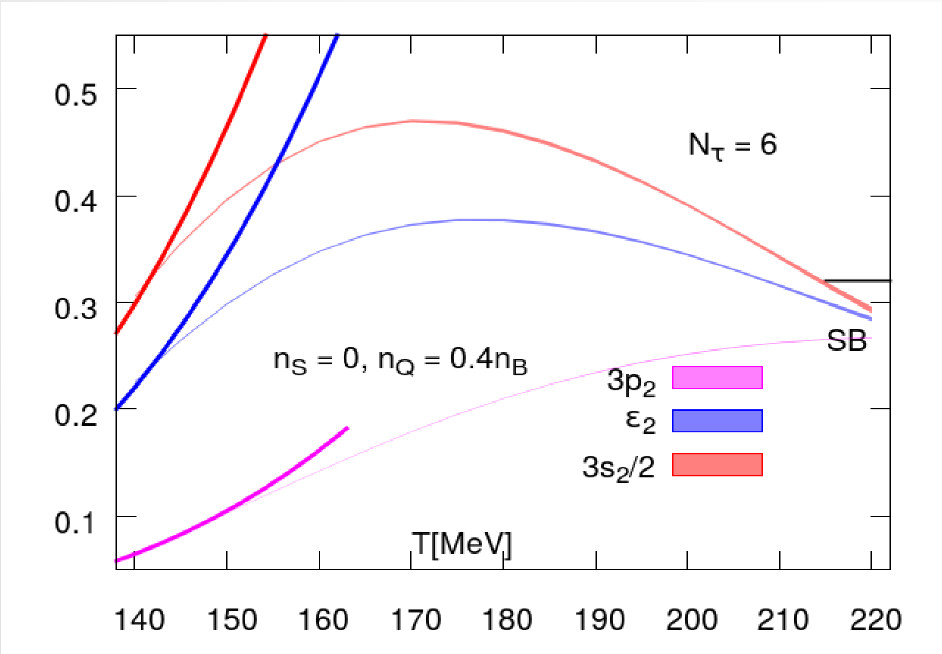
- We will use r = 0.40 throughout, which is the value for Pb-Pb collisions.
- Constrained case qualitatively similar to $mu_Q = mu_s = 0$ case, but values about 30-40% smaller.

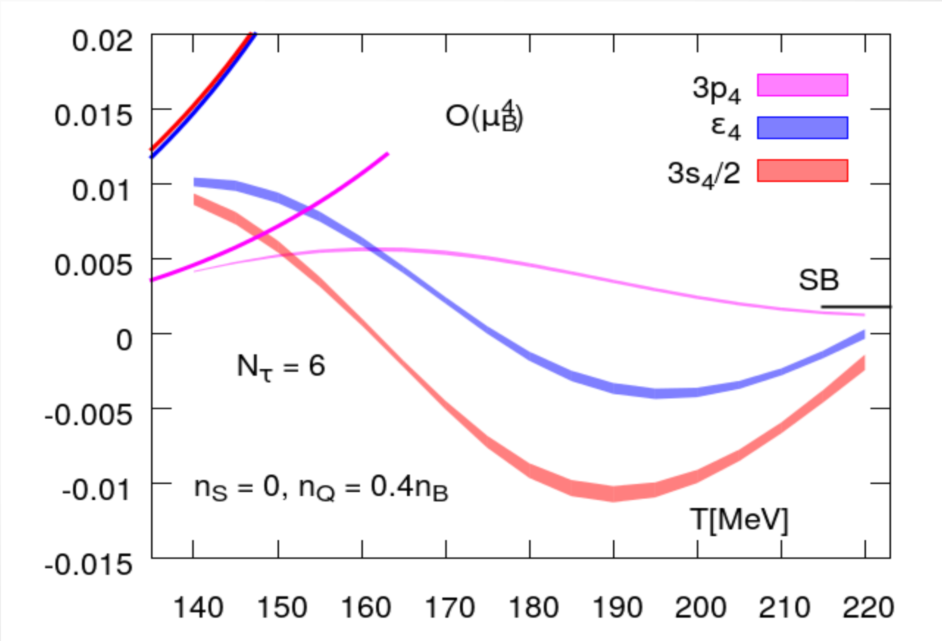
First derivatives

Just as for the pressure, we can Taylor-expand the energy density, and entropy density as well.

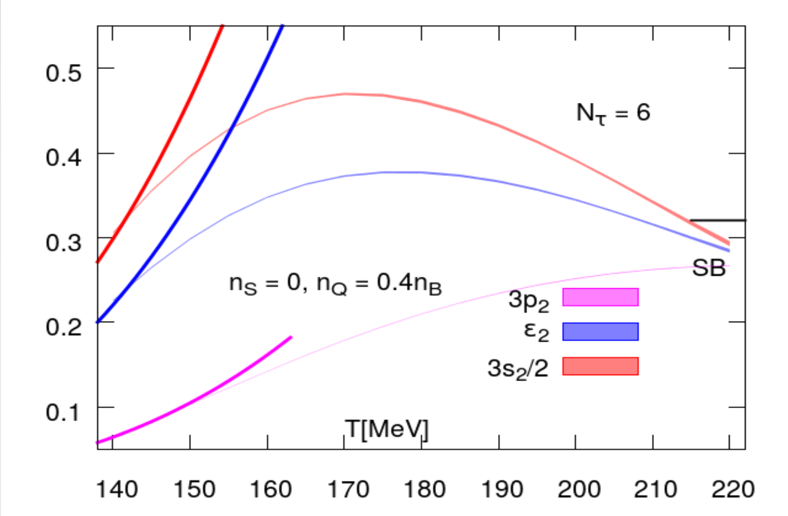
$$\frac{s}{T^3} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n \left\{T\frac{dc_n}{dT} + (4-n)c_n\right\}$$

