Excited-State Effects on Nucleon Form Factors

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work done in collaboration with
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Workshop on "Perspectives and Challenges in Lattice QCD"
Tata Institute of Fundamental Research, Mumbai, India
February 16-20, 2015

Introduction

- Proton radius puzzle:
  - why do *ep* scattering and muonic hydrogen results disagree so badly?

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- Possible explanations:
  - Pions too heavy
  - Lattice volumes too small
  - Excited-state effects
- Here: explore how to suppress excited-state effects

Form Factors

- eN scattering cross section parameterized in terms of Sachs form factors G_E, G_M via Rosenbluth formula

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \propto \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right)\right]\,, \quad \tau = \frac{Q^2}{4m_N^2}$$

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- Matrix element of vector current between nucleon states decomposed in terms of Dirac and Pauli form factors F_1, F_2 as

$$\langle N(p',s')|V_\mu|N(p,s)\rangle = \overline{u}(p',s')\left[\gamma_\mu F_1 + i\frac{\sigma_{\mu\nu}q_\nu}{2m_N}F_2\right]u(p,s)$$

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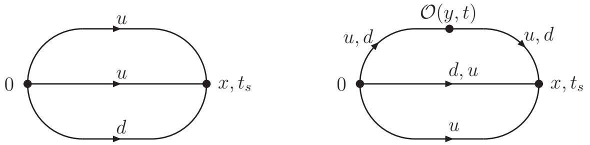
- Relationship given by

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Lattice Setup



- Measure two-point functions

$$C_2(\mathbf{p}, t) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}}\, \Gamma_{\beta\alpha}\, \langle \Psi^\alpha(\mathbf{x}, t)\overline{\Psi}^\beta(0)\rangle$$

  and three-point functions

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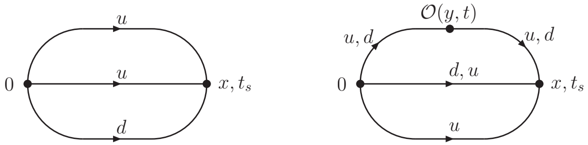

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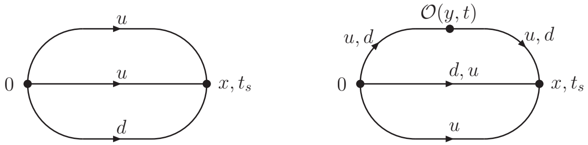
- Use extended propagator in the "fixed-sink" method (one additional inversion per value of t_s, but free choice of operator insertion and momentum transfer)

Lattice Setup



- Form ratios

$$R_{V_\mu}(\mathbf{q}, t, t_s) = \frac{C_{3,V_\mu}(\mathbf{q}, t, t_s)}{C_2(\mathbf{0}, t_s)} \sqrt{\frac{C_2(\mathbf{q}, t_s - t) C_2(\mathbf{0}, t) C_2(\mathbf{0}, t_s)}{C_2(\mathbf{0}, t_s - t) C_2(\mathbf{q}, t) C_2(\mathbf{q}, t_s)}}$$

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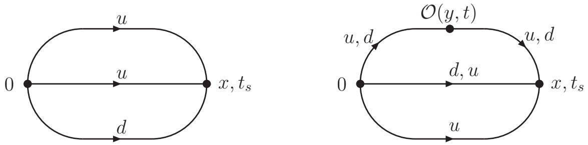

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- Extract Sachs form factors from

$$\mathrm{Re}\,[R_{V_0}] \xrightarrow{t,(t_s-t)\gg 0} \sqrt{\frac{m_N + E_{\mathbf{p}}}{2E_{\mathbf{p}}}} G_E(Q^2)$$

and

$$\mathrm{Re}\,[R_{V_i}]_{i=1,2} \xrightarrow{t,(t_s-t)\gg 0} \epsilon_{ij} p_j \frac{1}{\sqrt{2E_{\mathbf{p}}(m_N + E_{\mathbf{p}})}} G_M(Q^2)$$

