

Thermodynamic and Transport Properties in the Clustering of Color Sources Approach in Nuclear Collisions

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QCD @ High Density

TIFR

Mumbai, India

Jan. 27-30, 2015

Exploration of Hot QCD Matter The Next Decade

Berndt Muller
CERN Theory Institute (HIC10)
Aug.16- Sept.10, 2010

Which **properties of hot QCD matter** can we hope to determine from relativistic heavy ion data (RHIC and LHC, maybe FAIR) ?

Easy for
LQCD

$$T_{\mu\nu} \Leftrightarrow \varepsilon, p, s$$
$$c_s^2 = \partial p / \partial \varepsilon$$

Equation of state: spectra, coll. flow, fluctuations

Speed of sound: multiparticle correlations

$$\eta = \frac{1}{T} \int d^4x \langle T_{xy}(x) T_{xy}(0) \rangle$$

Shear viscosity: anisotropic collective flow

Hard for
LQCD

$$\hat{q} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle F^{a+*}(y^-) F_i^{a+}(0) \rangle$$
$$\hat{e} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle i\partial^- A^{a+}(y^-) A^{a+}(0) \rangle$$
$$\hat{e}_2 = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle F^{a+-}(y^-) F^{a+-}(0) \rangle$$

Momentum/energy diffusion:
parton energy loss, jet fragmentation

Easy for
LQCD

$$m_D = - \lim_{|x| \rightarrow \infty} \frac{1}{|x|} \ln \langle E^a(x) E^a(0) \rangle$$

Color screening: Quarkonium states

Color Strings

Multiparticle production at high energies is currently described in terms of color strings stretched between the projectile and target. Hadronizing these strings produce the observed hadrons.

The no. of strings grow with energy and the no. of participating nuclei and one expects that interaction between them becomes essential.

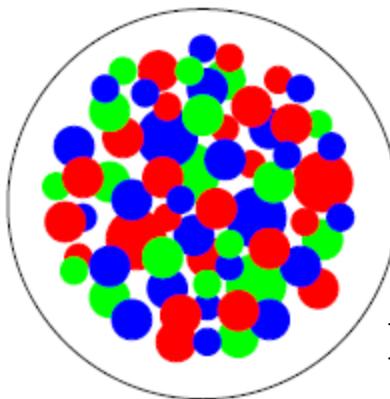
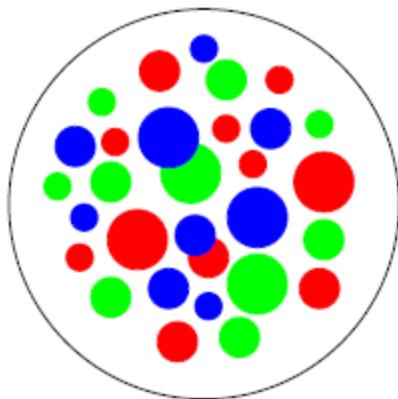
This problem acquires even more importance, considering the idea that at very high energy collisions of heavy nuclei (RHIC) may produce Quark-gluon Plasma (QGP).

The interaction between strings then has to make the system evolve towards the **QGP** state.

Clustering of Color Sources

De-confinement is expected when the density of quarks and gluons becomes so high that it no longer makes sense to partition them into color-neutral hadrons, since these would overlap strongly.

We have clusters within which color is not confined : De-confinement is thus related to cluster formation very much similar to cluster formation in percolation theory and hence a connection between percolation and de-confinement seems very likely.



Parton distributions in the transverse plane of nucleus-nucleus collisions

In two dimensions, for uniform string density, the percolation threshold for overlapping discs is: $\xi_c = 1.18$

$$\xi_c = 1.18$$

H. Satz, Rep. Prog. Phys. 63, 1511(2000).
H. Satz, hep-ph/0212046

Critical Percolation Density

Percolation : General

The general formulation of the percolation problem is concerned with elementary geometrical objects placed at random in a d -dimensional lattice. The objects have a well defined connectivity radius λ , and two objects are said to communicate if the distance between them is less than λ .



One is interested in how many objects can form a cluster of communication and, especially, when and how the cluster become infinite. The control parameter is the density of the objects or the dimensionless filling factor ξ . The percolation threshold $\xi = \xi_c$ corresponding to the minimum concentration at which an infinite cluster spans the space.

Percolation : General

It is well known that the percolation problem on large lattices displays the features of a system undergoing a second-order phase transition.

These characteristics include critical fluctuations, quantities which diverge, and quantities which vanish as the critical percolation probability is approached. These quantities are described by a finite number of critical exponents.

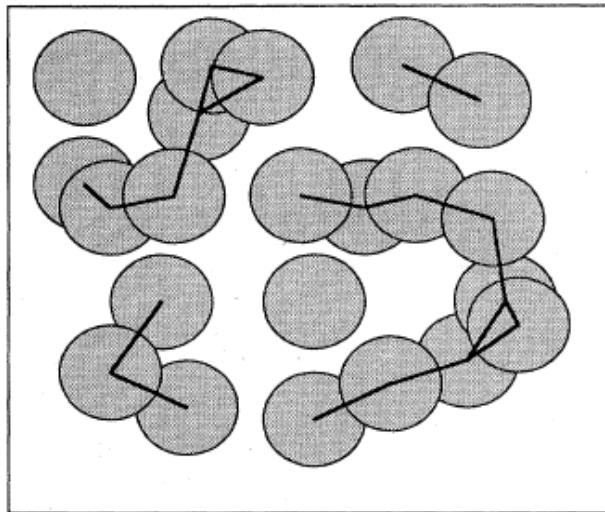
- * **Transition from liquid to gas**
- * **Normal conductor to a superconductor**
- * **Paramagnet to ferromagnet**

1. H. E. Stanley , Introduction to Phase Transitions and Critical Phenomena
2. D. Stauffer and A. Aharony, Introduction to Percolation Theory

Percolation, statistical topography, and transport in random media

M. B. Isichenko

Reviews of Modern Physics, Vol. 64, No. 4, October 1992



$$\xi = \pi n r^2$$

ξ is the percolation density parameter
 n is the density and r the radius of the disc
 ϕ is the fractional area covered by the cluster

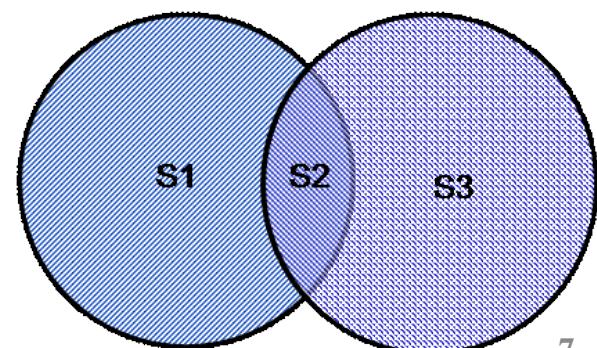
$$\phi = 1 - e^{-\xi}$$

ξ_c is the critical value of the percolation density at which a communicating cluster appears

For example at $\xi_c = 1.2$, $\phi \sim 2/3$

It means $\sim 67\%$ of the whole area is covered by the cluster

In the nuclear case it is the overlap area



Parton Percolation

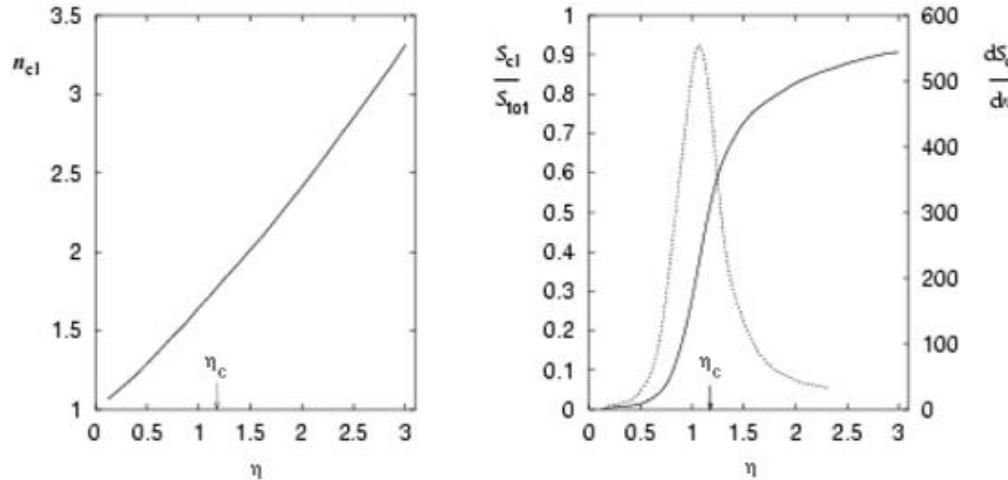


Figure 38: Average cluster density $n_{cl}(n)$ (left) and average fractional cluster size $S_{cl}(n)/S_{total}$ (right) as function of the overall density n of discs, for $r/R = 1/20$; in (b), the derivative of $S_{cl}(n)/S_{total}$ with respect to n is also shown (dotted line). The percolation point in the limit $r/R \rightarrow 0$ is indicated by n_p .