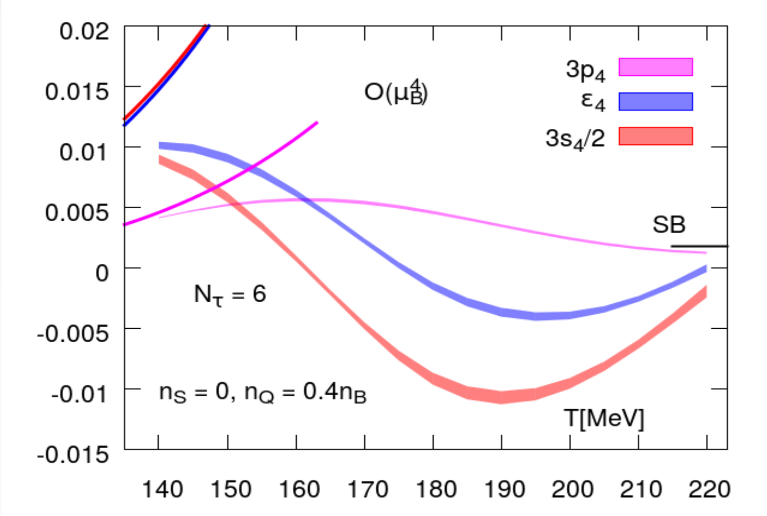
$$\frac{\varepsilon}{T^4} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n \left\{T\frac{dc_n}{dT} + 3c_n\right\}$$





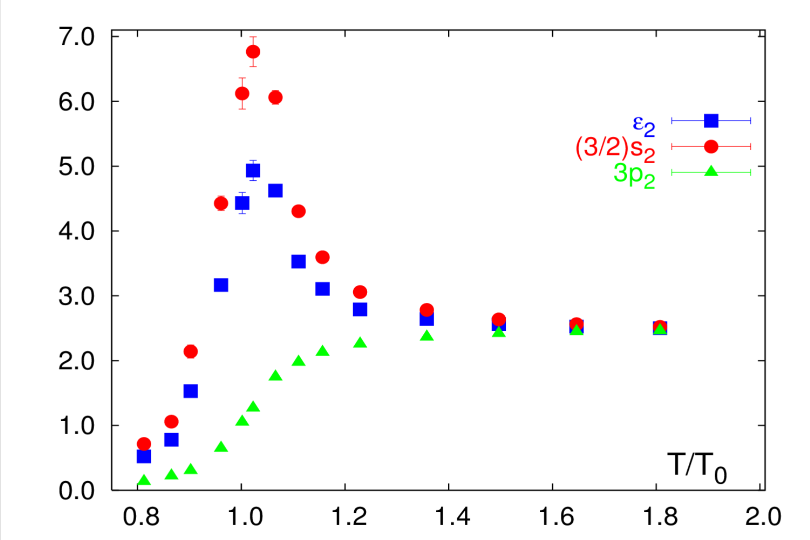
First derivatives

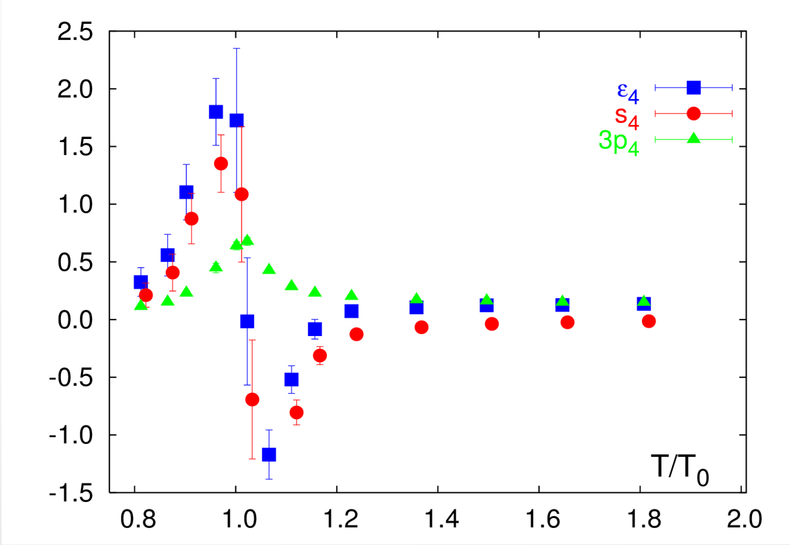




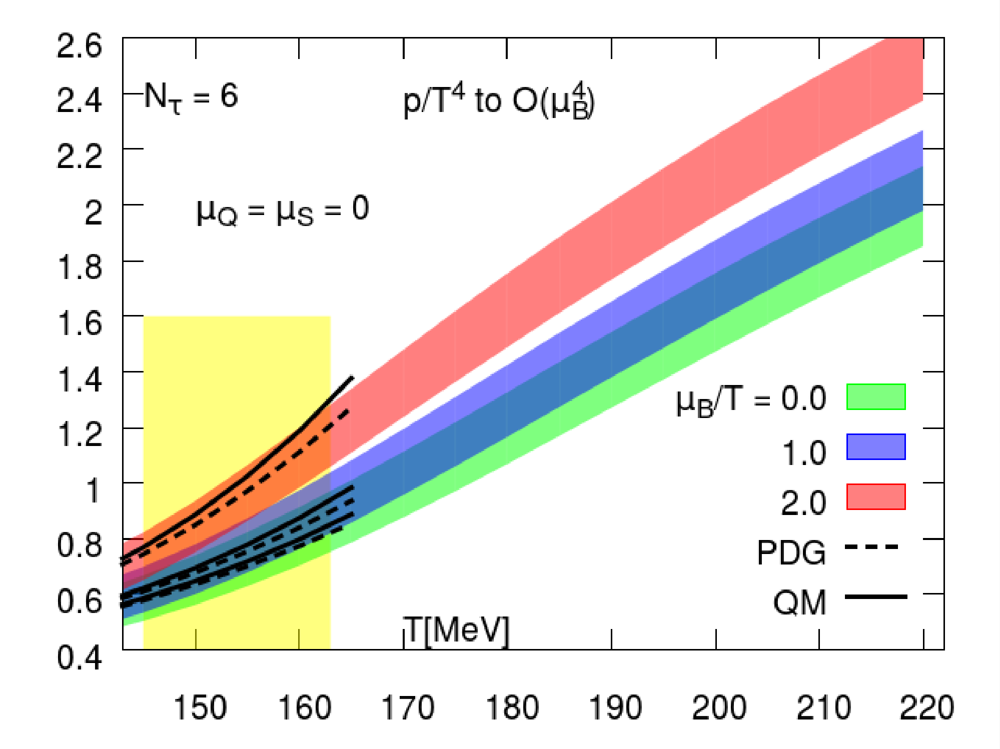
S.Ejiri *et al.* Phys. Rev. D73, 054506 (2006).