The Problem of Excited States

- Correlation functions have spectral decomposition

$$C_2(\mathbf{p}, t) = \sum_{n=1}^{\infty} Z_n(\mathbf{p}) \mathrm{e}^{-E_n(\mathbf{p})t}$$

where all states with quantum numbers compatible with a nucleon of momentum $\mathbf{p}$ contribute

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- In particular,
  - for $\mathbf{p} = 0$, the $N\pi\pi$ state with two pions in an S-wave contributes with $E \approx m_N + 2m_\pi \sim m_N$
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- For nucleons, the statistical noise grows exponentially like $\mathrm{e}^{(m_N - \frac{3}{2}m_\pi)t}$
- Hard to find a region of clean signal and good statistics!

Data Analysis Methods for Excited States

$$G_X^{\mathrm{eff}}(Q^2, t, t_s) = G_X(Q^2) + c_{X,1}(Q^2)\mathrm{e}^{-m_\pi t} + c_{X,2}(Q^2)\mathrm{e}^{-2m_\pi (t_s - t)} + \dots$$

- **Plateau method**: Identify plateaux in t
- Problem: Need large t_s, where signal-to-noise ratio is poor
- Observe systematic trend in t_s even for $t_s \sim 1.4$ fm

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- **Summation method:**

$$S_X(Q^2,t_s) = \sum_{t=0}^{t_s} G_X^{\mathrm{eff}}(\mathbf{q},t,t_s) \to c + t_s G_X(Q^2) + \mathcal{O}\left(\mathrm{e}^{-m_\pi t_s}\right)$$

- Advantage: Parametrically reduced excited state contamination – $m_\pi t_s$ instead of $m_\pi t$
- Disadvantage: Increased statistical errors

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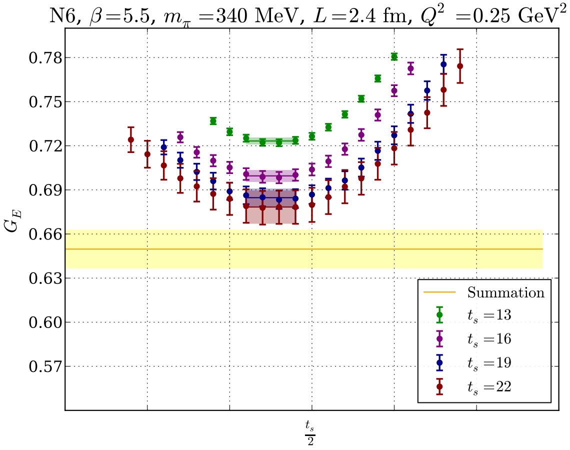
- **Excited-state fits:** Explicitly fit $G_X(Q^2, t, t_s)$ to leading excited-state contributions
  - as a function of t, $t_s - t$ at each t_s separately, or
  - as a function of t_s, t at all t_s simultaneously
- Advantage: Fully removes leading excited state contamination
- Disadvantage: Somewhat model-dependent, hard to assess trustworthiness of results

Excited State Contributions

$$G_X^{\rm eff}(Q^2,t,t_s) = G_X(Q^2) + c_{X,1}(Q^2)\mathrm{e}^{-m_\pi t} + c_{X,2}(Q^2)\mathrm{e}^{-2m_\pi(t_s-t)} + \ldots$$



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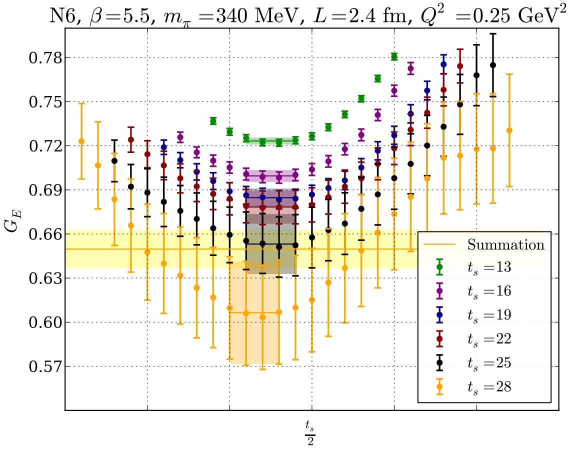

