Studying QCD plasma with lattice correlators

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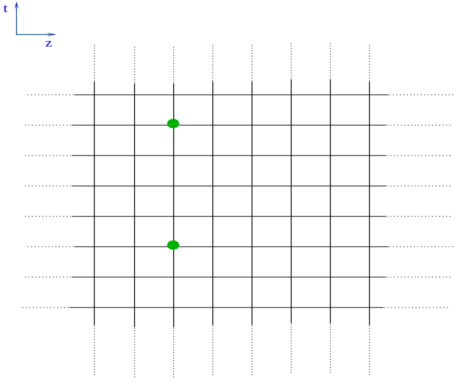
Correlator as probe

Two point functions of suitable operators:
way to find out excitations of QCD vacuum.



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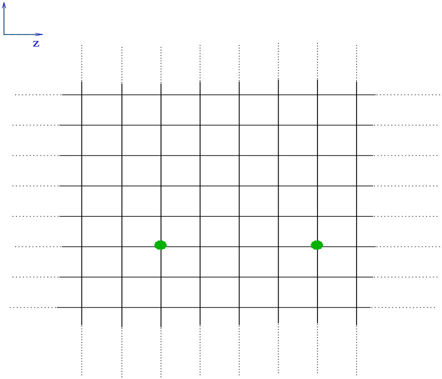
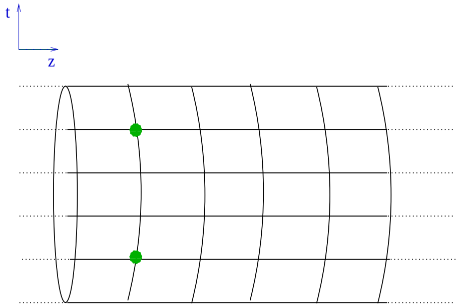


Thermal field theory on lattice

$$Z = \int_{(a)pbc} \mathcal{D}U\mathcal{D}(\psi, \bar{\psi}) \; e^{-\int_0^{1/T} \mathcal{L}(U,\psi,\bar{\psi})}$$



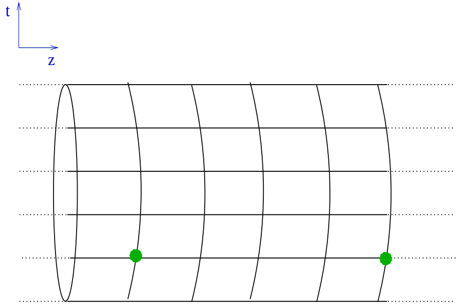
$$G(\Phi(\tau, \vec{x})\Phi^+(0, \vec{0})) = \langle \Phi(\tau, \vec{x})\Phi^+(0, \vec{0}) \rangle$$

periodicity: $G(\tau, 0) \;=\; G(\beta - \tau, 0)$

Short temporal extent makes analysis nontrivial.

Screening correlator

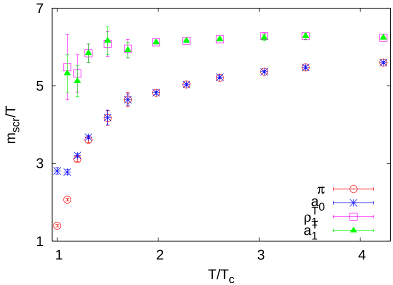
Techinques very similar to spectrum calculation can be used to study correlations in the spatial direction (take z, e.g.).

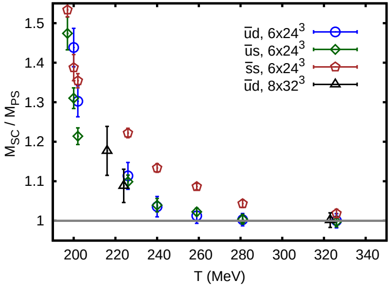


Similar to T=0 case, transfer matrix defined on z-slices
Screening masses give eigenvalues of z-slice transfer matrix.
Important informations about properties of the finite temperature theory.

