Magnetic response of isospin-asymmetric QCD matter

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Perspectives and Challenges in Lattice Gauge Theory
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  - lattice results at zero temperature, nonzero isospin density
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Matter in magnetic fields

Matter in magnetic fields (linear response)

- paramagnets: attracted by magnetic field
- diamagnets: repel magnetic field

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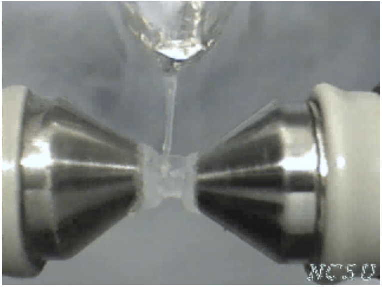


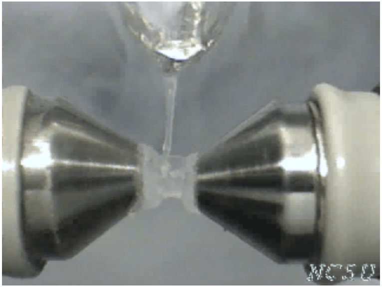
paramagnet: liquid oxygen

[NCSU physics demonstrations]

$\chi \approx 0.004$

Matter in magnetic fields (linear response)

- paramagnets: attracted by magnetic field
- diamagnets: repel magnetic field



paramagnet: liquid oxygen

[NCSU physics demonstrations]

$\chi \approx 0.004$

diamagnet: frog

[Ignobel prize '10]

$\chi \approx -0.00001$

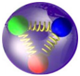
QCD matter in magnetic fields

- is the thermal/dense QCD medium para- or diamagnetic?
- what implications does the magnetic response have for phenomenology?

QCD matter in magnetic fields

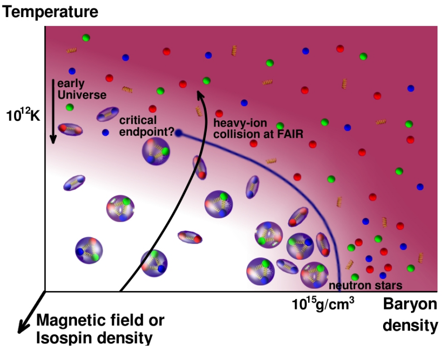
QCD and quark-gluon plasma

- elementary particle interactions:
  gravitational, $\underbrace{\text{electromagnetic, weak, strong}}_{\text{Standard Model}}$
- strong sector: Quantum Chromodynamics
- elementary particles: quarks ($\sim$ electrons) and gluons ($\sim$ photons)
  but: they cannot be observed directly
  $\Rightarrow$ *confinement* at low temperatures
- asymptotic freedom [Gross, Politzer, Wilczek '04]
  $\Rightarrow$ heating or compressing the system leads to *deconfinement*: quark-gluon plasma is formed
- transition between the two phases



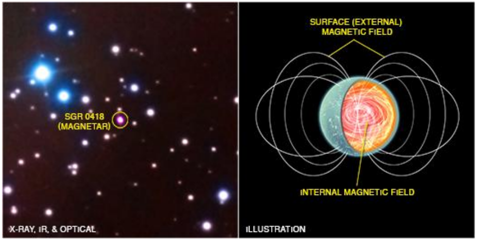
QCD phase diagram

- quark-gluon plasma in nature and in experiments
  - large T: early Universe, cosmological models
  - large ρ: neutron stars
  - large T and/or ρ: heavy-ion collisions



- additional, relevant parameters:
  - background magnetic field B
  - proton-neutron density asymmetry $\rho_I = \rho_p - \rho_n$

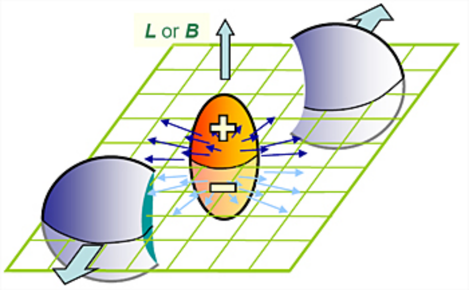
Example 1: neutron star



[Rea et al. '13]

