

# **Thermodynamic and Transport Properties in the Clustering of Color Sources Approach in Nuclear Collisions**

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*in collaboration with*

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**C. Pajares ( Universidale de Santiago de Compostela)**

**QCD @ High Density**  
**TIFR**  
**Mumbai, India**  
**Jan. 27-30, 2015**

# Exploration of Hot QCD Matter The Next Decade

**Berndt Muller**  
**CERN Theory Institute (HIC10)**  
**Aug.16- Sept.10, 2010**

Which **properties of hot QCD matter** can we hope to determine from relativistic heavy ion data (RHIC and LHC, maybe FAIR) ?

Easy for  
LQCD

$$T_{\mu\nu} \Leftrightarrow \varepsilon, p, s$$

**Equation of state:** spectra, coll. flow, fluctuations

$$c_s^2 = \partial p / \partial \varepsilon$$

**Speed of sound:** multiparticle correlations

$$\eta = \frac{1}{T} \int d^4x \langle T_{xy}(x) T_{xy}(0) \rangle$$

**Shear viscosity:** anisotropic collective flow

Hard for  
LQCD

$$\hat{q} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle F_i^{a+}(y^-) F_i^{a+}(0) \rangle$$

$$\hat{e} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle i \partial^- A^{a+}(y^-) A^{a+}(0) \rangle$$

$$\hat{e}_2 = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle F^{a+-}(y^-) F^{a+-}(0) \rangle$$

**Momentum/energy diffusion:**  
 parton energy loss, jet fragmentation

Easy for  
LQCD

$$m_D = - \lim_{|x| \rightarrow \infty} \frac{1}{|x|} \ln \langle E^a(x) E^a(0) \rangle$$

**Color screening:** Quarkonium states

# Color Strings

Multiparticle production at high energies is currently described in terms of color strings stretched between the projectile and target. Hadronizing these strings produce the observed hadrons.

The no. of strings grow with energy and the no. of participating nuclei and one expects that interaction between them becomes essential.

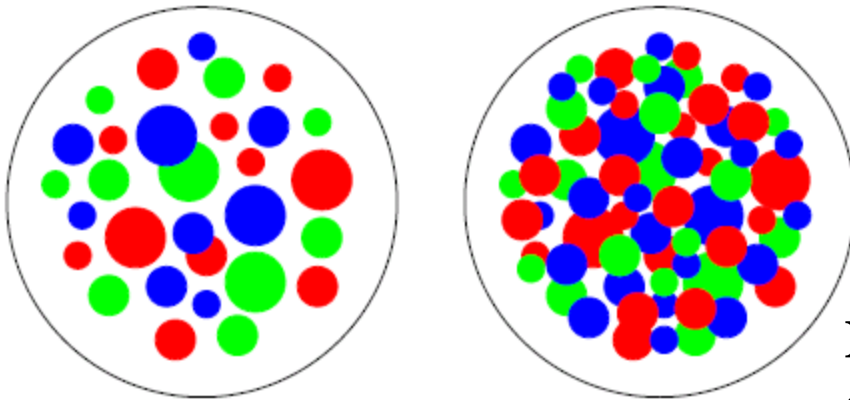
This problem acquires even more importance, considering the idea that at very high energy collisions of heavy nuclei (RHIC) may produce Quark-gluon Plasma (QGP).

The interaction between strings then has to make the system evolve towards the **QGP** state.

# Clustering of Color Sources

De-confinement is expected when the density of quarks and gluons becomes so high that it no longer makes sense to partition them into color-neutral hadrons, since these would overlap strongly.

We have clusters within which color is not confined : De-confinement is thus related to cluster formation very much similar to cluster formation in percolation theory and hence a connection between percolation and de-confinement seems very likely.



Parton distributions in the transverse plane of nucleus-nucleus collisions

In two dimensions, for uniform string density, the percolation threshold for overlapping discs is:  $\xi_c = 1.18$

H. Satz, Rep. Prog. Phys. 63, 1511(2000).  
H. Satz , hep-ph/0212046

**Critical Percolation Density**

# Percolation : General

The general formulation of the percolation problem is concerned with elementary geometrical objects placed at random in a  $d$ -dimensional lattice. The objects have a well defined connectivity radius  $\lambda$ , and two objects are said to communicate if the distance between them is less than  $\lambda$ .



One is interested in how many objects can form a cluster of communication and, especially, when and how the cluster becomes infinite. The control parameter is the density of the objects or the dimensionless filling factor  $\xi$ . The percolation threshold  $\xi = \xi_c$  corresponding to the minimum concentration at which an infinite cluster spans the space.

# Percolation : General

It is well known that the percolation problem on large lattices displays the features of a system undergoing a second-order phase transition.

These characteristics include critical fluctuations, quantities which diverge, and quantities which vanish as the critical percolation probability is approached. These quantities are described by a finite number of critical exponents.

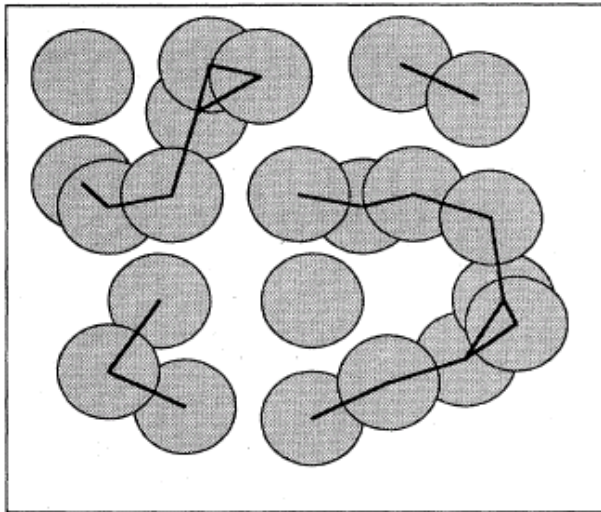
- \* **Transition from liquid to gas**
- \* **Normal conductor to a superconductor**
- \* **Paramagnet to ferromagnet**

1. H. E. Stanley , Introduction to Phase Transitions and Critical Phenomena
2. D. Stauffer and A. Aharony, Introduction to Percolation Theory

## Percolation, statistical topography, and transport in random media

M. B. Isichenko

Reviews of Modern Physics, Vol. 64, No. 4, October 1992



$$\xi = \pi n r^2$$

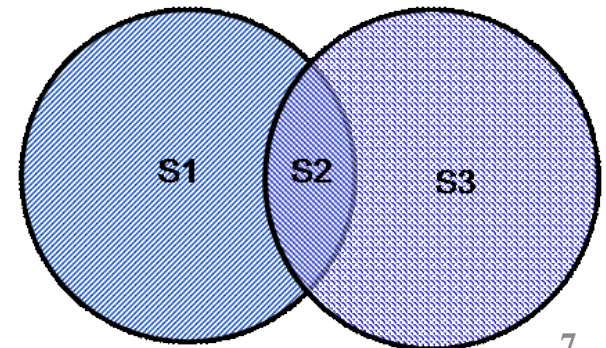
$\xi$  is the percolation density parameter  
 $n$  is the density and  $r$  the radius of the disc  
 $\phi$  is the fractional area covered by the cluster  
$$\phi = 1 - e^{-\xi}$$

$\xi_c$  is the critical value of the percolation density at which a communicating cluster appears

For example at  $\xi_c = 1.2$ ,  $\phi \sim 2/3$

It means  $\sim 67\%$  of the whole area is covered by the cluster

In the nuclear case it is the overlap area





# Parton Percolation

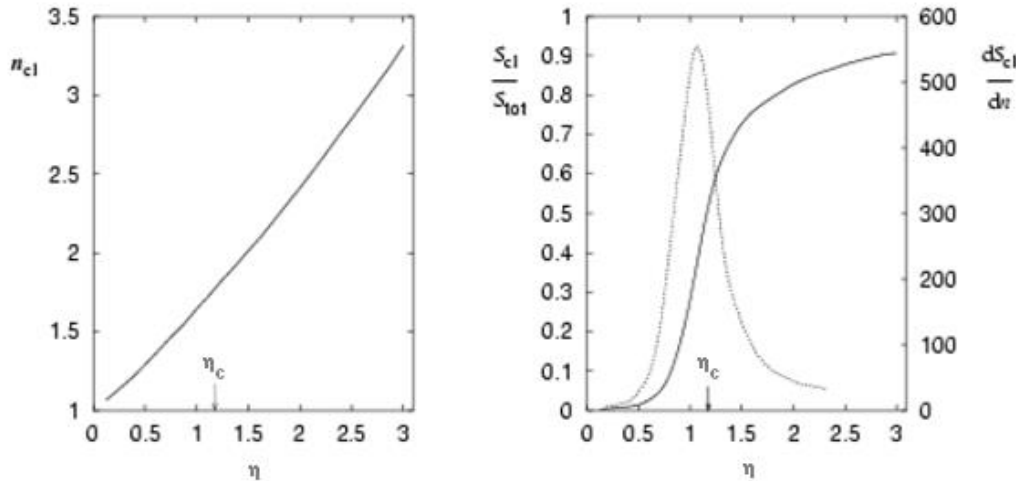


Figure 38: Average cluster density  $n_{cl}(n)$  (left) and average fractional cluster size  $S_{cl}(n)/S_{tot}(n)$  (right) as function of the overall density  $n$  of discs, for  $r/R = 1/20$ ; in (b), the derivative of  $S_{cl}(n)/S_{tot}(n)$  with respect to  $n$  is also shown (dotted line). The percolation point in the limit  $r/R \rightarrow 0$  is indicated by  $n_p$ .

