Spontaneous CP Violation in Quark Scattering from QCD Z(3) domains and its Implications

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Outline

1. $Z(N)$ and CP Violation
   - Confinement Deconfinement Transition
   - $Z(N)$ Symmetry
   - CP Violation
2. Implications of CP Violation
   - In Early Universe
   - Heavy Ion Collisions
3. Effect of Quarks
   - Explicit Breaking of $Z(N)$
   - Varying Quark Mass

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Pure Gauge Theory at Finite T

- Partition function

$$\mathcal{Z} = Tr\left(e^{-\beta H}\right) \propto \int_{A_\mu(\tau=0)}^{A_\mu(\tau=\beta)=A_\mu(\tau=0)} \mathcal{D}A_\mu exp\left(-S_E\right)$$

where

$$S_E = \int_0^\beta d\tau \int d^3x \frac{1}{4} Tr\left(F_{\mu\nu}F^{\mu\nu}\right)$$

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- In presense of a static test quark,

$$\mathcal{Z}_q \propto \int \mathcal{D}A_\mu exp\left(-S_E\right) Tr\Omega(\vec{x})$$

$$\Omega(\vec{x}) = \mathbf{P}exp\left(ig\int_0^\beta A_0(\vec{x},\tau)d\tau\right)$$

is called Thermal Wilson Loop.

Order Parameter

- **Polyakov loop** $L(x) = (1/N) Tr\Omega(\vec{x})$ L(x) Profile
- Also $\mathcal{Z} = e^{-\beta F}$

$$\Rightarrow \frac{\mathcal{Z}_q}{\mathcal{Z}} = e^{-\beta \Delta F_q} = \langle L(\vec{x}) \rangle$$

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$$\langle L^*(\vec{y}) L(\vec{x}) \rangle \propto e^{-\beta \Delta F_{q\bar{q}}(|\vec{x}-\vec{y}|)} \xrightarrow{|\vec{x}-\vec{y}| \to \infty} \langle L^*(\vec{y}) \rangle \langle L(\vec{x}) \rangle = |\langle L(\vec{x}) \rangle|^2$$

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- **Confining Phase**:-
  $|\vec{x} - \vec{y}| \longrightarrow \infty, \Delta F_{q\bar{q}} \left(|\vec{x} - \vec{y}| \right) \longrightarrow \infty, \Rightarrow |\langle L(\vec{x}) \rangle| \longrightarrow 0$

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- **Deconfining Phase**:- $|\vec{x} - \vec{y}| \longrightarrow \infty, \Delta F_{q\bar{q}} \left(|\vec{x} - \vec{y}| \right)$ is finite $\Rightarrow |\langle L(\vec{x}) \rangle| \neq 0.$

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Spontaneously Broken $Z(N)$ Symmetry

- Under SU(N), $\Omega(\vec{x}) \longrightarrow U(\vec{x},\beta)\Omega(\vec{x})U^{\dagger}(\vec{x},0)$ and $A_{\mu}(\vec{x},\tau) \longrightarrow U(\vec{x},\beta)A_{\mu}(\vec{x},\tau)U^{\dagger}(\vec{x},0) + \mathrm{i}U(\vec{x},\beta)\partial_{\mu}U^{\dagger}(\vec{x},0)$

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- However, if $U(\vec{x},\beta) = ZU(\vec{x},0)$; where $Z \in Z(N)$
  $Z = e^{i\phi}\mathbf{1}; \phi = 2\pi m/N; m = 0, 1 \ldots (N-1)$
- Then, $A'_\mu(\vec{x},\beta) = A'_\mu(\vec{x},0)$ but $L(\vec{x}) \longrightarrow Z\big(L(\vec{x})\big)$.

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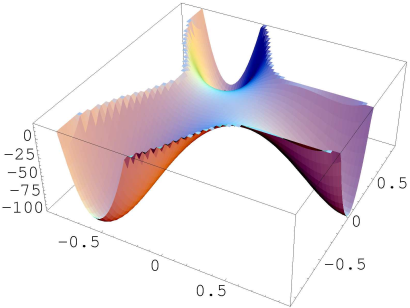
Degeneracy and Interfaces

- N -fold degeneracy of ground states.
- Domains with different $L(\vec{x})$ values will be formed.
- Interfaces exist between different domains.

Effective Potential

$$V(L) = \left(-\frac{b_2}{2}|L|^2 - \frac{b_3}{6}\left(L^3 + (L^*)^3\right) + \frac{1}{4}(|L|^2)^2\right) b_4 T^4$$

- [1]For $T > T_c$, second term leads to the three degenerate vacua corresponding to the three $\langle L(x) \rangle$ values.



[1]R.D. Pisarski, PRD 62,111501 (2000)

Effective Potential

Various parameters in the potential are fixed as [2]:-

- $b_3 = 2.0$ and $b_4 = 0.6061 \times 47.5/16$
- $b_2 = (1 - 1.11/x)\,(1 + 0.265/x)^2\,(1 + 0.300/x)^3 - 0.478$; where $x = T/T_c$ with $T_c \sim 182$ Mev
- As $T \to \infty, \langle L(x)\rangle \to y = b_3/2 + \frac{1}{2} \times \sqrt{b_3^2 + 4b_2\,(T = \infty)}$

Various quantities are rescaled as:-

- $L(x) \to L(x)/y$, $b_2 \to b_2/y^2$, $b_3 \to b_3/y$ and $b_4 \to b_4 y^4$
- $\langle L(x)\rangle \to 1$ as $T \to \infty$

[2] Dimitru and Pisarski, Phys, Lett. B 504, (2001); PRD 66, (2000); Nucl. Phys. A 698 (2002)

Explicit Breaking of $Z(N)$

- Fermion fields have anti-periodic boundary conditions
  $\psi(\vec{x}, 0) = -\psi(\vec{x}, \beta)$
- Under $Z(N)$ transformations,
  $\psi(\vec{x}, 0) \longrightarrow \psi'(\vec{x}, 0) = U^\dagger(\vec{x}, 0)\psi(\vec{x}, 0)$
  $\psi(\vec{x}, \beta) \longrightarrow \psi'(\vec{x}, \beta) = e^{-i\phi} U^\dagger(\vec{x}, 0)\psi(\vec{x}, \beta)$

[13] Dumitru et al PRD 70 (074001)

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$\psi(\vec{x},\beta) \longrightarrow \psi'(\vec{x},\beta) = e^{-i\phi}U^{\dagger}(\vec{x},0)\psi(\vec{x},\beta)$
- $Z(N)$ symmetry is explicitly broken.
- At the level of effective potential, the effect is studied by the addition of a linear term [13]

$$V(L) = \left(-\frac{b_2}{2}|L|^2 - \frac{b_3}{6}\left(L^3 + (L^*)^3\right) + \frac{1}{4}(|L|^2)^2\right)b_4T^4 - b_1\left(\frac{L+L^*}{2}\right)b_4T^4$$

[13] Dumitru et al PRD 70 (074001)

Continued..