Mesonic correlator in 2+1 flavor QCD

Mesonic screening correlators have been widely studied, since DeTar & Kogut ('87).



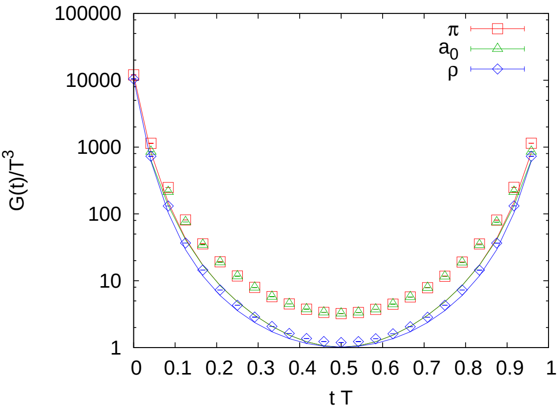


n_f=2+1 QCD, $m_\pi \sim 220$ MeV.

M. Cheng, N. Christ, S. Datta, A. Francis, et al., Eur.Phys.J.C71('11)1564

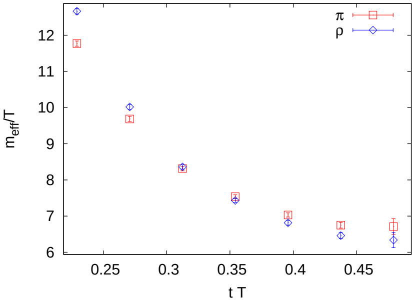
PS-S difference: $U_A(1)$ breaking?

Matsubara correlator for mesons



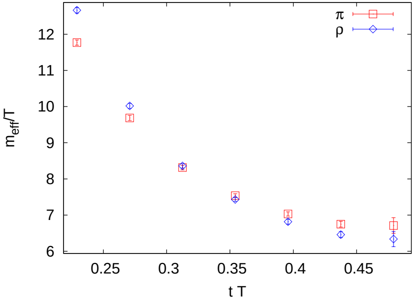
- Quenched theory, 1.5 T_c, $N_t = 24$, $m_q = 0$
- Chiral symmetry restoration seen
- ρ correlator $< 10\%$ away from free correlator
- More discrepancy in pion and scalar channel

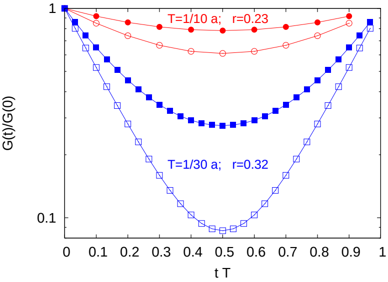
Matsubara correlator



No plateau in effective mass: description of thermal averaged properties.

Matsubara correlator





No plateau in effective mass: description of thermal averaged properties.

$$r = \mathrm{Avg}_t \frac{G^-(t) - G^+(\beta - t)}{G^-(t) + G^+(\beta - t)}$$

Apparent trend of chiral symmetry restoration in a system of noninteracting particles.

Matsubara and screening correlators

Matsubara correlators

- Analytically continued to retarded correlator: Can be connected to thermal observables.
- $\rho(q_0, \vec{q}) = \mathrm{Im}\ \mathrm{G^R(q_0, \tilde{q})}$ "spectral function"
- $G(\tau, \vec{q}) = \int_0^\infty \frac{dq_0}{\pi} \rho(q_0) \frac{\cosh(q_0(\beta/2-t))}{\sinh(p_0\beta/2)}$