**In two dimensions, for uniform string density, the percolation threshold for overlapping discs is:**

$$\boxed{\xi_c = 1.18} = \text{critical percolation density}$$

Satz, hep-ph/0007069

**The fractional area covered by discs at the critical threshold is:**

$$\boxed{1 - e^{-\xi_c}}$$

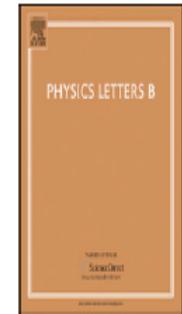




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Physics Letters B

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# Coherent center domains in SU(3) gluodynamics and their percolation at $T_c$

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## ABSTRACT

For SU(3) lattice gauge theory we study properties of static quark sources represented by local Polyakov loops. We find that for temperatures both below and above  $T_c$  coherent domains exist where the phases of the local loops have similar values in the vicinity of the center values  $0, \pm 2\pi/3$ . The cluster properties of these domains are studied numerically. We demonstrate that the deconfinement transition of SU(3) may be characterized by the percolation of suitably defined clusters.

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## Center Domains and their Phenomenological Consequences

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(Received 2 September 2012; revised manuscript received 6 November 2012; published 15 May 2013)

We argue that the domain structure of deconfined QCD matter, which can be inferred from the properties of the Polyakov loop, can simultaneously explain the two most prominent experimentally verified features of the quark-gluon plasma, namely its large opacity as well as its near ideal fluid properties.

# Color Strings + Percolation = CSPM

**Multiplicity and  $\langle p_T^2 \rangle$  of particles produced by a cluster of  $n$  strings**

**Multiplicity ( $\mu_n$ )**

$$\mu_n = F(\xi) N^s \mu_1$$

**Average Transverse Momentum**

$$\langle p_T^2 \rangle_n = \langle p_T^2 \rangle_1 / F(\xi)$$

$$F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}}$$

= **Color suppression factor**  
(due to overlapping of discs).

$$\xi = \frac{N^s S_1}{S_N}$$

**$\xi$  is the percolation density parameter.**

$N^s$  = # of strings  
 $S_1$  = disc area  
 $S_N$  = total nuclear overlap area

M. A. Braun and C. Pajares, Eur.Phys. J. C16,349 (2000)

M. A. Braun et al, Phys. Rev. C65, 024907 (2002)

# Percolation and Color Glass Condensate

Both are based on parton coherence phenomena.

**Percolation :** Clustering of strings

**CGC :** Gluon saturation

- ❑ Many of the results obtained in the framework of percolation of strings are very similar to the one obtained in the CGC.
- ❑ In particular , very similar scaling laws are obtained for the product and the ratio of the multiplicities and transverse momentum.
- ❑ Both provide explanation for multiplicity suppression and  $\langle p_t \rangle$  scaling with  $dN/dy$ .

Momentum  $Q_s$  establishes the scale in CGC with the corresponding one in percolation of strings

$$Q_s^2 = \frac{k \langle p_t^2 \rangle_1}{F(\xi)}$$

For large value of  $\xi$

$$Q_s^2 \propto \sqrt{\xi}$$

The no. of color flux tubes in CGC and the effective no. of clusters of strings in percolation have the same dependence on the energy and centrality.

This has consequences in the Long range rapidity correlations and the ridge structure.

# Results

## Bulk Properties

- ☐ Multiplicity
- ☐ pt distribution
- ☐ Particle ratios
- ☐ Elliptic flow
- ☐ Forward-Backward  
Multiplicity  
Correlations at RHIC

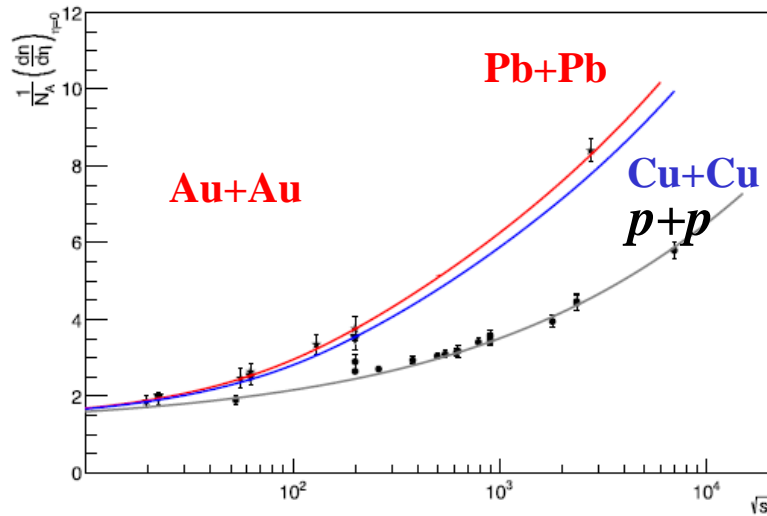
## Thermodynamics

- ☐ Temperature
- ☐ Energy Density
- ☐ Shear viscosity to  
Entropy density ratio
- ☐ Equation of State

**Determination of the Color Suppression Factor  $F(\xi)$   
from the Data**

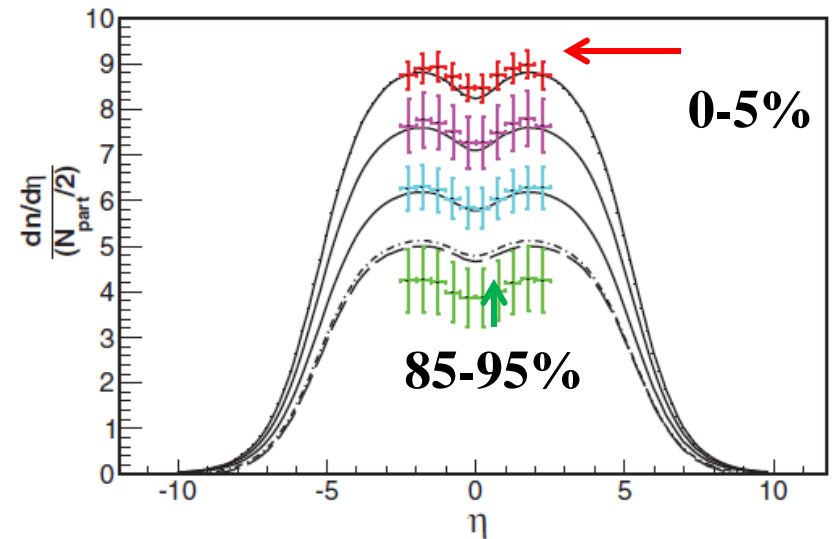
# Multiplicity

## Multiplicity dependence on $\sqrt{s}$



Phys. Lett., 715, 230 (2012)  
ALICE Coll. Phys. Rev. Lett., 106, 032301 (2011)

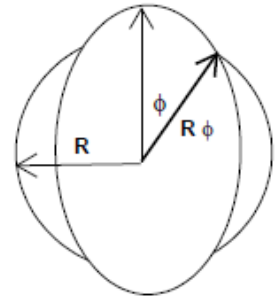
## Rapidity dependence of particle densities Pb+Pb at 2.76 TeV



Phys. Rev. C 86, 034909(2012)  
CMS Coll., JHEP, 08, 141 (2011).

# Elliptic flow at RHIC and LHC

- ❑ The cluster formed by the strings has generally asymmetric shape in the transverse plane. This azimuthal asymmetry is at the origin of the elliptic flow in percolation.
- ❑ The partons emitted inside the cluster have to pass a certain length through the strong color field and loose energy.
- ❑ The energy loss by the partons is proportional to the length and therefore the  $p_t$  of an particle will depend on the path length travelled and will depend on the direction of emission

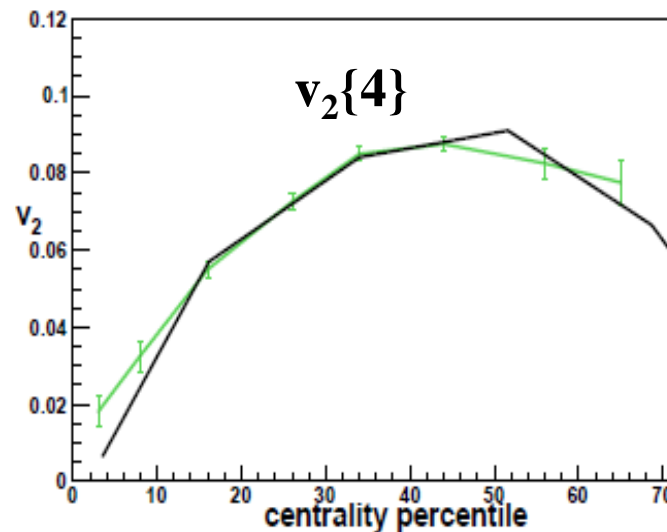
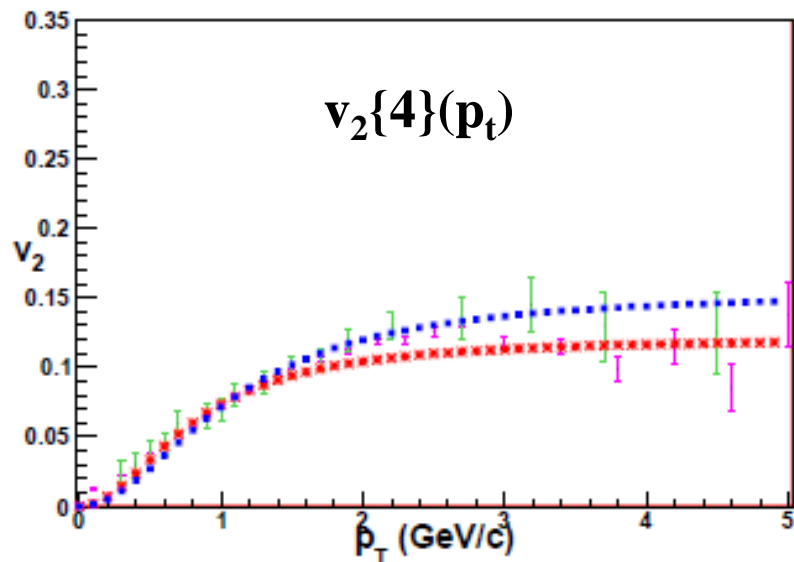