Metastable States

- Degeneracy is lifted, with $L(x) = 1$ being the true vaccum.
- $L(x) \neq 1$ states are thermodynamically metastable.
- Relevence to Cosmology and Heavy Ion Collisions.

- The value of b_1 can be related to the analytical estimates of the difference in the energies of the true and metastable vaccua [14]

$$\Delta V \sim \left(\frac{2}{3}\right)\left(\frac{N_l}{N^3}\right)\pi^2 T^4 \left(N^2 - 2\right) \sim 3T^4$$

- At $T = 400\ MeV$, the corresponding value of b_1 which gives the correct splitting is 0.645.

[14] V. Dixit and M. C. Ogilvie, Phys. Lett. B, 269, 353 (1991)

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CP Violation

- CP Violation in SM due to thermal effects of the phase of Wilson line [4].

$$A_0 \equiv g_s \left(A_0\right)_{SU(3)} + g_w \left(A_0\right)_{SU(2)} + g_Y \left(B_0\right)_{U(1)} .$$

- Computed the free energy, in perturbation theory.
- There are long lived metastable states.
- Metastable states are *not* CP self-conjugate. CP Violation!
- They then show that non-zero value of the Higgs field forces the phase of the Wilson line either to be zero or in the metastable minimum.

[4] KorthalAltes, Lee, Pisarski, PRL 73, 1754 (1994)

Localized Quark Solution

- Dirac eqn in $1+1$ dim Euclidean space is[5]:

$$\left[\gamma_e^0 \partial_0 \delta^{jk} + ig\gamma_e^0 A_0^{jk}(z) + \gamma_e^3 \partial_3\right]\psi_k = 0$$

where $\gamma_e^0 \equiv \gamma^0$ and $\gamma_e^3 \equiv i\gamma^3$ are Euclidean Dirac matrices.

- $\psi_{1,4}(z) = N \times exp\left[\int_z \left(\pi T - A_0(\zeta)\right) d\zeta\right] exp\left(-\pi i T \tau\right)$

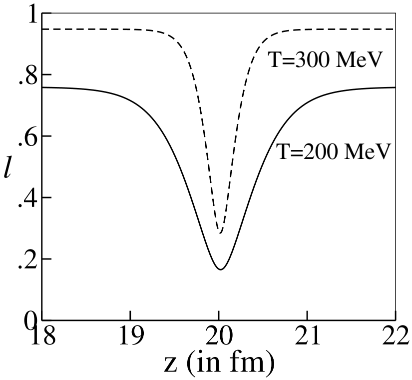
CP Conjugate and Density

- If ψ localizes then its CP conjugate $\gamma^0\gamma^2\psi^*$ does not.
- Density $(\psi^\dagger\psi)$ is static and localized.
- Qualitative discussion. No calculation of A_0 profile.

[5] Korthal Altes and Watson, PRL 75, 2799 (1995)

$L(x)$ Profile

- Profile of $L(x)$ for Polynomial Potential was calculated by energy minimization.[3] Gauge Profile
- Scattering of quarks and it's implication were discussed with no CP Violation.



[3] Layek, Mishra, Srivastava PRD 71, 070415 (2005)

Our Work

Atreya, Sarkar, Srivastava. (PRD85 (2012) 014009)

- We choose
$$A_0 = \frac{2\pi T}{g}\left(\frac{a}{3}\lambda_3 + \frac{b}{3}\lambda_8\right)$$
where λ_3 and λ_8 are Gell-Mann Matrices.
- a and b are fields depending on spatial coordinate only. ▸ L(x)

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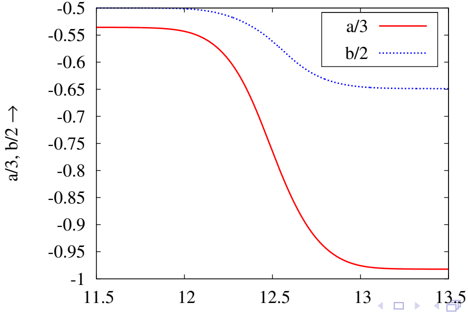
▸ Details



A_0 Profile

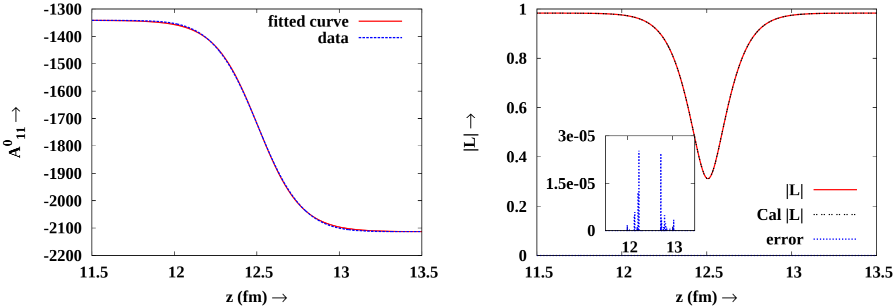
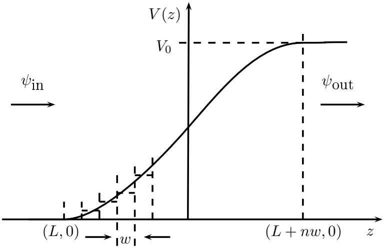


Figure : On Left: Corresponding A_0 Profile. Initial value is $(-1.5, -1.0)$.On Right: Plot of calculated $|L|$ and $|L|$ obtained from minimizing the energy.

Reflection and Transmission Coeff.

- We first approximated the profile by step function.
- For smooth profile we use the step potential approximation method.[6]
- Wavefunctions are matched at each step, relating ψ_j and ψ_{j+1}.
- The height of the j^{th} step potential is taken to be the mean value

$$V_j = \frac{[V(L+jw) + V(L+(j+1)w)]}{2}$$



[6] Kalotas and Lee, Am. J. Phys. 59, 48 (1991)

Continued..

- The incoming fermion wave-functions (ψ_j) and outgoing fermion wave-functions (ψ_{j+1}) at are

$$\psi_j(z) = A_j \begin{pmatrix} 1 \\ 0 \\ \frac{k_j}{E_j+m} \\ 0 \end{pmatrix} e^{ikz} + B_j \begin{pmatrix} 1 \\ 0 \\ \frac{-k_j}{E_j+m} \\ 0 \end{pmatrix} e^{-ik_j z},$$

$$\psi_{j+1}(z) = A_{j+1} \begin{pmatrix} 1 \\ 0 \\ \frac{k_{j+1}}{E_{j+1}+m} \\ 0 \end{pmatrix} e^{ik_{j+1} z} + B_{j+1} \begin{pmatrix} 1 \\ 0 \\ \frac{-k_{j+1}}{E_{j+1}+m} \\ 0 \end{pmatrix} e^{-ik_{j+1} z},$$

where $k_j = \sqrt{E_j^2 - m^2}$, and $E_j = E - V_j$.