$$t \sim \left| \frac{T - T_c}{T_c} + \frac{\mu^2}{T_c^2} \right|$$

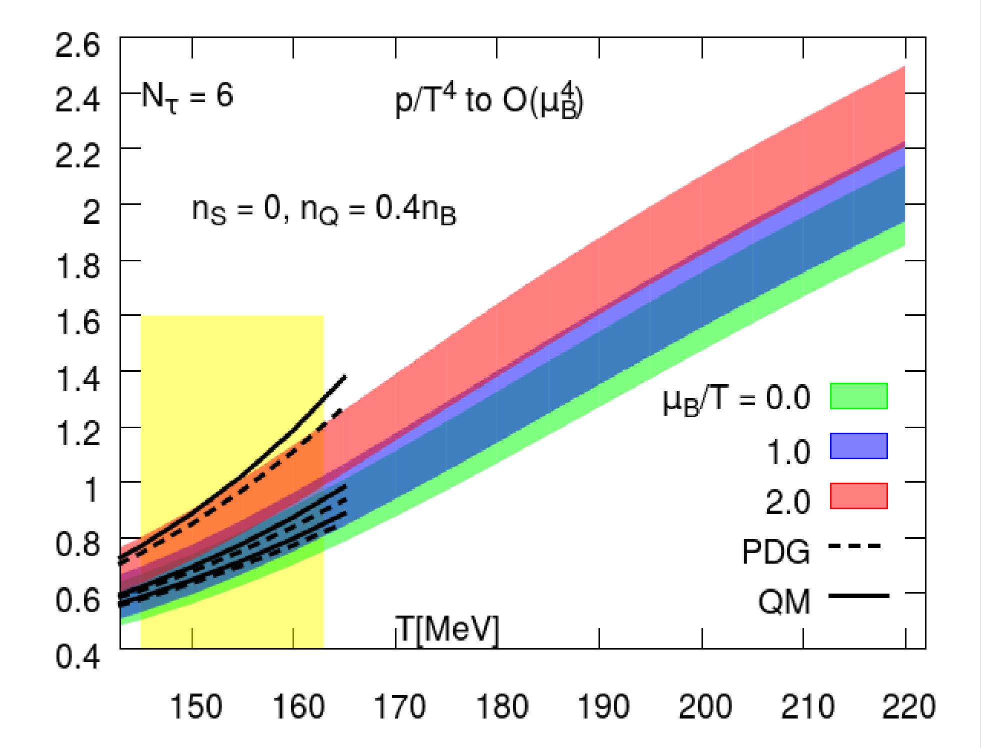




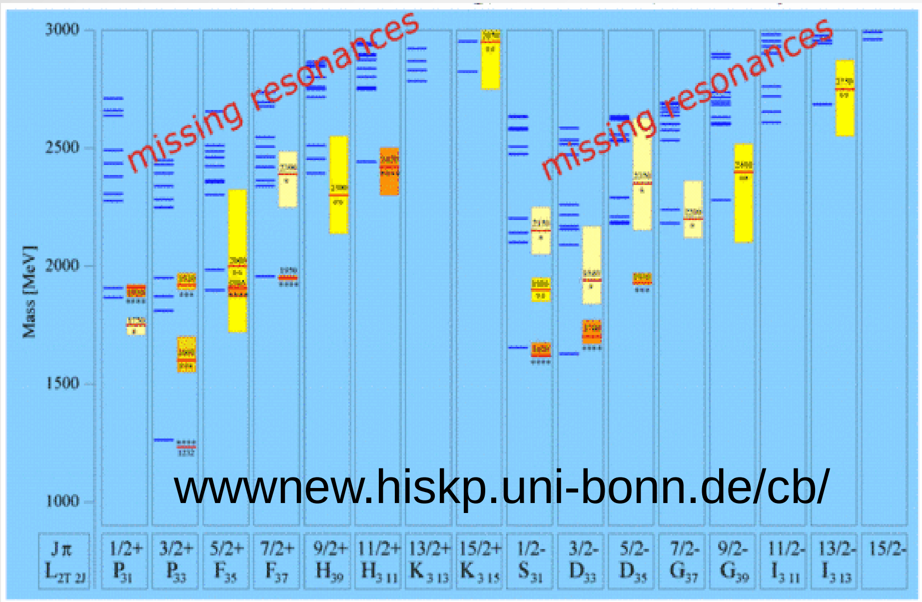
Putting everything together - I



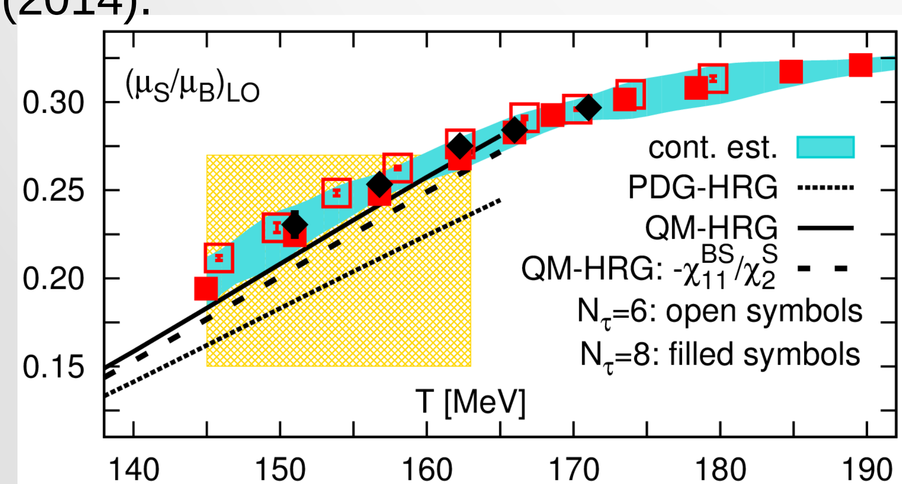
~10% corrections around the transition region up to $mu_B / T = 2.0$.

