Quark number susceptibility in mean field model : revisited with fluctuation-dissipation theorem.

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Ref: PhysRevD.90.054030 (2014)

Outline

- Introduction
- Effective Mean field model
- Exploration of Quark number susceptibility (QNS) with Fluctuation-Dissipation Theorem (FDT)
- An analytical technique to compute derivatives of mean fields
- Association of implicit depedences with quark line disconnected diagrams of non-perturbative QCD

Introduction

- The analysis of fluctuations is a powerful method for characterizing the thermodynamic properties of a system.
- Fluctuations are enhanced at the critical region hence an essential characteristic of phase transitions.

  [Asakawa *et. al.* PRL 85,(2000), Jeon & Koch, PRL 85 (2000)]
- Modifications in the magnitude of fluctuations : a phenomenological probe of deconfinement and chiral symmetry restoration in heavy-ion collisions. [Son and Stephanov PRL 88,(2002)]
- A measure of the intrinsic statistical fluctuations in a system close to thermal equilibrium is provided by the corresponding susceptibilities.
- Hence, fluctuations in a thermodynamic system may be explored by finding the dependence of susceptibilities on the thermal parameters, basically temperature T and chemical potential μ.

Introduction

- Quark Number Susceptibility (QNS) : Fluctuation of net quark number w.r.t. chemical potential.
- QNS may be used to identify the critical point in the QCD phase diagram. [Hatta and Ikeda PRD 67 (2003)]
- QNS have been studied extensively in lattice QCD. [Alton *et. al.* PRD 71 (2005), Gavai and Gupta, PRD 73 (2006), Borsányi *et. al.* JHEP (2012), HOTQCD PRD 86 (2012)]
- Observation : A suppression in the confined phase and increase with temperature near the transition region.
- The susceptibility near the transition temperature shows a strong increase with increasing quark chemical potential, leading to cusp-like structure.
- In effective model framework : QNS has been explored both in zero and non zero chemical potential. [Sasaki *et. al.* PRD75 (2007), Ghosh *et. al.* PRD77 (2006)]

Effective mean field models

- Use of effective QCD inspired models to understand the phase structure of Quantum Chromo Dynamics is a popular tool.
- NJL model :

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\partial\!\!\!/ - m_0 + \gamma_0\mu_q)\psi + \frac{G_s}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]$$

Successfully reproduces chiral symmetry breaking of QCD through a non vanishing chiral condensate $\sigma = \langle\bar{\psi}\psi\rangle$.

- PNJL model : Introduction of static background field $\rightarrow$ ties together the two aspects of QCD, *i.e.* the chiral symmetry breaking and the confinement-deconfinement transition.

$$\mathcal{L}_{\rm PNJL} = \bar{\psi}(iD\!\!\!\!/ - m_0 + \gamma_0\mu_q)\psi + \frac{G_s}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \mathcal{U}(\Phi, \bar{\Phi}, T)$$

where, $D^\mu = \partial^\mu - ig\mathcal{A}^\mu_a\lambda_a/2$ with $\mathcal{A}^\mu_a = \delta_{\mu 4}A^4_a$ and λ_a are Gell-Mann matrices.

- Only temporal component of the background field is considered.
- The Polyakov Loop L is given as:

$$\mathbf{L}(\vec{x},\tau) = e^{-i\int_0^\beta d\tau A_4(\vec{x},\tau)}$$

- Traced Polyakov loop $\Phi = \frac{1}{N_c}\mathrm{Tr}_c(\mathbf{L})$, $\quad \bar{\Phi} = \frac{1}{N_c}\mathrm{Tr}_c(\mathbf{L}^\dagger)$;
- Thermal average of Polyakov Loop, $<\Phi>\rightarrow$ order parameter for pure gluon theory [McLerran & Svetitsky, PRD 24 (1981)].
- $<\Phi>\sim e^{-\beta F_q}$; F_q is quark free energy.
- $F_q = \infty$ in confined phase $\Rightarrow$ Φ=0
- Deconfinement sets in $\rightarrow$ Φ has non zero value.
- In presence of dynamical quarks: indicator of phase transition.

Lagrangian to thermodynamic potential

1. The partition function :

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S}$$

2. Introduction of auxiliary field :

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}X e^{-S_{\rm MF}}$$

3. Integration over fermionic fields :

$$Z = \int \mathcal{D}X e^{-S_{\rm MF}(T,\mu)}$$

4. Saddle point approximation :

$$Z = \int \mathcal{D}X e^{-S_{\rm MF}} \approx e^{-S_{\rm MF}(T,\mu,X(T,\mu))}$$

5. Thermodynamic potential :

$$\Omega = -\frac{T}{V} \ln Z$$

From Ω to QNS :

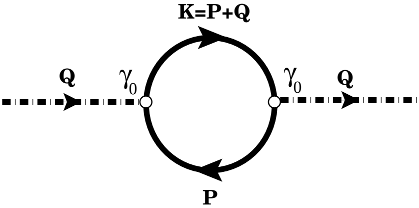
- Thermodynamic Potential is minimised w.r.t. the fields $\Phi, \bar{\Phi}, \sigma$. Solution of the simultaneous equations :

$$\frac{\partial \Omega}{\partial \Phi} = 0; \quad \frac{\partial \Omega}{\partial \bar{\Phi}} = 0; \quad \frac{\partial \Omega}{\partial \sigma} = 0$$

- The field value is used to evaluate the thermodynamic quantities : pressure, entropy, energy density, specific heat etc.
- QNS is usually obtained as the second order Taylor coefficient of pressure, when pressure is Taylor expanded in the direction of quark chemical potential.

QNS and FDT

- Temporal component of the vector correlator:



- Fluctuation Dissipation Theorem(FDT) $\rightarrow$ QNS from the temporal component of the vector correlator. [Kunihiro, PLB 271 (1991)]

$$\chi_q = -\lim_{q\to 0} Re\Pi_{00}(0, q = |\vec{Q}|)$$

$$\chi_q = \lim_{q\to 0} \beta \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{-2}{1-e^{-\beta\omega}} Im\Pi_{00}(\omega, q = |\vec{Q}|)$$

where, $\Pi_{00}(q_0 = \omega, q = |\vec{Q}|) = \mathrm{Tr}[\gamma_0 S_f(P+Q)\gamma_0 S_f(P)]$

QNS and FDT : Contd ....

- QNS obtained both from real part and imaginary part of Π_{00} :

$$\chi_q = 2\beta N_f N_c \int \frac{d^3p}{(2\pi)^3} \{ \frac{e^{\beta(E_p-\mu_q)}}{(1+e^{\beta(E_p-\mu_q)})^2} + \frac{e^{\beta(E_p+\mu_q)}}{(1+e^{\beta(E_p+\mu_q)})^2} \}$$

$$\Downarrow$$

$$\chi_q = \frac{\partial^2 \mathcal{P}}{\partial \mu_q^2}$$

- Starting from the temporal correlator with amputated external leg :

$$\frac{\partial^2 \mathcal{P}}{\partial \mu_q^2} = i \int \frac{d^4P}{(2\pi)^4} \mathrm{Tr}_{D,f,c}[\gamma_0 S_1(P) \gamma_0 S_1(P)].$$

[Ghosh *et. al.*, PRD 90 (2014)]

QNS in NJL model : Let's Revisit..