Continued..

- Apply boundary conditions at j^{th} step i.e at $z = L + jw$.

$$\begin{pmatrix} A_j \\ B_j \end{pmatrix} = M^{-1}(L+jw, k_j) \times M(L+jw, k_{j+1}) \begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix}$$

$$\text{where} \quad M(L+jw, q) = \begin{pmatrix} e^{ik_q(L+jw)} & e^{-ik_q(L+jw)} \\ \frac{e^{ik_q(L+jw)}k_q}{E_q+m} & -\frac{e^{-ik_q(L+jw)}k_q}{E_q+m} \end{pmatrix}$$

- On iteration we obtain the relation

$$\begin{pmatrix} A_{\text{in}} \\ B_{\text{in}} \end{pmatrix} = M^{-1}(L, k_{\text{in}})M(L, k_1)\ldots M^{-1}(L+nw, k_n)M(L+nw, k_{\text{out}}) \begin{pmatrix} A_{\text{out}} \\ 0 \end{pmatrix}$$

Continued..

- The reflection and transmission coefficients are then given by

$$R \equiv \left|\frac{J_{\rm ref}}{J_{\rm in}}\right| = \left|\frac{B_{\rm in}}{A_{\rm in}}\right| \tag{3a}$$

$$T \equiv \left|\frac{J_{\rm trans}}{J_{\rm in}}\right| = \left|\frac{A_{\rm out}}{A_{\rm in}}\right| \times r \tag{3b}$$

$$\text{where} \quad r = \left(\frac{k_{\rm out}}{k_{\rm in}}\right)\left(\frac{E+m}{E-V_{max}+m}\right). \tag{3c}$$

- For Charm $R_q = 0.00104992$ and $R_{\bar{q}} = 5.24229 \times 10^{-10}$.

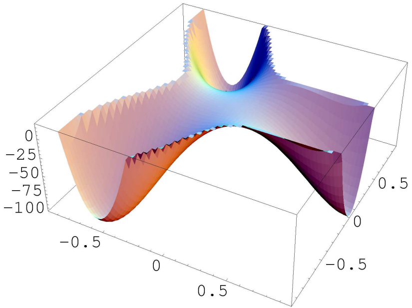
Logarithmic Potential

$$V\frac{a^3}{T^4} = -2(d-1)e^{-\sigma a/T}|L|^2 - \log\left[-|L|^4 + 8Re(L)^3 - 18|L|^2 + 27\right]$$

where[7]

- $\sigma = (425MeV)^2$ is the string tension,
- $T_d = 270MeV$ is the confinement temperature.
- a is the lattice constant with $a^{-1} = 272MeV$.
- For first order transition, $2(d-1)e^{-\sigma a/T_d} = 0.5153$.

[7] K. Fukushima, Phys. Lett. B591 (2004)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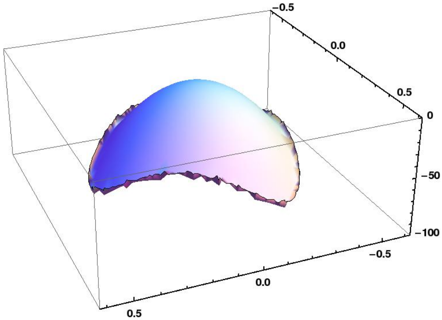
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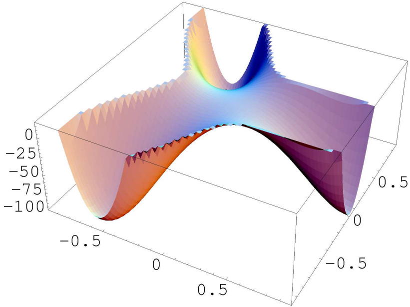
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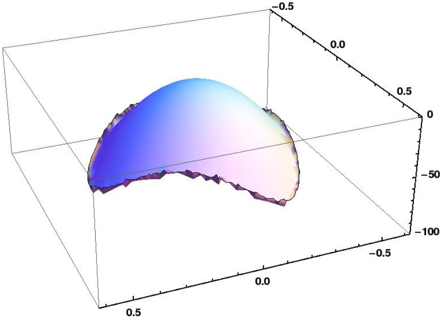
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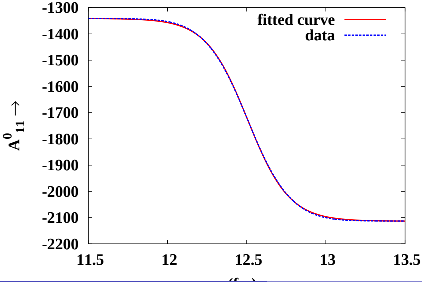
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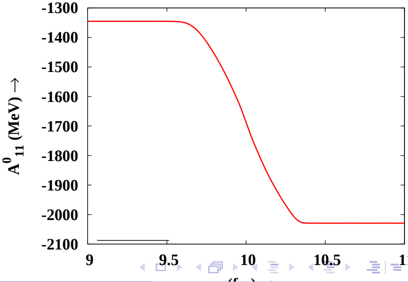
$A^0_{11} \rightarrow$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11
A^0_{11} (MeV) →

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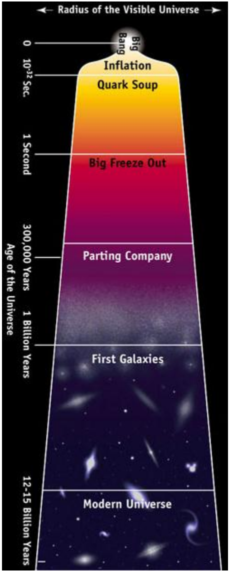
Evolution of Universe



Quark – Gluon Plasma

- Deconfined Phase; Free quarks and gluons

$\longleftarrow$ Hadronization

- Confined Phase; Hadron formation

$\longleftarrow$ Nucleosynthesis

- Helium Nuclei formed; Decoupling of Photons

$\longleftarrow$ Star Formation

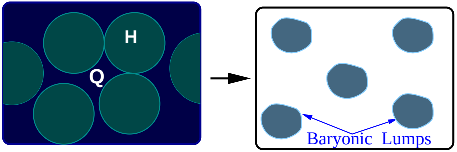
- Galaxy formation

$\longleftarrow$ Present Universe

Quark Nuggets

Atreya, Sarkar, Srivastava. (PRD90 (2014) 045010)

- Formation of stable quark nuggets during the phase transition [8].
- If QCD phase transition is first order, then the bubbles of Hadronic phase will form in the QGP Phase.
- As Universe cools, Hadronic bubbles will expand, and coalese.
- The QGP region will shrink, in the process trapping the baryons inside them.



[8] E. Witten, Phys.Rev. D32, (1984)

Why First Order?

