A Perturbative Randall Sundrum Cosmological Phase Transition

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Work with Jay Hubisz, Don Bunk
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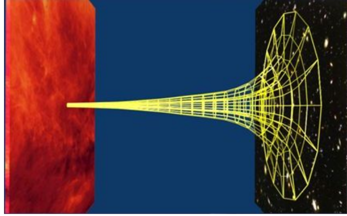
Motivation



- In large N gauge theories, evidence that confining transition is strongly first order
- Too strong: nucleation of true vacuum bubbles outpaced by Hubble dilution - trapped in false vacuum = empty universe
- Dual picture (Randall Sundrum model) exhibits same phenomenology

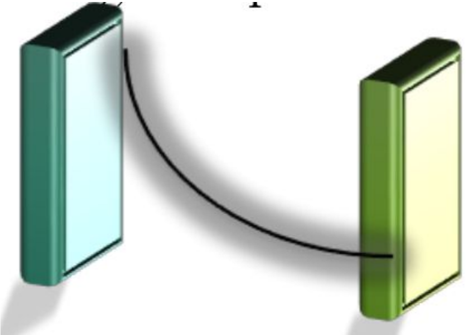
Problem

Perturbative Randall-Sundrum Models Stabilized by Goldberger-Wise Mechanism typically lead to an Empty Universe

Holographic Model at Zero Temperature

5 D Einstein-Scalar theory with extra spatial dimension, y (S_1/Z_2 orbifold)

$$S = \int d^5x\sqrt{g}\left[\frac{1}{2}(\partial_M\phi)^2 - V(\phi) - \frac{1}{2\kappa^2}\mathcal{R}\right] - \int d^5x\sqrt{g_0}\, V_0(\phi) \; - \int d^5x\sqrt{g_1}\, V_1(\phi)$$



Extra Dimension spans two branes y_0 and y_1

Brane Localized potentials contribute to total action

5D scalar minimally coupled to gravity	⟷	dual to coupling in CFT
VEV of scalar	⟷	CFT operator is sourced
y-dependence, e^{-ky}	⟷	Renormalisation group flow
Bulk Potential - Cosmological Const.	⟷	gives AdS curvature ($\mathcal{N}$ of CFT)
Φ dependence	⟷	beta function (VEV explicitly breaks Scale Invariance)

Coordinates and EOM

Metric ansatz: flat 4d slices $\rightarrow ds^2 = e^{-2A(\tilde{y})}\eta_{\mu\nu}\, dx^\mu dx^\nu - d\tilde{y}^2$

Take $A(\tilde{y}) = y\,,\quad G(y) = A'\big(\tilde{y}(y)\big)^2 \rightarrow$ $ds^2 = e^{-2y}\,\eta_{\mu\nu}\, dx^\mu dx^\nu - \frac{dy^2}{G(y)}$

Deviations from AdS encoded in G(y), Pure AdS, G=k

Equations of motion:

$$G = \frac{\frac{-\kappa^2}{6}V(\phi)}{1-\frac{\kappa^2}{12}\dot{\phi}^2}$$

$$\frac{\dot{G}}{G} = \frac{2\kappa^2}{3}\dot{\phi}^2$$

$$\ddot{\phi} = \left(4 - \frac{1}{2}\frac{\dot{G}}{G}\right)\dot{\phi} + \frac{1}{G}\frac{\partial V}{\partial \phi}$$

Eliminate G(y) in the scalar field EOM

Master Evolution equation :

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V(\phi)}{\partial \phi}\right)\left(1 - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

2 types of backreaction: Potential and Gravity

Effective Dilaton Action

- Introduction of IR brane/ deformation of geometry at y_1 is dual to spontaneous breaking of conformal symmetry

$$f^{-1} = \int_{y_0}^{y_1} \frac{e^y}{\sqrt{G}} dy$$

f is the dilaton

- Total value of the classical action can be expressed as a pure boundary term
- Replace the kinetic and potential terms for, φ using Einstein's equations, include singular terms at the brane and impose BCs

$$V_{\text{eff}} = e^{-4y_0}\left[V_0(\phi(y_0)) - \frac{6}{\kappa^2}\sqrt{G(y_0)}\right] + e^{-4y_1}\left[V_1(\phi(y_1)) + \frac{6}{\kappa^2}\sqrt{G(y_1)}\right]$$

- Entire effective potential= boundary term with brane localized potentials and jump conditions
- Depends only on asymptotic behavior of the geometry and the scalar field

