Lattice calculations for TMDs and DPDs

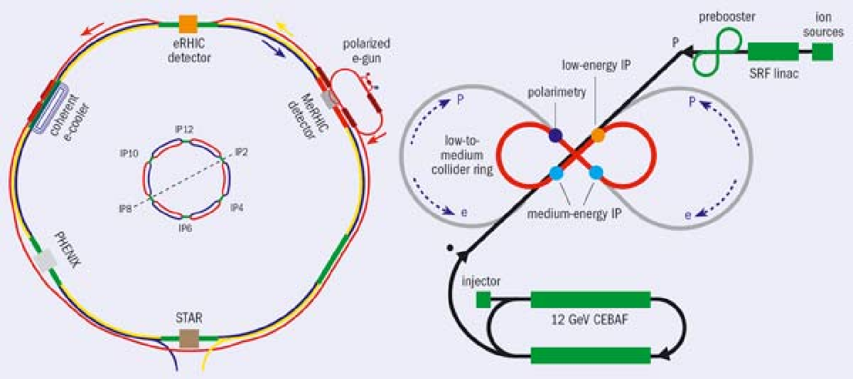
Andreas Schäfer, Regensburg

- Experimental relevance
- Analytic work
- Lattice simulations
  - Moments of TMDs
  - DPDs
  - X. Ji et als approach

In collaboration with: G. Bali, L. Castagnini, M. Diehl, M. Engelhardt, J. Gaunt, B. Gläßle, P. Hägler, X. Ji, B. Musch, D. Ostermeier, P. Plößl, C. Zimmermann, J. Zhang, ...

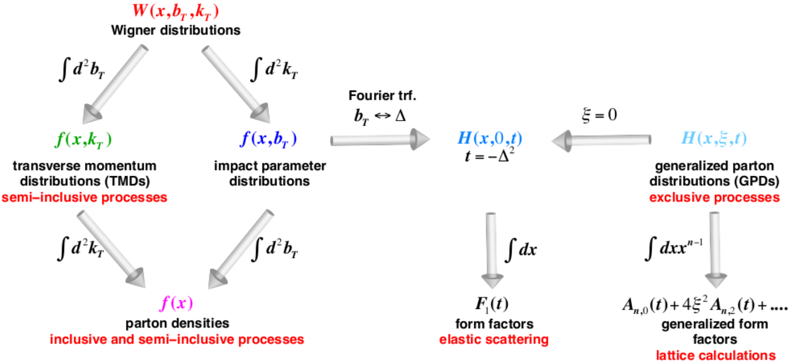
Experimental Relevance

Heavy Ion Collisions at RHIC will stop in a few years.
If there is a future for RHIC it will be eRHIC/EIC. JLab (ELIC/EIC) wants it too.



EIC has 3 selling points:
("understanding the glue that binds us all")

- spin-dependent and -independent PDFs and GPDs – e.g. $\Delta G(x, Q^2)$
- Single Spin Asymmetries and TMDs – Effects of gauge links
- e+A and saturation



The fundamental difference between TMDs and GPD: GPDs

$$\frac{1}{(2\pi)^2}\int d^2\Delta_\perp e^{\mathrm{i}b_\perp\cdot\Delta_\perp}\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle P_2|\,\bar{q}(-\frac{1}{2}z)\,\gamma^+[1+\vec{s}\cdot\vec{\gamma})\gamma_5]q(\frac{1}{2}z)\,|P_1\rangle\Big|_{z^+=0}^{z_\perp=0}$$

$$= \frac{1}{2}\Big[H - S^i\epsilon^{ij}b^j\frac{1}{m}E' - s^i\epsilon^{ij}b^j\frac{1}{m}\Big(E'_T+2\tilde{H}'_T\Big)$$

$$+ s^iS^i\Big(H_T-\frac{1}{4m^2}\Delta_b\tilde{H}_T\Big)+s^i(2b^ib^j-b^2\delta^{ij})S^j\frac{1}{m^2}\tilde{H}''_T\Big]$$

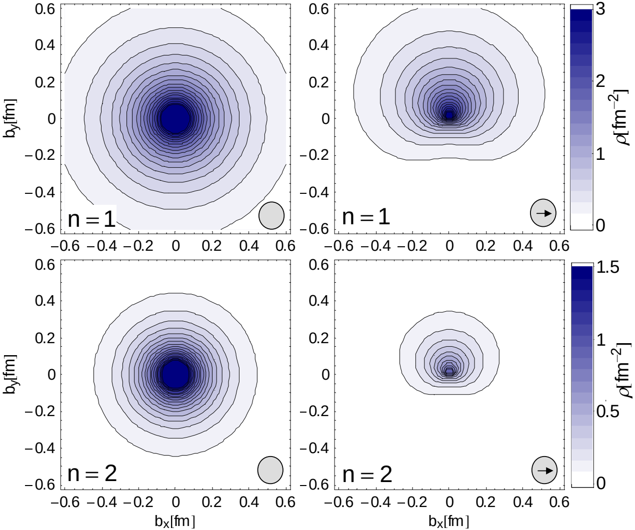
has a simple interpretation:

$S^i\epsilon^{ij}b^j$ coupling of proton spin to quark angular momentum

$s^i\epsilon^{ij}b^j$ coupling of quark spin to quark angular momentum

s^iS^i coupling of quark spin and proton spin

The pion first and second moment

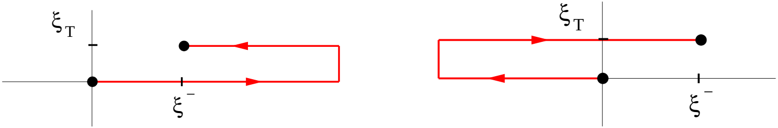


The matrix element contains in principle a gauge link

$$\langle P_2|\,\bar{q}(-\frac{1}{2}z)\;\Gamma\;U[-\frac{1}{2}z,\frac{1}{2}z]\;q(\frac{1}{2}z)\,|P_1\rangle$$

which is unity, however, in light-cone gauge. This is no longer true for non-zero $k_\perp$

gauge links for SIDIS and DY



for a pseudo T-odd quantity this leads to a minus sign

TMDs and SSAs test the very nature of local gauge theories !!