- possible quark core at center with high density, low temperature
- magnetars: extreme strong surface magnetic fields are measured
- nonzero isospin density: $\rho_n > \rho_p$

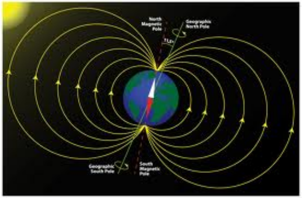
Example 2: heavy-ion collision



[STAR collaboration, '10]

- off-central collisions generate magnetic fields:
  strength controlled by $\sqrt{s}$ and impact parameter (centrality)
- strong (but very uncertain) time-dependence
- again $\rho_n > \rho_p$

Typical magnetic fields



- magnetic field of Earth 10^{-5} T

Typical magnetic fields

• magnetic field of Earth	10^{-5} T
• fridge magnet	10^{-3} T

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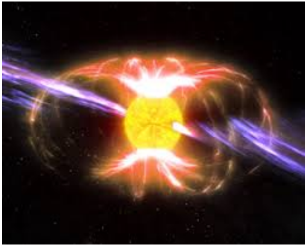

• magnetic field of Earth	10^{-5} T
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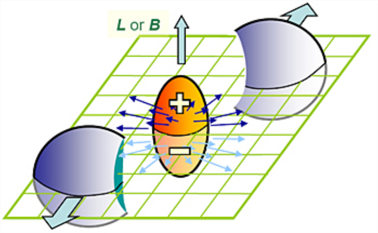
• magnetar surface [Duncan, Thompson '92]	10^{10} T
• magnetar core	10^{14} T?

Typical magnetic fields



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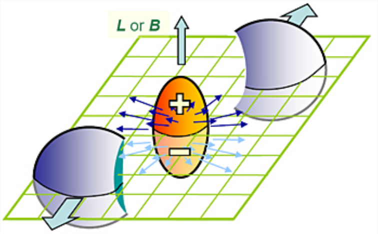

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convert: $e \cdot 10^{15}\ \mathrm{T} \approx 3m_\pi^2 \approx \Lambda_{\rm QCD}^2$

$\Rightarrow$ electromagnetic and strong interactions compete

Magnetic susceptibility

- simplification: constant background magnetic field B
- free energy density in background magnetic field

$$f(B) = -\frac{T}{V} \log \mathcal{Z}(B)$$

- magnetization

$$\mathcal{M} = -\frac{\partial f}{\partial(eB)}, \qquad \mathcal{M}|_{B=0} = 0$$

- susceptibility

$$\chi = \left.\frac{\partial \mathcal{M}}{\partial(eB)}\right|_{B=0} = -\left.\frac{\partial^2 f}{\partial(eB)^2}\right|_{B=0}$$

- sign distinguishes between
  - paramagnets ($\chi > 0$) enjoy magnetic field
  - diamagnets ($\chi < 0$) repel magnetic field

Objective

- calculate χ along the QCD phase diagram
  - advantageous to use μ instead of ρ, and μ_I instead of ρ_I (grand canonical approach)
- consider the free case (no strong interactions) $\rightarrow$ can be solved analytically

$$\chi^{\text{free}}(T, \mu, \mu_I)$$

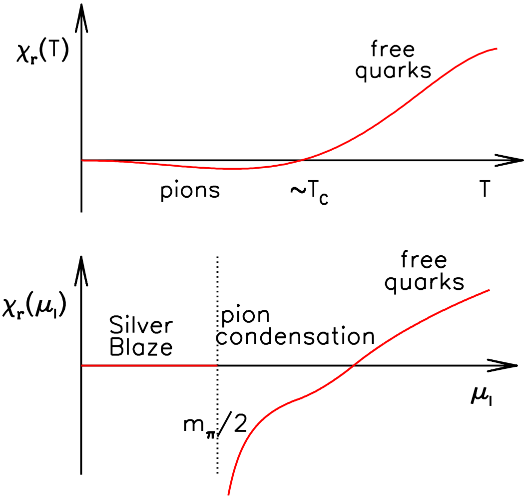
- consider full (non-perturbative) QCD $\rightarrow$ lattice simulations

$$\chi(T, \mu = 0, \mu_I)$$

  (sign problem at $\mu > 0 \ldots$)
  - asymptotic freedom: $\chi^{\text{free}} = \chi$ for high $T/\mu/\mu_I$
  - free case can give some insight

