# Supersymmetry and Duality

MAX-PLANCK-GESELLSCHAFT

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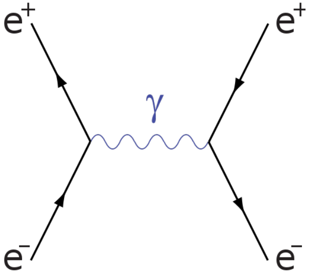
# QFTs from their perturbative expansion

Most of the talks we have heard in this conference approach particle physics from the point of view of perturbation theory: we have a collection of almost free fields, which interact via Feynman diagrams, in an order-by-order expansion.



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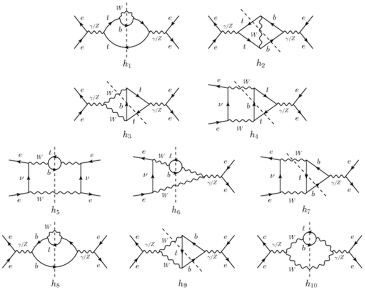

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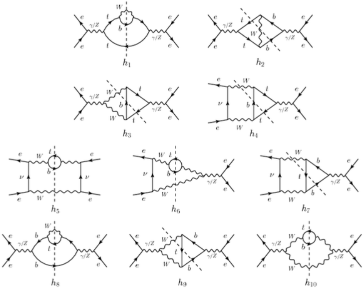

This is a fantastically useful and powerful approach.

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Supersymmetry is important to make progress here.
Historically $\mathcal{N}=4$ and $\mathcal{N}=2$ first, and then $\mathcal{N}=3$ and $\mathcal{N}=1$.

# Duality

Duality is the phenomenon in quantum field theory where the same quantum theory has more than one semiclassical description. This is possible since in quantum theory we are summing over fields, so the sum can be reexpressed in different variables

$$\int [D\phi_A]\mathcal{O}_a(x_1, \ldots, x_n)e^{iS_a(\phi_A, g_a)} = \int [D\phi_b]\mathcal{O}_b(x_1, \ldots, x_n)e^{iS_b(\phi_b, g_b)}$$

for some choice of classical theories $A$ and $B$ and a duality map between operators $\mathcal{O}_a$ and $\mathcal{O}_b$ and parameters $g_a$ and $g_b$.

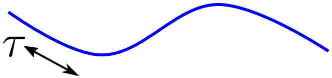
# $\mathcal{N}=4$ SYM theory in four dimensions

[Montonen, Olive '77], [Osborn '79] conjectured that $\mathcal{N}=4$ theories exhibit duality.

These theories has 16 supercharges. All $\mathcal{N}=4$ theories which are known are Lagrangian (Yang-Mills), and can be specified by giving a gauge group $G$, a complexified coupling $\tau=\theta+i/g^2$, and some extra discrete choices [Aharony, Seiberg, Tachikawa '13]:

$$\mathcal{L}=\frac{1}{g^2}\,\mathrm{Tr}(F\wedge\star F)+\theta\,\mathrm{Tr}(F\wedge F)+\ldots \tag{1}$$

The $\mathcal{N}=4$ theory is interacting and conformal for any finite $\tau$. Changing $\tau$ gives rise to a *marginal deformation* of this superconformal field theory (SCFT). The set of SCFTs connected by such marginal deformations is known as the *conformal manifold*.



# Symmetries and matter content

$\mathcal{N}=4$ theories in four dimensions have a global symmetry group $SU(4)_R$ ("$R$-symmetry"), and fields of spin-1, spin-$1/2$ and spin-0.

| Field | Spin | $SU(4)_R$ rep. | $U(N)$ rep |
|---|---|---|---|
| $A_\mu$ | 1 | **1** | **adj** |
| $\lambda_\alpha^\beta$ | $\frac{1}{2}$ | **4** | **adj** |
| $\Phi_i$ | 0 | **6** | **adj** |

In the vacuum we can have $\langle\Phi_i\rangle \neq 0$ without breaking supersymmetry or Lorentz invariance. For a generic such vacuum expectation value $U(N) \to U(1)^N$. (Coulomb branch.)

