Vector meson spectral function in a hot and dense medium within an effective QCD approach.

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29.01.15

Based on arXiv:1411.6407 [To appear in JHEP]

QCD 2015, TIFR

Outline:

- Introduction.
- Correlation function and its spectral representation.
- Effective QCD models: NJL & PNJL.
- Vector correlation and ring approximation.
- Results.
- Conclusions.

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QCD phase diagram:

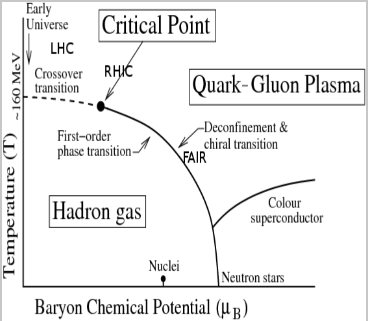

Figure : Phase Diagram of QCD

- Quantum Chromodynamics (QCD) exhibits a very rich phase structure at extreme conditions, *i.e.*, high temperature and/or high density.

Tools of studying QCD:

- Experiments like RHIC @ BNL and LHC @ CERN are exploring this noble state.
- New experiments are in the pipeline, as for example *FAIR* @ GSI, expected to run from 2018 onwards.
- Different methods: PQCD, LQCD & Effective QCD Models.
- Our work is based on Effective QCD Models, namely NJL and its Polyakov Loop extended version, PNJL.

What we have studied:

- The vector meson current-current correlation function and its spectral representation.
- It has been investigated with and without the isoscalar-vector interaction. The influence of the isoscalar-vector interaction is obtained using the ring summation (RPA).
- From the vector meson spectral function we obtained the dilepton production rate for a hot and dense medium.
- We also studied the conserved density fluctuation associated with temporal correlation function.

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Generalities:

- Mesonic correlation functions are constructed from meson currents and they can be of different types.
- Many properties of deconfined, strongly interacting matter are reflected in the structure of the correlation functions and its spectral representation.
- Here we will talk about dilepton rate and quark number susceptibility (QNS).
- Remember, the correlation functions and its spectral representation can be studied through many of the aforementioned methods to study QCD.

Correlation function:

- In general the correlation function in coordinate space:

$$\mathcal{G}_{AB}(t,\vec{x}) \equiv \mathcal{T}(\hat{A}(t,\vec{x})\hat{B}(0,\vec{0})) = \int \frac{d\omega}{2\pi} \int \frac{d^3\vec{q}}{(2\pi)^3} \; e^{i\omega t - i\vec{q}\cdot\vec{x}} \; \mathcal{G}_{AB}(\omega,\vec{q})$$

- Momentum space correlation function by FT:

$$\mathcal{G}_{AB}(\omega,\vec{q}) = \int dt \int d^3\vec{x} \; e^{-i\omega t + i\vec{q}\cdot\vec{x}} \; \mathcal{G}_{AB}(t,\vec{x})$$

Euclidean current-current correlation function:

- Thermal Current-Current Correlator in Eucledean time τ:

$$\begin{aligned}\mathcal{G}_M^E(\tau,\vec{x}) &= \langle \mathcal{T}(J_M(\tau,\vec{x})J_M^\dagger(0,\vec{0}))\rangle_\beta \\ &= T\sum_{n=-\infty}^{\infty}\int \frac{d^3q}{(2\pi)^3}\, e^{-i(\omega_n\tau+\vec{q}\cdot\vec{x})}\, \mathcal{G}_M^E(\omega_n,\vec{q})\end{aligned}$$

- Meson currents: $J_M = \bar{\psi}(\tau,\vec{x})\Gamma_M\psi(\tau,\vec{x}), \quad \Gamma_M{=}\mathbb{I},\gamma_5,\gamma_\mu,\gamma_\mu\gamma_5.$
- $\langle \quad \rangle_\beta$: Thermal average.
- Eucledean time: τ is restricted to the interval $[0,\ \beta = 1/T]$
- Matsubara modes $\omega_n = 2\pi nT$, $n = 0, 1, 2\cdots$

Spectral function:

- The momentum space correlator can be expressed as:

$$\mathcal{G}_M^E(\omega_n, \vec{q}) \;=\; -\int_{-\infty}^{\infty} d\omega \;\; \frac{\sigma_M(\omega, \vec{q})}{i\omega_n - \omega}$$

- The corresponding spectral function can be obtained through the analytic continution of $\mathcal{G}_M^E(\omega_n = \omega + i\epsilon)$:

$$\sigma_H(\omega, \vec{q}) \;=\; \frac{1}{\pi}\text{Im}\; \mathcal{G}_H^E(\omega + i\epsilon, \vec{q})$$

- $H = (00, ii, V)$ denotes (temporal, spatial, vector).