- Provides with an interface between two region of the universe while being in thermal equillibrium.
- Baryon excess in the collapsing domains is due to the baryon transport across the phase boundary.
- Not Possible in a cross-over or Second order transition.

[9] Gorham PRD 83, 123005; Astone et al arXiv:1306.5164
[10] Berilenkov et al arXiv:1304.7521
[3] Layek et al PRD 71, 070415 (2005)

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- Even now there are attempts to detect these objects [9].
- They have been proposed as the dark matter and dark energy candidates [10].
- $Z(N)$ interface provide us with an attractive alternative to the phase boundary as proposed by Witten.
- $Z(3)$ domain walls can lead to baryon inhomogeneity generation[3].
- Possibility of quark nuggets formation irrespective of the order of Phase transition.

[9] Gorham PRD 83, 123005; Astone et al arXiv:1306.5164

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Domain Wall Formation

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- For $Z(3)$ structures to form, we need a situation where the Universe goes form hadronic phase to the QGP phase: Inflation.

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- During inflation, universe cools drastically and matter is in Hadronic (confined) phase.
- Universe reheates, and transition from Hadronic (confined) phase to QGP (deconfined) phase occurs.
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Domain Wall Formation

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- $Z(3)$ structures are formed via standard Kibble Mechanism.
- Regions of true vaccum ($L = 1$) will expand, metastable vacua ($L \neq 1$) will shrink.
- Certain low energy inflationary models allow $Z(3)$ domains to survive till QCD transition epoch.

Scattering From Interfaces

- $Z(3)$ structures may survive till QCD transition scale if dynamics of these walls is friction dominated because of the non-trivial scattering of quarks across the wall.

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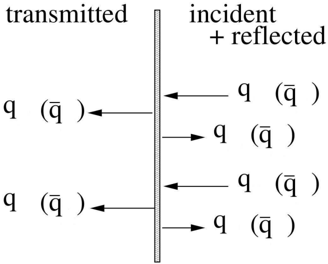
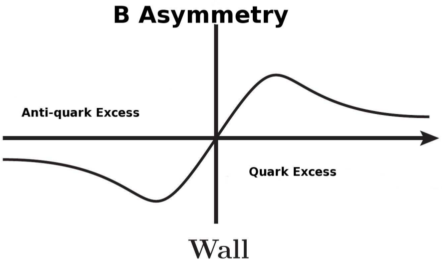
- $Z(3)$ structures may survive till QCD transition scale if dynamics of these walls is friction dominated because of the non-trivial scattering of quarks across the wall.
- Due to CP violating effects, quarks and anti-quarks scatter differently from interfaces.
- Results in segregation of Baryon number.





Evolution of Over-densities

- Total number of particle inside the wall

$$N_i = n_i V_i$$

$$\Rightarrow \dot{N}_i = \dot{n}_i V_i + n_i \dot{V}_i$$

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Inside quarks moving parallel to wall

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$$\overbrace{\textit{quarks moving from outside to inside}}$$

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Equations to be Solved

$$\dot{n}_i = \left(-\frac{2}{3} v_w T_w n_i + \frac{v_{rel}^o n_o T_{(-)} - v_{rel}^i n_i T_{(+)}}{6}\right)\frac{S}{V_i} - n_i \frac{\dot{V}_i}{V_i}$$

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$$\dot{n}_o = \left(\frac{2}{3}v_w T_w n_i - \frac{v^o_{rel} n_o T_{(-)} - v^i_{rel} n_i T_{(+)}}{6}\right)\frac{S}{V_o} + n_o \frac{\dot{V}_i}{V_o}$$

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$$R(t) = \frac{t}{N_d^{1/3}} - v_w (t - t_0)$$

where:-

- $\dot{n}_i$ and $\dot{n}_i$ are number densities inside and outside wall.
- v_w is the wall velocity.
- T_w, $T_{(-)}$ and $T_{(+)}$ are the Transmission coefficients for the quarks moving tranverse to v_w, towards the wall from inside and from outside respectively.

Baryon Anti-Baryon Segregation

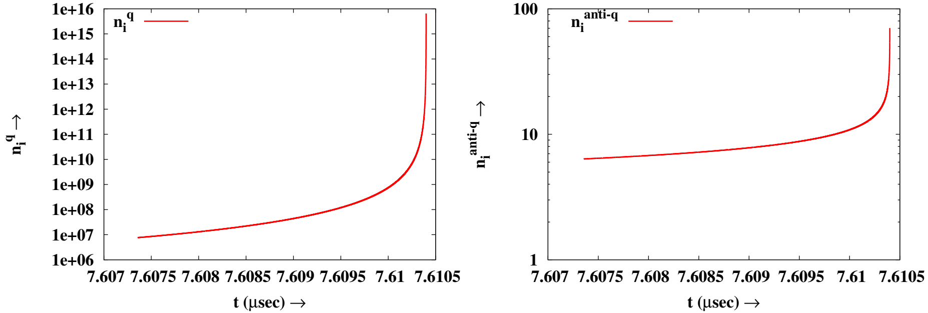

Figure : Evolution of number densities inside the domain wall. Left: For charm-quark. Right: For anti-charm.

Baryon Density Profile

$$\rho(R) = \frac{\dot{N}_i}{4\pi v_w R^2}$$

- $n_b \sim 10^{52} - 10^{53}$ for $R < 1\ m$.

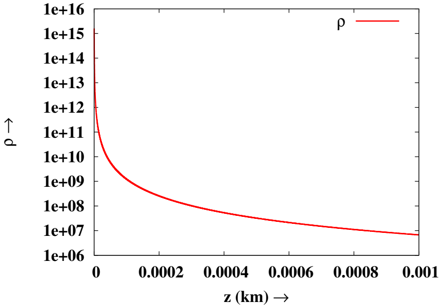

Figure : Baryon density left behind by collapsing wall.

Consequences

- Dark Matter candidates within the standard model of particle physics.
- Quark nuggets may act as the seed for Black hole formation[11].
- Important role in the structure formation.
- Inhomogeneties produced near QCD Phase transition can modify the dynamics of QCD phase transition[12].
- The over-densities which are produced near the electro-weak transition can alter the baryogenesis scenario.

[11] Lai and Xu, arXiv:0911.4777

[12] S. Sanyal PRD 67, 074009 (2003)

Outline

1. $Z(N)$ and CP Violation
   - Confinement Deconfinement Transition
   - $Z(N)$ Symmetry
   - CP Violation
2. Implications of CP Violation
   - In Early Universe
   - Heavy Ion Collisions
3. Effect of Quarks
   - Explicit Breaking of $Z(N)$
   - Varying Quark Mass

J/ψ Disintegration in QGP

Atreya, Bagchi, Srivastava. (PRC 90, 034912 (2014))

- QGP is supposed to be formed in Heavy Ion Collision experiments at RHIC and at LHC.
- QGP is the thermal system of quarks and gluons.
- Debye Screening of color charge. Debye radius $\propto T^{-1}$
- Matsui and Satz[3] proposed that due to this QGP medium, the $q\bar{q}$ potential in quarkonia (like J/ψ) meson will be Debye screened.
- If Debye screening length is smaller than the radius of J/ψ, then the potential will be completely screened and it will melt in the medium.
- This is the conventional mechanism of J/ψ melting.
- If the Debye length is larger, then the convetional mechanism of J/ψ melting does not work.