C.J. Bomhof, P.J. Mulders and F. Pijlman; EPJ C47 (2006) 147; arXiv hep-ph/0601171 'The construction of gauge-links in arbitrary hard processes'

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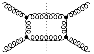
<td></td>
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$$\begin{aligned}\Phi_g \propto \Big\langle\, & \tfrac{1}{2N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\,] + \tfrac12\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,]\Big(\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c}\,\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]\dagger}]}{N_c}-\tfrac{3}{N_c^2}\Big)\\ &+\tfrac12\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\Big\{\tfrac12\,\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c}\mathcal{U}^{[-]}+\tfrac{1}{N_c^2}\mathcal{U}^{[+]}\Big\}\,]\\ &+\tfrac12\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\Big\{\tfrac12\,\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]\dagger}]}{N_c}\mathcal{U}^{[+]}+\tfrac{1}{N_c^2}\mathcal{U}^{[-]}\Big\}\,]\,\Big\rangle\\ \Delta_g \propto \Big\langle\, & \tfrac{1}{2N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[-]}\,] + \tfrac12\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,]\Big(\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c}\,\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]\dagger}]}{N_c}-\tfrac{3}{N_c^2}\Big)\\ &+\tfrac12\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\Big\{\tfrac12\,\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c}\mathcal{U}^{[-]}+\tfrac{1}{N_c^2}\mathcal{U}^{[+]}\Big\}\,]\\ &+\tfrac12\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\Big\{\tfrac12\,\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]\dagger}]}{N_c}\mathcal{U}^{[+]}+\tfrac{1}{N_c^2}\mathcal{U}^{[-]}\Big\}\,]\,\Big\rangle\end{aligned}$$

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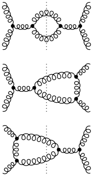
<td></td>
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$$\begin{aligned}\Phi_g \propto \Big\langle\, & \tfrac12\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,]\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c} + \tfrac12\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,]\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]\dagger}]}{N_c}\\ &-\tfrac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\Box]\dagger}\,] - \tfrac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]\dagger}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\Box]}\,]\,\Big\rangle\\ \Delta_g \propto \Big\langle\, & \tfrac12\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,]\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c} + \tfrac12\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,]\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]\dagger}]}{N_c}\\ &-\tfrac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\Box]\dagger}\,] - \tfrac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]\dagger}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\Box]}\,]\,\Big\rangle\end{aligned}$$

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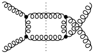
<td></td>
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$$\begin{aligned}\Phi_g \propto \Big\langle\, & \tfrac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\,] + \mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,]\Big(\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c}\,\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]\dagger}]}{N_c}-\tfrac{3}{N_c^2}\Big)\\ &+\tfrac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,] + \tfrac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,]\\ &+\tfrac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\Box]\dagger}\,] + \tfrac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]\dagger}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\Box]}\,]\,\Big\rangle\\ \Delta_g \propto \Big\langle\, & \tfrac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[-]}\,] + \mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,]\Big(\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]}]}{N_c}\,\tfrac{\mathrm{Tr}\,[\mathcal{U}^{[\Box]\dagger}]}{N_c}-\tfrac{3}{N_c^2}\Big)\\ &+\tfrac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,] + \tfrac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,]\\ &+\tfrac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\Box]\dagger}\,] + \tfrac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\Box]\dagger}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\Box]}\,]\,\Big\rangle\end{aligned}$$

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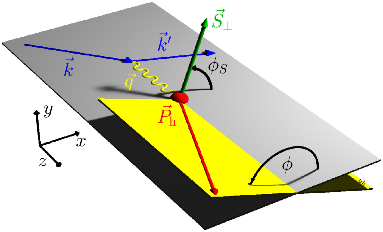
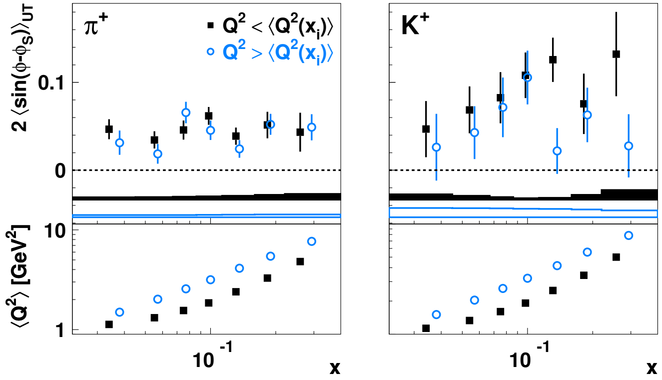
The gauge links result in single spin asymmetries (SSAs) in SIDIS

HERMES 0906.3918

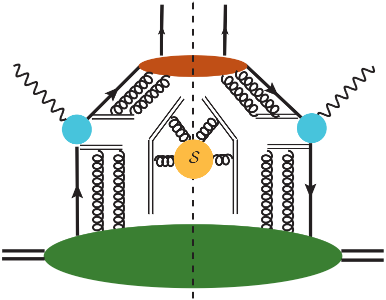




The experimental situation is complicated by strong evolution effects also from non-perturbative kinematics. Lattice input is urgently needed.

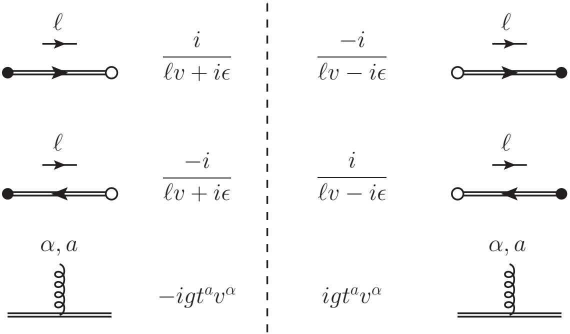
$$
\begin{aligned}
F(x, b; Q_f) &= F(x, b; Q_i)\exp\left\{-\int_{Q_i}^{Q_f}\frac{d\mu}{\mu}\left(\Gamma_{\mathrm{cusp}}\ln\frac{Q_f^2}{\mu^2}+\gamma^V\right)\right\} \\
&\times \left(\frac{Q_f^2}{Q_i^2}\right)^{-D(b;Q_i)} \\
\frac{dD}{d\ln\mu} &= \Gamma_{\mathrm{cusp}} \\
F(x, b; Q) &= F_{\mathrm{pert}}(x, b_*; Q)\exp(-S_{\mathrm{NP}}) \qquad \text{for } 1/b \leq \Lambda_{\mathrm{QCD}} \\
S_{\mathrm{NP}}^{\mathrm{pdf}}(b, Q) &= b^2\left(g_1^{\mathrm{pdf}}+\frac{g_2}{2}\ln\frac{Q}{Q_0}\right) \\
g_1^{\mathrm{pdf}} &= \frac{\langle k_\perp^2\rangle_{Q_0}}{4},
\end{aligned}
$$