In two dimensions, for uniform string density, the percolation threshold for overlapping discs is:

$$\xi_c = 1.18$$

= critical percolation density

Satz, hep-ph/0007069

The fractional area covered by discs at the critical threshold is:

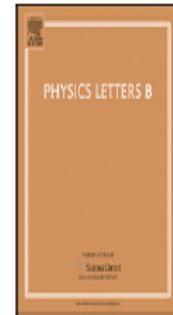
$$1 - e^{-\xi_c}$$



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Coherent center domains in SU(3) gluodynamics and their percolation at T_c

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ABSTRACT

For SU(3) lattice gauge theory we study properties of static quark sources represented by local Polyakov loops. We find that for temperatures both below and above T_c coherent domains exist where the phases of the local loops have similar values in the vicinity of the center values $0, \pm 2\pi/3$. The cluster properties of these domains are studied numerically. We demonstrate that the deconfinement transition of SU(3) may be characterized by the percolation of suitably defined clusters.

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Center Domains and their Phenomenological Consequences

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(Received 2 September 2012; revised manuscript received 6 November 2012; published 15 May 2013)

We argue that the domain structure of deconfined QCD matter, which can be inferred from the properties of the Polyakov loop, can simultaneously explain the two most prominent experimentally verified features of the quark-gluon plasma, namely its large opacity as well as its near ideal fluid properties.

Color Strings + Percolation = CSPM

Multiplicity and $\langle p_T^2 \rangle$ of particles produced by a cluster of n strings

Multiplicity (μ_n)

$$\mu_n = F(\xi) N^s \mu_1$$

Average Transverse Momentum

$$\langle p_T^2 \rangle_n = \langle p_T^2 \rangle_1 / F(\xi)$$

$$F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}}$$

= Color suppression factor
(due to overlapping of discs).

$$\xi = \frac{N^s S_1}{S_N}$$

N^s = # of strings

S_1 = disc area

S_N = total nuclear overlap area

ξ is the percolation density parameter.

M. A. Braun and C. Pajares, Eur.Phys. J. C16,349 (2000)

M. A. Braun et al, Phys. Rev. C65, 024907 (2002)

Percolation and Color Glass Condensate

Both are based on parton coherence phenomena.

Percolation : Clustering of strings

CGC : Gluon saturation

- Many of the results obtained in the framework of percolation of strings are very similar to the one obtained in the CGC.
- In particular , very similar scaling laws are obtained for the product and the ratio of the multiplicities and transverse momentum.
- Both provide explanation for multiplicity suppression and $\langle p_t \rangle$ scaling with dN/dy .

Momentum Q_s establishes the scale in CGC with the corresponding one in percolation of strings

$$Q_s^2 = \frac{k \langle p_t^2 \rangle_1}{F(\xi)}$$

For large value of ξ

$$Q_s^2 \propto \sqrt{\xi}$$

The no. of color flux tubes in CGC and the effective no. of clusters of strings in percolation have the same dependence on the energy and centrality.

This has consequences in the Long range rapidity correlations and the ridge structure.

Results

Bulk Properties

- Multiplicity
- pt distribution
- Particle ratios
- Elliptic flow
- Forward-Backward Multiplicity
- Correlations at RHIC

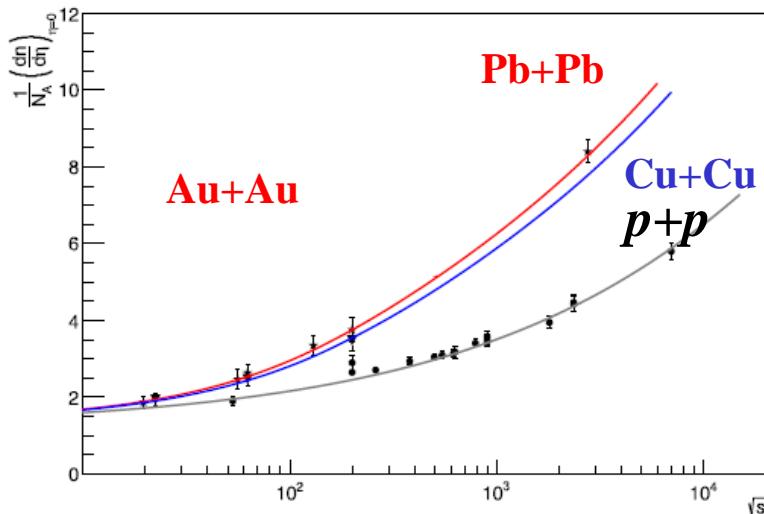
Thermodynamics

- Temperature
- Energy Density
- Shear viscosity to Entropy density ratio
- Equation of State

Determination of the Color Suppression Factor $F(\xi)$ from the Data

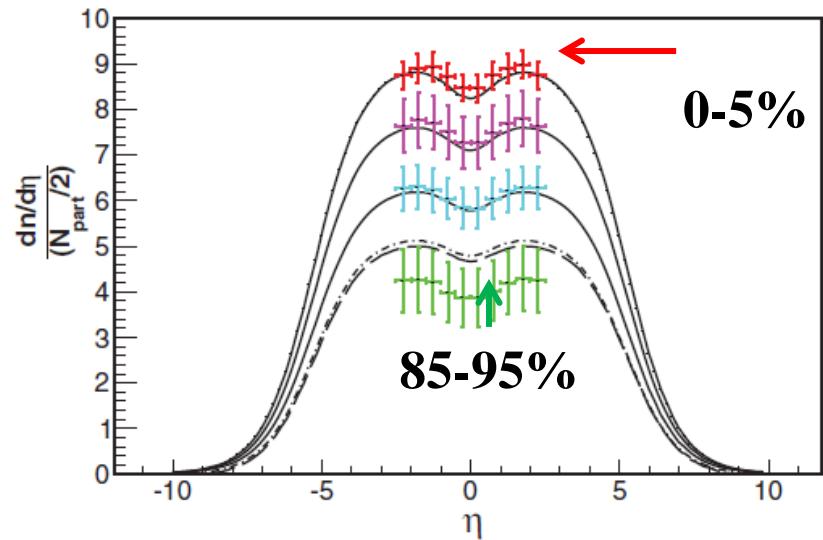
Multiplicity

Multiplicity dependence on \sqrt{s}



Phys. Lett., 715, 230 (2012)
ALICE Coll. Phys. Rev. Lett., 106, 032301 (2011)

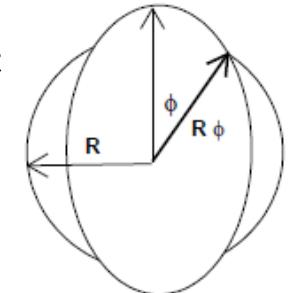
Rapidity dependence of particle densities Pb+Pb at 2.76 TeV



Phys. Rev. C 86, 034909(2012)
CMS Coll., JHEP, 08, 141 (2011).

Elliptic flow at RHIC and LHC

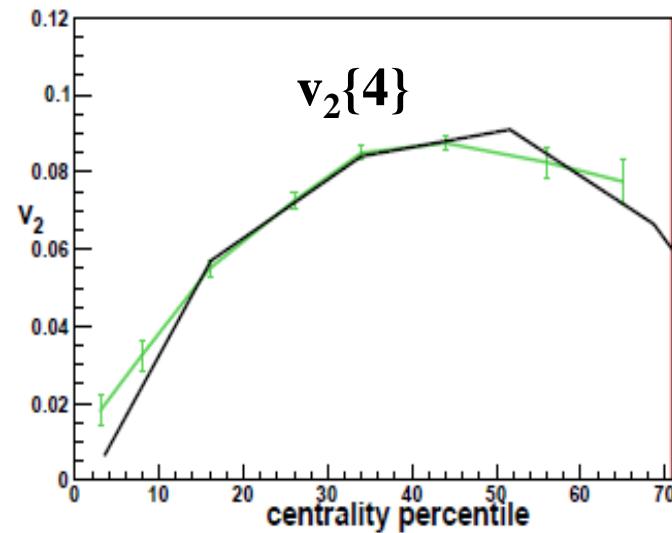
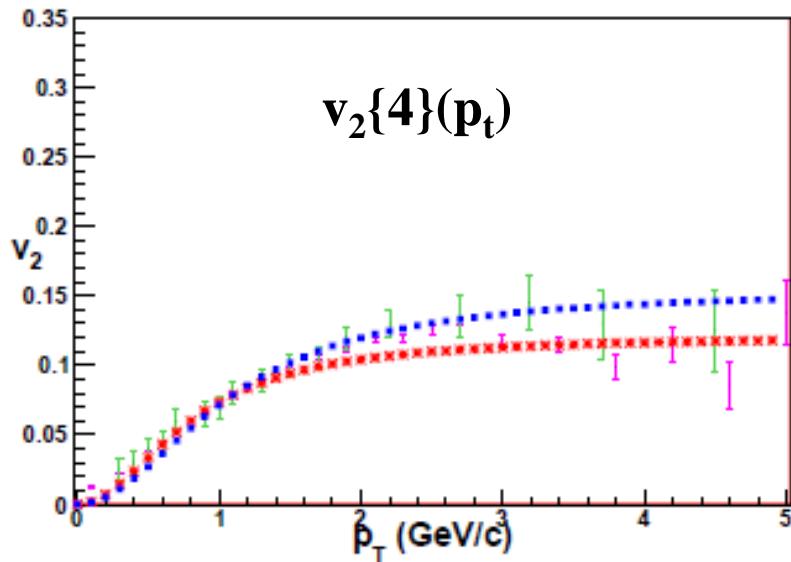
- The cluster formed by the strings has generally asymmetric shape in the transverse plane. This azimuthal asymmetry is at the origin of the elliptic flow in percolation.
- The partons emitted inside the cluster have to pass a certain length through the strong color field and loose energy.
- The energy loss by the partons is proportional to the length and therefore the p_t of an particle will depend on the path length travelled and will depend on the direction of emission



$$v_2 = \frac{2}{\pi} \int_0^\pi d\phi \cos 2\phi \left(\frac{R_\phi}{R} \right) \left(\frac{e^{-\xi} - F(\xi)^2}{2F(\xi)^3} \right) \frac{R}{R-1}$$

