Overlap fermions on HISQ gauge configurations

Nilmani Mathur

Department of Theoretical Physics

Tata Institute of Fundamental Research, India

Collaborators : S. Basak, S. Datta, A. Lytle, P. Majumdar, M. Padmanath

(Indian Lattice Gauge Theory Initiative)

Perspectives and challenges in Lattice Gauge Theory, TIFR

Overlap Fermions

- Some desirable features:

  - No $O(a)$ **error.**
  - $O(a^2)$ **errors are found to be small (quenched spectrum study)**
  - **The effective propagator :** $(1-\frac{1}{2}D)D(m)^{-1} = (D_c + ma)^{-1}$
  - $D_c = D/(1 - D/2)$ is chirally symmetric, *i.e.*, $\{\gamma_5, D_c\} = 0$.
  - $D_c + m$ is like in the continuum formalism.
  - Multi-mass algorithm (more than 20 masses -10-15% overhead
  - Renormalization may be relatively simple (e.g. with chiral Ward identity).

- Undesirable feature:

- *-- Cost*
- *Very costly to generate configurations without fixed topology*
- *See talk by Frommer for new developments*

2+1+1 flavoured HISQ configurations

MILC has generated a large number of configurations with Highly Improved Staggered Quarks

$\approx a$ (fm)	m_l/m_s	$N_s^3 \times N_t$	$M_\pi L$	M_π (MeV)	N_{lats}
0.15	1/5	$16^3 \times 48$	3.78	306.9(5)	1021
0.15	1/10	$24^3 \times 48$	3.99	214.5(2)	1000
0.15	1/27	$32^3 \times 48$	3.30	131.0(1)	1020
0.12	1/5	$24^3 \times 64$	4.54	305.3(4)	1040
0.12	1/10	$24^3 \times 64$	3.22	218.1(4)	1020
0.12	1/10	$32^3 \times 64$	4.29	216.9(2)	1000
0.12	1/10	$40^3 \times 64$	5.36	217.0(2)	1029
0.12	1/27	$48^3 \times 64$	3.88	131.7(1)	1000
0.09	1/5	$32^3 \times 96$	4.50	312.7(6)	1011
0.09	1/10	$48^3 \times 96$	4.71	220.3(2)	1000
0.09	1/27	$64^3 \times 96$	3.66	128.2(1)	235 + 467
0.06	1/5	$48^3 \times 144$	4.51	319.3(5)	1000
0.06	1/10	$64^3 \times 144$	4.25	229.2(4)	435 + 227
0.06	1/27	$96^3 \times 192$	3.95	135.5(2)	240

PHYSICAL REVIEW D 87, 054505 (2013)

2+1+1 flavoured HISQ configurations

MILC has generated a large number of configurations with Highly Improved Staggered Quarks

TABLE VIII. r_1/a, af_{p4s}, and am_{p4s} measured on the ensembles with physical strange- and charm-quark masses. These quantities are used to determine the lattice spacing, which is given in the next table. (Note that m_{p4s} is the quark mass corresponding to f_{p4s}.)

$10/g^2$	am_l	am_s	am_c	r_1/a	af_{p4s}	am_{p4s}
5.80	0.013	0.065	0.838	2.059(23)	0.12150(18)	0.02744(9)
5.80	0.0064	0.064	0.828	2.073(13)	0.12042(11)	0.02744(5)
5.80	0.00235	0.0647	0.831	2.089(8)	0.11948(6)	0.02762(3)
6.00	0.0102	0.0509	0.635	2.575(17)	0.09780(12)	0.02139(6)
6.00	0.00507	0.0507	0.628	2.585(19)	0.09614(14)	0.02111(7)
6.00	0.00507	0.0507	0.628	2.626(13)	0.09613(9)	0.02118(5)
6.00	0.00507	0.0507	0.628	2.614(9)	0.09605(7)	0.02113(4)
6.00	0.00184	0.0507	0.628	2.608(8)	0.09530(5)	0.02130(2)
6.30	0.0074	0.037	0.440	3.499(24)	0.07093(11)	0.01482(5)
6.30	0.00363	0.0363	0.430	3.566(14)	0.06953(6)	0.01467(3)
6.30	0.0012	0.0363	0.432	3.565(13)	0.06865(4)	0.01462(2)
6.72	0.0048	0.024	0.286	5.342(16)	0.04660(7)	0.00918(3)
6.72	0.0024	0.024	0.286	5.376(14)	0.04545(5)	0.00896(2)

PHYSICAL REVIEW D 87, 054505 (2013)

Can we combine these two?

Overlap over HISQ configurations

- to get best from both worlds!
- can we simulate light to heavy ?

A brief history of mixed action approaches

- Domain wall valence on ASQTAD sea : LHPC,
  R. Edwards et. al. (LHPC), Phys.Rev.Lett. 96 (2006) 052001

- Overlap valence on domain wall : Chi-QCD Phys.Rev. D82 (2010) 114501

- Overlap valence on clover sea : S. Durr et. al., PoS LAT2007 (2007) 113

- Overlap valence on twisted-mass fermion sea : K. Cichy, G. Herdoiza, and K. Jansen, Acta Phys.Polon.Supp.2 (2009) 497

Overlap fermions on 2+1+1 Flavors HISQ Configurations

- Lattices used for this study :

  HISQ gauge configurations from MILC

  24^3 x 64 , a = 0.12 fm, m_l/m_s = 1/5, $m_\pi L$ = 4.54, m_π = 305 MeV

  32^3 x 96 , a = 0.089 fm, m_l/m_s = 1/5, $m_\pi L$ = 4.5, m_π = 312 MeV

  48^3 x 144 , a = 0.058 fm, m_l/m_s = 1/5, $m_\pi L$ = 4.51, m_π = 319 MeV

  PHYSICAL REVIEW D 87, 054505 (2013) (MILC)

- **HYP smearing on gauge fields**
- **Both point source and coulomb gauge fixed wall source are used**
- **No of eigenvectors projected : 350 (a = 0.012 fm)**

  : 350 (a = 0.09 fm)

  : 75 (a = 0.058 fm)

Rest mass Vs Kinetic mass

Charm mass is tuned by meson kinetic mass and not from rest mass ……a la FermiLab formulation

El-khadra et al, PRD55, 3933(1997)

Expanding the energy momentum relation in powers of pa

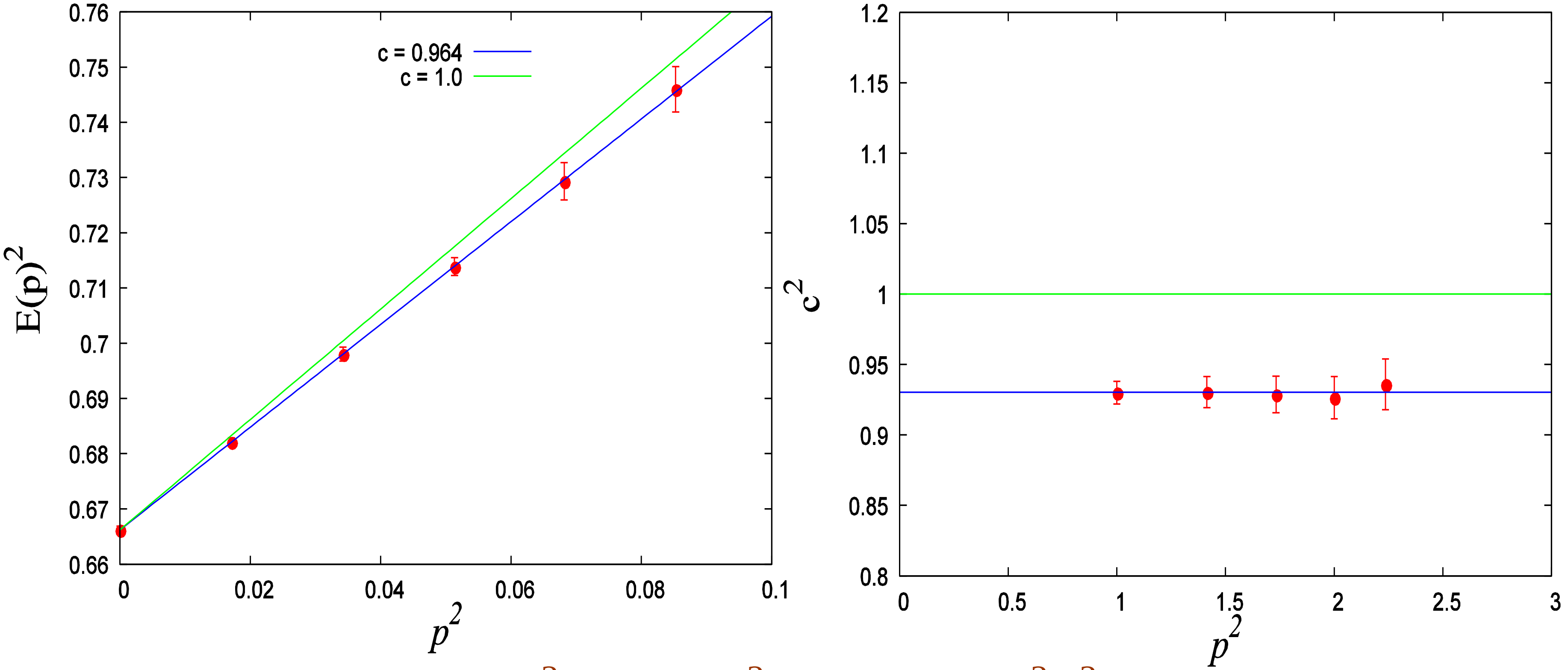
$$E(p)^2 = M_1^2 + \frac{M_1}{M_2}\mathbf{p}^2 + O(\mathbf{p}^4)$$

$$= M_1^2 + c^2 p^2 \qquad |\mathbf{p}| << m_0, 1/a$$

Rest mass : $M_1 = E(\mathbf{0})$

Kinetic mass : $M_2 = M_1/c^2$

Dispersion relation (at charm mass)



$$E^2(p) = E^2(p=0) + p^2c^2$$

Finite momentum wall source is used to project to particular momentum state which reduce errorbars substantially.