e.g., M. LeBellac, *Thermal Field Theory*

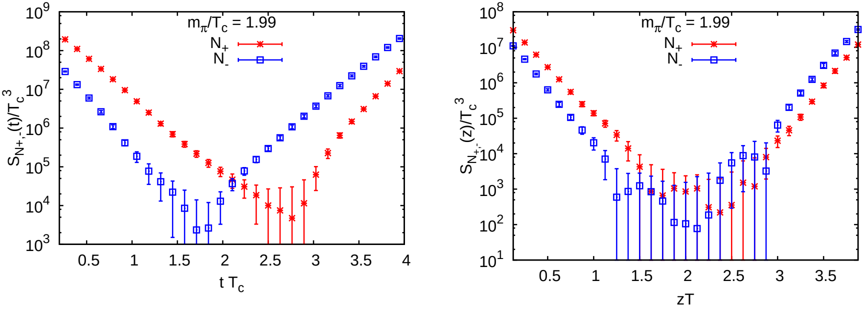
Screening correlators

- Useful for studying symmetries of the thermal transfer matrix.
- Debye screening, symmetry restoration, ...

  C. DeTar & J. Kogut, Phys. Rev. D 36 ('87) 2828.
- $G(z, \vec{p_\perp}) = \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}\, e^{ip_z z} \int_0^\infty dp_0\, \frac{\rho(p_0, \vec{p_\perp}, p_z)}{2p_0}$

Can also study chiral symmetry restoration by looking at nucleon and its parity partner.

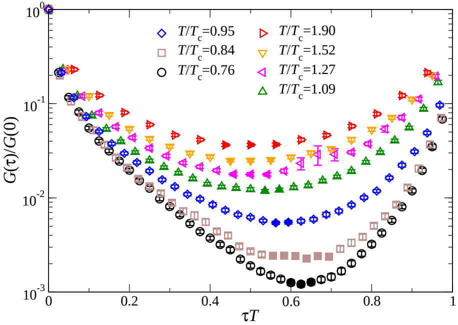
$$N^+(t)N^+(0) \xrightarrow{\text{large } t} C_+ e^{-m_{N^+} t} - C_- e^{-m_{N^-}(\beta - t)}$$



$$\frac{\mu_{N-} - \mu_{N+}}{M_{N-} - M_{N+}} \sim 0.68 \pm 0.12 \text{ at } 0.95\ T_c \text{ in gluon plasma.}$$

S. Datta, S. Gupta, Padmanath M., N. Mathur & J. Maiti, JHEP 1302 (2013) 145.

Matsubara correlator for nucleons



N_f=2+1, m_π = 384 MeV, anisotropic lattice

G. Aarts, C. Allton, S. Hands, B. Jäger, C. Praki, J-I. Skullerud, arXiv:1502.03603

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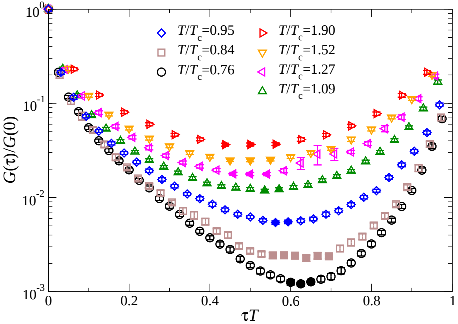