gauge links correspond to infinitely many soft gluons $\Rightarrow$ soft factors



$$\Phi^{[\Gamma]}(k,P,S;\ldots) \equiv \int \frac{d^4b}{(2\pi)^4}\, e^{ik\cdot b}\, \frac{\overbrace{\frac{1}{2}\left\langle P,S\right|\, \bar{q}(0)\,\Gamma\,\mathcal{U}[\mathcal{C}_b]\; q(b)\,\left|P,S\right\rangle}^{\equiv\, \widetilde{\Phi}^{[\Gamma]}_{\text{unsub}}(b,P,S;\ldots)}}{\widetilde{\mathcal{S}}(b^2;\ldots)}$$

For the eikonal lines one needs the eikonal Feynman rules



Lets assume that factorization proofs etc. will come along

- J. Collins: *Foundations of Perturbative QCD*

$$\widetilde{\mathcal{S}}(b^2;\ldots) = \sqrt{\frac{\tilde{S}_{(0)}(\vec{b}_T,+\infty,-\infty)\ \tilde{S}_{(0)}(\vec{b}_T,y_s,-\infty)}{\tilde{S}_{(0)}(\vec{b}_T,+\infty,y_s)}}$$

$\Rightarrow$ cancellation of rapidity divergences
Off-light-cone vector v^μ

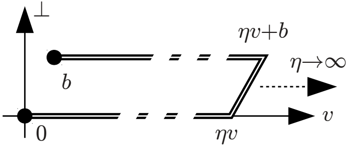
- Analysis within SCET: Echevarria, Idilbi and Scimemi 1111.4996
- J. C. Collins and T. C. Rogers, "Equality of Two Definitions for Transverse Momentum Dependent Parton Distribution Functions", 1210.2100

Lattice calculations for TMDs

"We" (i.e. M. Engelhardt, P. Hägler, B. Musch et al.) analyze on the lattice ratios for which soft factors as well as renormalization factors cancel

next problem: How to put an infinite gauge link on a finite lattice ?

solution: We study the length dependence of staple-shaped gauge links and find rapid convergence.



We simulate for spatial, not light-like separations, but the limit $\zeta \to \infty$ of

$$\hat{\zeta} \; := \; \frac{v \cdot P}{\sqrt{v^2}\sqrt{P^2}}$$

reproduces the light-cone behavior.

So far we always analyzed transverse b, i.e. b_T, but we could also set $b \cdot P \neq 0$ and get TMDs directly as functions of x.

The potential of this was realized independently by X. Ji, see 1305.1539 with whom we now collaborate on this.

$$
\begin{aligned}
q(x, \mu^2, P^z, a) &= \int \frac{dz}{4\pi} e^{\mathrm{i}zk^z} \langle P|\bar{\psi}(z)\gamma^z e^{-\mathrm{i}g \int_0^z dz' A^z(z')} \psi(0)|P\rangle \\
&+ O(\Lambda^2/(P^z)^2) + \ldots \\
&= \int_x^1 \frac{dy}{y} \, Z\left(\frac{x}{y}, \frac{\mu}{P^z}, a\mu\right) \, q(y, \mu^2)
\end{aligned}
$$

Who suggests to calculate quasi-PDFs, quasi-DAs, quasi-TMDs etc.

$\Gamma = 1, \gamma^5, \gamma^u, \gamma^\mu\gamma^5, \mathrm{i}\sigma_{\mu\nu}\gamma^5$; 32 independent amplitudes
$\tilde{A}_1(b^2, b\cdot P, v\cdot b/v\cdot P, -1/(m_N\hat{\zeta})^2, \eta v\cdot P), ..., \tilde{A}_{12}(...)$,
$\tilde{B}_1(...), ..., \tilde{B}_{20}(...)$
many of which are related to known objects. These amplitudes can be obtained for spatial correlators.

For SIDIS one has to perform the extrapolation $\eta v \cdot P \to \infty$ for DY $\eta v\cdot P \to -\infty$ as well as the limit $\hat{\zeta} \to \infty$ in both cases.

Furthermore, to relate the local matrix elements we define moments and leading terms of a Taylor expansion in $\vec{k}_T$.

$$f[m](n)(...) \ = \ \int_{-1}^{1} dx \ x^{m-1} \ \int d^2k_T \ \left(\frac{\vec{k}_T^2}{2m_N^2}\right)^n \ f(x, \vec{k}_T^2, ...)$$

Thus we obtain for fixed b_T !! "generalized shifts" which correspond for $b_T \to 0$ to the Boer Mulders shift which is "known" from phenomenological models of SSAs.

$$
\begin{aligned}
\langle \vec{k}_y \rangle_{TU}(\vec{b}_T^2; \hat{\zeta}, \eta v \cdot P) &= m_N \frac{\tilde{f}_{1T}^{\perp[1](1)}(\vec{b}_T^2; \hat{\zeta}, \ldots, \eta v \cdot P)}{\tilde{f}_1^{[1](0)}(\vec{b}_T^2; \hat{\zeta}, \ldots, \eta v \cdot P)} \\
&= -m_N \frac{\widetilde{A}_{12B}(-\vec{b}_T^2, 0, 0, -1/(m_N \hat{\zeta})^2, \eta v \cdot P)}{\widetilde{A}_{2B}(-\vec{b}_T^2, 0, 0, -1/(m_N \hat{\zeta})^2, \eta v \cdot P)} \\
&\overset{b_T \to 0}{\Longrightarrow} \left. \frac{\int dx \int d^2 k_T \, k_y \; \Phi^{[\gamma^+]}(x, \vec{k}_T, P, S; \ldots)}{\int dx \int d^2 k_T \; \Phi^{[\gamma^+]}(x, \vec{k}_T, P, S; \ldots)} \right|_{\vec{S}=(1,0)}
\end{aligned}
$$