- The spectral function is related to various physical quantities in hot and dense medium.

Dilepton rate from the spectral function:

- The differential dilepton production rate in terms of spectral function:

$$\frac{dR}{d^4x d^4Q} = \frac{5\alpha^2}{54\pi^2} \frac{1}{M^2} \frac{1}{e^{\omega/T} - 1} \sigma_V(\omega, \vec{q})$$

with, $\alpha = \frac{e^2}{4\pi}$; $Q \equiv (q_0 = \omega, \vec{q})$ and $q = |\vec{q}|$, invariant mass $M = \sqrt{\omega^2 - q^2}$.

Quark number susceptibility:

- QNS(χ_q) $\Rightarrow$ Response of quark number with μ.
- We note that χ_q is the density-density correlation i.e., $0-0$ component of the vector-vector current correlation:

$$
\begin{aligned}
\chi_q(T) &= \left.\frac{\partial\rho}{\partial\mu}\right|_{\mu=0} = \int d^4x \, \left\langle J_0(0,\vec{x}) J_0(0,\vec{0}) \right\rangle \\
&= \lim_{\vec{q}\to 0} \beta \int \frac{d\omega}{2\pi} \frac{-2}{1-e^{-\omega/T}} \mathrm{Im}\mathcal{G}_{00}(\omega,\vec{q}) \\
&= -\lim_{\vec{q}\to 0} \mathrm{Re}\, \mathcal{G}_{00}(0,\vec{q})
\end{aligned}
$$

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NJL model with vector type interaction:

- The Lagrangian we work with is the two flavour NJL model with isoscalar-vector type interaction:

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi + \frac{G_S}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \frac{G_V}{2}(\bar{\psi}\gamma_\mu\psi)^2$$

- $\bar{\psi} = (\bar{\psi_u}, \bar{\psi_d})$; $m = diag(m_u, m_d)$ with $m_u = m_d$ and $\vec{\tau} \rightarrow$ Pauli matrices.

- G_S and G_V denote coupling constants of the scalar type four-quark and vector type four-quark interactions respectively.

Corresponding thermodynamic potential:

- The thermodynamic potential for the NJL model:

$$\begin{aligned}\Omega_{\mathrm{NJL}} &= \frac{G_S}{2}\sigma^2 - \frac{G_V}{2}n^2 - 2N_f N_c \int_\Lambda \frac{d^3p}{(2\pi)^3} E_p \\ &- 2N_f N_c T \int \frac{d^3p}{(2\pi)^3} \left[\ln(1+e^{-(E_p-\tilde{\mu})/T}) + \ln(1+e^{-(E_p+\tilde{\mu})/T})\right]\end{aligned}$$

- $E_p = \sqrt{\vec{p}^2 + M_f^2}$ is the energy of a quark with flavor f.
- $M_f = m - G_S\sigma$ and $\tilde{\mu} = \mu - G_V n$.

PNJL model with vector type interaction:

- In PNJL model we have a couple of more mean fields in the form of the expectation value of the Polyakov Loop fields Φ and $\bar{\Phi}$.

$$\mathcal{L}_{\mathrm{PNJL}} = \bar{\psi}(i\gamma_\mu D^\mu - m)\psi + \frac{G_S}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \frac{G_V}{2}(\bar{\psi}\gamma_\mu\psi)^2 - \mathcal{U}[\Phi, \bar{\Phi}, T]$$

- $D^\mu = \partial^\mu - ig\mathcal{A}_a^\mu\lambda_a/2$, $\mathcal{A}_a^\mu$ being the $SU(3)$ background fields and λ_a's are the Gell-Mann matrices.

