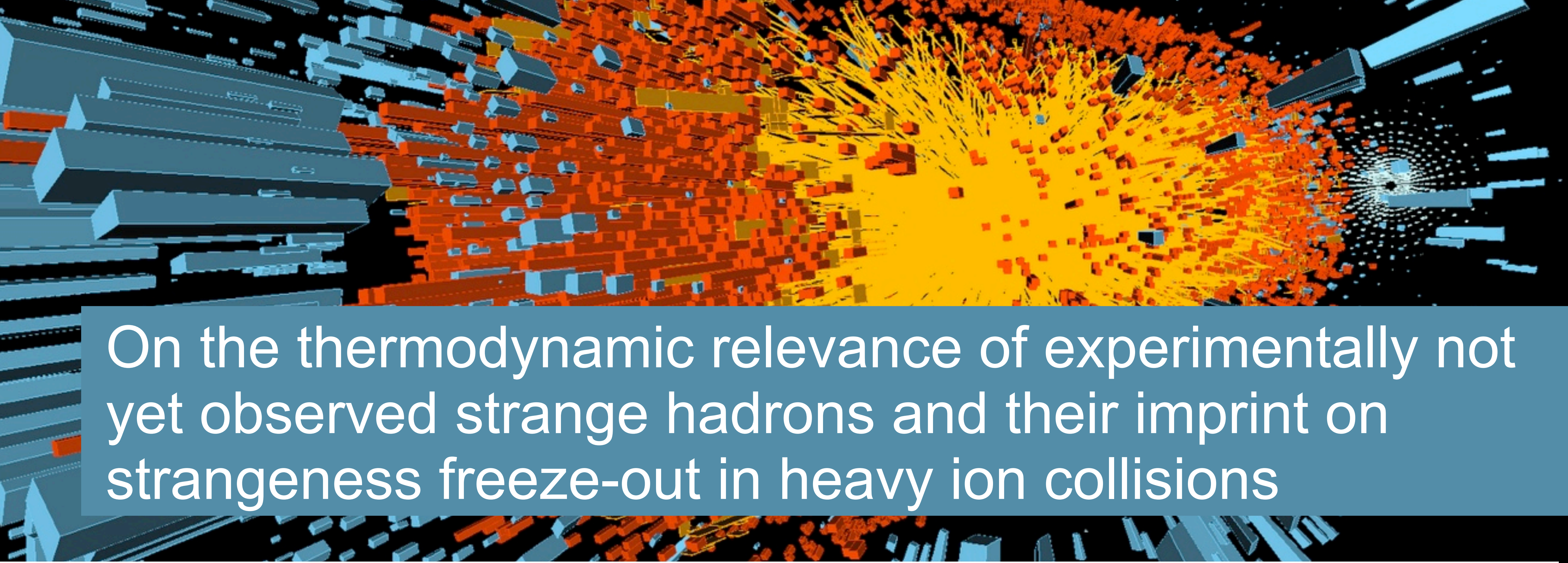

On the thermodynamic relevance of experimentally not yet observed strange hadrons and their imprint on strangeness freeze-out in heavy ion collisions

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BNL-Bi-CCNU Collaboration:

A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Meazawa, S. Mukherjee, H.Ohno, P. Petreczky, C. Schmidt, W. Soeldner, M. Wagner

Definitions

Expansion of the pressure:

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk,0}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$X = B, Q, S$: conserved charges

Lattice

$$\chi_n^X = \frac{\partial^n [p/T^4]}{\partial(\mu_X/T)^n}\Big|_{\mu_X=0}$$

generalized susceptibilities

$\Rightarrow$ only at $\mu_X = 0$!

Experiment

$$VT^3 \chi_2^X = \left\langle (\delta N_X)^2 \right\rangle$$

$$VT^3 \chi_4^X = \left\langle (\delta N_X)^4 \right\rangle - 3 \left\langle (\delta N_X)^2 \right\rangle^2$$

$$VT^3 \chi_6^X = \left\langle (\delta N_X)^4 \right\rangle - 15 \left\langle (\delta N_X)^4 \right\rangle \left\langle (\delta N_X)^2 \right\rangle + 30 \left\langle (\delta N_X)^2 \right\rangle^3$$

cumulants of net-charge fluctuations

$$\delta N_X \equiv N_X - \langle N_X \rangle$$

$\Rightarrow$ only at freeze-out $(\mu_f(\sqrt{s}), T_f(\sqrt{s}))$!

Motivations

Explore the QCD phase diagram

- Analyze higher order cumulants and test universal scaling behavior

  $\longrightarrow$ make prediction on the radius of convergence and possible experimental observables

Analyze freeze-out conditions

- Match various cumulant ratios of measured fluctuations to QCD

  $\longrightarrow$ determine freeze-out parameter

BNL-Bielefeld, PRL 109 (2012) 192302.

Identify the relevant degrees of freedom

- Compare (lattice) QCD fluctuations to various hadronic/quasiparticle models:

  $\longrightarrow$ deconfinement vs. chiral transition (melting of open strange/charm hadrons)

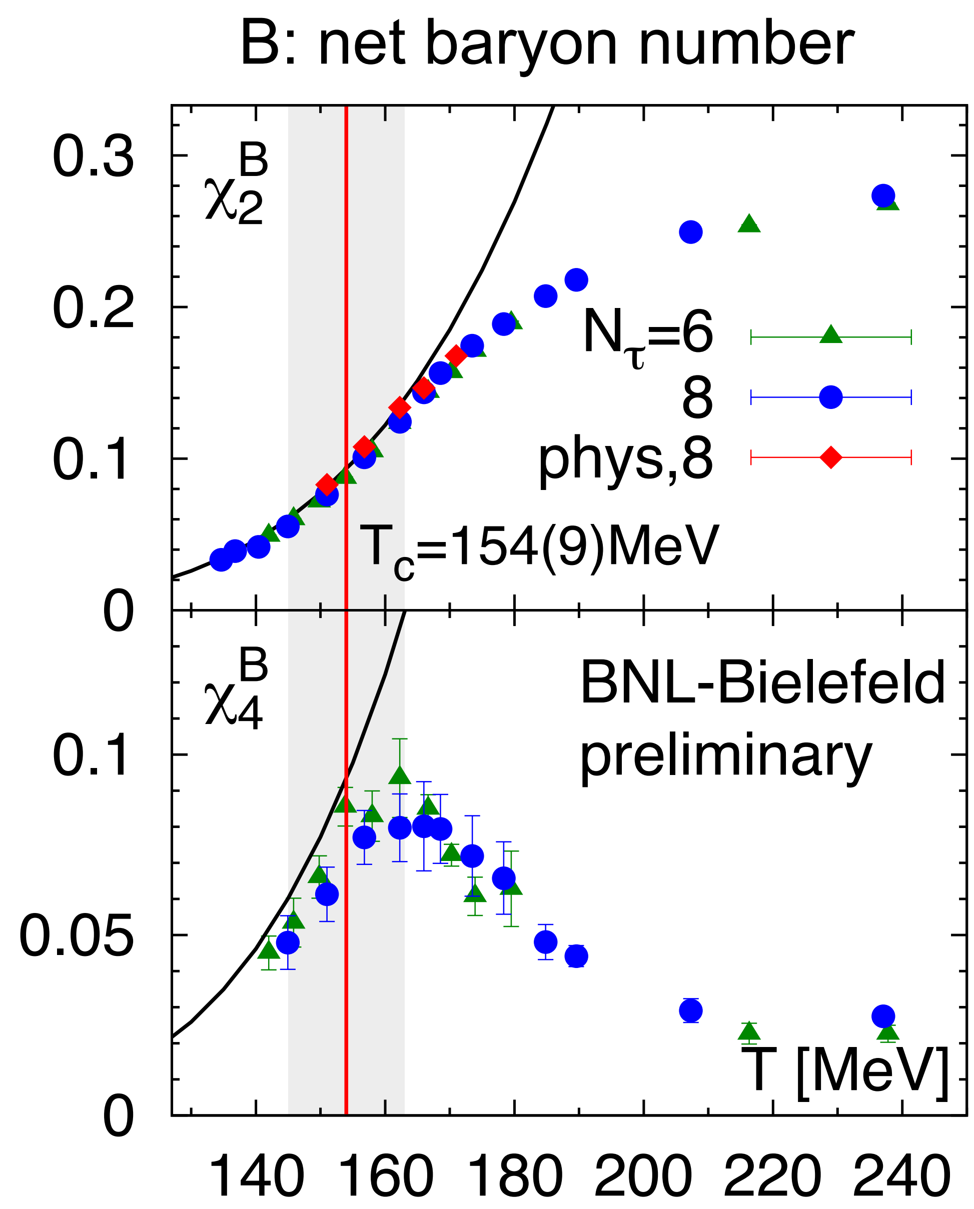
  $\longrightarrow$ evidence for experimentally not yet observed hadrons

BNL-Bielefeld, PRL 111 (2013) 082301;
BNL-Bielefeld-CCNU, PRL 113 (2014) 072001; PLB 737 (2010) 210

The lattice setup

Lattice parameters:

- (2+1)-flavor of highly improved staggered fermions (HISQ-action)
- a set of different lattice spacings ($N_\tau = 6, 8, 12$)
- two different pion masses: $m_\pi = 140, 160\ MeV$
- high statistics: $(10 - 30) \times 10^3$ configurations



$\Rightarrow$ statistical errors are under control for all 4th order cum

$\Rightarrow$ In general: find good agreement with HRG model for T<155 MeV

The lattice setup

Observables: traces of combinations of M^{-1} and $M^{(n)} = \partial^n M / \partial \mu^n$

$$\frac{\partial \ln Z}{\partial \mu} = \frac{1}{Z} \int \mathcal{D}U \, \mathrm{Tr}\left[M^{-1} M'\right] \, e^{\mathrm{Tr}\, \ln M} e^{-\beta S_G}$$

$$= \left\langle \mathrm{Tr}\left[M^{-1} M'\right] \right\rangle$$

$$\frac{\partial^2 \ln Z}{\partial \mu^2} = \left\langle \mathrm{Tr}\left[M^{-1} M''\right] \right\rangle - \left\langle \mathrm{Tr}\left[M^{-1} M' M^{-1} M'\right] \right\rangle + \left\langle \mathrm{Tr}\left[M^{-1} M'\right]^2 \right\rangle$$

Method: stochastic estimators with $N = 1500$ random vectors

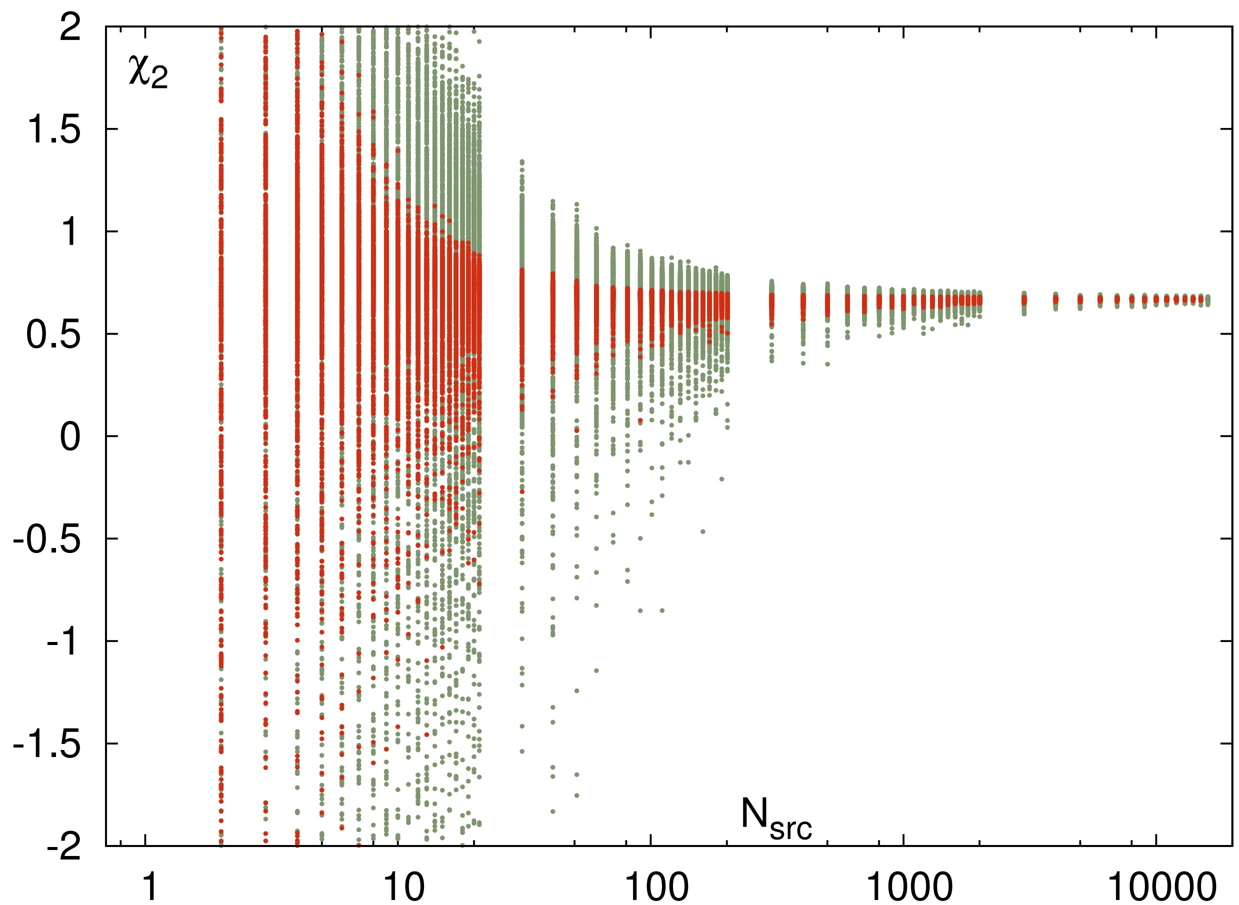
$$\mathrm{Tr}\left[Q\right] \approx \frac{1}{N} \sum_{i=1}^{N} \eta_i^\dagger Q \eta_i \quad \text{with} \quad \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \eta_{i,x}^\dagger \eta_{i,y} = \delta_{x,y}$$

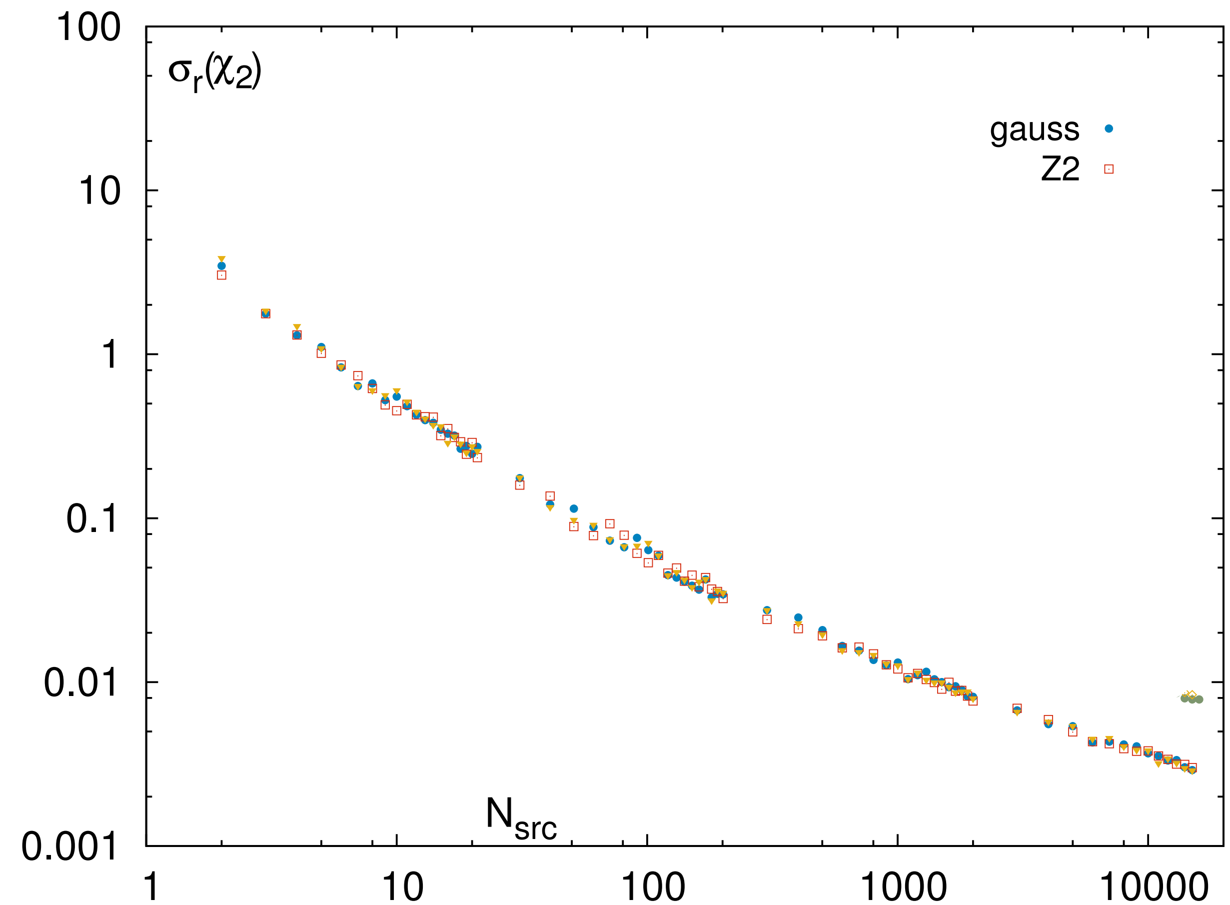
The lattice setup

Method: stochastic estimators with $N = 1500$ random vectors

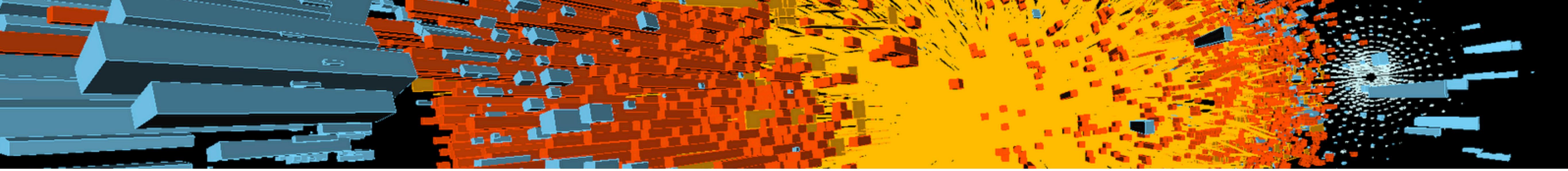
$$\mathrm{Tr}\,[Q] \approx \frac{1}{N}\sum_{i=1}^{N} \eta_i^\dagger Q \eta_i \qquad \text{with} \qquad \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \eta_{i,x}^\dagger \eta_{i,y} = \delta_{x,y}$$

unbiased estimator on a single configuration:





courtesy P. Steinbrecher



Freeze-out conditions in HIC

Make contact with experiment

Apply: initial conditions in HIC

- **strangeness neutrality:** $\langle N_S \rangle = 0$
- **isospin assymetry:** $\langle N_Q \rangle = r \langle N_B \rangle$

$r \approx 0.4$
for Au-Au
and Pb-Pb

expand in powers of μ_B, μ_Q, μ_S
solve for μ_Q, μ_S

$$\mu_Q(T,\mu_B) = q_1(T)\hat{\mu}_B + q_3(T)\hat{\mu}_B^3$$
$$\mu_S(T,\mu_B) = s_1(T)\hat{\mu}_B + s_3(T)\hat{\mu}_B^3$$

LO NLO

$\hat{\mu}_B = \mu_B/T$

two independent parameter
remain: $T^f,\ \mu_B^f$
need two ratios of cumulants from experiment to fix

$$\mu_Q^f \equiv \mu_Q(T^f, \mu_B^f)$$
$$\mu_S^f \equiv \mu_S(T^f, \mu_B^f)$$

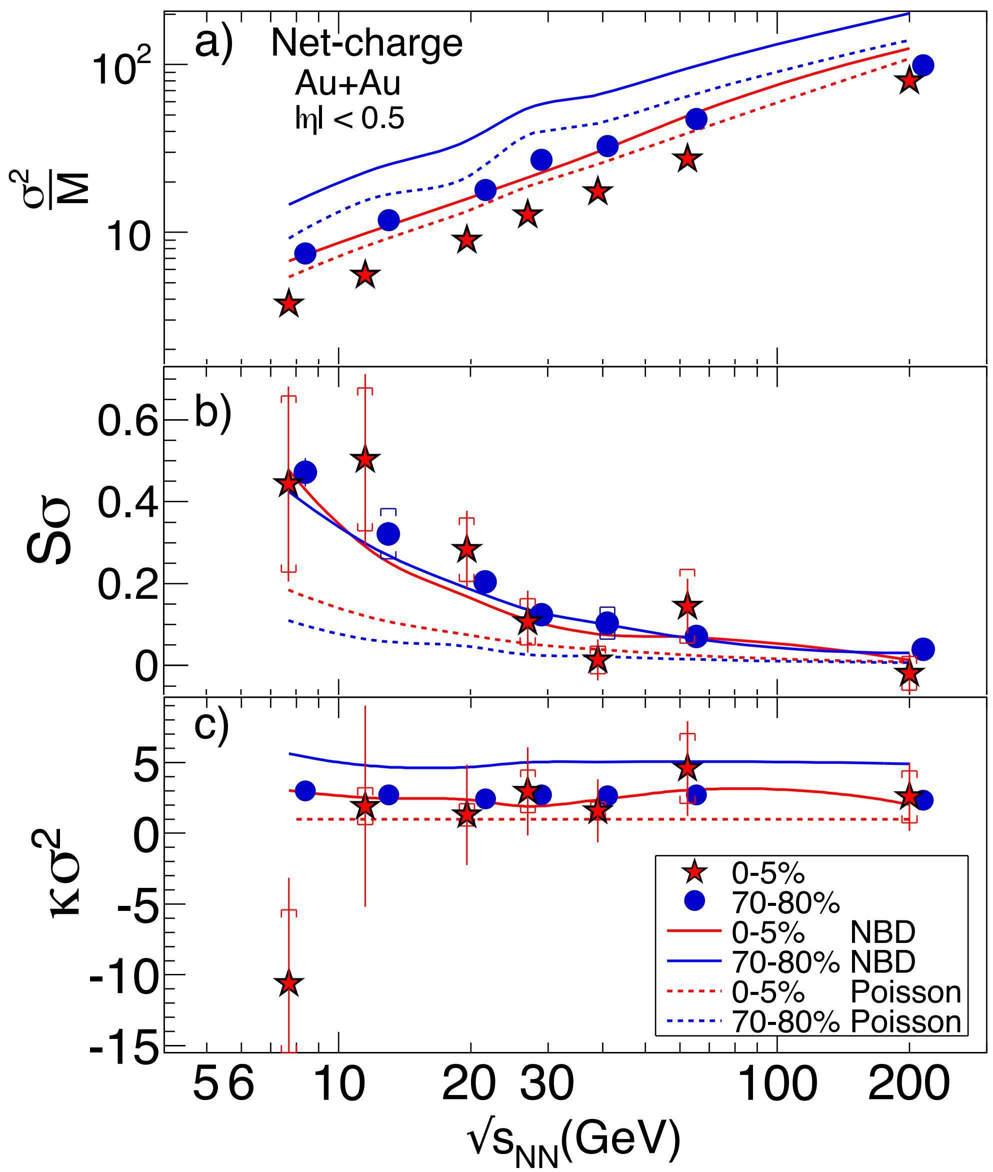
Make contact with experiment

- Need two ratios of cumulants to fix the remaining two freeze-out parameters T^f, μ_B^f

$\Rightarrow$ consider ratios of cumulants of **electric charge fluctuations**

BNL-Bielefeld, PRL 109 (2012) 192302.

STAR, PRL 113 (2014) 092301



Experiment: **Lattice:**

$$\frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = R_{12}^Q(T,\mu_B)$$

LO linear in μ_B, fixes μ_B^f (baryometer)

$$\frac{S_Q(\sqrt{s})\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\chi_3^Q(T,\mu_B)}{\chi_1^Q(T,\mu_B)} = R_{31}^Q(T,\mu_B)$$

LO independent of μ_B, fixes T^f (thermometer)

M: mean
σ: variance
S: skewness

χ_n: generalized susceptibilities

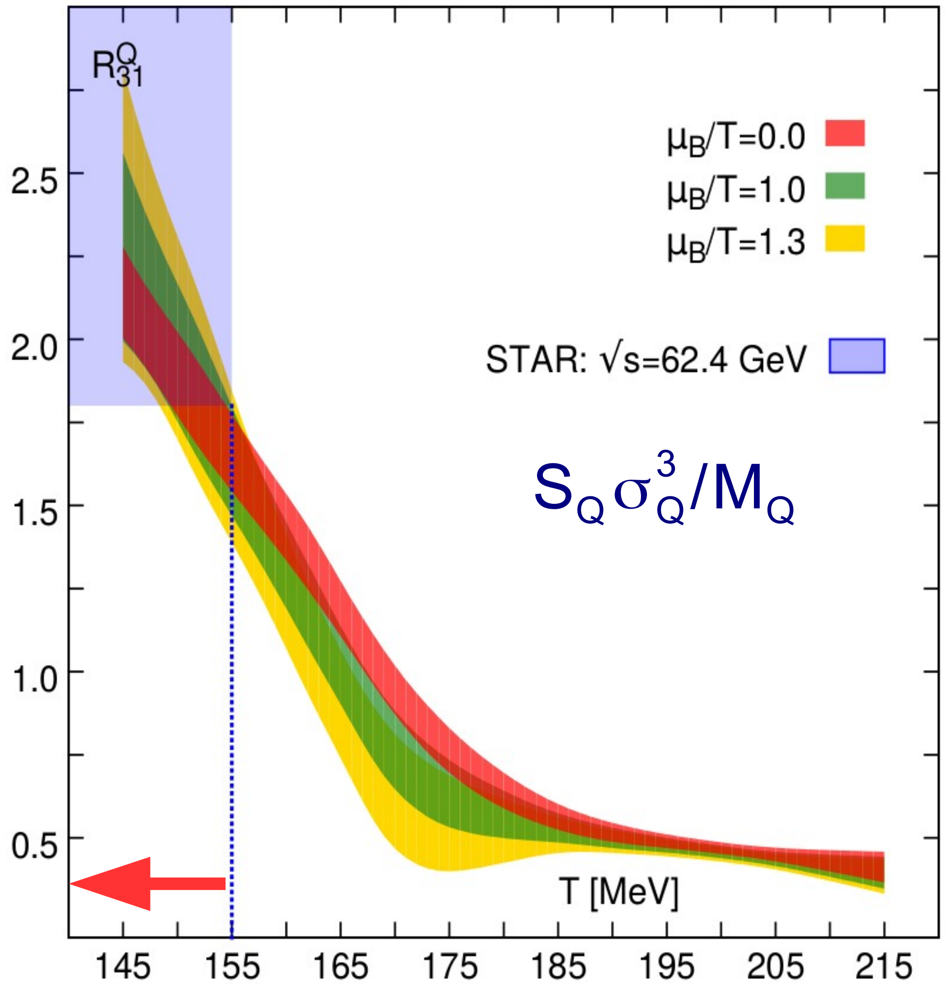
Make contact with experiment

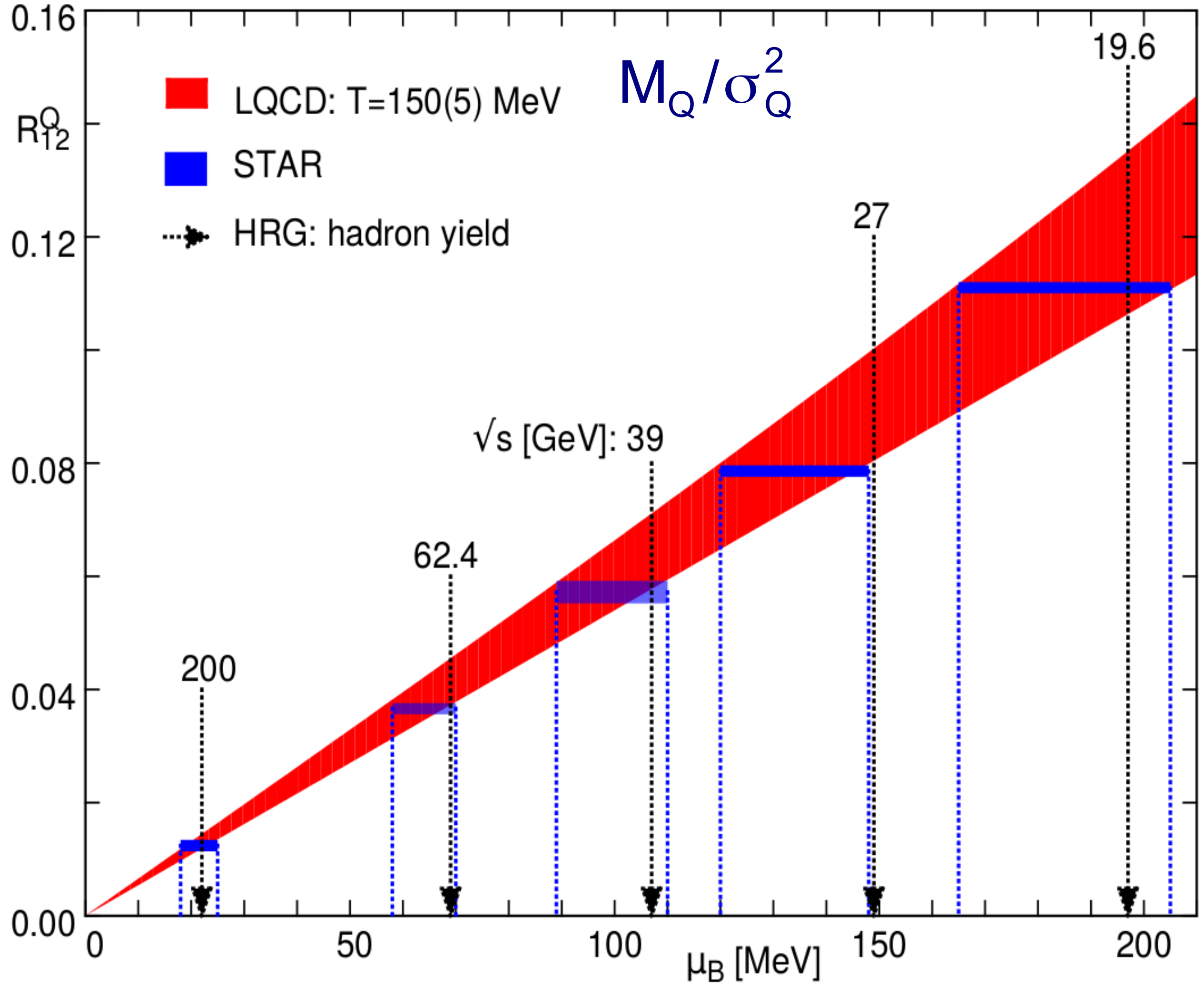
thermometer:

$$R_{31}^{Q}(T,\mu_B) = R_{31}^{Q,0} + R_{31}^{Q,2}\hat{\mu}_B^2$$

baryometer:

$$R_{12}^{Q}(T,\mu_B) = R_{12}^{Q,1}\hat{\mu}_B + R_{12}^{Q,3}\hat{\mu}_B^3$$

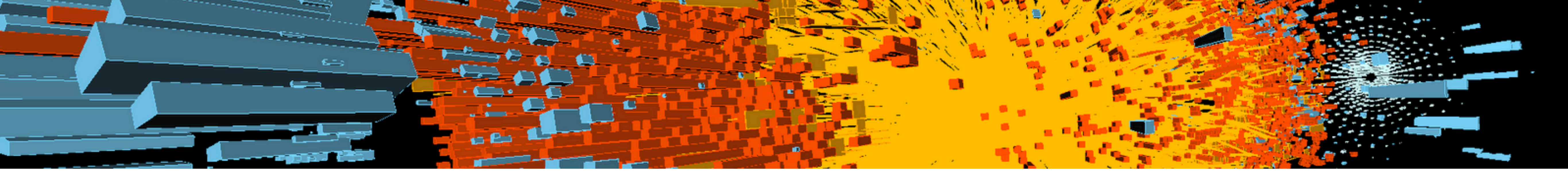




S. Mukherjee, CPOD'14

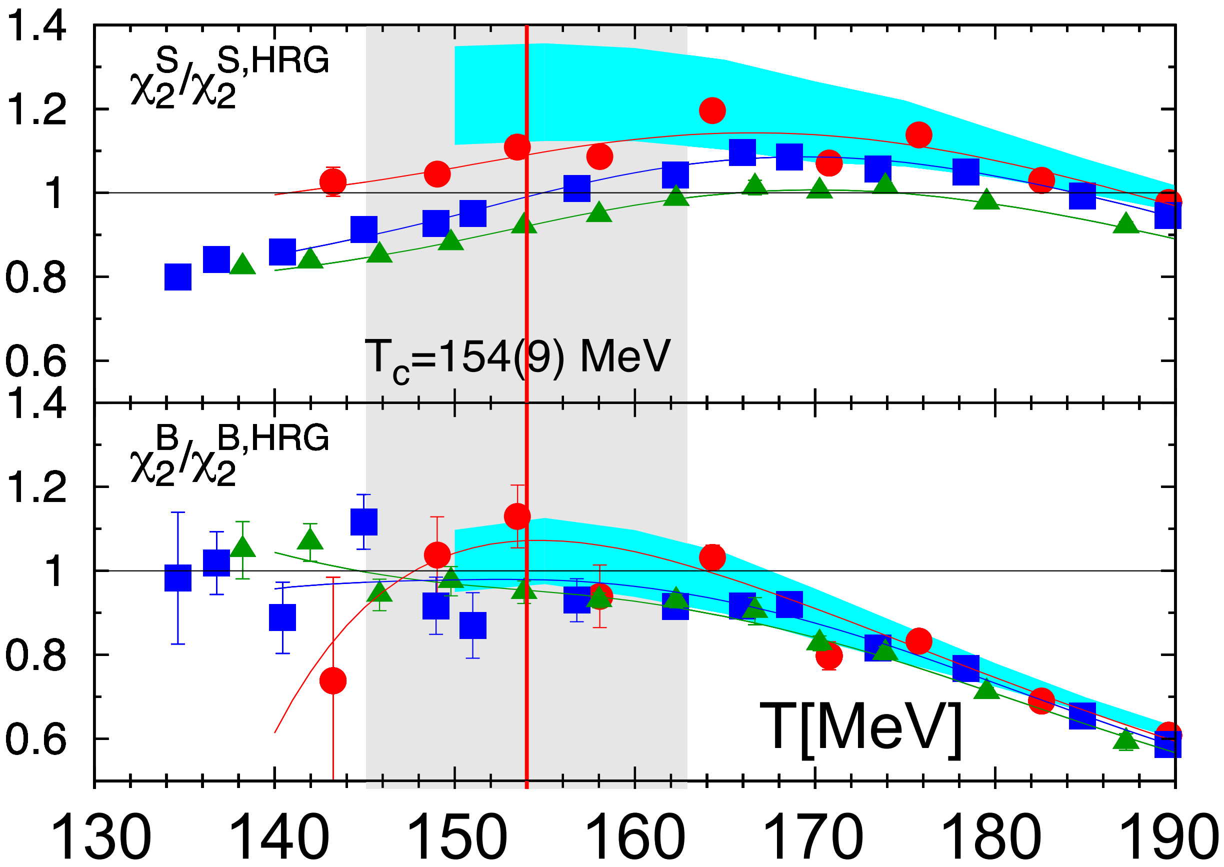
⇒ difficult to determine $\sqrt{s}$-dependence of T^f, need more precise experimental data from BES (BES-II)

⇒ need to check self-consistence using other ratios as input



Relevant degrees of freedom

Some notable differences in the strangeness sector



$N_\tau = 12$ HISQ-Action
$N_\tau = 8$
$N_\tau = 6$

cyan bands indicate continuum extrapolations

HotQCD, PRD 86 (2012) 034509
[arXiv: 1203.0784]

HRG:

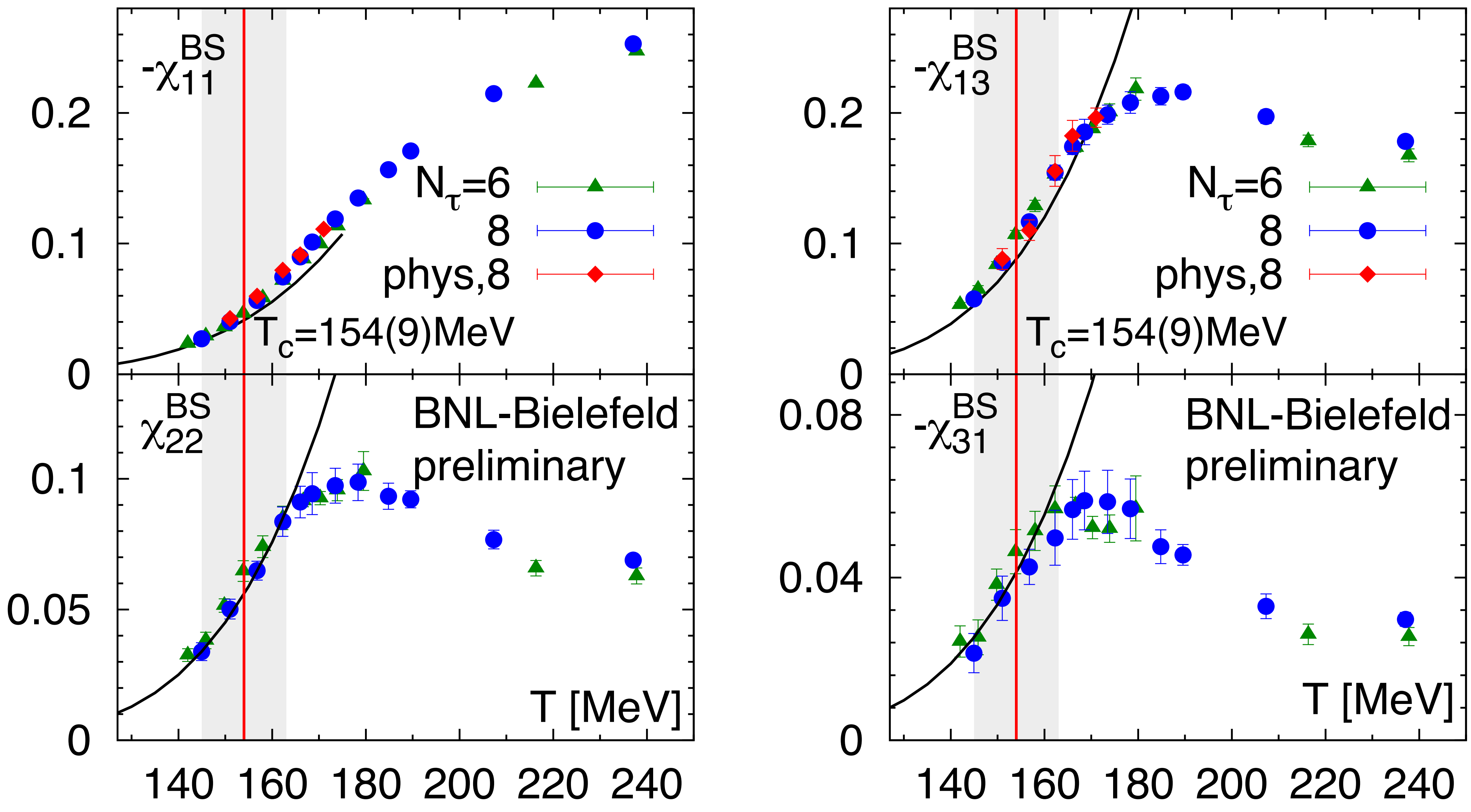
$$p^{\mathrm{HRG}} = \sum_{i\in\mathrm{mesons}} p^M_{m_i}(T,\mu_S) + \sum_{i\in\mathrm{baryons}} p^B_{m_i}(T,\mu_B,\mu_S)$$

hadron masses from PDG up to 2.5 GeV, and

$p^{M/B}_{m_i}$ pressure of free bosonic/fermionic quantum gas

(Lattice) QCD vs HRG

Also apparent in B,S off-diagonal cumulants



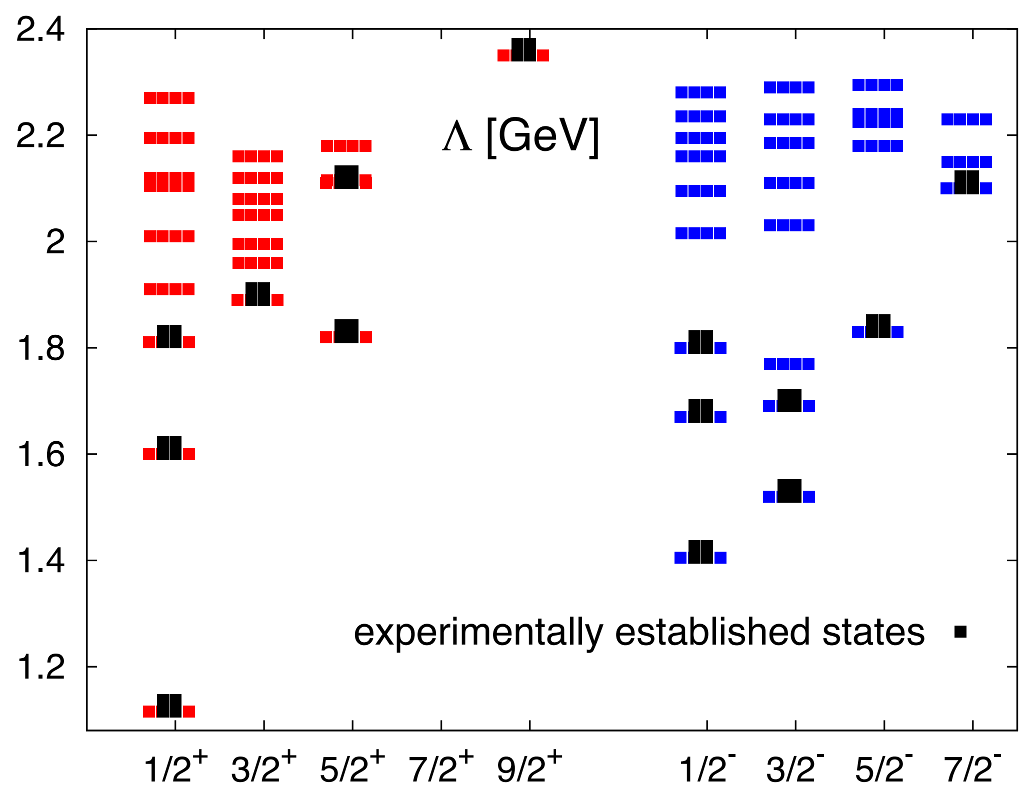
⇒ overshooting of HRG in the crossover region and below

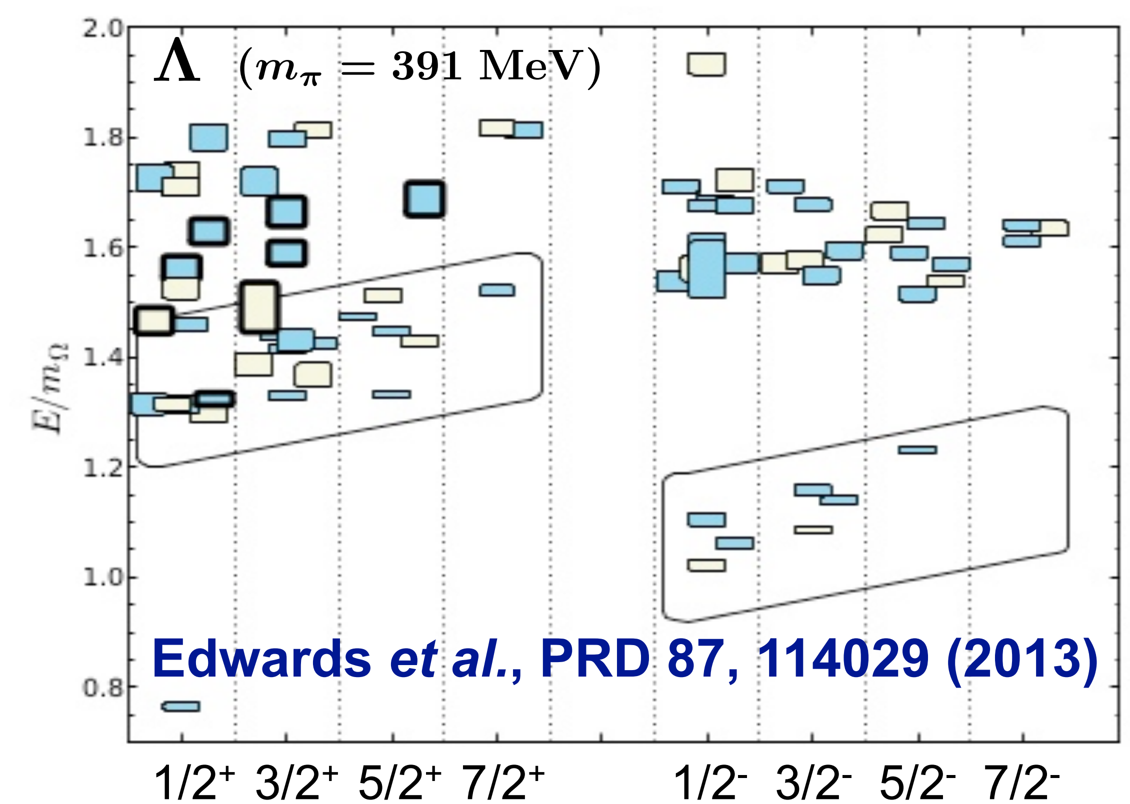
Are differences due to missing states in the PDG?

- Obvious in the charm sector
- How large could be the effect of missing states in the strange sector?

⇒ construct **QM-HRG**, including additional states predicted by Quark-Model

- **Use mesonic states from:** D. Ebert *et al.*, PRD 79, 114029 (2009)
- **Use baryonic states from:** S. Capstick and N. Isgur, PRD 34, 2809 (1986).





- Similar to the spectrum of strange baryons on the lattice

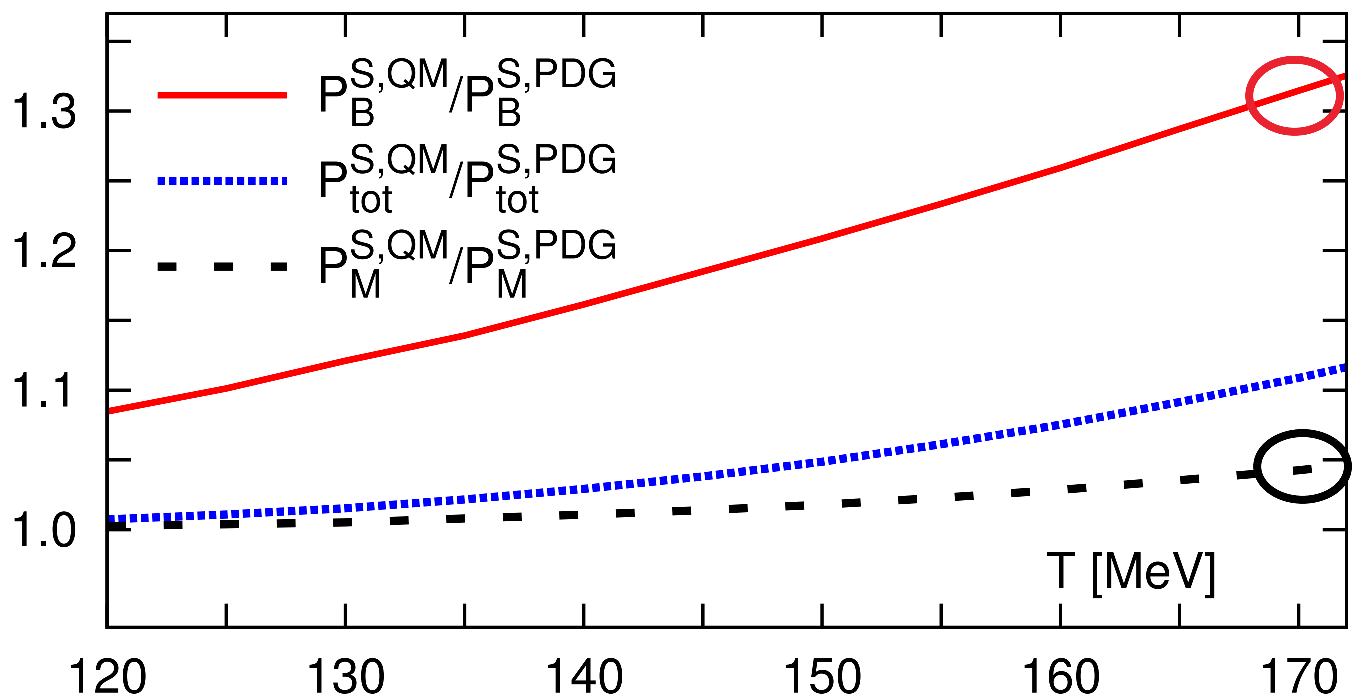
QM-HRG vs PDG-HRG

Partial pressure of strange mesons and baryons:

- Boltzmann approximation is used here and in the following

$$P^{S,X}_{M/B}(T,\vec{\mu}) = \frac{T}{2\pi^2}\sum_{i\in X} g_i \left(\frac{m_i}{T}\right)^2 K_2(m_i/T)\cosh(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S)$$

with $X = \mathrm{PDG}, \mathrm{QM}$ and $\hat{\mu}_q = \mu_q/T,\ q = B, Q, S$



⇒ open strange baryon sector experimentally much less known, additional baryons contribute up to 30% at T=170 MeV

⇒ open strange meson sector experimentally well known

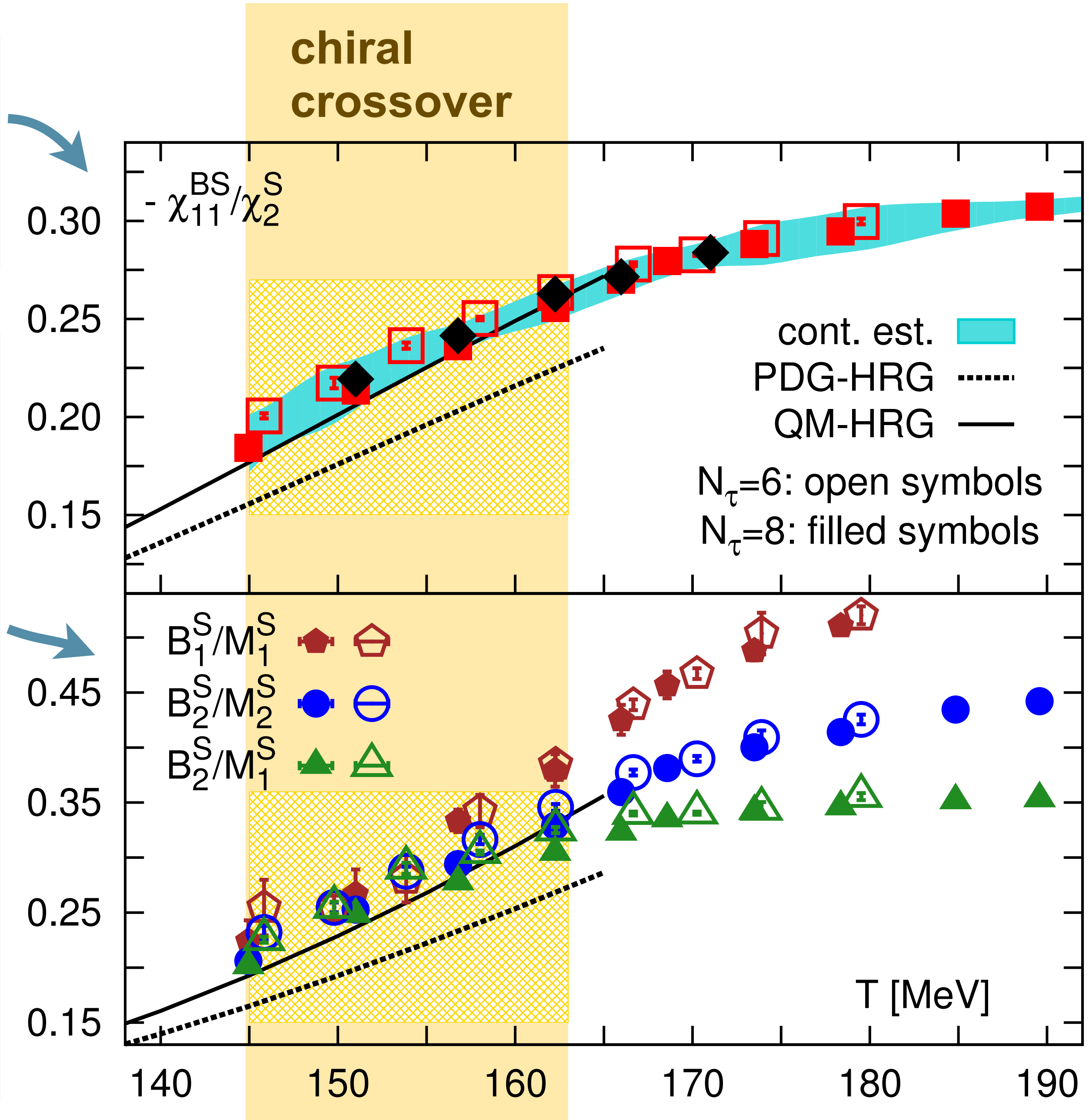
Evidence for more strange hadrons

- **BS-correlation χ_{11}^{BS}**
  at low T: weighted sum of partial pressure of strange baryons

- Different **linear combinations** of
  $$\chi_2^S, \chi_4^S, \chi_{11}^{BS}, \chi_{31}^{BS}, \chi_{22}^{BS}, \chi_{13}^{BS}$$
  are used to project onto partial pressure of strange baryons (B_i^S) and mesons (M_i^S) in the hadronic phase, e.g.

$$B_1^S = -\frac{1}{6}(11\chi_{11}^{BS} + 6\chi_{22}^{BS} + \chi_{13}^{BS})$$

$$B_2^S = \frac{1}{12}(\chi_4^S - \chi_2^S) - \frac{1}{3}(4\chi_{11}^{BS} - \chi_{13}^{BS})$$



⇒ **QM-PDG provides more accurate description of lattice data**

⇒ Re-confirmation of our previous findings [**PRL 111,082301**]: onset of melting of open strange hadrons consistent with chiral crossover

Separation of strangeness sectors

The pressure obtains contributions from 4 different strangeness sectors:

$$\frac{p^{HRG}}{T^4} = f_0(T) + f_M(T)\cosh(-\hat{\mu}_S) + f_{B,1}(T)\cosh(\hat{\mu}_B - \mu_S) + f_{B,2}(T)\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + f_{B,3}(T)\cosh(\hat{\mu}_B - 3\hat{\mu}_S)$$

$$\Rightarrow \left(\frac{p^{HRG}}{T^4}\right)_{S\neq 0} = M^{S=1} + B^{S=1} + B^{S=2} + B^{S=3}$$

$$\chi_2^S = (-1)^2 M^{S=1} + (-1)^2 B^{S=1} + (-2)^2 B^{S=2} + (-3)^2 B^{S=3}$$

$$\chi_{11}^{BS} = (-1) B^{S=1} + (-2) B^{S=2} + (-3) B^{S=3}$$

$$\vdots$$

Separation of strangeness sectors

Idea: separate strangeness sectors by making use of all diagonal strangeness fluctuations and baryon-strangeness correlations up to the 4th order

$$x_1\chi_{11}^{BS} + x_2\chi_{31}^{BS} + x_3\chi_2^{S} + x_4\chi_{22}^{BS} + x_5\chi_{13}^{BS} + x_6\chi_4^{S}$$

solve:

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & 1 & 1 & -1 & 1 \\ -2 & -2 & 4 & 4 & -8 & 16 \\ -3 & -3 & 9 & 9 & -27 & 81 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \begin{matrix} \leftarrow M^{S=1} \\ \leftarrow B^{S=1} \\ \leftarrow B^{S=2} \\ \leftarrow B^{S=3} \end{matrix}$$