I.Bautista, J. Dias de Deus , C. Pajares, arXiv 1102:3837
M. A. Braun , C. Pajares, Eur. J. Phys. C, 71 (2011) 1558

Elliptic flow at RHIC and LHC



Charged hadron elliptic flow

ALICE: 0-10% centrality Pb+Pb@2.76 TeV

Phys. Rev. Lett, 105 (2010) 252302

STAR: 0-10% centrality Au+Au@200 GeV

Phys. Rev. C 77, (2008) 054901

**Higher harmonics v_3 to v_8 have also been studied
in percolation approach**

M. A. Braun, C. Pajares, V. Vechernin,
arXiv:1204:5829

**Elliptic flow as a function
of centrality in Pb+Pb@2.76 TeV
Comparison with ALICE**

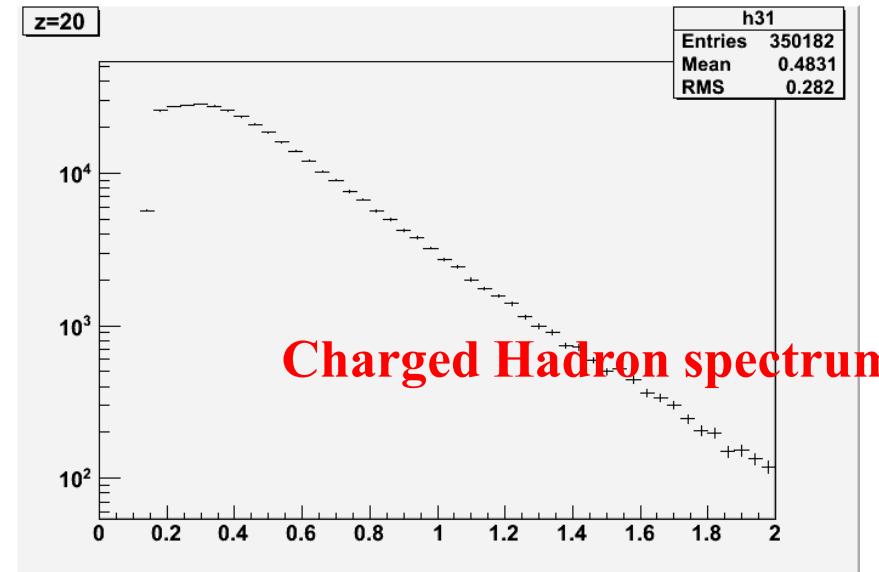
Using the p_T spectrum to calculate ξ

The experimental p_T distribution from pp data is used

$$\frac{d^2 N}{dpt^2} = \frac{a}{(p_0 + pt)^n}$$

$$\frac{d^2 N}{dpt^2}$$

a , p_0 and n are parameters fit to the data.



This parameterization can be used for nucleus-nucleus collisions to account for the clustering :

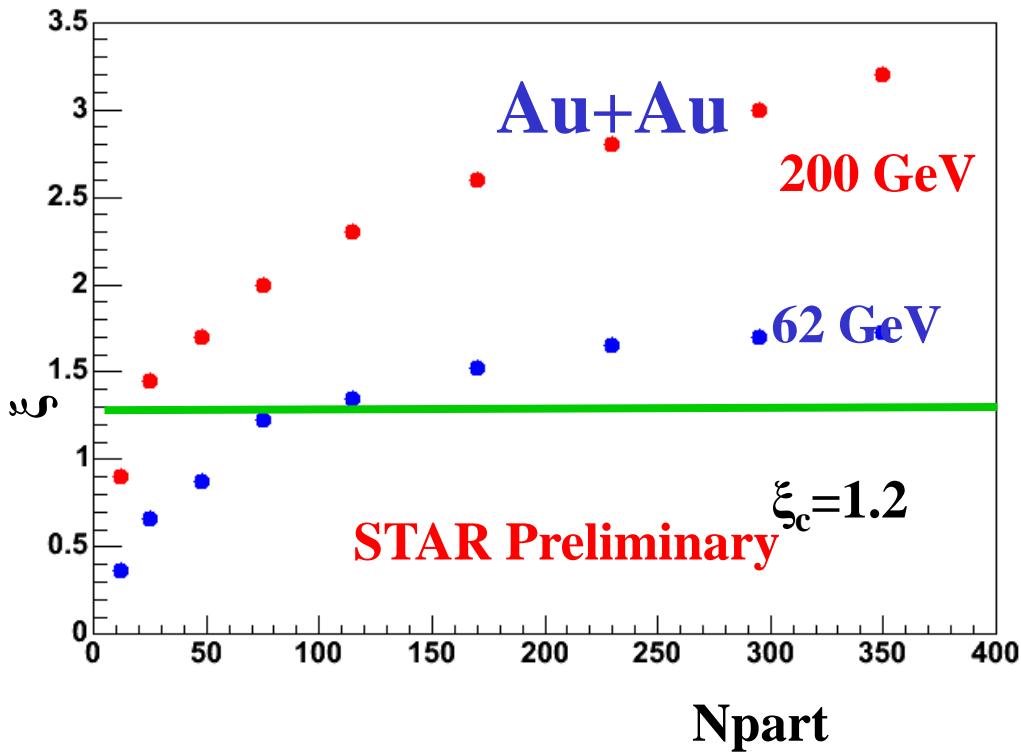
$$\frac{d^2 N}{dpt^2} = \frac{b}{\left(p_0 \sqrt{\frac{F(\xi_{pp})}{F(\xi_{AuAu})}} + pt \right)^n}$$

$$F(\xi)_{pp} = 1$$

$$F(\xi)_{AuAu} = 0.57$$

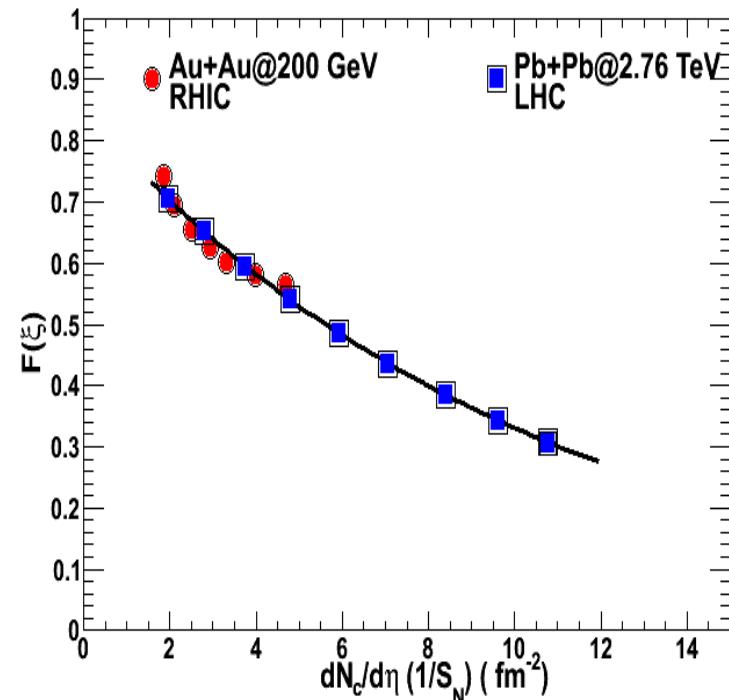
For central collisions

Percolation Density Parameter ξ



$$F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}}$$

Now the aim is to connect $F(\xi)$ with Temperature and Energy density



Using ALICE charged particle multiplicity
Phys. Rev. Lett. , 106, 032301 (2011).

Thermodynamic and Transport Properties

Schwinger Mechanism of Particle Production

p_t distribution of the produced quarks

$$\frac{dn}{d^2 p_\perp} \sim \exp\left(-\frac{\pi p_t^2}{k}\right)$$

k is the string tension

The tension of the macroscopic cluster fluctuates around its mean value because the chromoelectric field is not constant. Assuming a Gaussian form for these fluctuations one arrives at the probability distribution of transverse momentum.

