# Compact U(1) lattice gauge theory with non-perturbative gauge fixing

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# Introduction

- Investigate the continuum limit of a compact formulation of the lattice U(1) gauge theory in 4 dimensions using a novel regularization originally devised to tame the 'rough gauge' problem of lattice chiral gauge theories.
- Compact formulation of U(1) gauge theory with usual Wilson action has two different phases: weak coupling (with usual QED properties) and a strong coupling confining phase separated by a first order transition. To remove the regulator, there ought to be a continuous transition.
- Extended parameter space($g, \kappa, \tilde{\kappa}$) due to the inclusion of gauge-fixing term and mass counterterm along with the existing Wilson term.
- Existence of a continuous phase transition (FM-FMD) at which the *lgdof* decouple, and the U(1) gauge symmetry is restored (Bock et al 2000).

# The Regularization

- Action for the compact gauge-fixed U(1) theory, where the ghosts are free and decoupled:

$$S[U] = S_g[U] + S_{gf}[U] + S_{ct}[U].$$

where $S_g[U] = \frac{1}{g^2} \sum_{x\,\mu<\nu} (1 - \mathrm{Re}\, U_{\mu\nu x})$ is usual Wilson plaquette action.

- Golterman and Shamir proposed the gauge fixing term

$$S_{gf}(U) \equiv S_{HD}(\phi, U)|_{\phi_x = I}$$

$$S_{HD} = \frac{1}{2\alpha g^2}\left(\sum_{x,y} \phi_y^\dagger \Box_{yx}^\dagger \Box_{xy} \phi_y - \sum_x B_x B_x\right)$$

$$\Box_{xy}(U) = \sum_\mu \left(\delta_{y,x+\mu} U_{x\mu} + \delta_{y,x-\mu} U^\dagger_{x-\mu,\mu} - 2\delta_{yx}\right)$$

$$B_x = \sum_\mu \left(\frac{V_{x\mu} + V_{x-\mu,\mu}}{2}\right)^2, V_{x\mu} = \frac{1}{2i}\left(\phi_x^\dagger U_{x\mu} \phi_{x+\mu} - \mathrm{h.c.}\right)$$

# The Regularization : Gauge-fixing term

- The above satisfies the following properties:
  - näive continuum limit $\rightarrow S_{gf} = \frac{1}{2\alpha}(\partial_\mu A_\mu)^2 +$ irrelevant terms.
  - $S_{gf}$ has unique absolute minimum at $U_{x\mu} = I$.
  - $\phi$ fields decouple and gives the desired features in the phase diagram.
  - avoids Neuberger's theorem and Gribov copies.
- "Higgs picture" $S_{HD}(\phi, U)$ is equivalent to "vector picture" $S_{gf}(U)$ as they are related by a gauge transformation. (Bock et al 1999)
- In the vector picture $S_{gf}$, $V_{x\mu}$ reduces to $\mathbb{A}_{x\mu} \equiv \mathrm{Im}\, U_{x\mu}$.
- Defining $C_x \equiv -\sum_y \Box_{xy}(U)$ and $\bar{\kappa} \equiv \frac{1}{2\alpha g^2}$,

  the action can be written as $S_{gf} = \bar{\kappa} \sum_x S_x$,

  where $S_x = C_x^\dagger C_x - B_x^2$.

- Furthermore, $C$ can be broken down into its hermitian and anti-hermitian parts to obtain $S_x = S_x^{(1)} + S_x^{(2)}$, where

$$S_x^{(1)} = \left( \frac{C_x^\dagger - C_x}{2i} \right)^2$$

$$S_x^{(2)} = \left( \frac{C_x^\dagger + C_x}{2} + B_x \right) \left( \frac{C_x^\dagger + C_x}{2} - B_x \right)$$

- $S_x^{(1)} = \sum_\mu \Delta_\mu^b \mathbb{A}_{x\mu}$ gives the covariant gauge-fixing term in the näive continuum limit. This gives rise to numerous gauge-field configurations satisfying the gauge condition on the lattice a.k.a Gribov copies.
- $S_x^{(2)}$ should come to the rescue without any baggage of its own.