Analytical considerations in the free theory

Expectation for the susceptibility



- based on the free theory

Renormalization

- statement: χ undergoes additive renormalization
- proof: let's write down the *total* free energy density

$$f^{\rm total} = f^{\gamma} + f^{\rm matter} = \frac{B^2}{2} + f^{\rm matter}$$

  - photon wavefunction renormalizes:

$$A_\mu^2 = Z_q \cdot A_{\mu,r}^2, \qquad Z_q = 1 + \beta_1 q_r^2 \log \Lambda^2$$

  - due to $A_y = B \cdot x$ and QED Ward identity

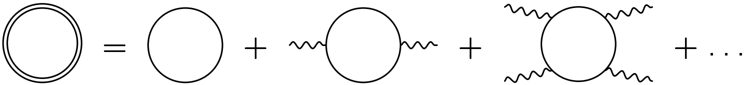
$$B^2 = Z_q \cdot B_r^2, \qquad q^2 = Z_q^{-1} \cdot q_r^2, \qquad qB = q_r B_r$$

  - β_1: lowest-order QED β-function coefficient (no internal γ)
  - matter part should cancel this divergence (no new divergences due to B in $f^{\rm total}$)

$$f^{\rm total} = \frac{B_r^2}{2} + \underbrace{\beta_1 (qB)^2 \log \Lambda + f^{\rm matter}}_{\text{should be finite}}$$

Renormalization

- consider free quarks (electric charge but no color charge)
- expand f^{matter} in B at $T = 0$: in terms of diagrams



- ▸ $\mathcal{O}(B^2)$ term is indeed $-\beta_1 \cdot (qB)^2 \log \Lambda$ [Schwinger '51], making f^{total} finite
- ▸ $\mathcal{O}(B^{4,6,\cdots})$ terms are finite
- ▸ background field method [Abbott '81]
- ▸ 'vacuum polarization diagram with magnetic field-legs'

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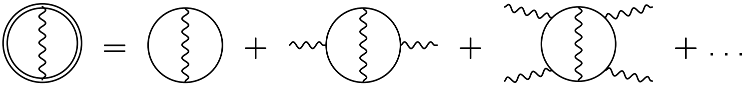

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- 2-loop contribution: $\beta_1 \to \beta_2 q^2$ [Abbott '81, Dunne '04]

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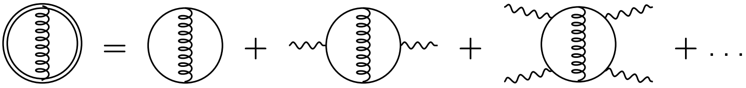

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- 2-loop contribution: $\beta_1 \to \beta_1 c_1 g^2$

Magnetic susceptibility

- *total* free energy density

$$f^{\mathrm{total}}(T=0,B) = \frac{B_r^2}{2} + \underbrace{\mathcal{O}((qB)^4)}_{\text{matter-related}}$$

  - so the susceptibility vanishes at $T=0$

$$\chi_r(T=0)=0$$

- at nonzero temperatures f^{matter} gets thermal $\mathcal{O}((qB)^2)$ contributions

$$\chi_r(T>0)\neq 0$$

Magnetic susceptibility at high T

- consider a free quark with charge q
- using Schwinger proper time regularization, the susceptibility reads [Bali, Bruckmann, Endrődi et al 1406.0269]

$$\chi_r^{\text{free}} = -N_c \cdot \beta_1 \cdot (q/e)^2 \int \frac{\mathrm{d}s}{s} e^{-m^2 s} \cdot \left\{ \Theta_4 \left[0, e^{-1/(4sT^2)} \right] - 1 \right\}$$

$$\xrightarrow{T\to\infty} N_c \cdot \beta_1 \cdot (q/e)^2 \log \left(\frac{T^2}{m^2} \right)$$

(see also [Elmfors et al. '94])

- QED is not asymptotically free ($\beta_1 = 1/12\pi^2 > 0$)
$\Rightarrow$ free quarks at high T are *para*magnetic