The Long walk in 5D

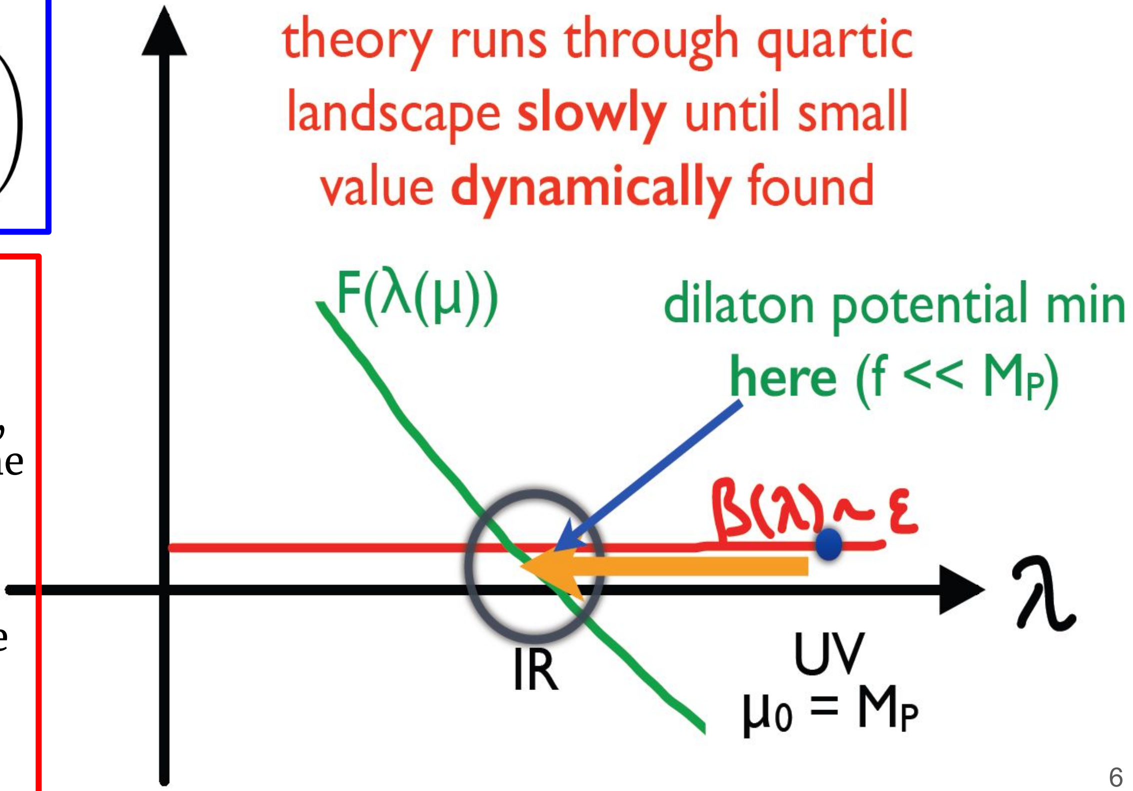
Want slow walking over large range of coupling in dual CFT

$$V(\phi) = -\frac{6k^2}{\kappa^2}\left(1 - \frac{\kappa^2}{3}\epsilon\phi^2\right)$$

AdS/CFT: Small dependence on scalar field value

Parametric suppression by small mass, no operators in potential with order one coefficients

Perform numerical solutions of non-linear zero temperature and finite temperature equations over large parameter space to calculate effective potentials