Corresponding thermodynamic potential:

- The thermodynamic potential for the PNJL model:

$$\begin{aligned}\Omega_{\rm PNJL} &= \mathcal{U}(\Phi,\bar{\Phi},T) + \frac{G_S}{2}\sigma^2 - \frac{G_V}{2}n^2 \\ &- 2N_f T\int\frac{d^3p}{(2\pi)^3}\ln\left[1+3\left(\Phi+\bar{\Phi}e^{-(E_p-\tilde{\mu})/T}\right)e^{-(E_p-\tilde{\mu})/T}+e^{-3(E_p-\tilde{\mu})/T}\right] \\ &- 2N_f T\int\frac{d^3p}{(2\pi)^3}\ln\left[1+3\left(\bar{\Phi}+\Phi e^{-(E_p+\tilde{\mu})/T}\right)e^{-(E_p+\tilde{\mu})/T}+e^{-3(E_p+\tilde{\mu})/T}\right] \\ &- \kappa T^4\ln[J(\Phi,\bar{\Phi})] - 2N_f N_c\int_\Lambda\frac{d^3p}{(2\pi)^3}E_p\end{aligned}$$

- Here we note that M_f and $\tilde{\mu}$ will now also depend on Φ and $\bar{\Phi}$ through σ and n.

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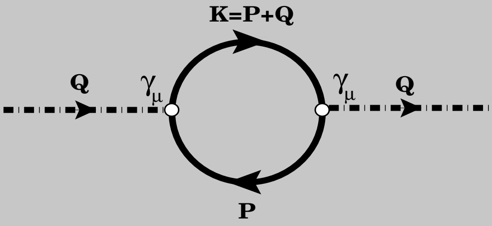
Vector correlation function in one loop:



- The current-current correlator in vector channel at one loop level:

$$\Pi_{\mu\nu}(Q) = \int \frac{d^4P}{(2\pi)^4} \mathrm{Tr}_{D,c}\left[\gamma_\mu S(P+Q)\gamma_\nu S(P)\right]$$

- $\mathrm{Tr}_{D,c}$ is trace over Dirac and colour indices respectively.

Vector correlation function in one loop (NJL & PNJL):

- The NJL effective quark propagator is given as:

$$S_{\mathrm{NJL}}(L) = [\gamma_\mu l^\mu - M_f + \gamma_0 \tilde{\mu}]^{-1}$$

- The PNJL one:

$$S_{\mathrm{PNJL}}(L) = [\gamma_\mu l^\mu - M_f + \gamma_0 \tilde{\mu} - i\gamma_0 \mathcal{A}_4]^{-1}$$

- The four momentum $L \equiv (l_0, \vec{l})$.

- The distribution function in NJL can be generalised to PNJL through the relation:

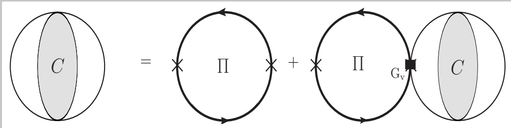
$$f(E_p \pm \tilde{\mu}) = \frac{\Phi e^{-\beta(E_p \pm \tilde{\mu})} + 2\bar{\Phi} e^{-2\beta(E_p \pm \tilde{\mu})} + e^{-3\beta(E_p \pm \tilde{\mu})}}{1 + 3\Phi e^{-\beta(E_p \pm \tilde{\mu})} + 3\bar{\Phi} e^{-2\beta(E_p \pm \tilde{\mu})} + e^{-3\beta(E_p \pm \tilde{\mu})}}$$

[Hansen *et al.*, PRD 75 (2007)].

Ring approximation(RPA):

- We have studied the properties of the vector meson current-current correlation function with and without the isoscalar-vector interaction.

- The influence of isoscalar-vector interaction on the vector meson correlator is obtained using the ring approximation(RPA).[Davidson *et al.*, PLB 359 (1995), zero temp.]



- The coupling constant G_V, which is considered to be a free parameter (as can't be fitted) in our calculation, comes into the picture.

DSE for resummed vector correlator at finite T & μ:

- The DSE for $C_{\mu\nu}$ within ring summation:

$$C_{\mu\nu} = \Pi_{\mu\nu} + G_V \Pi_{\mu\sigma} C^{\sigma}_{\nu},$$

where $\Pi_{\mu\nu}$ is one loop vector correlator.