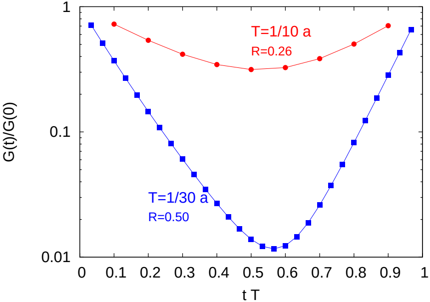

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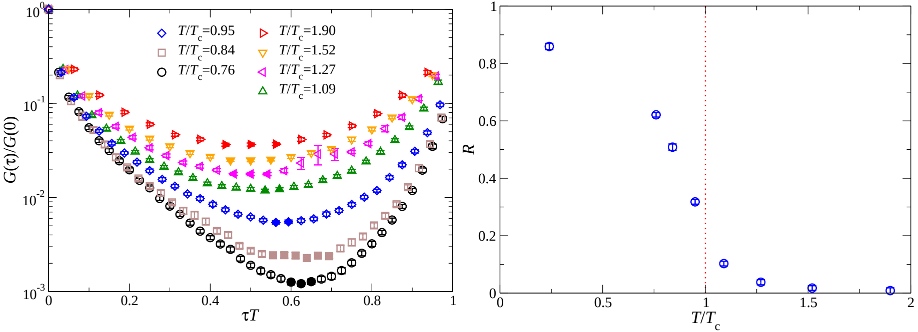

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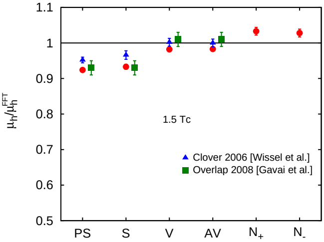
Screening length above T_c

Above T_c, chiral partners have equal screening masses, and screening masses quickly come close to the value for theory of free quarks. However, interesting patterns.



S. Datta, S. Gupta, Padmanath M., N. Mathur & J. Maiti, JHEP 1302 (2013) 145.

Perturbation theory: $\mu_{\rm meson} = 2\pi T + 0.14 g^2 T$

M. Laine and M. Vepsalainen, JHEP 02 ('04) 004.

Thermal observables from lattice

Lattice Matsubara correlators remain the only direct source for extracting thermal observables from QCD at moderately high temperatures.

- Viscosity of the plasma, from energy-momentum tensor correlators.

  Karsch & Wyld '87; Nakamura & Sakai '05; Meyer '06-'09.

- Thermal dilepton rate and electric conductivity of the plasma, from vector current correlators.

  Gupta '04; Aarts et al., '06–; Karsch et al., '02-'11;

- Heavy quark diffusion in plasma, in the static limit, from electric field correlators.

  Banerjee et al. '12; Francis et al. '13;

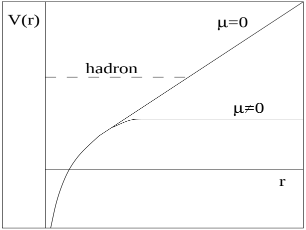
- Quarkonia in QGP.

Heavy quarkonia in gluon plasma

Heavy quarkonia ($\bar{c}c, \bar{b}b$): important probes of quark-gluon plasma

Qualitatively, Screening in plasma
$\Longrightarrow$ reduced binding between $\bar{c}c$

T. Matsui & H. Satz, Phys. Lett. B 178 (1986) 416.



What do we know about quarkonia in equilibrium plasma?

Phenomenology in heavy ion collisions much harder

- Production: color octet passing through plasma?
- Coherent energy loss
- Time evolution of plasma
- Regeneration

See, e.g., S. Datta, 1403.8151.

Lattice analysis

- A properly posed question: how do the quarkonia peaks in dilepton channel get modified at finite temperature?
- Calculate Matsubara correlator $G(t) = \langle \bar{c}\gamma_i c(t)\ \bar{c}\gamma_i c(0)\rangle$
- $G(t_i) = \sum_j K(\omega_j, t_i)\ \rho(\omega_j)\ \Delta\omega \qquad K(\omega_j, t_i) = \frac{1}{\pi}\ \frac{\cosh(\omega_j(\tau_i - 1/2T))}{\sinh(\omega_j/2T)}$
- Extraction of $\rho(\omega)$ Highly unstable in the discretized theory and with $G(\tau)$ of finite accuracy
- Bayesian methods: use prior information to keep in check uncontrolled directions in search space.
- "Maximum entropy method"

M. Jarell & J. Gubernatis, Phys.Rep. 269('96) 133.
M. Asakawa & T. Hatsuda, Prog. Pert. Nucl. Phys. 46('01) 459.

Widely used for charmonia.