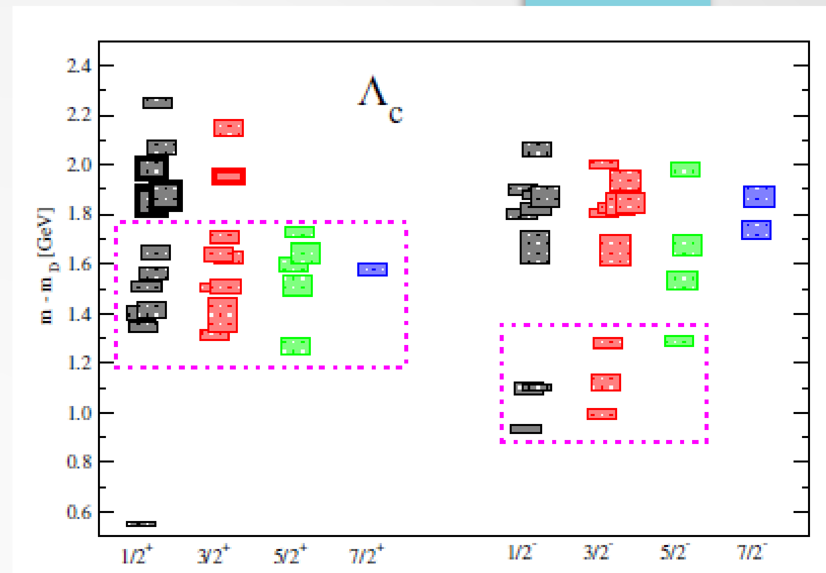


Additional resonances and the Quark Model



A.Bazavov *et al*. [HotQCD], PRL 113, 072001 (2014).



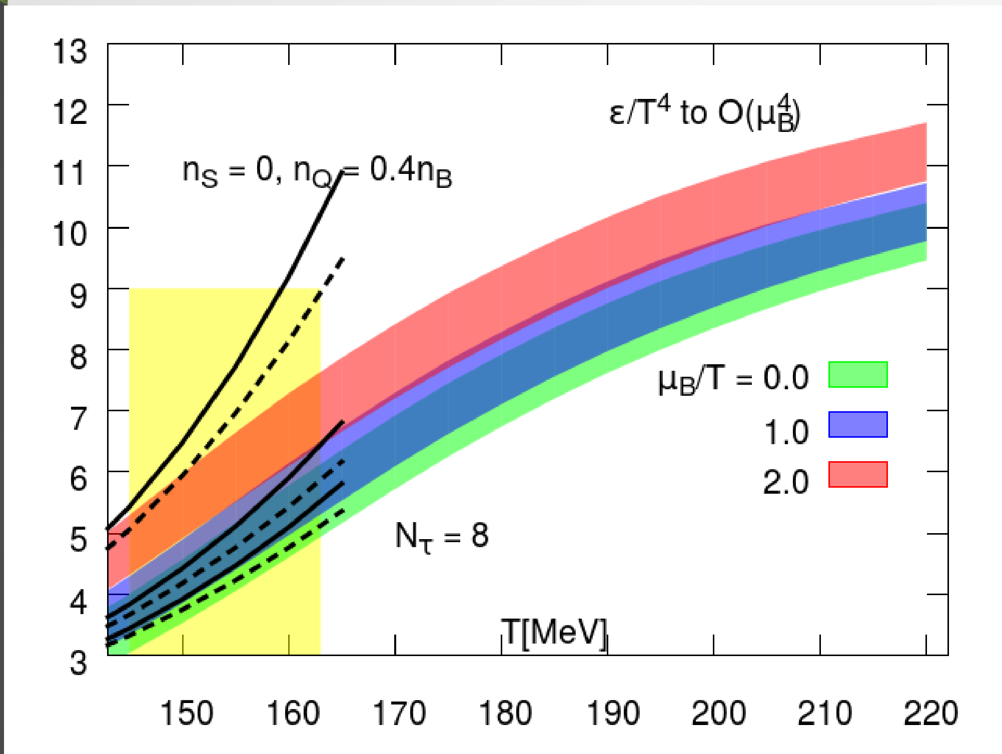


Padmanath *et al*. arXiv:1410.8791 [hep-lat]

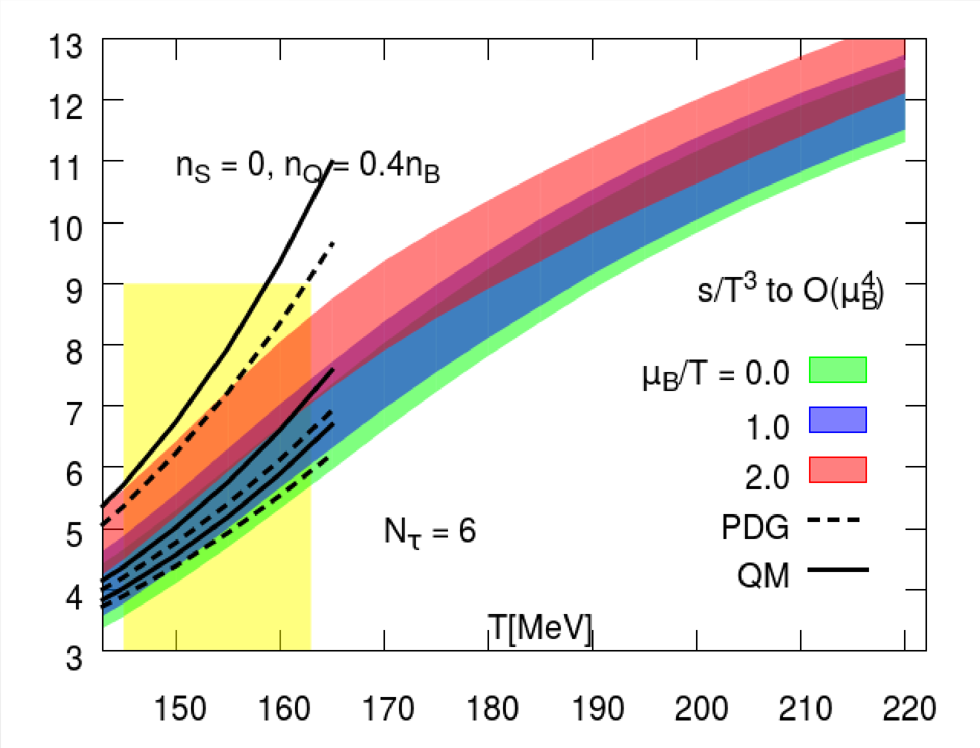
D.Ebert, R.Faustov & V.Galkin, Phys. Rev. D79, 114029 (2010).