- The general structure of the resummed vector correlator in medium reads (The master Eq.)[Islam *et al.*, arXiv:1411.6407]:

$$C_{\mu\nu} = \frac{\Pi_T}{1 - G_V \Pi_T} P^T_{\mu\nu} + \frac{\Pi_L}{1 - G_V \Pi_L} P^L_{\mu\nu},$$

$P^{L(T)}_{\mu\nu}$ are longitudinal (transverse) projecton operators.

Imaginary parts of the resummed vector correlator:

- The temporal component:

$$\mathrm{ImC}_{00} = \frac{\mathrm{Im}\Pi_{00}}{\left[1 - G_V\left(1 - \frac{\omega^2}{q^2}\right)\mathrm{Re}\Pi_{00}\right]^2 + \left[G_V(1 - \frac{\omega^2}{q^2})\mathrm{Im}\Pi_{00}\right]^2}$$

- The spatial component:

$$\mathsf{Im}C_{ii} = \mathsf{Im}C'_T + \mathsf{Im}C'_L,$$

$$\mathrm{Im}C'_T = \frac{\mathrm{Im}\Pi_{ii} - \frac{\omega^2}{q^2}\mathrm{Im}\Pi_{00}}{\left[1 + \frac{G_V}{2}\mathrm{Re}\Pi_{ii} - \frac{G_V}{2}\frac{\omega^2}{q^2}\mathrm{Re}\Pi_{00}\right]^2 + \frac{G_V^2}{4}\left[\mathrm{Im}\Pi_{ii} - \frac{\omega^2}{q^2}\mathrm{Im}\Pi_{00}\right]^2}$$

$$\mathrm{Im}C'_L = \frac{\frac{\omega^2}{q^2}\mathrm{Im}\Pi_{00}}{\left[1 - G_V\left(1 - \frac{\omega^2}{q^2}\right)\mathrm{Re}\Pi_{00}\right]^2 + \left[G_V(1 - \frac{\omega^2}{q^2})\mathrm{Im}\Pi_{00}\right]^2} = \frac{\omega^2}{q^2}\mathrm{Im}C_{00}$$

[Islam *et al.*, arXiv:1411.6407]

Resummed vector correlation and spectral function:

- The resummed vector correlator:

$$C_{\mu\nu} = \frac{\Pi_T}{1 - G_V \Pi_T} A^T_{\mu\nu} + \frac{\Pi_L}{1 - G_V \Pi_L} A^L_{\mu\nu}$$

- Corresponding resummed spectral function:

$$\sigma^R_V = \frac{1}{\pi}\Big[\mathrm{Im}C_{00} - \mathrm{Im}C_{ii}\Big].$$

- The imaginary parts (temporal & spatial) of one loop vector correlator are associated with an energy conserving delta function that imposes a finite limit of the quark loop momentum: $p_{\pm} = \frac{\omega}{2}\sqrt{1 - \frac{4M_f^2}{M^2}} \pm \frac{q}{2}$.
- For a given G_V and T, the resummed spectral function picks up continuous contribution above the threshold, $M^2 > 4M_f^2$.

Conserved density fluctuation in ring approximation:

- The real part of the resummed temporal correlation function:

$$\mathrm{Re}C_{00}(\omega,\vec{q}) = \frac{\mathrm{Re}\Pi_{00}(\omega,\vec{q}) + G_V\left(\frac{\omega^2}{q^2}-1\right)\left[(\mathrm{Re}\Pi_{00}(\omega,\vec{q}))^2 + (\mathrm{Im}\Pi_{00}(\omega,\vec{q}))^2\right]}{1+2G_V\left(\frac{\omega^2}{q^2}-1\right)\mathrm{Re}\Pi_{00}(\omega,\vec{q}) + \left(G_V(\frac{\omega^2}{q^2}-1)\right)^2\left[(\mathrm{Re}\Pi_{00}(\omega,\vec{q}))^2 + (\mathrm{Im}\Pi_{00}(\omega,\vec{q}))^2\right]}$$

- The resummed QNS in the ring approximation becomes:

$$\chi_q^R(T,\tilde{\mu}) = -\lim_{\vec{q}\to 0}\mathrm{Re}C_{00}(0,\vec{q}) = \frac{\chi_q(T,\tilde{\mu})}{1+G_V\chi_q(T,\tilde{\mu})}$$