Lattice spacings and tuning of charm and strange masses

Lattice spacings are calculated by Omega(sss) mass = 1672 GeV

48^3 x 144 : 0.0582(5) fm
32^3 x 96 : 0.0877(10) fm
24^3 x 64 : 0.1192(14) fm
which are quite consistent with lattice spacings determined by MILC

- Strange mass is tuned by setting pseudoscalar $\underline{s}s$ mass at 685 MeV

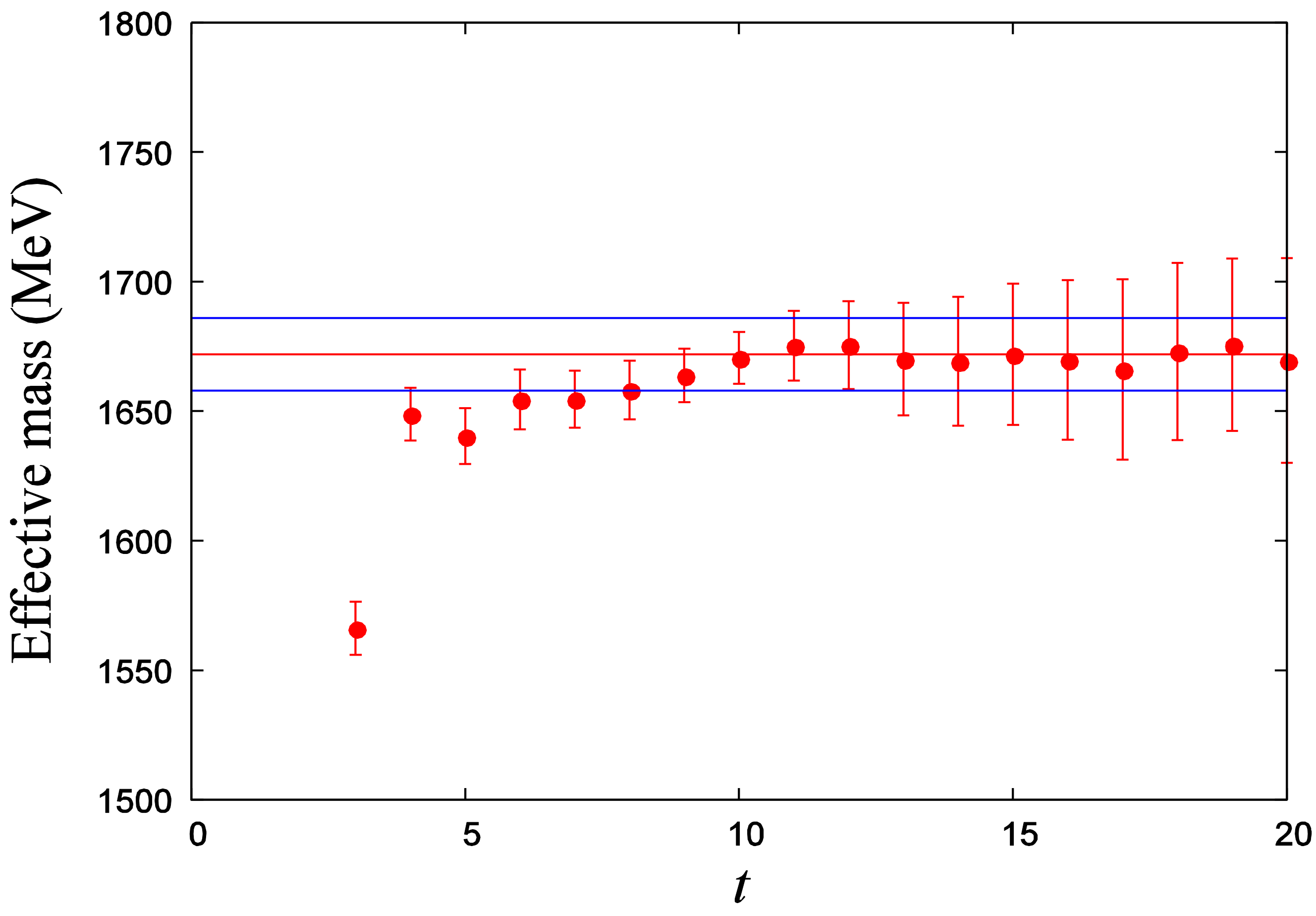
$$m_s a = 0.0738 \ (a = 0.0118\text{fm}) \\ = 0.048 \ (a = 0.0888 \text{ fm}) \\ = 0.028 \ (a = 0.0582\text{fm})$$

Taking m_s = 100 MeV
$m_s a = 0.0450$ (a = 0.0888fm),
= 0.0295(a = 0.0582fm)

- Charm mass is tuned by $\frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$

$$m_c a = 0.527 \ (a = 0.1192\text{fm}) \\ = 0.428 \ (a = 0.0888\text{fm}), \\ = 0.29 \ (a = 0.0582 \text{ fm})$$

Considering kinetic masses of mesons
(a la Fermilab formulation)



Omega(sss) effective mass

Lattice spacings and tuning of charm and strange masses

Lattice spacings are calculated by Omega(sss) mass = 1672 GeV

48^3 x 144 : 0.0582(5) fm
32^3 x 96 : 0.0877(10) fm
24^3 x 64 : 0.1192(14) fm
which are quite consistent with lattice spacings determined by MILC

- Strange mass is tuned by setting pseudoscalar $\underline{s}s$ mass at 685 MeV

$m_s a$ = 0.0738 (a = 0.0118fm)
= 0.048 (a = 0.0888 fm)
= 0.028 (a = 0.0582fm)

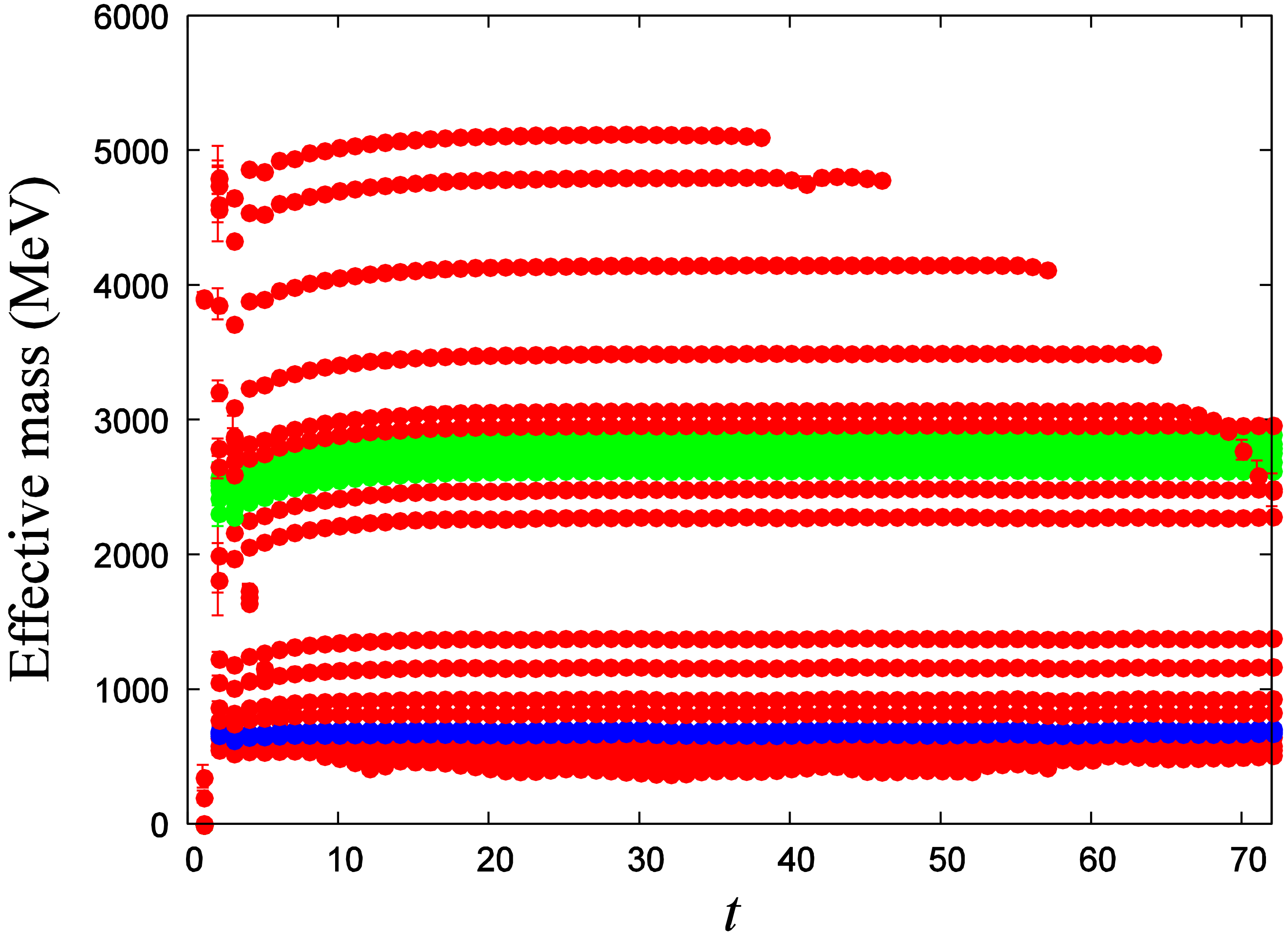
Taking m_s = 100 MeV
$m_s a$ = 0.0450 (a = 0.0888fm),
= 0.0295(a = 0.0582fm)

- Charm mass is tuned by $\frac{1}{4}\left(m_{\eta_c} + 3m_{J/\psi} \right)$

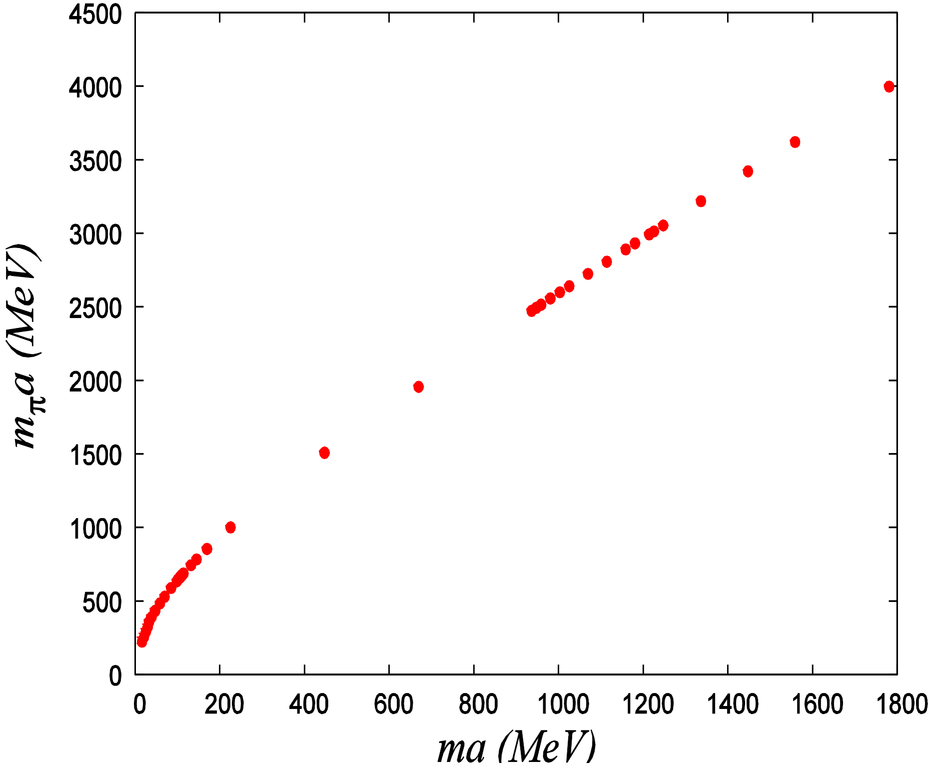
$m_c a$ = 0.527 (a = 0.1192fm)
= 0.428 (a = 0.0888fm),
= 0.29 (a = 0.0582 fm)