Thermal Distribution

$$\frac{dn}{d^2 p_\perp} \sim \exp\left(-\frac{\pi p_t^2}{T}\right)$$

$$T = \sqrt{\frac{\langle k \rangle}{2\pi}}$$

$$T = \sqrt{\frac{\langle p_t^2 \rangle_1}{2F(\xi)}}$$

**Cluster /Initial
Temperature**

Temperature

$$T = \sqrt{\frac{\langle p_t^2 \rangle_1}{2F(\xi)}}$$

At the critical percolation density

$$\xi_c = 1.2 \quad T_c = 167 \text{ MeV}$$

For Au+Au@ 200 GeV

0-10% centrality $\xi = 2.88$ $T \sim 195 \text{ MeV}$

PHENIX:

Temperature from direct photon
Exponential (consistent with thermal)

Inverse slope = $220 \pm 20 \text{ MeV}$

PRL 104, 132301 (2010)

Pb+Pb @ 2.76TeV for 0-5%

$T \sim 265 \text{ MeV}$

Temperature has increased by 35% from Au+Au @ 0.2 TeV

First Results from Pb+Pb Collisions@ 2.76 TeV at the LHC

Muller, Schukraft and Wyslouch, Ann. Rev. Nucl. Sci. Oct. 2012

ALICE : Direct Photon Measurement

$T = 304 \pm 51 \text{ MeV}$

QM 2012

Thermalization

- The origin of the string fluctuation is related to the stochastic picture of the QCD vacuum . Since the average value of color field strength must vanish, it cannot be constant and must vanish from point to point. Such fluctuations lead to the Gaussian distribution of the string.

H. G. Dosch, Phys. Lett. 190 (1987) 177

A. Bialas, Phys. Lett. B 466 (1999) 301

- The fast thermalization in heavy ion collisions can occur through the existence of event horizon caused by rapid deceleration of the colliding nuclei. Hawking-Unruh effect.

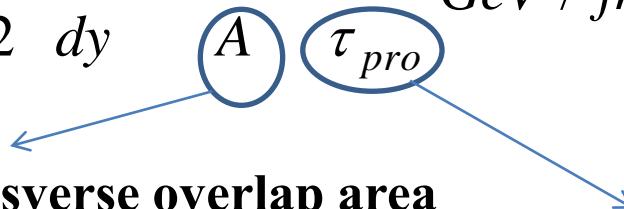
D. Kharzeev, E. Levin , K. Tuchin, Phys. Rev. C75, 044903 (2007)

H.Satz, Eur. Phys. J. 155, (2008) 167

Energy Density

Bjorken Phys. Rev. D 27, 140 (1983)

$$\varepsilon = \frac{3}{2} \frac{dN_c}{dy} \frac{\langle m_t \rangle}{A} \frac{1}{\tau_{pro}} \text{GeV / fm}^3$$



Transverse overlap area

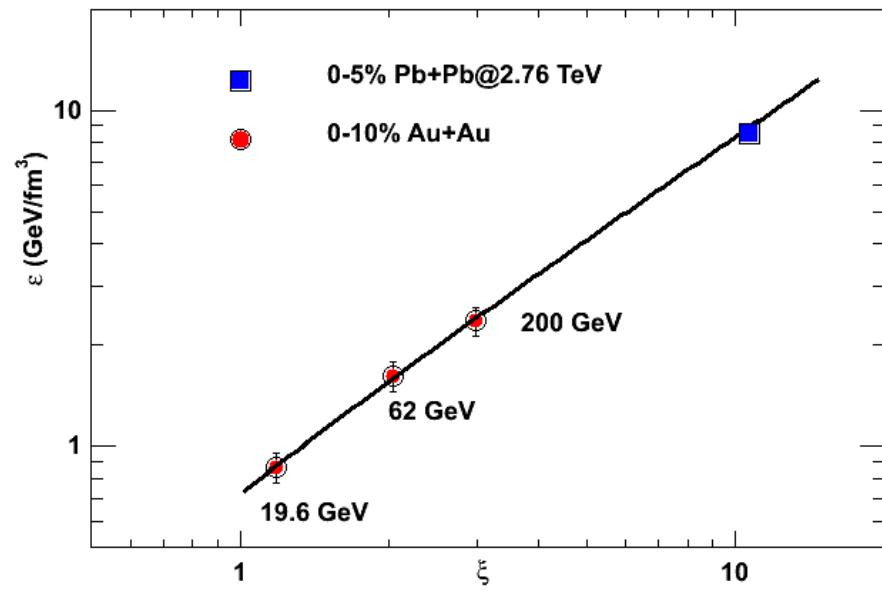
Proper Time

τ_{pro} is the QED production time for a boson which can be scaled from QED to QCD and is given by

$$\tau_{pro} = \frac{2.405\hbar}{\langle m_t \rangle}$$

STAR Coll., Phys. Rev. C 79, 34909 (2009)

Introduction to high energy
heavy ion collisions
C. Y. Wong



$\varepsilon \propto \xi$

J. Dias de Deus, A. S. Hirsch, C. Pajares ,
R. P. Scharenberg , B. K. Srivastava
Eur. Phys. J. C 72, 2123 (2012)

Having determined the initial temperature of the system from the data one would like to obtain the following quantities to understand the properties of QCD matter

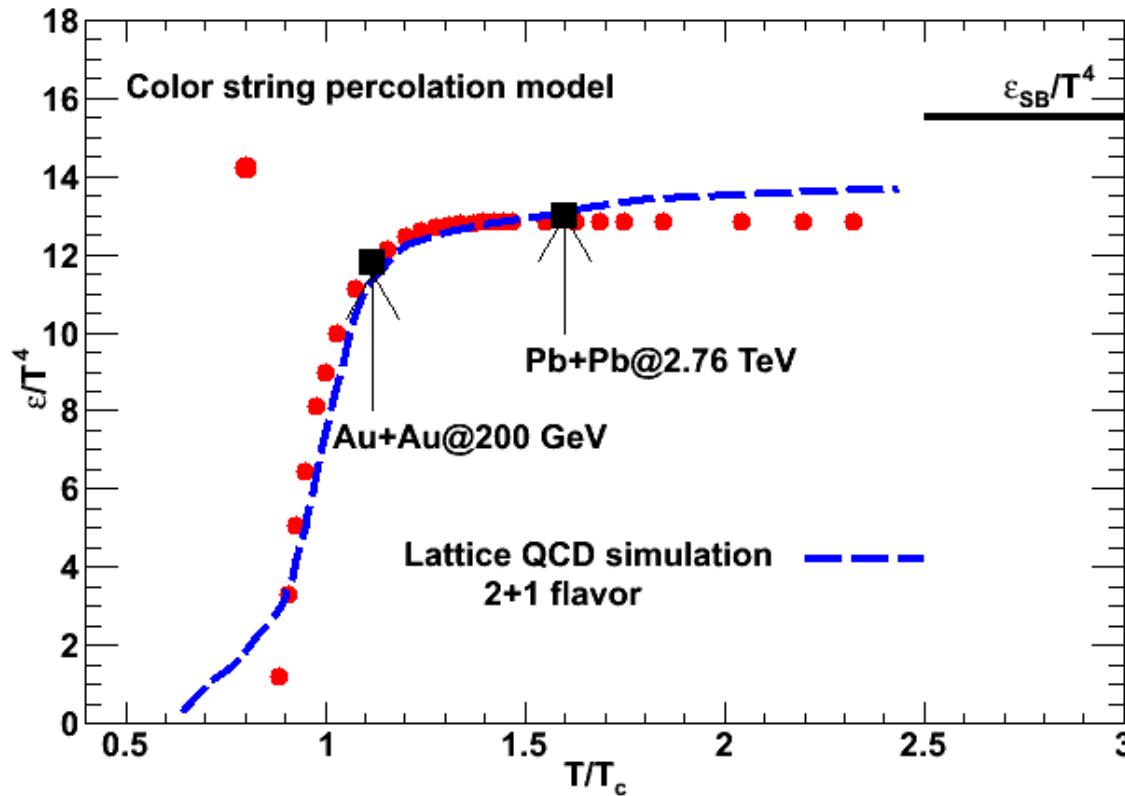
$$\varepsilon / T^4$$

Shear Viscosity

Equation of State

Energy Density

0-10% centrality



Lattice QCD : Bazavov et al, Phys. Rev. D 80, 014504(2009).