[3] Phys.Lett.B 178, 416 (1986)

J/ψ interaction with $Z(3)$ walls

- If there are $Z(3)$ domains, there will be a background A_0 profile.
- Then a J/ψ moving through the medium will interact with it.
- So c and $\bar{c}$ in J/ψ will experience different color forces depending on the color of quark and color composition of wall.
- This can change the color composition of J/ψ and also facilitates it's transition to higher excited states (like χ_c).
- As these states have size comparable or larger to Debye length, they will melt in the medium.
- This provides us an alternate mechanism of J/ψ melting.

Basic Assumptions

- We work in the rest frame of J/ψ with domain wall hitting the J/ψ with a velocity v along z-axis.
- Assume that there is no background vector potential, $A_i(z) = 0 \; i = 1, 2, 3$.
- Work with first order perturbation theory.
- The center of mass motion is not affected by the external perturbation.
- The interaction Hamiltonian is

$$\mathcal{H}_{int} = V^q(z'_1) \otimes \mathbb{1}^{\bar{q}} + \mathbb{1}^q \otimes V^{\bar{q}}(z'_2)$$

$$\text{with} \quad V^{q,\bar{q}}(z'_{1,2}) = gA_0'^{q,\bar{q}}(z'_{1,2})$$

Color Interaction

- The incoming state is a color singlet: $\frac{1}{\sqrt{3}}|r\bar{r} + b\bar{b} + g\bar{g}\rangle$.
- If the outgoing state is a singlet then the transition probability is identically zero as A_0 is traceless.

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- If the outgoing state is an octet state, it can be either of $|r\bar{b}\rangle$, $|r\bar{g}\rangle$, $|b\bar{g}\rangle$, $|b\bar{r}\rangle$, $|g\bar{b}\rangle$, $|g\bar{r}\rangle$, $\frac{1}{\sqrt{2}}|r\bar{r} - b\bar{b}\rangle$ and $\frac{1}{\sqrt{6}}|r\bar{r} + b\bar{b} - 2g\bar{g}\rangle$.
- Due to diagonal form of A_0 we get no transition to state like $|r\bar{g}\rangle$.
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- Only non-zero transition is for the two states with repulsive potential.
- We get the color part of transition probability as

$$\langle r\bar{r} - b\bar{b}|\mathcal{H}_{int}|\psi_{singlet}\rangle = A_0^r - A_0^b$$

$$\langle r\bar{r} + b\bar{b} - 2g\bar{g}|\mathcal{H}_{int}|\psi_{singlet}\rangle = A_0^r + A_0^b - 2A_0^g$$

where A_0^r, A_0^b and A_0^g are the diagonal componets of the matrix $A_0'\left(z_1'\right) - A_0'\left(z_2'\right)$. Details

Spatial Excitations

Cornell Potential

$$V\left(|\vec{r}_1 - \vec{r}_2|\right) = -\frac{\alpha_s}{|\vec{r}_1 - \vec{r}_2|} + \sigma|\vec{r}_1 - \vec{r}_2|$$

where α_s is the strong coupling constant and σ is the string tension.

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- In CM coordiantes, the transitional amplitude is

$$\int_{-\infty}^{\infty} \psi_j^* A_0^r \psi_i \, d\vec{r}_1 d\vec{r}_2 = \int_0^{\infty} \int_{-1}^{1} \int_0^{2\pi} \psi_n^*(r) Y_l^m \left(\cos\theta, \phi\right) A_0^r \psi_{100}(r) \; r^2 \; dr d(\cos\theta) d\phi.$$

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- No transition to a state which is symmetric under $\cos\theta \to -\cos\theta$.
- The excitation is possible to the first excited state of an octet (like an 'octet χ' state) which is more prone to melting in the medium.

Details

Transition Probability

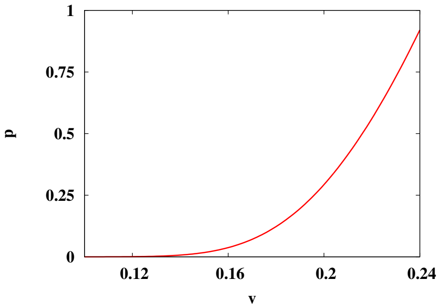


Figure : Transition Probability versus Energy.

- Probability increases dramatically for a slight increase in the energy.
- At higher energies, the perturbation theory breaks down and the results are not trustworthy. Details

Outline

Domain wall Profile

Atreya, Bagchi, Das, Srivastava. (PRD 90, 125016 (2014)

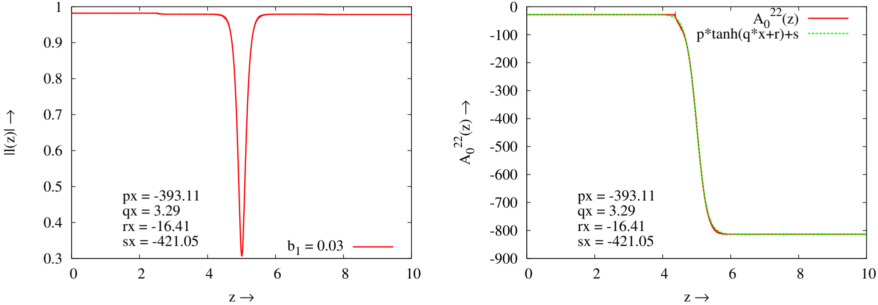


Figure : The domain wall profile for $b_1 = 0.03$ On Left:- $|L|$ profile. On Right:- A_0 profile.