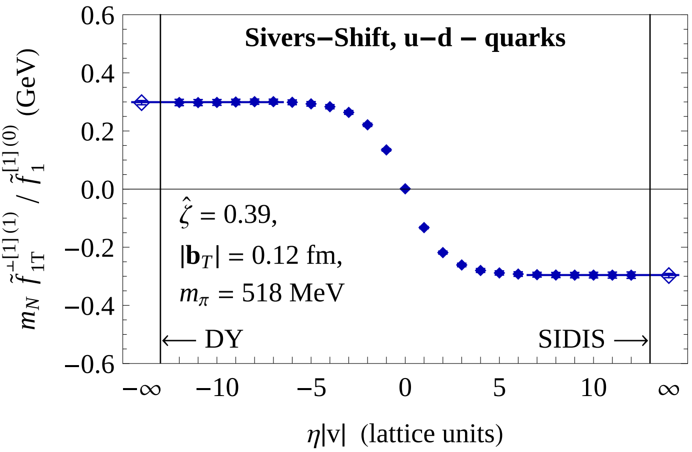
In leading twist: Average transverse momentum of a transversely polarized quark in an unpolarized hadron.

We used LHPC propagators for $N_f = 2 + 1$ MILC configurations

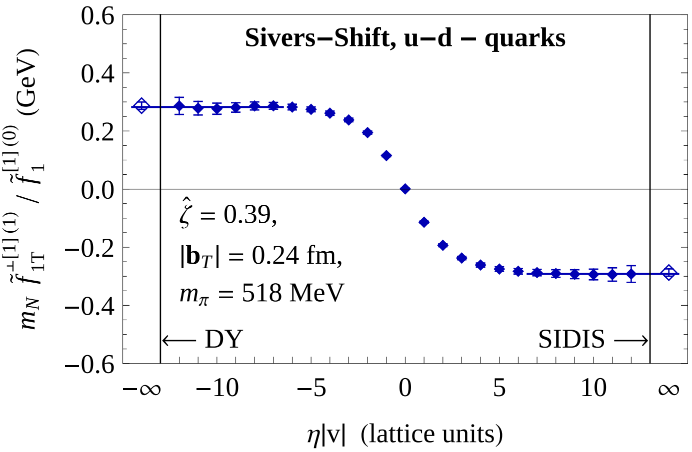
$\hat{m}_{u,d}$	$\hat{m}_s$	$\hat{L}^3 \times \hat{T}$	$10/g^2$	$a(fm)$	$m_\pi^{\mathrm{DWF}}(MeV)$	$m_N^{\mathrm{DWF}}(GeV)$	#conf.	#meas.
0.01	0.05	$28^3 \times 64$	6.76	0.11967(14)(99)	369.0(09)(35)	1.197(09)(12)	273	2184
0.01	0.05	$20^3 \times 64$	6.76	0.11967(14)(99)	369.0(09)(35)	1.197(09)(12)	658	5264
0.02	0.05	$20^3 \times 64$	6.79	0.11849(14)(99)	518.4(07)(49)	1.348(09)(13)	486	3888