$$v_2 = \frac{2}{\pi} \int_0^\pi d\phi \cos 2\phi \left( \frac{R_\phi}{R} \right) \left( \frac{e^{-\xi} - F(\xi)^2}{2F(\xi)^3} \right) \frac{R}{R-1}$$

I.Bautista, J. Dias de Deus , C. Pajares, arXiv 1102:3837  
M. A. Braun , C. Pajares, Eur. J. Phys. C, 71 (2011) 1558



# Elliptic flow at RHIC and LHC



## Charged hadron elliptic flow

ALICE: 0-10% centrality [Pb+Pb@2.76 TeV](#)

Phys. Rev. Lett, 105 (2010) 252302

STAR: 0-10% centrality [Au+Au@200 GeV](#)

Phys. Rev. C 77, (2008) 054901

**Higher harmonics  $v_3$  to  $v_8$  have also been studied in percolation approach**

M. A. Braun, C. Pajares, V. Vechernin,  
[arXiv:1204:5829](#)

**Elliptic flow as a function of centrality in [Pb+Pb@2.76 TeV](#)**  
**Comparison with ALICE**

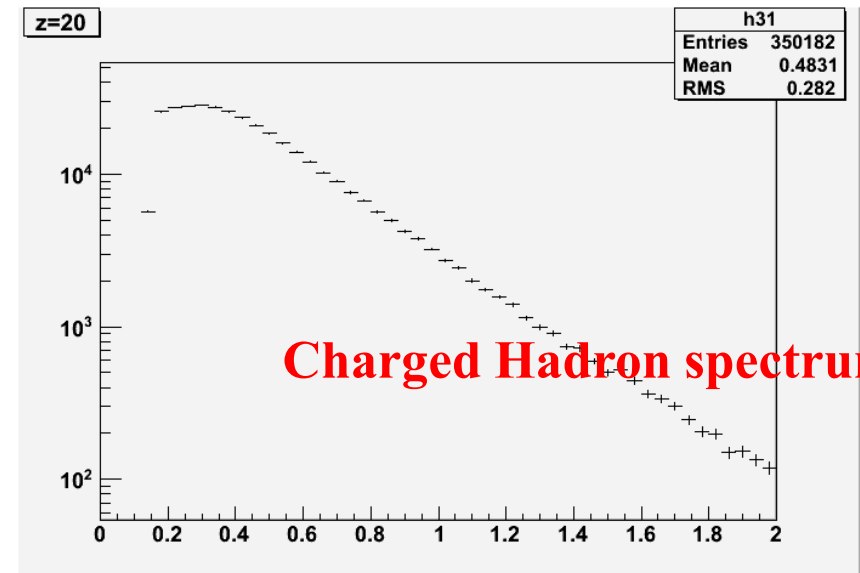
## Using the $p_T$ spectrum to calculate $\xi$

The experimental  $p_T$  distribution from pp data is used

$$\frac{d^2 N}{d p_T^2} = \frac{a}{(p_0 + p_T)^n}$$

$$\frac{d^2 N}{d p_T^2}$$

$a$ ,  $p_0$  and  $n$  are parameters fit to the data.



Charged Hadron spectrum

$p_T$  GeV/c

This parameterization can be used for nucleus-nucleus collisions to account for the clustering :

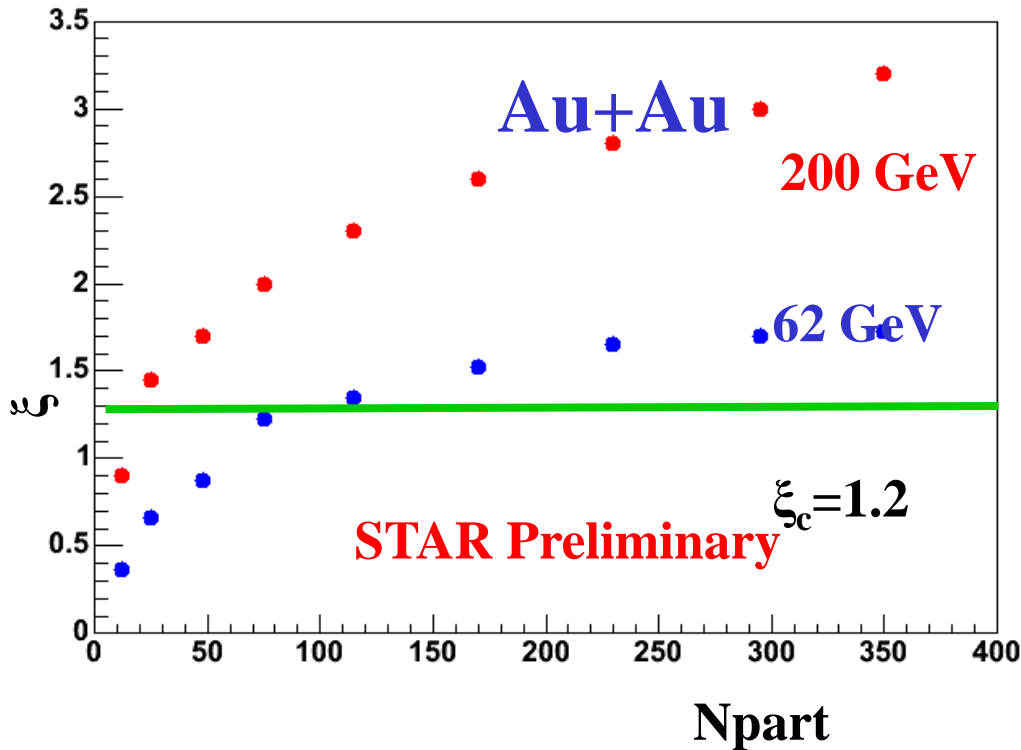
$$\frac{d^2 N}{d p_T^2} = \frac{b}{\left( p_0 \sqrt{\frac{F(\xi_{pp})}{F(\xi_{AuAu})}} + p_T \right)^n}$$

$$F(\xi)_{pp} = 1$$

$$F(\xi)_{AuAu} = 0.57$$

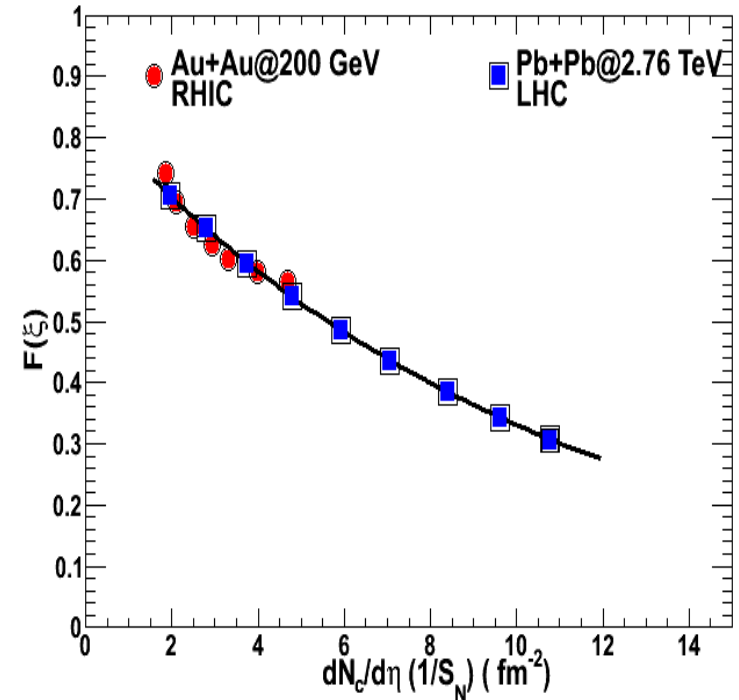
For central collisions

# Percolation Density Parameter $\xi$



STAR Coll., Nucleonika, 51, s109 (2006)

$$F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}}$$



Using ALICE charged particle multiplicity  
Phys. Rev. Lett. , 106, 032301 (2011).

Now the aim is to connect  $F(\xi)$  with Temperature and Energy density

# **Thermodynamic and Transport Properties**

# Schwinger Mechanism of Particle Production

$p_t$  distribution of the produced quarks

$$\frac{dn}{d^2 p_{\perp}} \sim \exp\left(-\frac{\pi p_t^2}{k}\right)$$

$k$  is the string tension

The tension of the macroscopic cluster fluctuates around its mean value because the chromoelectric field is not constant . Assuming a Gaussian form for these fluctuations one arrives at the probability distribution of transverse momentum.