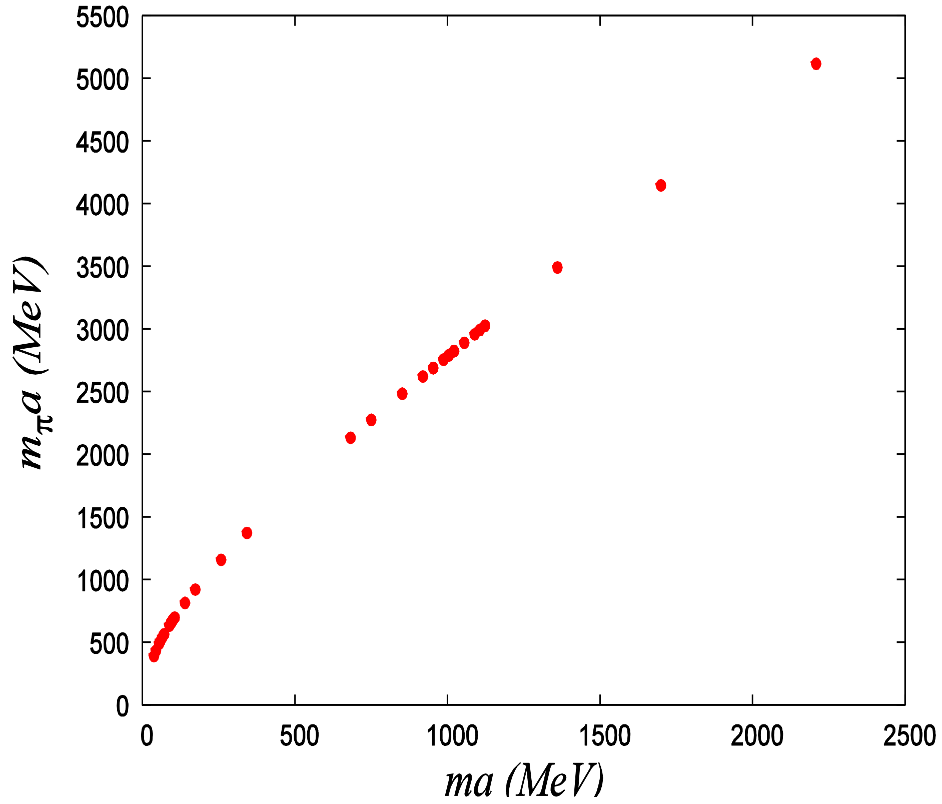
Considering kinetic masses of mesons
(a la Fermilab formulation)

Pseudo-scalar eff. masses

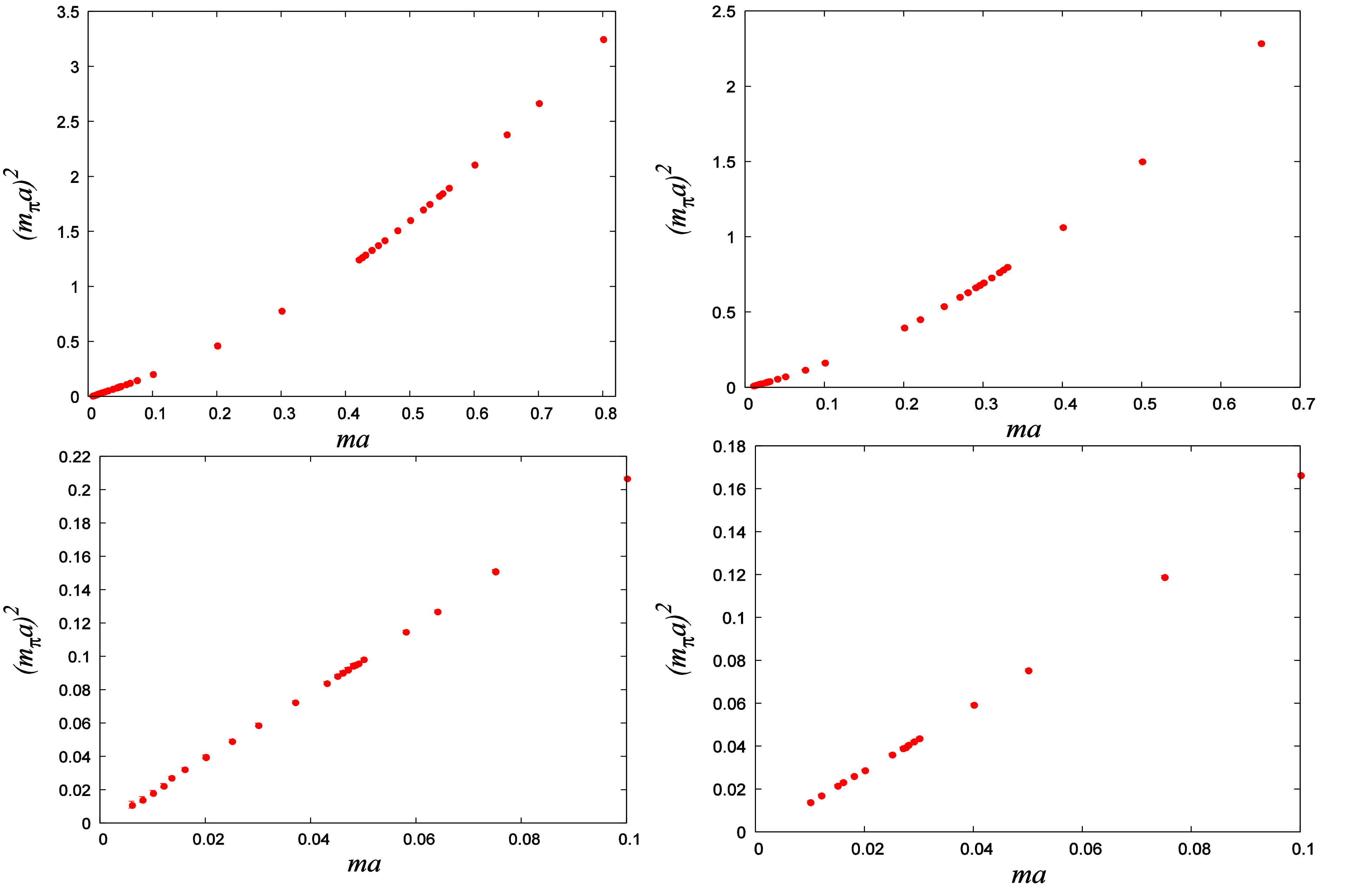


Pseudoscalar meson mass

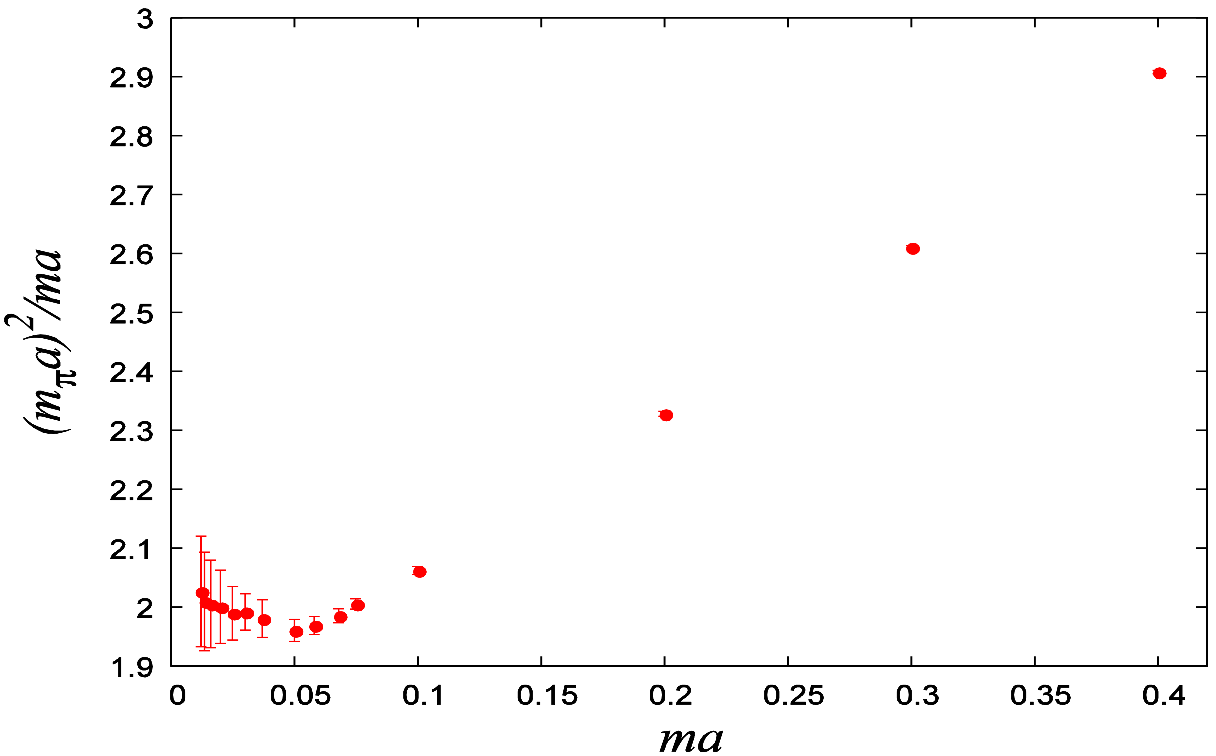




Pseudoscalar meson mass



Psedoscalar chiral log (32³ x 96, a= 0.09fm)