B. Musch, Ph. Hägler, M. Engelhardt, J.W. Negele, AS
1111.4249

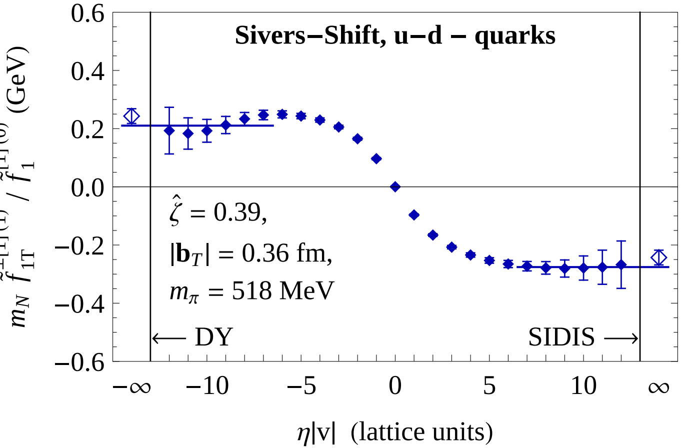
$\eta \to \pm\infty$



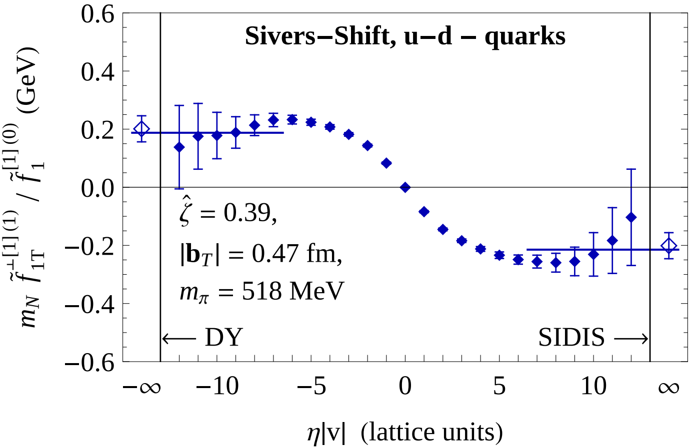
$\eta \to \pm\infty$



$\eta \to \pm\infty$

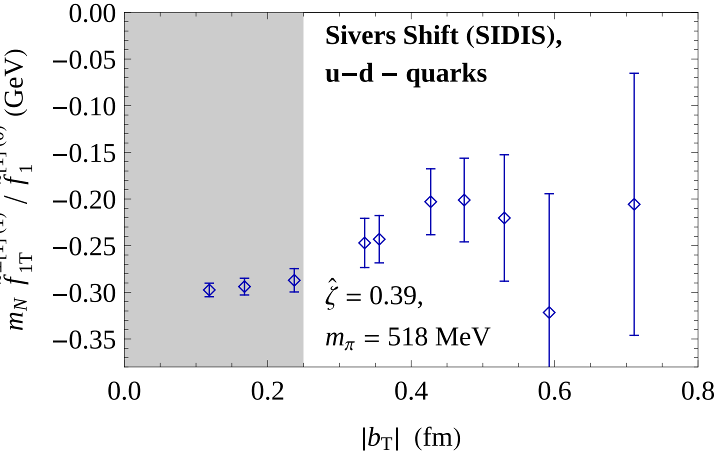


$\eta \to \pm\infty$

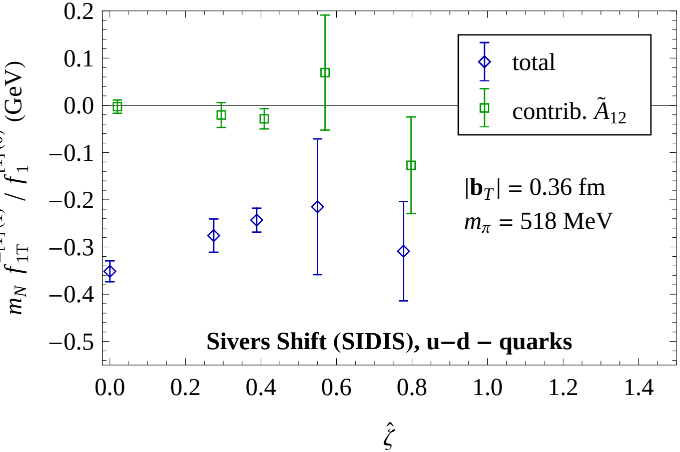


message: $\eta \to \pm\infty$ is no problem different b_T give additional information

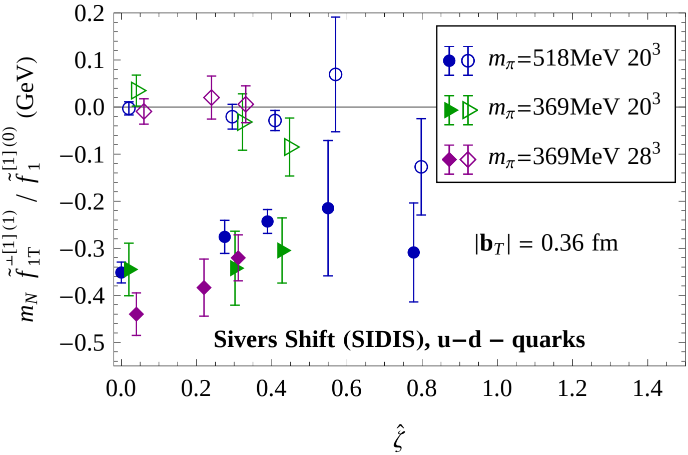
(for $b_T < 2a$ probably large cut-off effects.)



The crucial limit $\hat{\zeta} \to \infty$

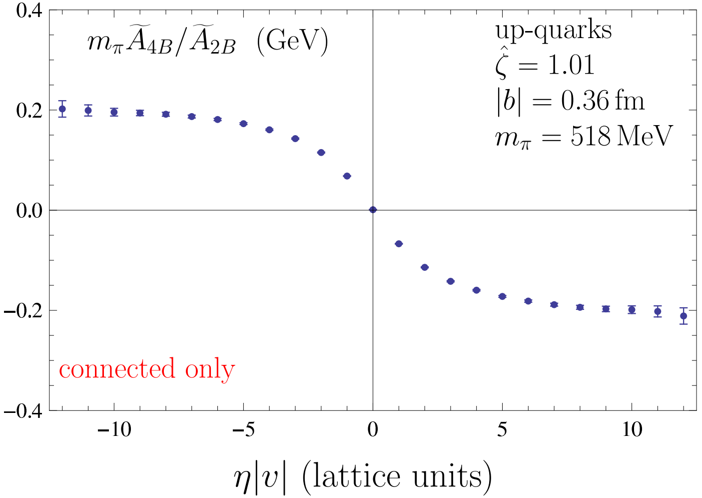


different ensembles

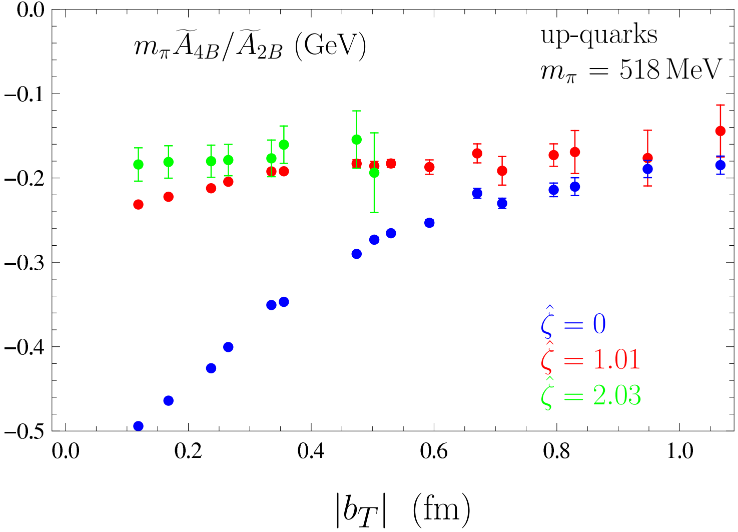


results for the pion; $\hat{\zeta} = \frac{v \cdot p}{v^2 m_\pi^2}$ is larger because m_π is small; also better statistics

