SAT+SMT 2016 - Day 3

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1. Prove: if $Mod(\mathcal{T})$ is singleton then $\mathcal{T}$ is complete.
2. Check satisfiability of the following formula

$$(f^4(a) \approx a \vee f^6(a) \approx a) \wedge f^3(a) \approx a \wedge f(a) \not\approx a$$

3. Run the quantifier elimination in $\mathcal{T}_s$ for the following formula

$$\exists x, y, z.\ s(s(x)) \approx s(y) \wedge s(s(s(z))) \approx y \wedge s(s(s(s(z)))) \not\approx s(s(x))$$

4. Check satisfiability of the following formula in $\mathcal{T}_{\mathbb{Z}}$ using Cooper's method

$$\exists x, y. 3x + 2y < 2 \wedge 2x + 3x \leq 1 \wedge -4x - 4y < -3$$

5. Show the following implication is valid using ODBMs

$$2 \leq y \wedge x - y \leq 2 \wedge y - z \leq -4 \wedge x + z \leq 6 \Rightarrow x \leq 4$$

6. Prove Theorem 2.5.
7. a. Add support of strict inequalities in Floyd-Warshall Algorithm.
   b. What is the complexity of the algorithm.
8. Prove commutativity of + in Presburger arithmetic using axioms of the theory.
9. Consider the theory of partial orders with signature $(\emptyset, \{<\})$

$$\forall x.\ x < x$$
$$\forall x, y.\ x < y \Rightarrow \neg y < x$$
$$\forall x, y, z.\ x < y \wedge y < z \Rightarrow x < z$$

   Note that the following formula will be unsatisfiable in difference logic. However, it is satisfiable in the theory of partial orders.

$$x < y \wedge u < z \wedge \neg z < x \wedge \wedge u < y$$

   Design a decision procedure for the theory of partial orders using a decision procedure for the difference logic as a subroutine.

10. Fourier-Motzkin algorithm for QF_LRA proceeds by eliminating variables one by one. After eliminating all the variables, if the input reduces to $\top$ then only the input is satisfiable. For each variable x, any conjunction of linear inequalities can be transformed into the following form.

$$\bigwedge_{j=1}^{m} s_j \leq x \wedge \bigwedge_{i=1}^{l} x \leq t_i \wedge \bigwedge_{k=1}^{n} u_k \leq 0$$

$$\Updownarrow$$

$$\bigwedge_{j=1}^{m} \bigwedge_{i=1}^{l} s_j \leq t_i \wedge \bigwedge_{k=1}^{n} u_k \leq 0$$

The above has no x.

a. Add support for equality, dis-equality, and strict inequalities

b. What is the complexity of Fourier-Motzkin algorithm?