Magnetic susceptibility at high μ or μ_I

- consider a free quark with charge q
- using Schwinger proper time regularization, the susceptibility reads [Bali, Bruckmann, Endrődi et al 1406.0269]

$$\chi_r^{\rm free} = -N_c \cdot \beta_1 \cdot (q/e)^2 \int \frac{\mathrm{d}s}{s} e^{-m^2 s} \cdot \left\{ \Theta_4\left[\frac{i\mu}{2T}, e^{-1/(4sT^2)}\right] - 1 \right\}$$

$$\xrightarrow{\mu\to\infty} N_c \cdot \beta_1 \cdot (q/e)^2 \log\left(\frac{\mu^2}{m^2}\right)$$

(see also [Elmfors et al. '94])

- QED is not asymptotically free ($\beta_1 = 1/12\pi^2 > 0$)
  $\Rightarrow$ free quarks at high μ/μ_I are *para*magnetic

Magnetic susceptibility at low energies

- low-energy regime of QCD: dominant degrees of freedom are pions (χPT)
  - so consider a free pion (charge $\pm e$)
- similarly as before: proper-time regularization
  [Bali, Bruckmann, Endrődi et al 1406.0269]

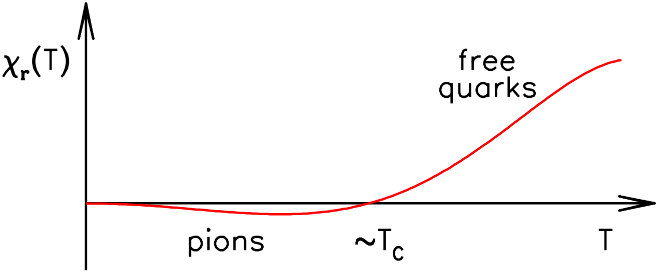
$$\chi_r^{\text{pion}}(T) = -\beta_1^{\text{scalar}} \underbrace{\int \frac{\mathrm{d}s}{s} e^{-m^2 s} \cdot \left\{ \Theta_3 \left[0, e^{-1/(4sT^2)} \right] - 1 \right\}}_{\text{finite and positive}}$$

(see also [Elmfors et al. '94])

  - scalar QED β-function $\beta_1^{\text{scalar}} = 1/48\pi^2 > 0$
    $\Rightarrow$ free pions are *dia*magnetic

Expectation for the susceptibility

- $\chi_r(T = 0) = 0$ due to renormalization prescription
- asymptotic freedom in QCD + no asymptotic freedom in QED
  $\Rightarrow \chi_r > 0$ for high temperatures
- expectation: pions are relevant at low energies
  $\Rightarrow \chi_r < 0$ for low temperatures



Results I: $T > 0$, $\mu = \mu_I = 0$

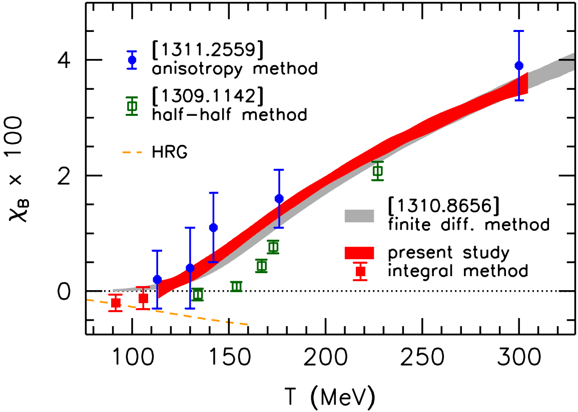
Lattice results

- direct lattice simulation at nonzero B is possible (no sign problem)
- complication: B in a finite periodic volume is quantized