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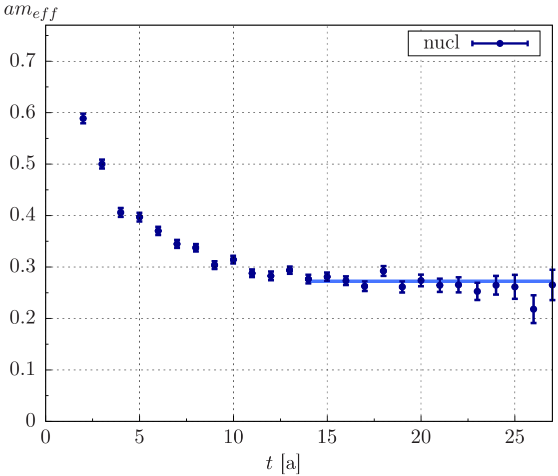
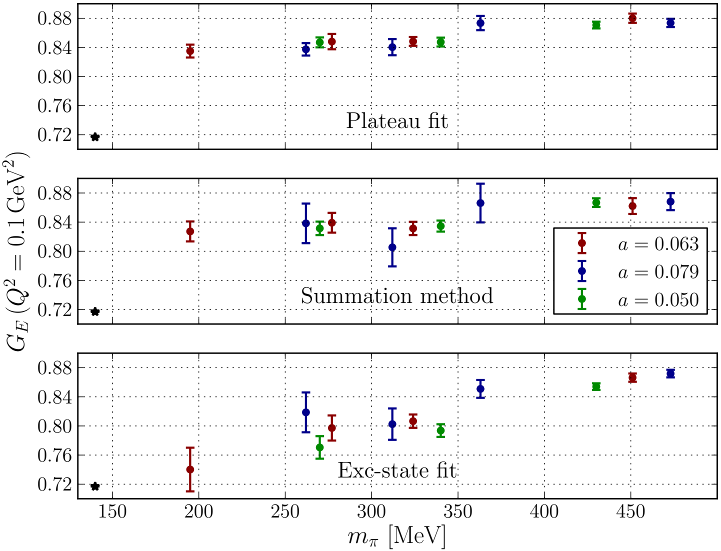
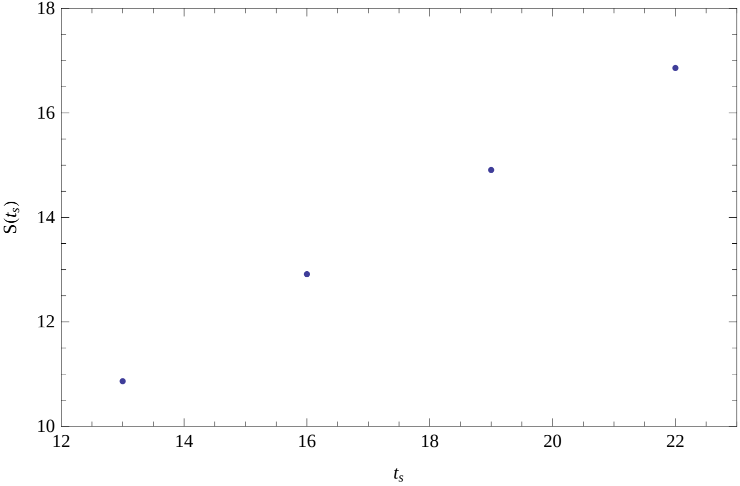