- In model calculations any response of a thermodynamic quantity to some external parameters should also account for the fact that the mean fields also depend on that parameter implicitly.
- Numerical computation of QNS from second order Taylor coefficient of Pressure w.r.t. chemical potential. [Ghosh *et. al.* PRD73 (2006), Mukherjee *et. al.* PRD75 (2007)] Implicit contribution already embedded $\Rightarrow \chi_q = \frac{d^2\mathcal{P}}{d\mu^2}$
- Analytic calculation of QNS from pressure: we need to consider the total derivative w.r.t. μ.
- Starting from pressure we can write;

$$\chi_q = \frac{d^2\mathcal{P}}{d\mu_q^2} = \frac{\partial^2\mathcal{P}}{\partial\mu_q^2} + \left[\frac{\partial}{\partial\mu_q}\left(\frac{\partial\mathcal{P}}{\partial\sigma}\right) + \frac{\partial}{\partial\sigma}\left(\frac{\partial\mathcal{P}}{\partial\mu_q}\right)\right] \cdot \frac{d\sigma}{d\mu_q} + \frac{\partial\mathcal{P}}{\partial\sigma} \cdot \frac{d^2\sigma}{d\mu_q^2} + \frac{\partial^2\mathcal{P}}{\partial\sigma^2} \cdot \left(\frac{d\sigma}{d\mu_q}\right)^2$$

Derivative of mean fields : Semi-analytical approach

- NJL model Lagrangian under mean field approximation:

$$\mathcal{L}_{\rm MF} = \bar{\psi}(i\partial\!\!\!/ - m_0 + \gamma_0 \mu_q + G_s\sigma)\psi - \frac{G_s}{2}\sigma^2$$

- Chiral condensate $\sigma = i\mathrm{Tr}(S_1)$, basically function of T, m_0 and μ_q
- S_1 is dressed propagator with modified mass $M = m_0 - G_s\sigma$.

$$S_1{}^{-1} = p\!\!\!/ - m_0 + \gamma_0\mu_q + G_s\sigma = S_0{}^{-1} + G_s\sigma$$

where, S_0 is bare propagator with current mass m_0.

- Ward identity at finite T and μ_q [Finite-Temperature Field Theory : Kapusta] :

bare insertion factor corresponding to S_0 is given by : $\dfrac{dS_0{}^{-1}}{d\mu_q} = \gamma_0$.

Semi-analytical approach : Contd....

- Applying Ward identity for S_1 and considering σ to be μ dependent, one can write;

$$\frac{d{S_1}^{-1}}{d\mu_q} = \gamma_0 + \left(G_s \frac{d\sigma}{d\mu_q} \right) \cdot \mathbb{1}_D \equiv \Gamma_0$$

  Γ_0 is the effective three-point function. [Ghosh *et. al* PRD 90,(2014)]

- $\sigma = i\mathrm{Tr}(S_1)$: transcendental equation.
- Impossible to extract closed form expression of σ solely in terms of T and μ_q.
- On the contrary the situation is markedly different for the derivatives of σ.

Field derivative : first order

Starting from $\sigma = i\mathrm{Tr}(S_1)$, one can show :

$$\frac{d\sigma}{d\mu_q} = i\mathrm{Tr}[S_1\Gamma_0 S_1] = -i\mathrm{Tr}(S_1\gamma_0 S_1) - i\mathrm{Tr}(S_1 G_s \frac{d\sigma}{d\mu_q} S_1)$$

Finally a compact expression :

$$\frac{d\sigma}{d\mu_q} = \frac{-i\mathrm{Tr}(S_1\gamma_0 S_1)}{1 + iG_s\mathrm{Tr}({S_1}^2)}$$

not of trancendental type !!

$\frac{d\sigma}{d\mu_q}$ is written in terms of T, μ_q and σ only.

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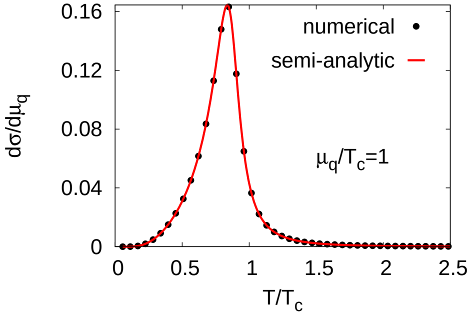
NJL model



Field derivative : second order

Similarly starting from

$$\frac{d\sigma}{d\mu_q} = i\mathrm{Tr}[S_1\Gamma_0 S_1] = i\mathrm{Tr}(S_1\gamma_0 S_1) + i\mathrm{Tr}(S_1 G\frac{d\sigma}{d\mu_q}S_1)$$

one further differentiation w.r.t. μ_q leads to;

$$\frac{d^2\sigma}{d\mu_q^2} = -2i\mathrm{Tr}[S_1\Gamma_0 S_1\Gamma_0 S_1] + i\mathrm{Tr}(S_1 G\frac{d^2\sigma}{d\mu_q^2}S_1)$$

We can write a compact expression :

$$\frac{d^2\sigma}{d\mu_q^2} = \frac{-2i\mathrm{Tr}(S_1\Gamma_0 S_1\Gamma_0 S_1)}{1 - iG\mathrm{Tr}({S_1}^2)}.$$

$\frac{d^2\sigma}{d\mu_q^2}$ is written in terms of T, μ_q, σ and $\frac{d\sigma}{d\mu_q}$ only.

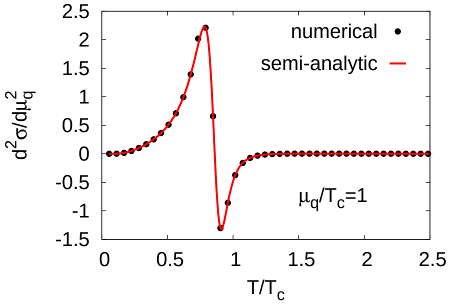
Field derivative : second order

$$\frac{d^2\sigma}{d\mu_q^2} = \frac{-2i[\mathrm{Tr}(\mathrm{S}_1\gamma_0\mathrm{S}_1\gamma_0\mathrm{S}_1) + 2(\mathrm{G}\frac{\mathrm{d}\sigma}{\mathrm{d}\mu_\mathrm{q}})\mathrm{Tr}({\mathrm{S}_1}^3\gamma_0) + (\mathrm{G}\frac{\mathrm{d}\sigma}{\mathrm{d}\mu_\mathrm{q}})^2\mathrm{Tr}({\mathrm{S}_1}^3)]}{1 - iG\mathrm{Tr}({S_1}^2)}.$$

Field derivative : second order

$$\frac{d^2\sigma}{d\mu_q^2} = \frac{-2i[\mathrm{Tr}(\mathrm{S}_1\gamma_0\mathrm{S}_1\gamma_0\mathrm{S}_1) + 2(\mathrm{G}\frac{\mathrm{d}\sigma}{\mathrm{d}\mu_\mathrm{q}})\mathrm{Tr}({\mathrm{S}_1}^3\gamma_0) + (\mathrm{G}\frac{\mathrm{d}\sigma}{\mathrm{d}\mu_\mathrm{q}})^2\mathrm{Tr}({\mathrm{S}_1}^3)]}{1 - iG\mathrm{Tr}({S_1}^2)}.$$