Maximum entropy method

- Maximize $\alpha S - \frac{1}{2}\chi^2$,
  $S = \sum_i \Delta\omega \left(\rho(\omega_i) - \rho_0(\omega_i) - \rho(\omega_i) \ln \frac{\rho(\omega_i)}{\rho_0(\omega_i)}\right)$
- $\rho(\omega) > 0$ for $\omega > 0$: parametrize $\rho = \rho_0 \; exp(a(\omega))$.
- Solution unique.
- Write $K(\omega_i, t_j) = U_{\omega_i, t_k} \; K^d_{t_k} V^T_{t_k, t_i}$
- Denoting columns of U as u^{t_i}, can expand

$$a(\omega) = \sum_{i=1}^{N_t} c_i u^i + \sum_{i=N_t+1}^{N_w} \tilde{c}_i \tilde{u}^i$$

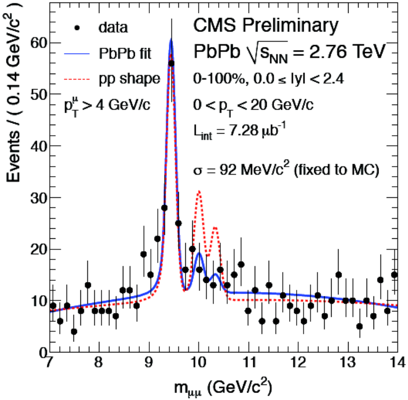
  where $(u^i, \tilde{u}^j)$ form an orthonormal basis in the N_w dimensional space.
- Now the maximization equation gives
  $\alpha c_i = K^d_i V^T_{it} C^{-1}_{tt'} (F(t') - G(t')), \qquad \tilde{c}_j = 0$
- So we can search for the solution in the N_t dimensional space (*or, singular space of K*).

R. K. Bryan, Eur. Biophys. J. 18 (1990) 165.

Charmonia in QGP

- First studies: 1S charmonia survive till quite deep in plasma, while the 1P states dissolve early.
  Datta, Karsch, Petreczky, Wetzorke ('04); Asakawa & Hatsuda ('04)
- Qualitatively similar results in recent dynamical studies.
  G. Aarts, et al., Phys.Rev.D 76('07)094513. S. Borsanyi, et al., arXiv:1401.5940.
- But correlators can be consistent with other scenarios.
  Umeda, 2007; Mocsy and Petreczky, 2007-2009.
- A recent, detailed quenched study found significant modification of J/ψ peak well before 1.5 T_c.
  H. Ding, et al., Phys. Rev. D 86 ('12) 014509.
- In the plasma, the vector correlator develops a transport peak at low ω, associated with heavy quark diffusion.
- Diffusion coeficient calculated on lattice using HQET.
  Francis, et al., arXiv:1311.3759.
  Banerjee, Datta, Gavai, Majumdar, Phys. Rev. D 85 (2012) 014510.
- Estimate from Ding et al. at 1.5 T_c consistent with the above: static approximation okay for charm at 1.5 T_c.

Bottomonia in QGP



Sharp $\Upsilon(1S)$ peak in 2.76 TeV Pb-Pb collision, but excited states of Υ highly suppressed.

S. Chatrchyan, et al., Phys. Rev. Lett. 109 (2012) 222301.

Bottomonia from lattice

- Study of bottomonia difficult as large discretization error $(m_b a \sim 1)$
- Study using NRQCD: $\Upsilon(1S)$ and $\eta_b(1S)$ survive till $> 2T_c$
  However, large width: $\sim$ 300 MeV at 1.5 T_c

  G. Aarts, et al., JHEP 1111 (2011), 103; JHEP 1312 (2013) 064.
- χ_{b_0}, χ_{b_1} drastically modified in the plasma.

  Aarts et al.
- Much smaller width of $\Upsilon(1S)$ found in a recent study.