Chiral behaviour



A Possible Scenario



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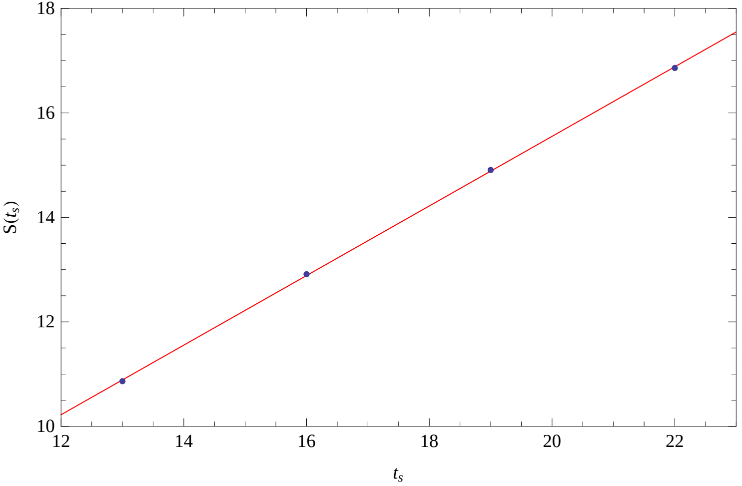

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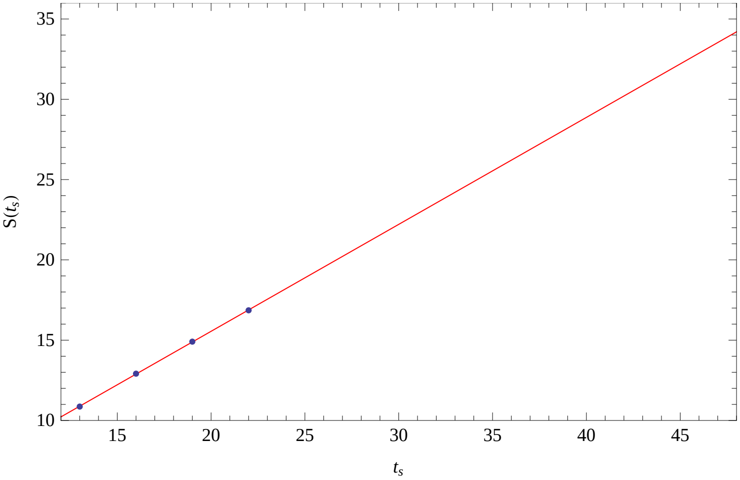

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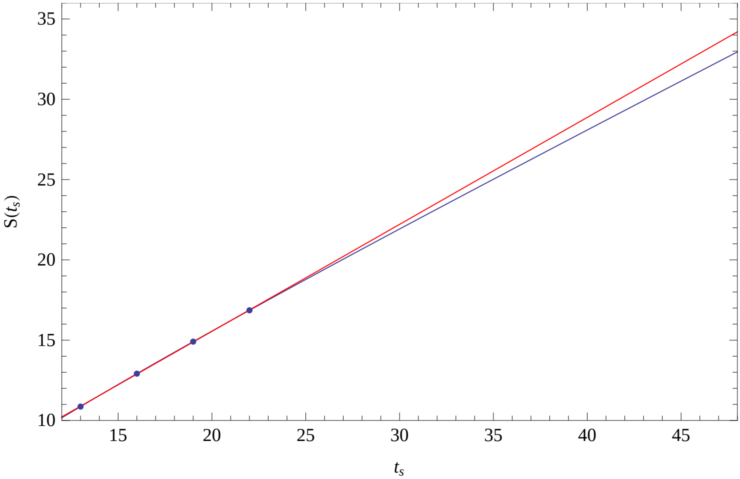