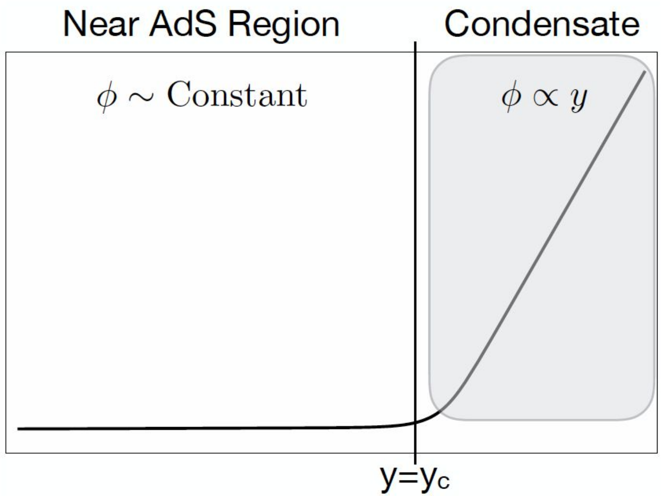


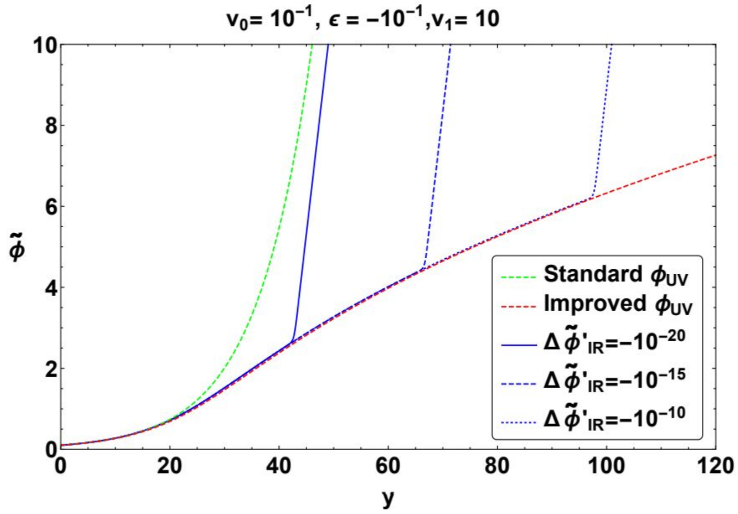
Scalar field evolution at T=0

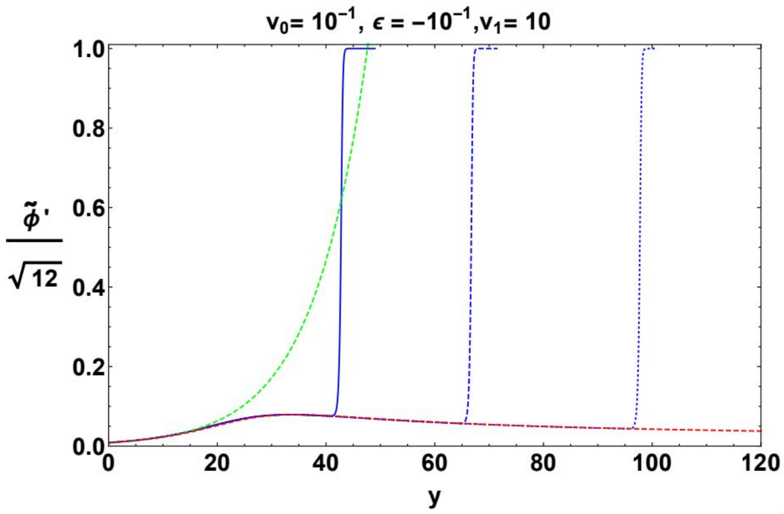
Slow evolution in UV

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V(\phi)}{\partial \phi}\right)\left(1 - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

IR behaviour universal:
linear growth= condensate “SOFT WALL”