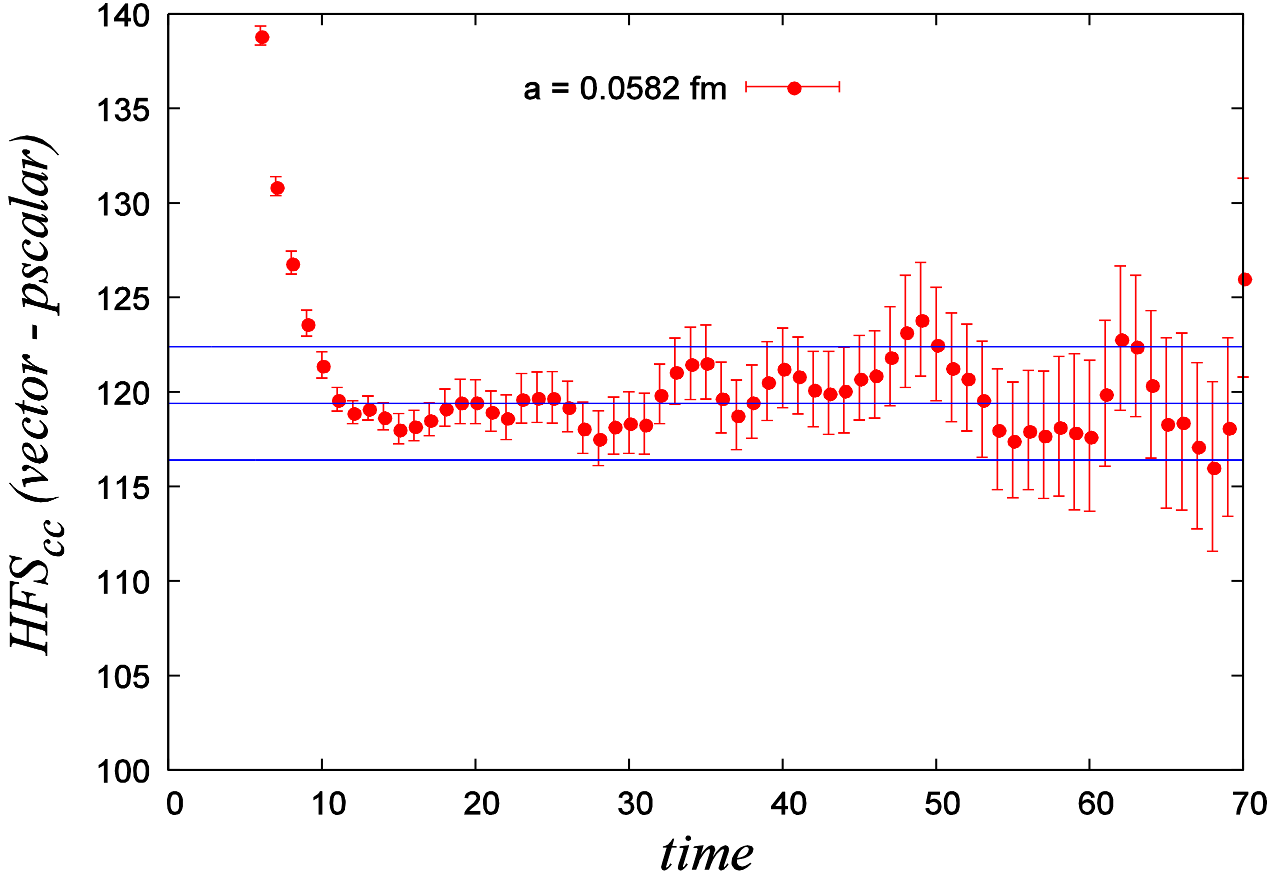


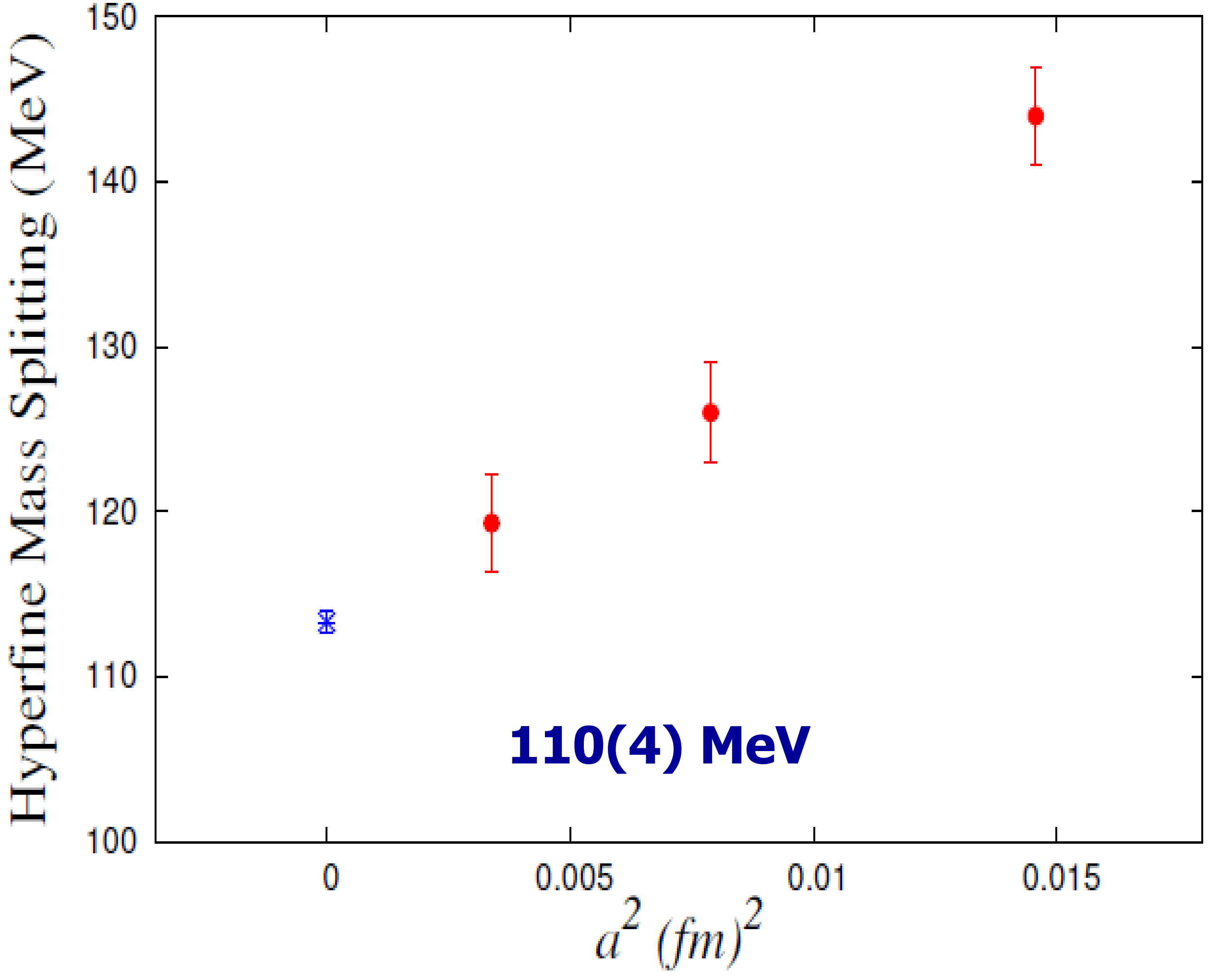
$$
\begin{aligned}
m_\pi^2 = 2B_0\hat{m}\Big\{ & 1 + \ln\left(\frac{m_\pi^2}{\mu^2}\right)\left[\frac{m_\pi^2}{(4\pi f)^2} - \frac{\tilde{\Delta}_{ju}^2(3\tilde{m}_X^2 - m_\pi^2)}{3(4\pi f)^2(\tilde{m}_X^2 - m_\pi^2)} + \frac{\tilde{\Delta}_{ju}^4\tilde{m}_X^2}{3(4\pi f)^2(\tilde{m}_X^2 - m_\pi^2)^2}\right] \\
& - \ln\left(\frac{\tilde{m}_X^2}{\mu^2}\right)\left[\frac{\tilde{m}_X^2}{3(4\pi f)^2} - \frac{2\tilde{\Delta}_{ju}^2\tilde{m}_X^2}{3(4\pi f)^2(\tilde{m}_X^2 - m_\pi^2)} + \frac{\tilde{\Delta}_{ju}^4\tilde{m}_X^2}{3(4\pi f)^2(\tilde{m}_X^2 - m_\pi^2)^2}\right] \\
& - \frac{16m_\pi^2}{f^2}\Big[L_4(\mu) + L_5(\mu) - 2L_6(\mu) - 2L_8(\mu)\Big] - \frac{32m_K^2}{f^2}\Big[L_4(\mu) - 2L_6(\mu)\Big] + \frac{a^2}{f^2}L_{ma^2}(\mu) \\
& - \left(\frac{32\Delta_{ju}^2}{f^2} + \frac{16\Delta_{rs}^2}{f^2}\right)\Big[L_4(\mu) - 2L_6(\mu)\Big] - \frac{\tilde{\Delta}_{ju}^2}{(4\pi f)^2} + \frac{\tilde{\Delta}_{ju}^4}{3(4\pi f)^2(\tilde{m}_X^2 - m_\pi^2)}\Big\}.
\end{aligned}
$$

arXiv:hep-lat/0611003v2

Charm Physics

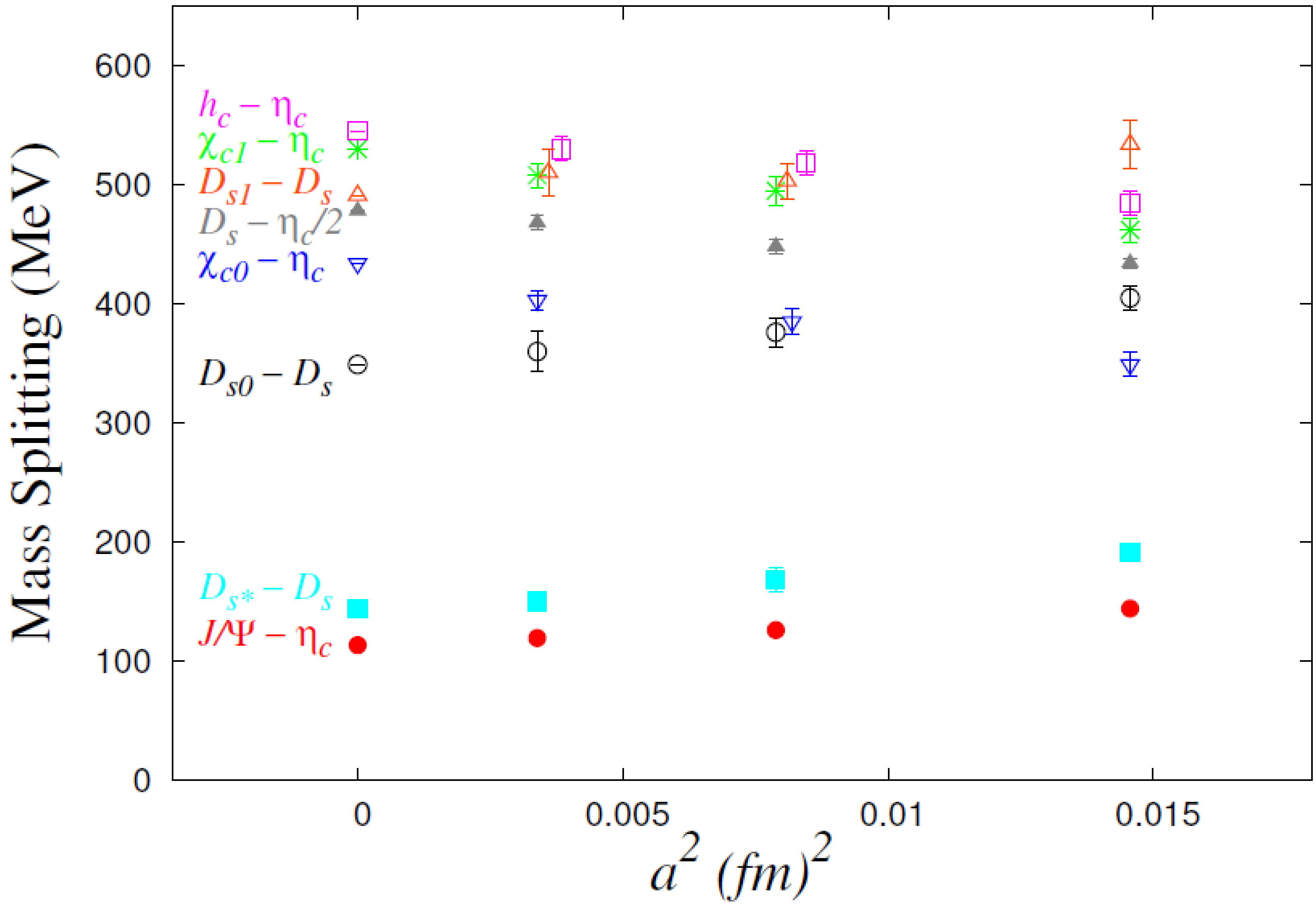
Effective mass for HFS (48^3 x 144, a= 0.0582fm)