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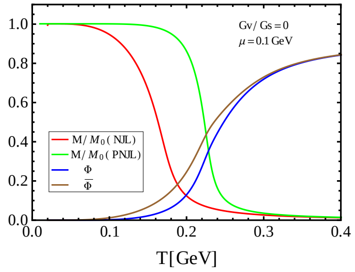
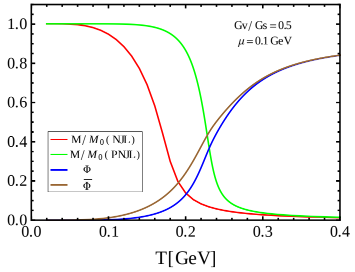
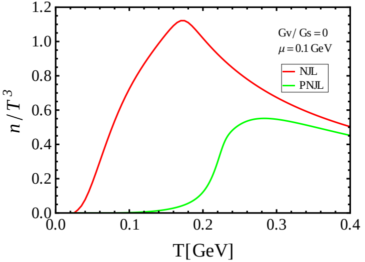
Variation of scaled quark mass & PL fields:

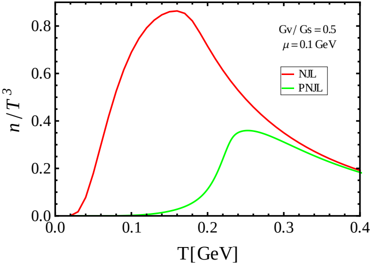




- Scaled quark mass decreases, approaches chiral limit at high T. PL fields increases, approaches unity.
- As $\mu \neq 0$, Φ & $\bar{\Phi}$ are different for a given T.
- At large T, PNJL becomes equivalent to NJL; also evident from the thermal distribution function.

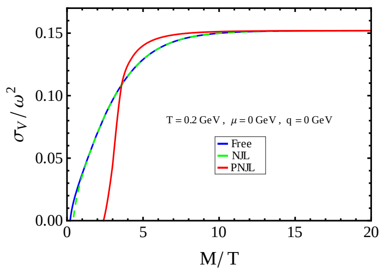
Variation of number density:

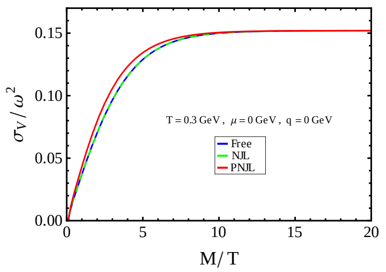




- For a given T, n is suppressed for PNJL as compared to NJL; due to the presence of PL fields.
- Presence of G_V decreases the n for both NJL & PNJL both; evident from the expression of modified quark chemical potential $(\tilde{\mu} = \mu - G_V n)$.

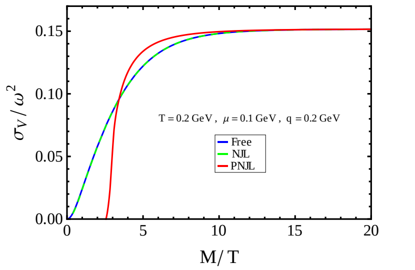
Scaled spectral function with $G_V = 0$:

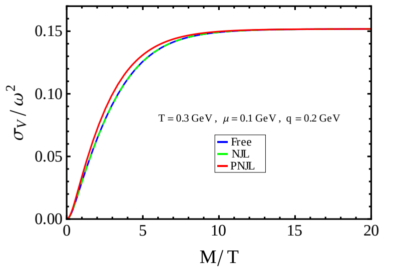




- At $T = 200$ MeV the spectral function in PNJL model has larger threshold than NJL model because of the larger quark mass.
- At $\vec{q} = 0$ and $\mu_q = 0$ the spectral function is proportional to $[1 - 2f(E_p)]$, $f(E_p)$ is the fermion distribution function.
- Presence of Polyakov Loop $\rightarrow$ suppression in $f(E_p)$, hence enhancement in spectral function.