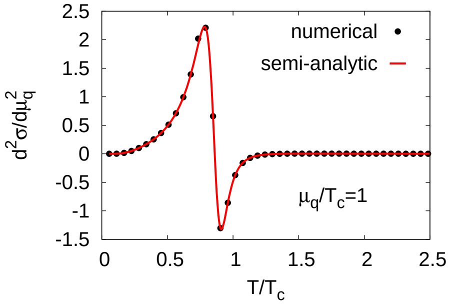
NJL model



Field derivative : second order

$$\frac{d^2\sigma}{d\mu_q^2} = \frac{-2i[\mathrm{Tr}(\mathrm{S}_1\gamma_0\mathrm{S}_1\gamma_0\mathrm{S}_1) + 2(\mathrm{G}\frac{\mathrm{d}\sigma}{\mathrm{d}\mu_\mathrm{q}})\mathrm{Tr}({\mathrm{S}_1}^3\gamma_0) + (\mathrm{G}\frac{\mathrm{d}\sigma}{\mathrm{d}\mu_\mathrm{q}})^2\mathrm{Tr}({\mathrm{S}_1}^3)]}{1 - iG\mathrm{Tr}({S_1}^2)}.$$

NJL model



- This method is quite general and can be carried upto any order.
  Generally, $\frac{d^n\sigma}{d\mu_q^n}$ can be expressed in terms of $T, \mu, \sigma, \frac{d\sigma}{d\mu_q}, \ldots\ldots\ldots$ $\ldots, \frac{d^{(n-1)}\sigma}{d\mu_q^{(n-1)}}$.

Returning to QNS from FDT

- Derivation of QNS using FDT $\Rightarrow$ Vector correlator needs modification.
- The thermodynamic potential $\Omega = -i\mathrm{Tr}\ln S_1^{-1} + \frac{G_s}{2}\sigma^2$
- Pressure in NJL model $\mathcal{P} = -\Omega$
- One can derive the relations; [Ghosh *et. al.* , PRD90 (2014)]

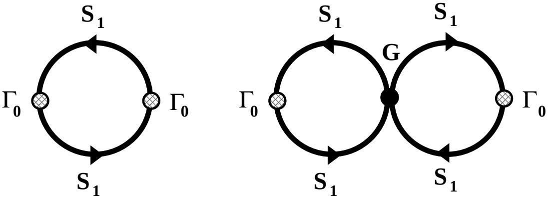
$$\frac{\partial^2 \mathcal{P}}{\partial \mu_q^2} = -i\mathrm{Tr}[\gamma_0 S_1 \gamma_0 S_1]; \qquad \frac{\partial}{\partial \mu_q}\left(\frac{\partial \mathcal{P}}{\partial \sigma}\right) = -iG_s\mathrm{Tr}[S_1\gamma_0 S_1]$$

$$\frac{\partial}{\partial \sigma}\left(\frac{\partial \mathcal{P}}{\partial \mu_q}\right) = -iG_s\mathrm{Tr}[S_1\gamma_0 S_1]; \qquad \frac{\partial^2 \mathcal{P}}{\partial \sigma^2} = G_s[-iG_s\mathrm{Tr}(S_1^2) - 1]$$

- Using these the QNS becomes: [Ghosh *et. al.* PRD90 (2014)]

$$\chi_q = \frac{d^2\mathcal{P}}{d\mu_q^2} = -i\mathrm{Tr}(\Gamma_0 S_1 \Gamma_0 S_1) - G_s(-i\mathrm{Tr}[S_1\Gamma_0 S_1])^2.$$

- In terms of diagramatic topology the terms in the R.H.S. can be viewed as :



- These are equivalent to the vector correlator in the NJL model in static limit or amputated legs.
- Inclusion of implicit μ_q dependences of the mean fields lead to *modified* correlators associated with the conserved density fluctuation.

QNS in NJL model : Result

At $\mu_q = 0$, $\frac{d\sigma}{d\mu_q} = 0 \Rightarrow$ Implicit contributions vanishes in QNS.

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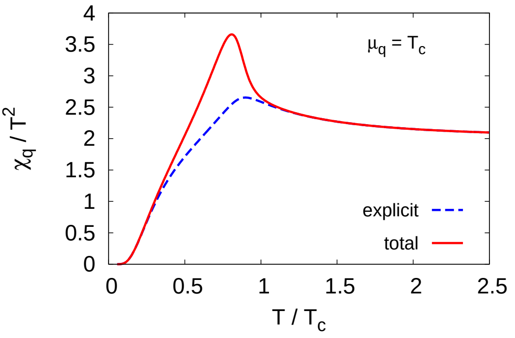
$\mu_q \neq 0$



Implicit contributions dominate close to the transition region where the change in the mean fields is most significant.

QNS in PNJL model

- Lagrangian of PNJL model in mean field approximation:

$$\mathcal{L}_{\rm PNJL} = \bar{\psi}(i\not{D} - m_0 + \gamma_0\mu)\psi + \frac{G_s}{2}\sigma^2 - \mathcal{U}[\Phi, \bar{\Phi}, T]$$

- No gluon-like quasi particles in PNJL model. We treat such bosonic fields as purely classical ones unlike the fermionic fields.
- The mean fields Φ and $\bar{\Phi}$ also depend on T and μ_q in a similar way as σ.
- Analytic forms of $\frac{d\Phi}{d\mu_q}$ and $\frac{d\bar{\Phi}}{d\mu_q}$ are difficult to extract within the present formalism.

Derivatives of mean fields in PNJL model

Mean field condition $\frac{\partial \Omega'}{\partial X} = 0$ with $X = \Phi, \bar{\Phi}, \sigma$.

$$\Omega' = \Omega - \kappa T^4 \ln[J(\Phi, \bar{\Phi})]$$ [Ghosh *et. al.* PRD 77 (2008)]

Start with:

$$\frac{d}{d\mu_q}\left(\frac{\partial \Omega'}{\partial X}\right) = 0$$

which immediately gives :

$$\frac{\partial}{\partial \mu_q}\left(\frac{\partial \Omega'}{\partial X}\right) + \frac{\partial}{\partial \Phi}\left(\frac{\partial \Omega'}{\partial X}\right) \cdot \frac{d\Phi}{d\mu_q} + \frac{\partial}{\partial \bar{\Phi}}\left(\frac{\partial \Omega'}{\partial X}\right) \cdot \frac{d\bar{\Phi}}{d\mu_q}$$
$$+ \frac{\partial}{\partial \sigma_u}\left(\frac{\partial \Omega'}{\partial X}\right) \cdot \frac{d\sigma_u}{d\mu_q} + \frac{\partial}{\partial \sigma_d}\left(\frac{\partial \Omega'}{\partial X}\right) \cdot \frac{d\sigma_d}{d\mu_q} = 0$$

Matrix Equation of the form $\mathbf{A} \cdot \mathbf{x} = \mathbf{B}$.