  S. Kim, P. Petreczky and A. Rothkopf, arXiv:1310.6461.
- Would be good to have a different kind of analysis. Even qualitative insight will be valuable.

Relativistic bottom in gluon plasma

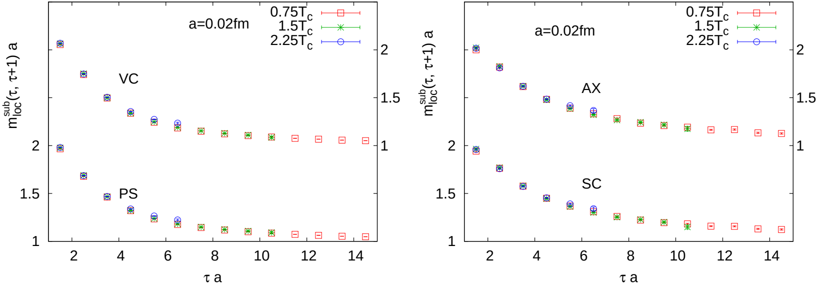
- NRQCD: discretization error difficult to control.
- We study relativistic bottom.
- Gluonic plasma: use sufficiently fine lattices, $1/a = 9.5$ and 12.7 GeV, and nonperturbatively $\mathcal{O}(a)$ improved bottom action.
- For thermal decay of bottomonia, thermal quarks not expected to be important.

  D. Kharzeev & H. Satz, Phys. Lett. B334 (1994) 155
- Fermilab method: mass to be tuned from dispersion relation, rotate fields $\Psi \to (1 + d\vec{\gamma} \cdot \vec{D})\Psi$.

  El-khadra, Kronfeld & Mackenzie, Phys.Rev. D 55 ('97) 3933.
- At a=1/9.7 GeV, $72^3 x N_t$ lattices with $N_t = 48$, 24, 20, 16 for $T/T_c = 0.75$, 1.5, 1.8, 2.25.
- Finer lattice: in progress.

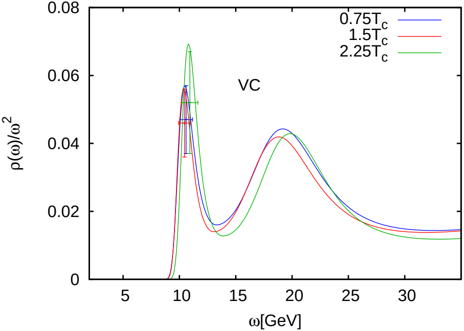
"Subtracted" Local masses

Diffusion peak at low $\omega \Longrightarrow$
Study "subtracted correlator" $G(\tau) - G(1/2T)$
"Local masses" : if correlation caused by a single stable particle, what is its mass?



No dramatic change on crossing T_c.

Spectral function using MEM

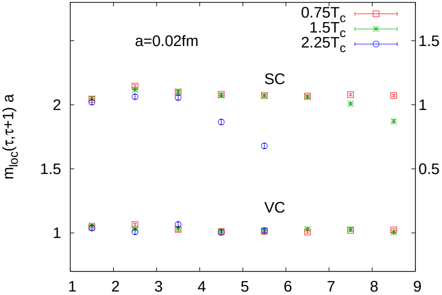


ρ using maximum entropy method and free theory result as prior information. Very little change up to 2.25 T_c.
Not suitable for extraction of width.

Smeared correlators

Use Gaussian smeared currents, to isolate the behavior of the lowest state

Note: this current does not directly connect to dilepton channel.



A (questionable) Breit-Wigner form gives $\Gamma < 0.1T$ for $\Upsilon(1S)$ at 1.5 T_c.