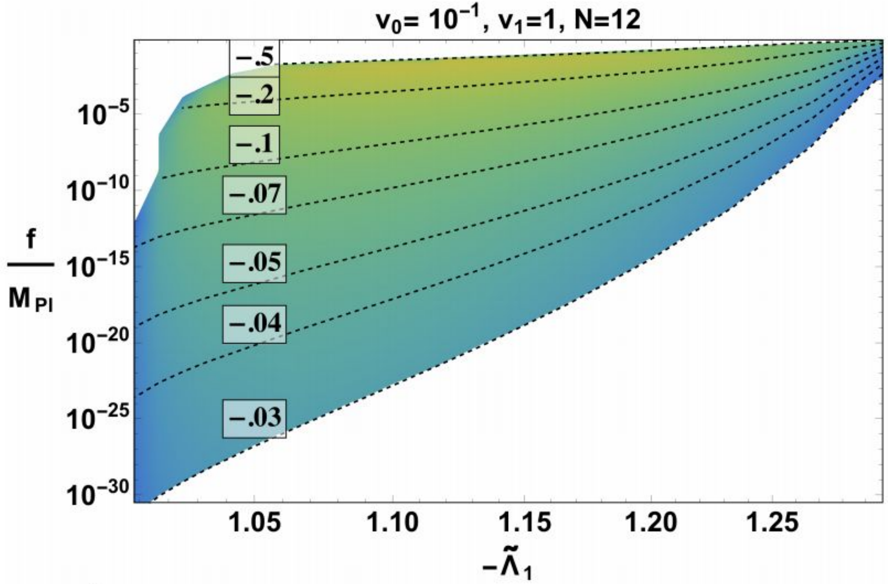




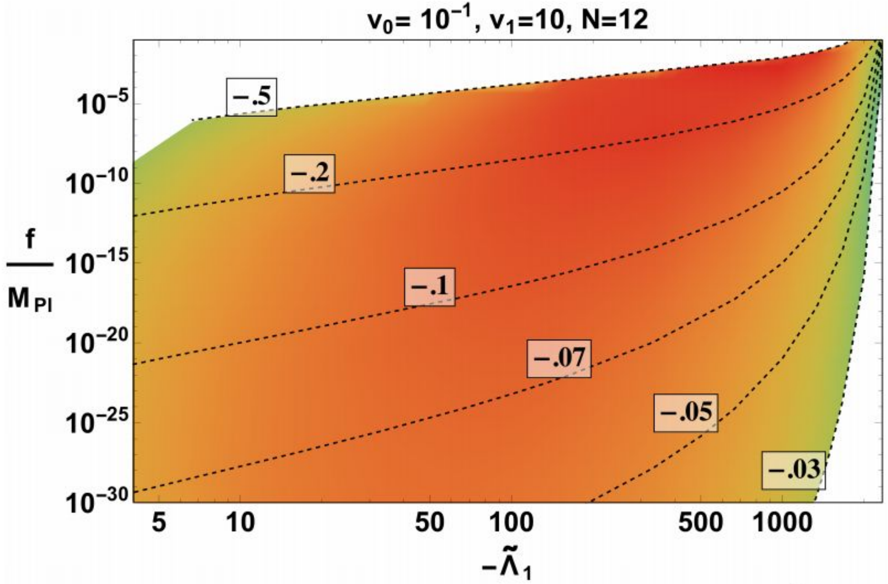


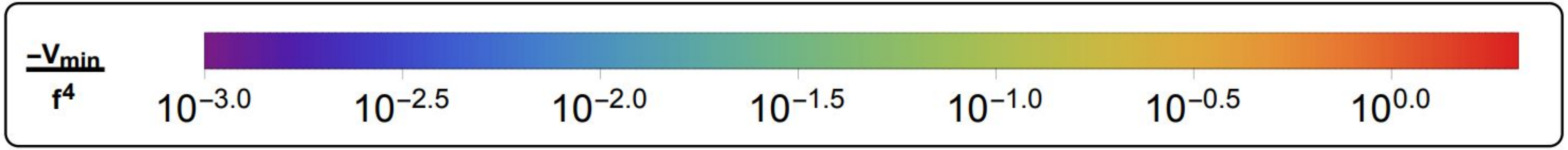
Results at Zero Temperature

Golberger-Wise



Soft-Wall Light Dilaton





Color shading is value of effective V at minimum in units of f^4
Boxed numbers are the values of epsilon (dimensionless scalar mass in units of curvature)

Finite Temperature

To model Finite T – compactify time dimension on circle $t \in [0, 1/T)$

Euclidean action = Helmholtz Free Energy

Metric modified to reflect symmetry of geometry

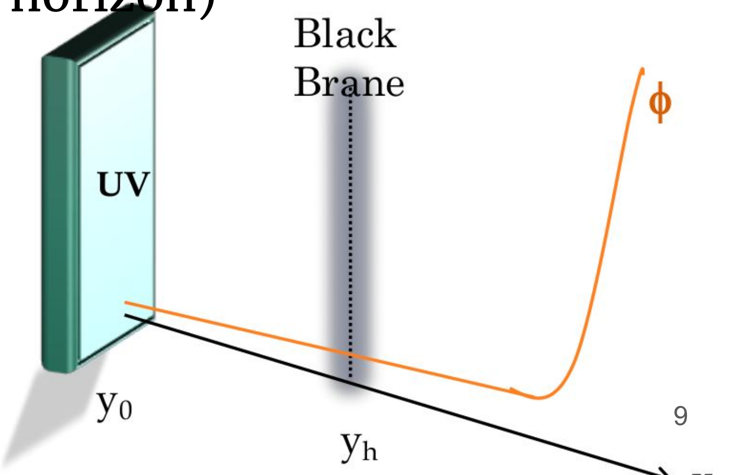
$$ds^2 = e^{-2y}\left[h(y)dt^2 + d\vec{x}^2\right] + \frac{1}{h(y)}\frac{dy^2}{G(y)}$$

At finite T, coordinate singularity at y_h (blackhole horizon)

No scalar VEV= AdS – Schwarzschild

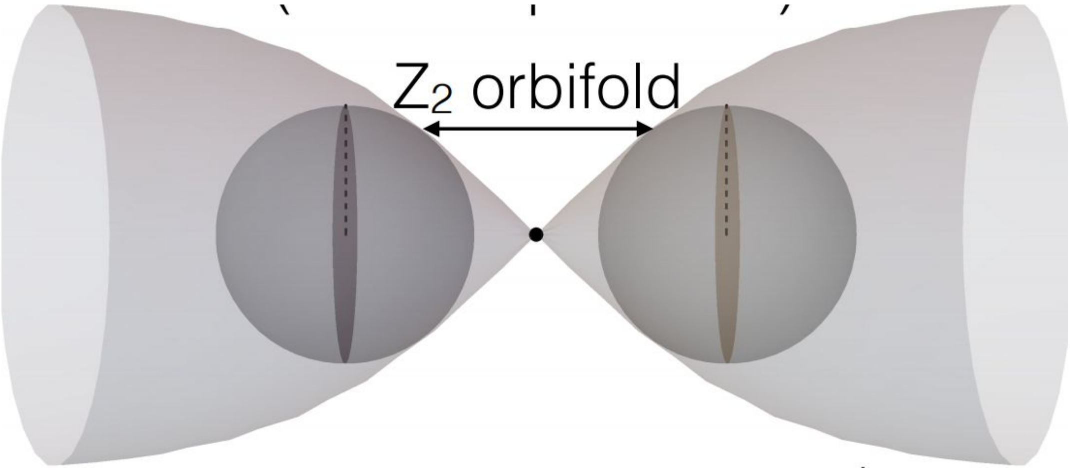
EOMs $$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V}{\partial \phi}\right)\left(1 - \frac{1}{4}\frac{\dot{h}}{h} - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

$$\frac{\ddot{h}}{\dot{h}} = 4 - \frac{\kappa^2}{3}\dot{\phi}^2 \qquad h(y_h)=0$$