**Thermal Distribution**

$$\frac{dn}{d^2 p_{\perp}} \sim \exp\left(-\frac{\pi p_t^2}{T}\right)$$

$$T = \sqrt{\frac{\langle k \rangle}{2\pi}}$$



$$T = \sqrt{\frac{\langle p_t^2 \rangle_1}{2F(\xi)}}$$

**Cluster /Initial  
Temperature**

# Temperature

$$T = \sqrt{\frac{\langle p_t^2 \rangle_1}{2F(\xi)}}$$

At the critical percolation density

$$\xi_c = 1.2 \quad T_c = 167 \text{ MeV}$$

For Au+Au@ 200 GeV

0-10% centrality  $\xi = 2.88$   **$T \sim 195 \text{ MeV}$**

**PHENIX:**

Temperature from direct photon  
Exponential (consistent with thermal)

Inverse slope =  **$220 \pm 20 \text{ MeV}$**

PRL 104, 132301 (2010)

**Pb+Pb @ 2.76 TeV for 0-5%**

**$T \sim 265 \text{ MeV}$**

**Temperature has increased by 35% from Au+Au @ 0.2 TeV**

**First Results from Pb+Pb Collisions@ 2.76 TeV at the LHC**

**Muller, Schukraft and Wyslouch, Ann. Rev. Nucl. Sci. Oct. 2012**

**ALICE : Direct Photon Measurement**

**$T = 304 \pm 51 \text{ MeV}$**

**QM 2012**

# Thermalization

- ❑ The origin of the string fluctuation is related to the stochastic picture of the QCD vacuum . Since the average value of color field strength must vanish, it cannot be constant and must vanish from point to point. Such fluctuations lead to the Gaussian distribution of the string.

H. G. Dosch, Phys. Lett. 190 (1987) 177

A. Bialas, Phys. Lett. B 466 (1999) 301

- ❑ The fast thermalization in heavy ion collisions can occur through the existence of event horizon caused by rapid deceleration of the colliding nuclei. Hawking-Unruh effect.

D. Kharzeev, E. Levin , K. Tuchin, Phys. Rev. C75, 044903 (2007)

H.Satz, Eur. Phys. J. 155, (2008) 167



# Energy Density

**Bjorken Phys. Rev. D 27, 140 (1983)**

$$\varepsilon = \frac{3}{2} \frac{dN_c}{dy} \frac{\langle m_t \rangle}{A \tau_{pro}} \text{GeV} / \text{fm}^3$$

**Transverse overlap area**

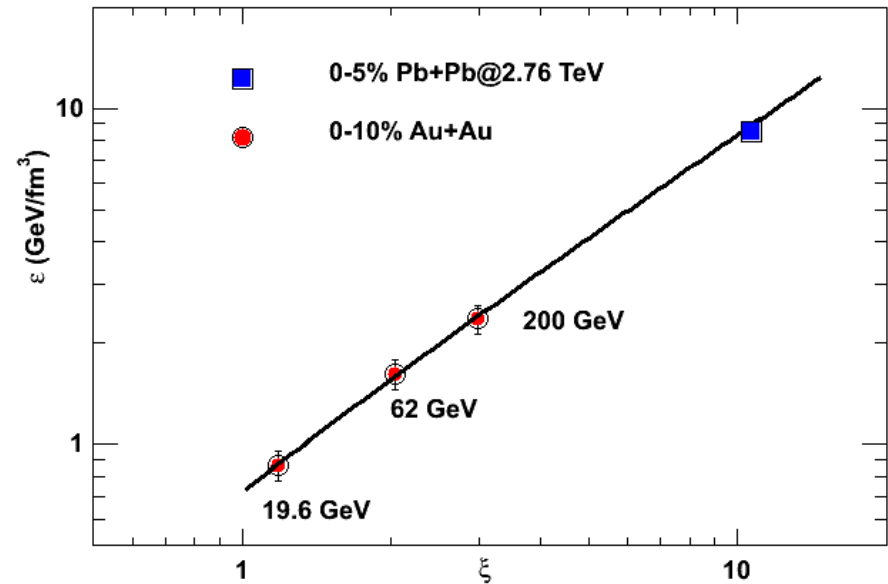
**Proper Time**

$\tau_{pro}$  is the QED production time for a boson which can be scaled from QED to QCD and is given by

$$\tau_{pro} = \frac{2.405\hbar}{\langle m_t \rangle}$$

**STAR Coll., Phys. Rev. C 79, 34909 (2009)**

**Introduction to high energy  
heavy ion collisions  
C. Y. Wong**



$$\varepsilon \propto \xi$$

**J. Dias de Deus, A. S. Hirsch, C. Pajares ,  
R. P. Scharenberg , B. K. Srivastava  
Eur. Phys. J. C 72, 2123 ( 2012)**

Having determined the initial temperature of the system from the data one would like to obtain the following quantities to understand the properties of QCD matter

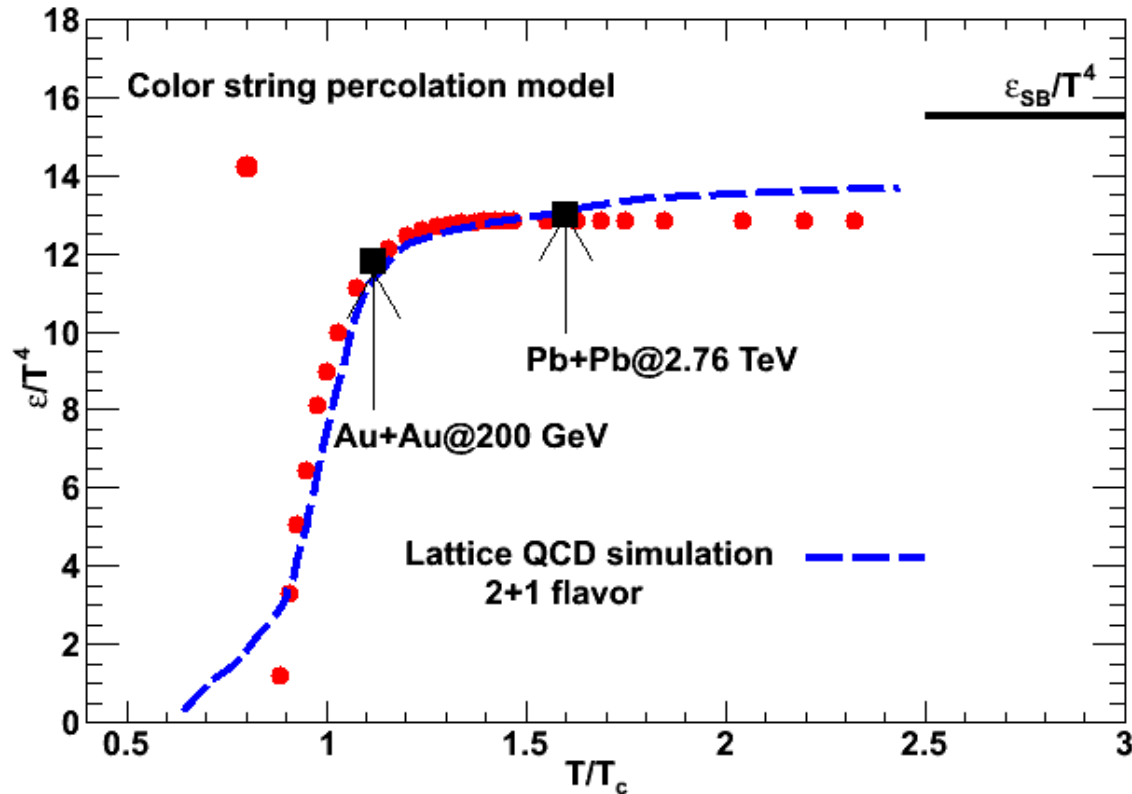
$$\varepsilon / T^4$$

Shear Viscosity

Equation of State

# Energy Density

0-10% centrality



Lattice QCD : Bazavov et al, Phys. Rev. D 80, 014504(2009).