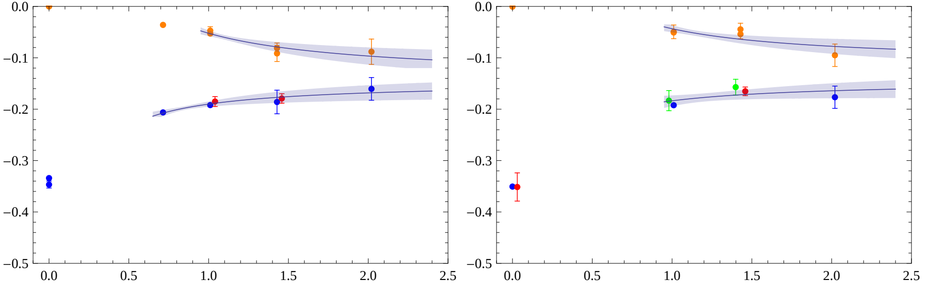
$\eta \to \pm\infty$



b_T dependence

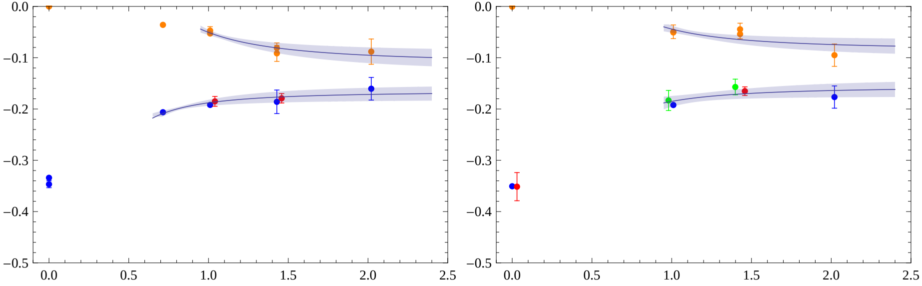


$\hat{\zeta} \to \infty$



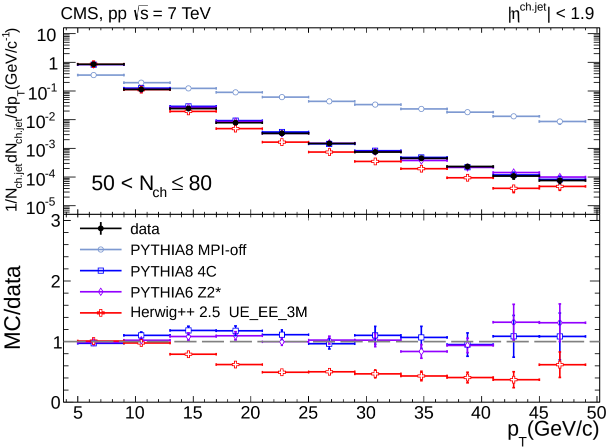
Fits of the ζ dependence for two $b_\perp$ assuming convergence with $1/\hat{\zeta}$.

$\hat{\zeta} \to \infty$

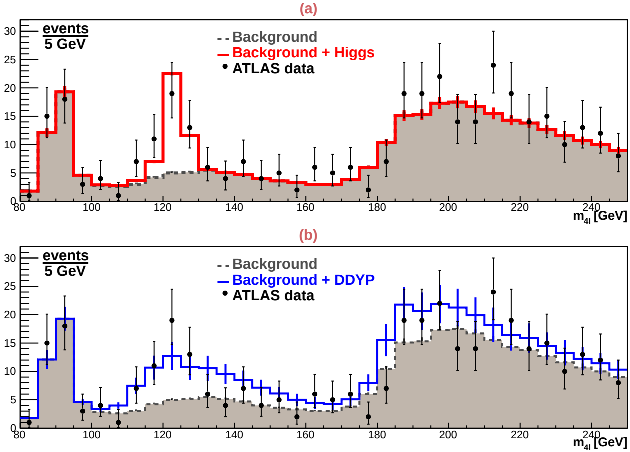


Fits of the ζ dependence for two $b_\perp$ assuming convergence with $1/\hat{\zeta}^2$.

DH (double-hard) reactions are important at LHC
CMS 1310.4554: Inclusive charged particle jets



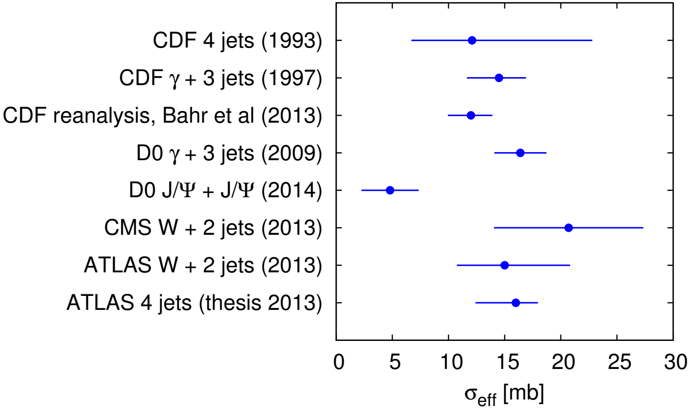
Krasny and Placzek 1501.04569



Normally the naive assumption

$$d\sigma_{DPS} = \frac{d\sigma_{SPS} d\sigma_{SPS}}{2\sigma_{eff}}$$

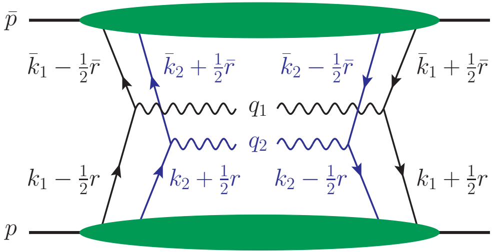
is made which might or might not be a good approximation



We have analyzed in detail the simplest case, Double Drell Yan in p+p.

M. Diehl, D. Ostermeier and A. Schäfer, "Elements of a theory for multiparton interactions in QCD," JHEP **1203** (2012) 089 [arXiv:1111.0910 [hep-ph]].

A few results for $q_1^2, q_2^2 \sim O(Q^2)$ and $q_{1,T}, q_{2,T} \sim O(q_T) \ll Q$.



(a)

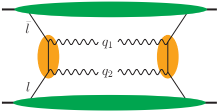
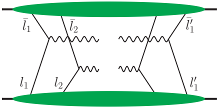
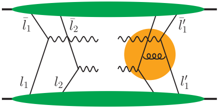
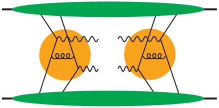
A surprisingly rich literature,e.g. Treleani (1982), Mekhfi (1985).