R. P. Scharenberg , B. K. Srivastava, A. S. Hirsch
Eur. Phys. J. C 71, 1510(2011)

Strongly Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions

Laszlo P. Csernai,^{1,2} Joseph I. Kapusta,³ and Larry D. McLerran⁴

¹*Section for Theoretical Physics, Department of Physics, University of Bergen, Allegaten 55, 5007 Bergen, Norway*

²*MTA-KFKI, Research Institute of Particle and Nuclear Physics, 1525 Budapest 114, P.O. Box 49, Hungary*

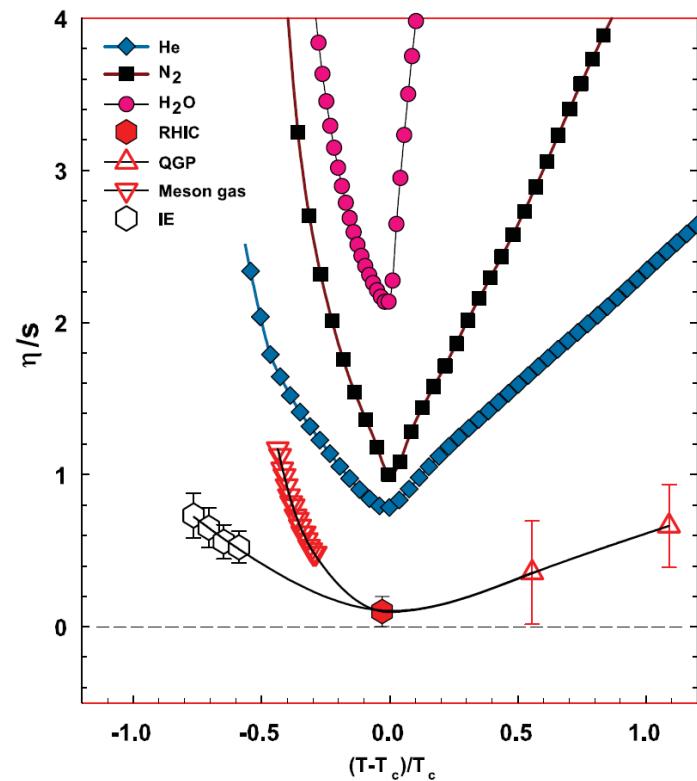
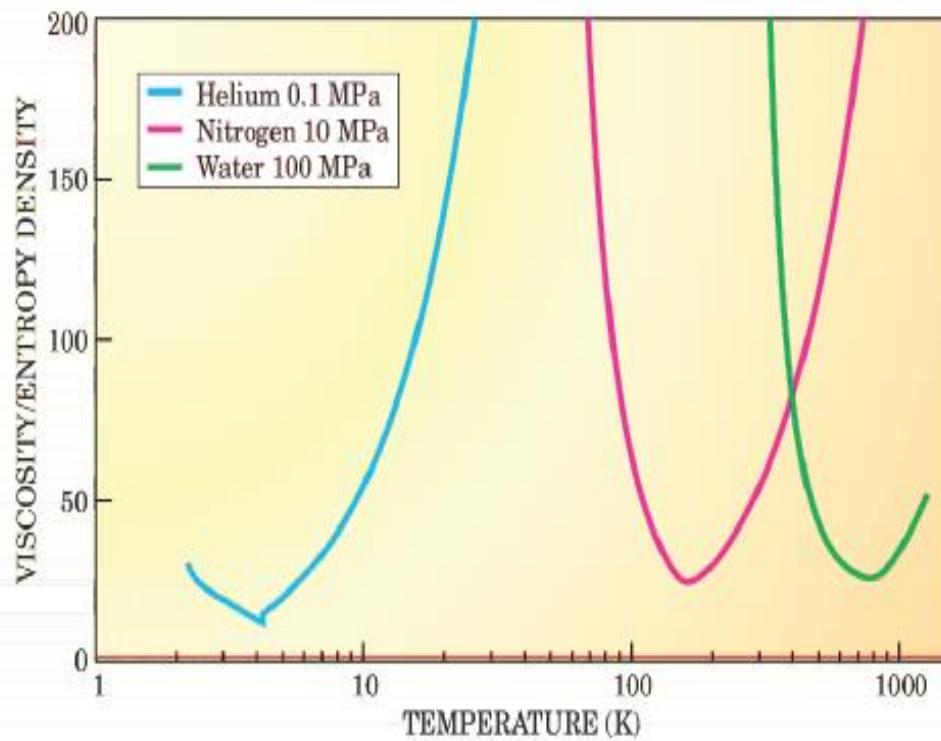
³*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*

⁴*Nuclear Theory Group and Riken Brookhaven Center, Brookhaven National Laboratory, Bldg. 510A, Upton, New York 11973, USA*

(Received 12 April 2006; published 12 October 2006)

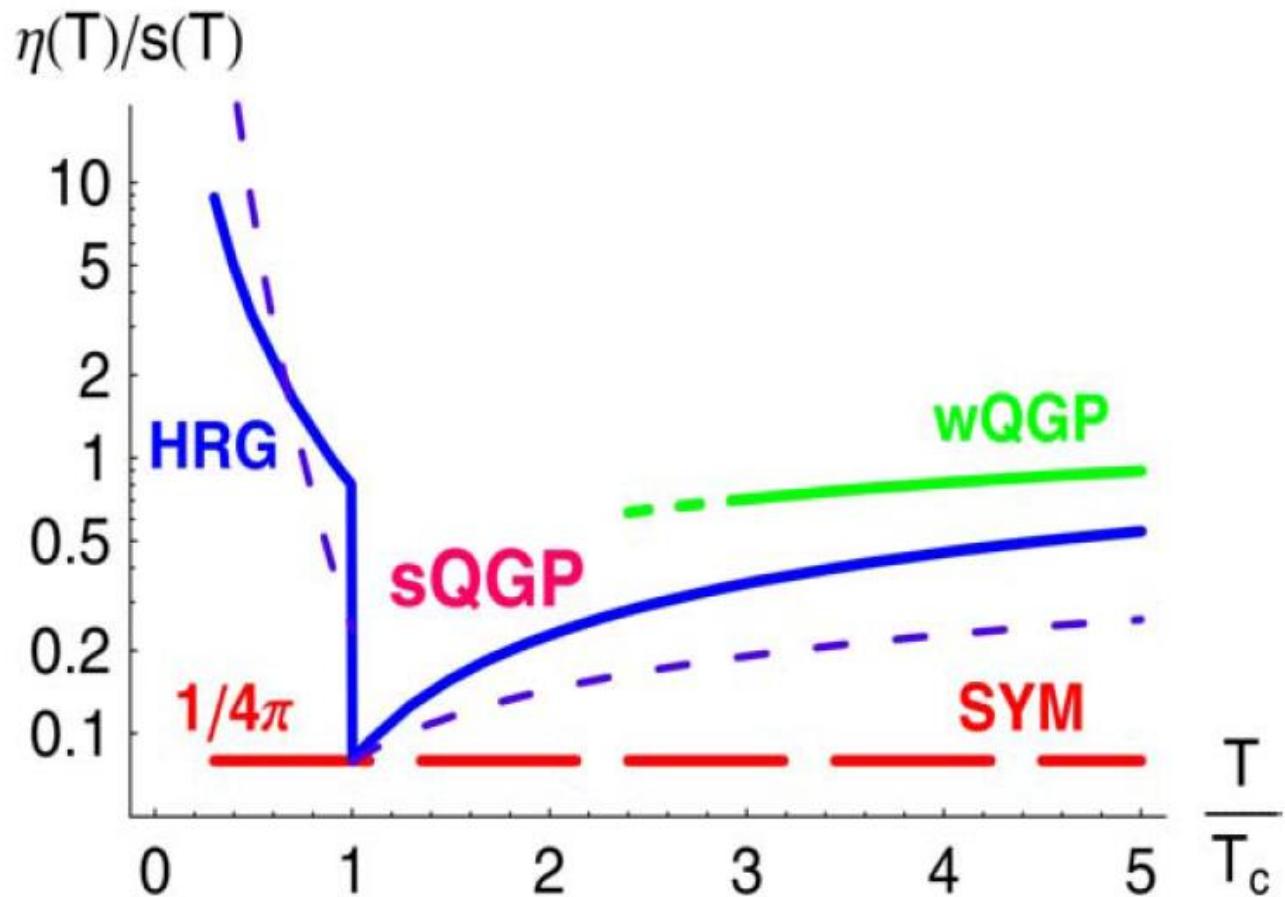
Substantial collective flow is observed in collisions between large nuclei at BNL RHIC (Relativistic Heavy Ion Collider) as evidenced by single-particle transverse momentum distributions and by azimuthal correlations among the produced particles. The data are well reproduced by perfect fluid dynamics. A calculation of the dimensionless ratio of shear viscosity η to entropy density s by Kovtun, Son, and Starinets within anti-de Sitter space/conformal field theory yields $\eta/s = \hbar/4\pi k_B$, which has been conjectured to be a lower bound for any physical system. Motivated by these results, we show that the transition from hadrons to quarks and gluons has behavior similar to helium, nitrogen, and water at and near their phase transitions in the ratio η/s . We suggest that experimental measurements can pinpoint the location of this transition or rapid crossover in QCD.