$$\Phi = qB \cdot L^2 = 2\pi N_b, \qquad N_b \in \mathbb{Z}$$

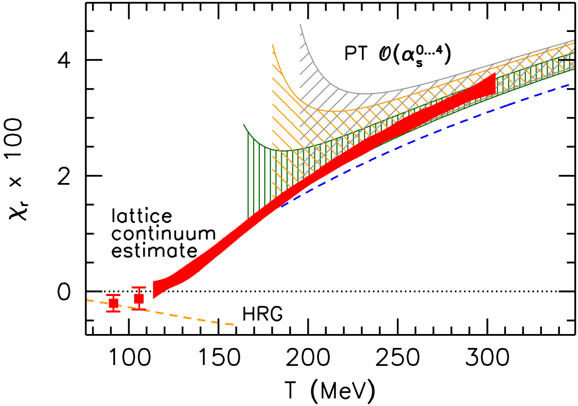
- ▸ χ is not directly accessible
- various methods to circumvent this problem
  [Bali et al 1303.1328] [DeTar et al 1309.1142]
  [Bonati et al 1307.8063] [Bali et al 1406.0269]
- lattice setup: stout smeared staggered quarks + Symanzik gauge action, physical pion mass, continuum estimate based on $N_t = 6, 8, 10$ [Bali et al 1406.0269]

Lattice results



- confirms the free-case prediction qualitatively
- quantitative agreement among the different approaches
  [Bali, Bruckmann, Endrődi et al 1406.0269]
- notice magnitude $\chi_r \approx 0.04$ at high T

Lattice results at $T > 0$



- transition from diamagnetism to paramagnetism slightly below T_c [Bali, Bruckmann, Endrődi et al 1406.0269]
- comparison to Hadron Resonance Gas model (low T) and to perturbation theory (high T)

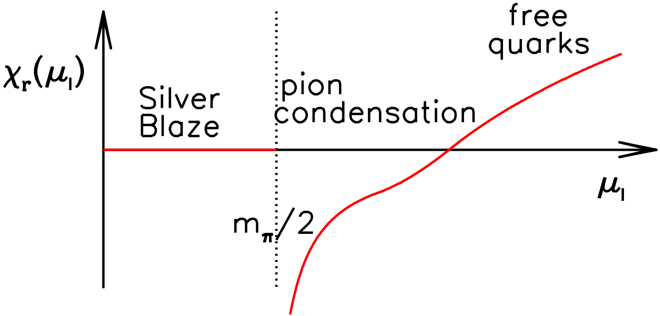
Results II: $T = \mu = 0$, $\mu_I \neq 0$

Isospin chemical potential

- $\mu_I = \mu_u = -\mu_d$ excites pions $\pi^+ = \bar{d}\gamma_5 u$
- at $T = 0$: to produce a pion costs $\mu_I = m_\pi/2$ energy
  - below $\mu_I = m_\pi/2$ nothing can happen: *Silver Blaze* region
  - above $\mu_I > m_\pi/2$: *Bose-Einstein condensation*
- there is a phase transition in between, characterized by various fermionic observables
  - quark condensate $\bar{\psi}\psi = \bar{u}u + \bar{d}d$
  - pion condensate $\pi = \bar{u}\gamma_5 d - \bar{d}\gamma_5 u$
  - isospin density $n_I = \bar{u}\gamma_0 u - \bar{d}\gamma_0 d$

Expectation for $\mu_I > 0,\ T = 0$

- $\chi_r(\mu_I < m_\pi/2) = 0$ due to renormalization prescription + Silver Blaze
- asymptotic freedom in QCD + no asymptotic freedom in QED $\Rightarrow \chi_r > 0$ for high μ_I
- pion condensation phase, superconducting $\Rightarrow \chi_r \to -\infty$ just above $\mu_I = m_\pi/2$



Isospin chemical potential on the lattice

- 2-flavor QCD

$$S_{\text{fermion}} = \bar{\psi} M \psi, \qquad \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

with fermion matrix [Kogut, Sinclair '02]

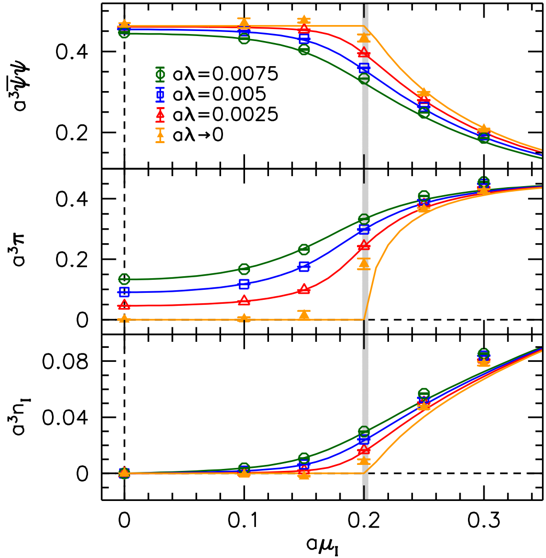
$$M = \begin{pmatrix} \not{D}(+\mu_I) + m & +\lambda\gamma_5 \\ -\lambda\gamma_5 & \not{D}(-\mu_I) + m \end{pmatrix}$$

