Mass Generation by non-perturbative technique

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Outline

Emergent Gravity

- **The idea is to explore 2-form field dynamics in $D = 5$.**
- **Form theory in D=5 : A_α dual to $B_{\alpha\beta}$(KR).**
- **A_α field $\Longrightarrow$ U(1) theory $\Longrightarrow$ $F_2 = dA_1$**
- **$B_{\alpha\beta}$ field $\Longrightarrow$ U(1) theory $\Longrightarrow$ $H_{\alpha\beta\gamma} = 3\nabla_{[\alpha}B_{\beta\gamma]}$ is invariant under the following U(1) gauge transformations** $\delta B_{\alpha\beta} = \sqrt{2\pi\alpha'}\left(\partial_\alpha\epsilon_\beta - \partial_\beta\epsilon_\alpha\right)$

$$S_B = -\frac{1}{12(8\pi g_s)\acute{a}^{3/2}}\int d^5x\sqrt{-g}H^2_{\alpha\beta\gamma} \tag{1}$$

$g_{\alpha\beta}$ **is the background metric.**

- **By bringing in connections, $\Gamma_{\alpha\beta}{}^{\gamma} = -\frac{1}{2}H_{\alpha\beta}{}^{\gamma}$**
- $\nabla_\mu \longrightarrow \mathcal{D}_\mu \qquad \Longrightarrow \qquad H_3 \longrightarrow \mathcal{H}_3$ **(geometric torsion).**

- **In $\mathcal{H}_3$ picture(NS − NS), $\mathcal{H}_{\mu\nu\lambda} = 3\mathcal{D}_{[\mu}\mathcal{B}_{\nu\lambda]} \neq 0$**
  but $\nabla_\alpha \mathcal{B}_{\mu\nu} = 0$

$$\begin{aligned}\mathcal{D}_\mu \mathcal{B}_{\nu\lambda} &= \partial_\mu \mathcal{B}_{\nu\lambda} - \Gamma^\rho_{\mu\nu}\mathcal{B}_{\rho\lambda} - \Gamma^\rho_{\mu\lambda}\mathcal{B}_{\nu\rho} - \mathbf{\Gamma}_{\mu\nu}{}^{\rho}\mathcal{B}_{\rho\lambda} - \mathbf{\Gamma}_{\mu\lambda}{}^{\rho}\mathcal{B}_{\nu\rho} \\ &= \nabla_\mu \mathcal{B}_{\nu\lambda} - \mathbf{\Gamma}_{\mu\nu}{}^{\rho}\mathcal{B}_{\rho\lambda} - \mathbf{\Gamma}_{\mu\lambda}{}^{\rho}\mathcal{B}_{\nu\rho} = \frac{1}{2}\mathcal{H}_{\mu\nu}{}^{\rho}\mathcal{B}_{\rho\lambda} + \frac{1}{2}\mathcal{H}_{\mu\lambda}{}^{\rho}\mathcal{B}_{\nu\rho}\end{aligned} \tag{2}$$

- $$\Big[\mathcal{D}_\mu, \mathcal{D}_\nu\Big]\mathbf{A}_\lambda = \Big(\mathcal{K}_{\mu\nu\lambda}{}^{\rho} + \mathcal{L}_{\mu\nu\lambda}{}^{\rho}\Big)\mathbf{A}_\rho + \mathcal{H}_{\mu\nu}{}^{\rho}\mathcal{D}_\rho \mathbf{A}_\lambda$$

- **Resulting curvature**[1]

$$\mathcal{K}_{\mu\nu\lambda}{}^{\rho} = \frac{1}{2}\partial_\mu \mathcal{H}_{\nu\lambda}{}^{\rho} - \frac{1}{2}\partial_\nu \mathcal{H}_{\mu\lambda}{}^{\rho} + \frac{1}{4}\mathcal{H}_{\mu\lambda}{}^{\sigma}\mathcal{H}_{\nu\sigma}{}^{\rho} - \frac{1}{4}\mathcal{H}_{\nu\lambda}{}^{\sigma}\mathcal{H}_{\mu\sigma}{}^{\rho}$$

- **Properties**

$$\mathcal{K}_{\mu\nu\lambda\rho} = -\mathcal{K}_{\nu\mu\lambda\rho} \qquad \mathcal{K}_{\mu\nu\lambda\rho} = -\mathcal{K}_{\mu\nu\rho\lambda} \qquad \mathcal{K}_{\mu\nu\lambda\rho} \neq \mathcal{K}_{\lambda\rho\mu\nu}$$

[1] A. K. Singh, K. Priyabrat Pandey, S. Singh and Supriya Kar, JHEP05, 033 (2013)

- $\mathcal{K}_{\mu\nu\lambda\rho} = \mathcal{K}^{(\mathbf{s})}_{\mu\nu\lambda\rho} + \mathcal{K}^{(\mathbf{ns})}_{\mu\nu\lambda\rho}$

  where 's' & 'ns' stands for Pair symmetric and Non-symmetric respectively.

- **where** $\mathcal{K}^{(\mathbf{s})}_{\mu\nu\lambda\rho} = \frac{1}{4}\mathcal{H}_{\mu\lambda}{}^{\sigma}\mathcal{H}_{\nu\sigma}{}^{\rho} - \frac{1}{4}\mathcal{H}_{\nu\lambda}{}^{\sigma}\mathcal{H}_{\mu\sigma}{}^{\rho}$

- $\mathcal{K}^{(\mathbf{s})}_{\mu\nu\lambda\rho} = -\mathcal{K}^{(\mathbf{s})}_{\nu\mu\lambda\rho} \quad , \mathcal{K}^{(\mathbf{s})}_{\mu\nu\lambda\rho} = -\mathcal{K}^{(\mathbf{s})}_{\mu\nu\rho\lambda} \quad , \mathcal{K}^{(\mathbf{s})}_{\mu\nu\lambda\rho} = \mathcal{K}^{(\mathbf{s})}_{\lambda\rho\mu\nu}$

- **Irreducible curvature scalar:** $\mathcal{K}^{(\mathbf{s})} = -\frac{1}{4}\mathcal{H}_{\mu\nu\lambda}\mathcal{H}^{\mu\nu\lambda}$

- **Pair non-symmetric curvature:** $\mathcal{K}^{(\mathbf{ns})}_{\mu\nu\lambda\rho} = \frac{1}{2}\mathcal{K}^{(\mathbf{as})}_{\mu\nu\lambda\rho} + \frac{1}{2\sqrt{2\pi\acute{\alpha}}}\mathcal{F}_{\mu\nu\lambda\rho}$

  where 'as' stands for pair anti-symmetric & Irreducible curvature $\quad \mathcal{F}_{\mu\nu\lambda\rho} = 4\sqrt{2\pi\alpha'}\mathcal{D}_{[\mu}\mathcal{H}_{\nu\lambda\rho]}$

- **tr** $\mathcal{K}^{(\mathbf{as})}_{\mu\nu\lambda\rho} = \mathcal{K}^{(\mathbf{as})}_{\nu\rho} = \mathcal{D}^{\delta}\mathcal{H}_{\delta\rho\nu}$

- **For an on-shell $\mathcal{B}_{\mu\nu}$ field,** $\Longrightarrow \mathcal{K}_{\mu\nu}{}^{\mathbf{(AS)}} = \mathbf{0}$.

$$\Longrightarrow \mathcal{K} \longrightarrow \left(\mathcal{K}^{\mathbf{(s)}}, \mathcal{F}_{\mathbf{4}}\right)$$

- **The resulting emergent theory**[2]

$$\mathcal{S}_{\mathbf{NP}} = \frac{\mathbf{1}}{\kappa'^{\mathbf{3}}} \int \mathbf{d^5x}\sqrt{-\mathbf{g}}\left(\mathcal{K}^{\mathbf{(s)}} - \frac{\mathbf{1}}{\mathbf{48}}\mathcal{F}_{\mathbf{4}}^{\mathbf{2}}\right) \tag{3}$$

where $\kappa'^{3} = \dfrac{1}{(8\pi^3 g_s)\alpha'^{3/2}}$

[2] S. Kar, arXiv:1610.07347 [hep-th] (2016)

- **The emergent curvature scalar $\mathcal{K}^{(\mathbf{s})}$, underlying a dynamical NS two form field describes a perturbation theory in first order.**

- **The curvature $\mathcal{F}_{\mathbf{4}}$ describes a non-perturbative quantum correction underlying three form dynamics in 5D.**

- **The geometric torsion dynamics, describes a non-perturbation theory in the second order, provided $\mathcal{B}_{\mu\nu}$ is on-shell in first order perturbation theory. Thus, the complete emergent theory described by the action $\mathcal{S}_{NP}$ is a non-perturbative quantum theory in 1.5 order.**

- **At the expense of this non-perturbative correction, one can recover torsion free geometry.**

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S. Kar and R. Nitish, arXiv:1611.04952v2

- **The non-perturbative theory of emergent gravity in $(4+1)$-dimensions underlying a geometric torsion in 1.5 order is described by the action**

$$\mathcal{S}_{NP} = \frac{1}{\kappa'^3} \int d^5x \sqrt{-g} \Big(\mathcal{K}^s - \frac{1}{48} \mathcal{F}_4^2 \Big) \tag{4}$$

- **The action has been shown to govern a NS field $\mathcal{B}_2$ dynamics in an emergent first order perturbation gauge theory and a local geometric torsion $\mathcal{H}_3$ in a second order non-perturbation theory.**

- **In terms of geometric forms,**

$$\mathcal{S}_{NP} = -\frac{1}{12} \int d^5x \sqrt{-g} \Big(\mathcal{H}_{\mu\nu\lambda} \mathcal{H}^{\mu\nu\lambda} + 6(\mathcal{D}_\mu \psi)(\mathcal{D}^\mu \psi) \Big) \tag{5}$$

$$= -\frac{1}{4}\int d^5x\sqrt{-g}\Big(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + 2(\mathcal{D}_\mu\psi)(\mathcal{D}^\mu\psi)\Big) \tag{6}$$

- **A geometric $\mathcal{F}_2$ in an emergent theory is given by**

$$\mathcal{F}_{\mu\nu} = \mathcal{D}_\mu A_\nu - \mathcal{D}_\nu A_\mu = F_{\mu\nu} + \mathcal{H}_{\mu\nu}{}^{\lambda} A_\lambda$$

where $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$. The Lorentz scalar for a generic two form field strength is given by

$$\mathcal{F}^2_{\mu\nu} = \Big(F^2_{\mu\nu} - \mathcal{K}^{(s)}_{\mu\nu} A^\mu A^\nu + \frac{\epsilon^{\mu\nu\lambda\rho\sigma}}{\sqrt{-g}} A_\mu F_{\nu\lambda}\mathcal{F}_{\rho\sigma}\Big) \tag{7}$$

- **Also, one can interpret symmetric second order curvature in terms of two form field strength as**

$$\mathcal{K}^{(s)}_{\mu\nu} = -\frac{1}{4}\mathcal{H}_{\mu\alpha\beta}\mathcal{H}^{\alpha\beta}{}_{\nu} = \Big(g_{\mu\nu}\mathcal{F}^2_2 + 2\mathcal{F}_{\mu\lambda}\mathcal{F}^{\lambda}{}_{\nu}\Big) \tag{8}$$

- **The effective non-perturbation dynamics is re-expressed as**

$$\mathcal{S}_{NP} = -\frac{1}{4}\int d^5x\sqrt{-g}\Big[F^2_{\mu\nu} - \mathcal{K}^{(s)}_{\mu\nu}A^\mu A^\nu + 2(\nabla_\mu\psi)^2\Big] + \int d^5x\Big(A_1 \wedge F_2 \wedge F_2 - \mathcal{B}_2 \wedge H_3\Big) \tag{9}$$

- **The emergent curvature $\mathcal{K}^{(s)}_{\mu\nu}$ appears to be a mass squared matrix.**

- **A count for the local degrees enforces $\mathcal{F}_4 = 0$ in the effective gauge theory. Thus, a geometric torsion turns out to be constant which defines a perturbative vacuum. However, in an emergent gravity, $\mathcal{F}_4 \neq 0$.**

- **Alternately the perturbative gauge vacuum may be realized in a gauge choice for $\mathcal{F}_4 = 0$. A constant $\mathcal{H}_3$ leads to a constant $\mathcal{K}^{(s)}_{\mu\nu}$ which is diagonalized. thus,**

$$\mathcal{K}^s_{\mu\nu} = m_1^2 g_{\mu\nu} \tag{10}$$

- **$\mathbf{m_1^2}$ is proportionality constant. It assigns a mass to $\mathbf{A}_\mu$ at the expense of a dynamical non-perturbative quantum correction. In fact, it helps to generate mass**

$$\mathbf{m_p} = \sqrt{\frac{\mathcal{K}^{(\mathbf{s})}}{\mathbf{d}}}$$

for a generic higher p-form field in a gauge theory in d-dimensions.

- **Also, the mass m_1 can be derived from a geometric two form. With a proportionality constant $\tilde{\mathbf{m}}_1^2$, $\mathcal{K}^{(\mathbf{s})}_{\mu\nu} = \tilde{\mathbf{m}}_1^2 \mathbf{g}_{\mu\nu}$ and the curvature scalar $\mathcal{K}^{(\mathbf{s})} = 3\mathcal{F}^2_{\mu\nu}$, implies**

$$\tilde{m}_1^2 = \frac{(d-2)}{d} \mathcal{F}^2_{\mu\nu} \tag{11}$$

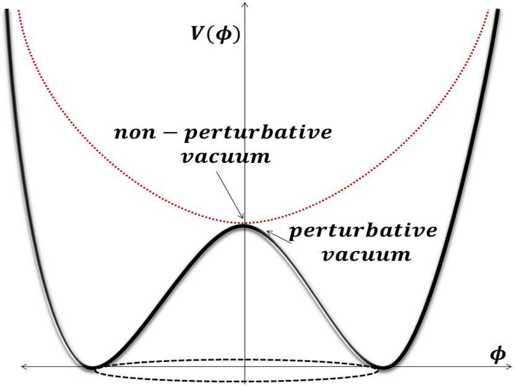
- **It can be checked that $m_1 = \tilde{m}_1$. and hence the mass of a one form is uniquely defined in a perturbation gauge theory using a non-perturbaive technique.**

- **Interestingly, the axion in a NP theory may be identified with a goldstone boson in a spontaneous local U(1) symmetry breaking phase of a perturbative vacuum. Then, the effective action in a weakly coupled gauge theory may be re-expressed as**

$$\begin{aligned} \mathcal{S}_{PG} &= -\frac{1}{4}\int d^5x\sqrt{-G}\Big(F_2^2 - m_1^2 A^2\Big) \\ &\quad - \int d^5x\Big(A_1\wedge F_2\wedge F_2 + B_2\wedge \mathcal{H}_3\Big) \end{aligned} \tag{12}$$

- $\mathcal{F}_2 \longrightarrow \mathbf{F}_2$ **in a perturbation gauge theory.**

- **A massless gauge field $\mathbf{A}_\mu$ in an emergent non-perturbation theory of gravity in 5D becomes massive at the expense of a NP dynamics.**
- **The NP-theory of emergent gravity is purely described by $\mathcal{H}_3$ and hence describes a torsion geometry. However, in a gauge choice $\mathcal{F}_4 = 0$, the emergent gravity describes a torsion free geometry sourced by a dynamical NS two form field.**



A non-perturbative stable vacuum may be viewed as a unstable vacuum in a perturbative theory.

- **A realization of perturbative vacuum within a NP-theory may be described for a two form KR field $B_{\mu\nu}$. It is given by**

$$\mathcal{S}_{PG} = -\frac{1}{12}\int d^5x\sqrt{-G}\Big(H_{\mu\nu\lambda}H^{\mu\nu\lambda} - m_2^2 B_2^2\Big) - \int d^5x\Big[\Big(\mathcal{F}_2 - B_2\Big)\wedge H_3\Big] \tag{13}$$

- **A massive Kalb-Ramond form field in a perturbation gauge theory is generated by a non-perturbative correction sourced by an axion in 5D.**

Summary

- **2-form dynamics may be describe in a 1st order perturbation theory in presence of torsion dynamics in 2 order which is believed to describe a non-perturbative correction.**

- **Decoupling of $\mathcal{F}_4$ describes a torsion free geometry.**

- **In 5D, $\mathcal{F}_4$ is dual to a dynamical axionic scalar (ψ) which is believed to be playing the role of Goldstone boson.**

Thank You