Spatial "screening" correlator

"Screening correlator": measures screening of $\bar{b}\Gamma b$ source.
For charmonia, rapid change after T_c in N_f=2+1 QCD

Karsch, Laermann, Mukherjee, Petreczky, PRD 85('12) 114501

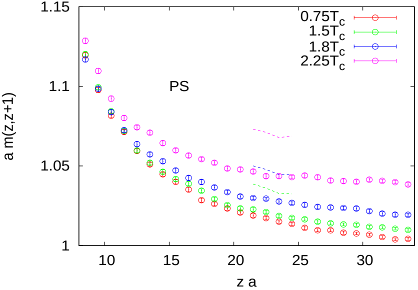
Similar results earlier in quenched theory.

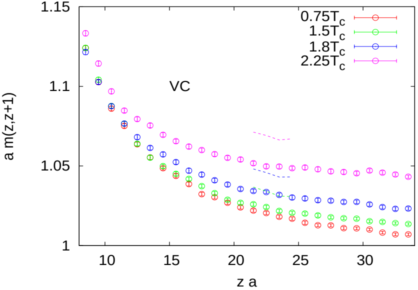
Datta, Karsch, Petreczky, Wetzorke, PRD ('04)

Difficult to interpret in terms of modification of the spectral function.
E.g., one can have a spectral function peak
$\delta(\omega^2 - A^2(T)\vec{p}^2) \Rrightarrow m_{\rm scr}(T) = m/A(T)$

Screening of $\bar{b}b$ charges: 1S

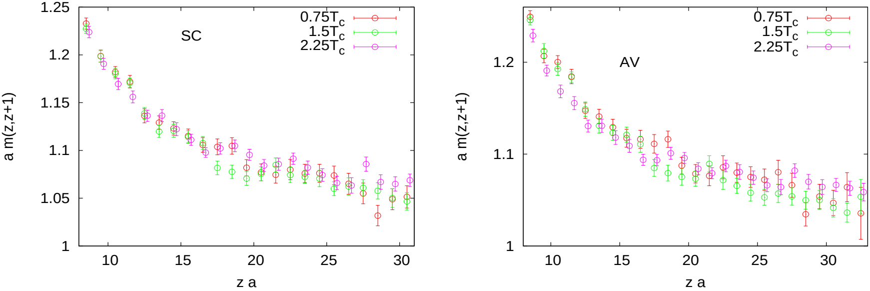
What does the screening correlator tell us about $\bar{b}b$ mesons?





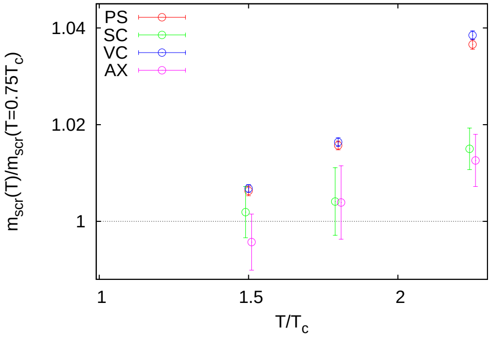
Very minor modification of screening mass at 1.5 T_c, however, clear temperature effect already at 1.8 T_c. Screening mass very different from free theory even at 2.25 T_c.

Screening of $\bar{b}b$ charges: 1P



Interestingly, the modifications of χ_b screening length are comparable to those of the 1S states η_b and Υ.

Combined plot of screening mass of $\bar{b}b$ sources



Discussion

- I discussed uses of both screening correlators and Matsubara correlators in studying properties of QCD medium.
- In particular, nucleon screening correlators were discussed. Their degeneracy pattern indicated precursor of chiral symmetry restoration in quenched theory just below T_c. Similar study has been reported recently with Matsubara correlators in N_f=2+1 QCD.
- At 1.5 T_c, $\mu_{N^+} = \mu_{N^-}$ is within 4% of free theory value, approaching it *from above*.
- We also presented results from an ongoing study of bottomonia in gluon plasma.
- $\Upsilon(1S)$ survives till above 2 T_c in gluonic plasma, with very little width at 1.5 T_c.
- Larger modification of the χ_b states, but survival of the peak in plasma till temperatures in excess of 1.5 T_c.