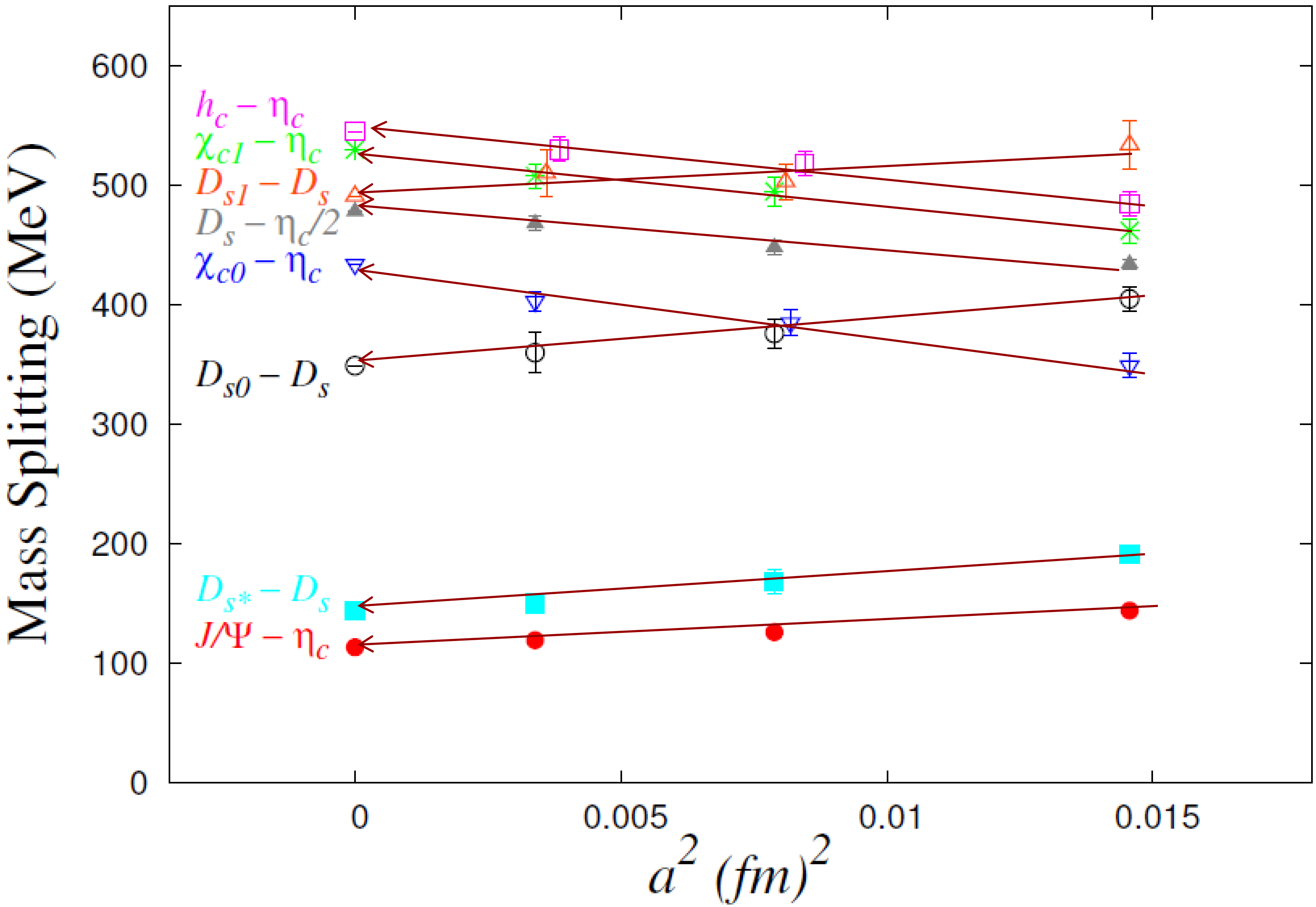


Hyperfine Mass Splitting (MeV)
150
140
130
120
110
100
0
0.005
0.01
0.015
a^2 (fm)^2
110(4) MeV

Meson mass splittings



Meson mass splittings



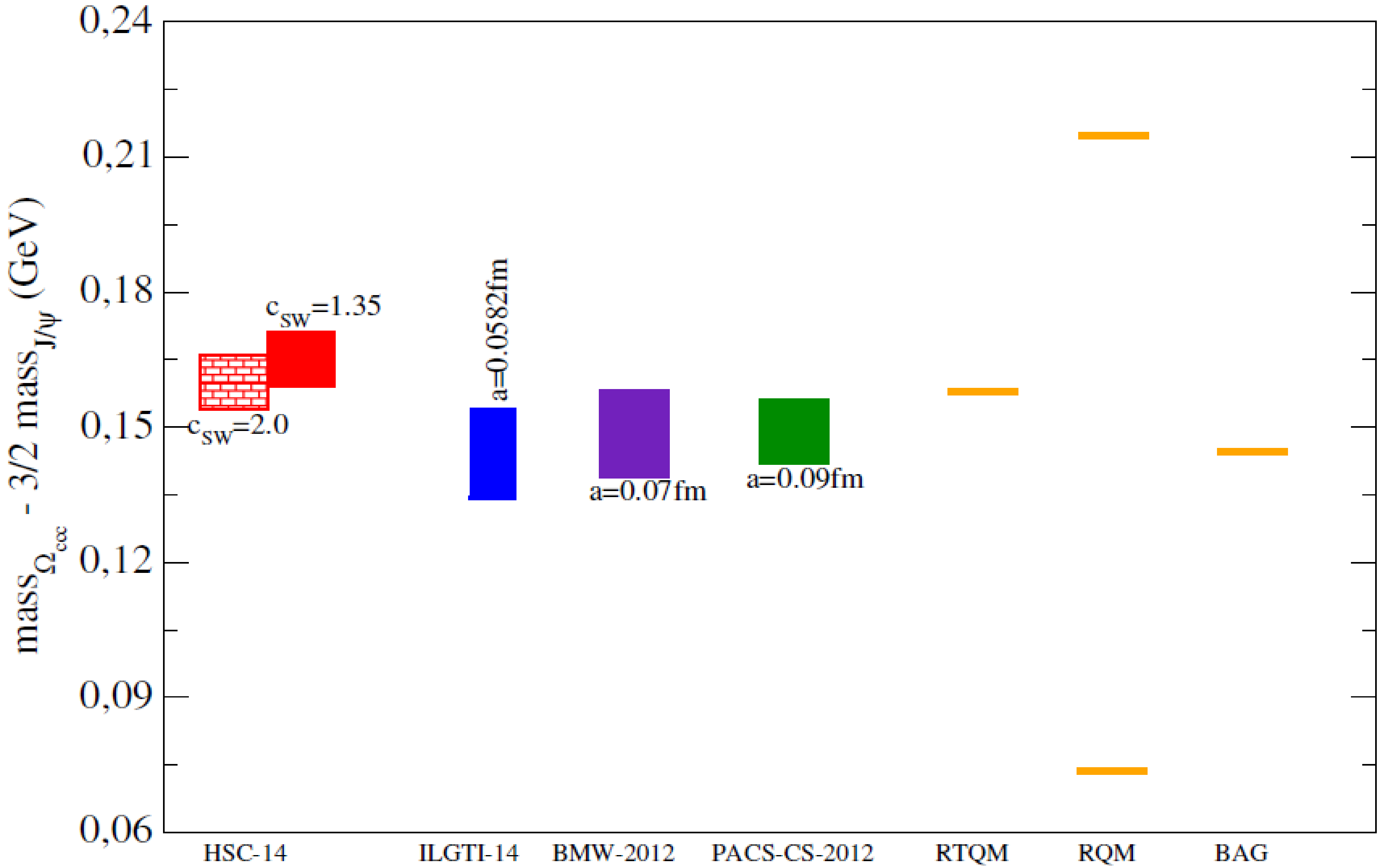
Triply -charmed baryon $\Omega_{ccc}(3/2^+)$