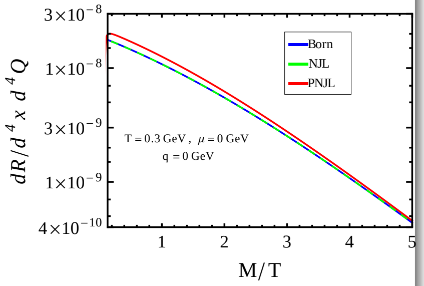
Scaled spectral function with $G_V = 0$:

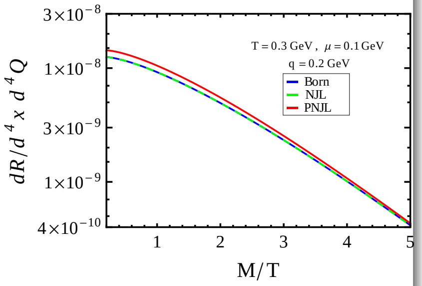




- Plots of the scaled spectral function for nonzero q and μ.

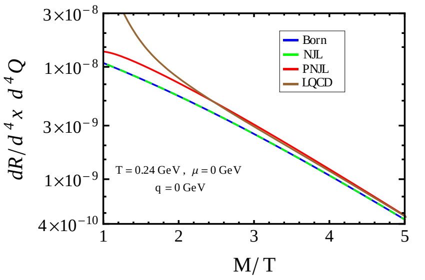
Dilepton rate for $T = 300$ MeV ($G_V = 0$):





- Dilepton rate in PNJL is enhanced as compared to Born or NJL case.
- This suggests that nonperturbative dilepton production rate is higher in a semi-QGP than the Born rate in a weakly coupled QGP.

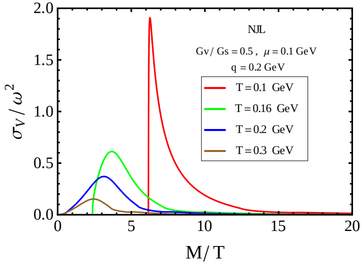
Dilepton rate for $T = 240$ MeV ($G_V = 0$):



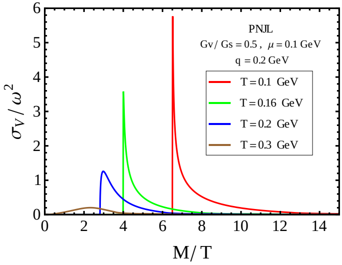
- The result is compared with the available quenched QCD results[Ding *et al.*, PRD 83 (2011)].

Scaled spectral function for different T with $G_V \neq 0$:

NJL



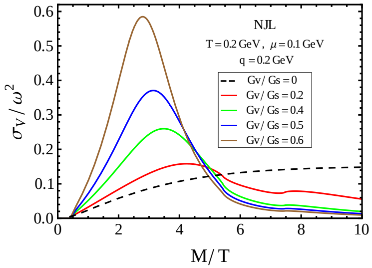
PNJL



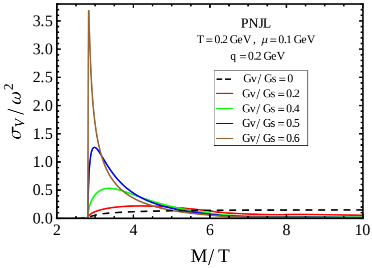
- The spectral function decreases with the increase of temperature.
- The spectral function in NJL model acquires a width earlier compared to PNJL.
- Peaks get smeared as we increase the temperature.

Scaled spectral function with diffrent $G_V(T = 200\ MeV)$:

NJL



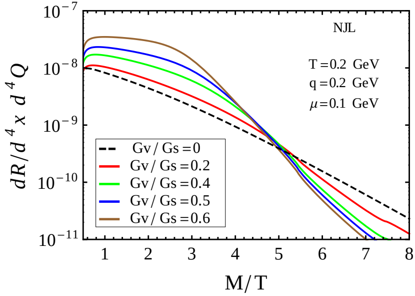
PNJL



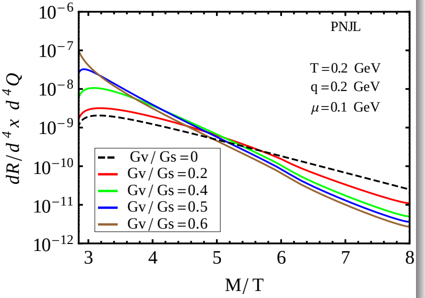
- Spectral strength increases with the increase of G_V.
- For a given G_V, spectral strength is always larger in PNJL model, due to the presence of the Polyakov Loop fields.