$$\mathbf{A} = \begin{pmatrix} \frac{\partial}{\partial \Phi}(\frac{\partial \Omega'}{\partial \Phi}) & \frac{\partial}{\partial \bar{\Phi}}(\frac{\partial \Omega'}{\partial \Phi}) & \frac{\partial}{\partial \sigma_u}(\frac{\partial \Omega'}{\partial \Phi}) & \frac{\partial}{\partial \sigma_d}(\frac{\partial \Omega'}{\partial \Phi}) \\ \frac{\partial}{\partial \Phi}(\frac{\partial \Omega'}{\partial \bar{\Phi}}) & \frac{\partial}{\partial \bar{\Phi}}(\frac{\partial \Omega'}{\partial \bar{\Phi}}) & \frac{\partial}{\partial \sigma_u}(\frac{\partial \Omega'}{\partial \bar{\Phi}}) & \frac{\partial}{\partial \sigma_d}(\frac{\partial \Omega'}{\partial \bar{\Phi}}) \\ \frac{\partial}{\partial \Phi}(\frac{\partial \Omega'}{\partial \sigma_u}) & \frac{\partial}{\partial \bar{\Phi}}(\frac{\partial \Omega'}{\partial \sigma_u}) & \frac{\partial}{\partial \sigma_u}(\frac{\partial \Omega'}{\partial \sigma_u}) & \frac{\partial}{\partial \sigma_d}(\frac{\partial \Omega'}{\partial \sigma_u}) \\ \frac{\partial}{\partial \Phi}(\frac{\partial \Omega'}{\partial \sigma_d}) & \frac{\partial}{\partial \bar{\Phi}}(\frac{\partial \Omega'}{\partial \sigma_d}) & \frac{\partial}{\partial \sigma_u}(\frac{\partial \Omega'}{\partial \sigma_d}) & \frac{\partial}{\partial \sigma_d}(\frac{\partial \Omega'}{\partial \sigma_d}) \end{pmatrix}$$

$$\mathbf{x} = (\frac{d\Phi}{d\mu_q}, \frac{d\bar{\Phi}}{d\mu_q}, \frac{d\sigma_u}{d\mu_q}, \frac{d\sigma_d}{d\mu_q})^{\mathrm{T}}$$

and

$$\mathbf{B} = (-\frac{\partial}{\partial \mu_q}(\frac{\partial \Omega'}{\partial \Phi}), -\frac{\partial}{\partial \mu_q}(\frac{\partial \Omega'}{\partial \bar{\Phi}}), -\frac{\partial}{\partial \mu_q}(\frac{\partial \Omega'}{\partial \sigma_u}), -\frac{\partial}{\partial \mu_q}(\frac{\partial \Omega'}{\partial \sigma_d}))^{\mathrm{T}}$$

$$\mathbf{A} = \begin{pmatrix} \frac{\partial}{\partial \Phi}(\frac{\partial \Omega'}{\partial \Phi}) & \frac{\partial}{\partial \bar{\Phi}}(\frac{\partial \Omega'}{\partial \Phi}) & \frac{\partial}{\partial \sigma_u}(\frac{\partial \Omega'}{\partial \Phi}) & \frac{\partial}{\partial \sigma_d}(\frac{\partial \Omega'}{\partial \Phi}) \\ \frac{\partial}{\partial \Phi}(\frac{\partial \Omega'}{\partial \bar{\Phi}}) & \frac{\partial}{\partial \bar{\Phi}}(\frac{\partial \Omega'}{\partial \bar{\Phi}}) & \frac{\partial}{\partial \sigma_u}(\frac{\partial \Omega'}{\partial \bar{\Phi}}) & \frac{\partial}{\partial \sigma_d}(\frac{\partial \Omega'}{\partial \bar{\Phi}}) \\ \frac{\partial}{\partial \Phi}(\frac{\partial \Omega'}{\partial \sigma_u}) & \frac{\partial}{\partial \bar{\Phi}}(\frac{\partial \Omega'}{\partial \sigma_u}) & \frac{\partial}{\partial \sigma_u}(\frac{\partial \Omega'}{\partial \sigma_u}) & \frac{\partial}{\partial \sigma_d}(\frac{\partial \Omega'}{\partial \sigma_u}) \\ \frac{\partial}{\partial \Phi}(\frac{\partial \Omega'}{\partial \sigma_d}) & \frac{\partial}{\partial \bar{\Phi}}(\frac{\partial \Omega'}{\partial \sigma_d}) & \frac{\partial}{\partial \sigma_u}(\frac{\partial \Omega'}{\partial \sigma_d}) & \frac{\partial}{\partial \sigma_d}(\frac{\partial \Omega'}{\partial \sigma_d}) \end{pmatrix}$$

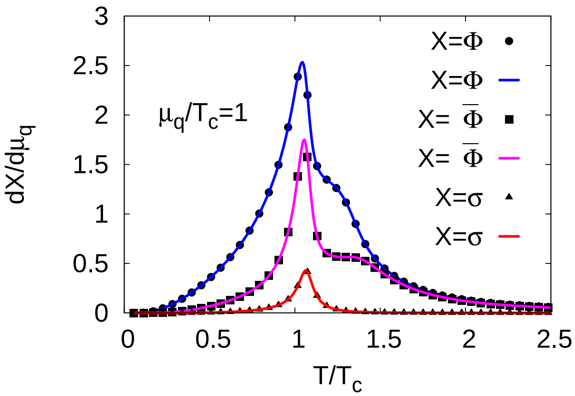
$$\mathbf{x} = (\frac{d\Phi}{d\mu_q}, \frac{d\bar{\Phi}}{d\mu_q}, \frac{d\sigma_u}{d\mu_q}, \frac{d\sigma_d}{d\mu_q})^{\mathrm{T}}$$

and

$$\mathbf{B} = (-\frac{\partial}{\partial \mu_q}(\frac{\partial \Omega'}{\partial \Phi}), -\frac{\partial}{\partial \mu_q}(\frac{\partial \Omega'}{\partial \bar{\Phi}}), -\frac{\partial}{\partial \mu_q}(\frac{\partial \Omega'}{\partial \sigma_u}), -\frac{\partial}{\partial \mu_q}(\frac{\partial \Omega'}{\partial \sigma_d}))^{\mathrm{T}}$$

Solution of $\frac{dX}{d\mu_q} \rightarrow$ using Cramer's rule or Gaussian elimination method.

Field Derivatives in PNJL model for $\mu_q \neq 0$



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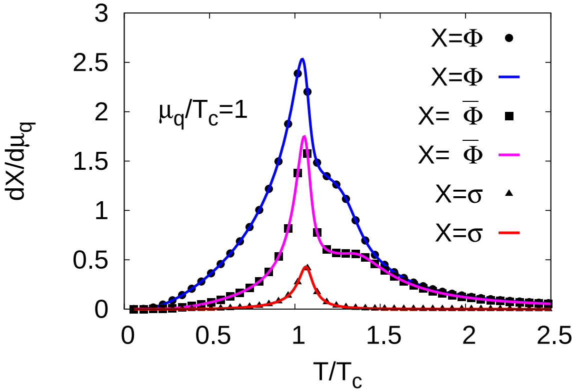


Effect of numerical inaccuracy is only through the calculation of mean fields. Expected to be stable at higher order.