Double hard interactions

Taking spin and color into account one has to distinguish many contributions. In the end cross sections are parameterized by the following four quark correlators ($a_i = q, \Delta q, \delta q$):

$$
\begin{aligned}
{}^1F_{a_1,a_2} &= \langle\!\langle\, (\bar{q}_3 \Gamma_{a_2} q_2)(\bar{q}_4 \Gamma_{a_1} q_1)\,\rangle\!\rangle, &\quad {}^1\tilde{F}_{a_1,a_2} &= \langle\!\langle\, (\bar{q}_4 \Gamma_{a_2} q_2)(\bar{q}_3 \Gamma_{a_1} q_1)\,\rangle\!\rangle \\
{}^1F_{a_1,\bar{a}_2} &= \langle\!\langle\, (\bar{q}_2 \Gamma_{\bar{a}_2} q_3)(\bar{q}_4 \Gamma_{a_1} q_1)\,\rangle\!\rangle, &\quad {}^1\tilde{F}_{a_1,\bar{a}_2} &= \langle\!\langle\, (\bar{q}_4 \Gamma_{\bar{a}_2} q_3)(\bar{q}_2 \Gamma_{a_1} q_1)\,\rangle\!\rangle \\
{}^1I_{a_1,\bar{a}_2} &= \langle\!\langle\, (\bar{q}_2 \Gamma_{\bar{a}_2} q_4)(\bar{q}_3 \Gamma_{a_1} q_1)\,\rangle\!\rangle, &\quad {}^1\tilde{I}_{a_1,\bar{a}_2} &= \langle\!\langle\, (\bar{q}_3 \Gamma_{\bar{a}_2} q_4)(\bar{q}_2 \Gamma_{a_1} q_1)\,\rangle\!\rangle
\end{aligned}
$$

If one inserts a complete set of states $\sum |X\rangle\langle X|$ and assumes that only the hadron in question contributes !?!?, one gets a sum of convolutions of generalized GPDs (GTMDs) in impact parameter space, respectively of product in momentum space

	$\frac{s d\sigma}{\prod_{i=1}^{2} dx_i \, d\bar{x}_i \, d^2\mathbf{q}_i}$	$\frac{s d\sigma}{\prod_{i=1}^{2} dx_i \, d\bar{x}_i}$
	$\frac{1}{\Lambda^2 Q^2}$	1
	$\frac{1}{\Lambda^2 Q^2}$	$\frac{\Lambda^2}{Q^2}$
	$\frac{1}{Q^4}$	$\frac{\Lambda^4}{Q^4}$
	$\frac{\Lambda^2}{Q^6}$	$\frac{\Lambda^4}{Q^4}$

Collins-Soper Equation (Sudakov factors)

(Only valid for small transverse separations of parton fields.)
All relevant one-gluon exchange graphs were calculated leading to a generalized Collins-Soper-Equation

$$\frac{d}{d\log\zeta}\begin{pmatrix}{}^1F\\{}^8F\end{pmatrix} = \left[G(x_1\zeta,\mu)+G(x_2\zeta,\mu)+K(\mathbf{z}_1,\mu)+K(\mathbf{z}_2,\mu)\right]\begin{pmatrix}{}^1F\\{}^8F\end{pmatrix} + \mathbf{M}(\mathbf{z}_1,\mathbf{z}_2,\mathbf{y})\begin{pmatrix}{}^1F\\{}^8F\end{pmatrix}$$

For small distances we have calculated the matrix $\mathbf{M}$ explicitly. G and K are just the usual CS-kernels.

A general solution can be given

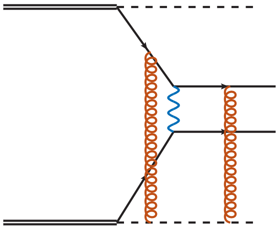
$$\begin{pmatrix} {}^{1}F(x_i,\mathbf{z}_i,\mathbf{y};\zeta,\mu) \\ {}^{8}F(x_i,\mathbf{z}_i,\mathbf{y};\zeta,\mu) \end{pmatrix} = \exp\big[-\mathcal{S}(x_1\zeta,\mathbf{z}_1,\mathbf{z}_2,\mu_0) - \mathcal{S}(x_2\zeta,\mathbf{z}_1,\mathbf{z}_2,\mu_0)\big]$$

$$\times \exp\left[\mathbf{M}(\mathbf{z}_1,\mathbf{z}_2,\mathbf{y}) \log \frac{\sqrt{x_1 x_2}\,\zeta}{\mu_0}\right] \begin{pmatrix} {}^{1}F^{\mu_0}(x_i,\mathbf{z}_i,\mathbf{y};\mu) \\ {}^{8}F^{\mu_0}(x_i,\mathbf{z}_i,\mathbf{y};\mu) \end{pmatrix}$$

$\zeta^2 = (2pv)^2/|v^2|$ is in physical processes of order Q

The problems with TMD factorization

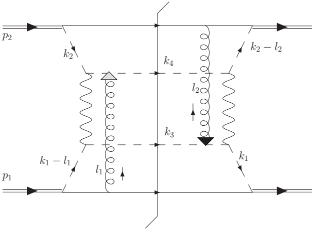
Mulders and Rogers (1001.2977), naive factorization breaks down when initial and final soft gluon exchange takes place simultaneously.



This is solved by the generalized TMD-factorization of Collins.

$$
\begin{aligned}
d\sigma \quad &\overset{!}{=} \quad \mathcal{H} \otimes \Phi_{H_1}^{[n_1,(\Box)]}(x_1, k_{1T}) \otimes \Phi_{H_2}^{[n_2,(\Box)]}(x_2, k_{2T}) \\
&\otimes \quad \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{k}_{3T} - \mathbf{k}_{4T}).
\end{aligned}
$$

But:

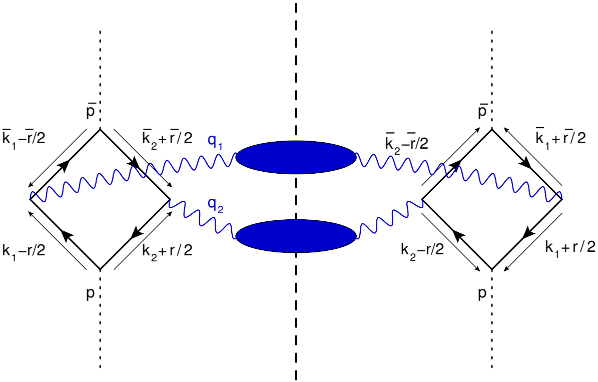


violates also generalized factorization. Perhaps a weaker form of universality might hold ??

No factorization $\Rightarrow$ lattice simulations make no sense

For a toy model: DDY with scalar quarks and gluons, factorization holds (probably)

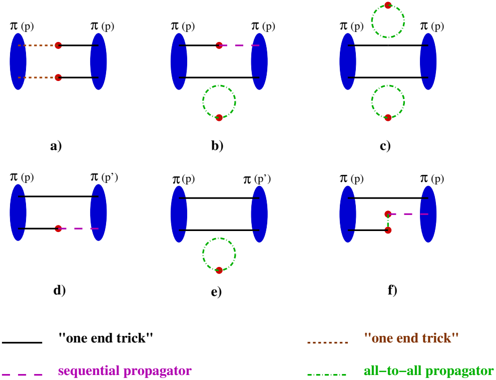
M. Diehl, D. Ostermeier, AS to be published.