Free Energy of BH solution - Conical Singularity

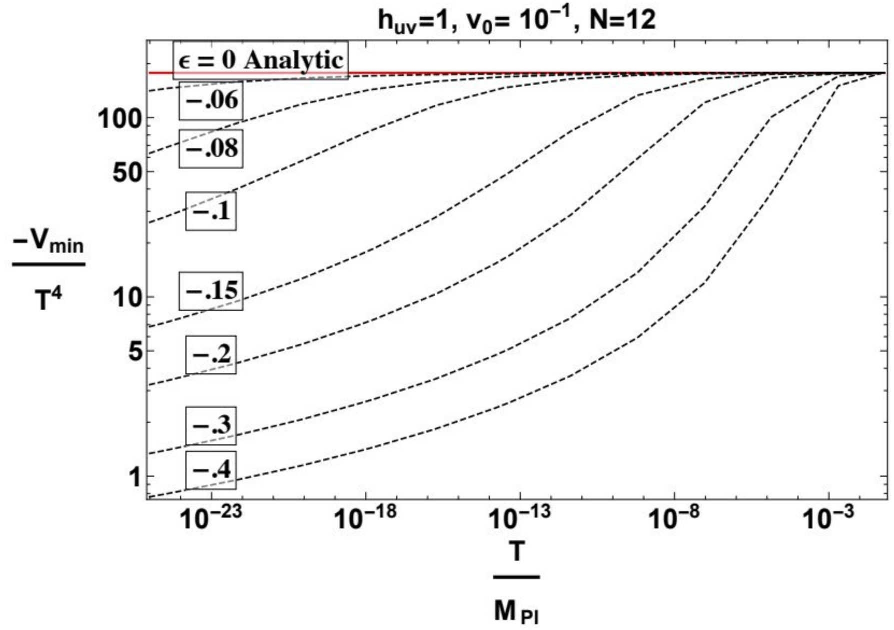
Near the horizon, $y \sim y_h$ geometry conically singular (out of equilibrium)

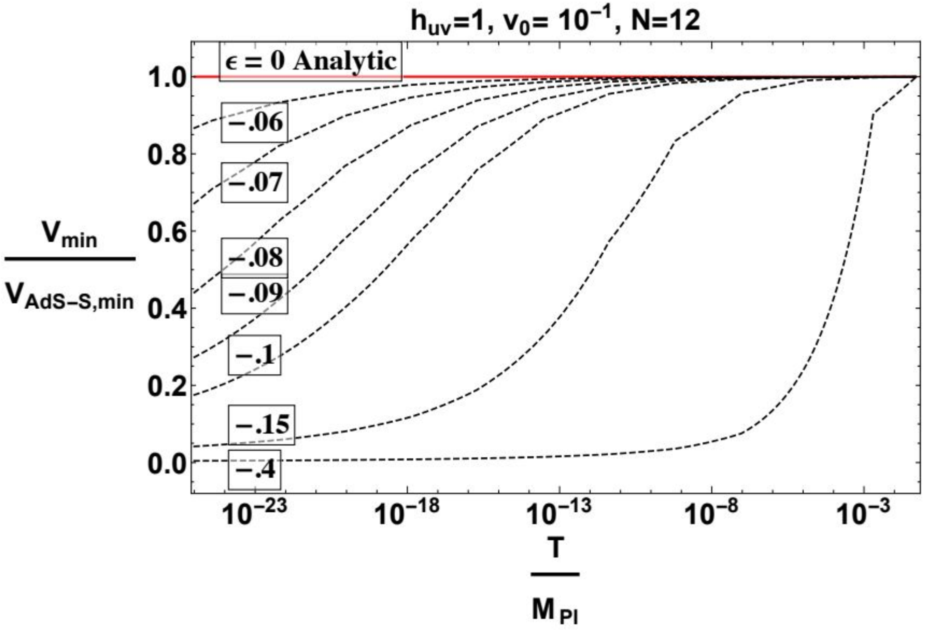


Free energy receives UV contribution and (regulated) entropy like contribution from near horizon geometry , *F = U−TS*

$$F = e^{-4y_0}\left[\sqrt{h(y_0)}V_0(\phi(y_0)) - \frac{6}{\kappa^2}h(y_0)\sqrt{G(y_0)}\right] - \frac{4\pi T}{\kappa^2}e^{-3y_h}$$

Results at Finite T





Very large differences compared to vanishing BR case BH suppressed gravitational BR - mostly due to potential BR

Bubble Nucleation

Universe starts in hot CFT phase (radiation domination) $\Rightarrow$ $$H^2 = \frac{8\pi G\rho}{3} \sim \frac{\pi^3 G N^2 T^4}{3}$$