One can intuitively write as in the case of the NJL model,

$$\frac{d\mathcal{P}}{d\mu_q} = \frac{\partial\mathcal{P}}{\partial\mu_q} + \frac{\partial\mathcal{P}}{\partial\sigma}\frac{d\sigma}{d\mu_q} + \frac{\partial\mathcal{P}}{\partial\Phi}\frac{d\Phi}{d\mu_q} + \frac{\partial\mathcal{P}}{\partial\bar{\Phi}}\frac{d\bar{\Phi}}{d\mu_q}$$

VdM term in the thermodynamic potential :

$$\mathcal{P} = -\Omega \neq -\Omega'$$

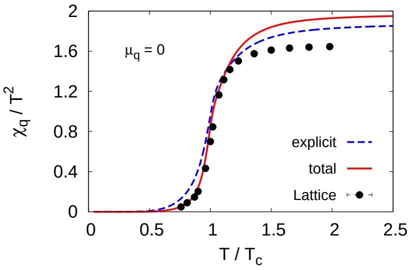
As a consequence : $\frac{\partial\mathcal{P}}{\partial\sigma} = 0$ but $\frac{\partial\mathcal{P}}{\partial\Phi} \neq 0$ and $\frac{\partial\mathcal{P}}{\partial\bar{\Phi}} \neq 0$.
Implicit contributions through Φ and $\bar{\Phi}$, exist even for first order derivative.
For second order :

$$\frac{d^2\mathcal{P}}{d\mu_q^2} = \frac{\partial^2\mathcal{P}}{\partial\mu_q^2} + 2\sum_{X=\sigma,\Phi,\bar{\Phi}} \frac{\partial^2\mathcal{P}}{\partial\mu_q\partial X}\cdot\frac{dX}{d\mu_q} + \sum_{X=\sigma,\Phi,\bar{\Phi}} \frac{\partial\mathcal{P}}{\partial X}\cdot\frac{d^2X}{d\mu_q^2}$$
$$+ \sum_{X,Y=\sigma,\Phi,\bar{\Phi}} \frac{\partial^2\mathcal{P}}{\partial X\partial Y}\cdot\frac{dX}{d\mu_q}\cdot\frac{dY}{d\mu_q}$$

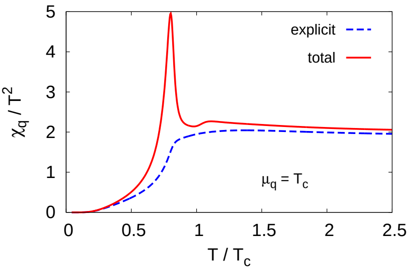
Implicit contributions to QNS survive even for $\mu_q = 0$, unlike NJL.

QNS revisited in PNJL model

$\mu_q = 0$



$\mu_q \neq 0$



Lattice data taken from Alton *et. al.* Phys. Rev D71,054508 (2005).

- $\mu_q = 0$: implicit contributions changes sign near T_c.
- $\mu_q \neq 0$: implicit contributions are strictly positive over the whole T range.

Diagrammatics in Non-perturbative QCD

Starting from [Gavai & Gupta, PRD 68, 034506 (2003)] :

$$\mathcal{Z}(T, \{m_f\}, \{\mu_f\}) = \int \mathcal{D}\mathcal{A} \prod_{f=u,d,\cdots} \det M(T, m_f, \mu_f) e^{-\mathcal{S}_{\mathcal{G}}(\mathcal{A})}$$

number of any given flavor :

$$n_f = \frac{T}{V} \frac{\partial}{\partial \mu_f} \ln \mathcal{Z} = \frac{T}{V} \langle \mathcal{O}_1^f \rangle$$

and flavor diagonal susceptibilities :

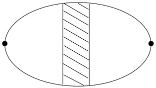
$$\chi_{ff} = \frac{T}{V} \frac{\partial^2}{\partial \mu_f^2} \ln \mathcal{Z} = \frac{T}{V} \langle \mathcal{O}_2^f \rangle + \frac{T}{V} \langle \mathcal{O}_1^f \cdot \mathcal{O}_1^f \rangle - \frac{T}{V} \langle \mathcal{O}_1^f \rangle^2$$

flavor off-diagonal susceptibilities :

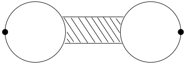
$$\chi_{fg} = \frac{T}{V} \frac{\partial^2}{\partial \mu_f^2} \ln \mathcal{Z} = \frac{T}{V} \langle \mathcal{O}_1^f \cdot \mathcal{O}_1^g \rangle - \frac{T}{V} \langle \mathcal{O}_1^f \rangle \cdot \langle \mathcal{O}_1^g \rangle$$

Diagrammatics in NPQCD : contd..

- At vanishing chemical potential : flavor diagonal susceptibilities have two types of contributions. [Gavai & Gupta, PRD 68, 034506 (2003)]
    1. quark-line connected contributions :



    2. quark-line disconnected contributions :



- Flavor off diagonal susceptibilities : only quark line disconnected contribution.

Specifically :

$$\chi_{uu} = \frac{T}{V}[\langle o_2^u \rangle + \langle o_1^u \cdot o_1^u \rangle - \langle o_1^u \rangle^2]$$

$$\chi_{dd} = \frac{T}{V}[\langle o_2^d \rangle + \langle o_1^d \cdot o_1^d \rangle - \langle o_1^d \rangle^2]$$

$$\chi_{ud} = \frac{T}{V}[\langle o_1^u \cdot o_1^d \rangle - \langle o_1^u \rangle \cdot \langle o_1^d \rangle]$$

the quark number and isospin number susceptibility can be written as:

$$\chi_q = \chi_{uu} + \chi_{dd} + 2\chi_{ud} \quad \text{and} \quad \chi_I = \chi_{uu} + \chi_{dd} - 2\chi_{ud}$$

In isospin symmetric limit the contribution of the disconnected diagrams of χ_{uu} and χ_{dd} are same as that of χ_{ud}.

- QNS will contain both quark line connected and disconnected contributions.
- INS will contain only quark line connected contributions.

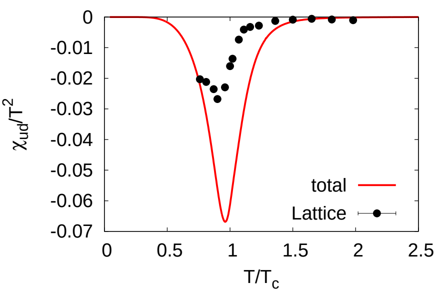
Returning to model

$$\chi_{ud} = \frac{\partial^2 \mathcal{P}}{\partial \mu_u \partial \mu_d} + \sum_{X=\sigma_u,\sigma_d,\Phi,\bar{\Phi}} \frac{\partial^2 \mathcal{P}}{\partial X \partial \mu_u} \cdot \frac{dX}{d\mu_d} + \sum_X \frac{\partial^2 \mathcal{P}}{\partial X \partial \mu_d} \cdot \frac{dX}{d\mu_u}$$
$$+ \sum_X \frac{\partial \mathcal{P}}{\partial X} \cdot \frac{d^2 X}{d\mu_u d\mu_d} + \sum_{X,Y=\sigma_u,\sigma_d,\Phi,\bar{\Phi}} \frac{\partial^2 \mathcal{P}}{\partial X \partial Y} \cdot \frac{dX}{d\mu_u} \cdot \frac{dY}{d\mu_d}$$