η/s for He, N₂ and H₂O



PRL 97, 152303 (2006)

Lacey et al., PRL 98, 092301 (2007)



Hirano and Gyulassy, Nucl. Phys. A 769, 71 (2006)

The viscosity can be estimated from kinetic theory to be

$$\eta \approx \frac{4}{15} \varepsilon(T) \lambda_{mfp} \approx \frac{1}{5} \frac{T}{\sigma_{tr}} \frac{s(T)}{n(T)}$$

$$\varepsilon(T) = \frac{3}{4} T s$$

$$\lambda_{tr} = \frac{1}{(n \sigma_{tr})}$$

$$\frac{n}{s} \approx \frac{T \lambda_{mfp}}{5}$$

Hirano & Gyulassy, Nucl. Phys. A769, 71(2006)

ε Energy density
s Entropy density
n the number density
 λ_{mfp} Mean free path

σ_{tr} Transport cross section
 $\sqrt{<pt>_1^2}$ Average transverse momentum of the single string

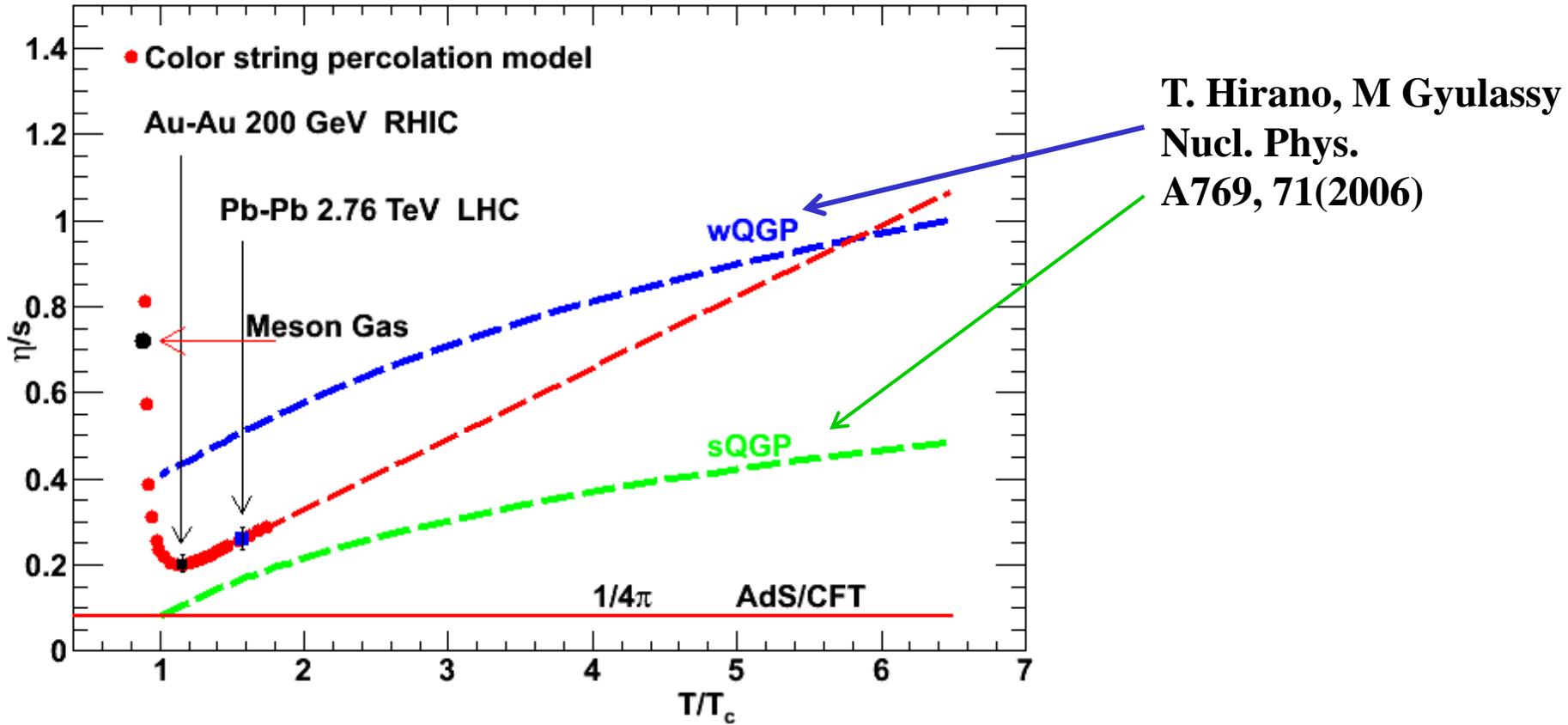
L is Longitudinal extension of the source 1 fm

$$\lambda_{mfp} = \frac{L}{1 - e^{-\xi}}$$

$$\frac{\eta}{s} \approx \frac{1}{5} \frac{L}{1 - e^{-\xi}} T$$

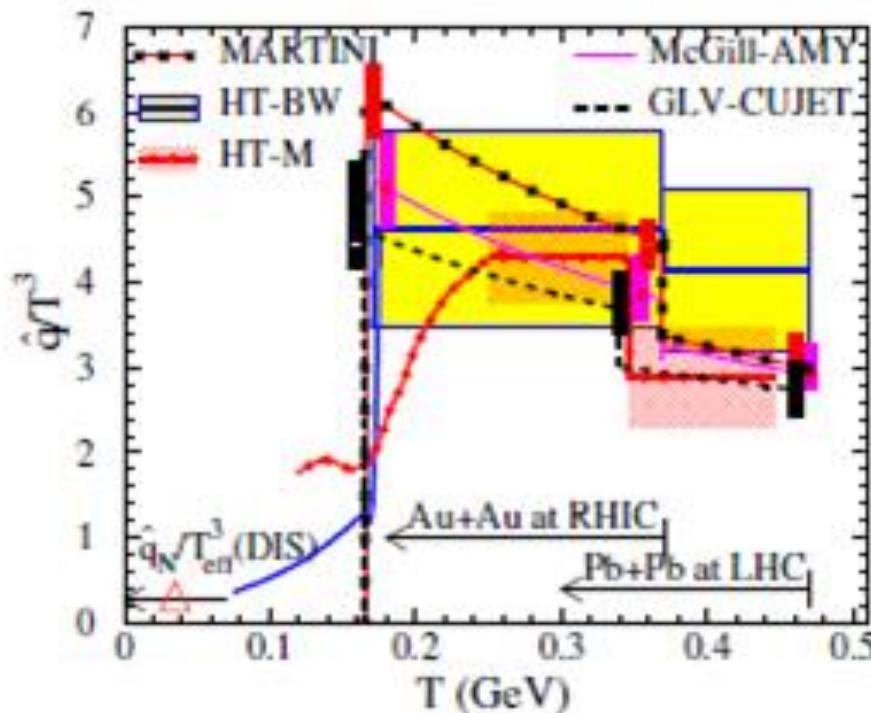
Shear viscosity to entropy density ratio

$$\frac{\eta}{s} \approx \frac{1}{5} \frac{L}{1-e^{-\xi}} T$$



J. Dias de Deus, A. S. Hirsch, C. Pajares, R. P. Scharenberg, B. K. Srivastava
Eur. Phys. J. C 72, 2123 (2012)

Jet Quenching Parameter



Jet Collaboration

Extracting the jet transport coefficient from jet quenching at RHIC and LHC

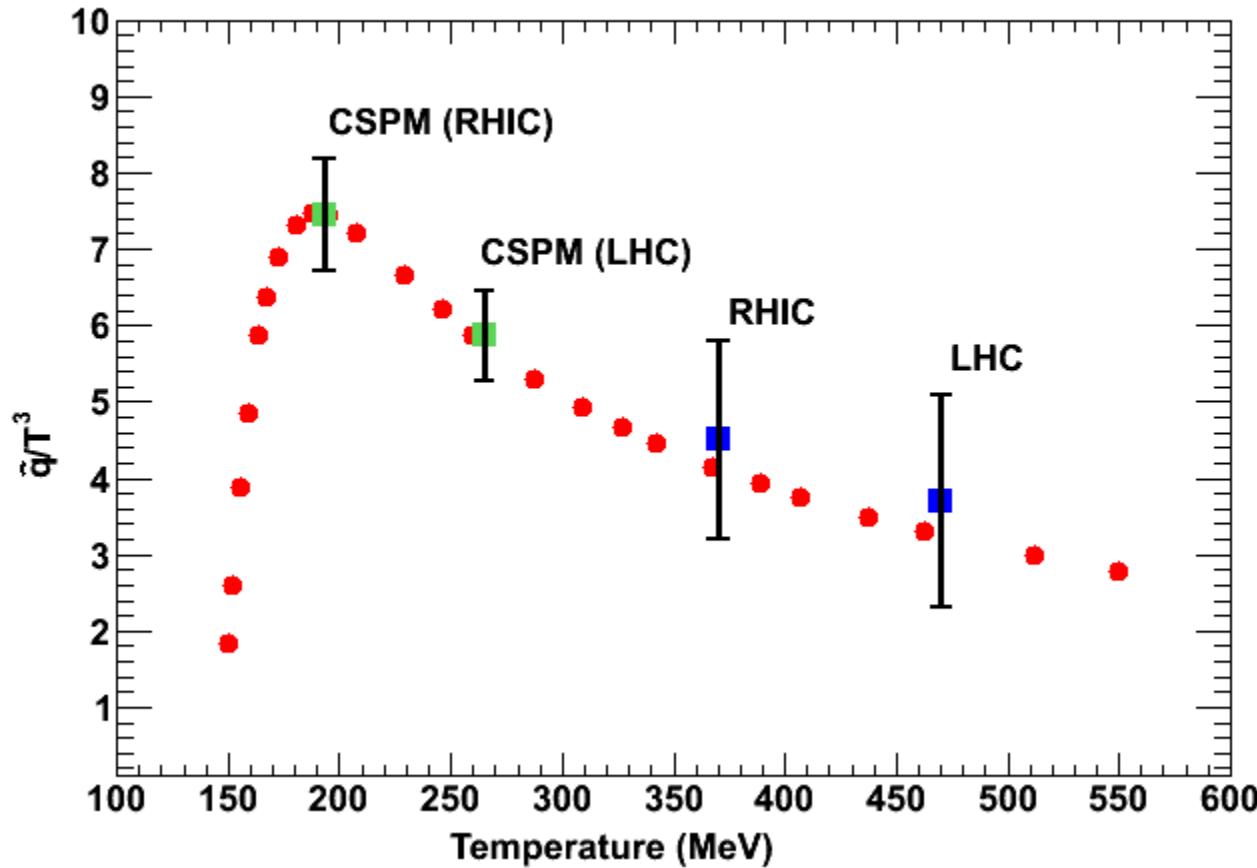
Phys. Rev. C 90 (2014) 014909

$$\frac{\hat{q}}{T^3} \approx \begin{cases} 4.5 \pm 1.3 & \text{RHIC 370 MeV} \\ 3.7 \pm 1.4 & \text{LHC 470 MeV} \end{cases}$$