Possibility of additional light and strange resonances beyond those listed in the PDG; see talk by C. Schmidt.

Putting everything together - II

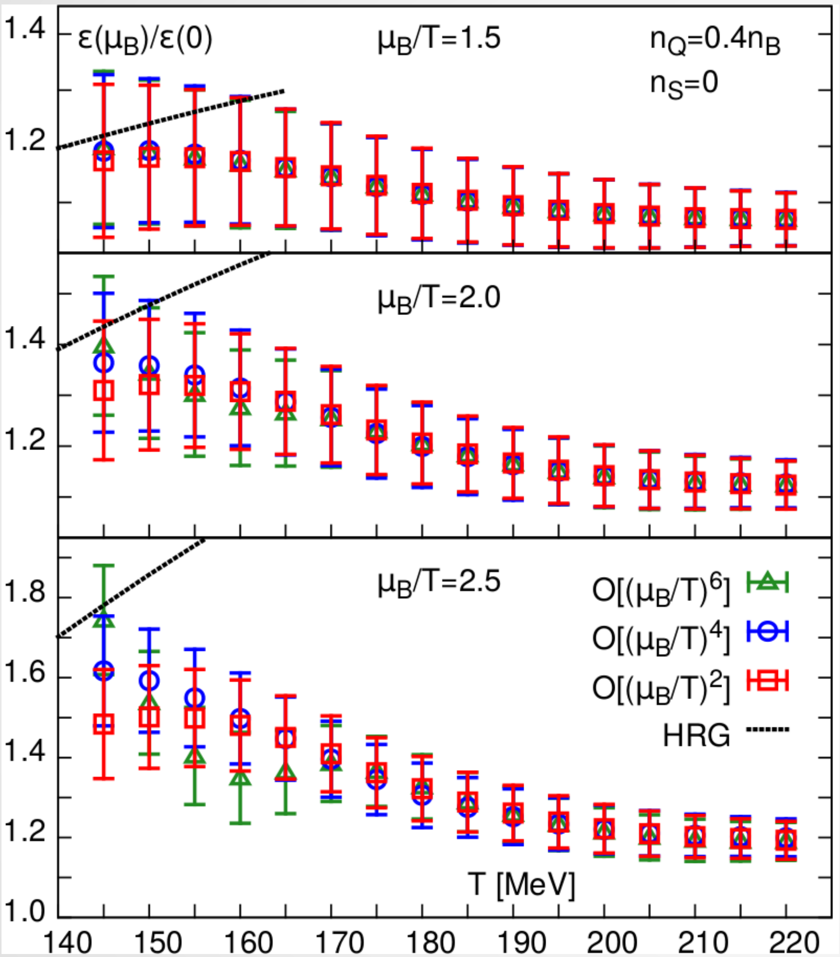


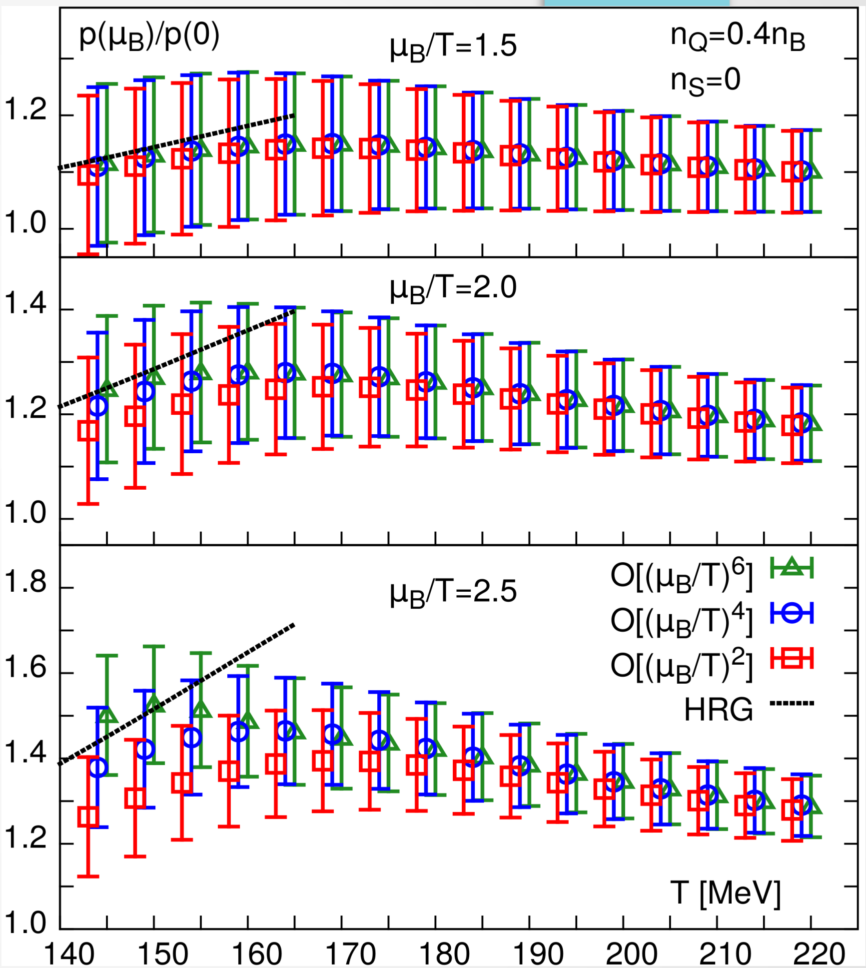
Higher derivatives expected to be more sensitive to higher-order corrections.



~10% corrections around the transition region up to $mu_B / T = 2.0$.

