Light Glueball masses using the Multilevel Algorithm

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Outlines

- Introduction
- Operators
- Computational Strategies
- Algorithm
- Results
- Conclusions

Introduction

- What are Glueballs ?
  - Stable low lying states in pure Yang-Mills theory are called glueballs. They are not elementary particles but composites.
- Glueball masses can be computed in lattice gauge theory simulations.
  - Monte-Carlo technique is used to evaluate the finite dimensional path-integral on the lattice.
- Masses obtained from correlators between gauge invariant sources
  - Sources are constructed by applying gauge invariant operators on the lattice gauge theory vacuum.

- Correlation functions of operators at a time separation Δt (in terms of the transfer matrix $\mathbb{T}$) is

$$C(\Delta t) = \langle O(\Delta t) O(0) \rangle = \frac{Tr(\mathbb{T}^{T-\Delta t} O(\Delta t) \mathbb{T}^{\Delta t} O(0))}{Tr(\mathbb{T}^{T})} \quad (1)$$

$$= \sum_{n>0} |\langle 0 | O(0) | n \rangle|^2 exp(-m_n \Delta t)$$

in large Δt limit

$$C(\Delta t) \sim exp(-m_1 \Delta t). \quad (2)$$

Also $\mathbb{T} = exp(-aH)$ T : temporal extent of lattice.

- Ground state glueball masses in a given symmetry channel can be extracted from the previous relation.
    - Glueballs are heavy (> 1 GeV). Correlators are strongly suppressed by the heavy mass.
    - Extraction of glueball masses at large temporal separation is extremely difficult, as the statistical noise dominates the signal.

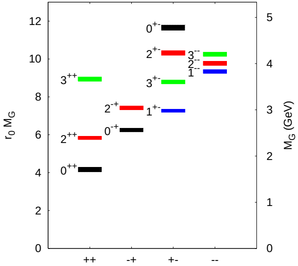
    - Alternative : Estimate effective masses from the correlators at short temporal separations

$$am_{eff} = ln\frac{\langle C(\Delta t)\rangle}{\langle C(\Delta t + 1)\rangle} \tag{3}$$

- The above alternative coupled with a finer lattice spacing in the temporal direction has been used by a lot of groups for computing the glueball spectrum.

Current Status

- Y. Chen, N. Mathur *et al.* used anisotropic lattices in which the temporal spacing is much smaller than that in the spatial directions. Exploiting the enhanced signal-to-noise ratio of the correlation functions at smaller temporal separations they obtained : arXiv:hep-lat/0510074



- They could follow the correlator up to a physical distance of about 0.6 fm.
- They also used variational techniques to construct glueball operators using different basis sets of Wilson loops, which have better ground state projection.
- Various groups used smearing methods to improve Signal-to-Noise ratio.
- We used improved methods to get better signal at larger physical distance.

Operators

- On lattice formulation rotational SO(3) symmetry is broken down to cubic O_h symmetry with 24 elements.
- In discrete symmetry notation irreps. of O_h are $A_1(1)$,$A_2(1)$,$E(2)$, $T_1(3)$, $T_2(3)$
- In continuum limit these representations corresponds to irreps D_J corresponding to spin J. Glueballs are identified with that spin (J) quantum number. Other quantum numbers for the glueballs are parity P and charge conjugation C.
- Wilson loops form the basis of irreducible representations of O_h.
- Linear combinations of the members of the basis span the irreducible representations of the symmetry channels.

- $A_1{}^{++}$ and E^{++} representations can be constructed using square Wilson loops in following way
  - $$A_1{}^{++}:$$
    $$\mathcal{A} = \mathbb{R}e(P_{xy} + P_{xz} + P_{yz})$$
  - $$E^{++}:$$
    $$\mathcal{E}_1 = \mathbb{R}e(P_{xz} - P_{yz}) \qquad \mathcal{E}_2 = \mathbb{R}e(P_{xz} + P_{yz} - 2P_{xy})$$

$$P_{ab} : \text{ Wilson loops in plane } ab \in \{x, y, z\}$$

  - Irreducible representations of O_h can be constructed using large basis sets of Wilson loops of different sizes and shapes.
  - In simplest case plaquettes in three spatial planes form the basis.
  - We choose large square Wilson loops to construct scalar and tensor glueball operators.