Asymmetric Profile

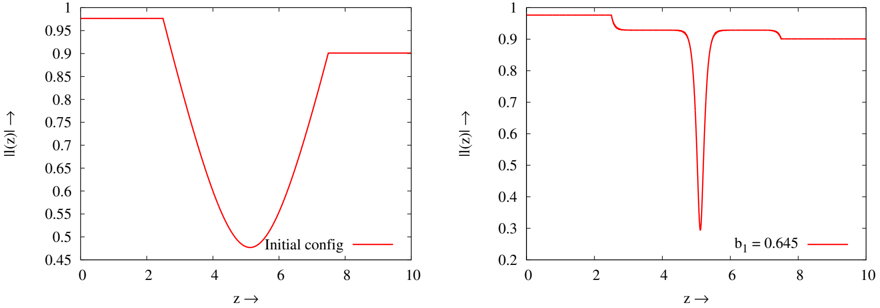
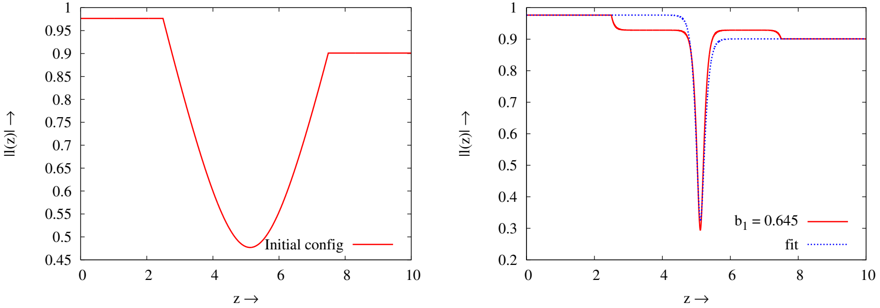


Figure : The domain wall profile for $b_1 = 0.645$ On Left:- Initial Condition. On Right:- Stable configuration.

Asymmetric Profile



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Asymmetric Profile?

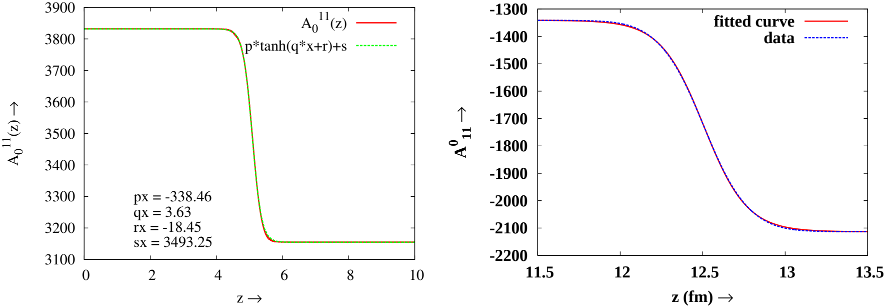

Figure : The domain wall profile for $b_1 = 0.645$. The fit is again $p * \tanh(q * z + r) + s$ function

Outline

Modeling the Quark Mass

- We know that $\langle L(x) \rangle = 0$ in the confined phase while it's non-zero in the deconfined phase.
- Also free (deconfined) quarks (as in QGP) have a dynamical mass which is very small ($m_{u,d} \sim 10\ MeV$) as compared to it's constituent mass ($\sim 350 MeV$) which is the mass of the quarks in hadrons.
- *Is there any connection between the two?*

[3] Layek et al PRD 71, 070415 (2005)

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- *Is there any connection between the two?*
- Proposal[3]:

$$m(x) = m_q + m_0 \left(l_0 - |L(x)| \right),$$

where $|L(x)|$ is the profile of $Z(3)$ domain wall, m_q is the dynamical quark mass and m_0 is the constituent quark mass. l_0 is the vaccum value of $|L(x)|$.

[3] Layek et al PRD 71, 070415 (2005)

Dirac Equation

$$\left[i\gamma^0\gamma^3\partial_3\delta^{jk} + \gamma^0 m(z)\delta^{jk} - gA_0^{jk}(z)\right]\psi_k(z) = E\psi_k(z).$$

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- The total potential now is

$$V(z) = gA_0^{jk}(z) - m_0\left(l_0 - |L(z)|\right).$$

- The space dependent part of quark mass appears as the potential in Dirac equation.
- The reflection is then calculated in the same way as the earlier case.

Asymmetric Reflection Coeffecients

	$b_1 = 0.03$	0.126	0.645
Left R_q	1.65437×10^{-6}	4.40706×10^{-6}	1.43314×10^{-10}
Right R_q	0.00003366	0.0141752	0.00394808
Left R_{aq}	2.25671×10^{-6}	1.85367×10^{-7}	2.07835×10^{-7}
Right R_{aq}	0.000376883	0.0820803	0.073885

Table : Reflection of Charm quark from the left and righ side for different b_1

Summary

- We have studied QCD $Z(3)$ interfaces at finite temp.
- Showed CP violating nature of $Z(3)$ domain walls and calculated Transmission Coeff.
- In context of early universe, we studied the evolution of baryon over-densities and discussed their effects.
- In context of heavy ion collisions, we showed that these $Z(3)$ structures can lead to the disintegration of J/ψ.
- We have also studied the effect of quarks on the spontaneous CP violation and calculated the reflection and transmission coeffecients from the asymmetric $l(x)$ profile.

Thank You !

Outline

Background A_0 Profile

- $L(x) = (1/3)\, Tr\left[\mathbf{P} exp\left(ig \int_0^\beta A_0(\vec{x},\tau) d\tau\right)\right]$
- For state corresponding to $L = 1, A_0 = 0$ is a solution trivially.

Background A_0 Profile

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- For state corresponding to $L = 1$, $A_0 = 0$ is a solution trivially.

Gauge Choice

- We choose

$$A_0 = \frac{2\pi T}{g}(a\lambda_3 + b\lambda_8)$$

a and *b* are constants, λ_3 and λ_8 are Gell-Mann Matrices

- *a* and *b* profiles are needed to get exact A_0 Profile.

Equations to be Solved

- On substituting and simplifying, we get

$$3L(x) = exp(i\alpha) + exp(i\beta) + exp(i\gamma)$$

- Two equations that are to be solved for *a* and *b* are:-

$$\cos(\alpha) + \cos(\beta) + \cos(\gamma) = 3|L|\cos(\theta)$$

$$\sin(\alpha) + \sin(\beta) + \sin(\gamma) = 3|L|\sin(\theta)$$

Where $\alpha = 2\pi\left(\frac{a}{3} + \frac{b}{2}\right)$, $\beta = 2\pi\left(\frac{a}{3} - \frac{b}{2}\right)$ and $\gamma = 2\pi(\frac{-2a}{3})$