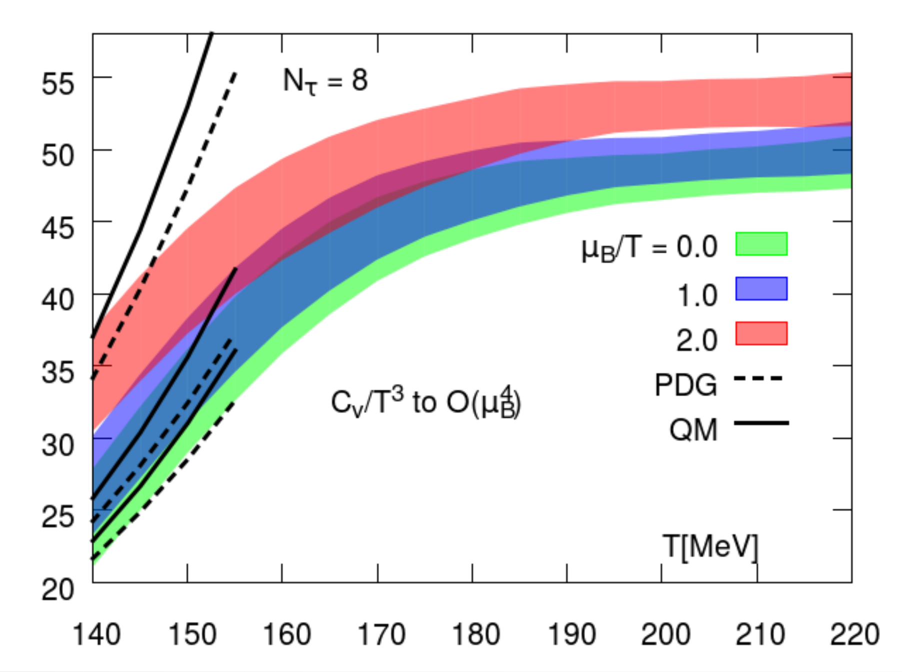
Range of extrapolation





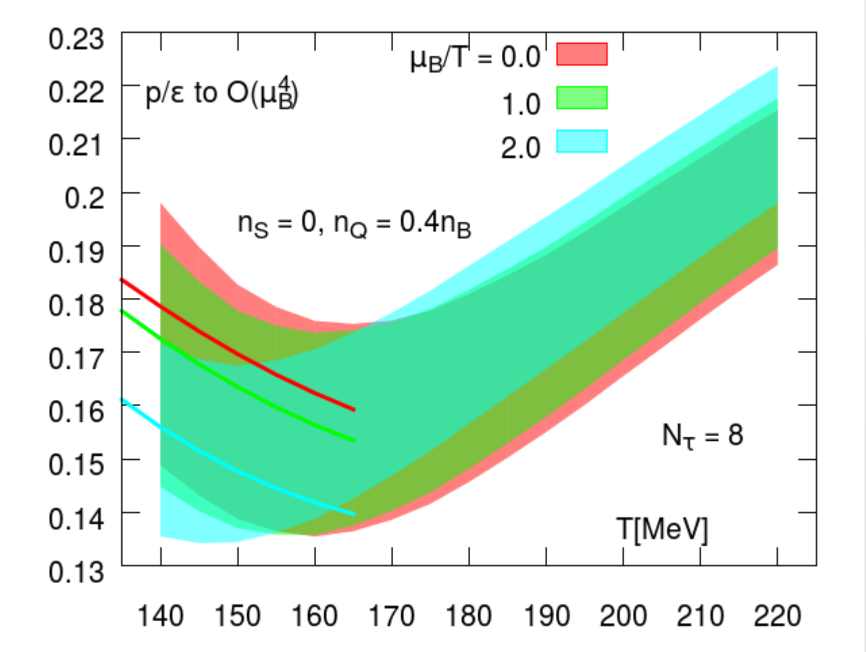
Different orders start to differ above $mu_B/T \sim 2.0$.

Specific heat and softest point

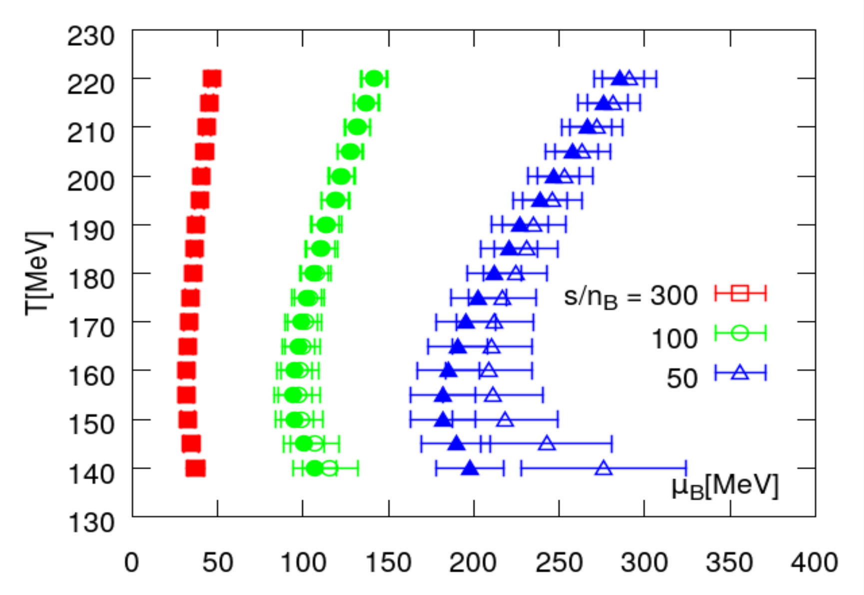


$$\frac{C_v}{T^3} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n \left\{ T^2 \frac{d^2 c_n}{dT^2} + 8T \frac{dc_n}{dT} + 12 c_n \right\}$$

Minimum in 'speed of sound' moves to lower T.