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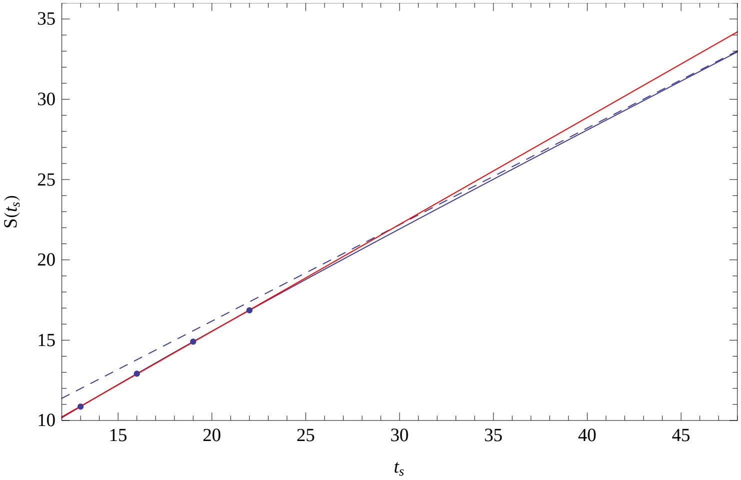

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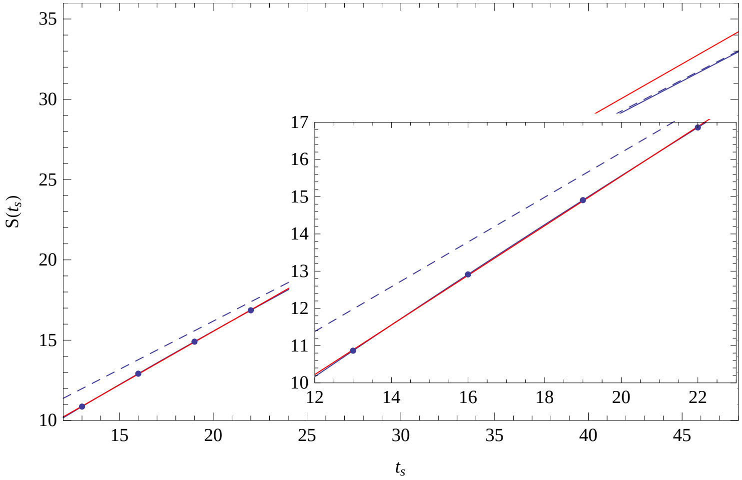

Momentum Dependence – Conventional Approach

- Parameterize each Sachs form factor as a dipole

$$G_X(Q^2) = \frac{G_X(0)}{\left(1 + \frac{Q^2}{M_X^2}\right)^2}$$

where $G_E(0) = 1,\ G_M(0) = \mu$

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- Charge radii and anomalous magnetic moment given by

$$\frac{1}{M_E^2} = \frac{r_E^2}{12} = \frac{r_1^2}{12} + \frac{\kappa}{8m_N^2} \qquad \frac{1}{M_M^2} = \frac{r_M^2}{12} = \frac{r_1^2 + \kappa r_2^2}{12(1+\kappa)}$$

where $\kappa = \mu - 1$

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where $\kappa = \mu - 1$

- Given $r_E \approx r_M$, can extract magnetic moment also using

$$\mu = \lim_{Q^2 \to 0} \frac{G_M(Q^2)}{G_E(Q^2)}$$

with a flat extrapolation

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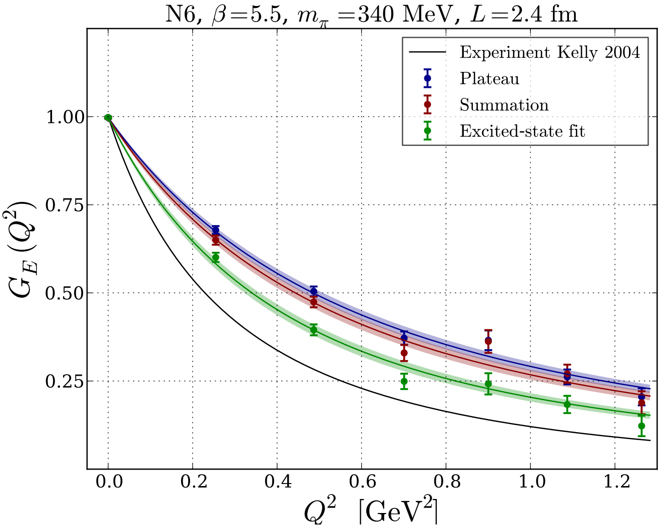

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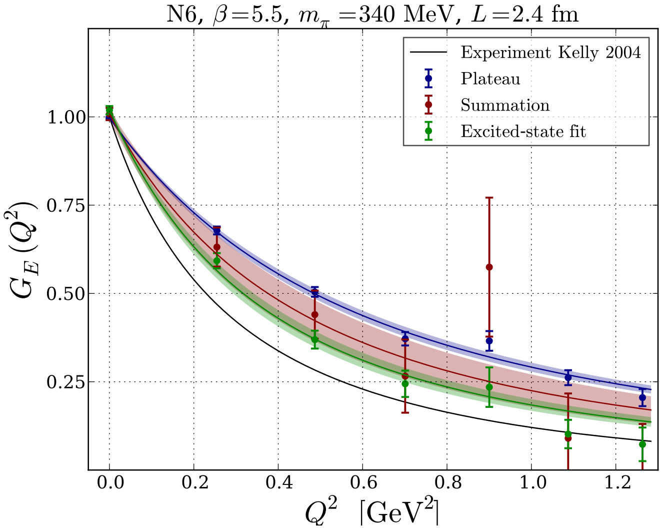

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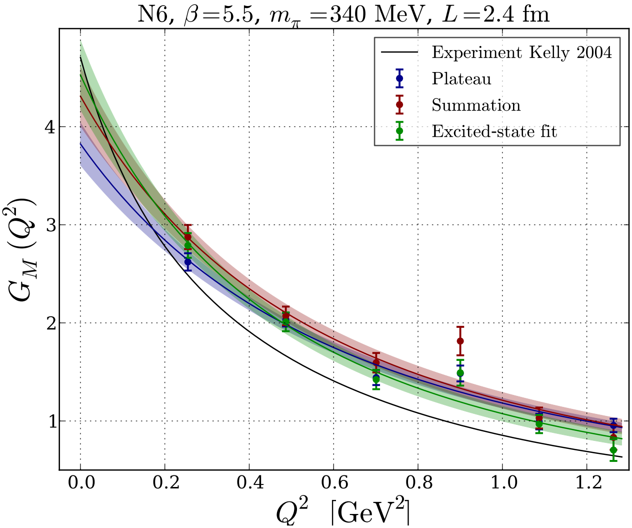

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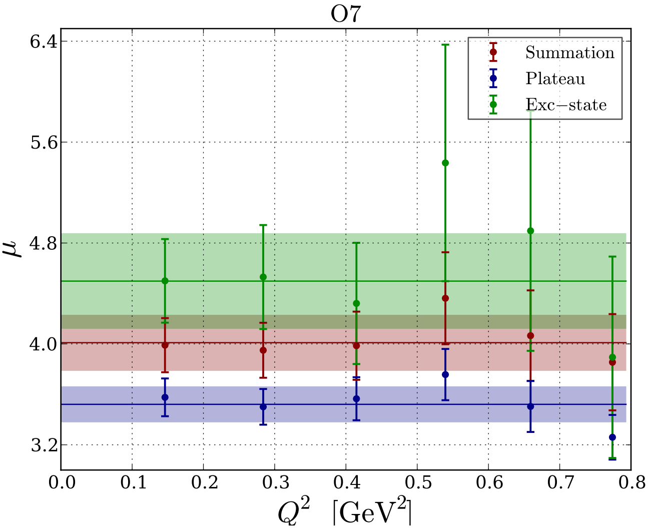
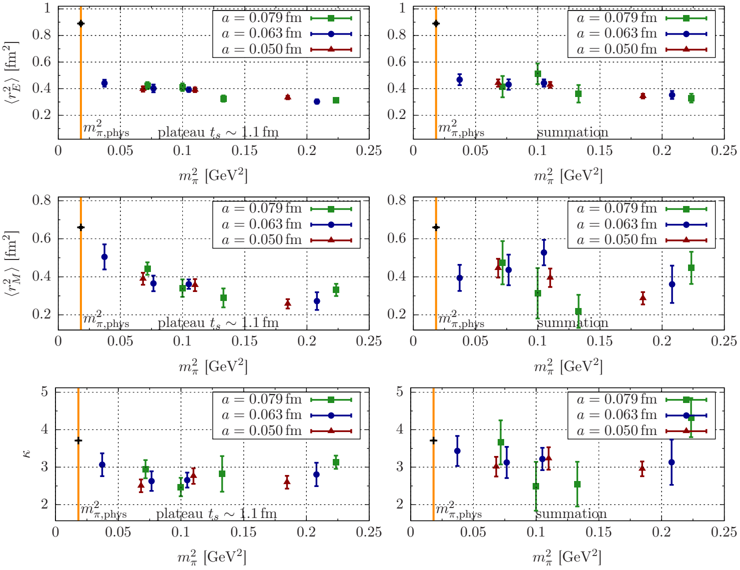