Returning to model

$$\chi_{ud} = \frac{\partial^2 \mathcal{P}}{\partial\mu_u \partial\mu_d} + \sum_{X=\sigma_u,\sigma_d,\Phi,\bar{\Phi}} \frac{\partial^2 \mathcal{P}}{\partial X \partial\mu_u} \cdot \frac{dX}{d\mu_d} + \sum_X \frac{\partial^2 \mathcal{P}}{\partial X \partial\mu_d} \cdot \frac{dX}{d\mu_u}$$
$$+ \sum_X \frac{\partial \mathcal{P}}{\partial X} \cdot \frac{d^2 X}{d\mu_u d\mu_d} + \sum_{X,Y=\sigma_u,\sigma_d,\Phi,\bar{\Phi}} \frac{\partial^2 \mathcal{P}}{\partial X \partial Y} \cdot \frac{dX}{d\mu_u} \cdot \frac{dY}{d\mu_d}$$

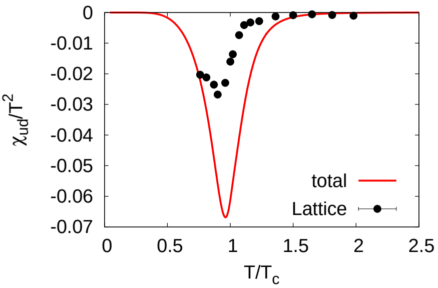
χ_{ud} in PNJL



Returning to model

$$\chi_{ud} = \frac{\partial^2 \mathcal{P}}{\partial\mu_u\partial\mu_d} + \sum_{X=\sigma_u,\sigma_d,\Phi,\bar{\Phi}} \frac{\partial^2 \mathcal{P}}{\partial X\partial\mu_u}\cdot\frac{dX}{d\mu_d} + \sum_X \frac{\partial^2 \mathcal{P}}{\partial X\partial\mu_d}\cdot\frac{dX}{d\mu_u}$$
$$+ \sum_X \frac{\partial \mathcal{P}}{\partial X}\cdot\frac{d^2X}{d\mu_u d\mu_d} + \sum_{X,Y=\sigma_u,\sigma_d,\Phi,\bar{\Phi}} \frac{\partial^2 \mathcal{P}}{\partial X\partial Y}\cdot\frac{dX}{d\mu_u}\cdot\frac{dY}{d\mu_d}$$

χ_{ud} in PNJL



- No flavor mixing term in pressure $\Rightarrow \frac{\partial^2 P}{\partial\mu_u\partial\mu_d} = 0$. Only implicit depedences contribute to χ_{ud}.

A possible conjecture

An important observation in χ_{ud}:

- model : solely through implicit dependences.
- QCD : solely through quark line disconnected diagrams.

implicit dependences $\Leftrightarrow$ disconnected diagrams

If the conjecture is correct then at $\mu_u = \mu_d = 0$:

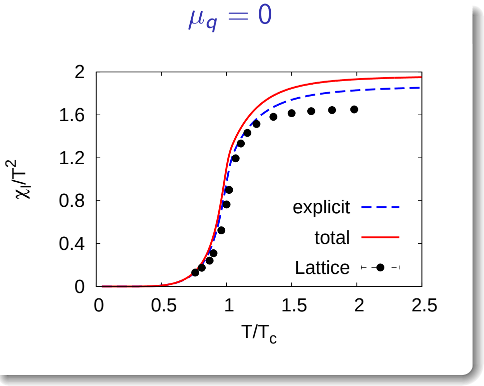
- Implicit contributions will add up in case of QNS.
  $\chi_q = \chi_{uu} + \chi_{dd} + 2\chi_{ud}$ will have explicit as well as implicit contribution.
- Implicit contributions will cancel in case of INS
  $\chi_I = \chi_{uu} + \chi_{dd} - 2\chi_{ud}$ will have only explicit contribution.

$$\chi_{uu} = \frac{\partial^2 P}{\partial \mu_u^2} + 2\sum_X \frac{\partial^2 P}{\partial \mu_u \partial X} \cdot \frac{dX}{d\mu_u} + \sum_{X,Y} \frac{\partial^2 P}{\partial X \partial Y} \cdot \frac{dX}{d\mu_u} \cdot \frac{dY}{d\mu_u} + \sum_X \frac{\partial P}{\partial X} \cdot \frac{d^2 X}{d\mu_u^2}$$

$$\chi_{dd} = \frac{\partial^2 P}{\partial \mu_d^2} + 2\sum_X \frac{\partial^2 P}{\partial \mu_d \partial X} \cdot \frac{dX}{d\mu_d} + \sum_{X,Y} \frac{\partial^2 P}{\partial X \partial Y} \cdot \frac{dX}{d\mu_d} \cdot \frac{dY}{d\mu_d} + \sum_X \frac{\partial P}{\partial X} \cdot \frac{d^2 X}{d\mu_d^2}$$

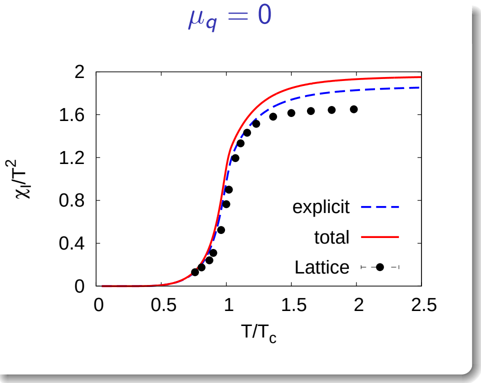
$$\chi_{ud} = \frac{\partial^2 \mathcal{P}}{\partial \mu_u \partial \mu_d} + \sum_X \frac{\partial^2 \mathcal{P}}{\partial \mu_u \partial X} \cdot \frac{dX}{d\mu_d} + \sum_X \frac{\partial^2 \mathcal{P}}{\partial \mu_d \partial X} \cdot \frac{dX}{d\mu_u} + \sum_{X,Y} \frac{\partial^2 \mathcal{P}}{\partial X \partial Y} \cdot \frac{dX}{d\mu_u} \cdot \frac{dY}{d\mu_d} + \sum_X \frac{\partial \mathcal{P}}{\partial X} \cdot \frac{d^2 X}{d\mu_u d\mu_d}$$

INS in PNJL model



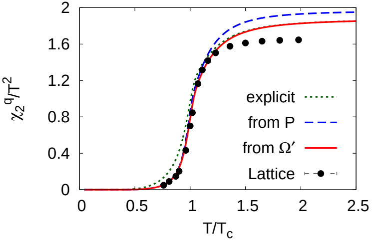
- INS has also non-trivial implicit contributions which are always positive, unlike QNS.
- $\frac{\partial \mathcal{P}}{\partial X} \neq 0$ for $X = \Phi, \bar{\Phi}$.