# The Regularization : Gauge-fixing term

- $\left(\frac{C_x^\dagger + C_x}{2} + B_x\right)$ is positive. With $\left(\frac{C_x^\dagger + C_x}{2} - B_x\right) = \sum_\mu \left(D^{(1)}_{x,\mu} + D^{(2)}_{x,\mu}\right)$, where

$$D^{(1)}_{x,\mu} = \left(I - \tfrac{1}{4}\left(U_{x\mu} + U_{x-\mu,\mu} + \mathrm{h.c.}\right)\right)^2 ,$$
$$D^{(2)}_{x,\mu} = \tfrac{1}{2} I - \tfrac{1}{8}\left(U^\dagger_{x\mu} U_{x-\mu,\mu} + U_{x\mu} U^\dagger_{x-\mu,\mu} + \mathrm{h.c.}\right),$$

  its positivity is evident due to construction in the first term and unitarity of link variables in the latter $\Rightarrow S_x^{(2)}$ is a positive definite quantity.
- Observe that $S_x^{(2)} = 0$ when $U_{x\mu} = I$ and this is a unique minimum. As a result, $S_{gf}$ now has a unique minimum at $U_{x\mu} = I$ thus bypassing the problem of Gribov copies.
- Expansion of $S_x^{(2)}$ gives the lowest order term to be of six dimensions $\Rightarrow$ irrelevant.
- $S_x^{(2)}$ also breaks BRST invariance explicitly, since $S_{gf}$ cannot be written as the square of a gauge-fixing condition, thus avoiding Neuberger's theorem.

# The Regularization : Mass Counterterm

- Validity of weak coupling perturbation theory around $g = 0$ or $\tilde{\kappa} = \infty$ together with perturbative renormalizability helps to determine the form of the counterterms to be present in $S_{ct}$. (Bock et al 2000)
- It turns out that the most important gauge counterterm is the gauge field mass counterterm, given by,

$$S_{ct} = -\kappa \sum_{\mu x} \left( U_{\mu x} + U_{\mu x}^{\dagger} \right) .$$

- It alone leads to a continuous phase transition that recovers gauge symmetry.

# Constant Field Approximation

- The classical potential, obtained as leading order term in the perturbative expansion of $U_{\mu x} = \exp igA_{\mu x}$ with constant field approximation around $U_{\mu x} = 1$, is

$$V_{cl} = \kappa \left[ g^2 \sum_\mu A_\mu^2 \right] + \frac{g^4}{2\alpha} \left[ \left( \sum_\mu A_\mu^2 \right) \left( \sum_\mu A_\mu^4 \right) \right]$$

- For $\kappa > 0$, the gauge boson is massive and $V_{cl}$ has a minimum at $A_\mu = 0$. A broken phase called FM phase.

  For $\kappa < 0$, the minimum of $V_{cl}$ shifts to a nonzero value:

$$A_\mu = \pm \left( \frac{\xi |\kappa|}{3g^2} \right)^{\frac{1}{4}} \qquad \text{for all} \quad \mu$$

  implying an unusual phase with broken rotational symmetry in addition to the broken gauge symmetry – directional ferromagnetic phase (FMD).

  For $\kappa = 0 \equiv \kappa_c$, the gauge boson becomes massless with the minimum of $V_{cl}$ still being the same $\Rightarrow$ phase transition at this point.

# Observables

- We used the following observables (for a $L^4$-lattice):

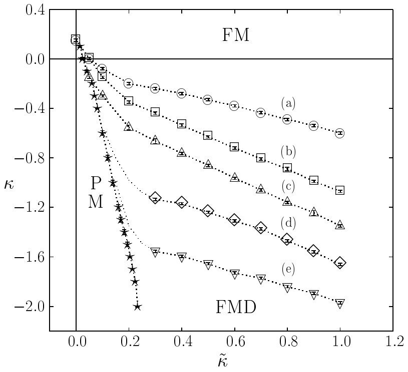
$$
\begin{aligned}
E_P &= \frac{1}{6L^4} \left\langle \sum_{x,\mu<\nu} \mathrm{Re}\, U_{\mu\nu x} \right\rangle \\
E_\kappa &= \frac{1}{4L^4} \left\langle \sum_{x,\mu} \mathrm{Re}\, U_{\mu x} \right\rangle \\
V &= \left\langle \sqrt{\frac{1}{4} \sum_{\mu} \left( \frac{1}{L^4} \sum_{x} \mathrm{Im}\, U_{\mu x} \right)^2} \right\rangle .
\end{aligned}
$$

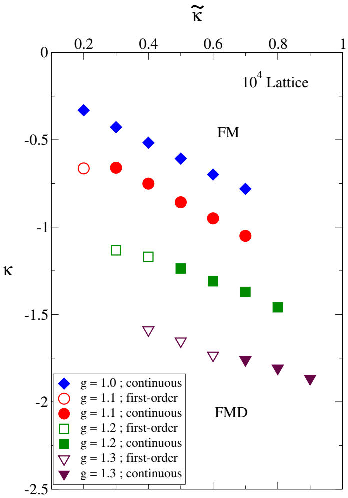
where $E_P$ and $E_\kappa$ though not order parameters, signal phase transitions with sharp changes by varying $\kappa$ for a fixed $g$ and $\tilde{\kappa}$.

- $V$ behaves like an order parameter as it takes a non-zero value only in the FMD phase.

# Numerical Simulations

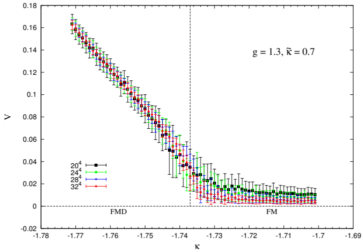
- Earlier simulations have been done with the above action by Bock et al and Basak et al but in very small lattice sizes.
- For small values of $g$, usual perturbative results have been obtained at the FM-FMD phase transition like the restoration of gauge symmetry by the decoupling of *lgdofs* and zero gauge boson mass.
- Continuous phase transition has been observed by varying $\kappa$ for even small values of $\tilde{\kappa}$.
- This is a preliminary work to explore interesting regions of the phase diagram.
- For large $g$, a first-order transition has been observed at the FM-FMD transition for small values of $\tilde{\kappa}$.
- This phase transition has to be accessed from the FM phase since the FMD phase is a phase with broken rotational symmetry.
- We have used 4-hit Metropolis method to generate configurations for $10^4, 16^4, 20^4, 24^4, 28^4$ and $32^4$ lattices.

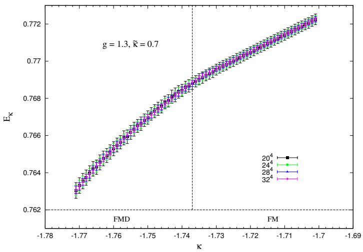


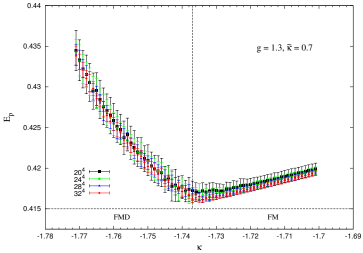



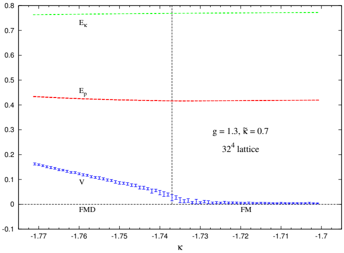

- Ref: S. Basak, A. K. De and T. Sinha, Phys. Lett. B 580 (2004) 209

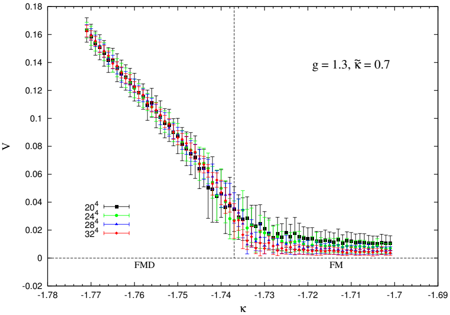
# Preliminary numerical results: Continuous transition



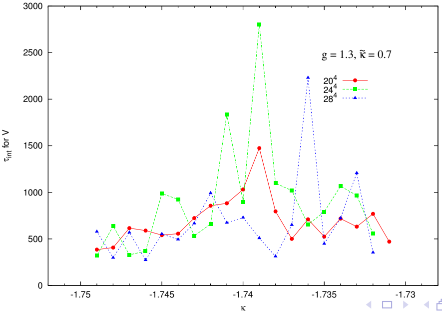



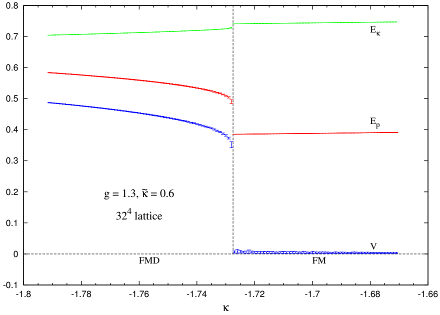


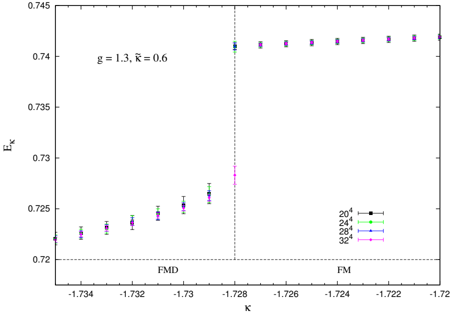
g = 1.3, κ̃ = 0.7
V
κ
FMD
FM
20^4
24^4
28^4
32^4