For successful phase transition Bubble nucleation must outpace dilution $\Rightarrow$ $$\frac{\Gamma}{V} \gtrsim H^4$$

Nucleation rate proceeds like exponential Of Euclidean bubble action (Coleman) $\Rightarrow$ $$\frac{\Gamma}{V} \approx T_c^4 e^{-S_E}$$

up to mult. order 1 factors

T_c is critical temperature (typically close to f)

Criteria for successful phase transition $\Rightarrow$ $$S_E \lesssim 4 \log\left(\frac{M_{\rm Pl}}{f}\right)$$

Bubble Nucleation Actions

Action for bubble very hard to calculate generally - several approximations

Bubbles can be big -O(3) symmetry (wrap time direction)

Or small - O(4) symmetry (neglect compactification)

Surface energy

Volume term

$$S_E^{O(4)} = S_4 = 2\pi^2 \int r^3 \left[\frac{\mathcal{N}}{2} \left(\vec{\nabla} f \right)^2 + V(f,T) \right] dr$$

$$S_E^{O(3)} = S_3/T = \frac{4\pi}{T} \int r^2 \left[\frac{\mathcal{N}}{2} \left(\vec{\nabla} f \right)^2 + V(f,T) \right] dr$$

Canonical Normalization for dilaton: $\mathcal{N} = 3N^2/2\pi^2$

Bubble Nucleation

Thick wall Appx : When the bubble radius,R ~ thickness of bubble and surface energy ~ R

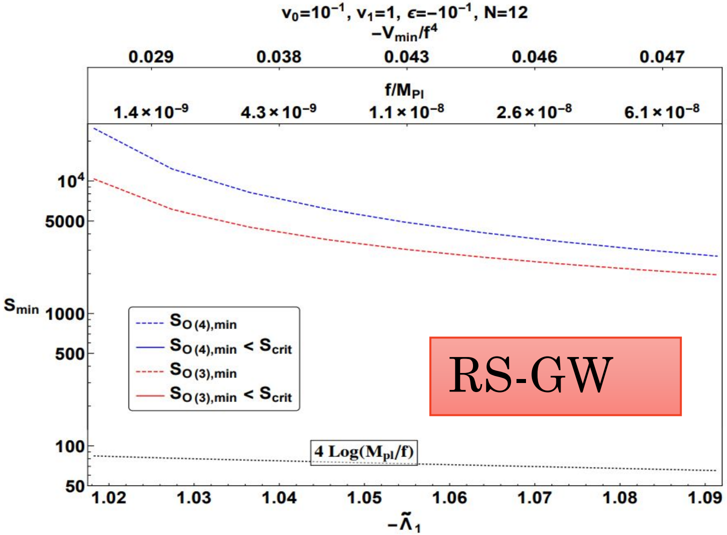
Thin wall Appx: Bubble radius >> thickness of bubble

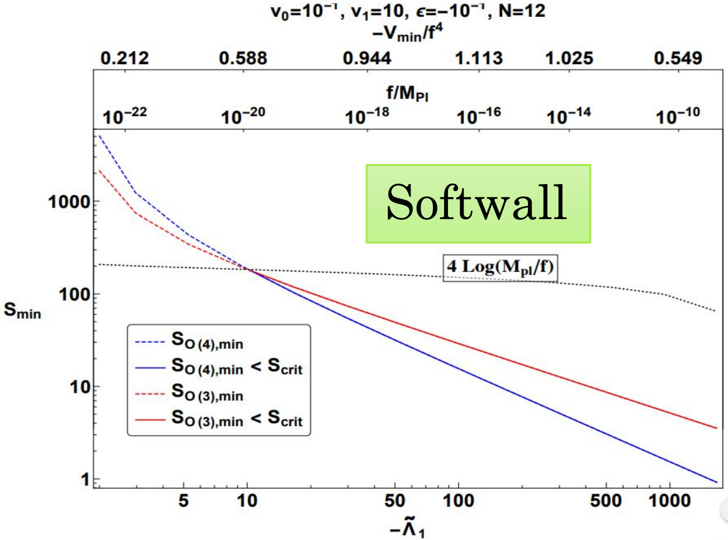
Thick wall actions are smaller Whichever appx gives smaller action “wins”

$$S_3/T(\text{min}) = \frac{4\pi}{3}\frac{\mathcal{N}^{3/2} f^3}{\sqrt{2|\bar{V}|}}$$