Initial time of $\tau_0 \sim 0.6 \text{ fm/c}$

A. Majumdar, B. Muller , X.-N. Wang , Phys. Rev. Lett. 99 (2007) 192301
J. Casalderrey-Solana, X. –N. Wang , Phys. Rev. C 77 (2008) 024902

Jet Quenching Parameter



$$\frac{\eta}{s} \approx \frac{3}{2} \frac{T^3}{\hat{q}}$$

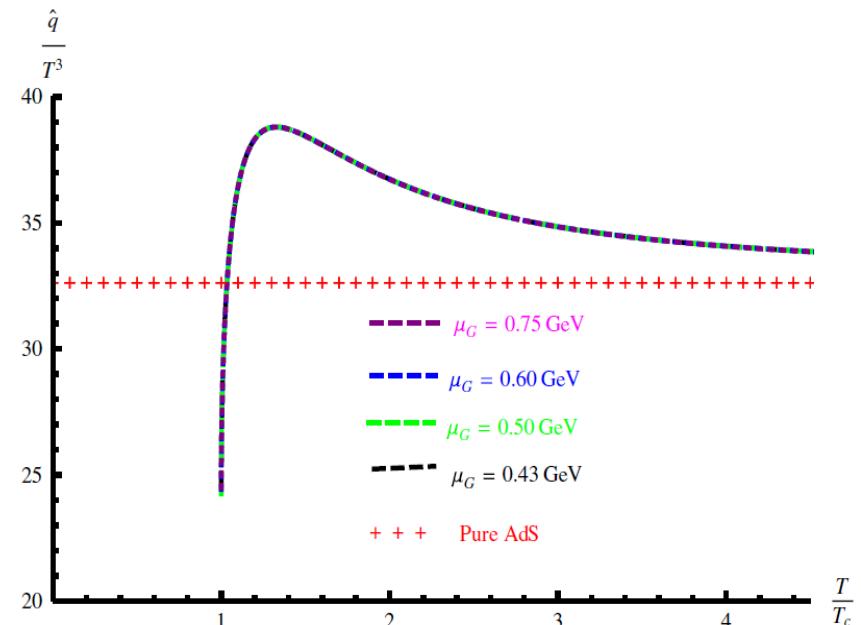
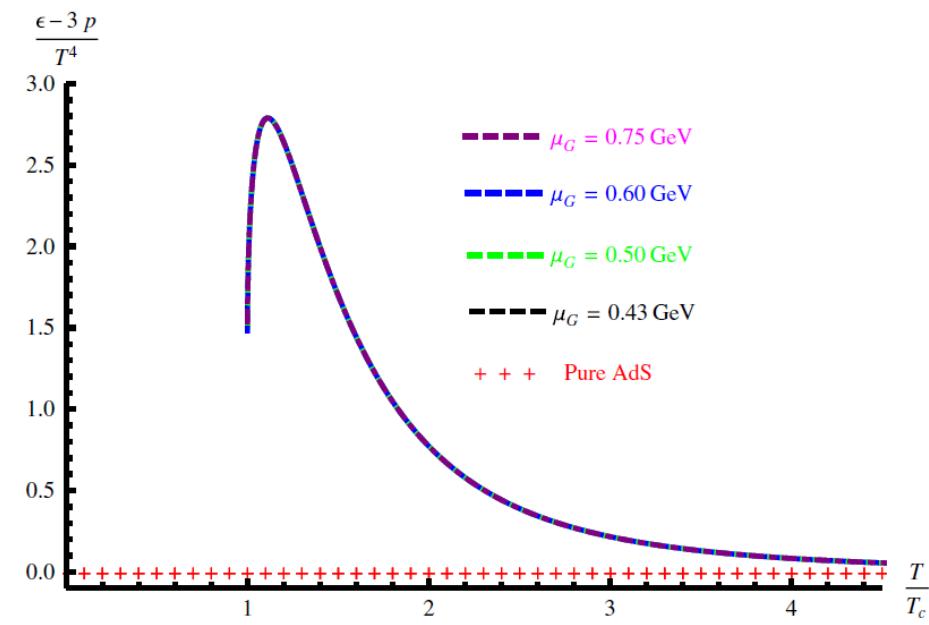
J. Casalderrey-Solana, X. -N. Wang , Phys. Rev. C 77 (2008) 024902

Thermodynamic results from LQCD -> In Quasi particle approach

- Bluhm et al., Phys. Rev. C 84, 025201 (2011)
- Plumari et al., Phys. Rev. D 84, 094004 (2011)
- Khvorostukhin et al, Nucl. Phys. A 845 , 106 (2010)
- Marty et al., Phys. Rev. C88, 045204 (2013)

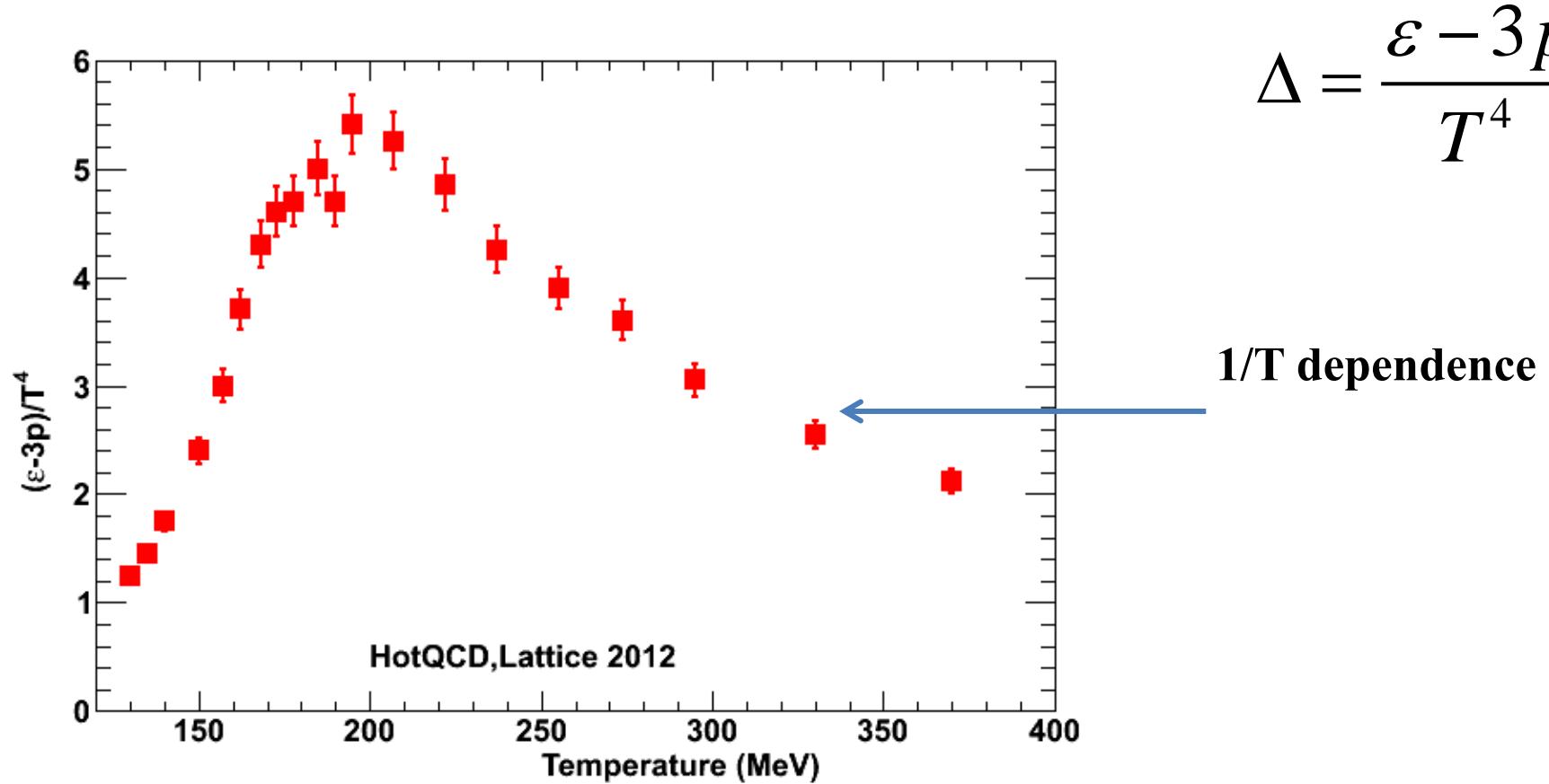
Dynamical Holographic Model

- Enhancement of Jet Quenching around Phase Transition
- Results agree well with Lattice for a pure gluon system
- Both $\frac{(\varepsilon - 3p)}{T^4}$ and $\frac{\hat{q}}{T^3}$ show a peak around the critical temperature



How to obtain the pressure ?