**R. P. Scharenberg , B. K. Srivastava, A. S. Hirsch**  
**Eur. Phys. J. C 71, 1510( 2011)**

## Strongly Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions

Laszlo P. Csernai,<sup>1,2</sup> Joseph I. Kapusta,<sup>3</sup> and Larry D. McLerran<sup>4</sup>

<sup>1</sup>*Section for Theoretical Physics, Department of Physics, University of Bergen, Allegaten 55, 5007 Bergen, Norway*

<sup>2</sup>*MTA-KFKI, Research Institute of Particle and Nuclear Physics, 1525 Budapest 114, P. O. Box 49, Hungary*

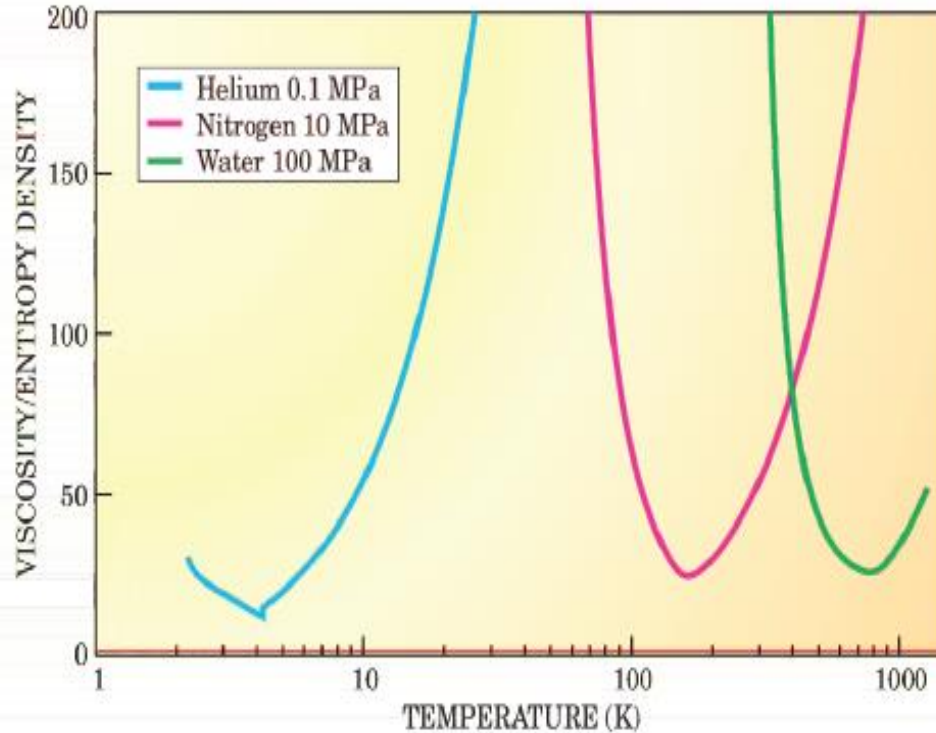
<sup>3</sup>*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*

<sup>4</sup>*Nuclear Theory Group and Riken Brookhaven Center, Brookhaven National Laboratory, Bldg. 510A, Upton, New York 11973, USA*

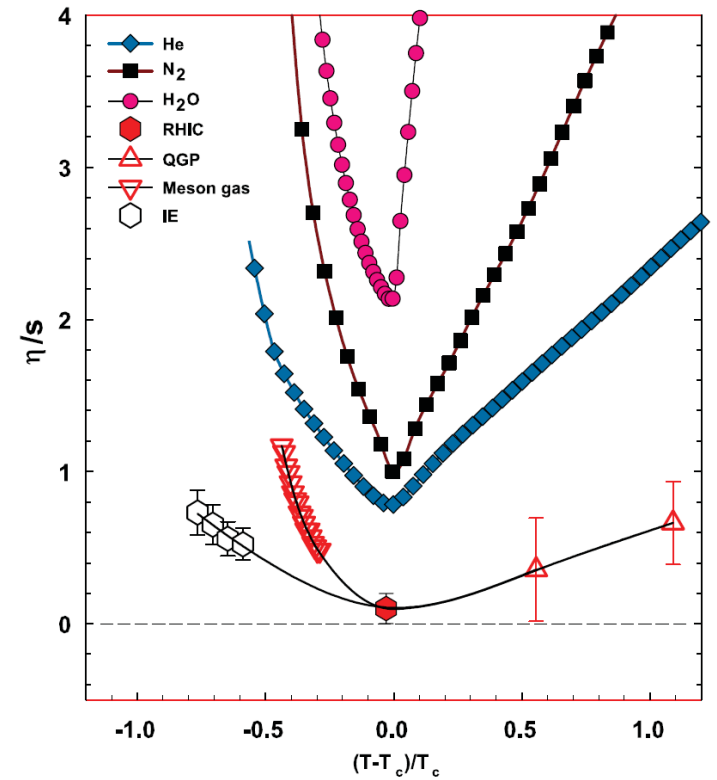
(Received 12 April 2006; published 12 October 2006)

Substantial collective flow is observed in collisions between large nuclei at BNL RHIC (Relativistic Heavy Ion Collider) as evidenced by single-particle transverse momentum distributions and by azimuthal correlations among the produced particles. The data are well reproduced by perfect fluid dynamics. A calculation of the dimensionless ratio of shear viscosity  $\eta$  to entropy density  $s$  by Kovtun, Son, and Starinets within anti-de Sitter space/conformal field theory yields  $\eta/s = \hbar/4\pi k_B$ , which has been conjectured to be a lower bound for any physical system. Motivated by these results, we show that the transition from hadrons to quarks and gluons has behavior similar to helium, nitrogen, and water at and near their phase transitions in the ratio  $\eta/s$ . We suggest that experimental measurements can pinpoint the location of this transition or rapid crossover in QCD.

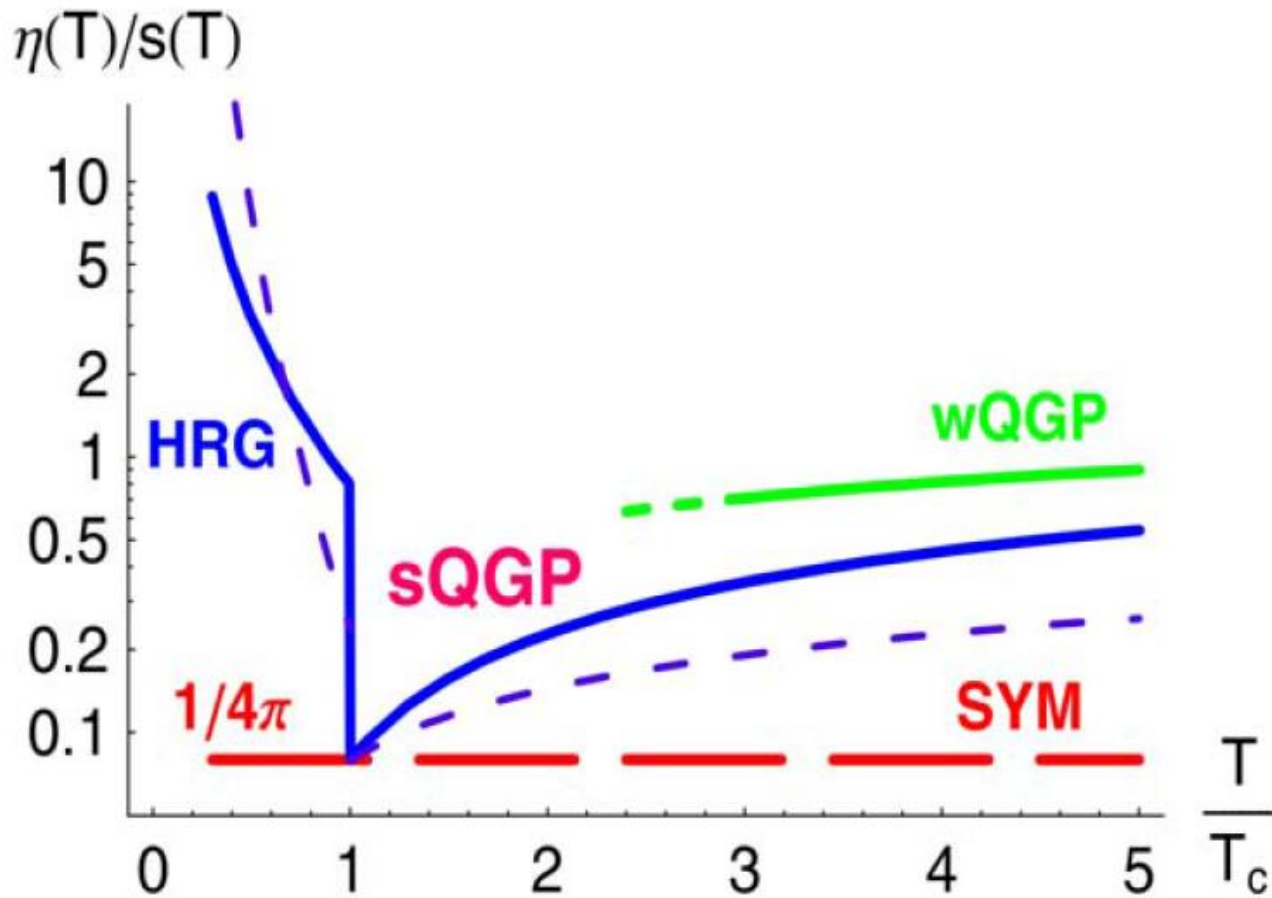
# $\eta/s$ for He, N<sub>2</sub> and H<sub>2</sub>O



PRL 97, 152303 (2006)



Lacey et al., PRL 98, 092301 (2007)



Hirano and Gyulassy, Nucl. Phys. A 769, 71 (2006)

# The viscosity can be estimated from kinetic theory to be

$$\eta \approx \frac{4}{15} \varepsilon(T) \lambda_{mfp} \approx \frac{1}{5} \frac{T}{\sigma_{tr}} \frac{s(T)}{n(T)}$$

$$\varepsilon(T) = \frac{3}{4} Ts$$

$$\lambda_{tr} = \frac{1}{(n \sigma_{tr})}$$

$$\frac{n}{s} \approx \frac{T \lambda_{mfp}}{5}$$

Hirano & Gyulassy, Nucl. Phys. A769, 71(2006)

$\varepsilon$  Energy density

$s$  Entropy density

$n$  the number density

$\lambda_{mfp}$  Mean free path

$\sigma_{tr}$  Transport cross section

$\sqrt{\langle pt \rangle_1^2}$  Average transverse momentum of the single string

