



भारतीय प्रौद्योगिकी संस्थान हैदराबाद  
Indian Institute of Technology Hyderabad

# Testing GR from line of sight Shapiro delay using multi-messenger signals

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Workshop on gamma-ray bursts, NCRA

## Collaborators

Emre Kahya (Istanbul Technical Univ.)  
Richard Woodard (Univ. of Florida)

# Line of Sight Shapiro Delay

## References:

- 0804.3804 SD, Kahya, Woodard
- 1001.0725 Kahya
- 1510.08828 SD, Kahya
- 1602.04779 Kahya, SD
- 1612.02532 SD, Kahya

## Related/Similar work by other authors:

- 1602.01566 Wu et al
- 1604.06668 Liu et al
- 1604.02566 Luo et al (Lanikea supercluster)
- 1601.00180 Li et al
- 1602.04460 Li et al
- 1601.03636 Nusser (cosmological)
- 1606.00458 Takahashi
- 1703.09935 Wu et al



# Shapiro Delay

Irwin Shapiro PRL, 13, 789 (1964)

## FOURTH TEST OF GENERAL RELATIVITY

Irwin I. Shapiro

Lincoln Laboratory,\* Massachusetts Institute of Technology, Lexington, Massachusetts  
(Received 13 November 1964)

Recent advances in radar astronomy have made possible a fourth test of Einstein's theory of general relativity. The test involves measuring the time delays between transmission of radar pulses towards either of the inner planets (Venus or Mercury) and detection of the echoes. Because, according to the general theory, the speed of a light wave depends on the strength of the gravitational potential along its path, these time delays should thereby be increased by almost  $2 \times 10^{-4}$  sec when the radar pulses pass near the sun.<sup>1</sup> Such a change, equivalent to 60 km in distance, could now be measured over the required path length to within about 5 to 10% with presently obtainable equipment.<sup>2</sup>

Measurements over last 5 decades at all scales from solar system to binary pulsars

Used as tests of GR and also as an astrophysics probe to measure masses of neutron stars in binary systems

# Formula for Shapiro Delay

## Time delay due to light traveling around a single mass [ edit ]

For a signal going around a massive object, the time delay can be calculated as the following:[citation needed]

$$\Delta t = -\frac{2GM}{c^3} \log(1 - \mathbf{R} \cdot \mathbf{x})$$

Here  $\mathbf{R}$  is the [unit vector](#) pointing from the observer to the source, and  $\mathbf{x}$  is the unit vector pointing from the observer to the gravitating mass  $M$ . The dot denotes the usual Euclidean [dot product](#).

Using  $\Delta x = c\Delta t$ , this formula can also be written as

$$\Delta x = -R_s \log(1 - \mathbf{R} \cdot \mathbf{x}),$$

which is the extra distance the light has to travel. Here  $R_s$  is the [Schwarzschild radius](#).

In [PPN parameters](#),

$$\Delta t = -(1 + \gamma) \frac{R_s}{2c} \log(1 - \mathbf{R} \cdot \mathbf{x}),$$

which is twice the Newtonian prediction (with  $\gamma = 0$ ).[3]

Source: wikipedia

# First Shapiro Delay Measurement

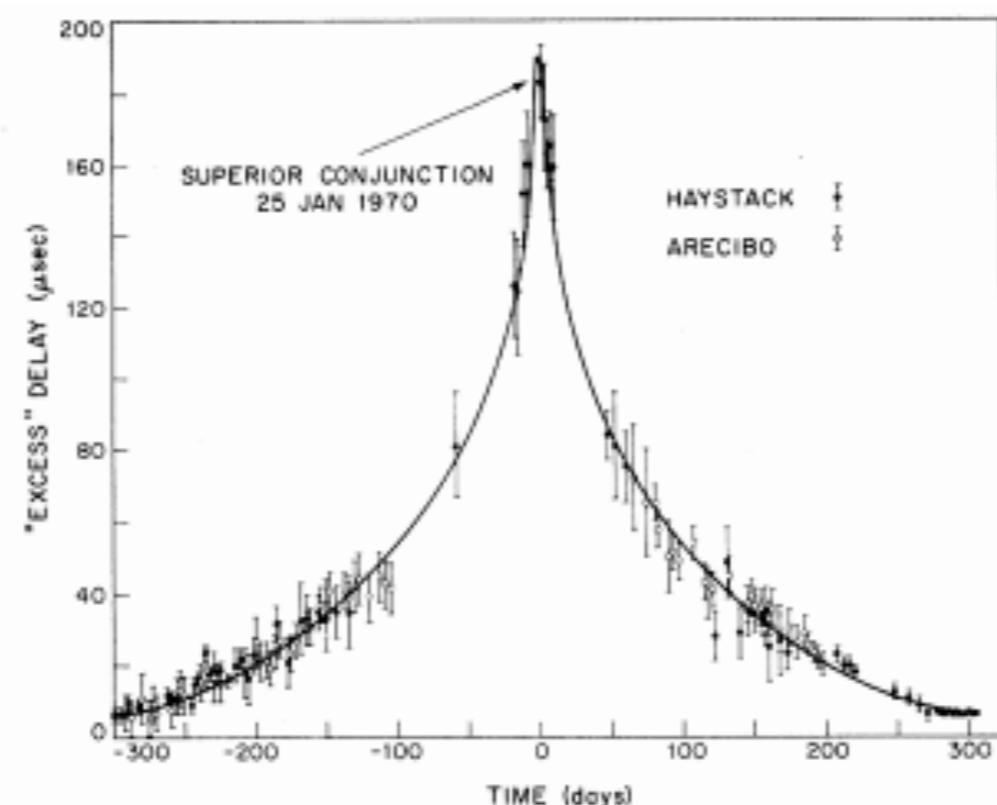
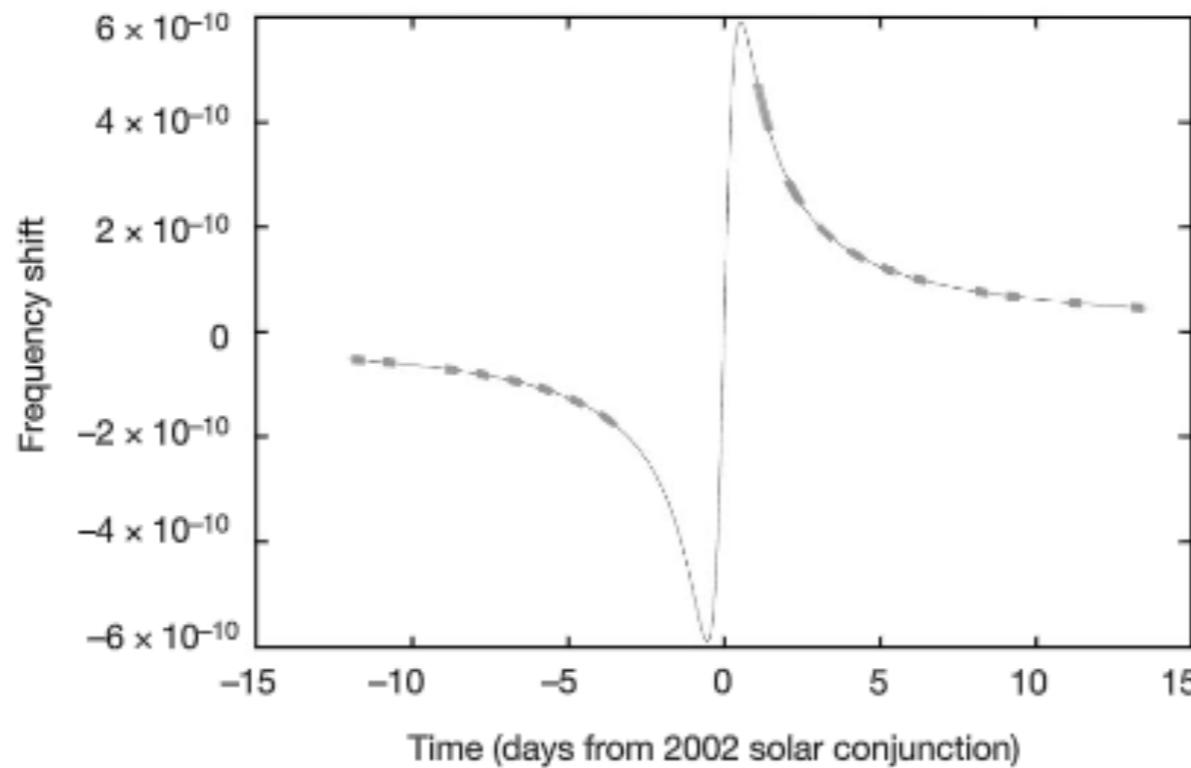