# Duality and validity of the classical description

In a given weak coupling description one has

- $A_\mu$ spin-1 fields and their superpartners. In the Coulomb branch we have $U(1)^N$ with a bunch of massive $W$ bosons, with mass proportional to $\sqrt{\tau^{-1}} \langle \Phi \rangle \sim g \langle \Phi \rangle$.
- Monopoles: classical solitons with their collective degrees of freedom, with mass proportional to $\sqrt{\tau} \langle \Phi \rangle \sim 1/g \langle \Phi \rangle$.

**Conjecture:** when $g \to \infty$ a new classical description emerges where the monopoles are the fundamental degrees of freedom, the original $W$ bosons can be understood as solitons, and $g_{\text{eff}} = 1/g$.

# $\mathcal{N}=4$ SYM theory in four dimensions

[Montonen, Olive '77] argue that if such an effective description is possible, it must be in terms of a gauge group ${}^L G$:

$$\mathcal{T}(G,\tau) = \mathcal{T}({}^L G, -\frac{1}{\tau})\,. \tag{2}$$

${}^L G$ is the Langlands dual to $G$:

| $G$ | ${}^L G$ |
|---|---|
| $U(N)$ | $U(N)$ |
| $SO(2N)$ | $SO(2N)$ |
| $SU(N)$ | $SU(N)/\mathbb{Z}_N$ |
| $SO(2N+1)$ | $USp(2N)$ |

# $\mathcal{N}=4$ SYM theory in four dimensions

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Note that in the last example even the *algebra* changes: the notion of gauge group is useful but not fundamental!

# Duality in $\mathcal{N}=1$

Interestingly, a similar story holds for $\mathcal{N}=2$ and $\mathcal{N}=1$ theories. In the last few years, starting with [Argyres, Seiberg '07], we have understood how to construct large classes of theories where we have a first principles way of understanding the physics at every "$g \to \infty$" limit:

- For $\mathcal{N}=2$ in [Gaiotto '09].
- For $\mathcal{N}=1$ in [Gaiotto, Razamat '15] and [I.G.-E., Heidenreich '16].

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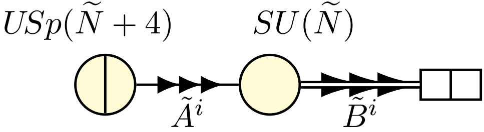
The tool that allows us to do this is string theory: we embed the theories in string theory, and using standard properties of string theory the question of field theory duality becomes a question about geometry, which we can solve.

# A simple example: $\mathbb{C}^3/\mathbb{Z}_3$ phase I

| | $USp(\tilde{N}+4)$ | $SU(\tilde{N})$ | $SU(3)$ | $U(1)_R$ | $\mathbb{Z}_3$ |
|---|---|---|---|---|---|
| $\tilde{A}^i$ | $\overline{\square}$ | $\square$ | $\square$ | $\frac{2}{3}-\frac{2}{\tilde{N}}$ | $1$ |
| $\tilde{B}^i$ | $1$ | $\overline{\square\square}$ | $\square$ | $\frac{2}{3}+\frac{4}{\tilde{N}}$ | $-2$ |

(here $\tilde{N} \in 2\mathbb{Z}$). Graphically we can represent this as




$$W = \epsilon_{ijk}\,\mathrm{Tr}(\tilde{A}^i\tilde{A}^j\tilde{B}^k)$$

# A simple example: $\mathbb{C}^3/\mathbb{Z}_3$ phase II

| | $SO(N-4)$ | $SU(N)$ | $SU(3)$ | $U(1)_R$ | $\mathbb{Z}_3$ |
|---|---|---|---|---|---|
| $A^i$ | $\overline{\square}$ | $\square$ | $\square$ | $\frac{2}{3}+\frac{2}{N}$ | $1$ |
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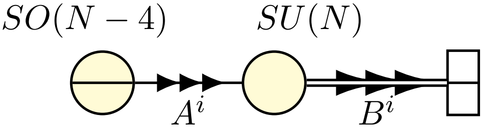