INS in PNJL model

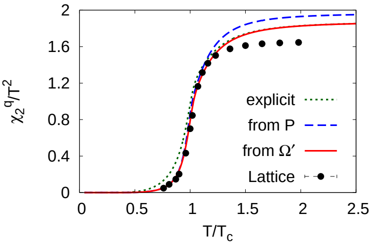


- INS has also non-trivial implicit contributions which are always positive, unlike QNS.
- $\frac{\partial \mathcal{P}}{\partial X} \neq 0$ for $X = \Phi, \bar{\Phi}$.

- Our claimed conjecture seems to be perfectly fine if we start all of our calculations from logarithm of partition function or full thermodynamic potential $\Omega' = \Omega - \kappa T^4 \ln[J(\Phi, \bar{\Phi})]$.
- Then mean field condition will give $\frac{\partial \Omega'}{\partial X} = 0$. The base line is set to associate implicit dependences and quark line disconnected diagrams.



χ_2^q/T^2
2
1.6
1.2
0.8
0.4
0
0
0.5
1
1.5
2
2.5
T/T_c
explicit
from P
from Ω′
Lattice



χ_2^q/T^2
2
1.6
1.2
0.8
0.4
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1
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T/T_c
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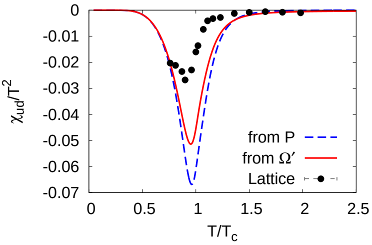
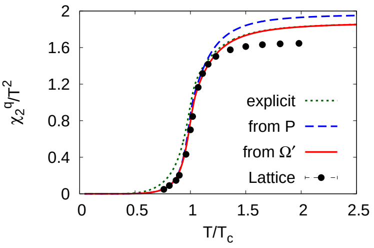
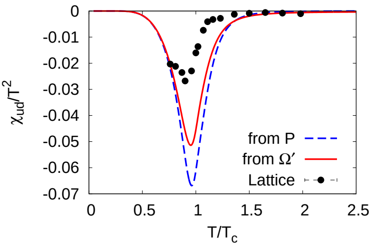
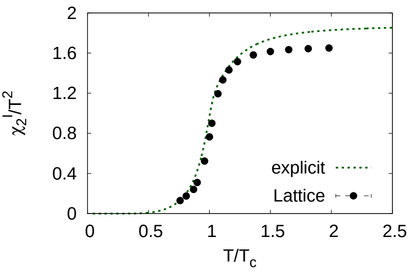
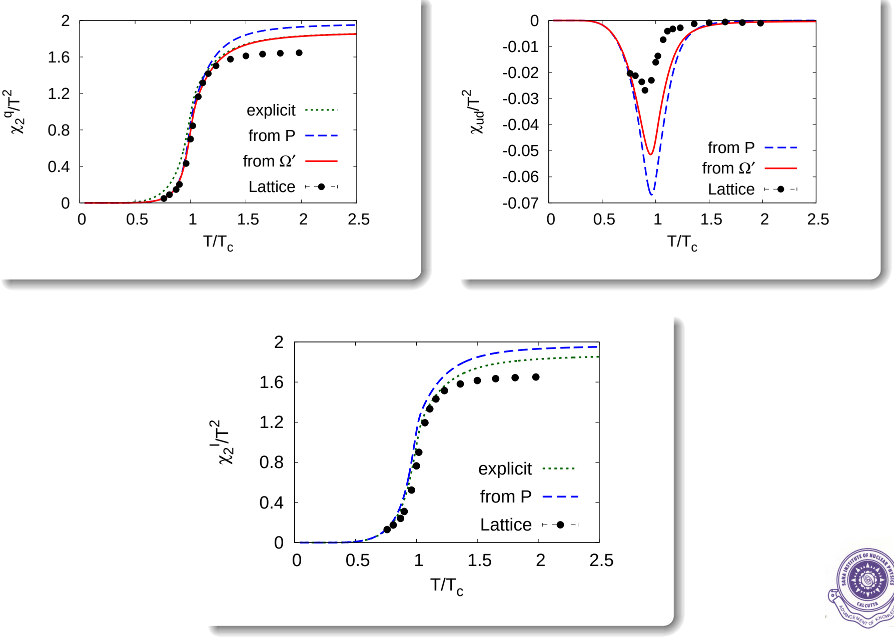


χ_{ud}/T^2
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
0
0.5
1
1.5
2
2.5
T/T_c
from P
from Ω′
Lattice

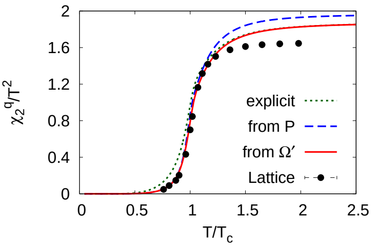

χ_2^q/T^2
T/T$_c$
explicit
from P
from Ω′
Lattice


χ_{ud}/T^2
T/T$_c$
from P
from Ω′
Lattice


χ_2^I/T^2
T/T$_c$
explicit
Lattice



χ_2^q/T^2
χ_{ud}/T^2
χ_2^I/T^2
T/T$_c$
explicit
from P
from Ω′
Lattice



explicit
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χ_2^q/T^2
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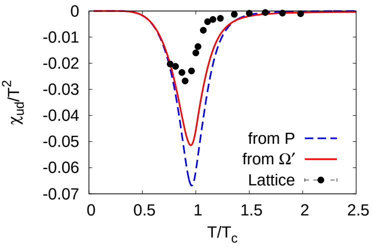

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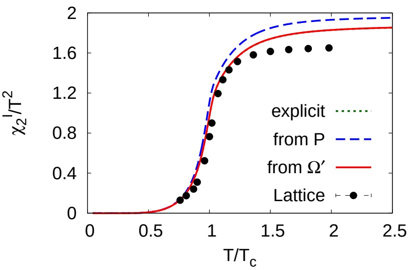
T/T$_c$



explicit
from P
from Ω′
Lattice
χ_2^I/T^2
T/T$_c$

More on association

4^{th} order : quark line connected diagrams does not have the cusp like structure, rather behaves like order parameter [Gavai & Gupta, PRD 72 (2005)].

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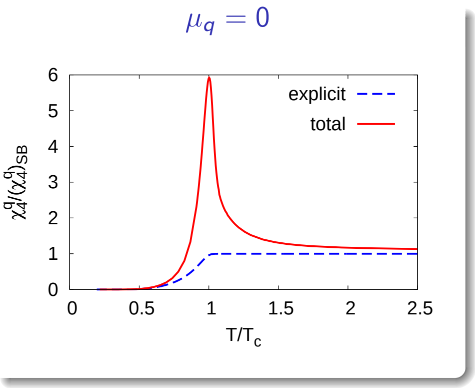

explicit contribution never goes beyond SB limit $\Rightarrow$ cusp like structure is coming solely due to implicit contributions.

Summary

- QNS is calculated using FDT both In NJL and PNJL model.
- Effect of implicit μ_q dependence of mean fields plays a major role.
- Static correlator is modified with this implicit derivatives.
- As a side result an analytical tool is proposed for calculating the derivatives of mean field.
- A baseline is set for the association of implicit dependences to quark line disconnected diagrams.