The Solutions

```
(* r=1, θ=0 *)
Solve[{Cos[2*π*(a/3+b/2)] + Cos[2*π*(a/3-b/2)] + Cos[2*π*(-2*a/3)] == 3*L1[1, 0],
   Sin[2*π*(a/3+b/2)] + Sin[2*π*(a/3-b/2)] + Sin[2*π*(-2*a/3)] == 3*L2[1, 0]},
  {a, b}, InverseFunctions → True] // N

{{b → -2., a → -3.}, {b → -2., a → 0.}, {b → -2., a → 3.}, {b → -1., a → -1.5},
 {b → -1., a → 1.5}, {b → 0., a → -3.}, {b → 0., a → 0.}, {b → 0., a → 3.},
 {b → 1., a → -1.5}, {b → 1., a → 1.5}, {b → 2., a → -3.}, {b → 2., a → 0.}, {b → 2., a → 3.}}

(* r=1, θ=2*π/3 *)
Solve[{Cos[2*π*(a/3+b/2)] + Cos[2*π*(a/3-b/2)] + Cos[2*π*(-2*a/3)] == 3*L1[1, 2*π/3],
   Sin[2*π*(a/3+b/2)] + Sin[2*π*(a/3-b/2)] + Sin[2*π*(-2*a/3)] == 3*L2[1, 2*π/3]},
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(* r=1, θ=4*π/3 *)
Solve[{Cos[2*π*(a/3+b/2)] + Cos[2*π*(a/3-b/2)] + Cos[2*π*(-2*a/3)] == 3*L1[1, 4*π/3],
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```

Proceedure for Intermediate Values

- We start from $\theta = 0$ vaccum and choose one value.
- The variation of the gauge field (A_0) should be continuous.

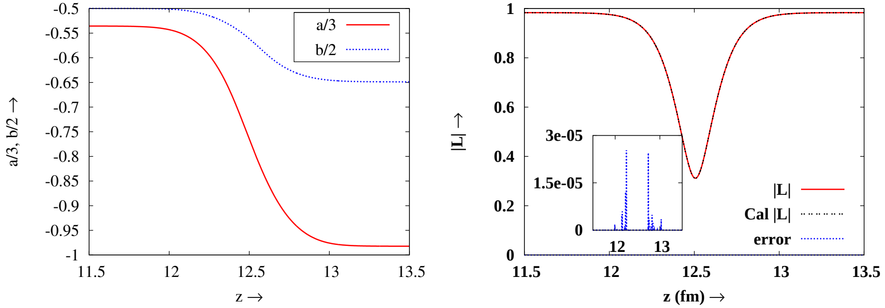
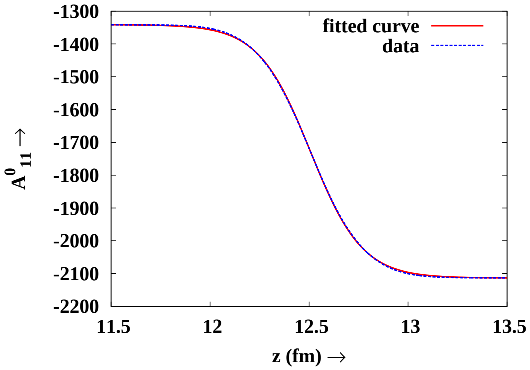


Figure : On left: Variation of *a* and *b* across the domain wall. On right: Plot of calculated $|L|$ and the one obtained from minimizing the energy.

A_0 Profile



- Profile was fitted to $A_0(x) = p\tanh(qx + r) + s$.
- Parameters are $p = -378.27$, $q = 7.95001$, $r = -49.7141$, $s = -1692.48$.
- The difference between the two profiles is extremely small.

back

Outline

Propagating Solutions

- We are interested in the propagating solutions.
- Need to solve Dirac Equation in Minkowski space with background gauge field.
- The background gauge field profile comes from the finite temperature filed theory formulated in Euclidean space.
- How to justify?

Propagating Solutions

- We are interested in the propagating solutions.
- Need to solve Dirac Equation in Minkowski space with background gauge field.
- The background gauge field profile comes from the finite temperature filed theory formulated in Euclidean space.
- How to justify?
- Start with Dirac Equation in Euclidean space.
- Do the analytic continuation of the full equation to Minkowski space.
- Using that equation we calculate the reflection and transmition coefficient.

Numerical Method

- We use step potential approximation method.[1]
- The potential in j^{th} bin is taken to be the

$$V_j = \frac{[V(L + jw) + V(L + (j+1)w)]}{2}$$

- Wavefunctions are matched at each step, relating ψ_j and ψ_{j+1}.

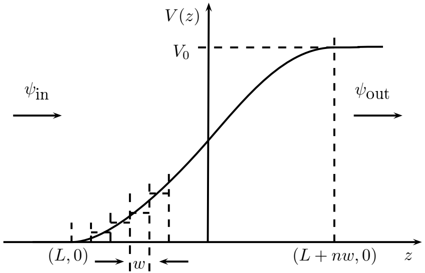


Figure : Potential ($V(z)$) approximated by n step potentials.

Outline

4 CP Violation
- Gauge Profile
- Calculation of Reflection Coeff.
- Results

5 J/ψ Disintegration
- Formalism
- Numerical Results

With Step Potential

- We first approximate the entire profile by a single step function.
- The reflection coeff. is given by

$$R = \frac{(1-r)^2}{(1+r)^2}; \text{where } r = \frac{q}{k}\frac{(E+m_0)}{(E-V_0+m_0)}$$

- $V_0 = -gA_0$ is the potential as seen by the incoming fermion.
- CP violating effect is larger for heavier quarks.

	m (MeV)	E (GeV)	R_q	$R_{\bar{q}}$
u	2.5	3.0	1.72×10^{-7}	1.92×10^{-8}
d	5.0	3.0	6.76×10^{-7}	7.54×10^{-8}
s	100	3.0	2.83×10^{-4}	3.14×10^{-5}
c	1270	3.0	0.14	0.006

Using Exact Profile

- Calculated using Mathematica and FORTRAN.
- For c quark $R = 0.001$ and for $\bar{c}$ we get $R = 5.24 \times 10^{-10}$
- As a check, we shrank the profile and compared with step potential.

Shrinking Factor	Reflection Coeff
No shrinking	0.001
0.5	0.01
0.05	0.11
0.005	0.12
Step Potential	0.14

Table : Table for the reflection coefficients when the profile is shrunk. Results approach the step potential as the profile gets narrower.

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Outline

Basic Assumptions

- We work in the rest frame of J/ψ.
- Gauge Potential is chosen to be in diagonal gauge,

$$A_0 = a\lambda_3 + b\lambda_8.$$

- Consider the domain wall coming and hitting the J/ψ with a velocity v along z-axis.

$$A_0(z) \rightarrow A'_0(z') = \gamma\left(A_0(z) - vA_3(z)\right)$$
$$A_3(z) \rightarrow A'_3(z') = \gamma\left(A_3(z) - vA_0(z)\right)$$
$$z = \gamma\left(z' + vt'\right)$$

- Assume that there is no background vector potential, $A_i(z) = 0 \; i = 1, 2, 3$.

Perturbation Theory

- Color electric field due to t' dependence A_0.

$$E_{induced} = -\frac{\partial A'_3}{\partial t'} \propto v^2 << 1.$$

- We use first order time dependent perturbation theory

$$\mathcal{A}_{ij} = \delta_{ij} - \ i \int_{t_i}^{t_f} \langle \psi_j | \mathcal{H}_{int} | \psi_i \rangle e^{-i(E_j - E_i)t} dt.$$

- The interaction Hamiltonian is

$$\mathcal{H}_{int} = V^q(z'_1) \otimes \mathbb{1}^{\bar{q}} + \mathbb{1}^q \otimes V^{\bar{q}}(z'_2)$$

$$\text{with} \quad V^{q,\bar{q}}(z'_{1,2}) = g A_0'^{q,\bar{q}}(z'_{1,2})$$