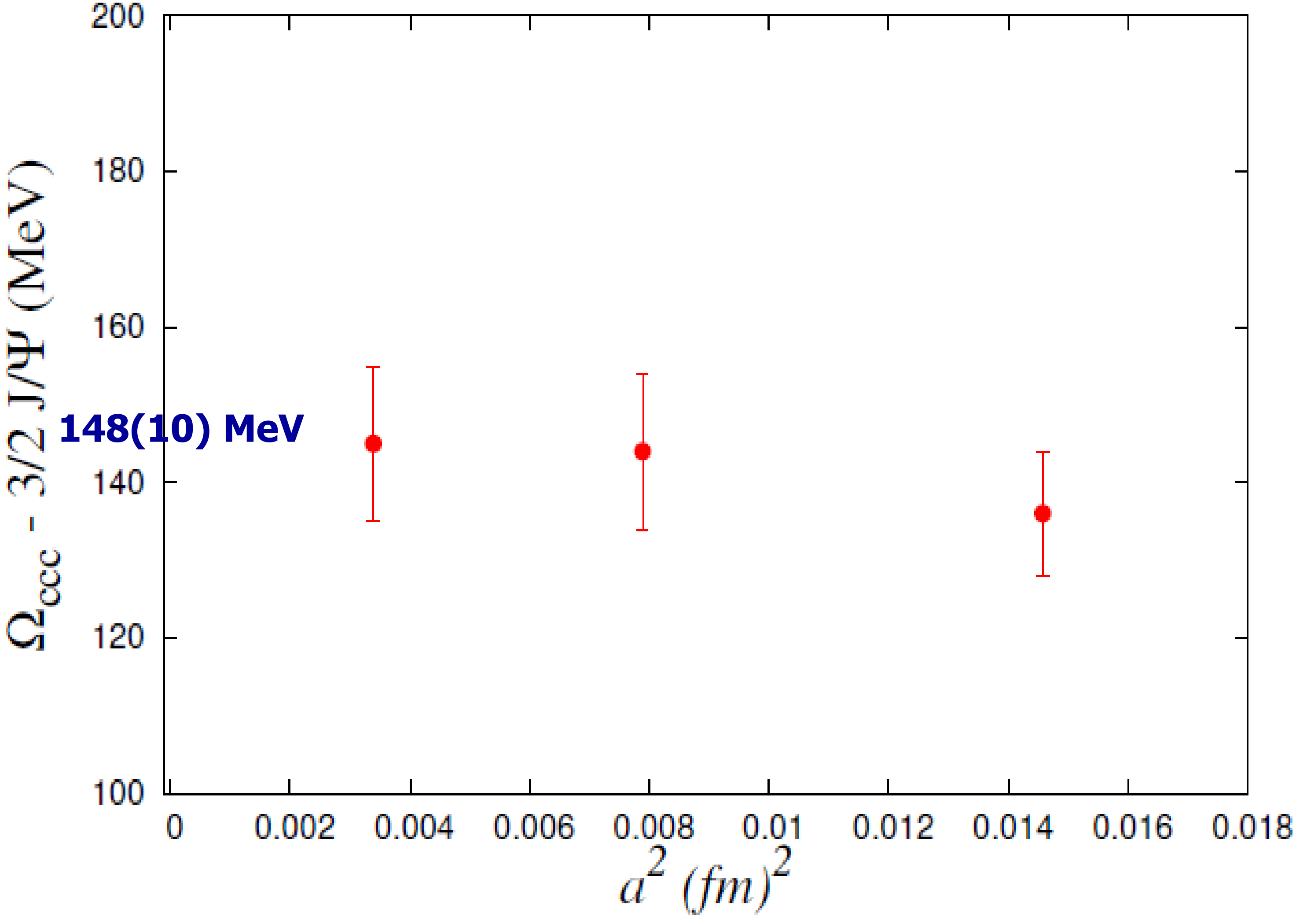
- Δ (uuu) s exist
- Ω (sss) exists
- Why not Ω (ccc) ?
- Charmonia analogue of baryons
- The triply-charmed baryons may provide a new window for understanding the structure of baryons –Bjorken 1976
- Production : don't know really (Just may be possible in Super Belle? LHCb – need extensive search due to background)

$$\Omega_{ccc}^{++} \to \Omega_{sss}^{-} + 3\mu^{+} + 3\nu_{\mu}$$

$$\Omega_{ccc}^{++} \to \Omega_{sss}^{-} + 3\pi^{+}$$

Triply-charmed $\Omega_{ccc}(3/2^+)$ baryon

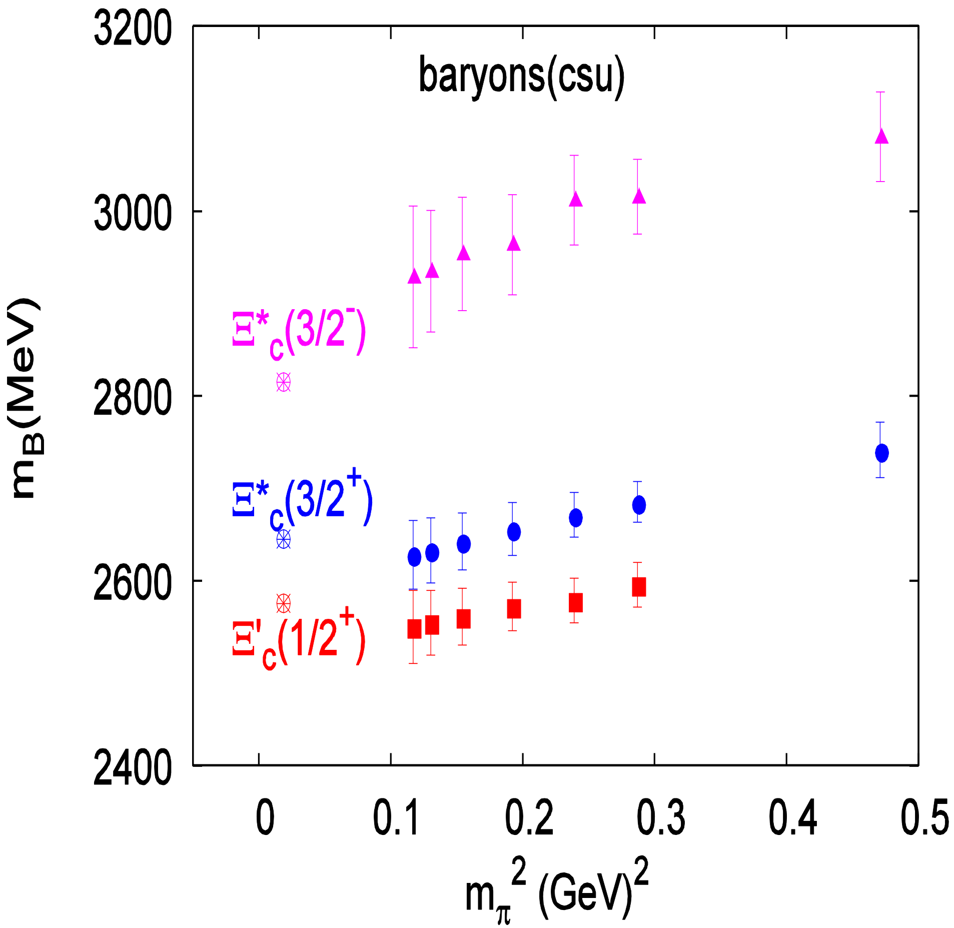


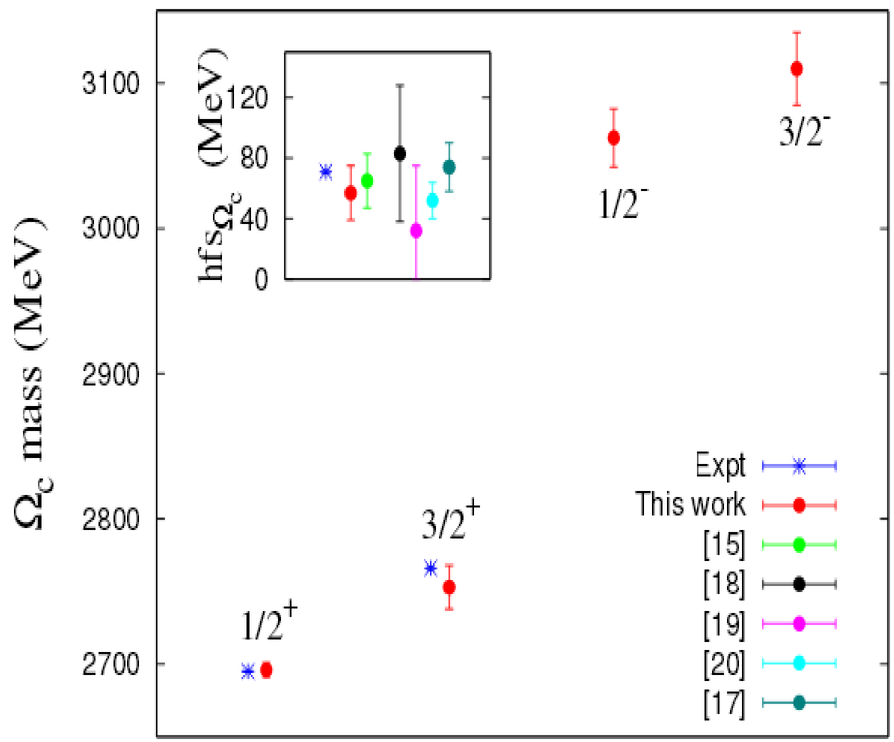


148(10) MeV
Ω_{ccc} - 3/2 J/Ψ (MeV)
a^2 $(fm)^2$
200
180
160
140
120
100
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018

Heavy baryons (Singly charmed)

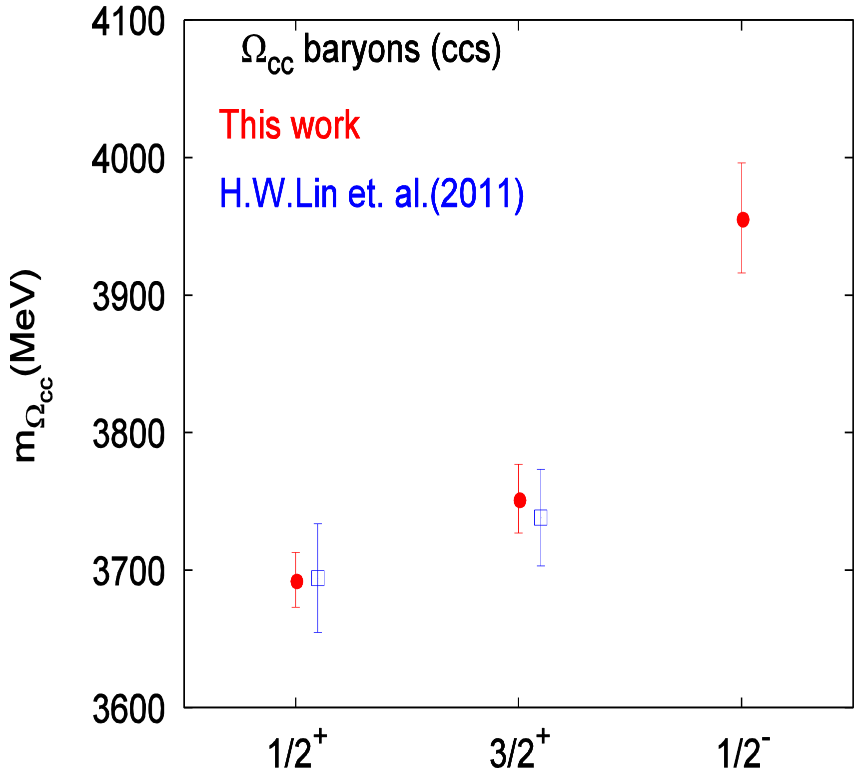
(32^3 x 96, a = 0.09fm)