Chiral Extrapolation – Conventional Approach



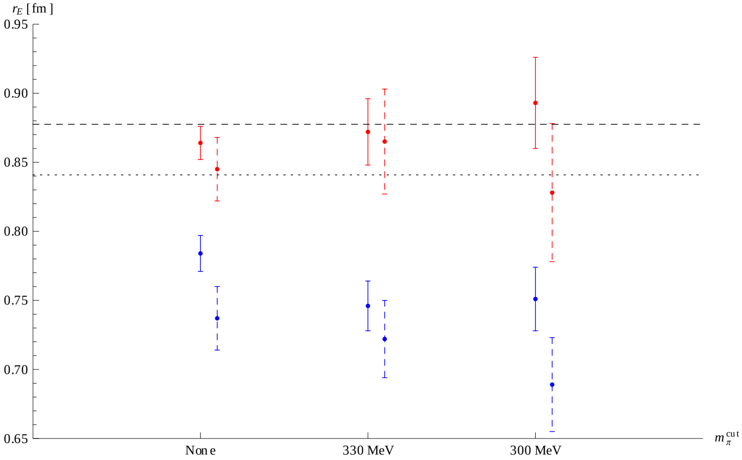
Momentum Dependence – Direct Approach in χPT

- Simultaneously fit $G_E(Q^2, m_\pi)$ and $G_M(Q^2, m_\pi)$ to the corresponding formulae from χPT including the ρ (and optionally the Δ)
- Can include lattice artifacts using

$$G_E(Q^2, m_\pi, a) = G_E^\chi(Q^2, m_\pi) + aQ^2\, G_E^a$$
$$G_E(Q^2, m_\pi, a) = G_M^\chi(Q^2, m_\pi) + a\, G_M^a$$

  taking into account that $G_E(0)$ is O(a)-improved already
- Use experimental values of chiral limit LECs (leaving 4 fit parameters)
- Alternatively use ensemble values of m_N, m_ρ
- Impose cuts on m_π and/or Q^2 to check convergence

Pion-Mass Dependence – Direct Approach in χPT



Blue – Summation method   Red – Two-state fits

Summary

- Systematic trend in G_E plateau values persists to $t_s \sim 1.4$ fm
- Even with summation method, G_E systematically too high
- Considering only the largest values of t_s brings summation method closer to experiment at the expense of large statistical errors
- Excited-state fits indicate a possible reason:
  - with small gap m_π, approach to plateau is very slow
  - summed ratios still receive sizeable corrections
- Serious reduction of noise required for further clarification
- Chiral behaviour may be better describable using a direct χPT fit to the form factor data rather than dipole fits followed by chiral extrapolations

The end

Thank you for your attention