- Zero momentum operators at time slice t is

$O_i(t) = 1/L^{3/2} \sum_x O_i(x,t)$ x : points on a time-slice

$$O(t) = 1/L^{3/2} \sum_x O(x,t) \tag{4}$$

where $O(x,t) = \mathcal{W}[U] - \langle \mathcal{W}[U] \rangle$

$\mathcal{W}[U]$: Wilson loops on time-slice t.

Strategies

- R. Gupta *et al.* showed that optimal size of Wilson loops for computing glueball correlators was $r_0 \times r_0$ (where $r_0 = 0.5fm$).
  Phys. Rev. D 43 (1991) 2301

- To reduce excited state contamination and improve Signal-to-Noise ratio at large temporal separation
  - We constructed glueball operators from large Wilson loops of dimension $r_0 \times r_0$.
  - Extracted masses from correlators with fit range between **0.5 - 1.0** fm.
  - We used improved algorithm to get signals at these physical distances.

Algorithm

- We used Cabibbo-Marinary heatbath for $SU(3)$:
  3 Over-relaxation steps for every heatbath steps.
  - Heatbath algorithm updates single link at a time, keeping all other field variables fixed.
  - This is something like bringing the chosen link variable in touch with an infinite heatbath.
  - Over-relaxation is used to minimize the autocorrelation between consecutive measurements.
- The method we have used is particularly useful in theories with mass gap, where the distant regions of the theory are uncorrelated as the correlation length is finite.

Multilevel Technique

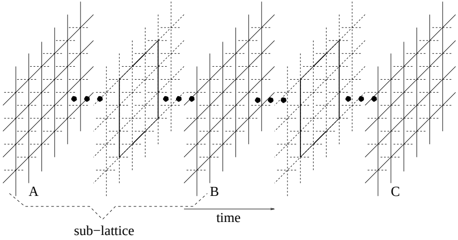


Figure: Multilevel Scheme

- Slice lattice along temporal direction by fixing spatial links (A,B & C in fig.) and compute intermediate expectation values of Glueball operators by performing sub-lattice updates.
- Intermediate values are first constructed by averaging over sub-lattices with boundaries. Full expectation values – by averaging over the intermediate values with different boundaries.

- This method uses the locality property of the Wilson gauge action.

- On top of the Multilevel algorithm we used Multihit technique
  - Variance reduction technique, replacing link variables with the averaged link:

$$\overline{U} = \frac{\int dU \exp[(\frac{\beta}{N})ReTr(US)]U}{\int dU \exp[(\frac{\beta}{N})ReTr(US)]} \tag{5}$$

  - This averaging can be done using Monte-Carlo method.
    We used semi-analytic method due to de Forcrand and Roiesnel, which is order of magnitude faster than Monte-Carlo method.

Phys. Lett. B 151 (1985) 77.

Simulation Parameters

- Parameters for the Scalar Channel :

Lattice Size	β	(r_0/a)	sub−lattice thickness	iupd	loop size
$10^3 \times 18$	5.7	2.922(9)	3	30	2×2
$12^3 \times 18$	5.8	3.673(5)	3	25	3×3
$16^3 \times 20$	5.95	4.898(12)	4	50	5×5

- Parameters for the Tensor Channel :

Lattice Size	β	(r_0/a_t)	sub−lattice thickness	iupd	loop size
$12^3 \times 18$	5.8	3.673(5)	3	70	3×3
$12^3 \times 20$	5.95	4.898(12)	5	100	5×5
$12^3 \times 20$	6.07	6.033(17)	5	100	5×5

Results

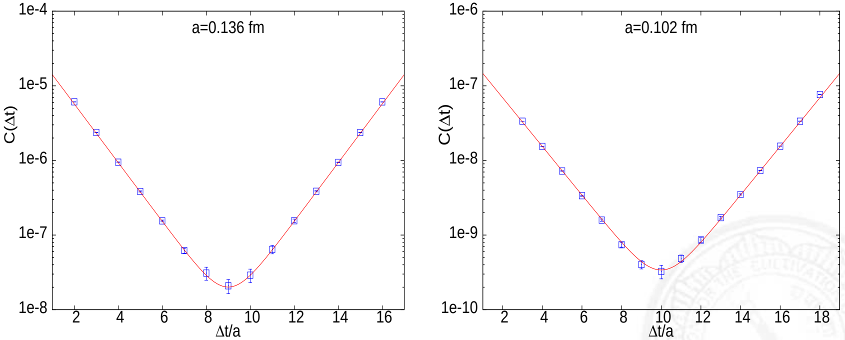


Figure: Scalar Glueball Correlators

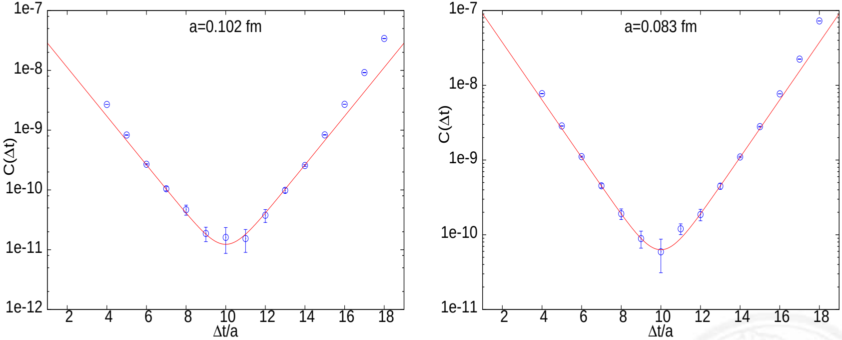


Figure: Tensor Glueball Correlators

Effective Mass plots

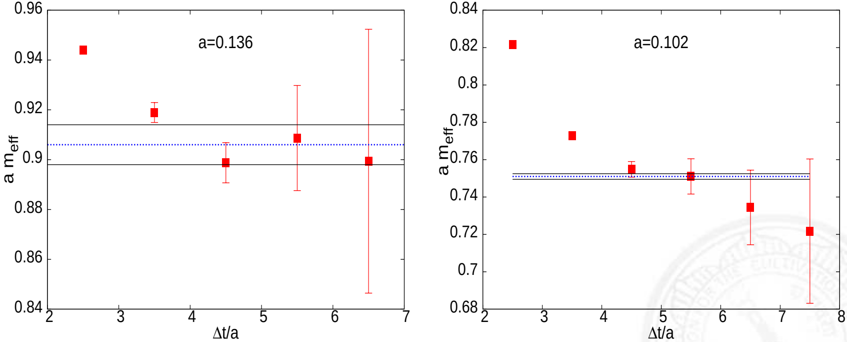


Figure: Effective mass plots for scalar glueballs

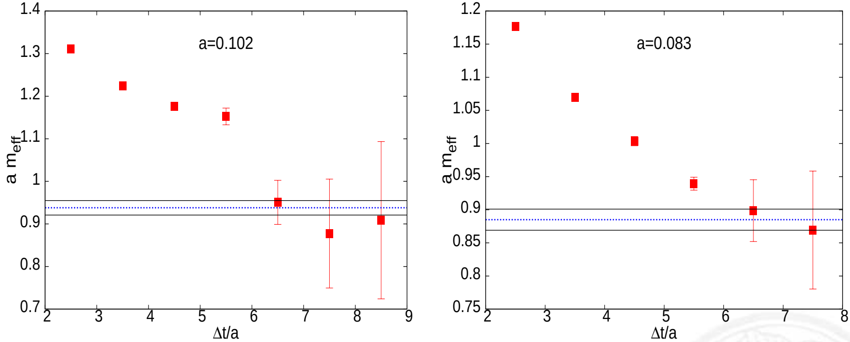


Figure: Effective mass plots for tensor glueballs

Fits

- We used 10 sweeps over entire lattice for each measurement to remove autocorrelation.
- We have fitted the correlators to the form :
  - $$C(\Delta t) = A\left(e^{-m\Delta t} + e^{-m(T-\Delta t)}\right) \tag{6}$$

m : glueball mass T : temporal extent of lattice
Fits to data folded about $T/2$. (We used periodic b.c on lattice)
Routine : "non-linear model fit" of Mathematica.