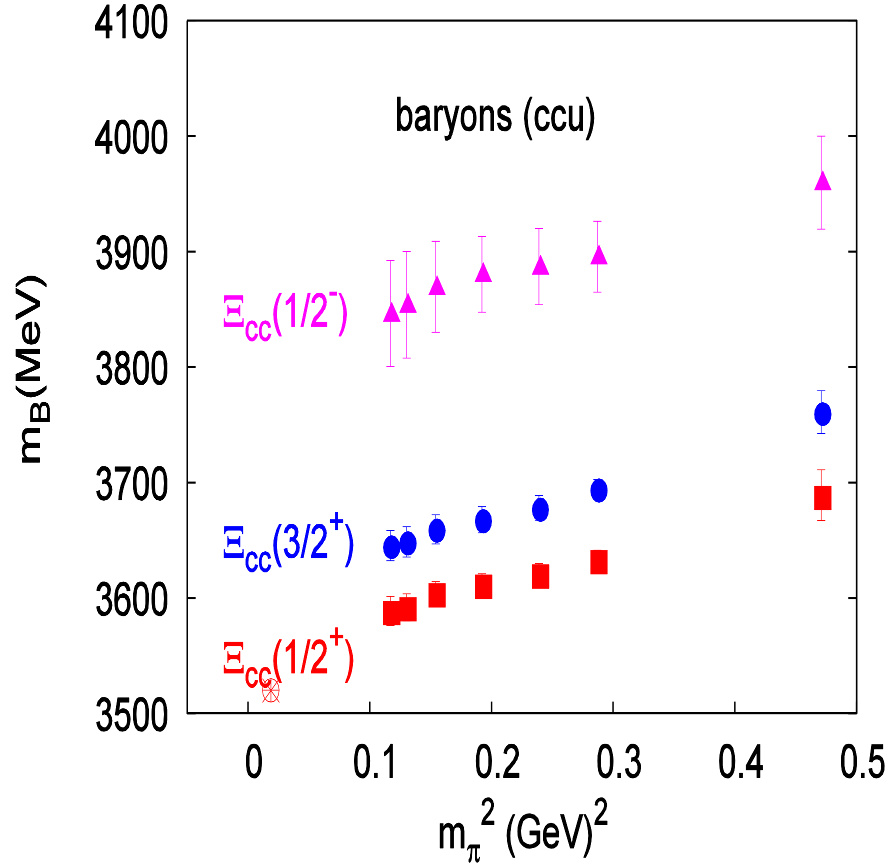




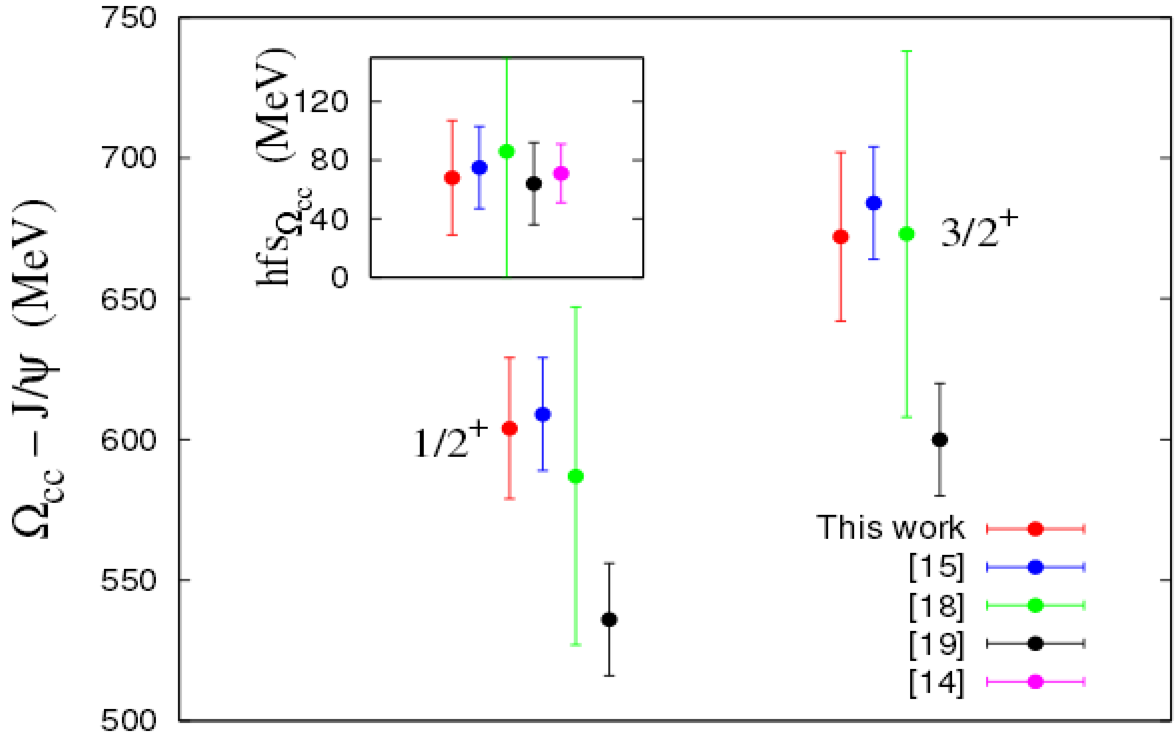
Heavy baryons (Doubly charmed)

(32^3 x 96, a = 0.09fm)





Doubly-charmed Ω baryons



Decay constants

- $\langle 0|A_\mu|D_s(p)\rangle = f_{D_s}\, p^\mu$
- $\langle 0|V_\mu|D_s^*(p,\lambda)\rangle = f_{D_s^*}\, M_{D_s^*}\, \varepsilon_\mu^\lambda$

$$\{A_\mu, V_\mu\} = \{Z_A \bar{s}\gamma_\mu\gamma_5 c, Z_V \bar{s}\gamma_\mu c\}$$

- **From PCAC :**

$$M_{\rm PS}^2 f_{\rm PS} = (\mu_1 + \mu_2)|\langle 0|P^1(0)|{\rm PS}\rangle|$$

$\mu_{1,2}$ are the bare quark masses

$$Z_m Z_P = 1$$

$$f_{D_s} = \frac{(m_c + m_s)}{m_{D_s}^2}\sqrt{2A\, m_{D_s}}$$

$$x \equiv |\langle 0|P|D_s\rangle|,\ \overline{2A} = x^2/m_{D_s}$$

Both i) point-point propagators
and ii) wall-point with wall-wall propagators were utilized

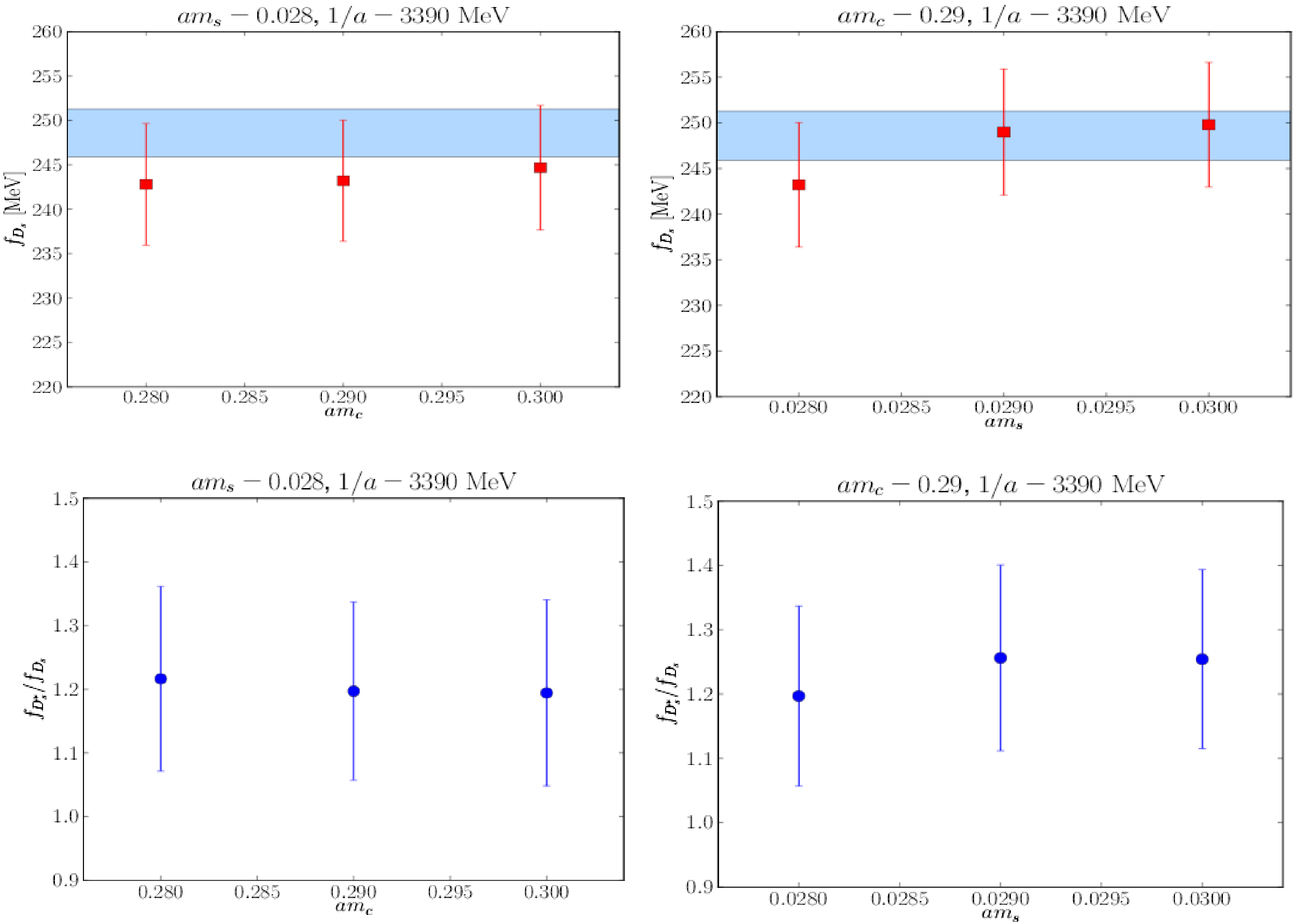
The ratio $\frac{f_{D^*s}}{f_{Ds}}$

- $\langle 0|\bar{c}(0)\gamma_\mu\gamma_5 q(0)|D_q(p)\rangle = f_{D_q} p_\mu$
- $\langle 0|\bar{c}(0)\gamma_\mu q(0)|D_q^*(p,\lambda)\rangle = f_{D_q^*} m_{D_q^*} e_\mu^\lambda$
- $m_{D_s} f_{D_s} = Z_A |\langle 0|A4|D_s\rangle|$
- $Z_A = \frac{m_{D_s} f_{D_s}}{\sqrt{2A\, m_{D_s}}}$
- $f_{D_s^*} = \frac{Z_A}{m_{D_s^*}} |\langle 0|V|D_s^*\rangle|$