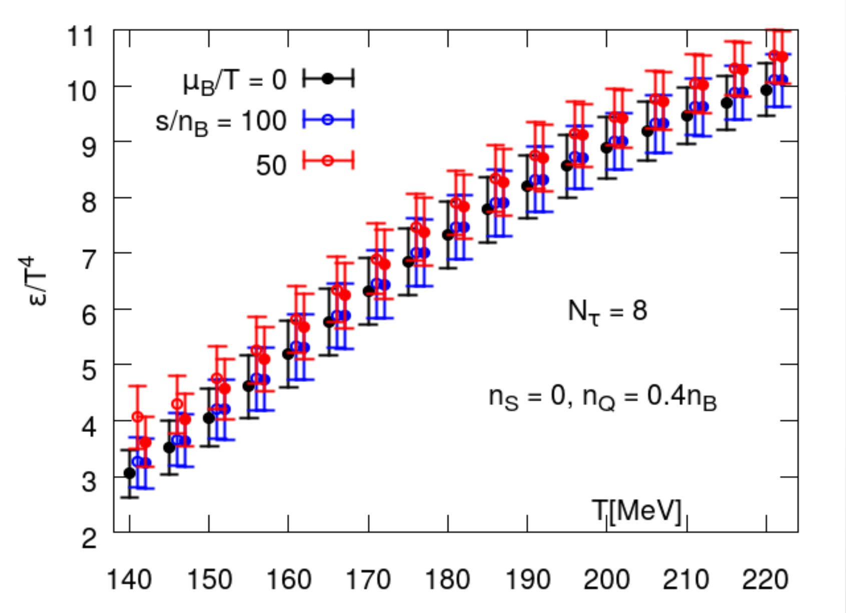
Equation of state at fixed s/n_B

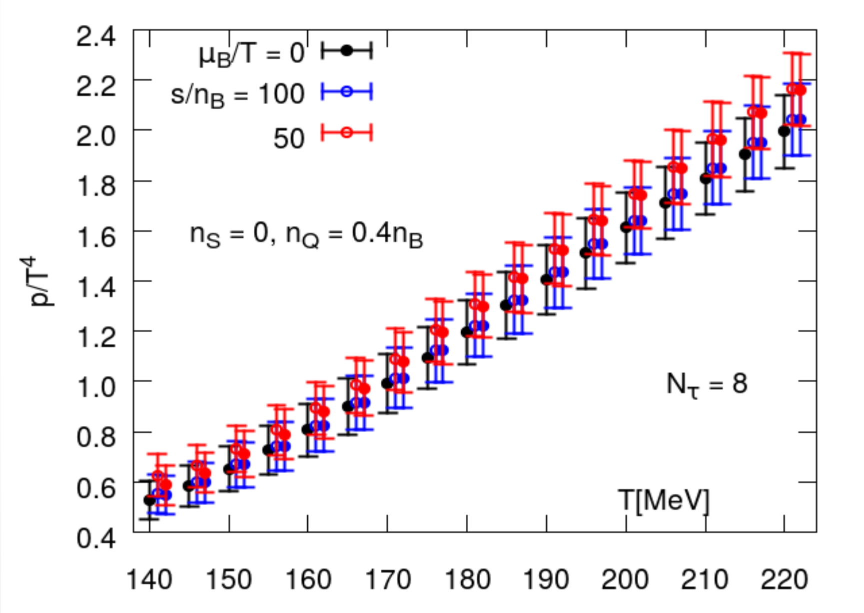


s/n_B=constant gives mu_B for a given T.

$$s_0 + s_2 \left(\frac{\mu_B}{T}\right)^2 = K n_{B_1} \frac{\mu_B}{T}$$

Quadratic to lowest order.





Freeze-out curve

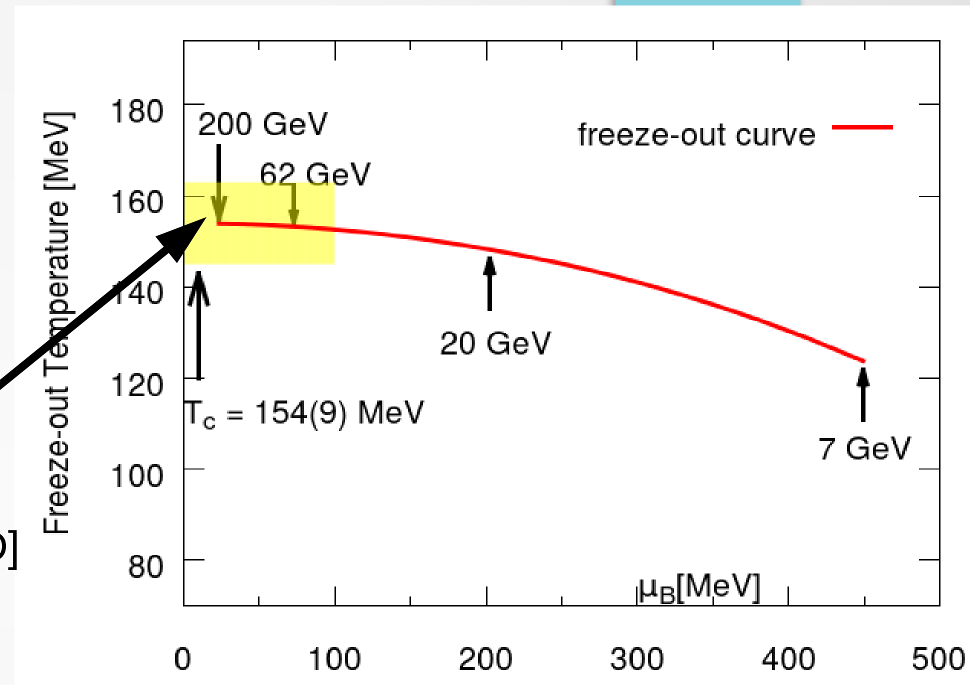
$$T^f = \frac{T^f_\infty}{1 + \exp\left(1.176 - \frac{\ln x}{0.45}\right)}$$

Andronic *et al.* Nucl.Phys. A772 (2006) 167-199.

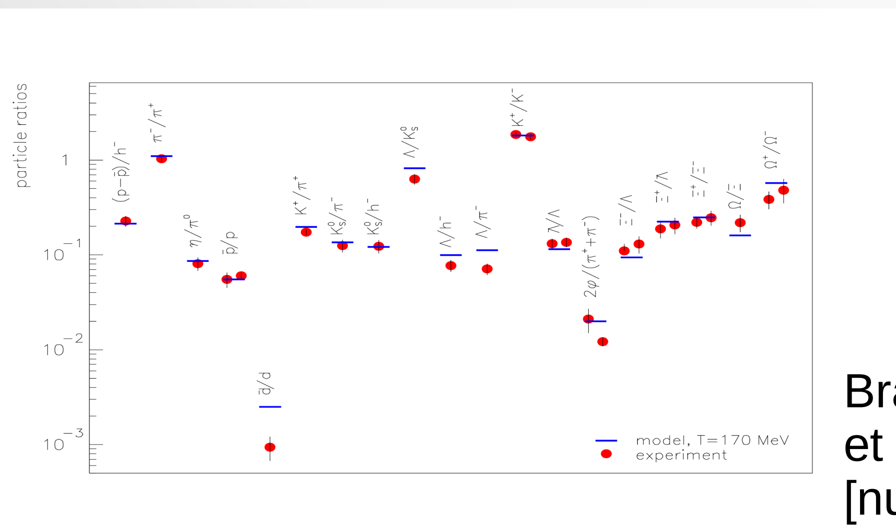
$$x = s_{NN}^{1/2} \text{ in GeV}$$

$$T^f_\infty = 154\,\text{MeV}$$

A.Bazavov *et al.* [HotQCD] Phys. Rev. D85 054503 (2012).