$L$  is Longitudinal extension of the source 1 fm

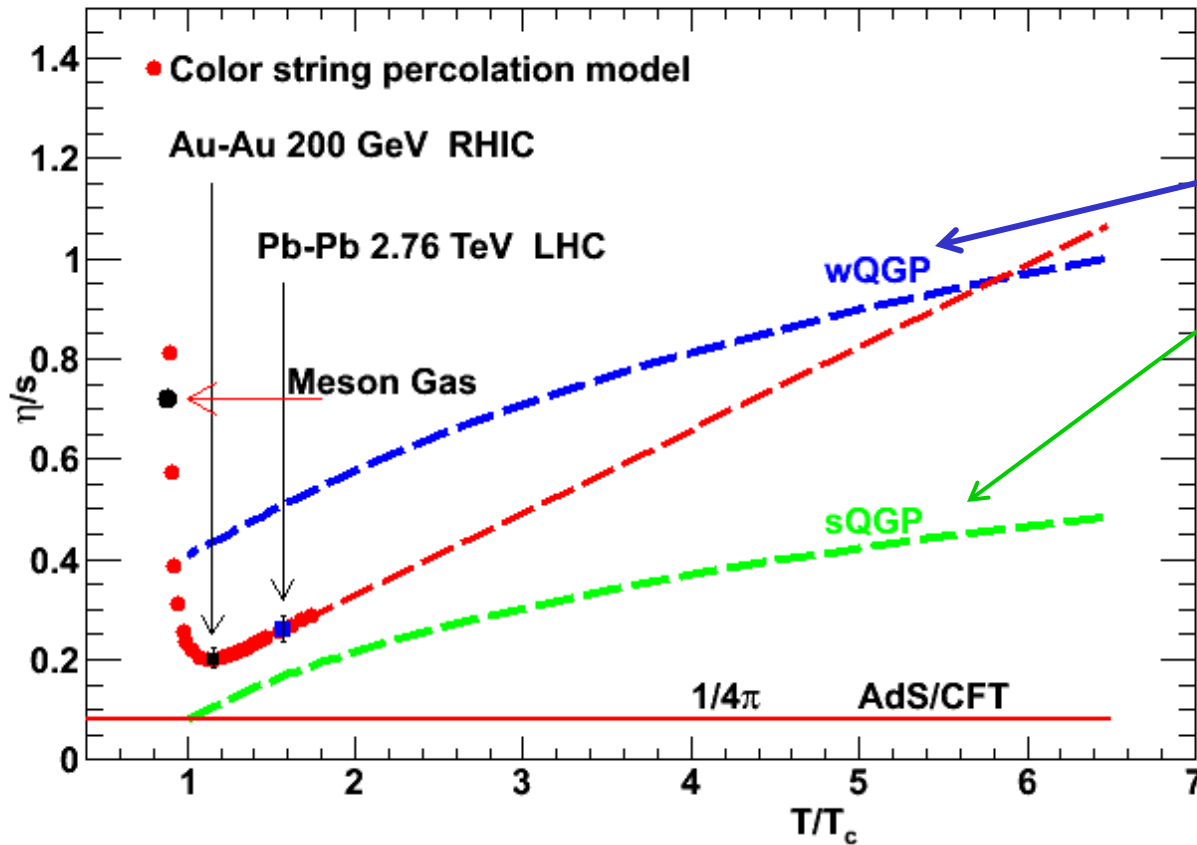
$$\lambda_{mfp} = \frac{L}{1 - e^{-\xi}}$$

$$\frac{\eta}{s} \approx \frac{1}{5} \frac{L}{1 - e^{-\xi}} T$$



# Shear viscosity to entropy density ratio

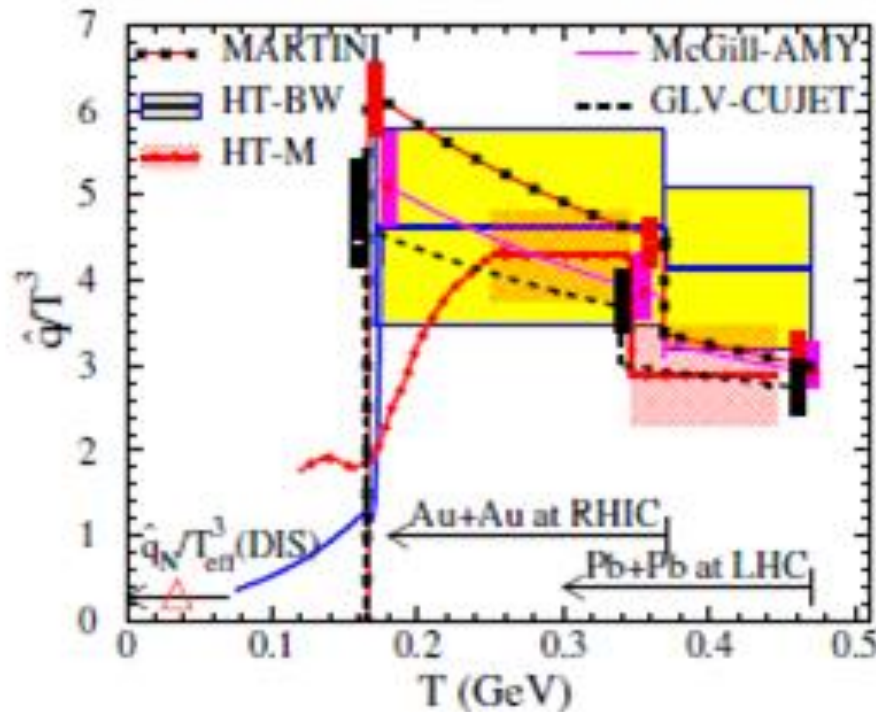
$$\frac{\eta}{s} \approx \frac{1}{5} \frac{L}{1 - e^{-\xi}} T$$



T. Hirano, M Gyulassy  
Nucl. Phys.  
A769, 71(2006)

J. Dias de Deus, A. S. Hirsch, C. Pajares, R. P. Scharenberg, B. K. Srivastava  
Eur. Phys. J. C 72, 2123 (2012)

# Jet Quenching Parameter



## Jet Collaboration

Extracting the jet transport coefficient from jet quenching at RHIC and LHC

Phys. Rev. C 90 (2014) 014909

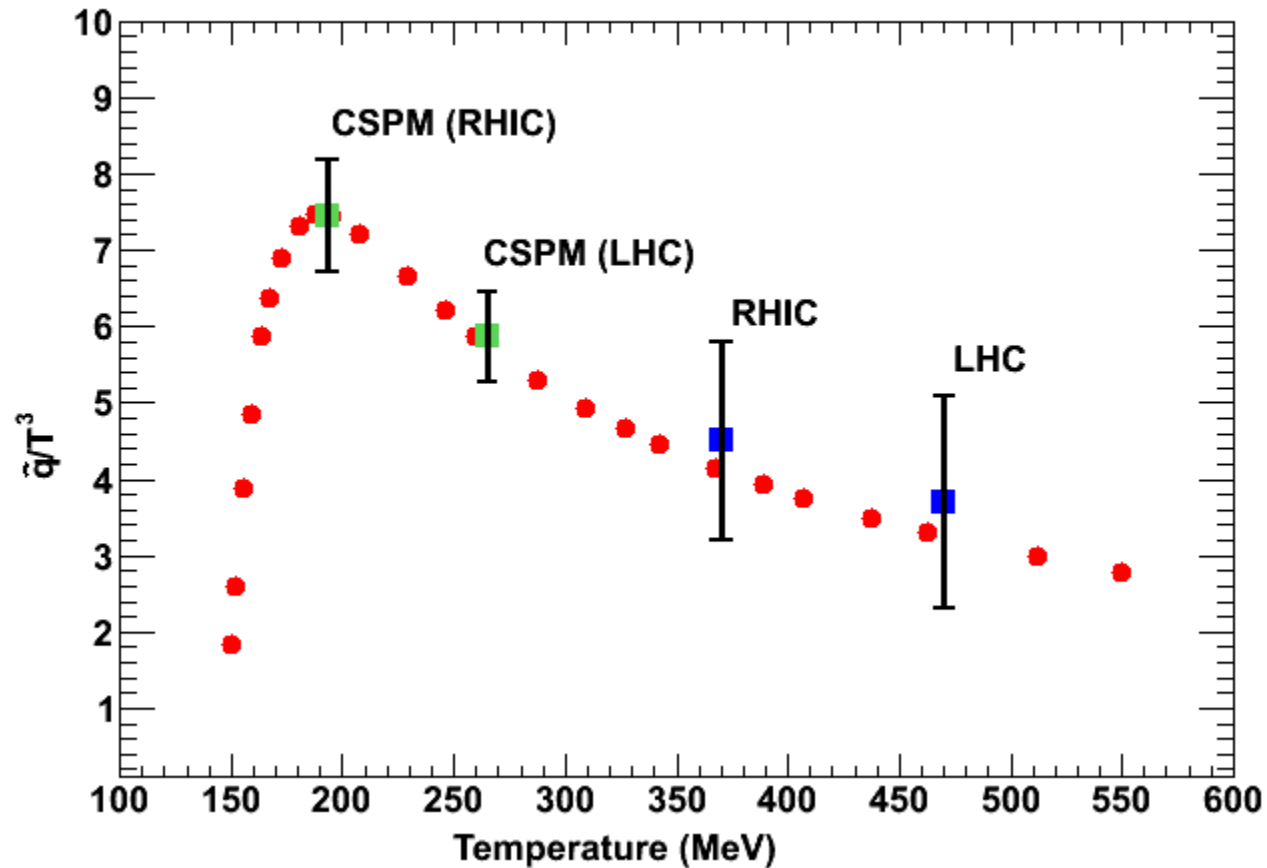
$$\frac{\hat{q}}{T^3} \approx \begin{array}{ll} 4.5 \pm 1.3 & \text{RHIC 370 MeV} \\ 3.7 \pm 1.4 & \text{LHC 470 MeV} \end{array}$$