- symmetry breaking pattern

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \xrightarrow{m \neq 0} \mathrm{SU}(2)_V \xrightarrow{\mu_I \neq 0} \mathrm{U}(1)_{\tau_3} \xrightarrow{\lambda \neq 0} \phi$$

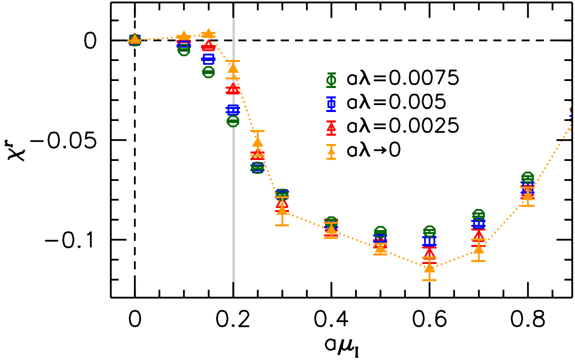
- explicit breaking λ is necessary to observe spontaneous symmetry breaking of $\mathrm{U}(1)$ (in finite volume)
  - simulate at $\lambda \neq 0$ and extrapolate $\lambda \to 0$ at end
- lattice setup: rooted staggered quarks + plaquette gauge action, $m_\pi a = 0.402(5)$ [Endrődi 1407.1216]

Observables at nonzero μ_I [Endrődi 1407.1216]



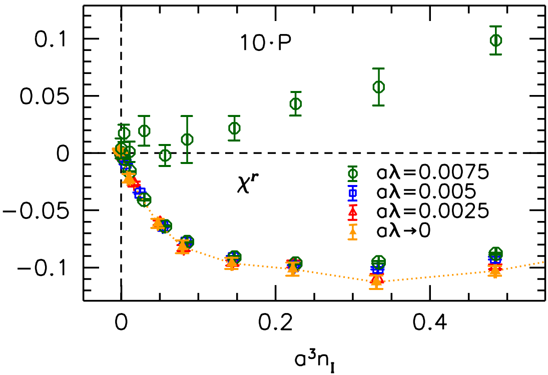
- extrapolation $\lambda \to 0$: use χPT [Splittorff et al. '02]

Susceptibility at nonzero μ_I [Endrődi 1407.1216]



- again Silver Blaze up to $m_\pi/2$ (as should be for any observable)
- strong diamagnetism, as predicted by the free-case argument

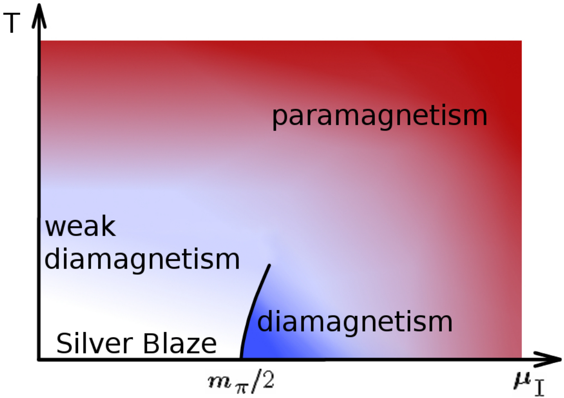
Perfect diamagnetism?



- condensed pions form a superconducting state
  $\rightarrow$ should expel magnetic field completely ($\chi_r = -\infty$)
  - no real superconductivity due to pionic interactions plus finite volume
  - at $\mu_I > m_\pi/2$ deconfinement also sets in, pions dissolve
- altogether, χ_r remains finite

Implications

Magnetic phase diagram



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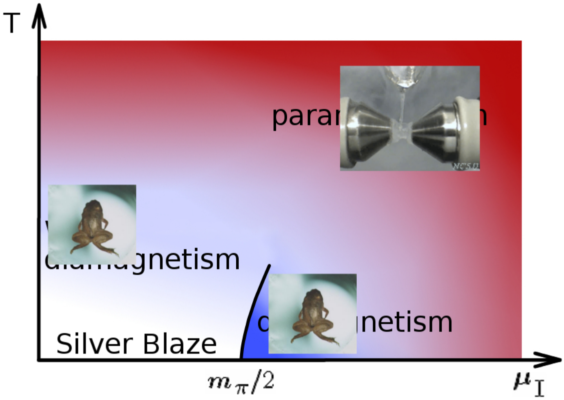

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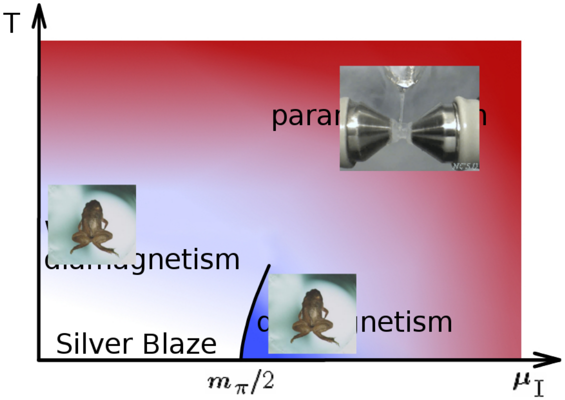
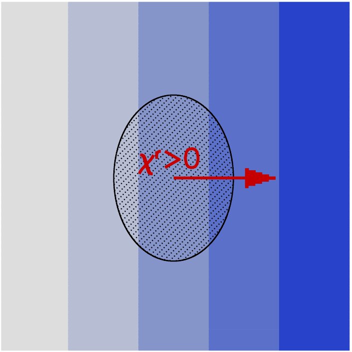


- strong paramagnetism at low μ_I, high T
  $\Rightarrow$ affect elliptic flow in HIC
  [Bali, Bruckmann, Endrődi, Schäfer 1311.2559]
- strong diamagnetism at $T = 0$, high μ_I
  $\Rightarrow$ impact on inner core of magnetars [Endrődi 1407.1216]

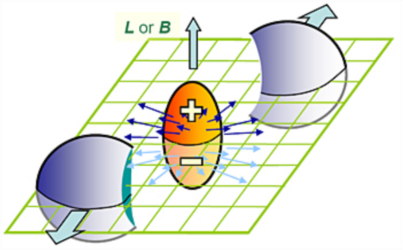
Paramagnetism and inhomogeneous fields

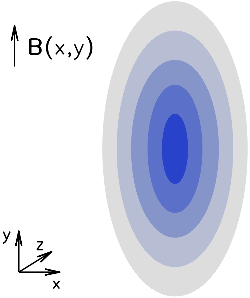


- $-\partial^2 f^r / \partial(eB)^2 = \chi^r > 0$
  $\Rightarrow$ free energy f^r minimized in the region where B is maximal

Implication I: heavy ion collisions

- strong paramagnetism at high T
  $\rightarrow$ free energy minimal where B is *maximal*
- non-uniform magnetic fields in HIC [Deng et al '12]





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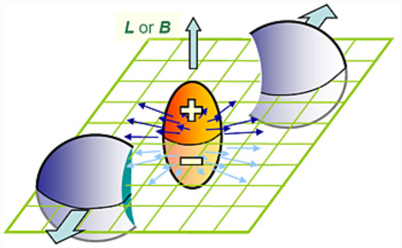
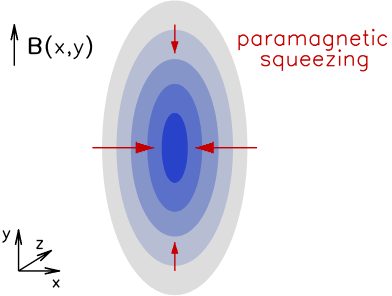
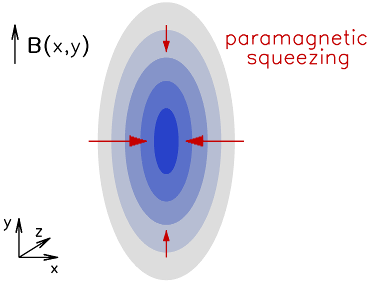