Color Interaction

- The color interaction can be written as

$$\langle \psi_{out} | \mathcal{H}_{int} | \psi_{singlet} \rangle = \langle \psi_{out} | g A_0'^{q}(z_1') \otimes \mathbb{1}^{\bar{q}} | \psi_{singlet} \rangle + \langle \psi_{out} | \mathbb{1}^{q} \otimes g A_0'^{\bar{q}}(z_2') | \psi_{singlet} \rangle .$$

- The incoming state is a color singlet

$$|\psi_{singlet}\rangle = \frac{1}{\sqrt{3}} \left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}^{q} \otimes \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}^{\bar{q}} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}^{q} \otimes \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}^{\bar{q}} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}^{q} \otimes \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}^{\bar{q}} \right]$$

- If the outgoing state is a singlet then each of the above term on the RHS is identically zero, $\mathcal{A}_{ij} = 1$ for ground state $(i = j)$.

Color Octets

- If the outgoing state is an octet state, it can be either of $|r\bar{b}\rangle$, $|r\bar{g}\rangle$, $|b\bar{g}\rangle$, $|b\bar{r}\rangle$, $|g\bar{b}\rangle$, $|g\bar{r}\rangle$, $\frac{1}{\sqrt{2}}|r\bar{r} - b\bar{b}\rangle$ and $\frac{1}{\sqrt{6}}|r\bar{r} + b\bar{b} - 2g\bar{g}\rangle$.
- Due to diagonal form of A_0 we get no transition to state like $|r\bar{g}$.
- Only non-zero transition is for the two states with repulsive potential.
- We get the color part of transition probability as

$$\langle r\bar{r} - b\bar{b}|\mathcal{H}_{int}|\psi_{singlet}\rangle = A_0^r - A_0^b$$

$$\langle r\bar{r} + b\bar{b} - 2g\bar{g}|\mathcal{H}_{int}|\psi_{singlet}\rangle = A_0^r + A_0^b - 2A_0^g$$

where A_0^r, A_0^b and A_0^g are the diagonal componets of the matrix $A_0'\left(z_1'\right) - A_0'\left(z_2'\right)$. Back

Spatial Excitations

- For spatial part, we need the potential between c and $\bar{c}$

$$V\left(|\vec{r}_1 - \vec{r}_2|\right) = -\frac{\alpha_s}{|\vec{r}_1 - \vec{r}_2|} + \sigma|\vec{r}_1 - \vec{r}_2|$$

where α_s is the strong coupling constant and σ is the string tension.

- As potential is central, we go to the centre of mass coordinates.

$$\vec{R}_{cm} = \frac{\vec{r}_1' + \vec{r}_2'}{2} \quad \text{and}$$

$$\vec{r} = \vec{r}_1' - \vec{r}_2',$$

- We write J/ψ wavefunction as $\Psi(\vec{R}_{cm})\psi(\vec{r})$.

Spatial Excitations

- Assuming that the centre of mass motion is not affected by the external perturbation, we get

$$\Psi(\vec{R}_{cm}) = \exp^{-i\vec{K}_{cm}.\vec{R}_{cm}} \quad \text{and}$$

$$\psi(r,\theta,\phi) = \psi(r)\, Y_l^m\,(\cos\theta,\phi)$$

. The perturbation is then

$$A_0^r = \gamma A_0^{11}\left[\gamma(r\cos\theta + vt')\right] - \gamma A_0^{11}\left[\gamma(-r\cos\theta + vt')\right]$$

- The transitional amplitude then gives

$$\int_{-\infty}^{\infty} \psi_j^* A_0^r \psi_i \; d\vec{r}_1 d\vec{r}_2 = \int_0^{\infty}\int_{-1}^{1}\int_0^{2\pi} \psi_n^*(r)\, Y_l^m\,(\cos\theta,\phi)\, A_0^r \; \psi_{100}(r)\; r^2\; dr d(\cos\theta) d\phi.$$

Spatial Excitations

- Under $\cos\theta \to -\cos\theta$, $A_0^r \to -A_0^r$ and ψ_i does not change.
- So if $Y_l^m(\cos\theta,\phi) = Y_l^m(-\cos\theta,\phi)$ then transition probability is zero.
- No transition to a state which is symmetric under $\cos\theta \to -\cos\theta$.
- The excitation is possible to the first excited state of an octet (like an 'octet χ' state).
- As the excited state will have a radius larger than the ground state it is more prone to melting in the medium. Back

Outline

4. CP Violation
   - Gauge Profile
   - Calculation of Reflection Coeff.
   - Results

5. J/ψ Disintegration
   - Formalism
   - Numerical Results

Charmonium Wavefunctions

- The radial wavefunction is the solution of radial part of Schrödinger eqn with the potential

$$V(r) = -\frac{\alpha_s}{r} + \sigma r + \frac{l(l+1)}{2\mu r^2}$$

where μ is the reduced mass.

- We used energy minimization to get the wavefunction.
- Check:- wavefunction and binding energy of the hydrogen atom.
- $m_c = 1.3 GeV$, $\alpha_s = 0.3$, and $\sigma = 0.16\ GeV^2$ [9].
- The strong coupling is chosen such that $N/g^2 = 0.8$.

[9] F. Giannuzzi and M. Mannarelli, Phys.Rev. D80, 054004 (2009)

Charmonium Wavefunctions

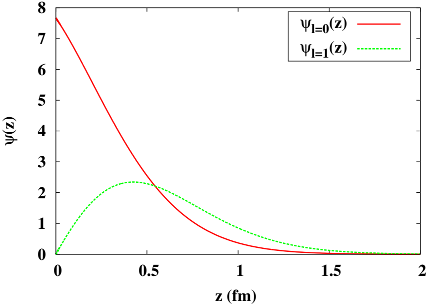

Figure : Wavefunctions for J/ψ ($l = 0$) and χ ($l = 1$) states.

- The binding energies are $E_{j/\psi} = 0.5\ GeV$ and $E_{\chi_c} = 0.83\ GeV$
- Radius of $J/\psi \sim 0.5\ fm$ while that for $\chi \sim 1.0\ fm$.

Transition Probability

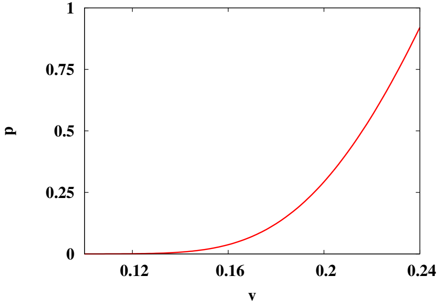

Figure : Transition Probability versus Energy.

- Probability increases dramatically for a slight increase in the energy.
- At higher energies, the perturbation theory breaks down and the results are not trustworthy. Back