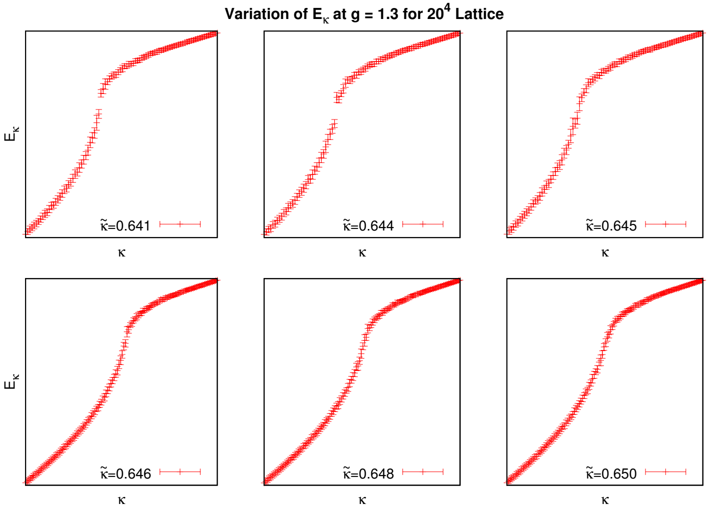
g = 1.3, κ̃ = 0.7
τ_int for V
κ
20^4
24^4
28^4

# Preliminary numerical results: First order transition



# Identification of tricritical point



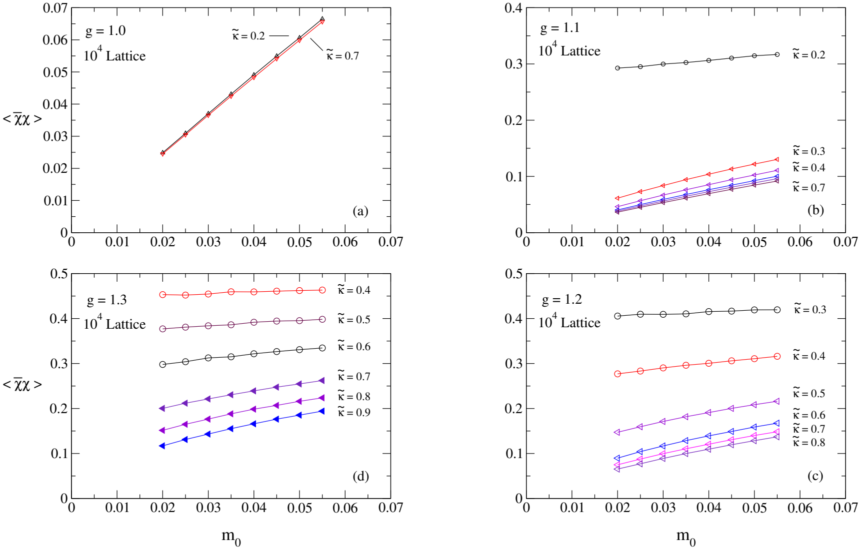
# Calculation of Quenched chiral condensate

- Using quenched staggered fermions, we have measured the chiral condensates

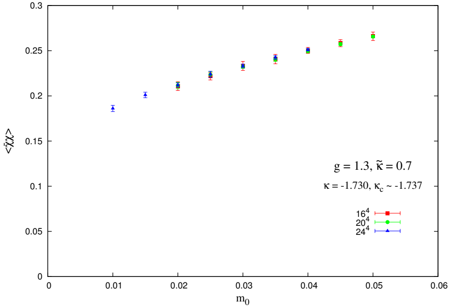
$$\langle \bar{\chi}\chi \rangle_{m_0} = \frac{1}{L^4} \sum_x \langle M^{-1}_{xx} \rangle$$

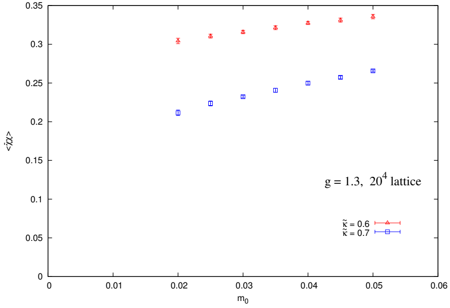
as a function of vanishing fermionic bare mass $m_0$ where $M$ is the fermion matrix. The chiral condensates were computed with the Gaussian noise estimator method.
- Measurement is near the phase transition from the FM phase.
- We observe a chiral phase transition near $g \sim 1.1$.
- We need to determine whether this coincides with the tricitical point at the FM-FMD transition.

# Preliminary results: Quenched chiral condensate



# Preliminary results: Quenched chiral condensate



# Conclusions

- This regularization scheme allows us to take a continuum limit, hitherto difficult in pure U(1) theory, which gives us known physics at weak coupling.
- Approach to the continuum limit in the strong coupling phase needs to be looked at carefully.
- This scheme of abelian gauge-fixing is crucial for the overall success of the gauge-fixing approach to chiral gauge theories.

# Thank You...