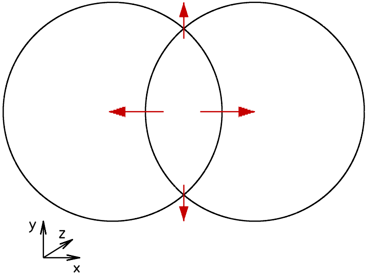


- free energy minimization squeezes QCD matter anisotropically
  [Bali, Bruckmann, Endrődi, Schäfer 1311.2559]

Squeezing versus elliptic flow

- elliptic flow: anisotropic pressure gradients due to initial geometry

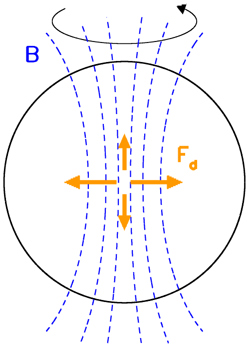




- competition between squeezing and elliptic flow
- crude estimate: squeezing contributes $5 - 50\%$, depending on beam energy [Bali, Bruckmann, Endrődi, Schäfer 1311.2559]
- need more sophisticated models foc realistic comparison, ongoing work

Implication II: magnetized neutron stars

- strong diamagnetism at high μ_I
  $\rightarrow$ free energy minimal where B is *minimal*
- in magnetars: outward force

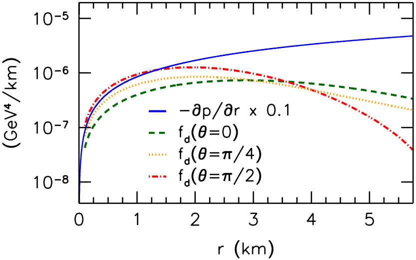


- poloidal field configuration [Bocquet et al '95]:
  component towards equator is larger
- consider typical pressure profile estimate [Glendenning]
- F_d can be 10% of the pressure gradient [Endrődi 1407.1216]

  impact on e.g. convective processes in the core

Implication II: magnetized neutron stars

- strong diamagnetism at high μ_I
  $\to$ free energy minimal where B is *minimal*
- in magnetars: outward force



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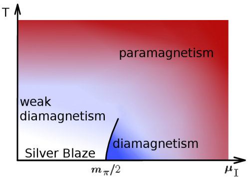
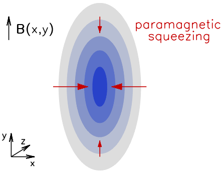
Summary

- B significantly affects the thermal/dense QCD medium

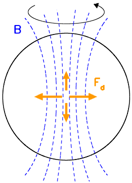
  - ‘magnetic phase diagram’



  - possible implication for heavy-ion collisions: paramagnetic squeezing



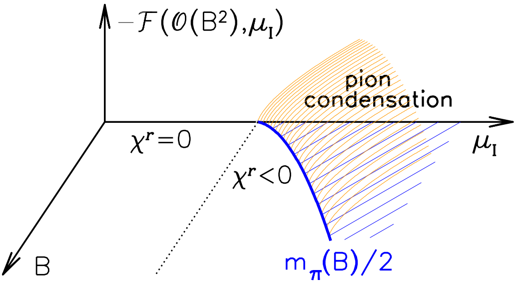
  - possible implication for neutron stars: anisotropic outward force



Backup

Susceptibility and BEC

- qualitative understanding of strong diamagnetism from a different point of view:
  - pion mass in magnetic field $m_\pi(B) = \sqrt{m_\pi^2 + eB}$
  - expect condensation threshold $\mu_{I,crit} = m_\pi(B)/2$
  - automatically ensures a large negative χ_r



Magnetic field and magnetic induction

- distinguish between external field and induction

$$B = H + \mathcal{M} \cdot e$$

- here we work with a field B traversing the medium

$$\mathcal{M} \approx \chi_r \cdot (eB)$$

  - finding out H for small fields

$$H = (1 - e^2\chi_r) \cdot B \quad \rightarrow \quad \frac{B}{H} = \frac{1}{1 - (4\pi\alpha)\chi_r}$$

(then complete Meissner effect $\leftrightarrow \chi_r = -\infty$)