FIG. 1. Typical sample of post-fit residuals for Earth-Venus time-delay measurements, displayed relative to the "excess" delays predicted by general relativity. Corrections were made for known topographic trends on Venus. The bars represent the original estimates of the measurement standard errors. Note the dramatic increase in accuracy that was obtained with the radar-system improvements incorporated at Haystack just prior to the inferior conjunction of November 1970.

Shapiro et al, 1971  
PRL 26,1132

Delay  $\sim 200$   
microsecond

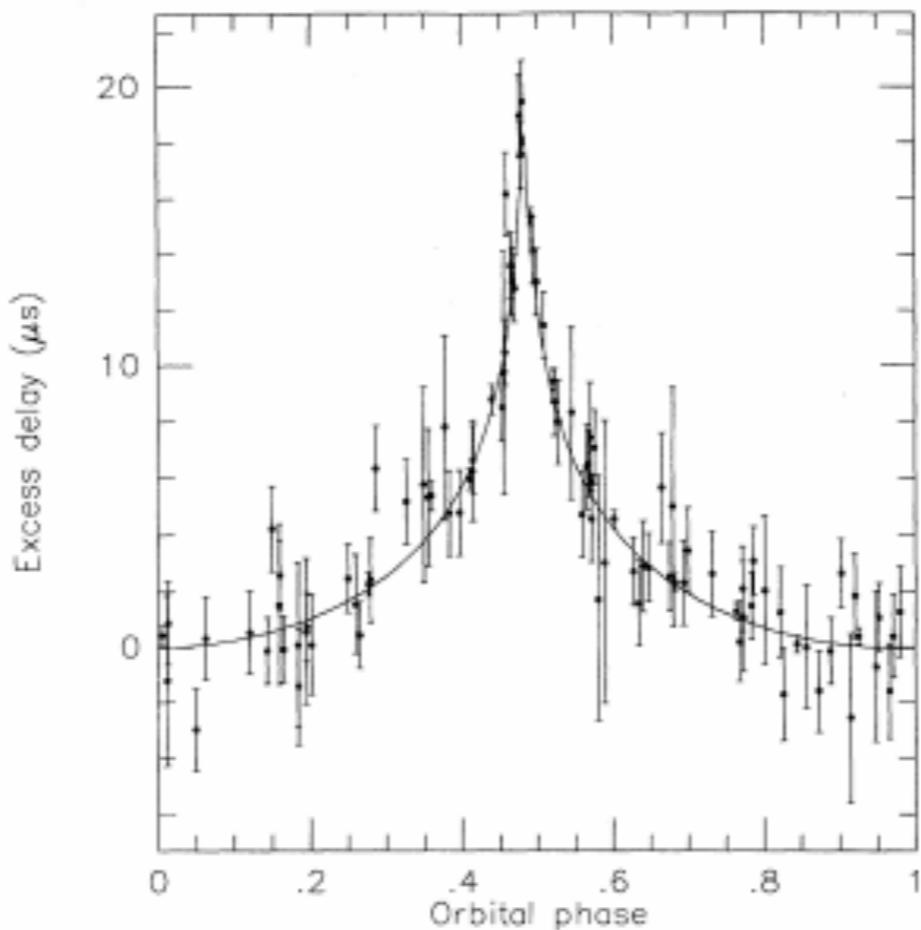
# Shapiro Delay Measurements



Cassini satellite

Bertotti et al Nature  
2003

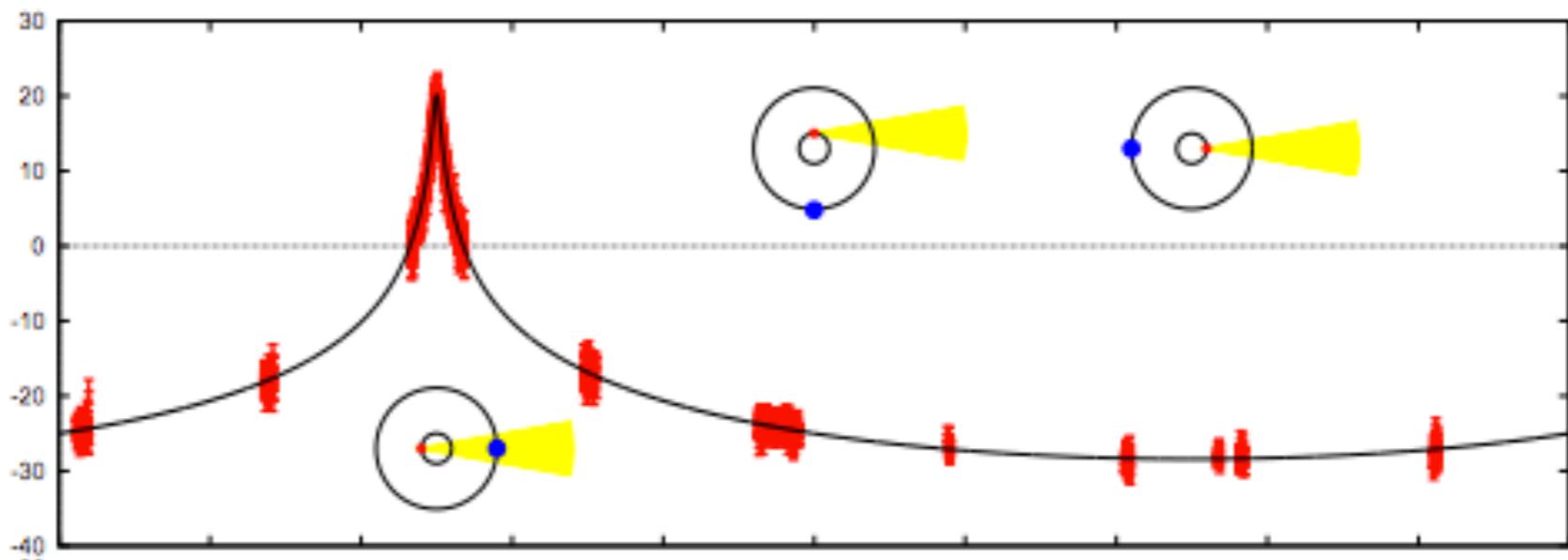
# Shapiro Delay in Binary Pulsars



J.H. Taylor, Nobel Prize  
lecture 1994 ,  
Rev. Mod. Phys. 66,711

FIG. 8. Measurements of the Shapiro time delay in the PSR 1855+09 system. The theoretical curve corresponds to Eq. (10), and the fitted values of  $r$  and  $s$  can be used to determine the masses of the pulsar and companion star.

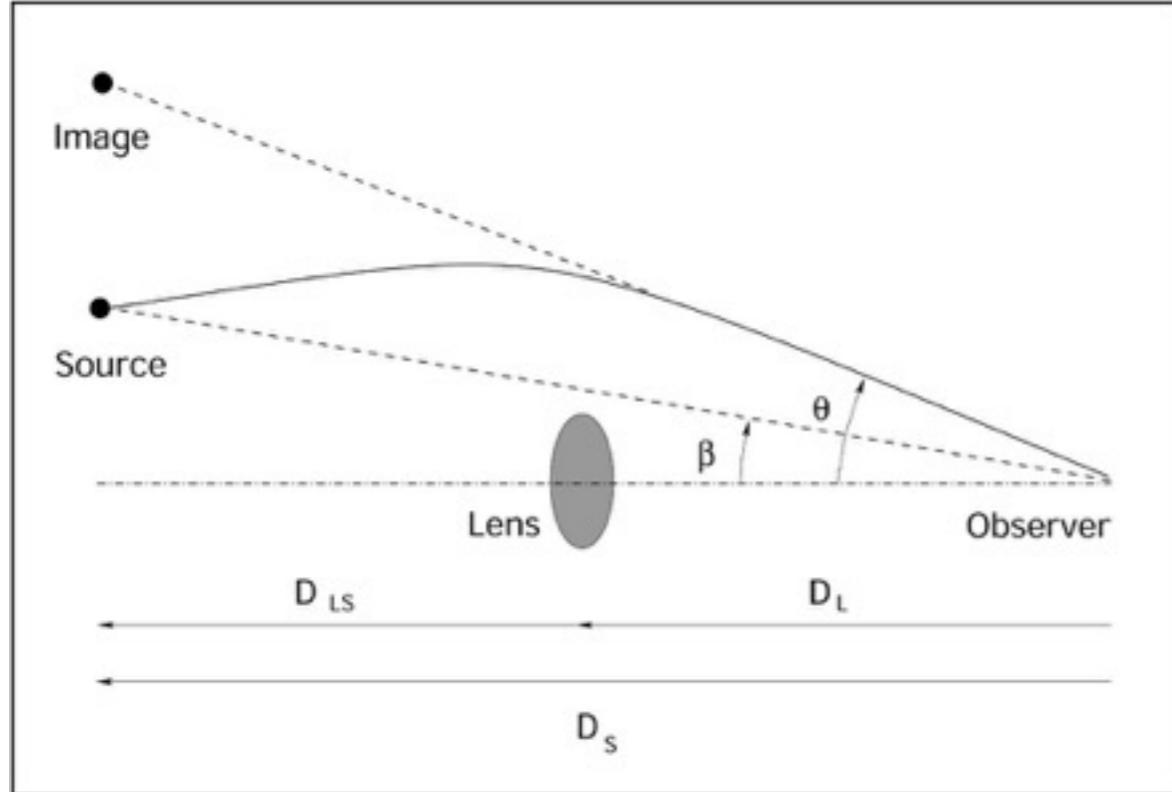
# Mass Measurement using Shapiro Delay



arxiv:1010.5788

Discovery of 2 solar mass Neutron Star PSRJ1614-2230

# Effect of Shapiro Delay in Time Delay Measurements of Lensed Quasars



0.08 x 0.08 arc minute  
ESA/Hubble

Each contribution to the total time-delay writes as:

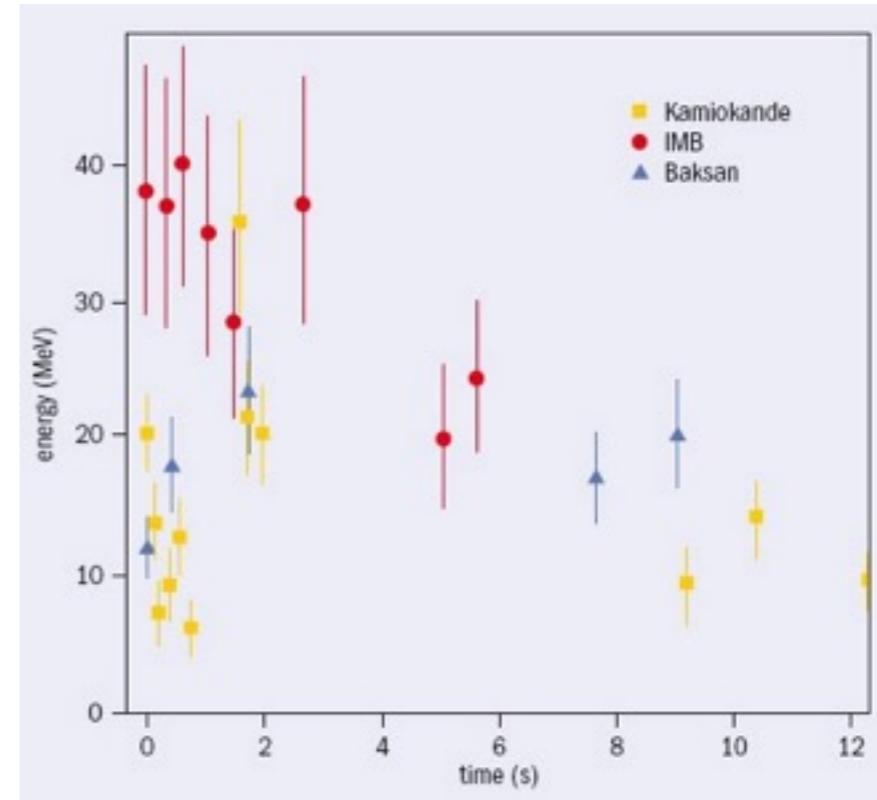
$$t_{\text{geom}}(\vec{\theta}) = (1 + z_L) \frac{D_L D_S}{c D_{LS}} (\vec{\theta} - \vec{\beta})^2,$$

$$t_{\text{grav}}(\vec{\theta}) = (1 + z_L) \frac{8\pi G}{c^3} \nabla^{-2} \Sigma(\vec{\theta}).$$

Time-delays between lensed images for a variable source used to measure Hubble constant

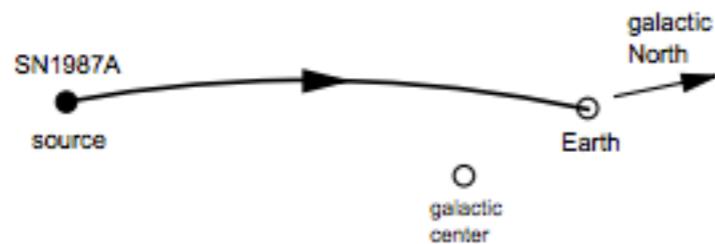
Similar idea for GWs proposed in  
1602.05882

# Birth of multi-messenger Astronomy



**Detection of neutrinos from SN 1987 A (@50 kpc) followed by flash of optical light (4 hours later)**  
IMB, Kamiokande, Baksan (disputed)  
2002 Nobel Prize to Masatoshi Koshiba

# Shapiro delay for neutrinos



## New Precision Tests of the Einstein Equivalence Principle from SN1987A

Michael J. Longo

*University of Michigan, Ann Arbor, Michigan 48109*  
(Received 14 September 1987)



As is shown below, the gravitational field of our galaxy causes a significant time delay,  $\approx 5$  months, in the transit time of photons from SN1987A. (This is the delay relative to the transit time expected if the gravitation of the galaxy could be “turned off.”) The fact that the arrival time of the neutrinos from SN1987A was the same as that for the first optical photons from the supernova to within several hours allows an accurate comparison of the general-relativistic time delay of the photons and neutrinos. The arrival time of the neutrinos is known to

PRL 60, 173 (1988)

Also, Krauss & Tremaine (1988)  
same issue of PRL next paper

**Only direct proof that neutrinos are affected by gravity and obey equivalence principle (to within 0.2%)**

# More results from SN 1987A

## Shapiro Delay CP invariant (LoSecco 1988)

The test of the equivalence principle pointed out by Longo and by Krauss and Tremaine can be easily extended to comparing the infall velocities of matter and antimatter. The very close coincidence in arrival times for neutrinos and antineutrinos places strong constraints on the coupling of gravitational interactions to matter and antimatter. The relative difference in gravitational delay is less than  $5 \times 10^{-6}$ .

## Non-0 neutrino mass does not change the delay (Bose & McGlinn 1988)

QED corrections to Shapiro delay can explain anomalous events seen in Mt Blanc detector.

(Franson, arXiv:1111.6986)

# Shapiro delay For GWs

*Constraints on the photon mass and charge and test of equivalence principle from GRB 990123* 629

As

$$\frac{\delta t_\gamma(\gamma_{\text{ray}}) - \delta t_{\text{opt}}}{\delta t_\gamma} = \frac{1}{2}(\gamma_\gamma - \gamma_{\text{opt}}) < 20/9 \times 10^7$$

(from the observed delay of 20 seconds)

This gives

$$\gamma_\gamma - \gamma_{\text{opt}} \leq 4 \times 10^{-7} \quad (7)$$

Thus  $\gamma_{\text{ray}}$  and optical photons 'see' the same gravitationally induced time delay to about 4 parts in  $10^7$  and the difference between gamma and radio photons is about one part in  $10^3$  (as here  $\delta t \sim 1\text{ day}$ ). If future detectors are able to register simultaneously neutrino and gravitational waves during gamma rays bursts, all the above formulae would give similar constraints on their properties and limits on violation of EEP for them also.

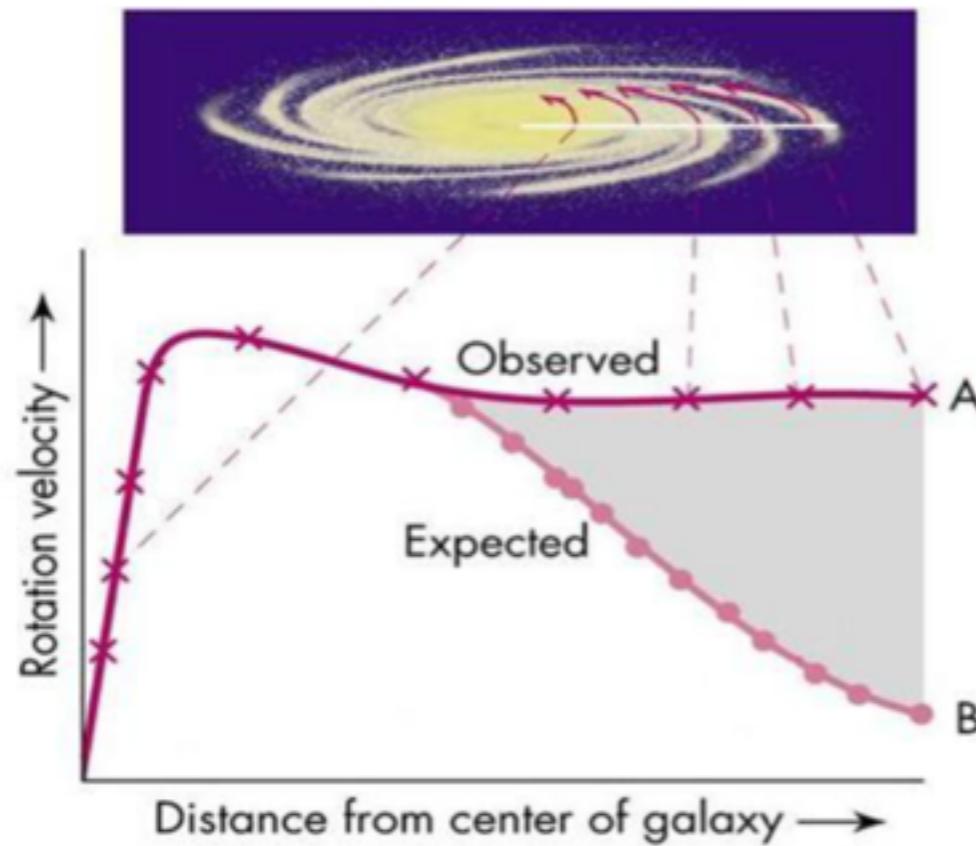


First proposed test by C. Sivaram (1999)

Bulletin of Astronomical Society of India 27,627

Gravitational waves gravitate due to  
a static potential at infinity.

# Galactic Rotation Curves



Conventional interpretation is most of mass of galaxy made up dark matter haloes.

Milgrom noticed (1983):  $\longrightarrow$  MOND

- Need for D.M. arises below a fixed acceleration scale ( $10^{-8} \text{ m/s}^2$ )

$$a = a_{\text{newt}} (a_{\text{newt}}/a_0)^{-1/2} \text{ for } a < a_0$$

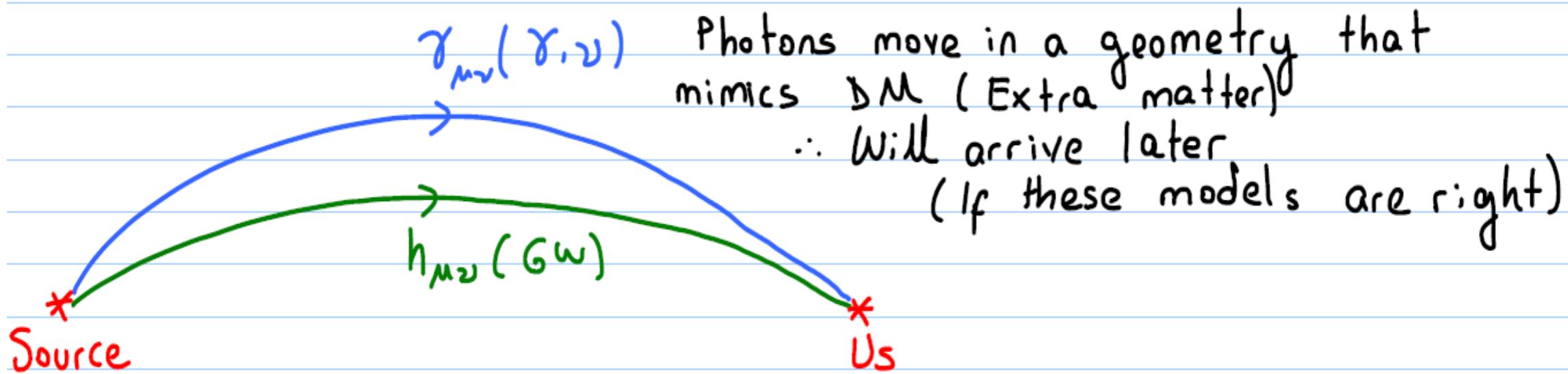
- Explains flat rotation curve and Tully-Fisher relation

$$L \propto v^4$$

## No-go theorem (Soussa/Woodard 2003)

Cannot construct metric theory of MOND and agree with solar system tests of GR and explain lensing without dark matter

**IDEA:** Two metrics  $\gamma_{\mu\nu}$ :  $\gamma$ 's  $\omega$ 's follow (Constructed to mimic DM observations)  
 $g_{\mu\nu}$ : GW follow (No DM)



∴ Arrival times will be different (Shapiro delays)  
Q: How big is the difference? ( $\mu$ s or observable)

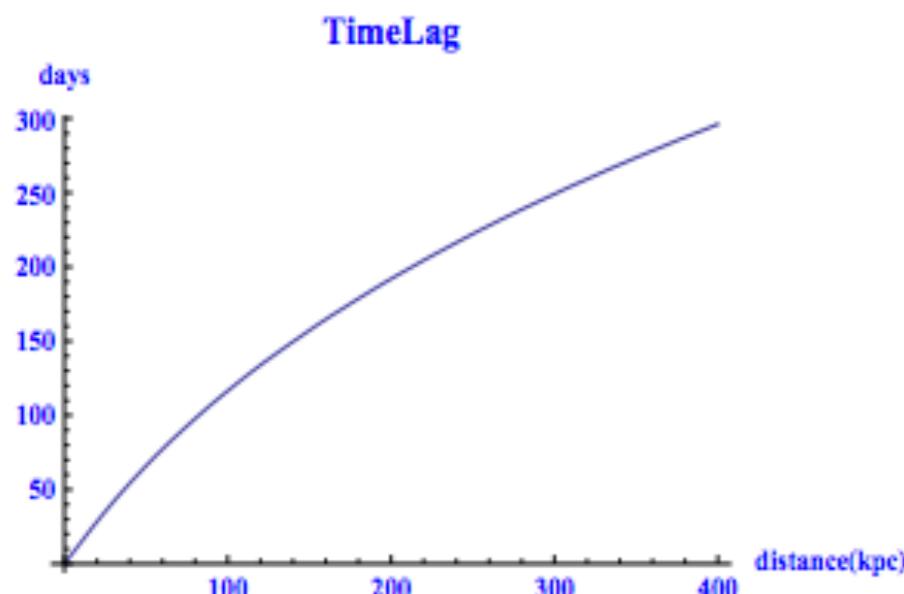
# Differential Shapiro delay in relativistic MOND

For a whole class of modified gravity models which avoid dark matter :

- Shapiro Delay for light/neutrinos = Potential of visible + dark matter.
- Shapiro Delay for gravity waves = Potential of visible matter only.

Profile	GRB 070201	SN 1987a	Sco-X1
Isothermal	742 days	78.2 days	4.98 days
NFW	804 days	74.8 days	4.88 days
Moore	811 days	74.5 days	4.97 days

arXiv:0804.3804



arXiv:1001.0725

FIG. 1: Shapiro delays for sources located in Milky Way.

# First GW Detection : *GW150914*

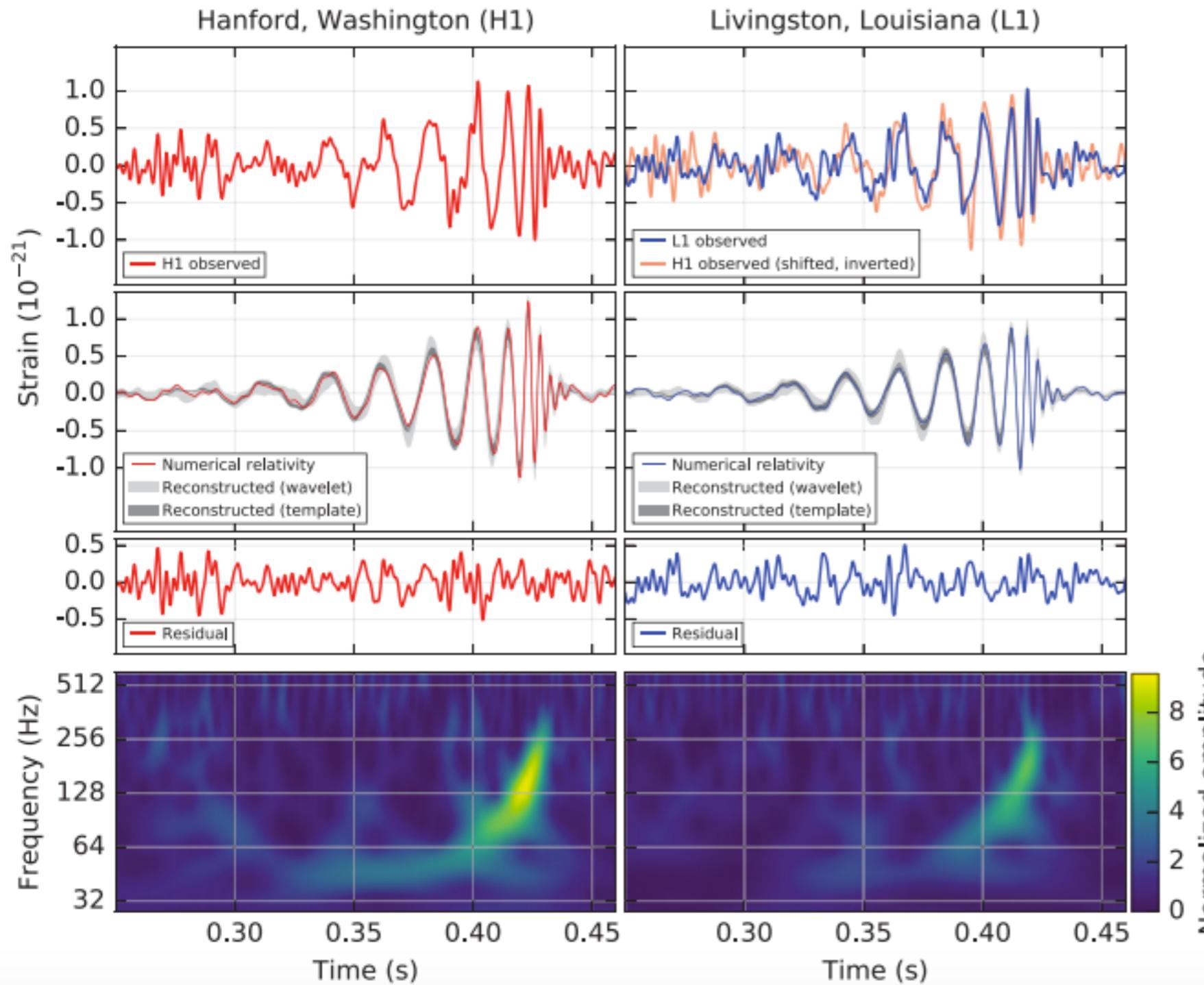


TABLE I. Source parameters for GW150914. We report median values with 90% credible intervals that include statistical errors, and systematic errors from averaging the results of different waveform models. Masses are given in the source frame; to convert to the detector frame multiply by  $(1+z)$  [90]. The source redshift assumes standard cosmology [91].

Primary black hole mass	$36^{+5}_{-4} M_{\odot}$
Secondary black hole mass	$29^{+4}_{-4} M_{\odot}$
Final black hole mass	$62^{+4}_{-4} M_{\odot}$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	$410^{+160}_{-180}$ Mpc
Source redshift $z$	$0.09^{+0.03}_{-0.04}$

LIGO-VIRGO  
Collaboration  
1602.03837

# E-M followup of GW150914

arXiv:1602.08492

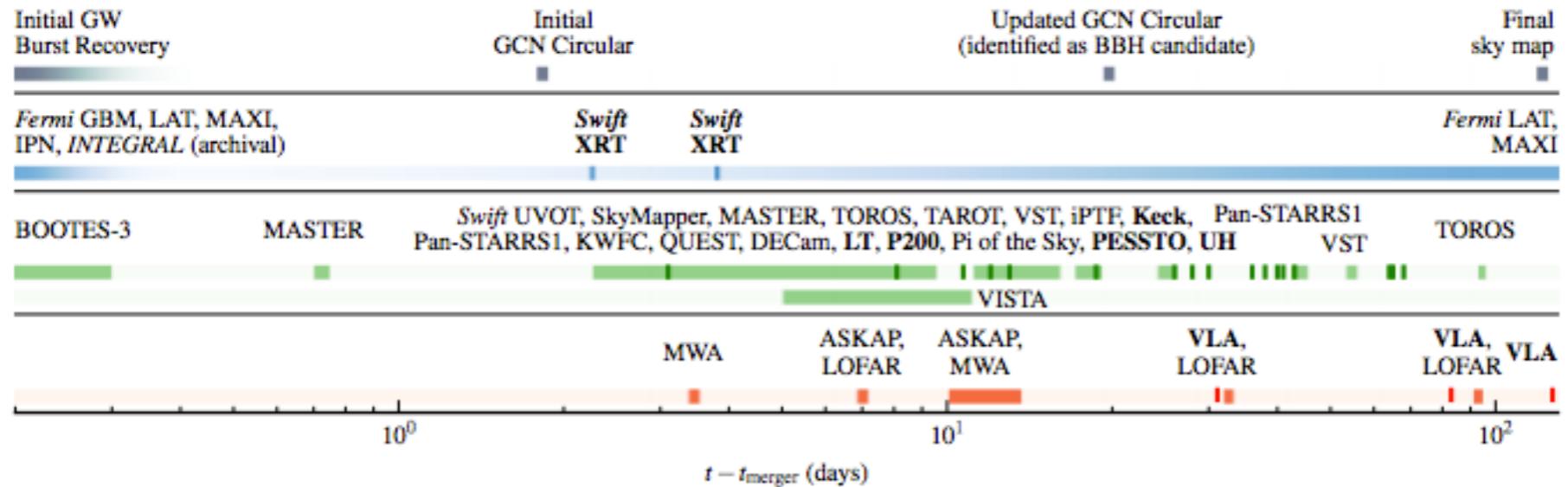


Figure 1. Timeline of observations of GW150914, separated by band and relative to the time of the GW trigger. The top row shows GW information releases. The bottom four rows show high-energy, optical, near-infrared, and radio observations, respectively. Optical spectroscopy and narrow-field radio observations are indicated with darker tick marks and boldface text. Table 1 reports more detailed information on the times of observations made with each instrument.

Table 1. Summary of Tiled Observations

Facility/ Instrument	Band <sup>a</sup>	Depth <sup>b</sup>	Time <sup>c</sup>	Area <sup>d</sup> (deg <sup>2</sup> )				Contained Probability (%)	GCN
				cWB	LB	BSFR <sup>e</sup>	LALIn		
Gamma-ray									
Fermi LAT	20 MeV- 300 GeV	$1.7 \times 10^{-9}$	(every 3 sec)	—	100	100	100	100	18799
Fermi GBM	8 keV-40 MeV	$0.7-5 \times 10^{-7}$	(archival)	—	100	100	100	100	18359
INTEGRAL	75 keV-1 MeV	$1.3 \times 10^{-7}$	(archival)	—	100	100	100	100	18354
IPN	15 keV-10 MeV	$1 \times 10^{-7}$	(archival)	—	100	100	100	100	—
X-ray									
MAXI/GSC	2-20 keV	$1 \times 10^{-10}$	(archival)	17900	95	89	92	84	18033
Swift XRT	0.3-10 keV	$5 \times 10^{-12}$ (pA)	2.3, 1, 1	—	0.6	0.08	0.04	0.05	18321
		$2-4 \times 10^{-12}$ (LMC)	3.4, 1, 1	41	1.2	1.9	0.16	0.26	18366
Optical									
DECam	i, r	$i < 22.5, r < 21.5$	3.9, 5, 22	100	38	34	14	31	18344, 18350
iPTF	R	$R < 20.4$	3.1, 3, 1	130	2.8	2.5	0.0	0.2	18307
KWFC	i	$i < 18.8$	3.4, 1, 1	24	0.0	1.2	0.0	0.1	18361
MASTER	C	$c < 19.9$	-3.3, 7, 7	730	80	36	89	80	18333, 18360, 18605, 18621
Pan-STARRS1	i	$i < 19.2-20.8$	3.2, 21, 42	430	28	29	2.0	42	18325, 18343, 18362, 18394
La Silla	g, r, z	$r < 21$	3.8, 3, 0.1	80	23	16	6.2	37	18347
QUEST	i, r	$i < 19.1, r < 17.1$	2.4, 2, 3	30	9.1	7.9	1.5	19	18349
SkyMapper	i, r	$i < 19.8$ (pA)	2.3, 1, 1	—	3	0.7	1.0	0.1	18331
Swift UVOT	v	$v < 18.8$ (LMC)	3.4, 1, 1	—	—	—	—	—	18366
TAROT	C	$R < 18$	2.8, 5, 14	30	15	3.5	1.6	1.9	18332, 18348
TOROS	C	$r < 21$	2.8, 7, 99	0.6	0.08	0.0	0.0	0.0	18338
VST/ESO	r	$r < 22.4$	2.9, 6, 50	90	29	19	14	30	18326, 18397
Near Infrared									
VISTA/ESO	Y, J, H, K <sub>S</sub>	$J < 20.7$	4.8, 1, 7	70	15	6.8	10	8.0	18355
Radio									
ASKAP	863.5 MHz	5-15 mJy	7.5, 2, 6	270	82	28	44	27	18363, 18669
LOFAR	145 MHz	12.5 mJy	6.8, 3, 90	300	27	1.5	0.0	0.1	18364, 18424, 18890
MWA	118 MHz	200 mJy	3.8, 2, 8	2800	97	72	86	86	18348

<sup>a</sup>Band: optical: visual, radio: flux density limit (Jy); <sup>b</sup>Depth: flux density limit (Jy); <sup>c</sup>Time: time since trigger (days); <sup>d</sup>Area: survey area (deg<sup>2</sup>); <sup>e</sup>BSFR: broad-band survey flux density limit (Jy)

GW150914 followed up by 62 EM followup teams and also in neutrinos (low+highE) and cosmic rays

GW and EM counterpart from NS/BH mergers should help definitely rule out (or confirm) relativistic MOND theories

# Shapiro delay From GW150914

## Constraints on frequency-dependent violations of Shapiro delay from GW150914

Emre O. Kahya, Shantanu Desai

(Submitted on 15 Feb 2016 (v1), last revised 16 Mar 2016 (this version, v3))

On 14th September 2015, a transient gravitational wave (GW150914) was detected by the two LIGO detectors at Hanford and Livingston from the coalescence of a binary black hole system located at a distance of about 400 Mpc. We point out that GW150914 experienced a Shapiro delay due to the gravitational potential of the mass distribution along the line of sight of about 1800 days. Also, the near-simultaneous arrival of gravitons over a frequency range of about 100 Hz within a 0.2 second window allows us to constrain any violations of Shapiro delay and Einstein's equivalence principle between the gravitons at different frequencies. From the calculated Shapiro delay and the observed duration of the signal, frequency-dependent violations of the equivalence principle for gravitons are constrained to an accuracy of  $\mathcal{O}(10^{-9})$

Comments: 3 pages, accepted for publication in Phys. Lett. B. This paper is dedicated to the memory of Prof. Steven Detweiler

Subjects: General Relativity and Quantum Cosmology (gr-qc); Cosmology and Nongalactic Astrophysics (astro-ph.CO); High Energy Astrophysical Phenomena (astro-ph.HE)

Journal reference: Phys. Lett. B 756, 265 (2016)

Similar paper by Wu et al, 2016 PRD 1602.01566  
with same conclusions

# Recent line-of-sight Shapiro delay calculations

## Constrains on violation of PPN gamma parameter

TABLE I: Upper bounds on the differences of the  $\gamma$  values from the Shapiro time delay measurements.

Author (year)	Source	Measengers	Gravitational field	$\Delta\gamma$	References
Krauss & Tremaine (1988)	Supernova 1987A	eV photons and MeV neutrinos	Milky Way	$5.0 \times 10^{-3}$	[3]
Longo (1988)	Supernova 1987A	eV photons and MeV neutrinos	Milky Way	$3.4 \times 10^{-3}$	[4]
	Supernova 1987A	7.5–40 MeV neutrinos	Milky Way	$1.6 \times 10^{-6}$	[4]
Gao et al. (2015)	GRB 090510	MeV–GeV photons	Milky Way	$2.0 \times 10^{-8}$	[5]
	GRB 080319B	eV–MeV photons	Milky Way	$1.2 \times 10^{-7}$	[5]
Wei et al. (2015)	FRB 110220	1.2–1.5 GHz photons	Milky Way	$2.5 \times 10^{-8}$	[8]
	FRB/GRB 100704A	1.23–1.45 GHz photons	Milky Way	$4.4 \times 10^{-9}$	[8]
Tingay & Kaplan (2016)	FRB 150418	1.2–1.5 GHz photons	Milky Way	$(1-2) \times 10^{-9}$	[9]
Nusser (2016)	FRB 150418	1.2–1.5 GHz photons	Large-scale structure	$10^{-12}-10^{-13}$	[16]
Wei et al. (2016a)	Blazar Mrk 421	keV–TeV photons	Milky Way	$3.9 \times 10^{-3}$	[10]
	Blazar PKS 2155-304	sub TeV–TeV photons	Milky Way	$2.2 \times 10^{-6}$	[10]
Wang et al. (2016)	Blazar PKS B1424-418	MeV photons and PeV neutrino	Virgo Cluster	$3.4 \times 10^{-4}$	[11]
	Blazar PKS B1424-418	MeV photons and PeV neutrino	Great Attractor	$7.0 \times 10^{-6}$	[11]
Wei et al. (2016b)	GRB 110521B	keV photons and TeV neutrino	Laniakea supercluster of galaxies	$1.3 \times 10^{-13}$	[6]
Wu et al. (2016a)	GW 150914	35–150 Hz GW signals	Milky Way	$\sim 10^{-9}$	[14]
Yang & Zhang (2016)	Crab pulsar	8.15–10.35 GHz photons	Milky Way	$(0.6-1.8) \times 10^{-15}$	[12]
Wu et al. (2016b)	GRB 120308A	Polarized optical photons	Laniakea supercluster of galaxies	$1.2 \times 10^{-10}$	This paper
	GRB 100826A	Polarized gamma-ray photons	Laniakea supercluster of galaxies	$1.2 \times 10^{-10}$	This paper
	FRB 150807	Polarized radio photons	Laniakea supercluster of galaxies	$2.2 \times 10^{-16}$	This paper