Initial time of  $\tau_0 \sim 0.6 \text{ fm/c}$

A. Majumdar, B. Muller, X.-N. Wang, Phys. Rev. Lett. 99 (2007) 192301

J. Casalderrey-Solana, X.-N. Wang, Phys. Rev. C 77 (2008) 024902

# Jet Quenching Parameter



$$\frac{\eta}{s} \approx \frac{3}{2} \frac{T^3}{\hat{q}}$$

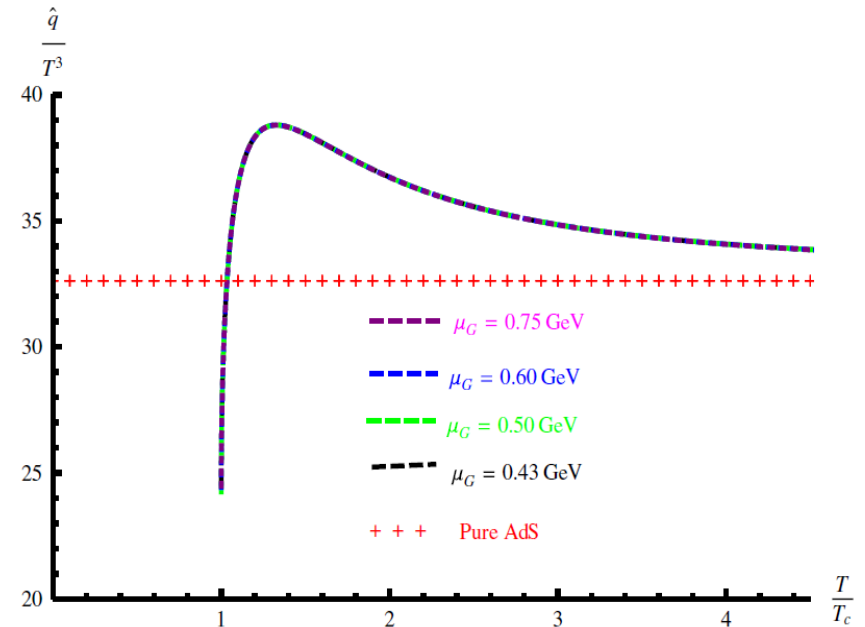
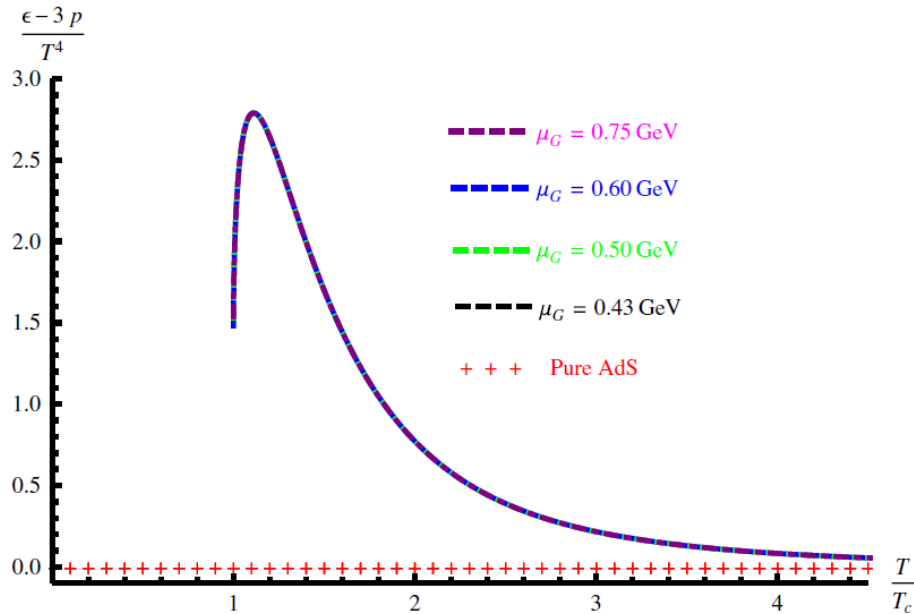
J. Casalderrey-Solana, X. -N. Wang , Phys. Rev. C 77 (2008) 024902

## **Thermodynamic results from LQCD -> In Quasi particle approach**

- ❑ Bluhm et al., Phys. Rev. C 84, 025201 (2011)**
- ❑ Plumari et al., Phys. Rev. D 84, 094004 (2011)**
- ❑ Khvorostukhin et al, Nucl. Phys. A 845 , 106 (2010)**
- ❑ Marty et al., Phys. Rev. C88, 045204 (2013)**

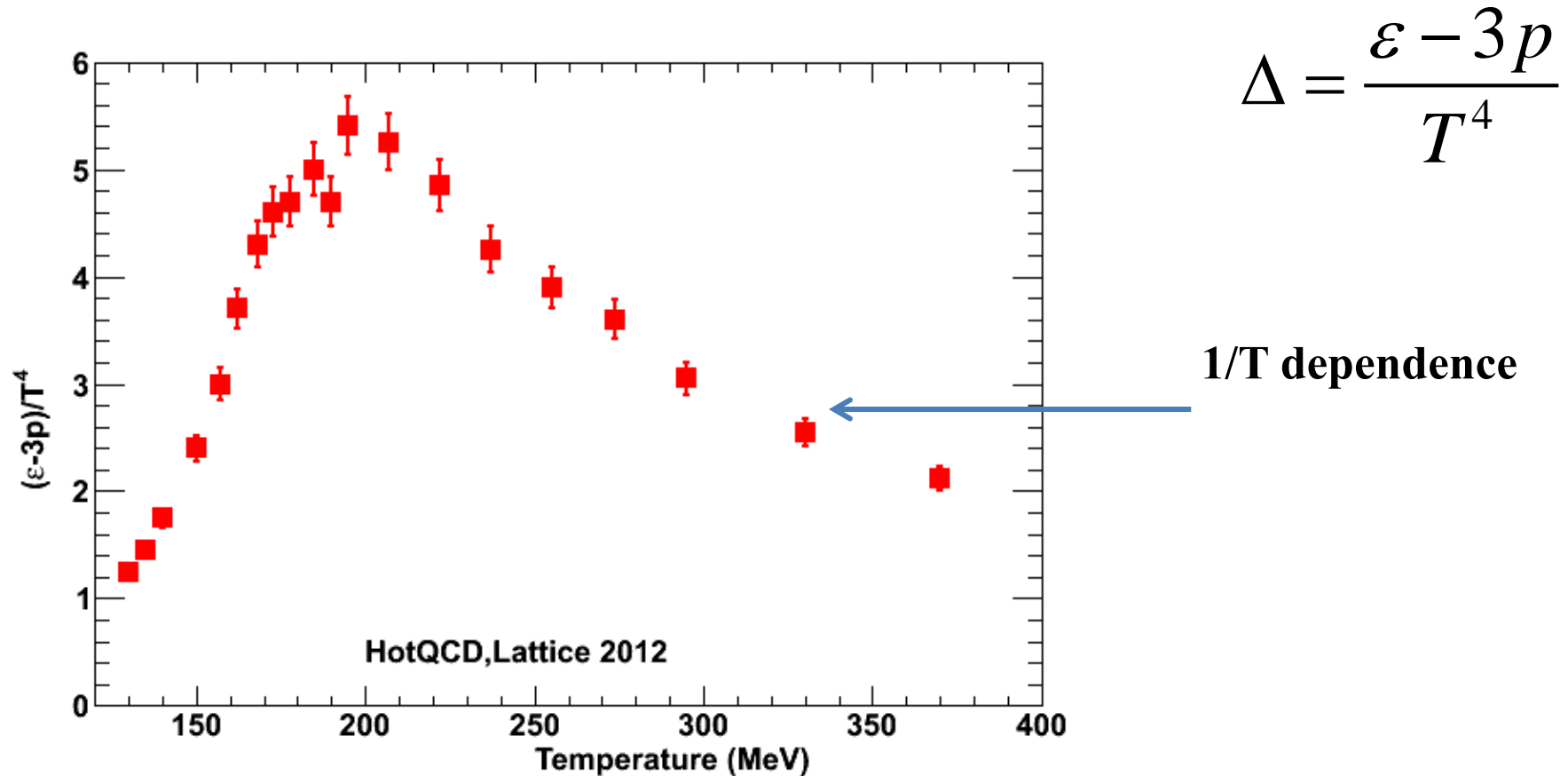
# Dynamical Holographic Model

- ❑ Enhancement of Jet Quenching around Phase Transition
- ❑ Results agree well with Lattice for a pure gluon system
- ❑ Both  $\frac{(\varepsilon - 3p)}{T^4}$  and  $\frac{\hat{q}}{T^3}$  show a peak around the critical temperature