Dilepton rate with diffrent $G_V (T = 200 \text{ MeV})$:

NJL



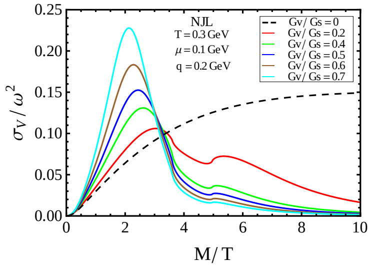
PNJL



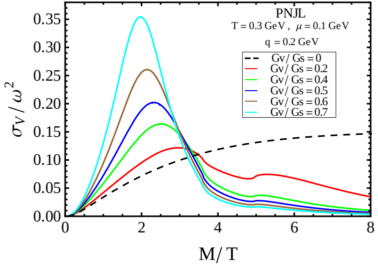
- The differential dilepton production rate increases with the increase of G_V.
- Dilepton production rate is always larger in PNJL model than NJL for a given G_V.

Scaled spectral function with diffrent $G_V(T = 300\ MeV)$:

NJL



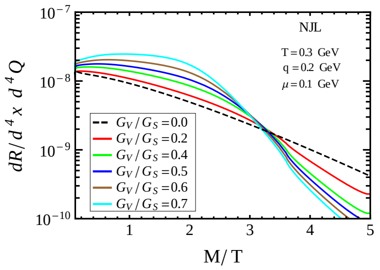
PNJL



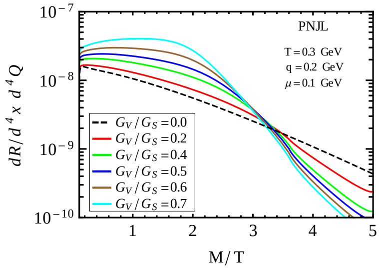
- Spectral strength decreases with the increase of temperature.

Dilepton rate with diffrent $G_V(T = 300\ MeV)$:

NJL



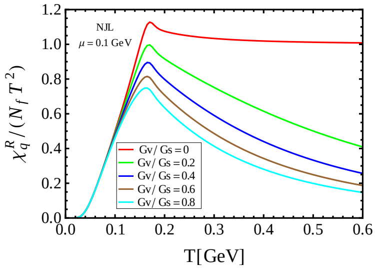
PNJL



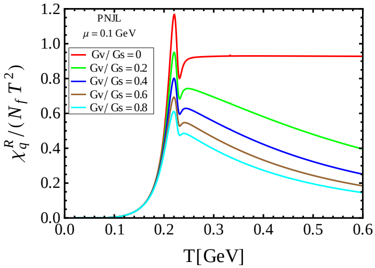
- Corresponding differential dilepton production rate.

Quark number susceptibility (QNS):

NJL



PNJL



- The resummed susceptibility gets suppressed as G_V is increased.
- Compressibility of the system decreases with the increase of $G_V \rightarrow$ Susceptibility gets suppressed.

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Conclusions:

- Vector meson current-current correlation and spectral function are investigated in the framework of the mean field models namely, NJL & PNJL with and without isoscalar-vector interaction extension.
- The influence of the isoscalar-vector interaction is obtained using ring approximation (RPA).
- Then dilepton rate from a hot and dense matter and QNS have been studied using the resummed vector correlator.
- It has been observed that the dilepton rate gets enhanced in low invariant mass region when the vector coulping is nonzero.
- QNS gets suppressed as the vector coupling is increased.

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Thank You

Backup Slide No:1

Temporal part:

$$\mathrm{Im}\Pi_{00}(\omega,\vec{q}) \;=\; \frac{N_f N_c}{4\pi}\int_{p_-}^{p_+} p\, dp\, \frac{4\omega E_p - 4E_p^2 - M^2}{2E_p q}\left[f(E_p-\tilde{\mu}) + f(E_p+\tilde{\mu}) - 1\right]$$

Spatial part:

$$\mathrm{Im}\Pi_{ii}(\omega,\vec{q}) \;=\; \frac{N_f N_c}{4\pi}\int_{p_-}^{p_+} p\, dp\, \frac{4\omega E_p - 4p^2 + M^2}{2E_p q}\left[f(E_p-\tilde{\mu}) + f(E_p+\tilde{\mu}) - 1\right]$$

Dilepton rate for $T = 200$ MeV: