QCD thermodynamics from the lattice

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Perspectives and Challenges in Lattice Gauge Theory,

TIFR, Mumbai, February 16-20.

Outline

- Focus on the “perspectives” part i.e. what lattice QCD has told us so far about the QCD phase transition and the quark-gluon plasma.

- Will try to cover the following:-

    1. The QCD phase diagram at $mu_B=0$.
    2. The question of $U(1)_A$ restoration.
    3. Equation of state at $mu_B = 0$.
    4. Fluctuations and their applications.
    5. Equation of state at $mu_B > 0$.

The phase diagram at a glance

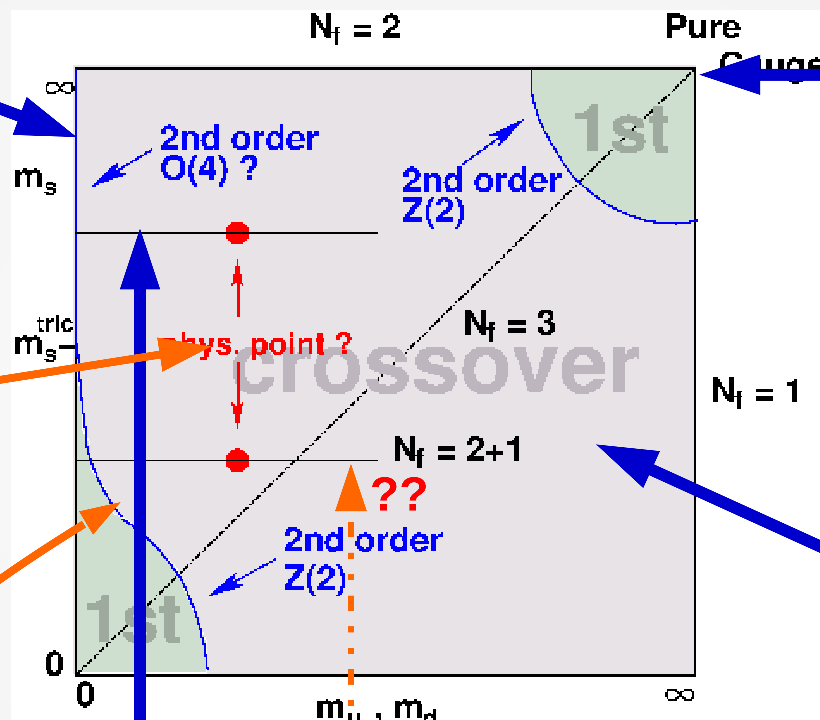
Ding *et al.* PoS LATTICE (2013)

Ejiri *et al.* PRD (2009)

T_c = 154(9) MeV

A.Bazavov *et al.* PRD '12

Y.Aoki *et al.* JHEP '06, '09



G.Boyd, J.Engels, F.Karsch *et al.* Nucl.Phys.B, '96

Bernard *et al.* PRD (2006)

Cheng *et al.* PRD (2006)

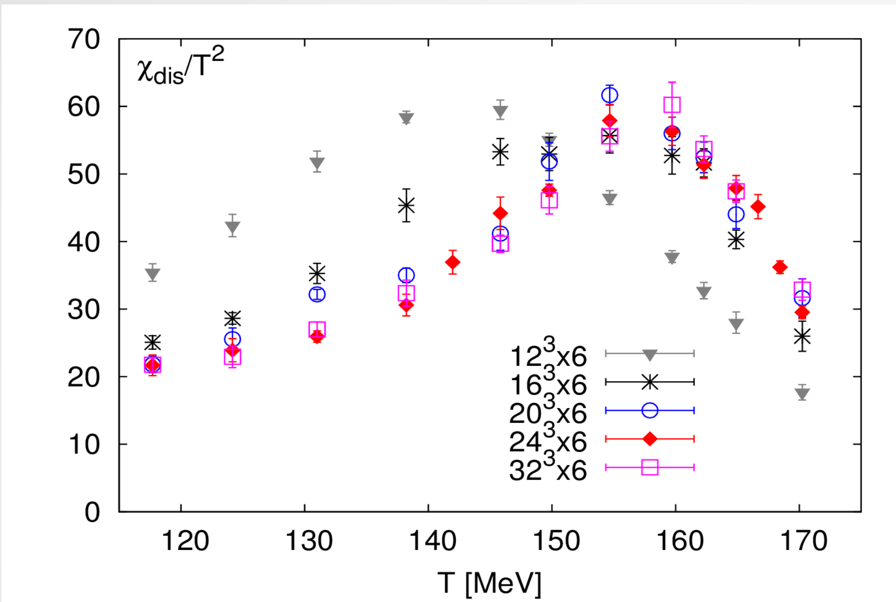
Y.Aoki *et al.* Nature (2006)

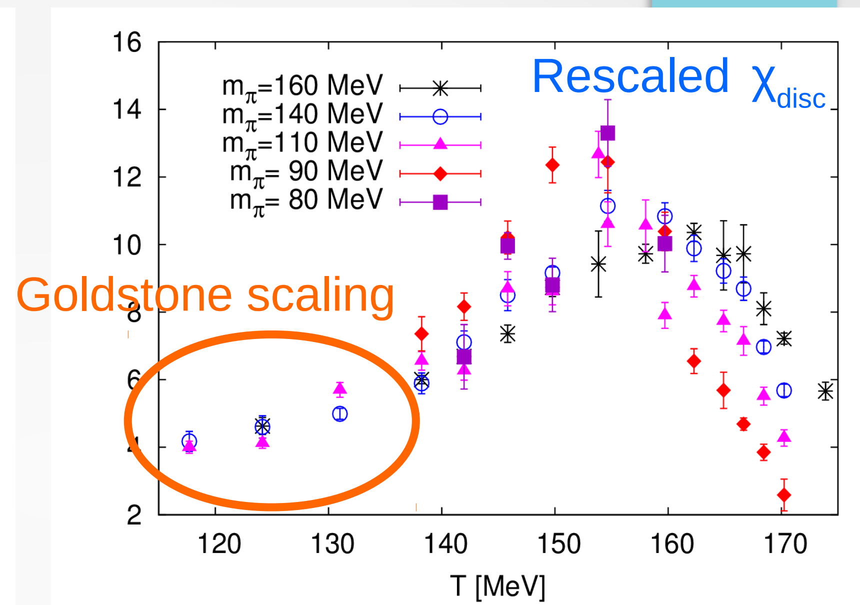
HISQ, m_E< 80 MeV

Ding *et al.* PoSLattice 2013

Clover-imp. Wilson [X.-Y.Jin et al. PRD (2014)]

The two-flavor chiral limit



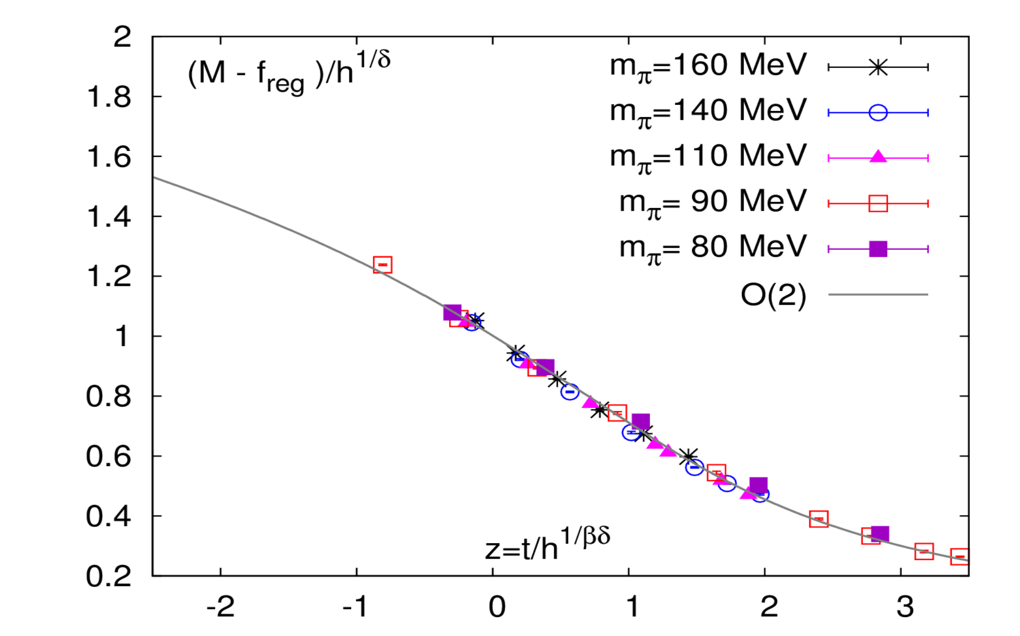


H.-T.Ding, Lattice 2013

N_τ=6 simulations with the HISQ action for various volumes and light quark masses, keeping the strange quark mass fixed at its physical value.

The rescaled susceptibility, $(m_l/m_s)^{1/2}\chi_{disc}$, is independent of the quark mass, indicating that Goldstone modes contribute [P.Hasenfratz and H.Leutwyler, Nucl.Phys.B (1990)].

The two-flavor chiral limit



H.-T.Ding, Lattice 2013

Transition expected to belong to the 3-dim. O(4) universality class in the chiral limit.

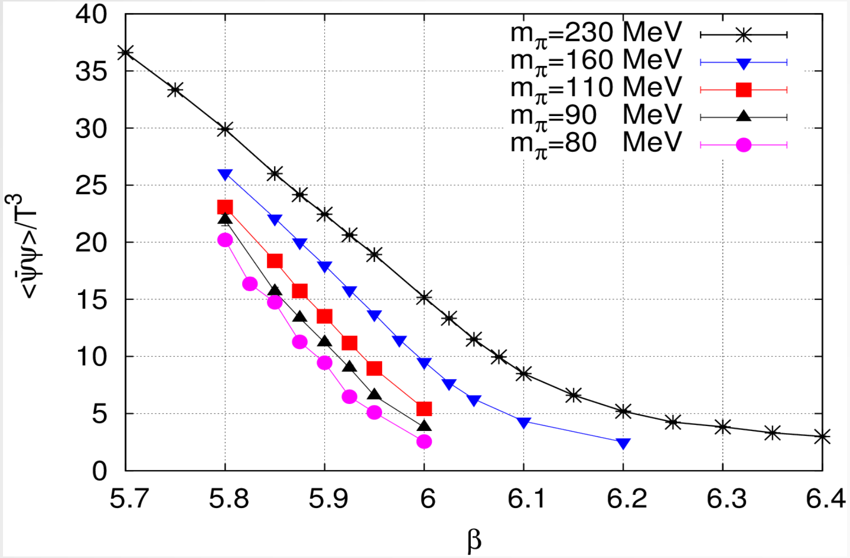
Observe scaling with O(n) critical exponents (n=2 for staggered fermions).

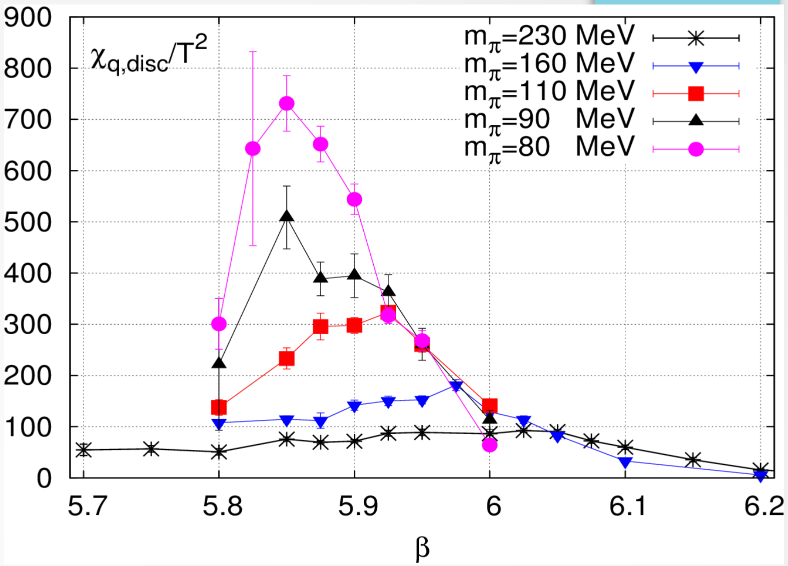
$$z = t/h^{1/\beta\delta}$$

$$M = h^{1/\delta} f_G(z) + f_{reg}, \quad f_{reg} = \left(a_0 + a_1 \frac{T - T_c}{T_c}\right) \frac{m_l}{m_s}, \qquad t = \frac{1}{t_0} \frac{T - T_c}{T_c}, \qquad h = \frac{H}{h_0}.$$

D. Toussaint, PRD '97; J. Engels et al. ,'00, '01, '03.

The three-flavor transition



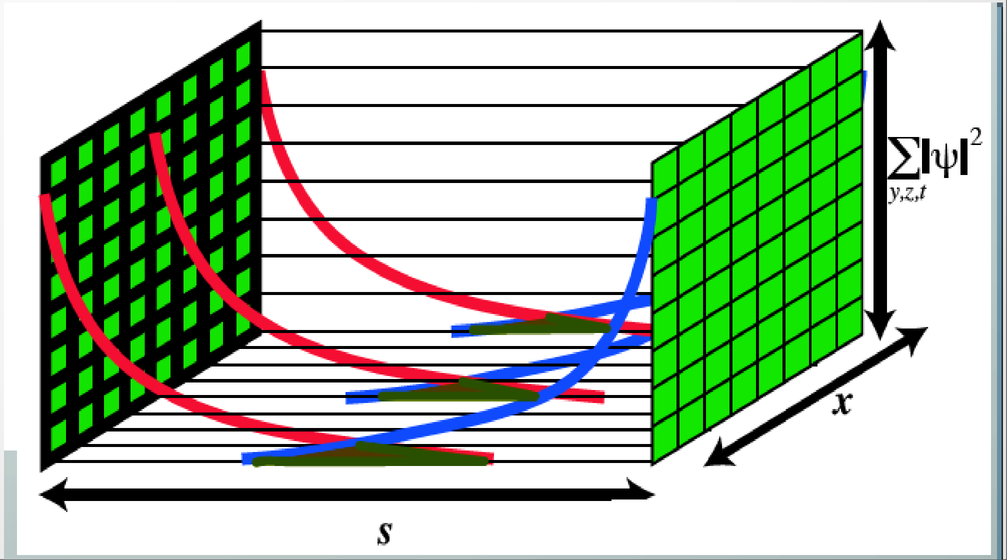


H.-T.Ding *et al.* PoS LATTICE2013, 157 (2014).

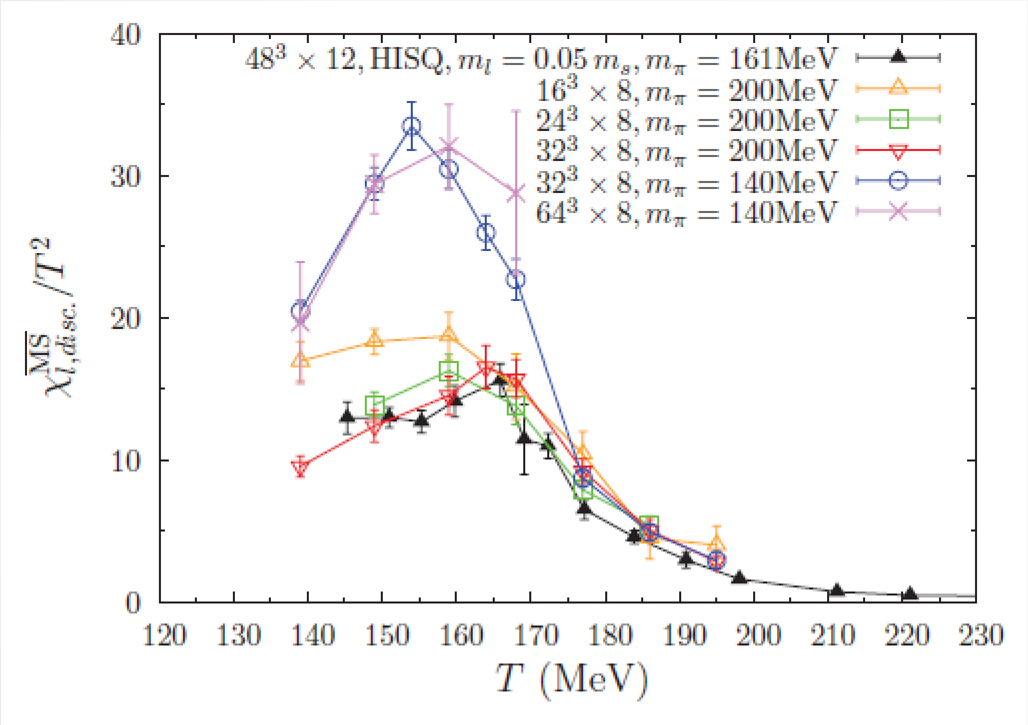
- 2nd-order transition belonging to the Z(2) universality class for $m_\pi = m_\pi^{(crit.)}$.
- No discontinuity observed in the chiral condensate down to m_π=80 MeV. No volume scaling of χ_{disc} either.
  - Similar results also obtained by the BMW collaboration [G.Endrodi *et al.* PoS Lattice 182 (2007).
  - However, a similar calculation done using the Wilson action yields a different result [X.-Y.Jin *et al.* Phys. Rev. **D91** 014508 (2014)].

$U(1)_A$ symmetry restoration

- The use of a chiral action allows one to study questions related to topology.
- $U(1)_A$ restoration can alter the symmetry group, and hence the order, of the phase transition [Pisarski & Wilczek, Phys.Rev. D29, 338 (1984)]
- Domain-wall fermions: Five-dimensional fermions with a four-dimensional, chiral, low-energy spectrum.
- Gauge fields remain four-dimensional.
- When the fifth dimension is finite, residual chiral symmetry breaking m_{res}.



Crossover transition temperature

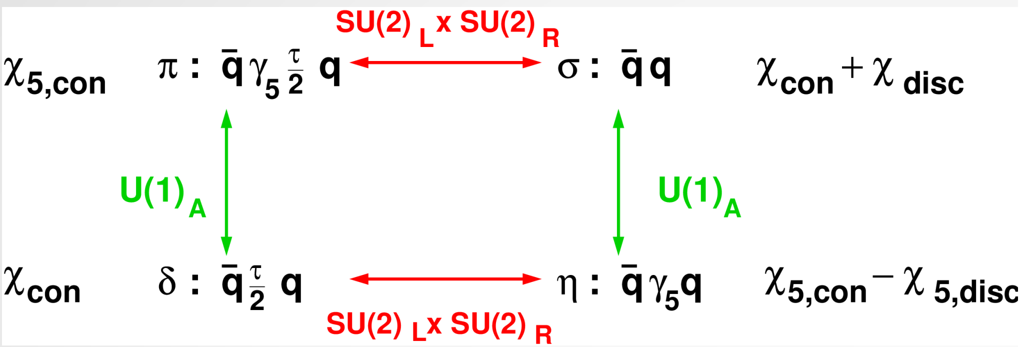


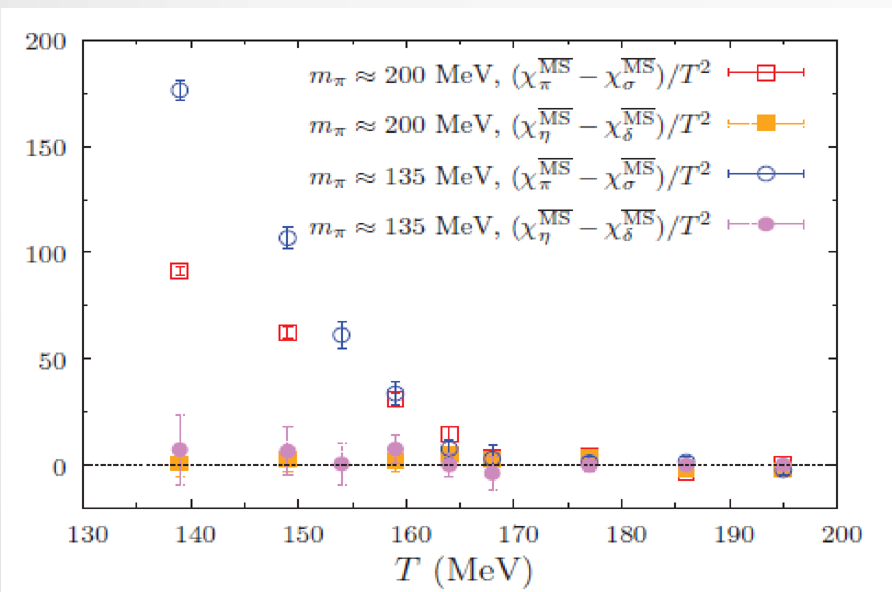
RBC/LLNL, PRL 113, 8, 082001 (2014).

Disconnected chiral susceptibility, χ_{disc}, peaks around T=155 MeV for physical quark masses, in agreement with the staggered (HISQ) results.

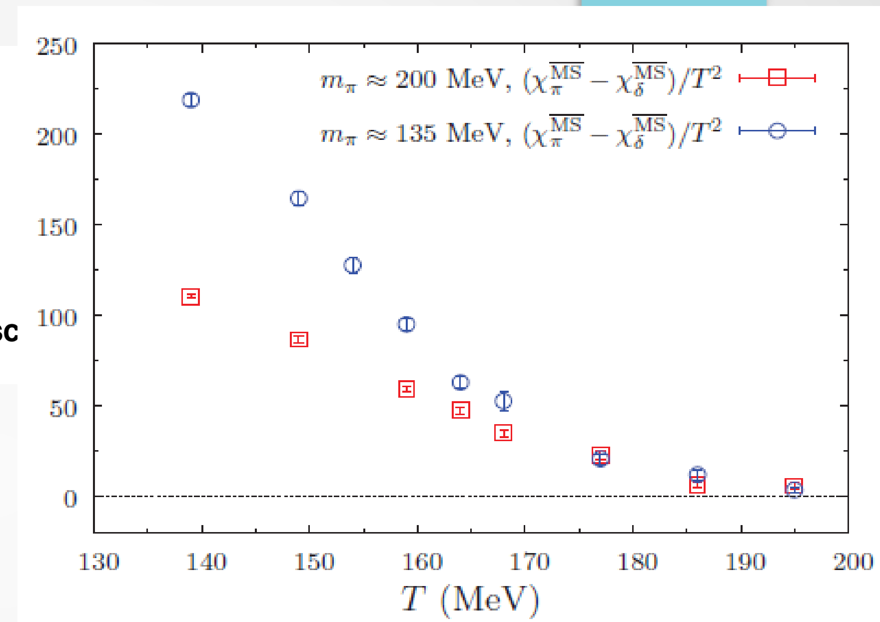
- Full flavor symmetry group $SU(3)_V$ x $SU(3)_A$.
- No doublers, different hadronic tastes, taste-splitting, etc.
- Satisfies Ward identities similar to the continuum identities.
- Satisfies an index theorem.

$U(1)_A$ versus $SU(2)_V$ x $SU(2)_A$





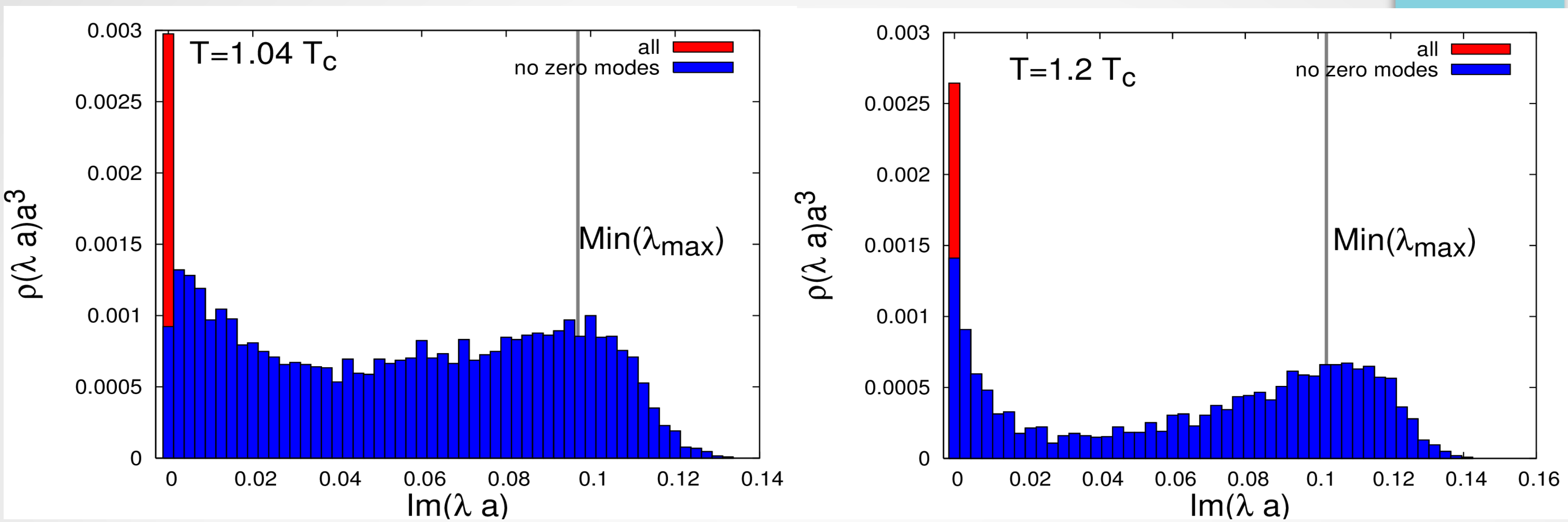
χSB difference zero for $T_c \approx 165$ MeV and above.



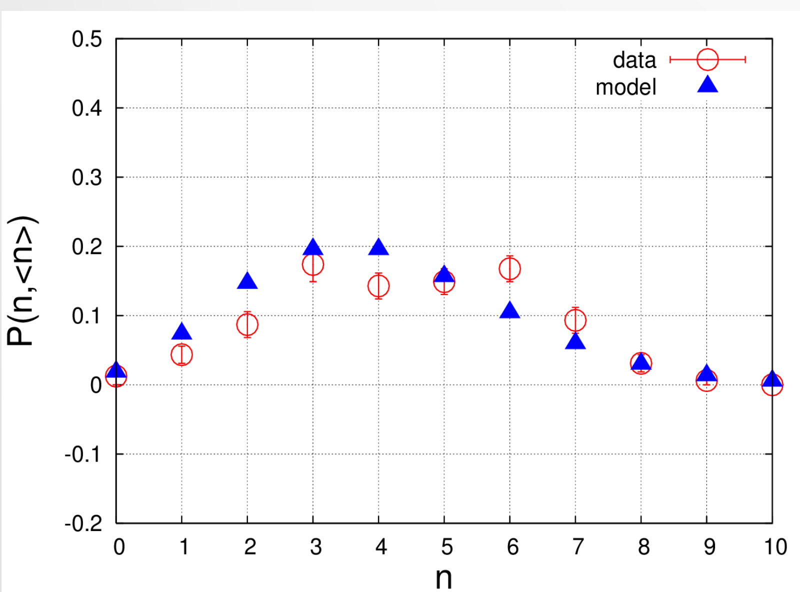
RBC/LLNL, PRL 113, 8, 082001 (2014).

$U(1)_A$-breaking difference many standard deviations away from zero even above T_c.

Studying $U_A(1)$ through Dirac eigenvalues



S.Sharma & V.Dick, LATTICE 2013



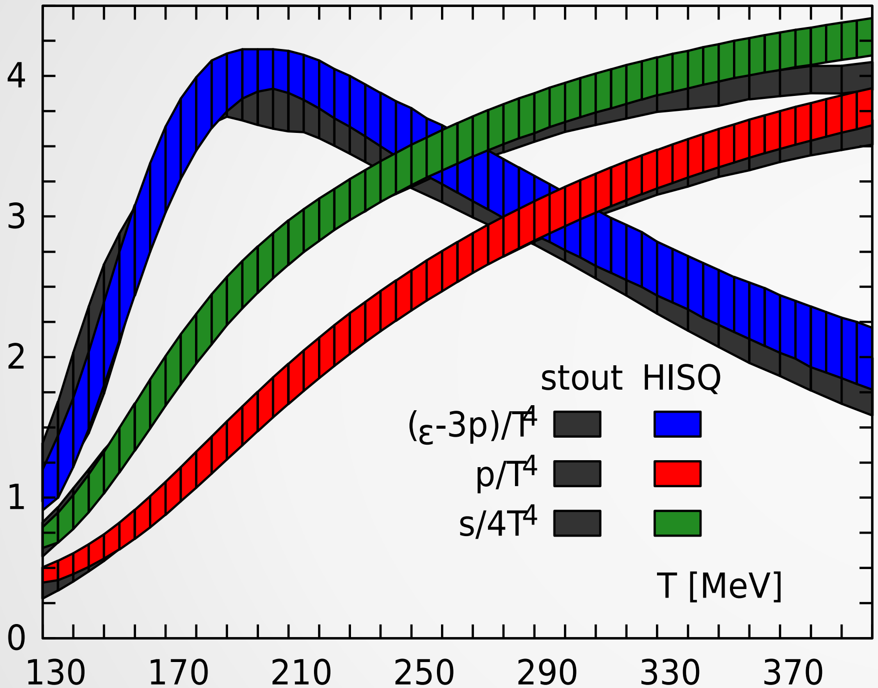
- Density of zero modes is non-zero even above T_c.
  - Similar results also obtained by H.Ohno *et al.*, LATTICE 2013.
- Distribution of the topological charge Poissonian, resembles a dilute gas of instanton—anti-instanton pairs.

The QCD equation of state

- Fundamental property of QCD, related to the question of the degrees of freedom at a given temperature and density.
- Basic input in modelling the hydrodynamic evolution of the fireball created in heavy-ion collisions.
- Resummed perturbation theory does not work for $T <\approx 2T_c$. Need a non-perturbative way of calculating such as lattice QCD, for example.

Equation of state: What we know

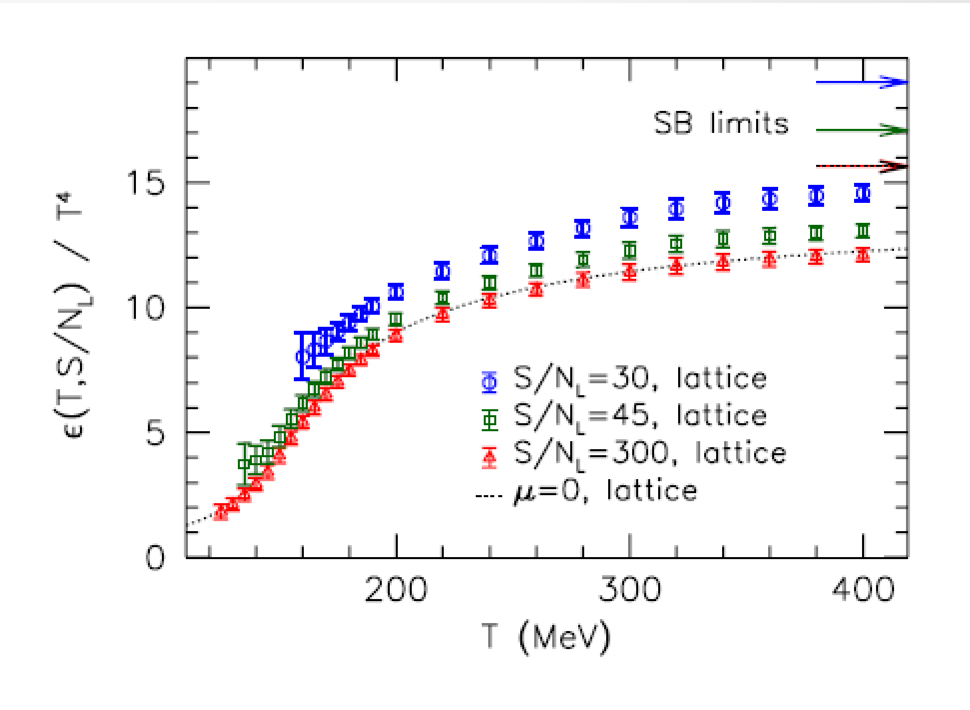
0^{th}-order equation of state



A. Bazavov et al. [HotQCD collaboration] Phys. Rev. D90, 054903 (2014).

S. Borsanyi et al. [BMW collaboration], JHEP 1011, 077 (2010).

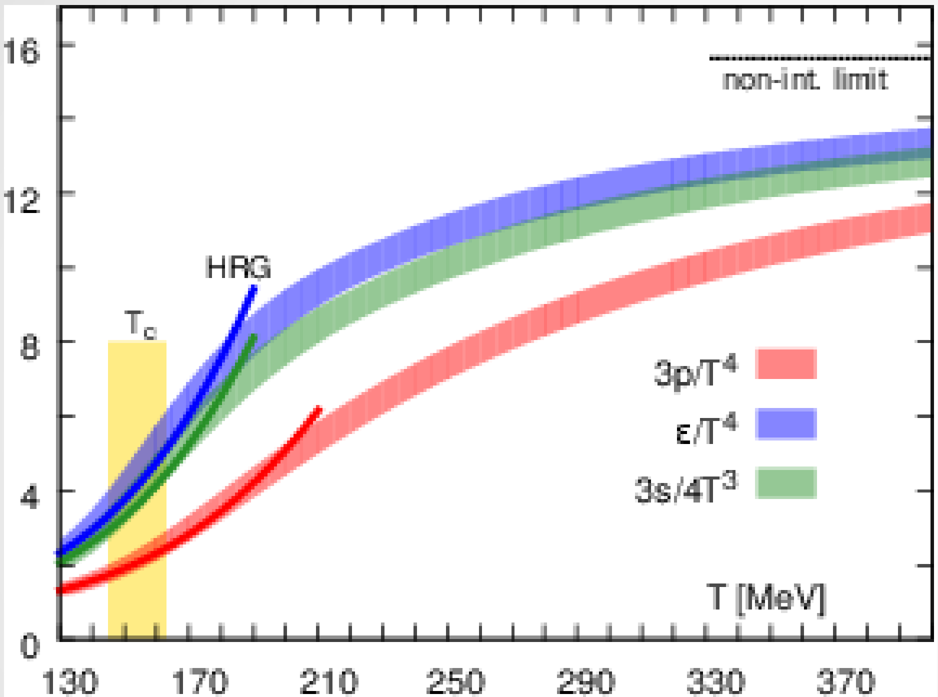
2^{nd}-order equation of state

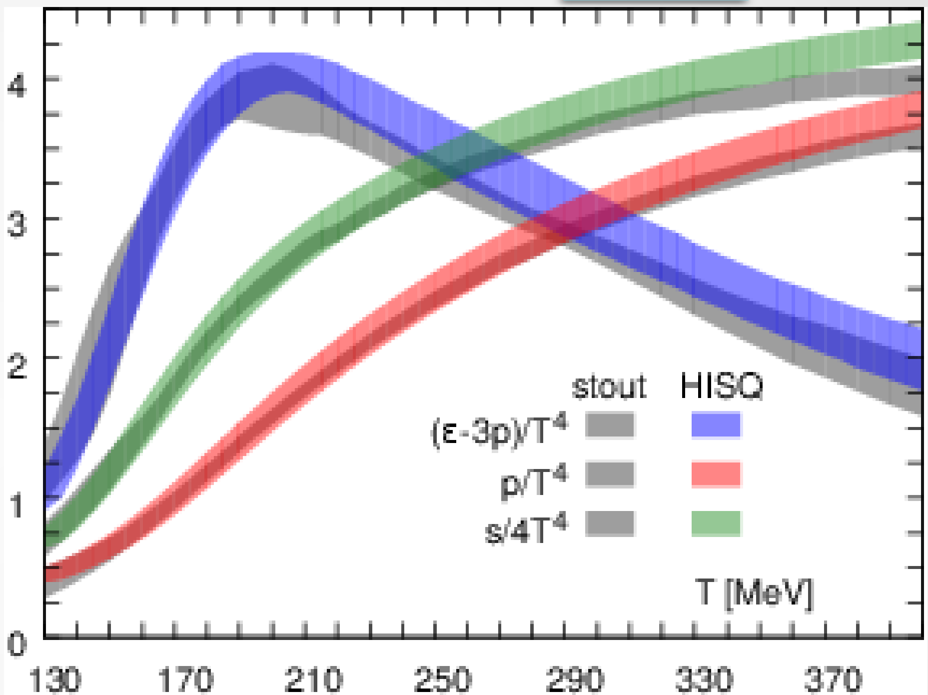


S. Borsanyi et al. [BMW collaboration], JHEP 1208, 053 (2012).

Here we will present results for a 4^{th}-order equation of state.

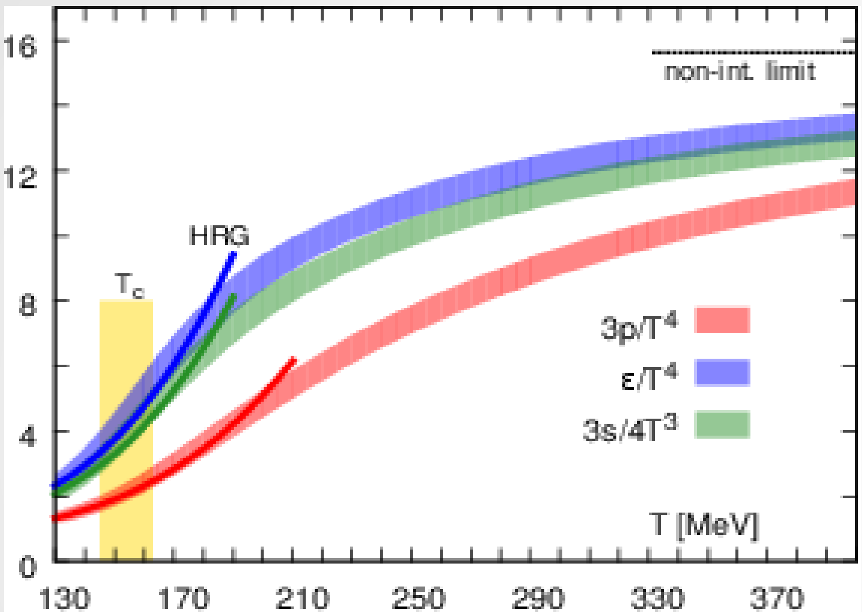
HotQCD continuum equation of state

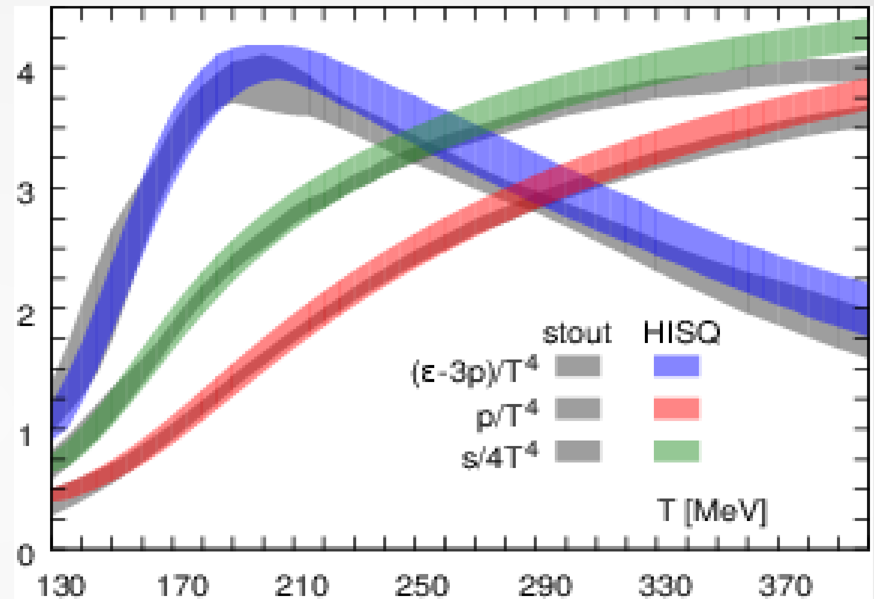




- 2% error from scale setting; taken into account in the above results.
- Slight remnant discrepancy at high temperatures; stout results ~7% less than ours.

HotQCD continuum equation of state

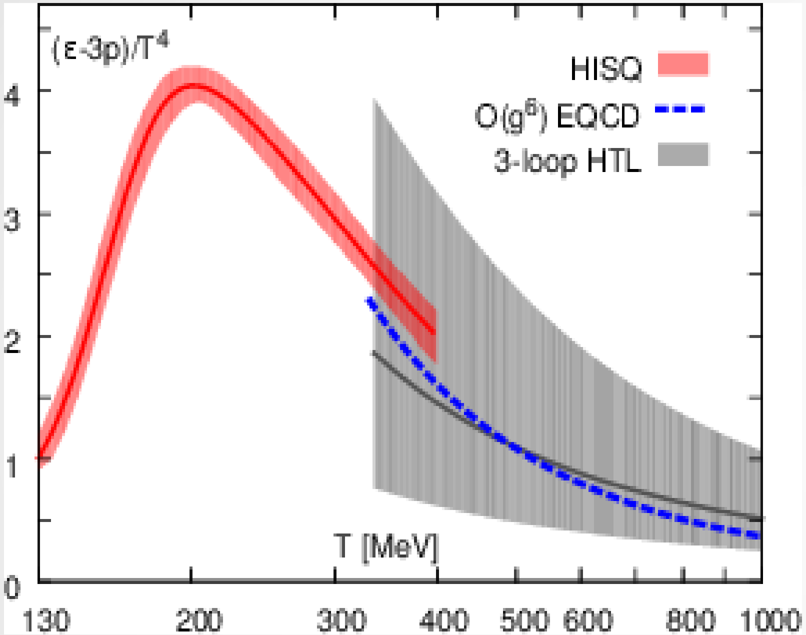


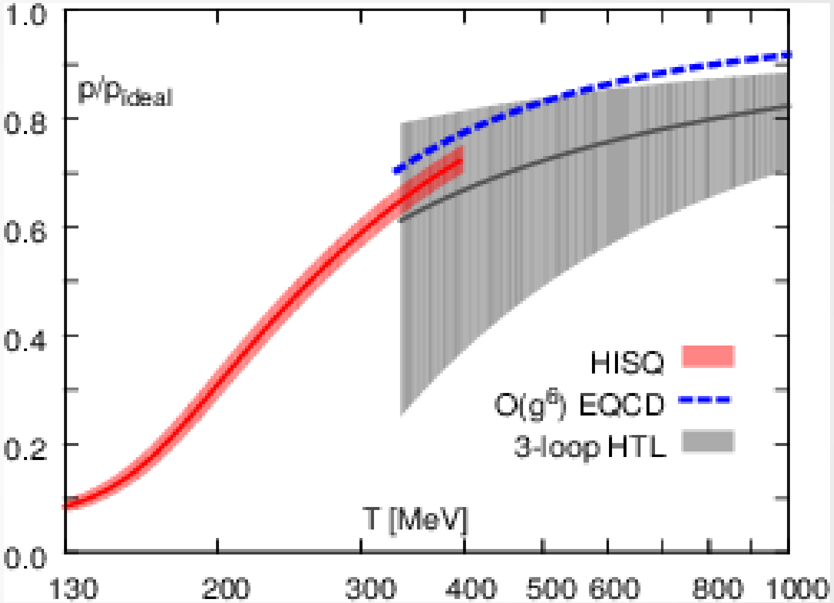


- We have also provided an analytic parametrization of the pressure [arXiv:1407.6387]:

$$\frac{p}{T^4} = \frac{1+\tanh\left(c_t\,(t-t_0)\right)}{2}\,\frac{p_{\rm id} + a_n/\bar{t} + b_n/\bar{t}^2 + c_n/\bar{t}^3 + d_n/\bar{t}^4}{1 + a_d/\bar{t} + b_d/\bar{t}^2 + c_d/\bar{t}^3 + d_d/\bar{t}^4}$$

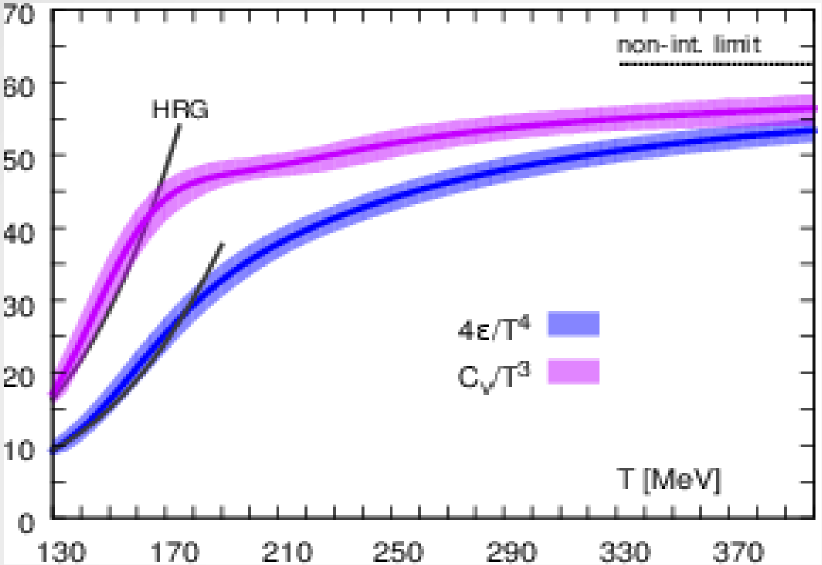
Comparison with perturbation theory





- 3-loop HTL: Haque, Bandopadhyay, Anderson, Mustafa, Strickland *et al*., JHEP 1405, 027 (2014).
- O(g6) EQCD: M.Laine & Y.Schroder, Phys.Rev. D73, 085009 (2006).

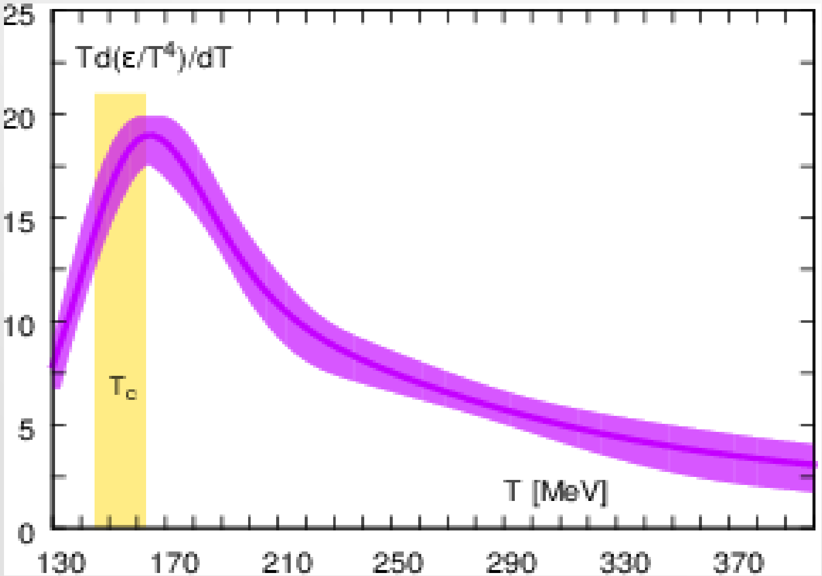
Speed of sound and specific heat



Second-order phase transition should exhibit a peak in the specific heat.

$$\frac{\epsilon}{T^4} = e_0 + e_1\left(\frac{T-T_c}{T_c}\right) + \mathcal{O}\left(|T-T_c|^{1-\alpha}\right)$$

But α = -0.21 for the 3d O(4) model.

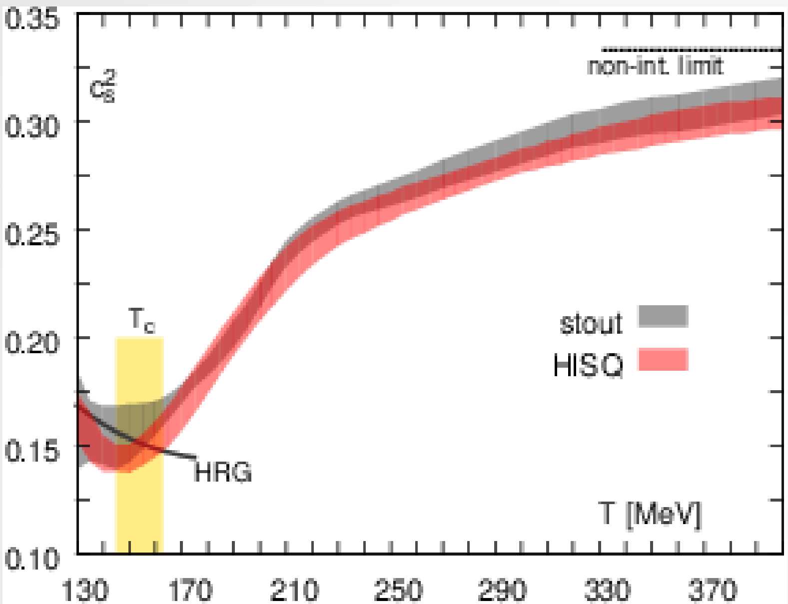


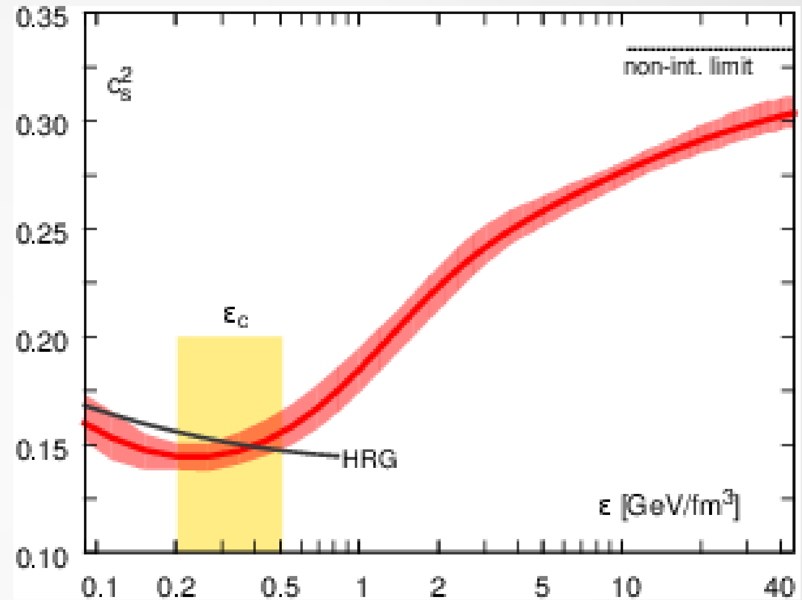
Construct a “subtracted” specific heat

$$T\frac{\mathrm{d}\epsilon/T^4}{\mathrm{d}T} \equiv \frac{\overline{C}_V}{T^3}$$

And indeed, we do see a peak near the crossover temperature.

Speed of sound and specific heat

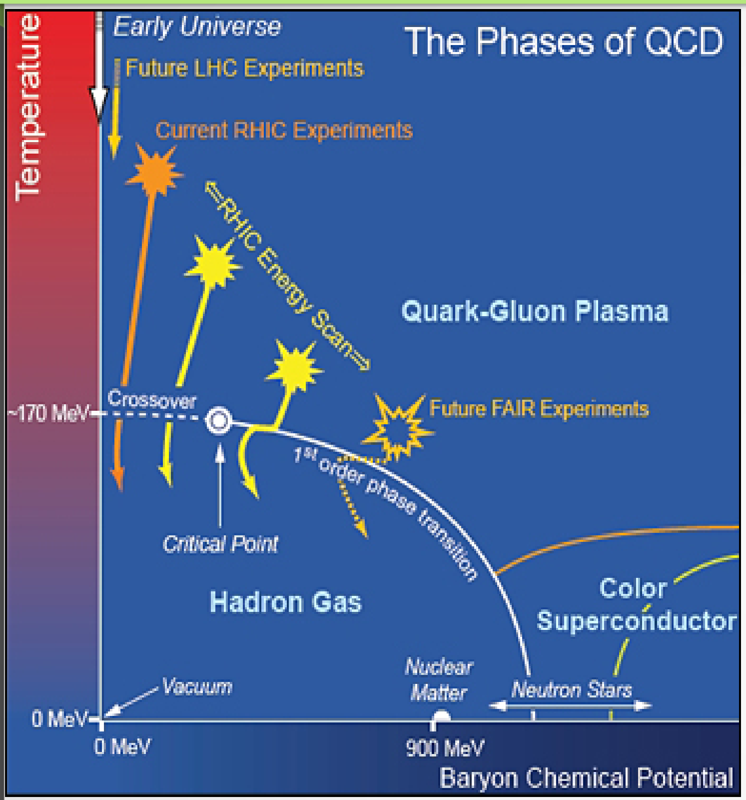




The “softest point” of the equation of state occurs just near the crossover and is not very different from HRG expectation.

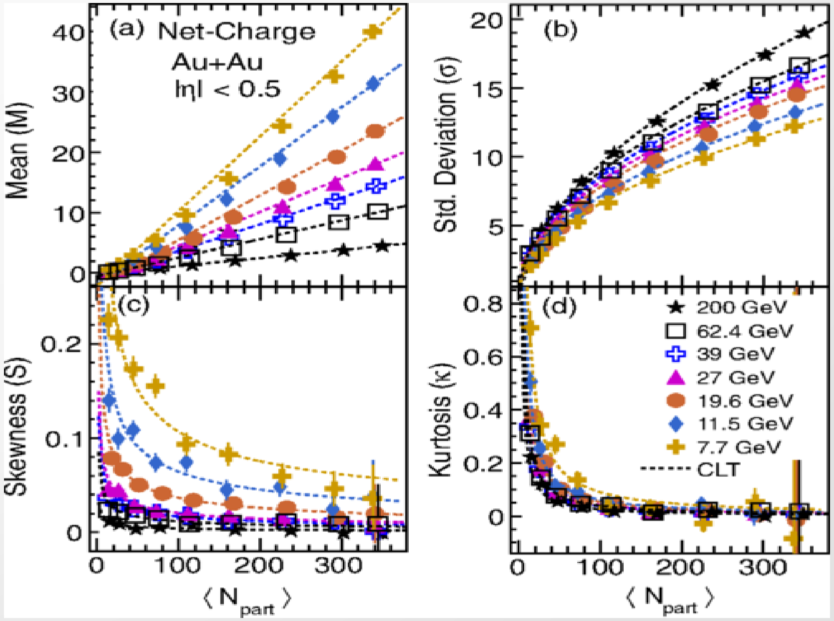
The energy density at the softest point is 180-500 GeV/fm^3. For comparison, ε_{proton}~ 450 GeV/fm^3.

Beam energy scan at RHIC



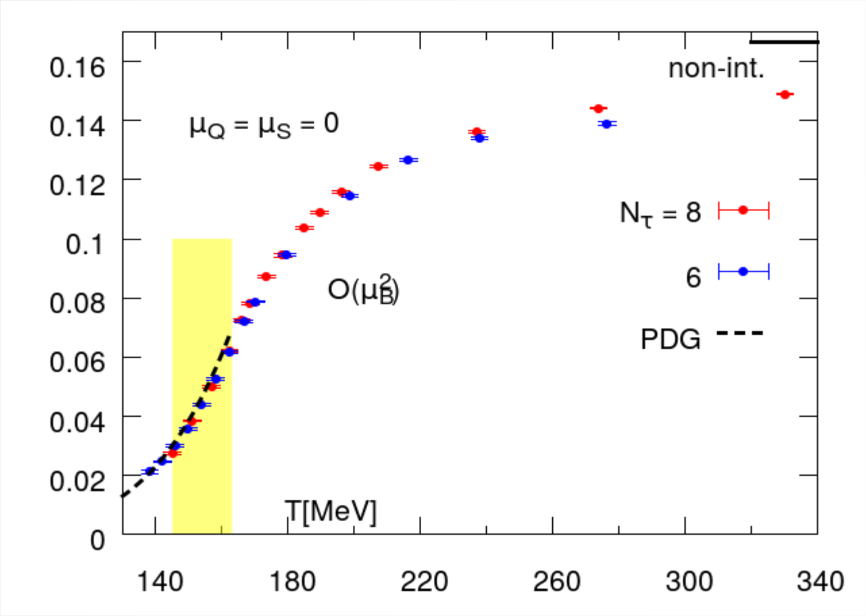
- Possible existence of a critical point at large mu_B.
- Search up to mu_B ~ 400-450 MeV.

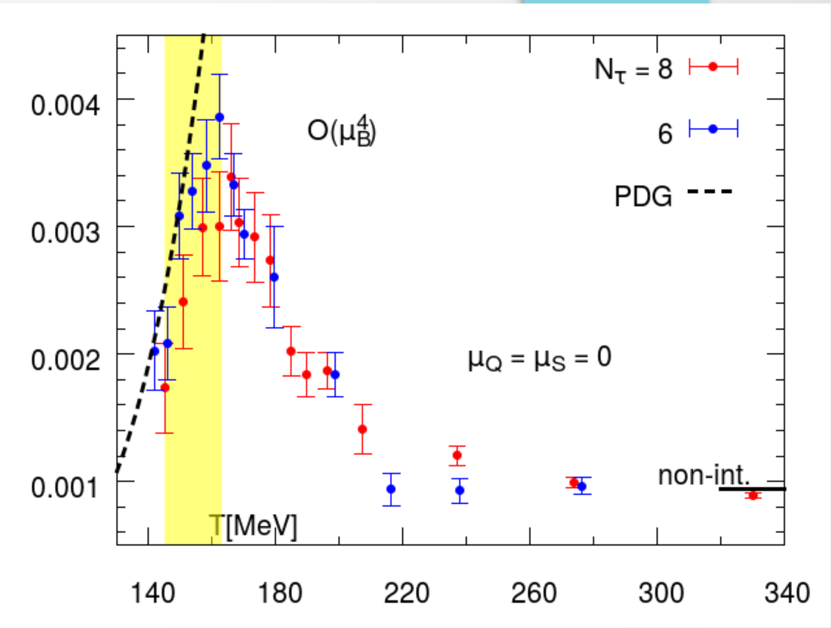
- Look at higher moments of net charge (electric, baryon number,...) distributions.



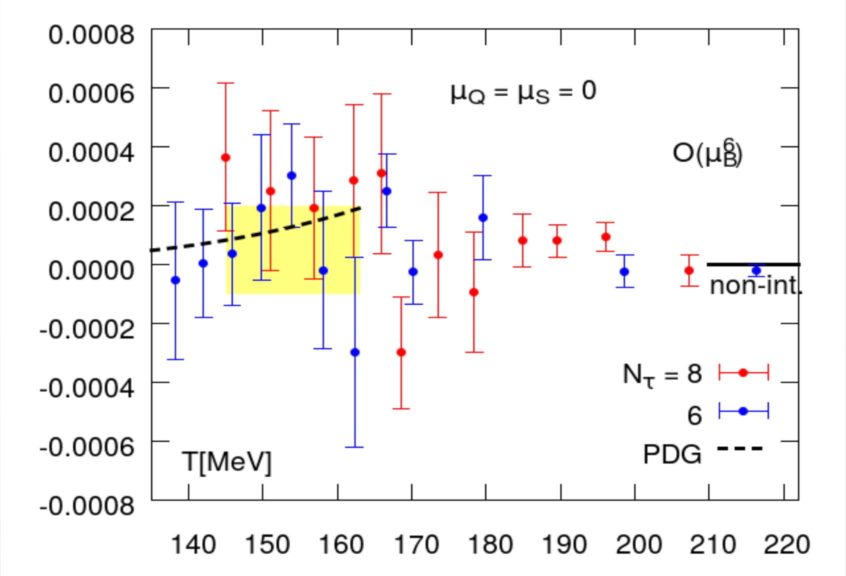
STAR collaboration, PRL 113, 0902301 (2014).

The method of Taylor expansions





$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}}{i!\,j!\,k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$



6th-order coefficients noisy, but not more than ~5% of the 4th-order coefficients.

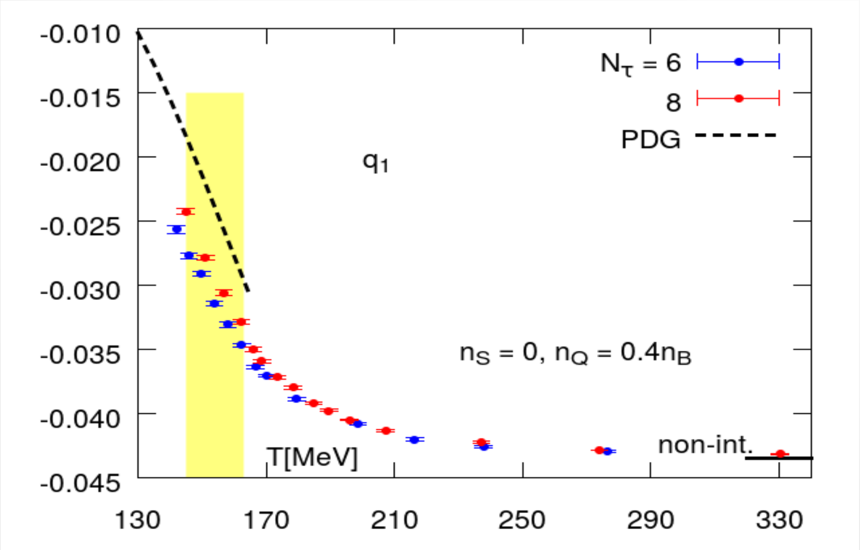
Initial conditions in heavy-ion collisions

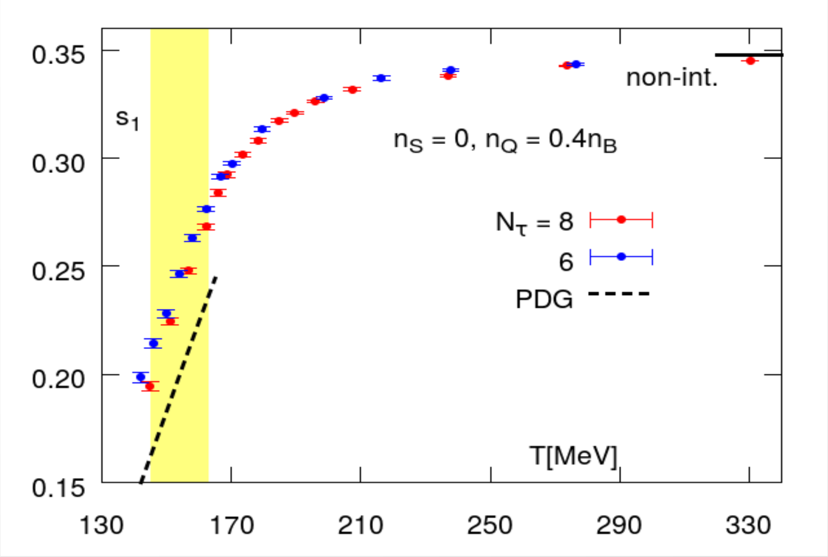
Fix mu$_Q$ and mu$_S$ from initial conditions:

- N_S = 0 (no valence strange quarks),
- r = $N_p/(N_p+N_n)$ = const. (fixed Z-to-A ratio).

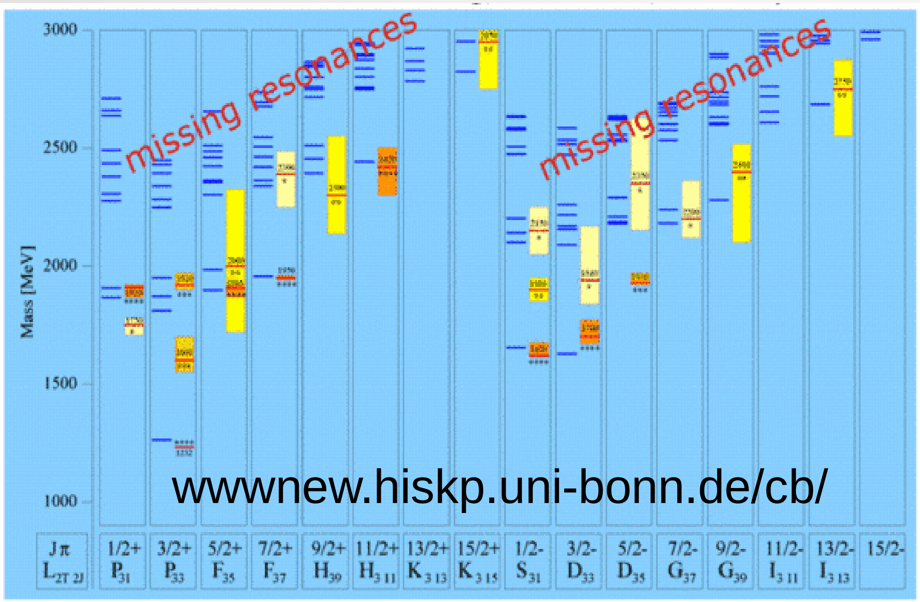
$$\frac{\mu_Q}{T} = q_1 \frac{\mu_B}{T} + q_3 \left(\frac{\mu_B}{T}\right)^3 + \ldots$$

$$\frac{\mu_S}{T} = s_1 \frac{\mu_B}{T} + s_3 \left(\frac{\mu_B}{T}\right)^3 + \ldots$$

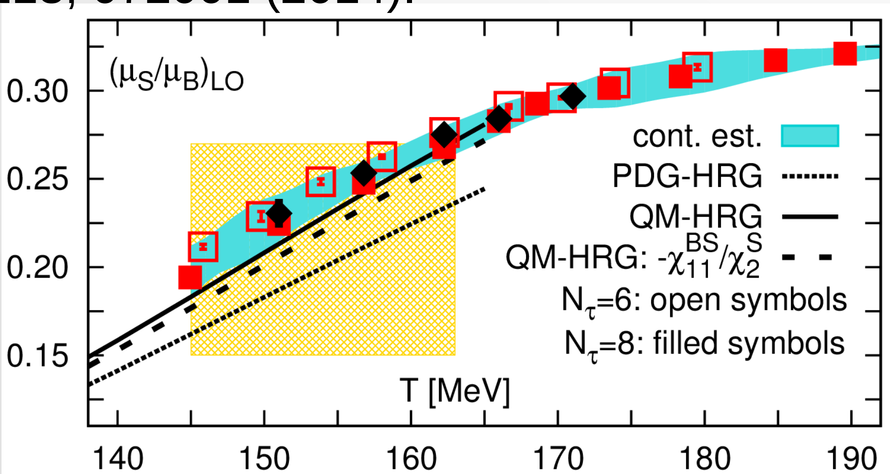


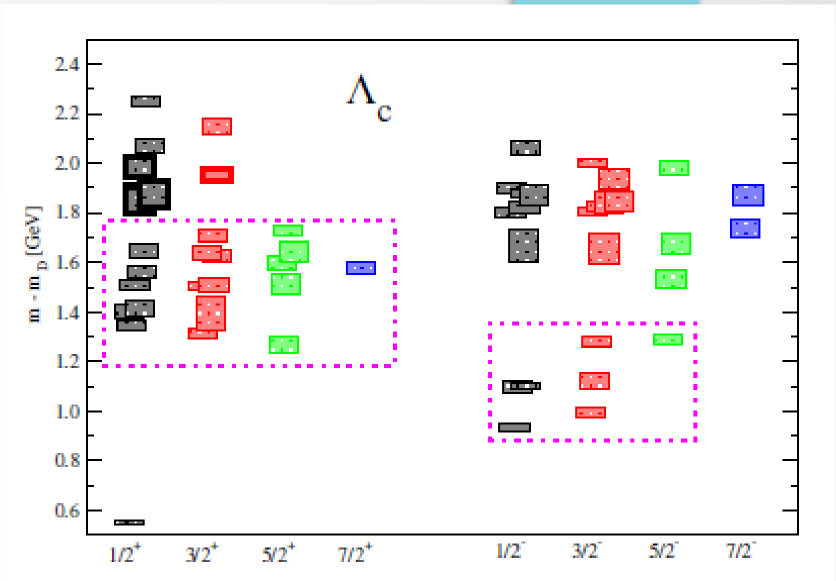


Additional resonances and the Quark Model



A.Bazavov *et al*.[BNL-Bielefeld-CCNU], PRL 113, 072001 (2014).



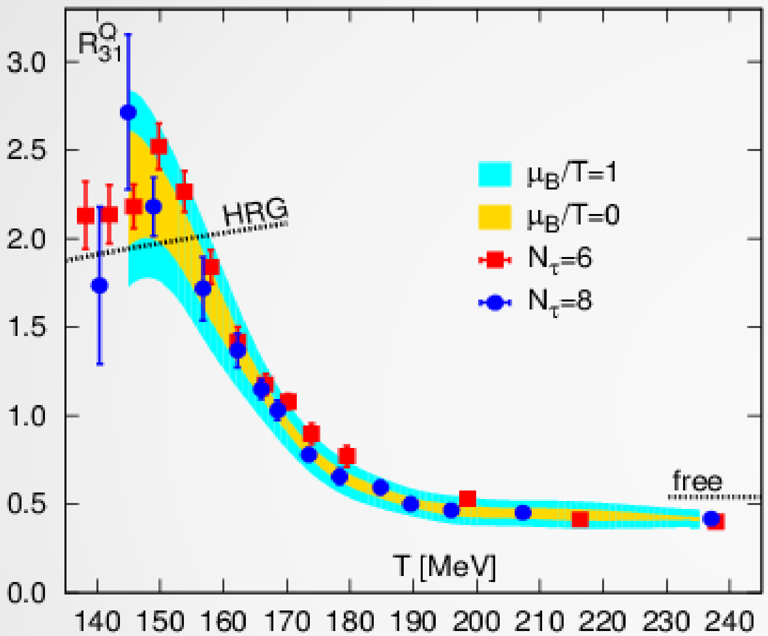


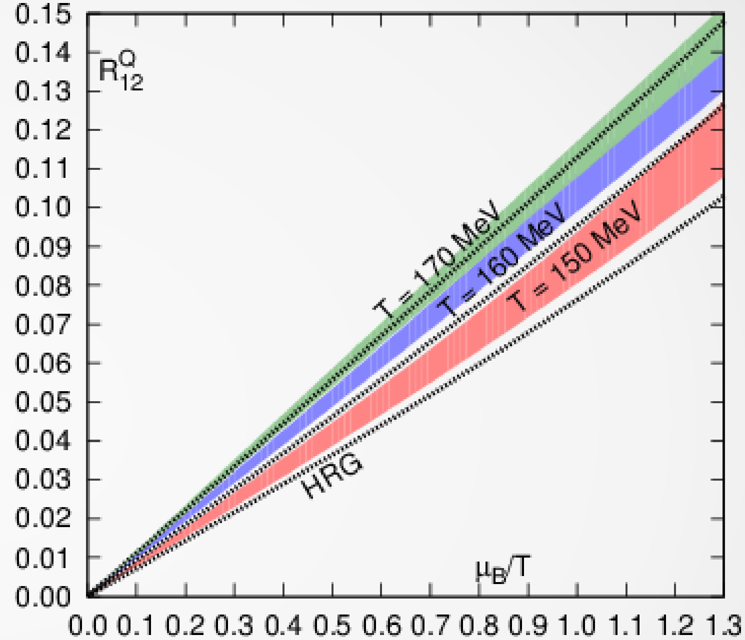
Padmanath *et al*. arXiv:1410.8791 [hep-lat]

D.Ebert, R.Faustov & V.Galkin, Phys. Rev. D79, 114029 (2010).

Possibility of additional light and strange resonances beyond those listed in the PDG; see talk by S. Mukherjee.

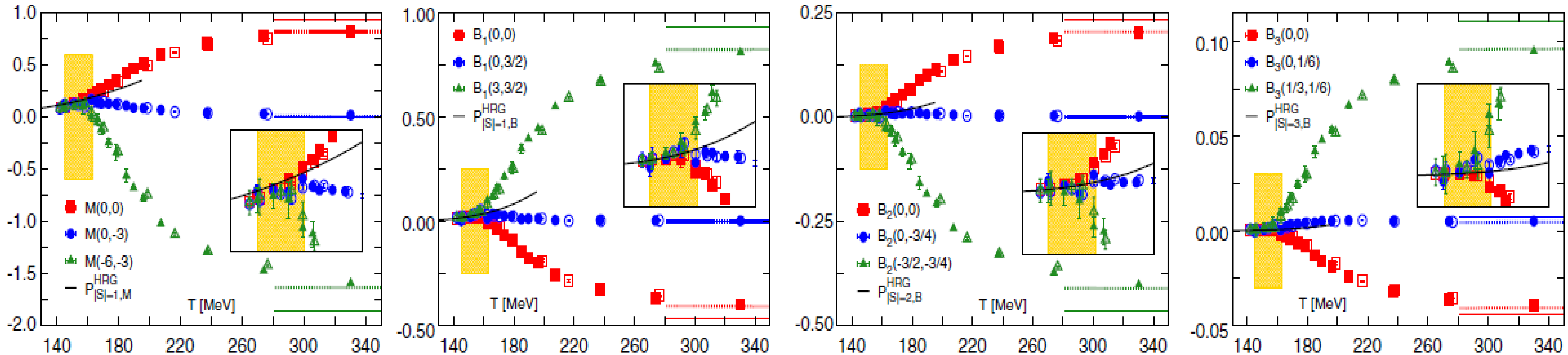
Freeze-out conditions from lattice QCD





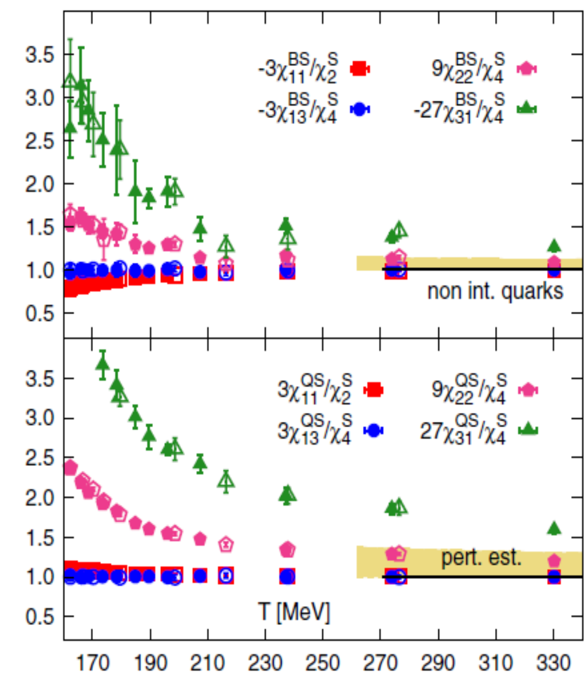
- Determine freeze-out parameters in experiments from *ab initio* lattice calculations through ratios of fluctuations.
- Still in preliminary stage. Need to account for systematics.
- Preliminary study: L. Kumar [STAR], CPOD 2013 (2013), 047.

Strangeness-carrying degrees of freedom



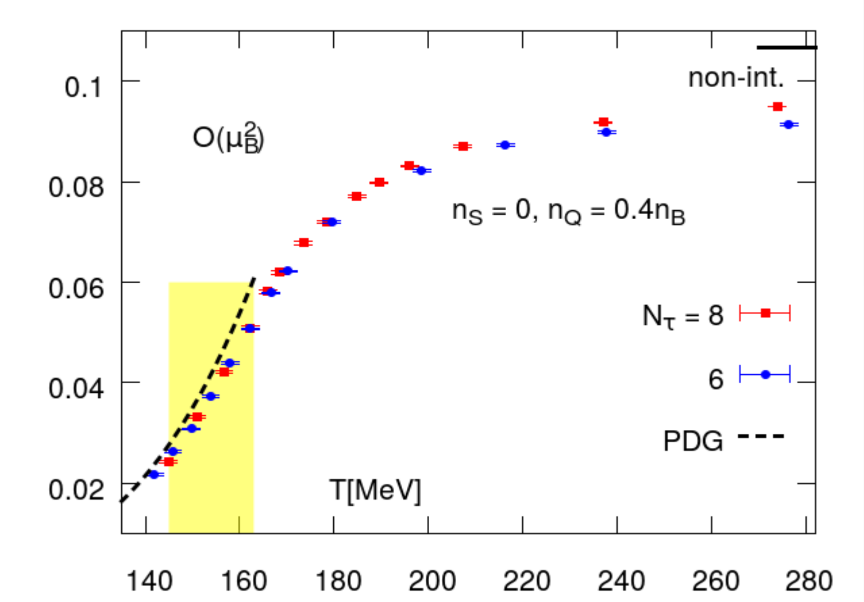
BNL-Bielefeld, PRL 111 (2013)

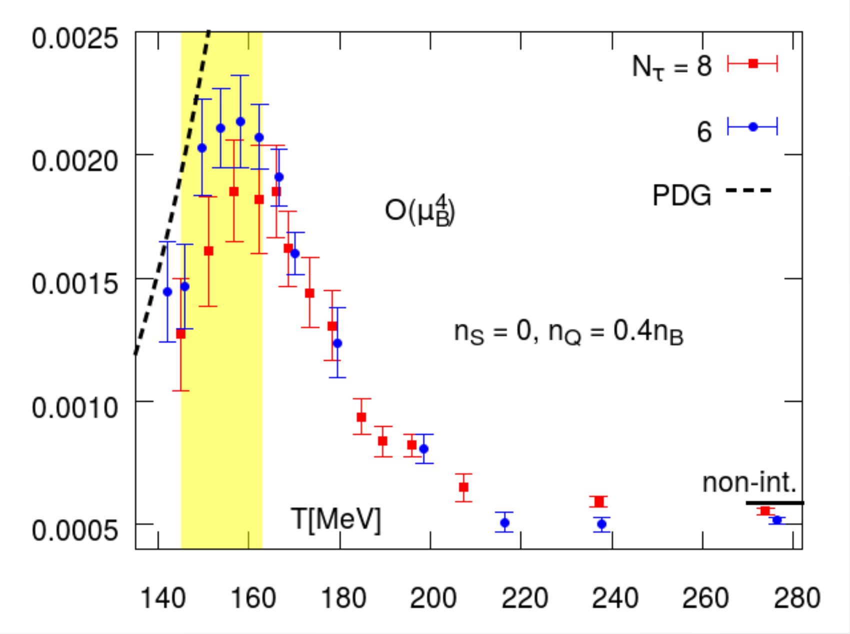
$$
\begin{aligned}
P_S^{\mathrm{HRG}}(\hat{\mu}_B, \hat{\mu}_S) = {} & P^{\mathrm{HRG}}_{|S|=1,M} \cosh(\hat{\mu}_S) \\
& + P^{\mathrm{HRG}}_{|S|=1,B} \cosh(\hat{\mu}_B - \hat{\mu}_S) \\
& + P^{\mathrm{HRG}}_{|S|=2,B} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
& + P^{\mathrm{HRG}}_{|S|=3,B} \cosh(\hat{\mu}_B - 3\hat{\mu}_S),
\end{aligned}
$$

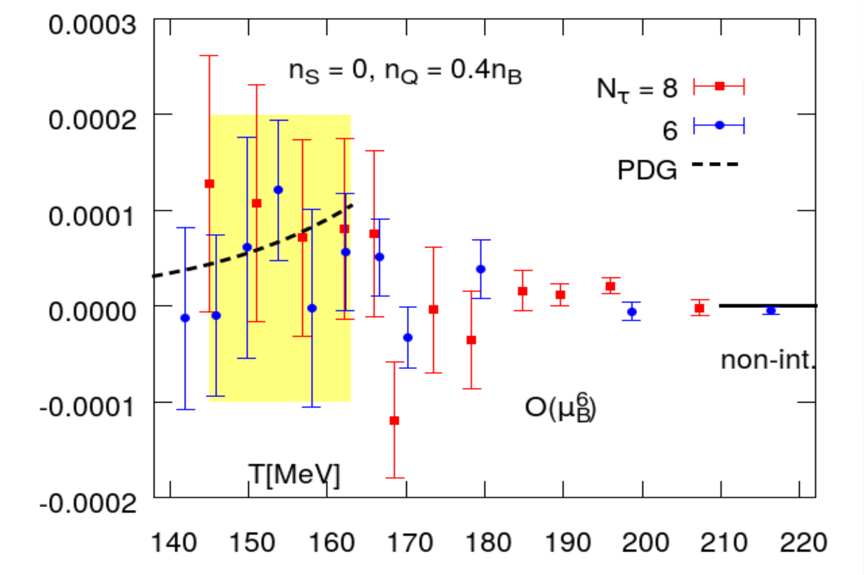


All open strange mesons and baryons dissociate in the crossover region

Susceptibilities: Constrained case

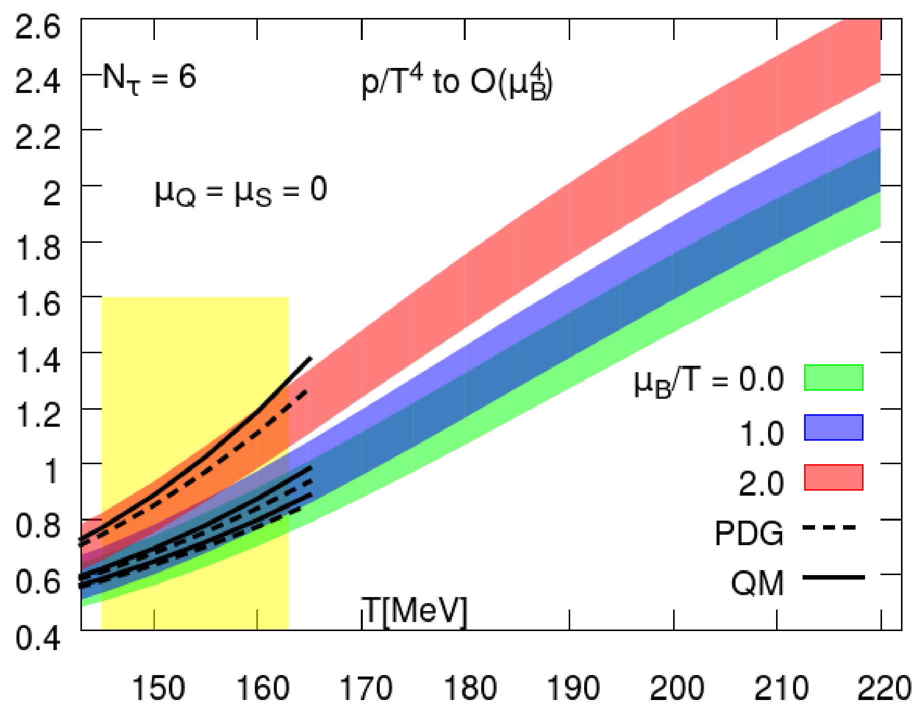




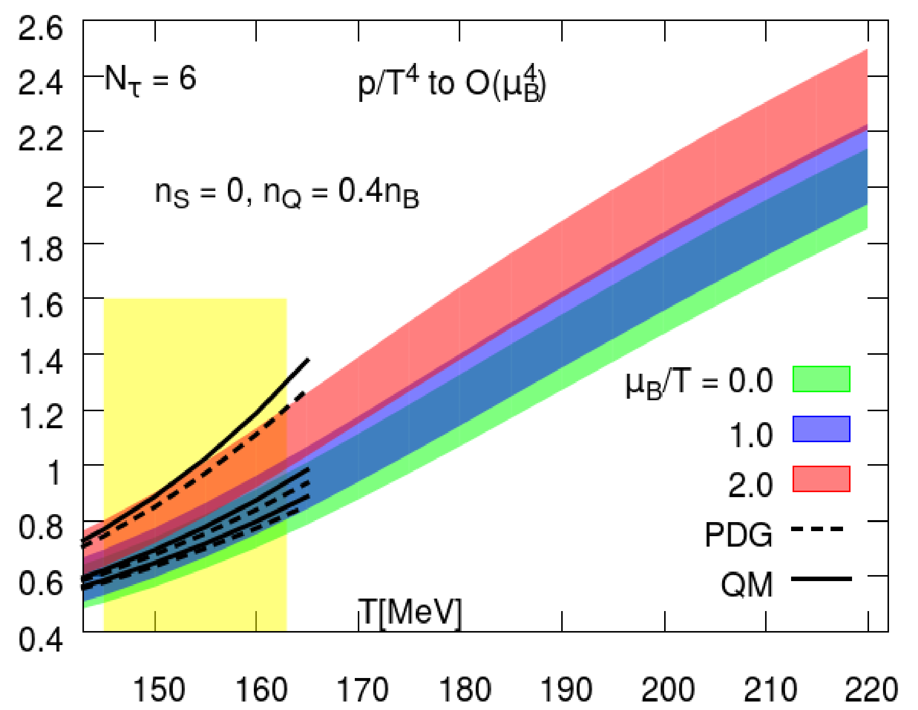


- We will use r = 0.40 throughout, which is the value for Pb-Pb collisions.
- Constrained case qualitatively similar to $mu_Q = mu_S = 0$ case, but values about 30-40% smaller.

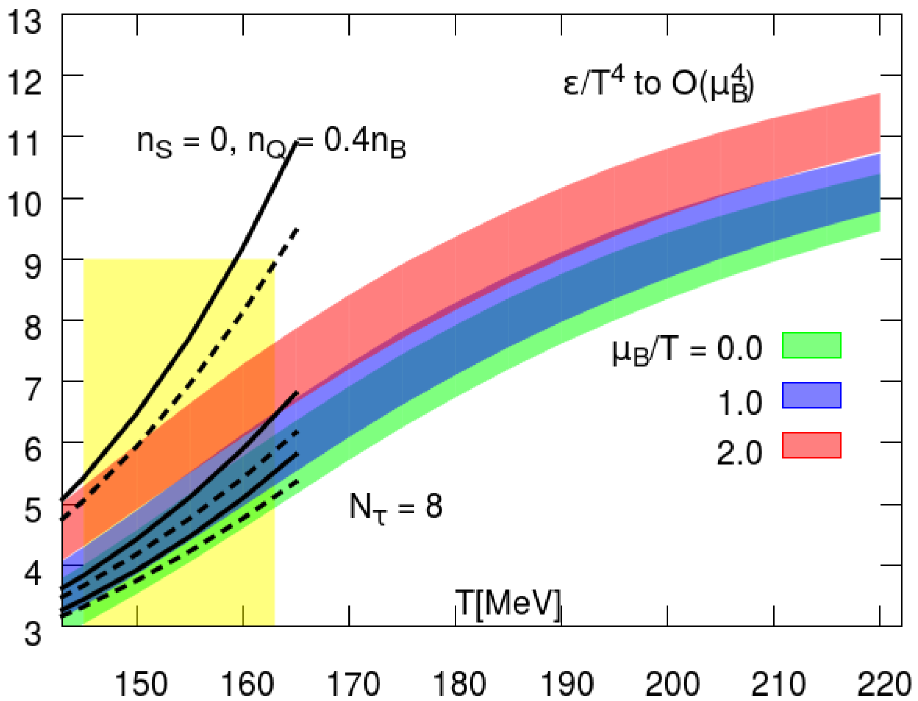
Putting everything together - I



~10% corrections around the transition region up to mu_B / T = 2.0.

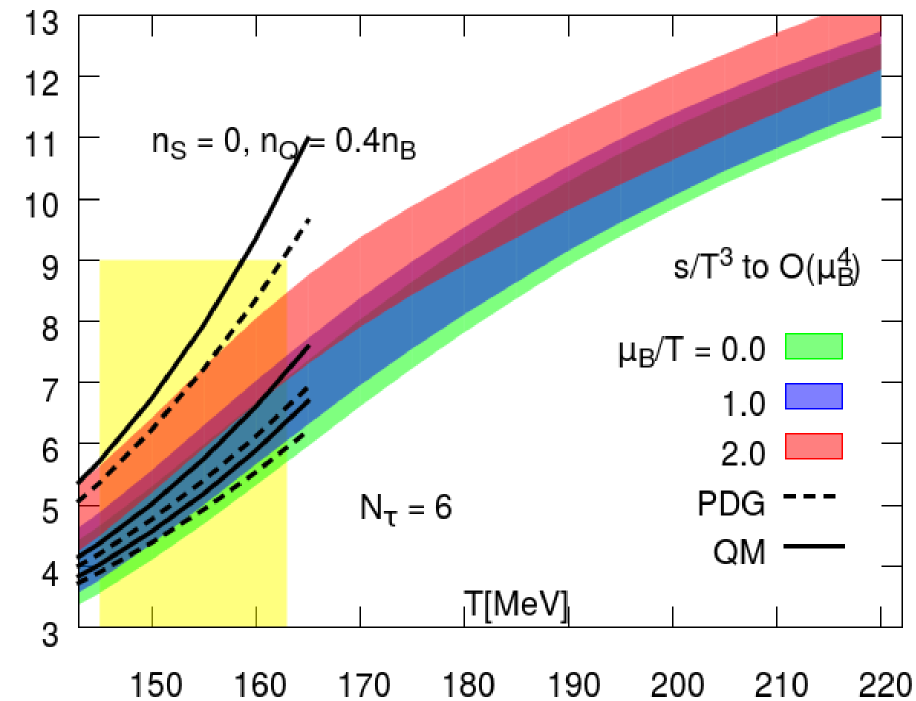


Putting everything together - II



$$\frac{\varepsilon}{T^4} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n \left\{ T\frac{dc_n}{dT} + 3c_n \right\}$$

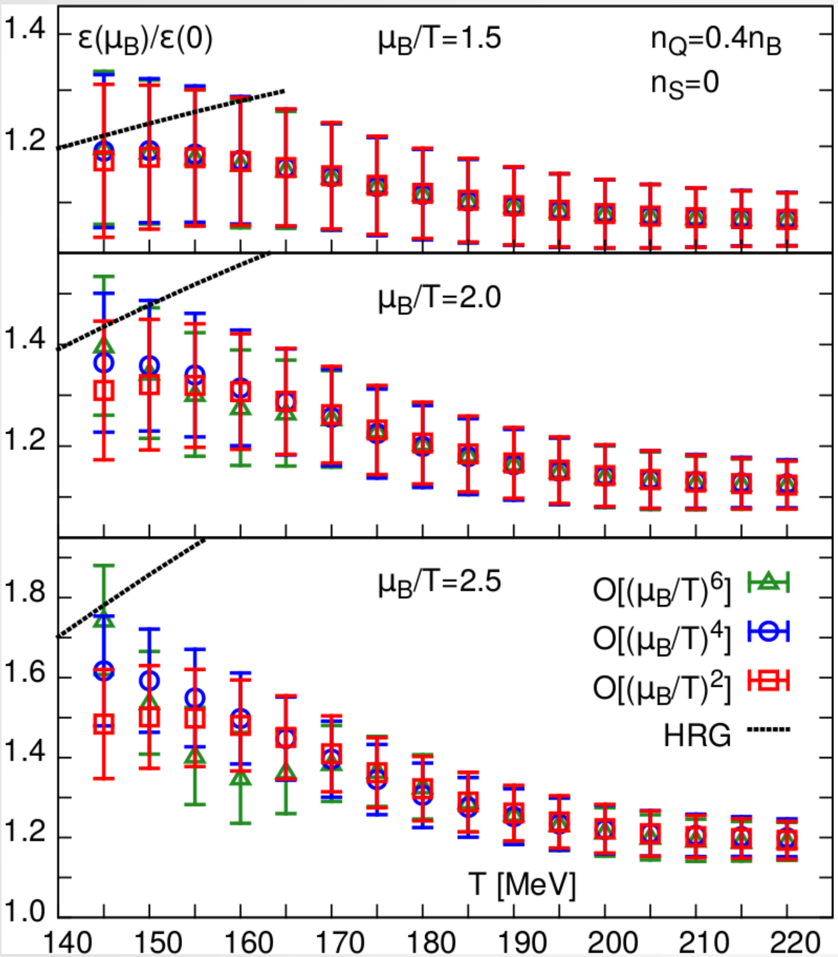
$$\frac{s}{T^3} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n \left\{ T\frac{dc_n}{dT} + (4-n)c_n \right\}$$

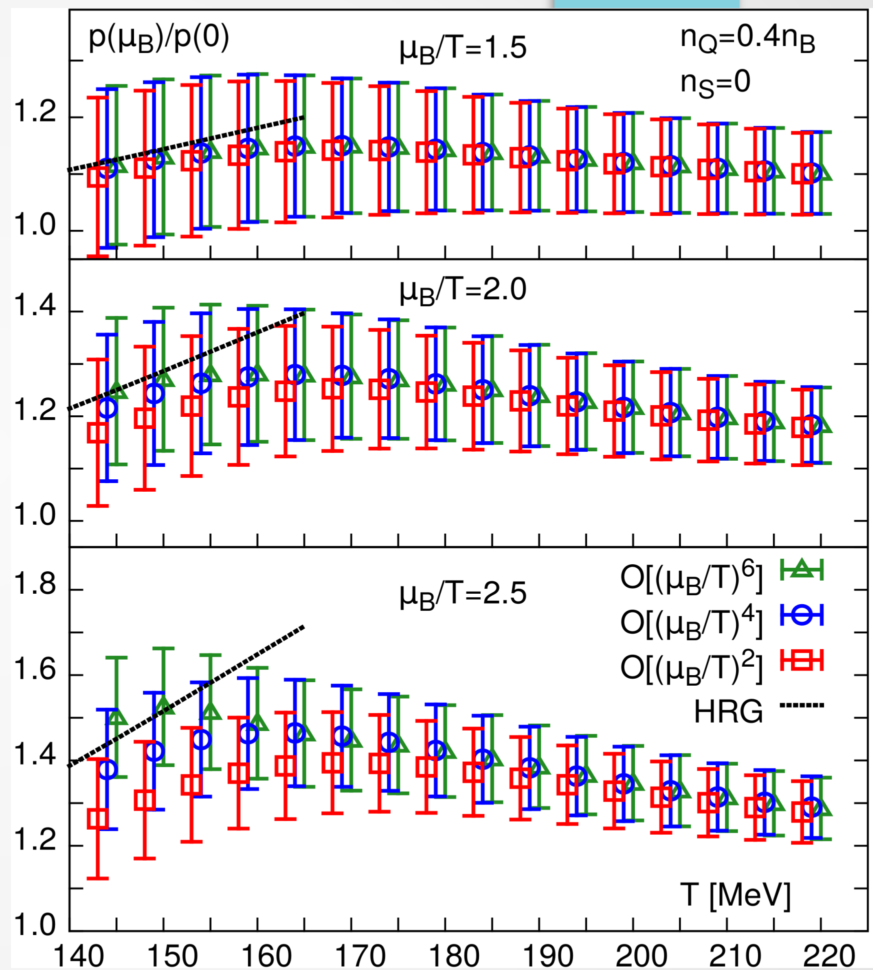


~10% corrections around the transition region up to mu_B / T = 2.0.

Higher derivatives expected to be more sensitive to higher-order corrections.

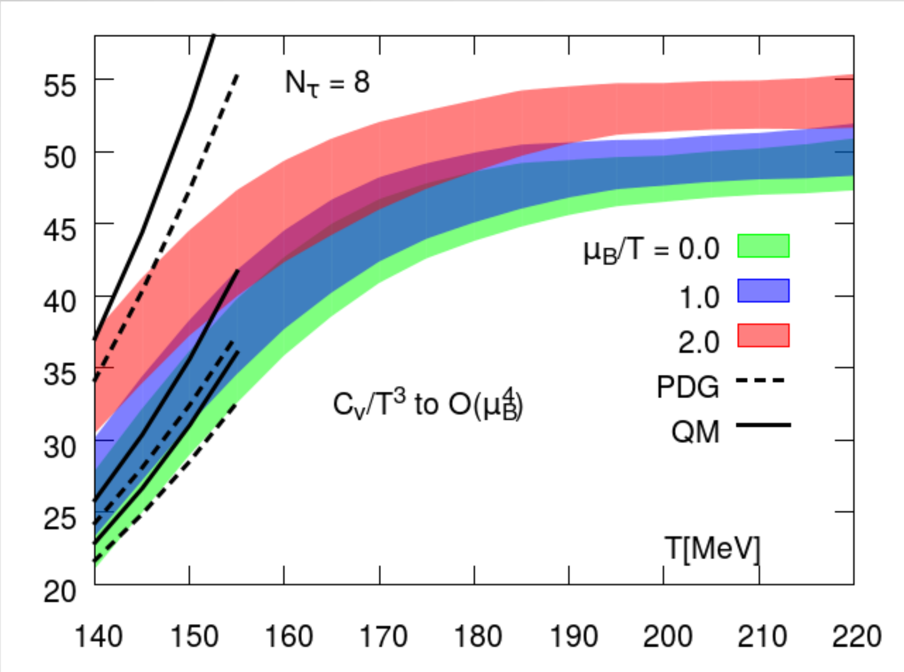
Range of extrapolation



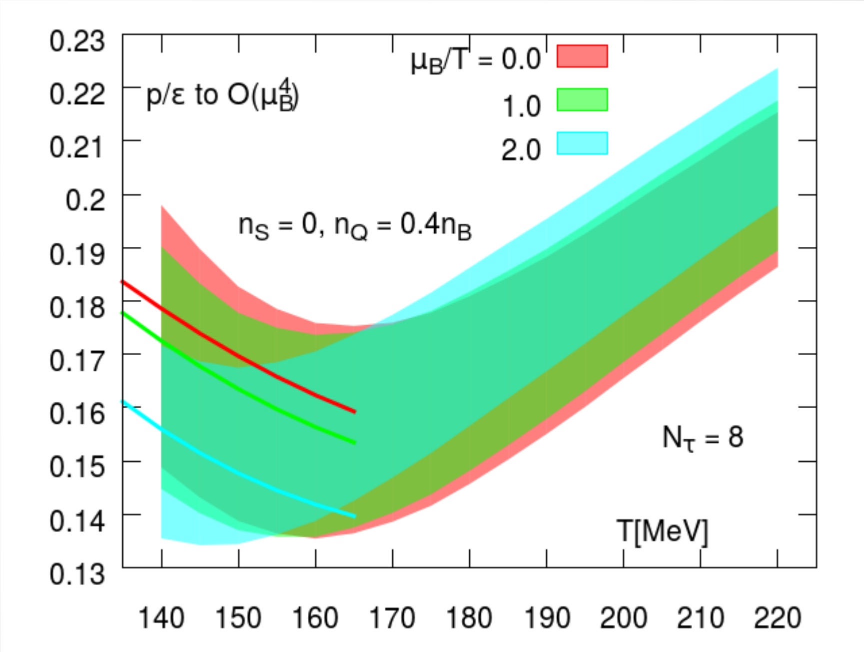


Different orders start to differ above $mu_B/T \sim 2.0$.

Specific heat and softest point

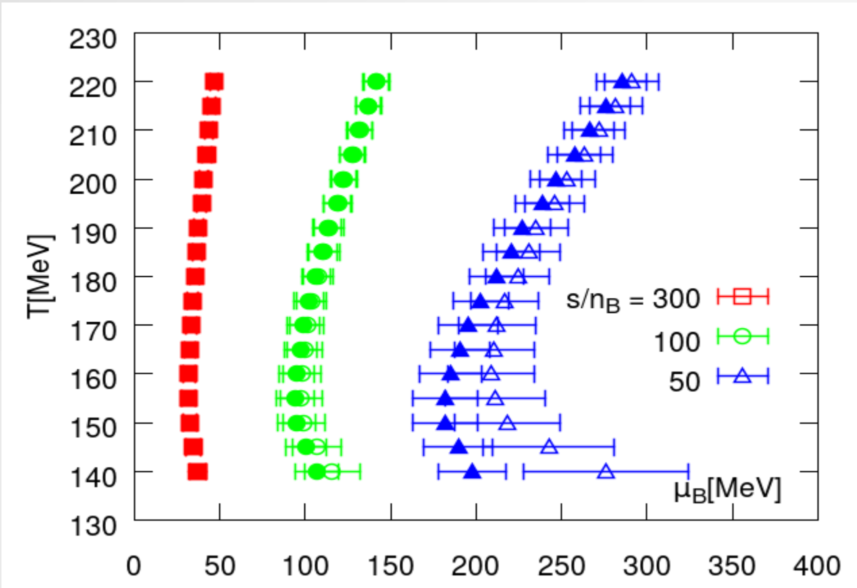


$$\frac{C_v}{T^3} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n \left\{ T^2 \frac{d^2 c_n}{dT^2} + 8T \frac{dc_n}{dT} + 12 c_n \right\}$$



Minimum in 'speed of sound' moves to lower T.

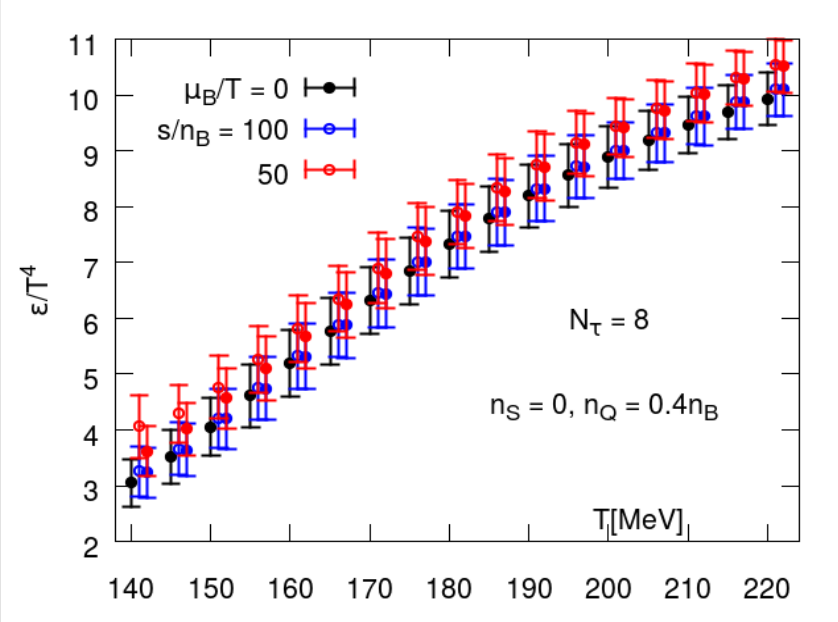
Equation of state at fixed s/n_B

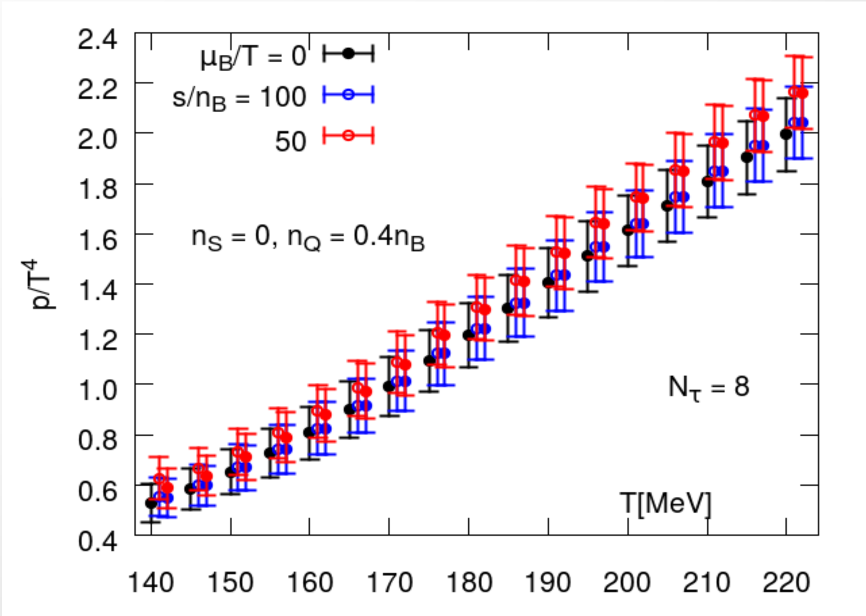


s/n_B=constant gives mu_B for a given T.

$$s_0 + s_2 \left(\frac{\mu_B}{T}\right)^2 = K n_{B_1} \frac{\mu_B}{T}$$

Quadratic to lowest order.





Freeze-out curve

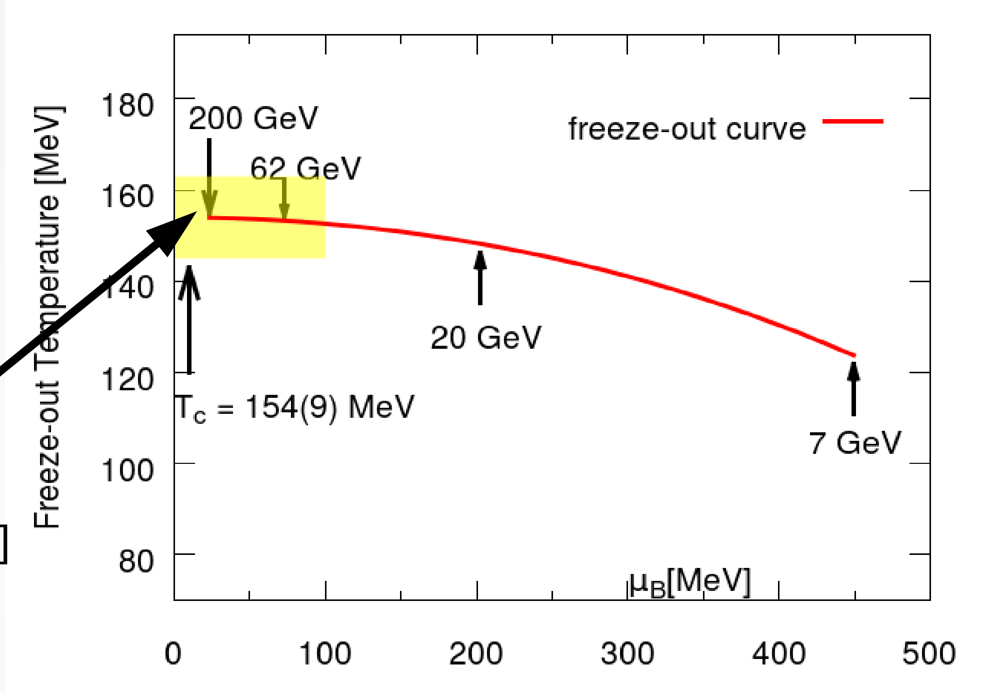
$$T^f = \frac{T^f_\infty}{1 + \exp\left(1.176 - \frac{\ln x}{0.45}\right)}$$

Andronic *et al.* Nucl.Phys. A772 (2006) 167-199.

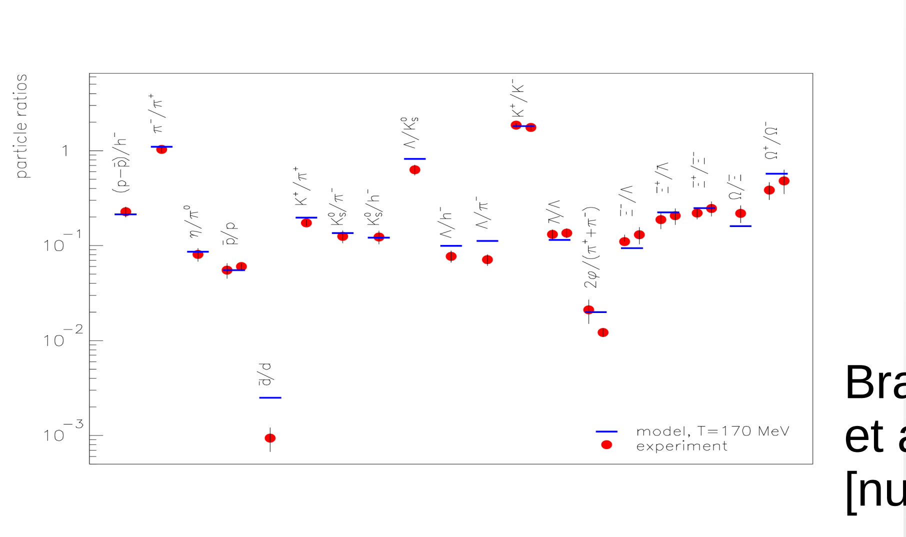
$$x = s_{NN}^{1/2} \text{ in GeV}$$

$$T^f_\infty = 154\,\text{MeV}$$

A.Bazavov *et al.* [HotQCD] Phys. Rev. D85 054503 (2012).

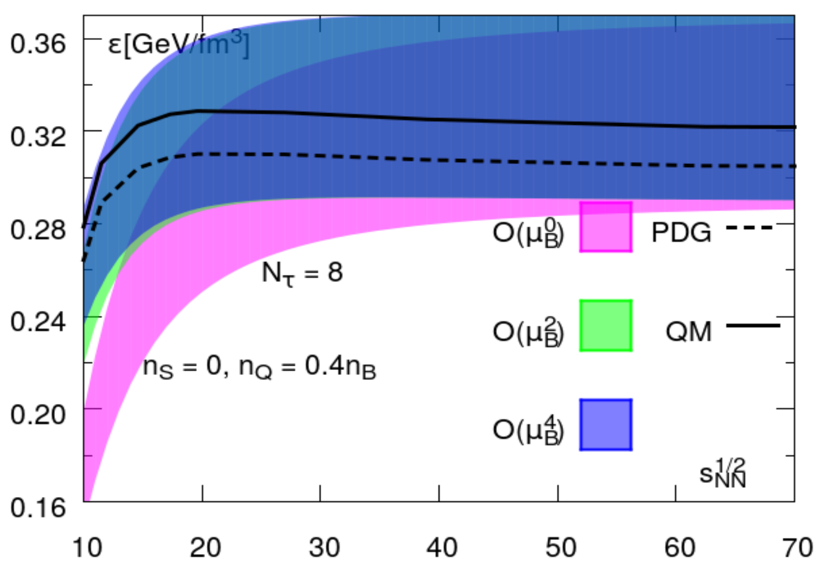


$$\mu_B^f = \frac{1.303}{1 + 0.286x}\ \text{GeV}$$

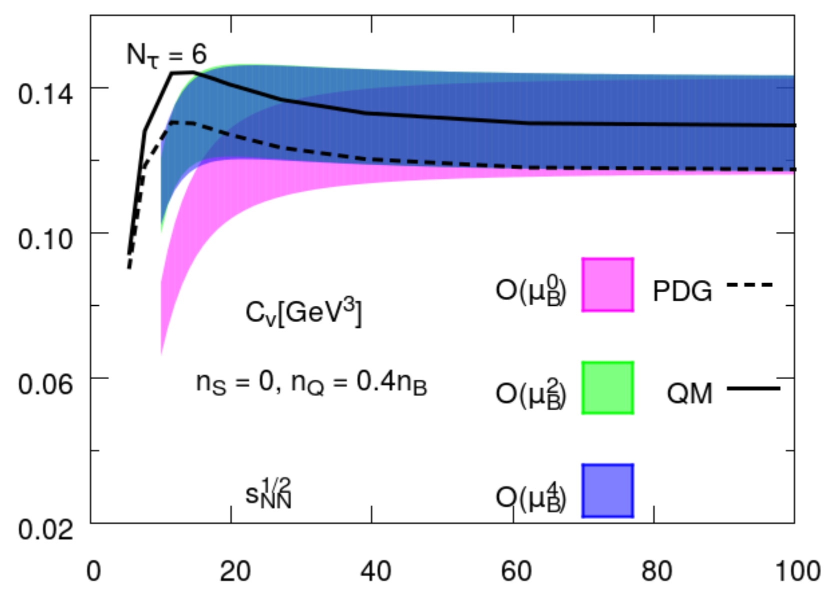


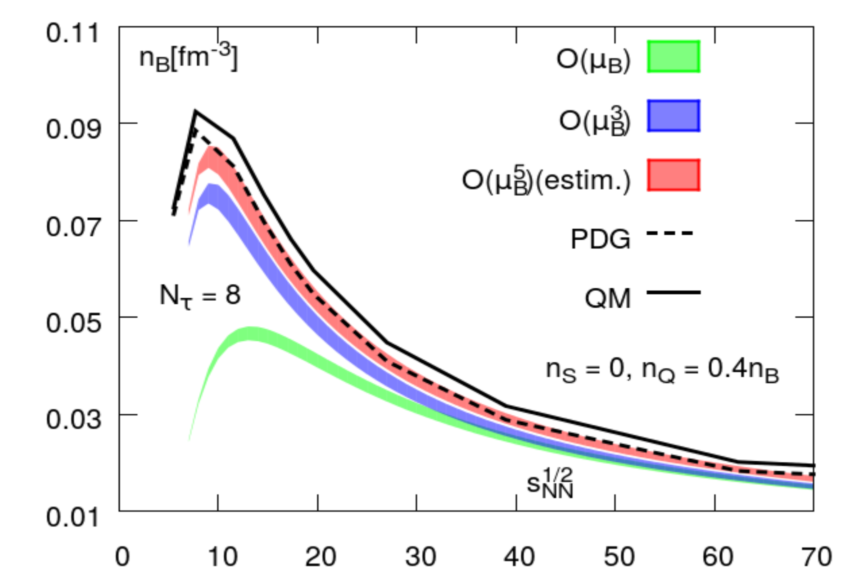
Braun-Munzinger *et al.* in Hwa, R.C. (ed.) et al.: Quark gluon plasma* 491-599 [nucl-th/0304013].

Freeze-out equation of state



Fourth-order expansion valid down to beam energies ~20 GeV.





Conclusions

- Lattice QCD has proven to be very useful in furthering our knowledge about the QCD phase transition and the quark-gluon plasma.
- So far, lattice results seem to confirm the Pisarski-Wilczek picture regarding the QCD phase diagram, though we cannot yet give exact numbers for all parts of the diagram.
- Studies with chiral fermions show that $U_A(1)$ remains broken above the chiral transition. Above Tc, UA(1) is broken by near-zero modes coming from instanton—anti-instanton pairs.
- Equation of state is a key input in hydrodynamic modelling. $\mu=0$ useful at LHC and RHIC 200 GeV runs, whereas $\mu>0$ useful for the Beam Energy Scan programs.
- Fourth-order Taylor expansion can provide an equation of state valid upto $\sqrt{s}$ ~ 20 GeV. With higher orders we should be able to push this to even lower CoM energies (unless the expansion breaks down).