It is better to calculate the ratio $\frac{f_{D^*s}}{f_{Ds}}$ where the effect of various normalization factors and mixed action effect will be smaller

$$\frac{Z_A\, f_{D_s^*}}{Z_V\, f_{D_s}} \approx \frac{f_{D_s^*}}{f_{D_s}} \qquad Z_A/Z_V = 1$$

massless chiral fermions



$am_s = 0.028,\ 1/a = 3390$ MeV
f_{D_s} [MeV]
260
255
250
245
240
235
230
225
220
0.280
0.285
0.290
0.295
0.300
am_c
$am_c = 0.29,\ 1/a = 3390$ MeV
f_{D_s} [MeV]
0.0280
0.0285
0.0290
0.0295
0.0300
am_s
$am_s = 0.028,\ 1/a = 3390$ MeV
$f_{D_s^*}/f_{D_s}$
1.5
1.4
1.3
1.2
1.1
1.0
0.9
am_c
$am_c = 0.29,\ 1/a = 3390$ MeV
$f_{D_s^*}/f_{D_s}$
am_s

Mixed action effects

- Mixed actions : Overlap valence on HISQ sea
- In leading order of mixed action partially quenched staggered chiral perturbation theory with chiral valence quarks :

$$m_{vv'}^2 = B_{\rm ov}(m_v + m_{v'})$$
$$m_{ss'}^2 = B_{\rm HISQ}(m_s + m_{s'}) + a^2\Delta_t$$
$$m_{vs}^2 = B_{\rm ov} m_v + B_{\rm HISQ} m_s + a^2(\Delta_{\rm mix} + \Delta'_{\rm mix})$$

$$\Delta_{\rm mix} = 16 C_{\rm mix}/f^2$$

- For chirally symmetric valence, it is like partial quenching with one extra parameter in valence-sea mass (Chen, O'Connell, Walker-Loud, hep-lat/0611003, arXiv:0706.0035)

Mixed action effects

- **Mixed actions : Overlap valence on HISQ sea**

$$\delta m^2(m_v) \equiv m_{vs}^2 - m_{ss}^2/2 = B_{\rm ov} m_v + a^2(\Delta_{\rm mix} + \Delta'_{\rm mix})$$

Wilsonized staggered propagator :

$$G_{\psi_s}(x,y) = \Omega(x)\Omega^\dagger(y) \times G_\chi(x,y)$$

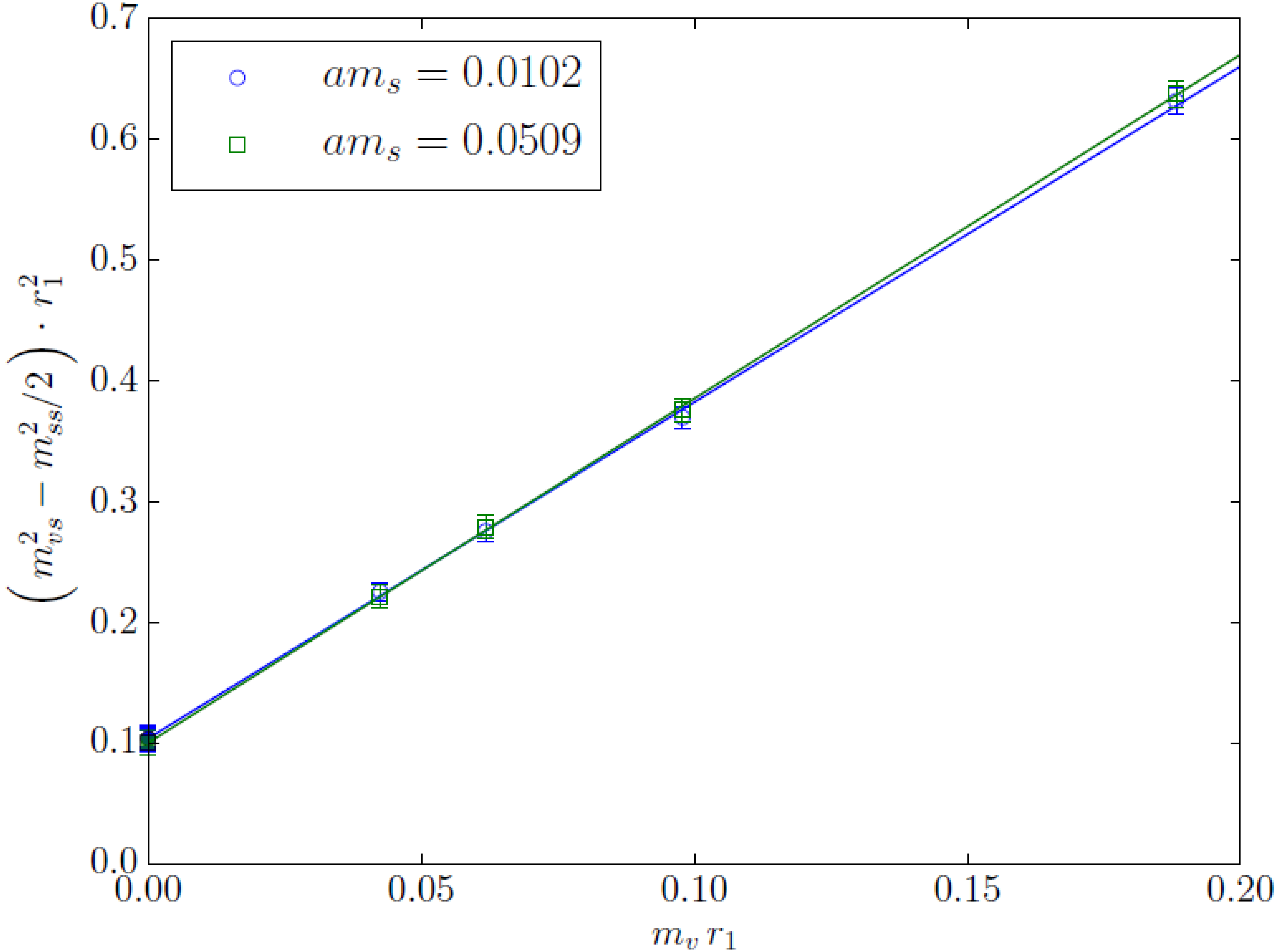
Kawamoto-Smit transformation

$$\Omega(x) = \prod_\mu (\gamma_\mu)^{x_\mu}$$

Fitting form :

$$C_{vs}^\Gamma(t) \sim [A + (-1)^t B] \cosh(m_{vs}(t - T/2))$$

Mixed action effects



Mixed action effects

- **Mixed actions : Overlap valence on HISQ sea**

$$r_1^2 a^2 (\Delta_{\rm mix} + \Delta'_{\rm mix}) = 0.104(9)$$

$$a^2 \Delta_{\rm mix} \simeq (140\ {\rm MeV})^2 \quad (a = 0.121\ {\rm fm}).$$

About half than domain-wall on asqtad!

Summary and outlook

- ✓ Overlap valence on 2+1+1 flavour HISQ configurations **is a promising approach** to do lattice QCD simulation with light, strange and charm quark together in same lattice formulation.
- ✓ However, we found that the dispersion relation with overlap fermions, at charm mass, is not better than that of clover fermions found in literature.
- ✓ Kinetic masses of mesons are used instead of pole masses to tune charm quark mass. Dispersion relation improved at kinetic masses.
- ✓ Preliminary results are encouraging, particularly, the hyperfine splitting for charmonium. We are studying meson and baryon spectra in details.
- ✓ We are also studying heavy-light decay constants. Necessary renormalization constant calculations are ongoing.
- ✓ We also calculated the mixing parameter for this mix action approach, and found that its effects is smaller than domain wall on Asqtad approach.

Acknowledgement :

- Computations : ILGTI-TIFR BG/P
- Thanks to MILC collaboration (particularly, S. Gottlieb) for giving access to HISQ configurations

$$D(0) = [1 + \gamma_5 \epsilon(H)]$$

$$\{\gamma_5, D\} = D\gamma_5 D$$

$$\{\gamma_5, D^{-1}(0)\} = \gamma_5$$

$$\tilde{D}^{-1}(0) \equiv [D^{-1}(0) - 1/2]$$

$$\{\gamma_5, \tilde{D}^{-1}(0)\} = 0.$$

$$D(0,\rho) = 1 + \gamma_5 \varepsilon = 1 + \gamma_5 \frac{H_W(\rho)}{\sqrt{H_W{}^2(\rho)}} \approx 1 + \gamma_5 H_W \sum_{i=1}^{n} \frac{b_i}{H_W^2 + c_i}$$

$$D_W(x,y) = \left[\delta_{x,y} - \kappa \sum_{\mu} \{(r - \gamma_\mu) U_\mu(x) \delta_{y,x+\hat{\mu}} + (r + \gamma_\mu) U_\mu^\dagger(x - a\hat{\mu}) \delta_{y,x-\hat{\mu}}\}\right]$$

$$\kappa \equiv \frac{1}{2(-\rho) + 8}$$

$$\kappa > \kappa_c \text{ and } \rho < 2r$$

$$r = 1 \text{ and } \rho = 1.368$$

$$D(m) = \rho D + ma(1 - \frac{1}{2}D) = \rho + \frac{m}{2} + (\rho - \frac{m}{2})\gamma_5 \epsilon(H)$$

$$\hat{\psi} = (1 - \tfrac{1}{2}D)\psi$$

$$D_c = \rho D / (1 - \tfrac{1}{2}D)$$

Overlap with Deflation **arXiv:1005.5424**

$$D(m,\rho)X^{H}_{L,R} = \eta_{L,R} - \sum_{i=1}^{n}(1\mp\gamma_5)\,|\,i\rangle\langle i\,|\,\eta_{L,R}\rangle$$

where,

$$D(0,\rho)\,|\,i\rangle = \lambda_i\,|\,i\rangle; \quad D(0,\rho)\gamma_5\,|\,i\rangle = \lambda_i^{*}\gamma_5\,|\,i\rangle$$

Therefore,

$$X^{H}_{L,R} = D^{-1}(m,\rho)\eta_{L,R} - X^{L}_{L,R}$$

where,

$$X^{L}_{L,R} = \sum_{i=1}^{n}\left[\frac{|\,i\rangle\langle i\,|\,\eta_{L,R}\rangle}{\rho\lambda_i + m(1-\lambda_i/2)} \mp \frac{\gamma_5\,|\,i\rangle\langle i\,|\,\eta_{L,R}\rangle}{\rho\lambda_i^{*} + m(1-\lambda_i^{*}/2)}\right]$$

and

$$\mathrm{X} = (\mathrm{X}^{H}_{L} + X^{H}_{R}) + (\mathrm{X}^{L}_{L} + X^{L}_{R})$$ except for the zero modes.