# How to obtain the pressure ?

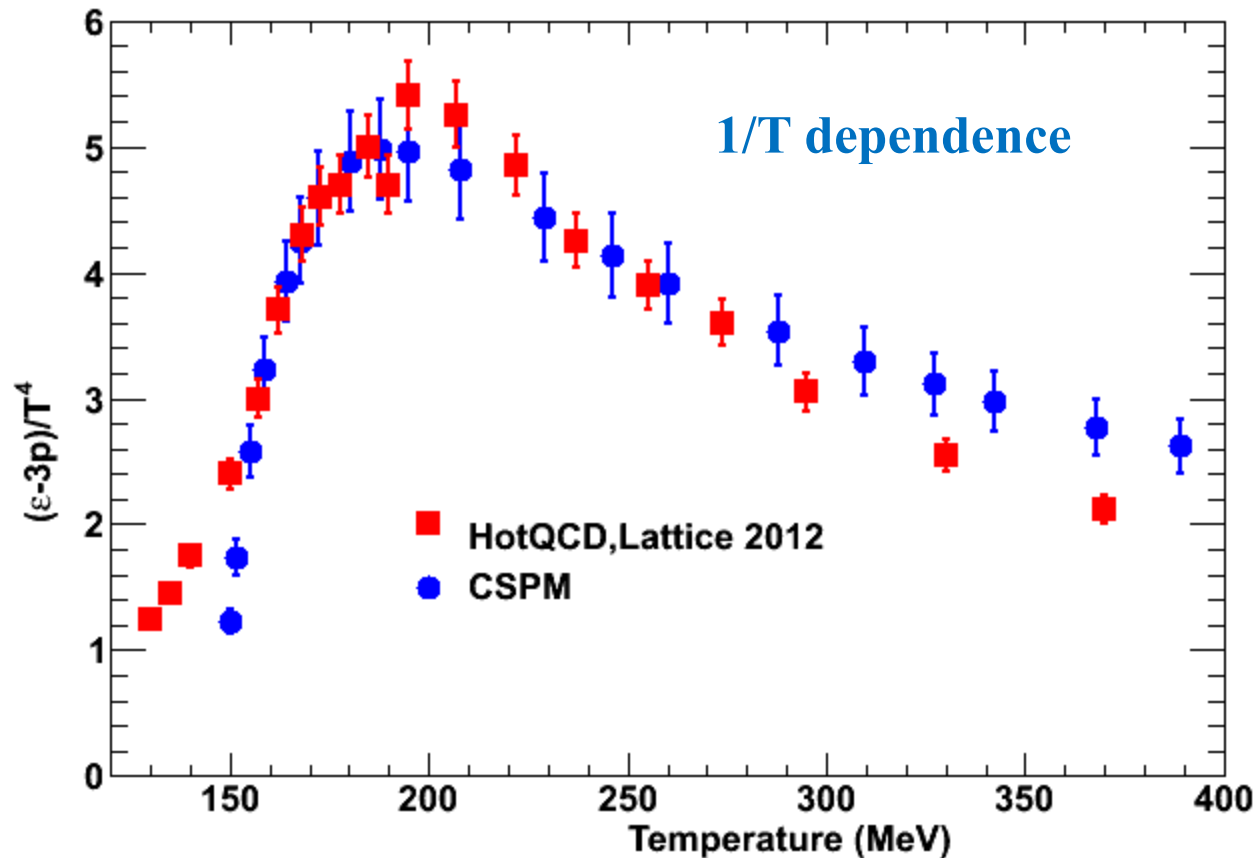
## Interaction Measure from Lattice QCD Calculation



P. Petreczki ( HotQCD Collaboration), Lattice 2012  
Cairns, Australia

**Ansatz :**  $\Delta = 1 / (\eta / s)$  Magnitude and functional dependence  
same as in LQCD

## Interaction Measure

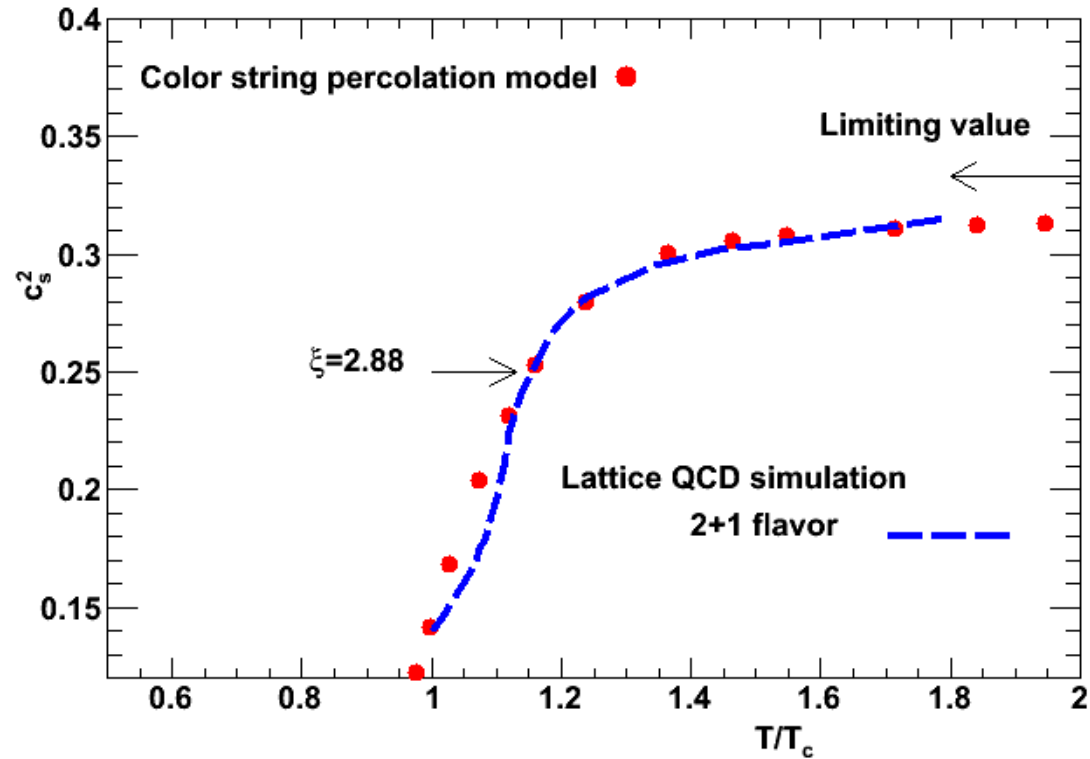




# Sound Velocity

$$\Delta = \frac{\varepsilon - 3p}{T^4}$$

$$C_s^2 = \frac{dp}{d\varepsilon} = \varepsilon \frac{dp / \varepsilon}{d\varepsilon} + \frac{p}{\varepsilon}$$



$$C_s^2 = (-0.33) \left( \frac{\xi e^{-\xi}}{1 - e^{-\xi}} - 1 \right) + 0.0191(\Delta / 3) \left( \frac{\xi e^{-\xi}}{(1 - e^{-\xi})^2} - \frac{1}{1 - e^{-\xi}} \right)$$

# Summary

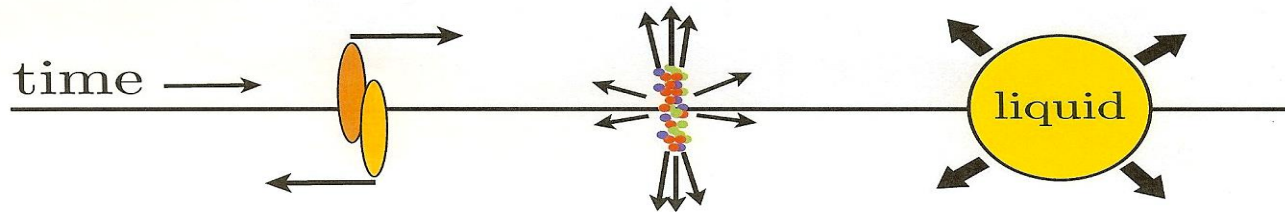
- ❑ The Clustering of Color Sources leading to the Percolation Transition may be the way to achieve de-confinement in High Energy collisions.
- ❑ The Percolation framework provide us with a microscopic partonic structure which explains the early thermalization.
- ❑ The minimum in  $\eta/s$  can be studied as a function of the beam energy at RHIC that could locate the critical point/crossover in QCD phase diagram as seen in substances like He, N<sub>2</sub> and H<sub>2</sub>O.
- ❑ A further definitive test of clustering phenomena can be made at LHC energies by comparing *h-h* and A-A collisions.
- ❑ The important message is that the percolation/clustering has a physical basis although it can not be deduced directly from QCD.

*Thank You*

**Extras**

# DIFFERENCE IN FLOW AND THERMALIZATION

Hydrodynamics & thermalization in heavy ion collisions



Basic theoretical questions:

- How long is  $t_{\text{hydro}}$ ?
- How long is  $t_{\text{therm}}$ ?
- How is  $t_{\text{therm}}$  correlated with  $t_{\text{hydro}}$ ?

( Au- Au  
example)

$$t_{\text{hydro}} \sim 0.35 \text{ fm/c} \quad t_{\text{therm}} \sim 1.05 \text{ fm/c}$$

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