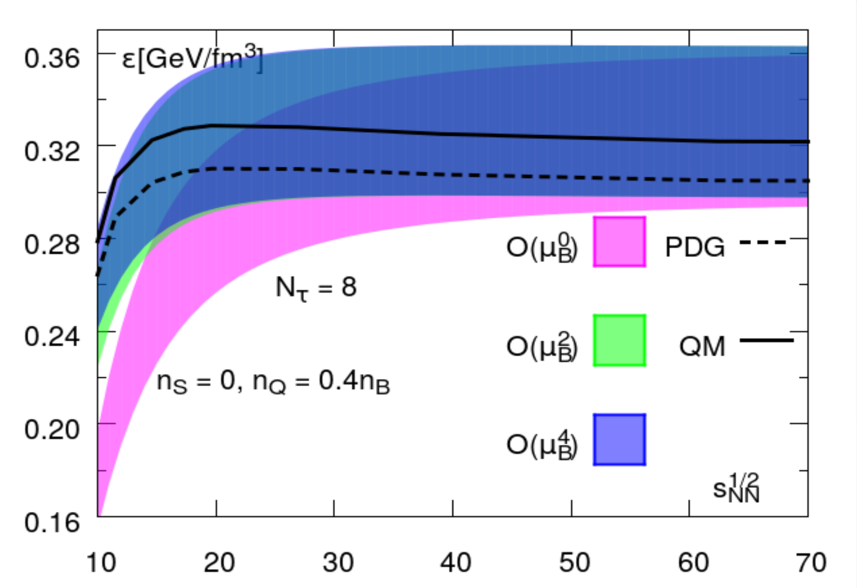


$$\mu_B^f = \frac{1.303}{1 + 0.286x}\ \text{GeV}$$

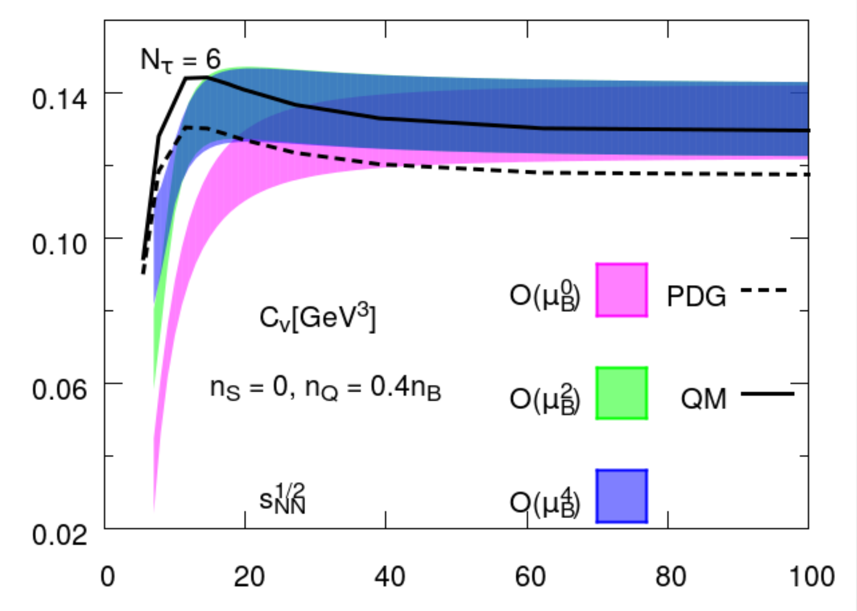


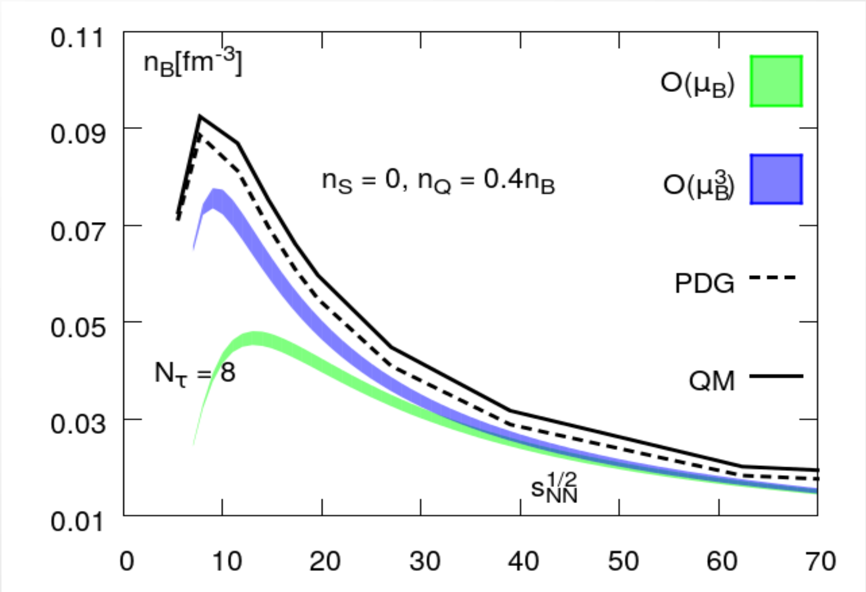
Braun-Munzinger *et al.* in Hwa, R.C. (ed.) et al.: Quark gluon plasma* 491-599 [nucl-th/0304013].

Freeze-out equation of state



Fourth-order expansion valid down to beam energies ~ 20 GeV.





Conclusions

- Need robust QCD inputs to extract physics from heavy-ion collisions.
- Lattice QCD has provided such inputs in the past and can continue to do so in the future.
- Equation of state is a key input in hydrodynamic modelling. $\mu=0$ useful at LHC and RHIC 200 GeV runs, whereas $\mu>0$ useful for the Beam Energy Scan programs.
- Fourth-order Taylor expansion can provide an equation of state valid upto $\sqrt{s} \sim 20$ GeV. With higher orders we should be able to push this to even lower CoM energies (unless the expansion breaks down).