First preliminary lattice results of the Hamburg-Regensburg collaboration for DPDs in the pion

M. Diehl, G. Bali, L,. Castagnini, S. Collins, M. Engelhardt, J. Gaunt, B. Gläßle, J. Najjar

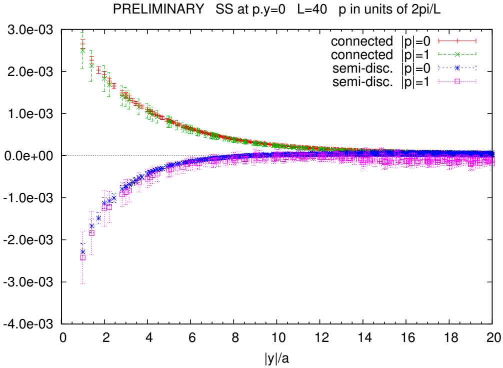
calculated (to be calculated) contributions



Summary of $N_f = 2$ data so far analyzed

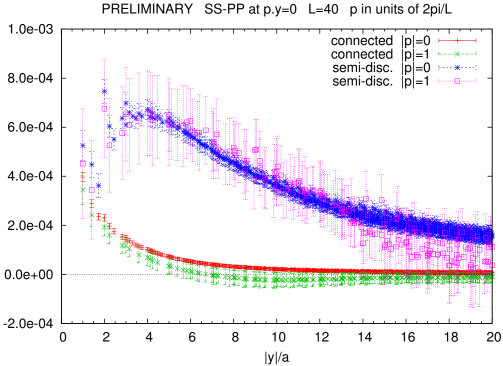
β	κ	m_π [MeV]	Volume	correlators	Configs	momenta
5.29	0.13632	285	$32^3 \times 64$	4-point fct.	375	(000), (001) (010), (100)
				3-point fct.	1965	(000), (001) (010), (100) (011), (101) (110), (111) (002)
5.29	0.13632	285	$40^3 \times 64$	3-point fct.	1995	(000),(001) (010), (100) (011), (101) (110), (111) (11-1),(1-1-1) (002)

preliminary !!



The correlator $\langle \pi^+ | \bar{u} u(\vec{y}_T) \bar{d} d(\vec{0}_T) | \pi^+ \rangle$.

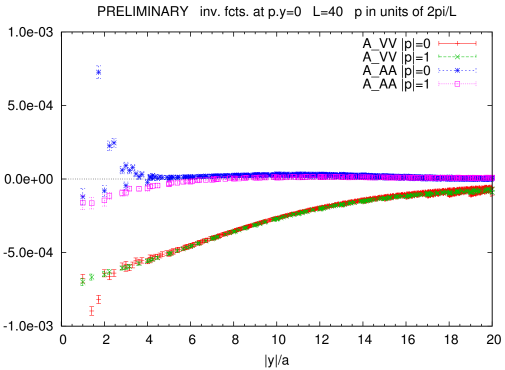
preliminary !!



The difference of correlators SS-PP

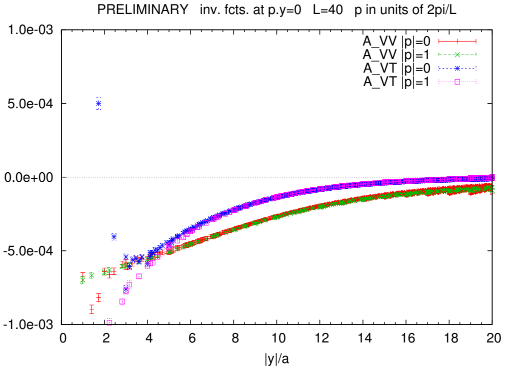
We find this to be strongly quark mass dependent
$(m_{ud} \to m_s \to m_c)$!!

preliminary !!



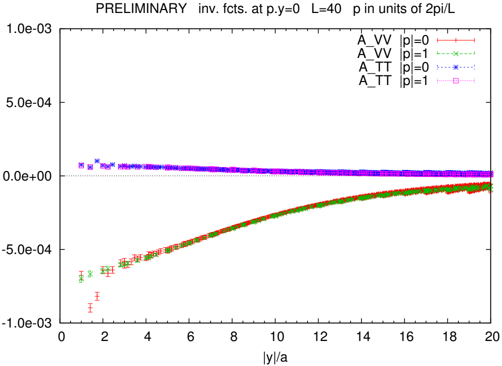
The correlators $\langle\pi^+|\bar{u}\gamma_\mu u(\vec{y}_T)\bar{d}\gamma_\mu d(\vec{0}_T)|\pi^+\rangle$ and $\langle\pi^+|\bar{u}\gamma_\mu\gamma_5 u(\vec{y}_T)\bar{d}\gamma_\mu\gamma_5 d(\vec{0}_T)|\pi^+\rangle$.

preliminary !!



The vector and tensor correlators.

preliminary !!



The vector and tensor correlators.

Next step: Physical interpretation ?!?

- The same information can be obtained from the $p \neq 0$ data. Works in many cases, but there are still some puzzles.
- We have data for $m_u = m_d$, m_s and m_c which shows significant systematic trends ?!?
- We have to construct phenomenological models using lattice input to estimate whether MPIs for pion collisions are significant
- All of this has to be repeated for the nucleon.

Conclusions

- Transverse momentum dependent physics is much, much more difficult than collinear one
- Therefore, the formalism is not yet well established
  $\Rightarrow$ If you succeed you will become very well known.
- Progress is urgently needed for EIC and LHC
- We have calculated Boer Mulders shifts etc. on the lattice for nucleon and pion.
  - Early convergence for $\hat{\zeta} \to \infty$!!
  - All Boer-Mulders functions are in fact looking alike
- We have also calculated DPDs for pions with physical quark masses.
  - for many channels we get clear correlations for, e.g., the AA channel it is basically zero
  - the phenomenological effects if similarly large correlations in the proton have for LHC physics have to be analyzed.
  - after the pion DPDs are understood one has to proceed to the nucleon