Thank you!

Back-Up

Introduction of VdM term

$$Z = \int \prod_x \mathcal{D}\mathbf{L}(x)\ e^{-\mathcal{S}}$$

where, $\mathcal{S}$ is the action of the system and $\mathcal{D}\mathbf{L}$ being the SU(3) Haar measure. Now, after transforming the integration variable from the Wilson line $\mathbf{L}$ to Polyakov loop Φ and its conjugate $\bar{\Phi}$ one obtains:

$$\begin{aligned} Z &= \int \prod_x \mathcal{D}\Phi(x)\mathcal{D}\bar{\Phi}(x)\ J[\Phi(x), \bar{\Phi}(x)]\ e^{-\mathcal{S}} \\ &= \int \prod_x \mathcal{D}\Phi(x)\mathcal{D}\bar{\Phi}(x)\ e^{-\mathcal{S} + \sum_x \ln J[\Phi(x), \bar{\Phi}(x)]} \end{aligned}$$

$$\begin{aligned} Z &\approx e^{-\mathcal{S} + N \ln J}\big|_{\Phi=\Phi_{min}, \bar{\Phi}=\bar{\Phi}_{min}} \\ &= e^{-\beta V \mathcal{U} + N \ln J} = e^{-\beta V(\mathcal{U} - \frac{NT}{V} \ln J)} \equiv e^{-\beta V \mathcal{U}'} \end{aligned}$$

Derivation of pressure

$$
\begin{aligned}
P &= T\frac{d}{dV}\ln Z \\
&= T\frac{d}{dV}(-\beta V\Omega') \\
&= -\Omega' - V\frac{d\Omega'}{dV} \\
&= -\Omega' - V(\frac{\partial\Omega'}{\partial V} + \sum_{\alpha}\frac{\partial\Omega'}{\partial X_\alpha}\cdot\frac{dX_\alpha}{dV}) \\
&= -\Omega' - V\frac{\partial\Omega'}{\partial V} \\
&= -\Omega' - V\frac{\partial}{\partial V}(\Omega - \frac{NT}{V}\ln J) \\
&= -(\Omega - \frac{NT}{V}\ln J) - V(\frac{\partial\Omega}{\partial V} + \frac{NT}{V^2}\ln J)
\end{aligned}
$$

Ω being an intensive quantity, we are left with $P = -\Omega$.

number density in nonperturbative QCD

$$\begin{aligned} n_f &= \frac{T}{V}\frac{\partial}{\partial \mu_f}\ln \mathcal{Z} = \frac{T}{V}\frac{1}{\mathcal{Z}}\frac{\partial \mathcal{Z}}{\partial \mu_f} \\ &= \frac{T}{V}\frac{1}{\mathcal{Z}}\int \mathcal{D}\mathcal{A}\ M_f' \prod_{f'\neq f} \det M_{f'}\ e^{-\mathcal{S}_{\mathcal{G}}(\mathcal{A})} \\ &= \frac{T}{V}\frac{1}{\mathcal{Z}}\int \mathcal{D}\mathcal{A}\ \mathrm{Tr}(M_f' M_f^{-1}) \prod_{f'} \det M_{f'}\ e^{-\mathcal{S}_{\mathcal{G}}(\mathcal{A})} \\ &= \frac{T}{V}\langle \mathrm{Tr}(M_f' M_f^{-1})\rangle = \frac{T}{V}\langle \mathcal{O}_1 \rangle. \end{aligned}$$

where, we have used,

$$M'(x) = \frac{\partial}{\partial x}\det M(x) = \mathrm{Tr}(M' M^{-1})\det M(x).$$

Every M_f' corresponds to an insertion of γ^0 in continuum and each M_f^{-1} is a quark propagator of flavor f.

diagonal susceptibility in nonperturbative QCD

$$
\begin{aligned}
\chi_{ff} &= \frac{T}{V}\frac{\partial^2}{\partial\mu_f^2}\ln\mathcal{Z} \\
&= \frac{T}{V}\frac{1}{\mathcal{Z}}\int\mathcal{DA}\ \mathrm{Tr}(M_f''M_f^{-1} - M_f'M_f^{-1}M_f'M_f^{-1})\prod_{f'}\det M_{f'}\ e^{-\mathcal{S}_{\mathcal{G}}(\mathcal{A})} \\
&+ \frac{T}{V}\frac{1}{\mathcal{Z}}\int\mathcal{DA}\ \mathrm{Tr}(M_f'M_f^{-1})\cdot\mathrm{Tr}(M_f'M_f^{-1})\prod_{f'}\det M_{f'}\ e^{-\mathcal{S}_{\mathcal{G}}(\mathcal{A})} \\
&- \frac{T}{V}\Big(\frac{1}{\mathcal{Z}}\int\mathcal{DA}\ \mathrm{Tr}(M_f'M_f^{-1})\prod_{f'}\det M_{f'}\ e^{-\mathcal{S}_{\mathcal{G}}(\mathcal{A})}\Big)^2 \\
&= \frac{T}{V}\langle\mathrm{Tr}(M_f''M_f^{-1} - M_f'M_f^{-1}M_f'M_f^{-1})\rangle \\
&+ \frac{T}{V}\langle\mathrm{Tr}(M_f'M_f^{-1})\cdot\mathrm{Tr}(M_f'M_f^{-1})\rangle - \frac{T}{V}\langle\mathrm{Tr}(M_f'M_f^{-1})\rangle^2 \\
&= \frac{T}{V}\langle\mathcal{O}_2\rangle + \frac{T}{V}\langle\mathcal{O}_1\cdot\mathcal{O}_1\rangle - \frac{T}{V}\langle\mathcal{O}_1\rangle^2
\end{aligned}
$$

off-diagonal susceptibility in NPQCD

$$
\begin{aligned}
\chi_{fg} &= \frac{T}{V}\frac{\partial^2}{\partial\mu_f\partial\mu_g}\ln\mathcal{Z} \\
&= \frac{T}{V}\frac{1}{\mathcal{Z}}\int\mathcal{D}\mathcal{A}\ \mathrm{Tr}(M_f' M_f^{-1})\cdot\mathrm{Tr}(M_g' M_g^{-1})\prod_{f'}\det M_{f'}\ e^{-\mathcal{S}_\mathcal{G}(\mathcal{A})} \\
&- \frac{T}{V}\left(\frac{1}{\mathcal{Z}}\int\mathcal{D}\mathcal{A}\ \mathrm{Tr}(M_f' M_f^{-1})\prod_{f'}\det M_{f'}\ e^{-\mathcal{S}_\mathcal{G}(\mathcal{A})}\right) \\
&\times \left(\frac{1}{\mathcal{Z}}\int\mathcal{D}\mathcal{A}\ \mathrm{Tr}(M_g' M_g^{-1})\prod_{f'}\det M_{f'}\ e^{-\mathcal{S}_\mathcal{G}(\mathcal{A})}\right) \\
&= \frac{T}{V}\langle\mathcal{O}_1^f\cdot\mathcal{O}_1^g\rangle - \frac{T}{V}\langle\mathcal{O}_1^f\rangle\cdot\langle\mathcal{O}_1^g\rangle.
\end{aligned}
$$