$$S_4(\text{min}) = \pi^2 \frac{\mathcal{N}^2 f^4}{2|\bar{V}|}$$

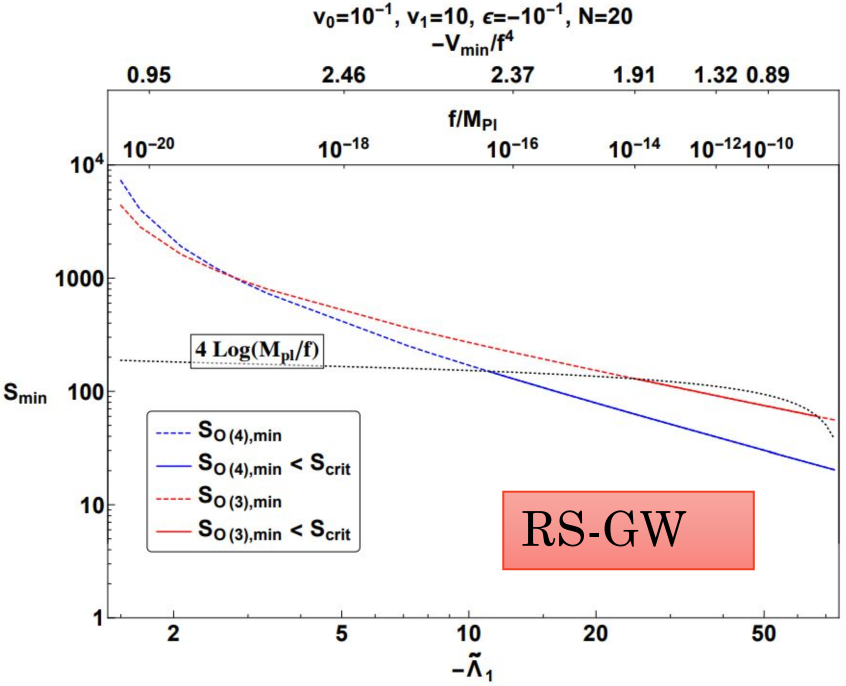
$$|\bar{V}| \approx F_{\text{min}}(T) - V_{\text{dilaton}}(f)$$

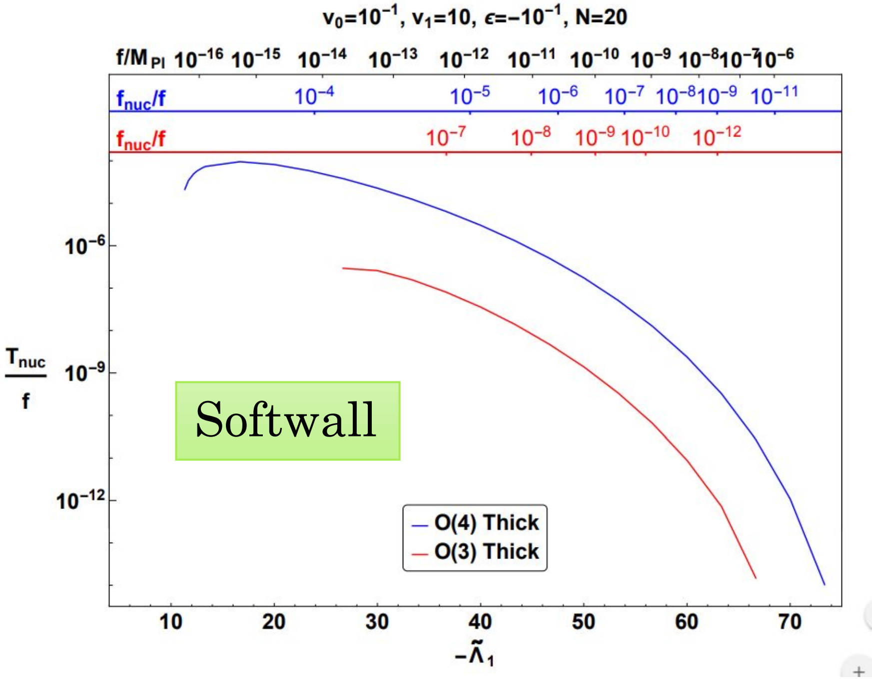




Largest N?

We have done an incomplete by-hand scan and found N=20 completes with TeV scale f, mass² ~ 0.1





Nucleation temperature and nucleation f value becoming tiny at threshold - don't tunnel to true min

Gravitational Waves

Bubbles break spacetime symmetries and create turbulence in hot plasma - Source GR waves