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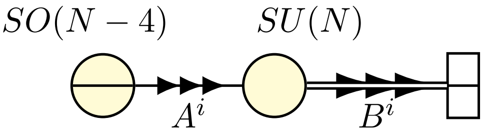


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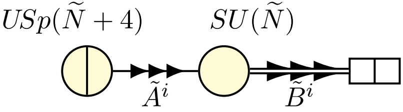
**Note:** For $N=5$ this is $SU(5)$ with three generations of $\mathbf{5}$ and three of $\overline{\mathbf{10}}$. [Lykken, Poppitz, Trivedi '98]

# Duality

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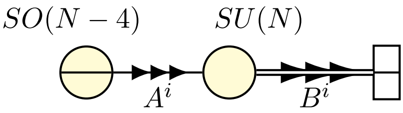
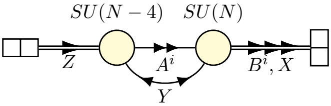
is dual to




$$W = \epsilon_{ijk}\,\mathrm{Tr}(A^iA^jB^k)$$

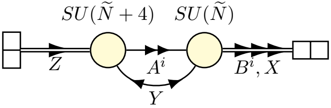
Global anomalies, the moduli spaces, SCIs and the spectrum of operators match if $\boxed{\tilde{N} = N-3}$. [I.G.-E., Heidenreich, Wrase '12]

# A more involved example: $\mathcal{C}(dP_1)$




$$W = \epsilon_{ij} B^i A^j Y + \tfrac{1}{2}\epsilon_{ij} X A^i Z A^j$$

is dual to

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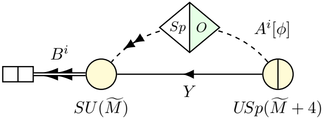
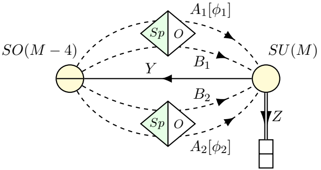
is dual (*trial?*, *plural?*) to



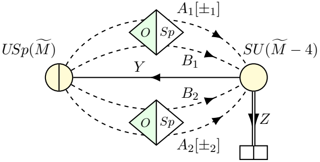

$$W = \varepsilon_{ij} A^i B^j Y$$

# A generic case

For $\mathcal{C}(dP_2)$, **every** duality phase includes isolated CFT factors:




$W = Y(B_1 - B_2) + A_1 A_2 Z$

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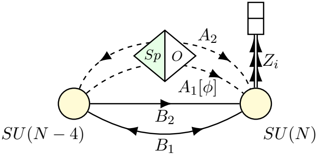
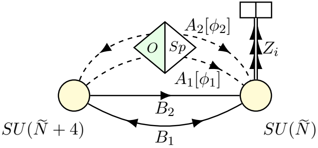



$W = A_1 B_1 Z_1 + A_2 B_2 Z_2 + B_1 B_2 Z_3$

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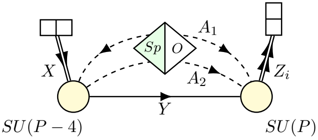
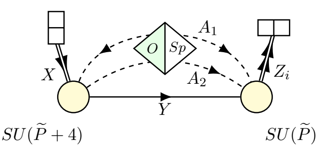



$W = A_1 Y Z_1 + X A_2 Y Z_2$

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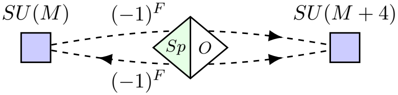
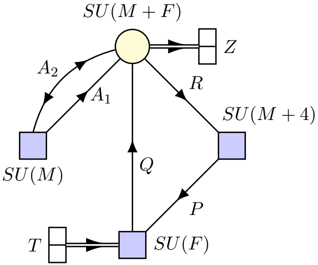
What do we know about these isolated SCFTS?

# Deconstructing isolated SCFTs

The class of theories I just discussed are particularly nice, in that the isolated SCFTs can be described as the IR fixed point of an ordinary Lagrangian theory:




is the (interacting) IR fixed point of




$$W = A_1 A_2 Z + PQR + TQ^2 Z$$

# Lagrangian vs. non-Lagrangian

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I will now briefly review one recent amusing example where we can prove that there is no suitable RG flow from a weakly coupled theory. (Caveat: I am only considering flows that have the same amount of susy as the IR fixed point. It is not possible at this stage to say anything about potential flows with accidental susy enhancement in the IR.)

# Back to the textbooks

Most of those who have studied extended susy have at some point wondered why (in the absence of gravity) we only talk about $\mathcal{N} \in \{0, 1, 2, 4\}$. The standard answers are:

- $\mathcal{N} > 4$ would imply the existence of a particle with helicity greater than 1. (Nothing wrong with this, but let us avoid sugra since we are thinking about ordinary SCFTs.)
- $\mathcal{N} = 3$ is equivalent to $\mathcal{N} = 4$: the minimal CPT invariant $\mathcal{N} = 3$ multiplet is the $\mathcal{N} = 4$ multiplet, and its interactions are as in the $\mathcal{N} = 4$ theory.

# Better answers

**These aren't very good answers!** They assume the existence of a Lagrangian description, which is very limiting if we are thinking about SCFTs. We have only learned the proper answer (for SCFTs) recently:

- Unitary SCFTs with $\mathcal{N} > 4$ have no stress-tensor multiplet. [Cordova, Dumitrescu, Intriligator '16]
- $\mathcal{N} = 3$ theories **do** exist, they are isolated. [I.G.-E., Regalado '15]

# Back to Montonen-Olive duality in $\mathcal{N}=4$

In the Coulomb branch ($U(N) \to U(1)^N$) we had

- Massive $W$ bosons, with mass proportional to $\sqrt{\tau^{-1}}\,\langle\Phi\rangle \sim g\,\langle\Phi\rangle$.
- Monopoles, with mass proportional to $\sqrt{\tau}\,\langle\Phi\rangle \sim 1/g\,\langle\Phi\rangle$.

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In fact, there is a *symmetry* ($\mathcal{S}$) exchanging the $W$ bosons and the monopoles at $g=1$.

The evidence from field theory for $\mathcal{S}$ is rather circumstantial, but when the system is embedded in string theory the existence of the symmetry becomes very obvious.

# $\mathcal{N}=3$ from $\mathcal{N}=4$

Using this symmetry we can construct $\mathcal{N}=3$ theories, intuitively:

$$\{\mathcal{N}=3\} = \frac{\{\mathcal{N}=4\}}{\mathcal{S}}\,. \tag{3}$$

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Some remarks:

- This was very hand-wavy, but the intuition can be made precise in string theory. [I.G.-E., Regalado '15]
- $\mathcal{S}$ is a symmetry only when $g = 1$, so the marginal deformation of the $\mathcal{N} = 4$ theory is frozen out: $\mathcal{N} = 3$ theories are stuck at strong coupling. I.e. the conformal manifold is a point, and there is no weak coupling regime.

# Conclusions

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# Conclusions

- Thinking about duality, it is very natural to generalize what "weak coupling" means to include isolated SCFTs with weakly gauged global symmetries.
- Such isolated SCFTs are not rare, we now know how to construct and analyse various infinite families.
- We cannot always access these theories via a Lagrangian, but the theories still exist. In some cases, like $\mathcal{N}=3$, one can show that no $\mathcal{N}=3$ Lagrangian exists.
- Technology for analysing these isolated SCFTs is in active development. (I.e. how to compute the spectrum of operators and correlators, at least in some subsectors.)