Wu et al arxiv:1703.09935

Also, SD & Kahya (using improved Shapiro delay calculation to Crab Pulsar)

$$\Delta\gamma < 2.4 \times 10^{-15}$$

# Arrival time differences between gravitational waves and electromagnetic signals due to gravitational lensing

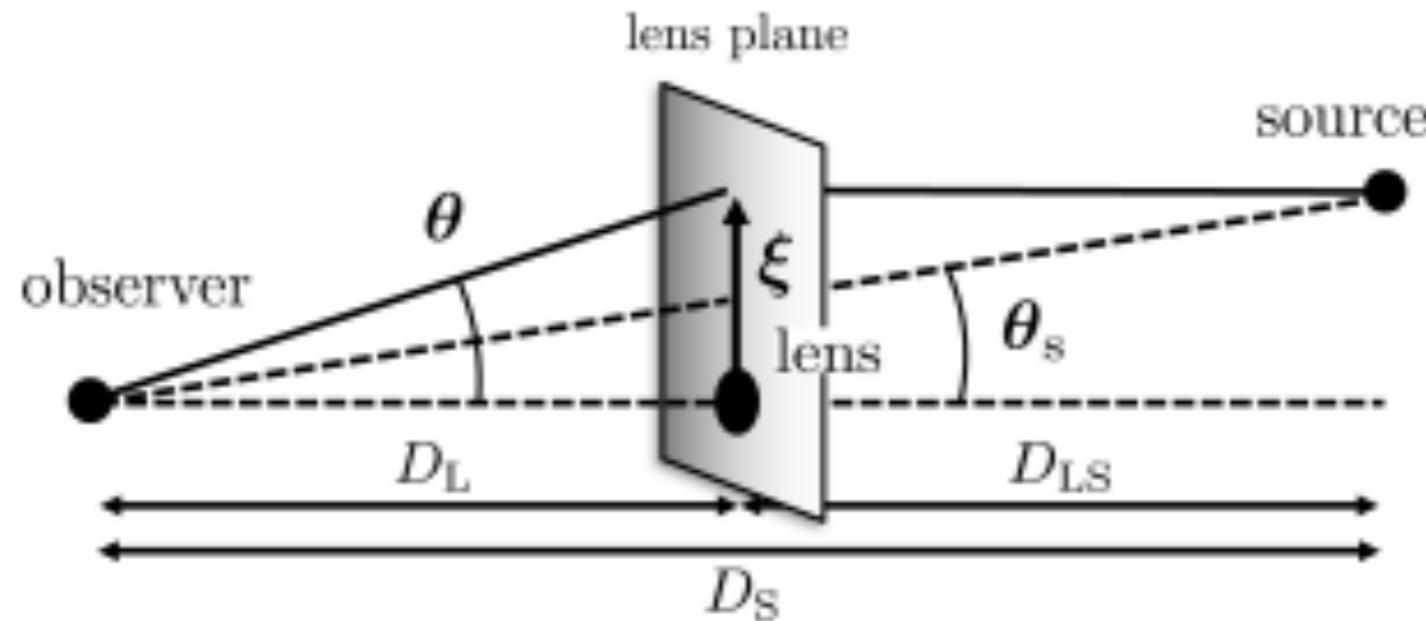
Ryuichi Takahashi

(Submitted on 1 Jun 2016 (v1), last revised 30 Dec 2016 (this version, v3))

In this study, we demonstrate that general relativity predicts arrival time differences between gravitational wave (GW) and electromagnetic (EM) signals caused by the wave effects in gravitational lensing. The GW signals can arrive *earlier* than the EM signals in some cases if the GW/EM signals have passed through a lens, even if both signals were emitted simultaneously by a source. GW wavelengths are much larger than EM wavelengths; therefore, the propagation of the GWs does not follow the laws of geometrical optics, including the Shapiro time delay, if the lens mass is less than approximately  $10^5 M_\odot (f/\text{Hz})^{-1}$ , where  $f$  is the GW frequency. The arrival time difference can reach  $\sim 0.1 \text{ s} (f/\text{Hz})^{-1}$  if the signals have passed by a lens of mass  $\sim 8000 M_\odot (f/\text{Hz})^{-1}$  with the impact parameter smaller than the Einstein radius; therefore, it is more prominent for lower GW frequencies. For example, when a distant super massive black hole binary (SMBHB) in a galactic center is lensed by an intervening galaxy, the time lag becomes of the order of 10 days. Future pulsar timing arrays including SKA (the Square Kilometre Array) and X-ray detectors may detect several time lags by measuring the orbital phase differences between the GW/EM signals in the SMBHBs. Gravitational lensing imprints a characteristic modulation on a chirp waveform; therefore, we can deduce whether a measured arrival time lag arises from intrinsic source properties or gravitational lensing. Determination of arrival time differences would be extremely useful in multimessenger observations and tests of general relativity.

Comments: Revised version, 10 pages, 7 figures, accepted for publication in ApJ

arXiv:1606.00458



arXiv:1606.00458  
Takahashi

FIG. 1.— The lensing configuration of an observer, a lens, and a source. The angular diameter distances to the lens and the source are  $D_L$  and  $D_S$ , respectively. The distance between them is  $D_{LS}$ . The waves from the source are scattered on the lens plane at a point  $\xi$ . The incoming wave direction is  $\theta (= \xi/D_L)$ , and the angular source position is  $\theta_s$ .

## Geometric optics approximation breaks down when

$$\lambda > GM/c^2 \quad \text{or} \quad M \leq 10^5 M_\odot (f/\text{Hz})^{-1}$$

No Shapiro delay seen in that case

Estimated time delay for ground-based detectors  $\sim 0.11 \text{ (f/Hz)}^{-1} \text{ sec}$

# Open Question in multi-messenger astronomy with GW

If gravitational waves experience no Shapiro delay for

$$M \leq 10^5 M_{\odot} (f/Hz)^{-1}$$

For a source which simultaneously emits GWs and neutrinos/photons, will they reach the same time due to the cumulative effect (or not) of all masses along the line of sight?

# Backup

Distance  $\sim 400$  Mpc  $\Rightarrow$  Shapiro delay  $\sim 1800$  days

within a 0.2 second window the near-simultaneous arrival of gravitons over a freq range  $\sim 200$  Hz

Constrain EEP bwn the gravitons at different freqs.

Freq-dep violations of EEP for gravitons constrained to be  $O(10^{-9})$

Shapiro delay calculation becomes much more difficult

Other uncertainties additional to DM profies

Multiple galaxies on GW's way to us

Cosmological effect  $\rightarrow$  arXiv:1601.03636 (Adi Nusser)

an increase on the estimate of Wei et.al. arXiv:1512.07670

# (Backup) Time-Delay Calculation

Calculate Shapiro Delay for various dark matter potentials

*Geodesic Equations:*  $\chi^\mu + \Gamma_{\rho\sigma}^\mu \chi^\rho \chi^\sigma = 0$

For isothermal halo model:

$$\Delta t = \frac{\varepsilon \Delta x}{c} \left[ 1 + \frac{\alpha}{2} \ln \left( \frac{r_L}{r_S} \right) - \sqrt{\beta - \alpha^2} \tan^{-1} \left( \frac{\sqrt{\beta - \alpha^2}}{\beta + \alpha} \right) \right]$$

$$\alpha \equiv \frac{\vec{x}_L \cdot \Delta \vec{x}}{\Delta x^2} \quad \text{and} \quad \beta \equiv \frac{r_L^2}{\Delta x^2}$$

$$\begin{aligned} r_S &= 8.0 \text{ kpc}, r_L = 50.9 \text{ kpc}, \Delta x = 51.4 \text{ kpc} \\ \Rightarrow \alpha &= -0.9775, \beta = 0.9793 \end{aligned}$$

$$\Delta t \big|_{SN1987a} = -78 \text{ days}$$

Neutrinos from SN 1987A should arrive 78 days later than gravitational waves in relativistic MOND theories