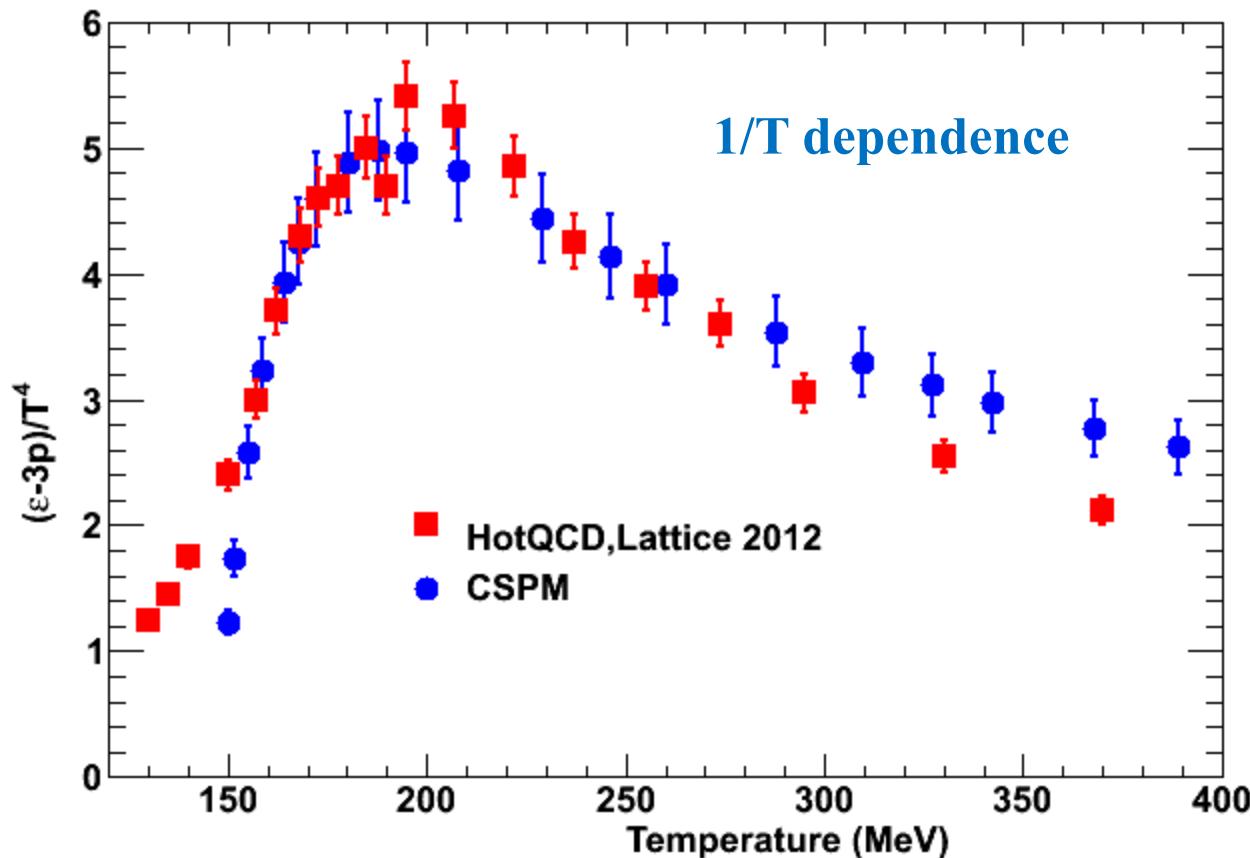
Interaction Measure from Lattice QCD Calculation



Ansatz : $\Delta = 1 / (\eta / s)$

**Magnitude and functional dependence
same as in LQCD**

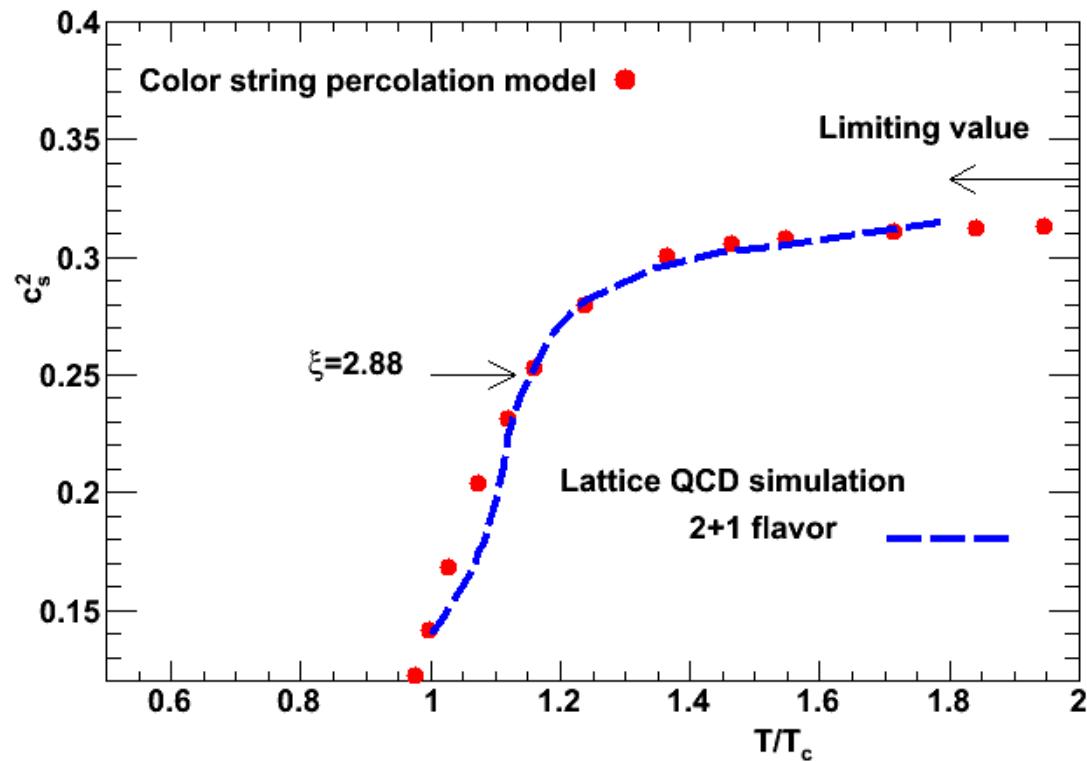
Interaction Measure



$$\Delta = \frac{\varepsilon - 3p}{T^4}$$

Sound Velocity

$$C_s^2 = \frac{dp}{d\varepsilon} = \varepsilon \frac{dp/\varepsilon}{d\varepsilon} + \frac{p}{\varepsilon}$$



$$C_s^2 = (-0.33) \left(\frac{\xi e^{-\xi}}{1 - e^{-\xi}} - 1 \right) + 0.0191(\Delta / 3) \left(\frac{\xi e^{-\xi}}{(1 - e^{-\xi})^2} - \frac{1}{1 - e^{-\xi}} \right)$$

Summary

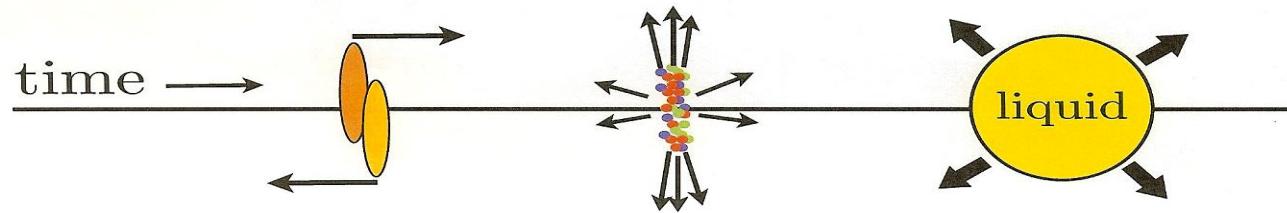
- The Clustering of Color Sources leading to the Percolation Transition may be the way to achieve de-confinement in High Energy collisions.
- The Percolation framework provide us with a microscopic partonic structure which explains the early thermalization.
- The minimum in η/s can be studied as a function of the beam energy at RHIC that could locate the critical point/crossover in QCD phase diagram as seen in substances like He, N₂ and H₂O.
- A further definitive test of clustering phenomena can be made at LHC energies by comparing *h-h* and A-A collisions.
- The important message is that the percolation/clustering has a physical basis although it can not be deduced directly from QCD.

Thank You

Extras

DIFFERENCE IN FLOW AND THERMALIZATION

Hydrodynamics & thermalization in heavy ion collisions



Basic theoretical questions:

- How long is t_{hydro} ?
- How long is t_{therm} ?
- How is t_{therm} correlated with t_{hydro} ?

(Au-Au
example)

$$t_{\text{hydro}} \sim 0.35 \text{ fm/c} \quad t_{\text{therm}} \sim 1.05 \text{ fm/c}$$

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