- We computed the effective mass from the correlator as

$$am_{\text{eff}} = -\log \frac{\langle C(\Delta t + 1)\rangle}{\langle C(\Delta t)\rangle} \tag{7}$$

Mass

- Mass and range in scalar channel :

Lattice	β	fit-range	ma	$\chi^2/d.o.f$
$10^3 \times 18$	5.7	5-9	0.952(11)	0.066
$12^3 \times 18$	5.8	6-9	0.906(8)	0.03
$16^3 \times 20$	5.95	5-10	0.7510(15)	0.02

- Mass and range in tensor channel :

Lattice	β	fit-range	ma	$\chi^2/d.o.f$
$12^3 \times 18$	5.8	4-7	1.585(54)	1.64
$12^3 \times 20$	5.95	6-10	0.938(17)	0.12
$12^3 \times 20$	6.07	6-10	0.885(16)	1.6

- We cross-check our data with existing data by M. Teper *et al.* (JHEP 0406 (2004) 067) :

- Scalar channel:

β	5.7	5.8	5.95
am	0.941(25) 0.969(18)	0.909(15) 0.945(21)	0.743(12)
$am\ \binom{\text{this}}{\text{work}}$	0.952(11)	0.906(8)	0.7510(15)

- Tensor channel :

β	5.8	5.95	6.07
am	1.52(5) / 1.57(6)	1.148(19)	0.913(13)
$am\ \binom{\text{this}}{\text{work}}$	1.585(54)	0.938(17)	0.885(16)

Algorithmic Gain

- We compared the performance of our algorithm with naive method
  - runs for the same computer time using both methods:
  - Scalar Channel :

Lattice	run-time (mins)	$\frac{error_{naive}}{error_{multilevel}}$	gain(time)
$10^3 \times 18$	3850	5.7	32
$6^3 \times 18$	1000	5.5	30
$8^3 \times 24$	1100	18	324

  - Tensor Channel :

Lattice	run-time (mins)	$\frac{error_{naive}}{error_{multilevel}}$	gain(time)
$6^3 \times 18$	12000	27	729
$8^3 \times 30$	5775	20	400
$10^3 \times 30$	15000	-	-

Systematic Error

- Error reduction techniques only reduce statistical error.
- There are systematic errors as well .
- Most important among them are finite volume effects.
- In our lattice with small physical volumes we encountered them.
- for $\beta = 5.8$ the tensor glueball was lighter than scalar glueball in small lattice volumes.
- To avoid this finite volume effects we choose our lattice volumes such that $mL > 9$ in all cases.

Discussions

- Glueballs are expected to survive in theory with quarks. In that case there is a possibility of mixing of glueballs states with mesons with same quantum numbers.

- Possible mixing of glueball states with mesons of same quantum numbers, complicates it's unambiguous identification.

- The Particle Data Group (PDG) has listed few candidate glueball resonances

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  - $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $f_J(2220)$ etc.

Conclusions

- Extraction of glueball masses from correlators is a difficult problem in lattice QCD due to very low signal to noise ratio.
- In this work we presented a new method, based on multilevel technique.
- The multilevel algorithm is very efficient for calculating quantities with very small expectation values. Operators in the tensor channel have zero expectation values and are therefore ideal for direct evaluation. For scalar operators we have subtracted the non-zero VEVs from the operators to get the connected correlators directly.
- We improve upon the existing error bars on the masses in the scalar and tensor channel.

Future Plans

- In lattice calculation lattice spacing a gives rise to another systematic error known as lattice discretization error.
- Main purpose of using lattice gauge theory calculation for glueball is to find glueball masses in continuum limit.
- To minimize the discretization error and to get continuum limit glueball masses we are continuing our calculations on finer lattices with large lattice volumes.

Thank You!

Backups

- Candidate resonances

Name	Mass[MeV/c^2]	Width[MeV/c^2]	Decays
$f_0(1370)$	1200-1500	200-500	$\pi\pi, K\overline{K}, \eta\eta$
$f_0(1500)$	1500-1510	100-110	$\pi\pi, K\overline{K}, \eta\eta$
$f_0(1710)$	1700-1730	125-140	$\omega\omega, K\overline{K}, \pi\pi, \eta\eta$
$f_J(2220)$	2225-2235	15-30	$\gamma\pi\pi, \eta\eta'$

- Spin contents of different representations

irreps.	Spin contents
A_1	0, 4, 6, 8 ...
A_2	3, 6, 7, 9 ...
E	2, 4, 5, 6 ...
T_1	1, 3, 4, 5 ...