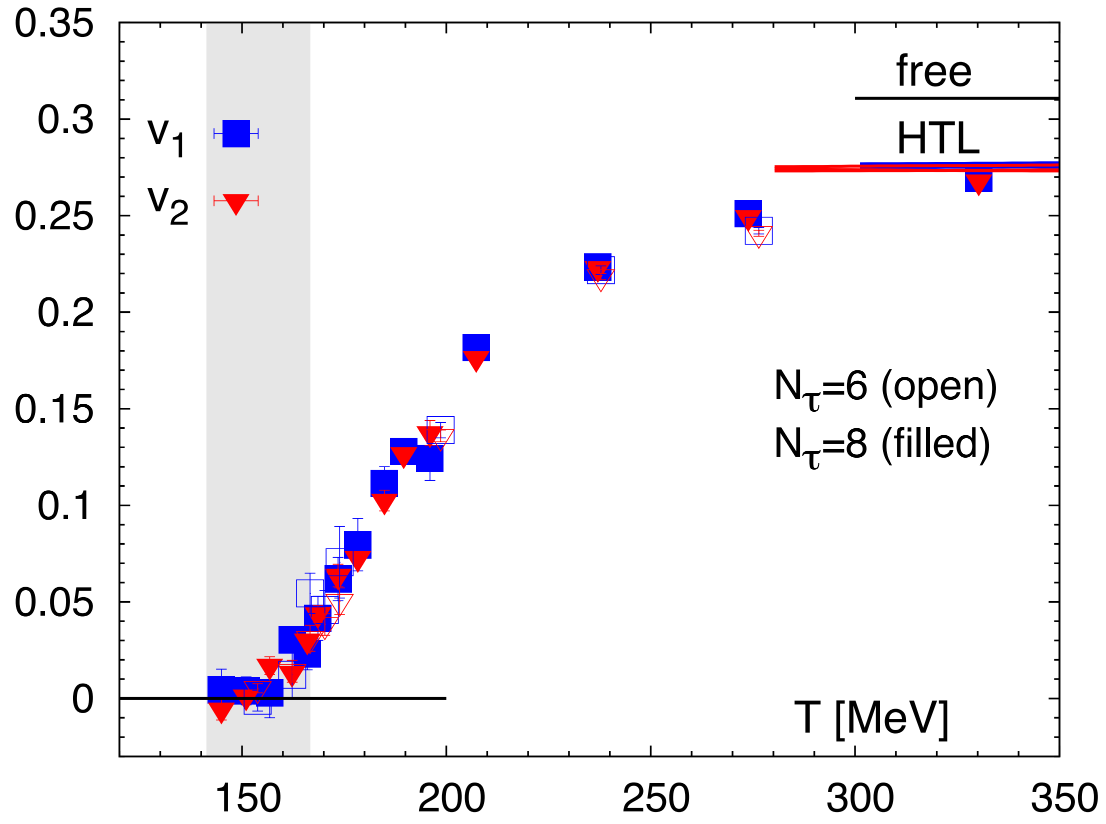
Rank=4

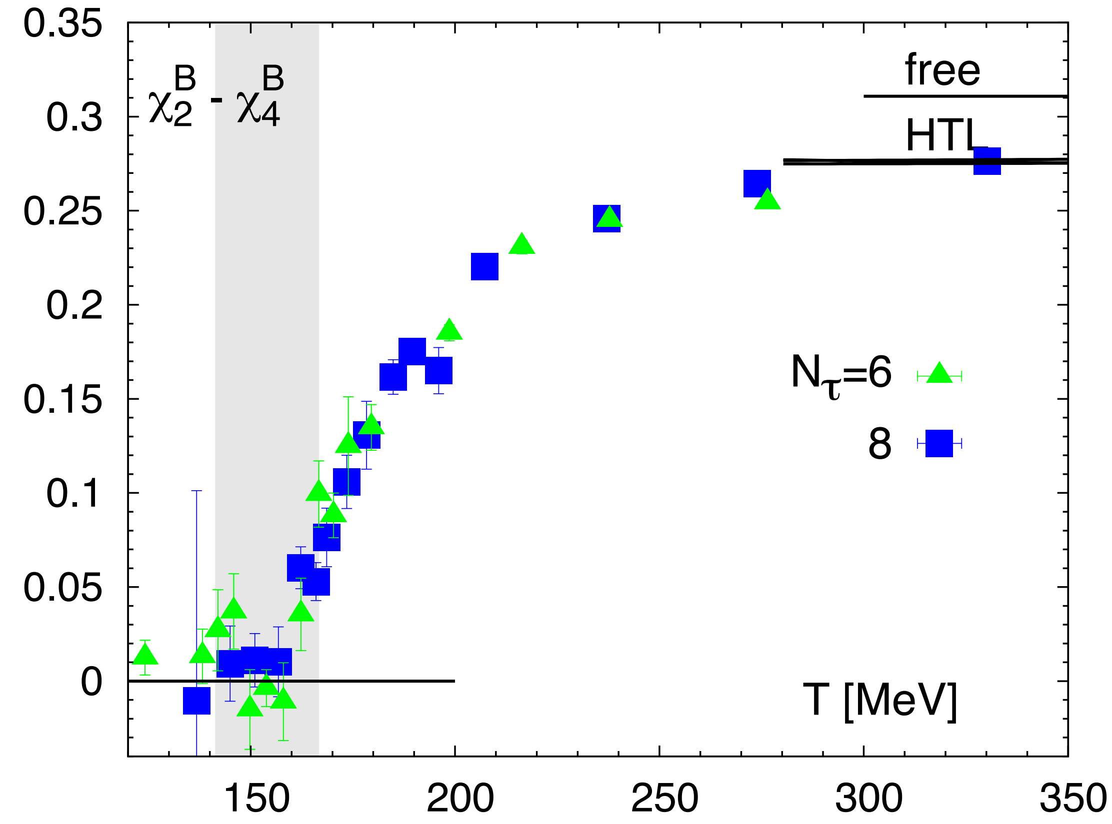
$\Rightarrow$ dim (Kernel)=2, spanned by v_1, v_2

$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

$$v_2 = \frac{1}{3}(\chi_2^S - \chi_4^S) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

Separation of strangeness sectors





- strange baryons carry baryon number 1
- partial pressure from strange particles is hadronic

- all baryons carry baryon number 1

$\Rightarrow$ indicator for the validity of the HRG

- at $T \lesssim 160\ \mathrm{MeV}$ we find reasonable agreement with the HRG (within 20%)
- at $T \gtrsim 160\ \mathrm{MeV}$ deviations from HRG become large

Separation of strangeness sectors

solving 4 inhomogenous systems

$\Rightarrow$ the solutions are translations of the kernel

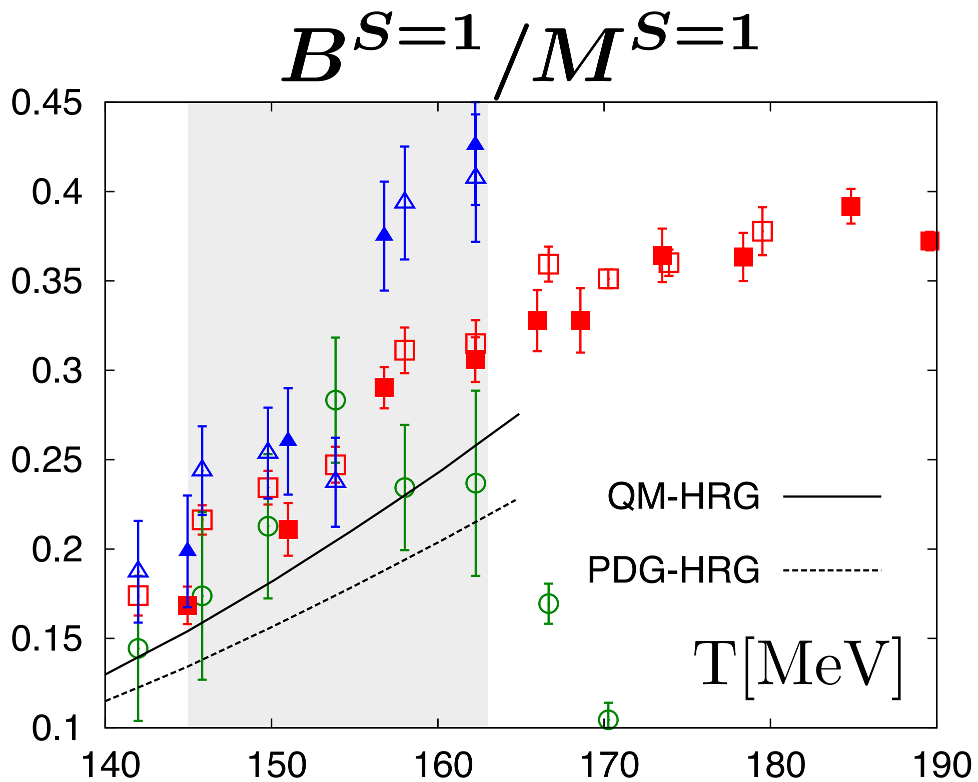
$$M^{S=1}(c_1,c_2) = \chi_2^S - \chi_{22}^{BS} + c_1 v_1 + c_2 v_2 \quad (1)$$

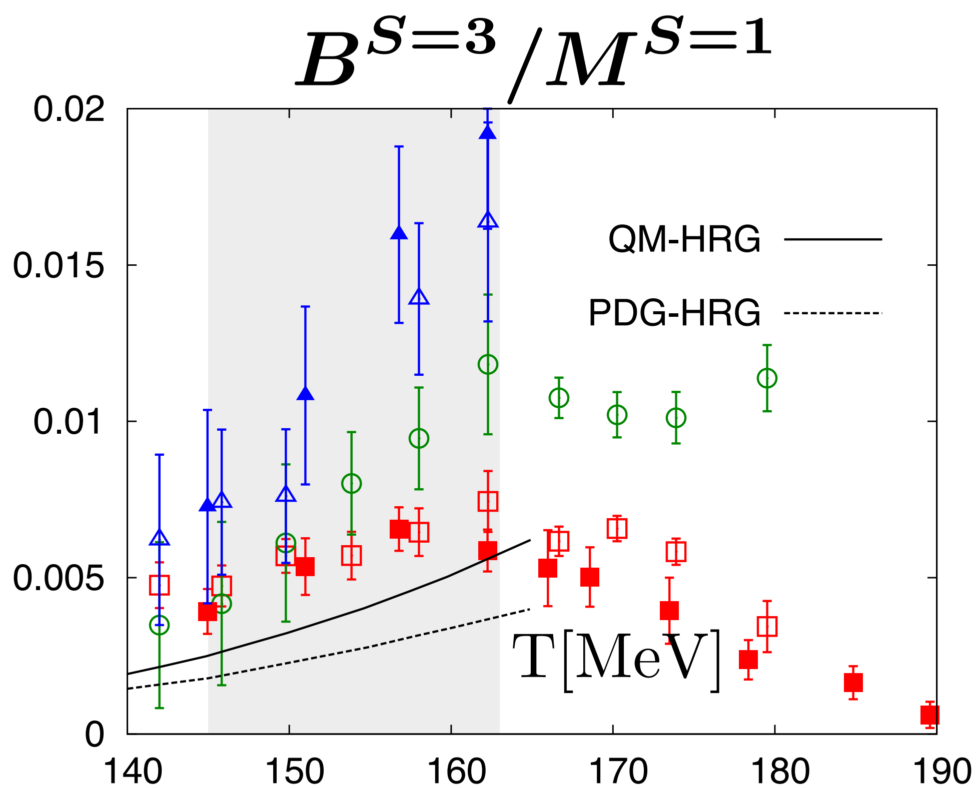
$$B^{S=1}(c_1,c_2) = \frac{1}{2}\left(\chi_4^S - \chi_2^S + 5\chi_{13}^{BS} + 7\chi_{22}^{BS}\right) + c_1 v_1 + c_2 v_2 \quad (2)$$

$$B^{S=2}(c_1,c_2) = -\frac{1}{4}\left(\chi_4^S - \chi_2^S + 4\chi_{13}^{BS} + 4\chi_{22}^{BS}\right) + c_1 v_1 + c_2 v_2 \quad (3)$$

$$B^{S=3}(c_1,c_2) = \frac{1}{18}\left(\chi_4^S - \chi_2^S + 3\chi_{13}^{BS} + 3\chi_{22}^{BS}\right) + c_1 v_1 + c_2 v_2 \quad (4)$$

Separation of strangeness sectors





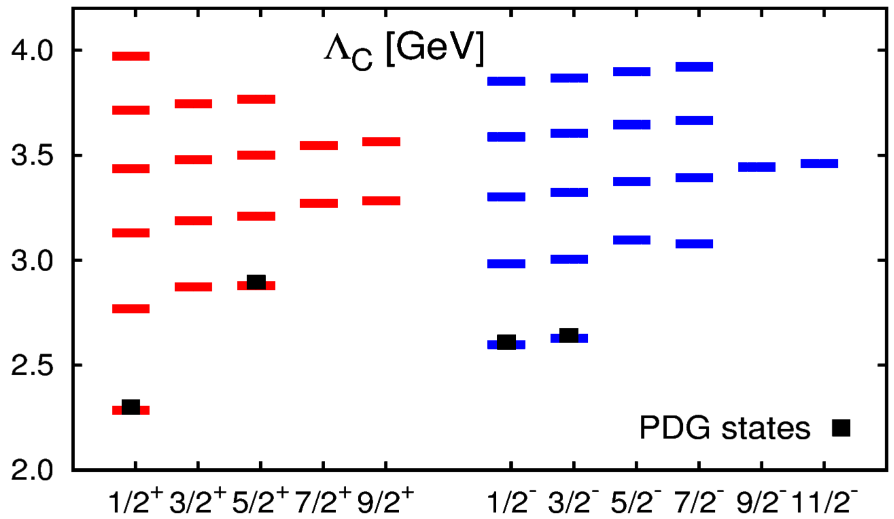
78% of the partial pressure of strange baryons are from $B^{S=1}$

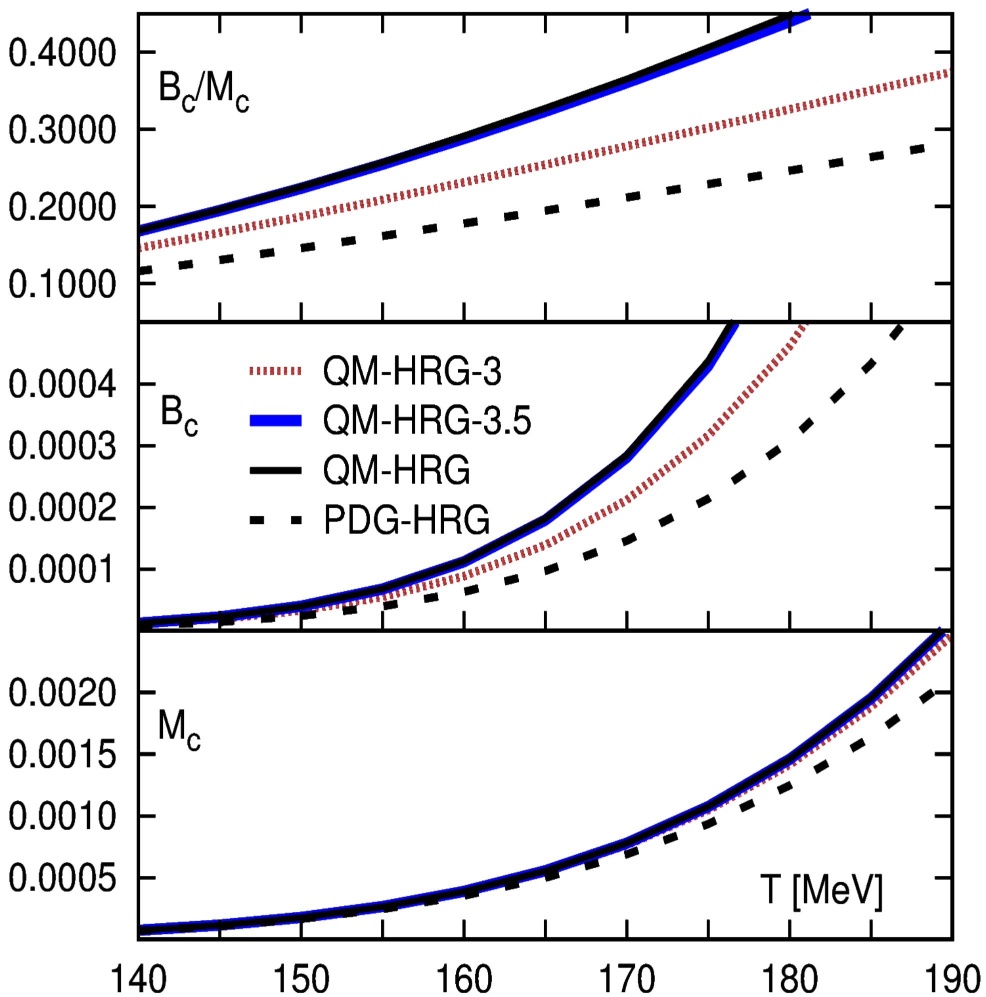
20% of the partial pressure of strange baryons are from $B^{S=2}$

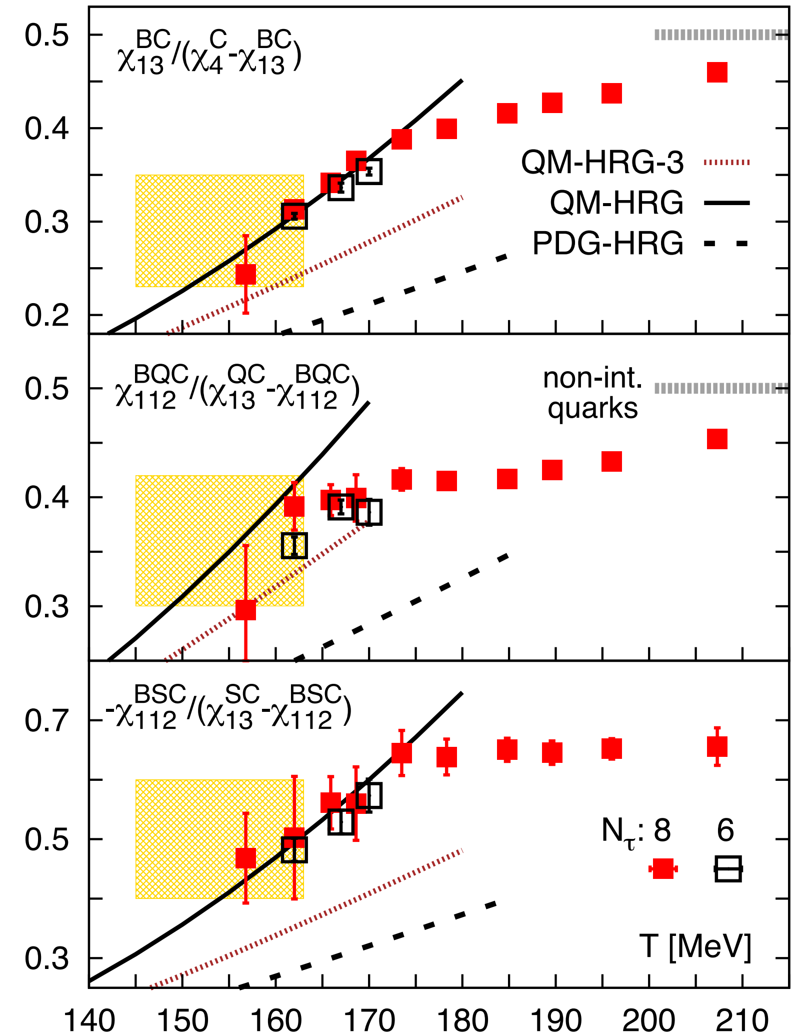
2% of the partial pressure of strange baryons are from $B^{S=3}$

melting of all open strange baryons start at the chiral crossover

QM-HRG for the charm sector





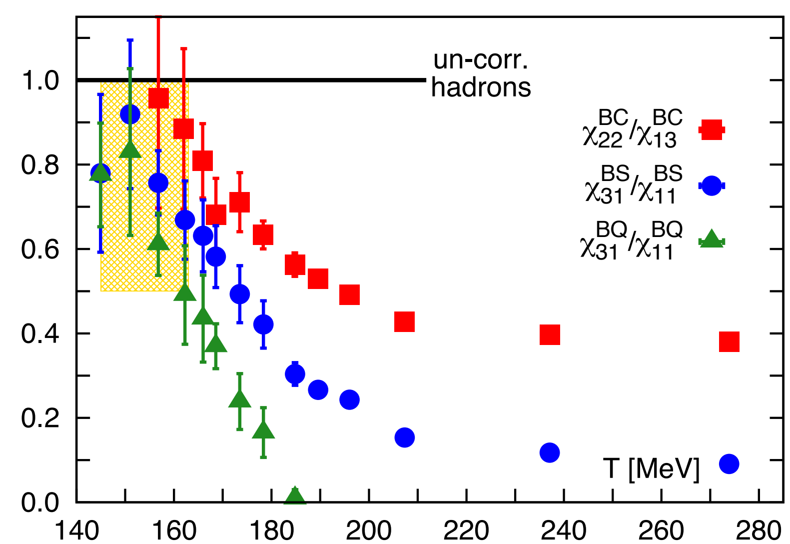


charmed baryons to charmed mesons

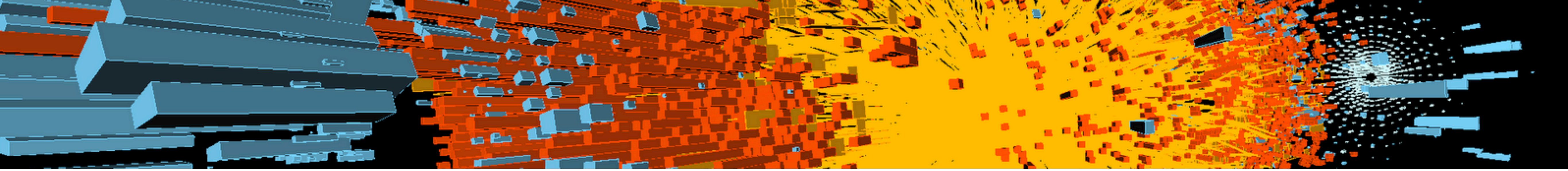
charged charm baryons to charmed mesons

strange charm baryons to charmed mesons

⇒ evidence for more charmed hadrons



⇒ melting of open charm hadrons start at chiral crossover



coming back to ...

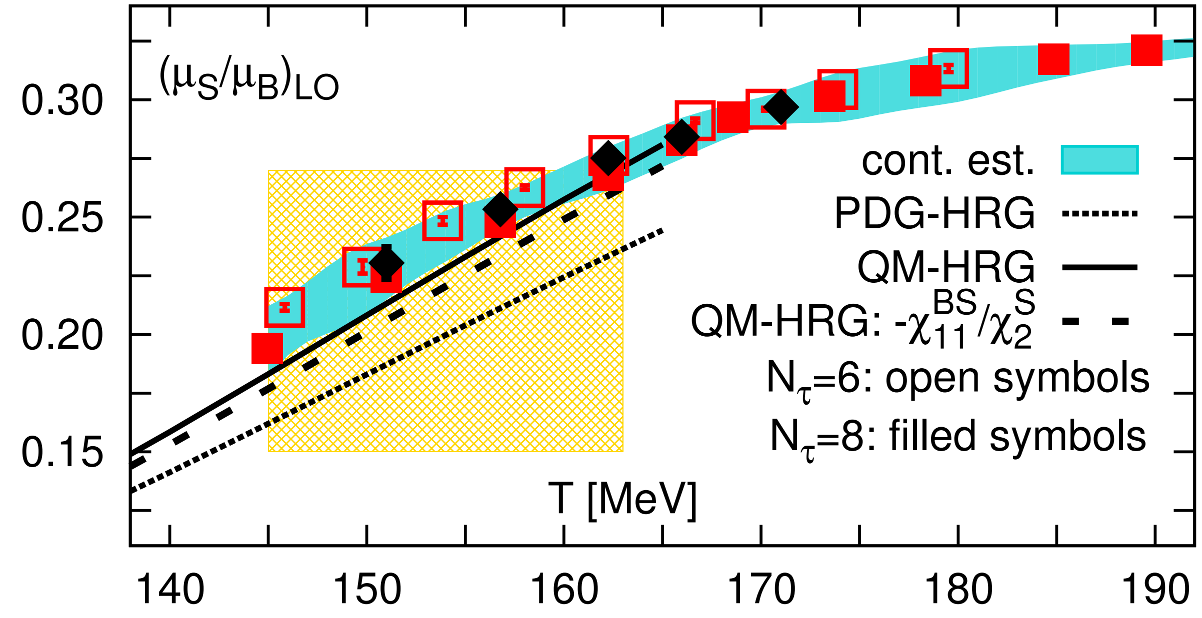
Freeze-out conditions in HIC

Now determined from relative yields of open strange hadrons.

Implications for freeze-out conditions

- in order to make contact to experiment: apply strangeness neutrality constraint $n_S = 0$ (and iso-spin $n_Q/n_B = 0.4$)

$\Rightarrow T, \mu_B, \mu_S, \mu_Q$ not independent



(Lattice) QCD: unique expansion

$$(\mu_S/\mu_B)(T) = s_1(T) + s_3(T)\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$\Rightarrow \left(\frac{\mu_S}{\mu_B}\right)_{\mathrm{LO}} \equiv s_1(T) = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S}\frac{\mu_Q}{\mu_B}$$

- small corrections from $\mu_Q > 0$
- neglect NLO contribution, known to be small for $\mu_B \lesssim 200 MeV$ [PRL 111,082301]

HRG: $\mu_S(T, \mu_B)$ depends on relative abundance of open strange baryons and mesons

additional strange baryons
$\Rightarrow$ larger μ_S
(at fixed T, μ_B)

Implications for freeze-out conditions

Experiment:

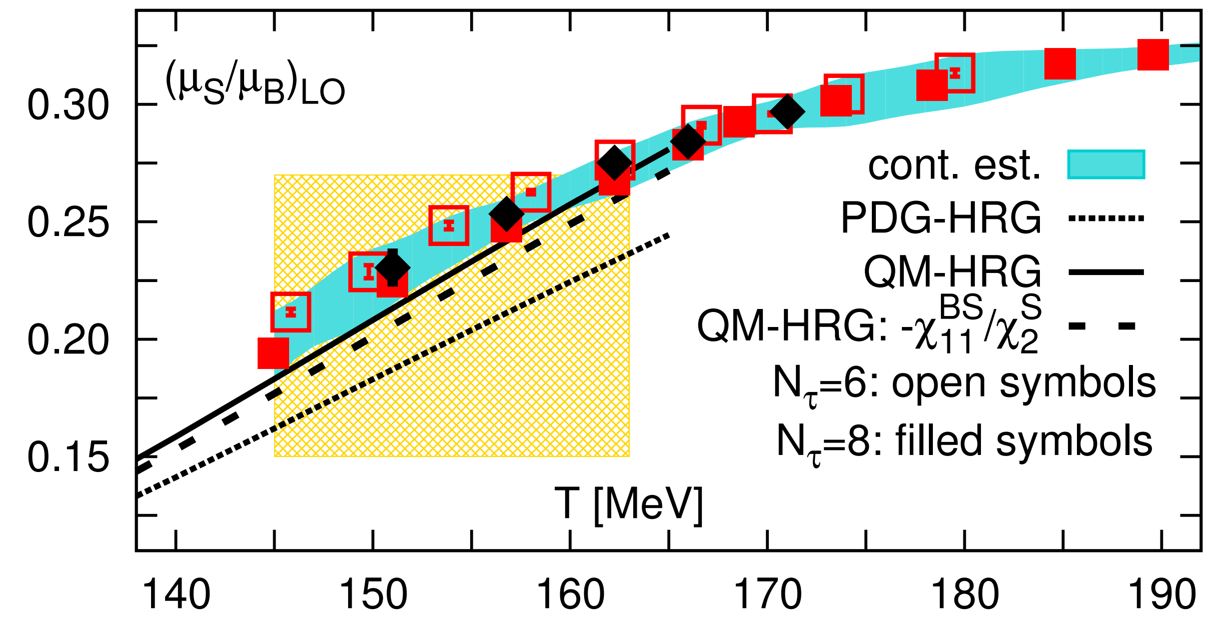
relative yields of strange anti-baryons to baryons at freeze-out are controlled by freeze-out parameter (T^f, μ_B^f, μ_S^f)

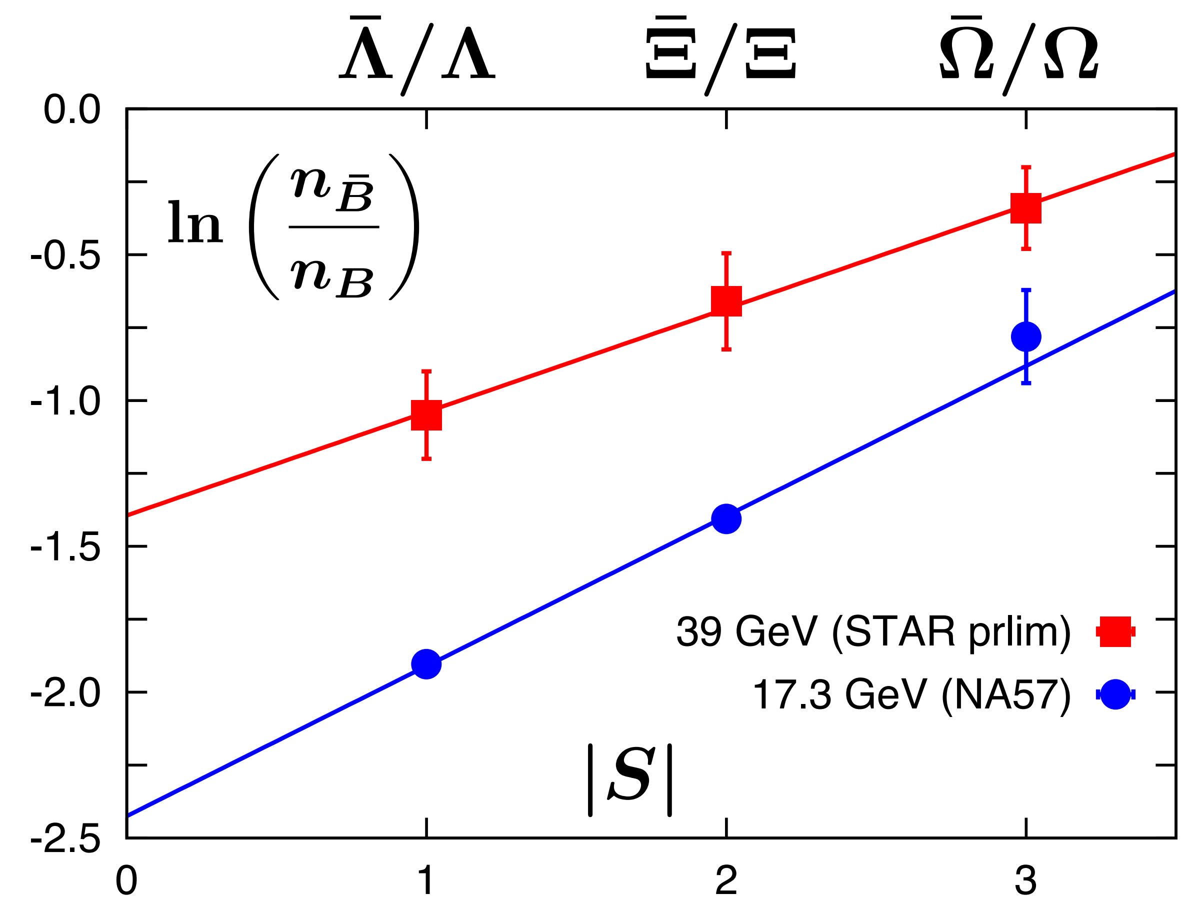
$$\frac{n_{\bar{B}}}{n_B} = \exp\left\{-\frac{2\mu_B^f}{T^f} + \frac{2\mu_S^f}{T^f}|S|\right\}$$

$$= \exp\left\{-\frac{2\mu_B^f}{T^f}\left(1 - \frac{\mu_S^f}{\mu_B^f}|S|\right)\right\}$$

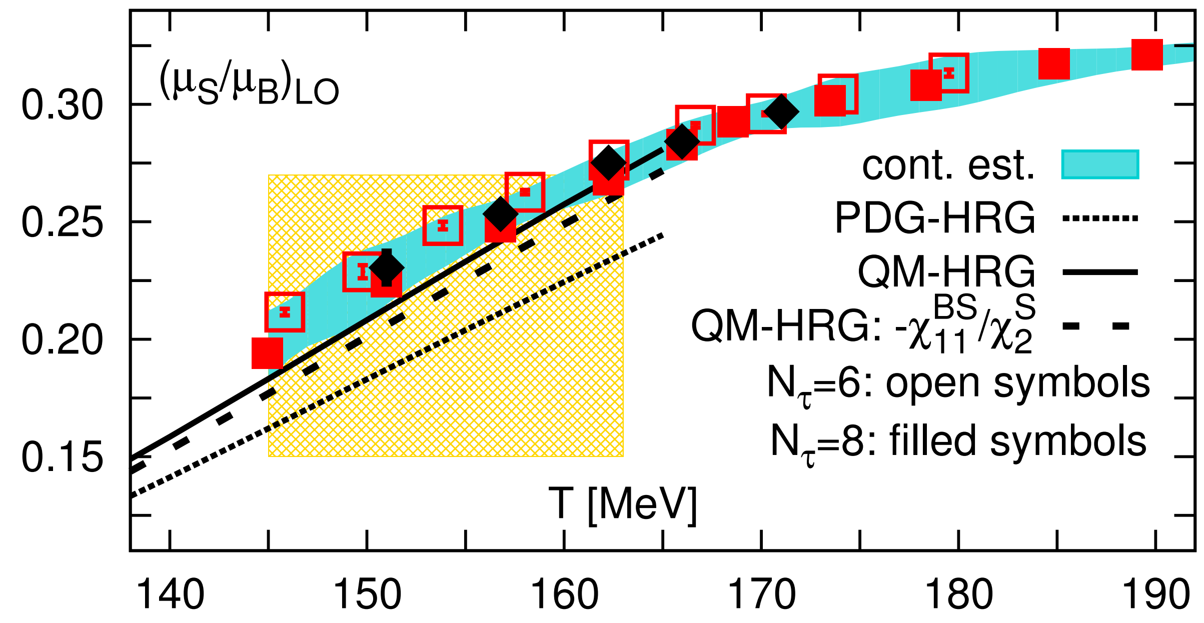
⇒ μ_B^f/T^f and μ_S^f/μ_B^f can be obtained by fitting experimentally measured values of $\bar{\Lambda}/\Lambda$, $\bar{\Xi}/\Xi$ and $\bar{\Omega}/\Omega$

⇒ fit function independent of details of open strange hadron spectrum



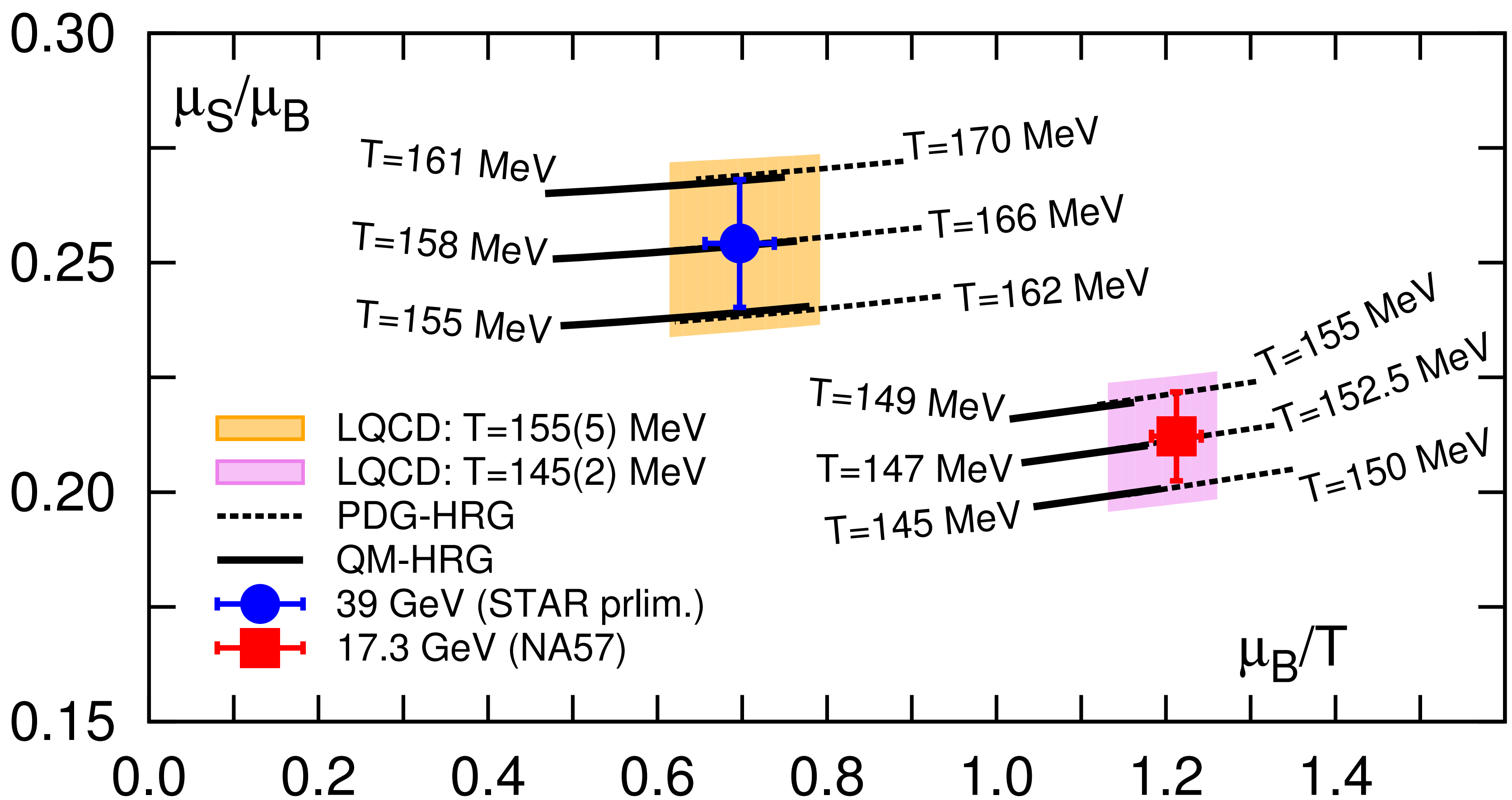


Implications for freeze-out conditions



Comparison:

vary T^f in order to mach μ_B^f/T^f and μ_S^f/μ_B^f



⇒ QM-HRG in agreement with (lattice) QCD

⇒ QM-HRG yields 5-8 MeV smaller freeze-out temperature than PDG-HRG

Summary and Conclusions

- ratios of fluctuations of conserved charges can be used to determine the freeze-out parameter in HIC

- partial pressure of strange baryons from QM-HRG and PDG-HRG differ by about 30% at T=170 MeV (even more for charmed baryons)

  $\Rightarrow$ additional strange/charm baryons are thermodynamically relevant in the crossover region

- in the crossover region the QM-HRG provides a more accurate description of (lattice) QCD w.r.t. the conventionally used PDG-HRG

  $\Rightarrow$ evidence for additional, experimentally not yet observed, strange/charm baryons

- presence of additional strange hadrons get imprinted in the yields of ground state strange hadrons

  $\Rightarrow$ significant reductions of freeze-out temperature of (5-8) MeV if determined from relative yields of strange hadron (μ_S/μ_B)