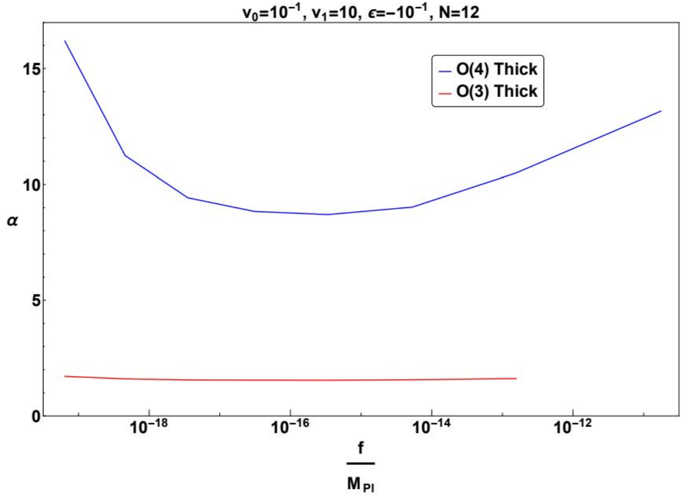
Characterized by two features

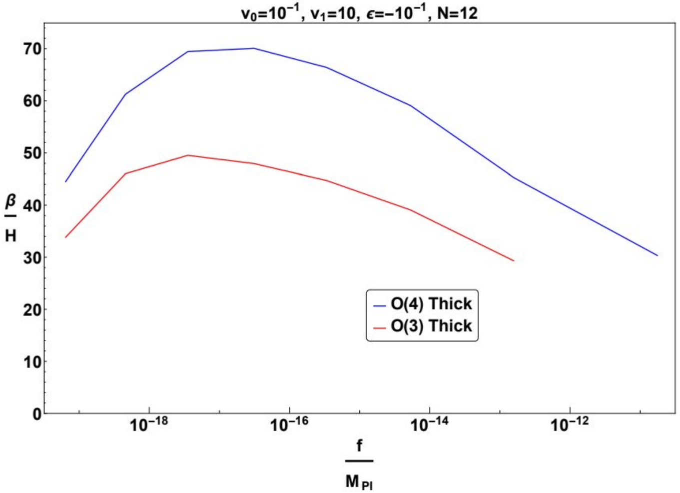
Latent heat release:

$$\alpha = \frac{V_{T=0}(f_n)}{V_T(T=T_n)} - 1$$

Transition time scale:

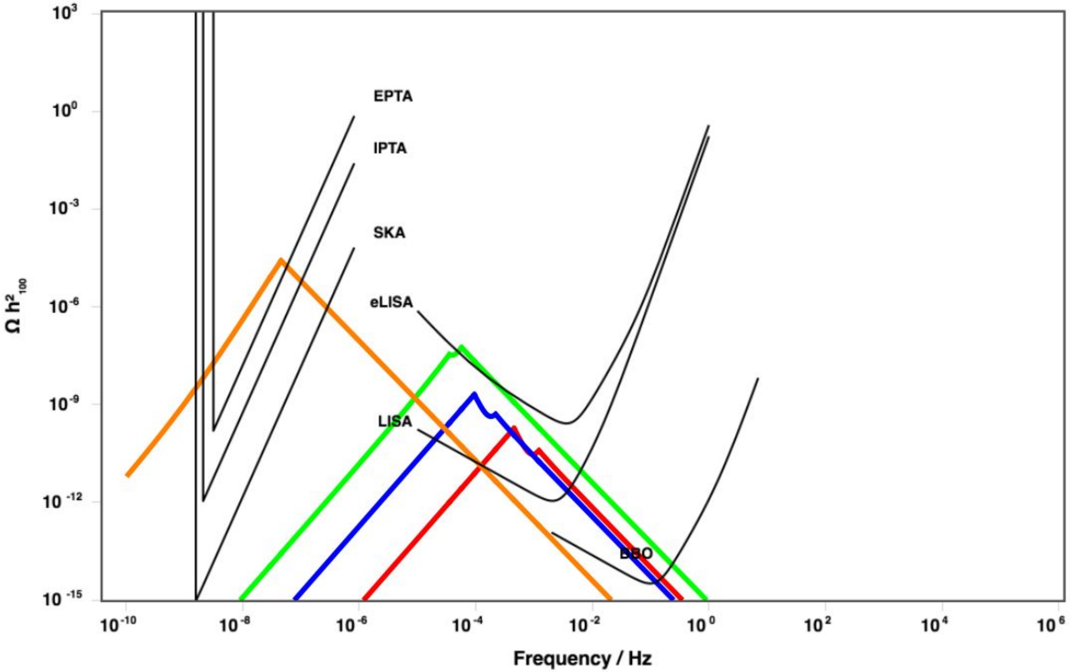
$$\frac{\beta}{H^*} = T^* \left.\frac{dS}{dT}\right|_{T^*}$$





Gravitational Waves

Observational Possibility because of large α and β/H



Dashed lines are signals: various values of N (N=6 (red), then 9 (blue),12 (green), to orange N=20) $\epsilon = 0.1,\ v_0 = 0.1,\ v_1 = 10,\ \text{and}\ f = 1\ \text{TeV}$

Conclusions

1. We have performed a study of zero and finite temperature Randall Sundrum models including arbitrary backreaction (scalar BR on V and gravitational BR on metric)

2. Numerical solutions yield shape of dilaton/radion effective potential at zero and finite temperature - large deviations from picture in literature

3. Results used to estimate bubble action - rate of true vacuum nucleation in early universe cosmology (natural spontaneous breaking of ~ conformal invariance)

4. Large values of N (small 5D gravity coupling) yield acceptably fast nucleation rate - successful RS cosmology

5. Large N TeV-scale transition potentially observable at LISA