Consequences of generalized residual $\mathbb{Z}_2 \times \mathbb{Z}_2$ in scaling neutrino Majorana mass matrix

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Motivation

- The oscillation data, the bounds on the sum of the light neutrino masses and $\beta\beta_{0\nu}$ decay experiments severely constrain the flavour structures of the light neutrino mass matrix.

- Admissible mass matrices satisfying above constraints can be tested further based on their predictions about the yet unresolved issues such as the:
  (i)*hierarchy* of light neutrino masses,
  (ii)*octant of* θ_{23}, and particularly,
  (iii)*CP violation* in the leptonic sector.

- Moreover, if neutrino is a Majorana particle, the *prediction of Majorana phases* will also serve as an added ingredient to discriminate different models.

- We address these issues using the approach of a *generalized* $\mathbb{Z}_2 \times \mathbb{Z}_2$ *residual symmetry* for a scaling neutrino Majorana mass matrix.

Residual Symmetry

- An arbitrary light Majorana neutrino mass matrix M_ν enjoys $\mathbb{Z}_2 \times \mathbb{Z}_2$ residual symmetry[1] which is a remnant of some high energy flavour group.

- A linear unitary transformation of the $\nu_{L\alpha}$ fields $\nu_{L\alpha} \to G_{\alpha\beta}\nu_{L\beta}$ leads to an invariance of effective neutrino Majorana mass term

$$-\mathcal{L}^{\nu}_{mass} = \frac{1}{2}\bar{\nu}^C_{L\alpha}(M_\nu)_{\alpha\beta}\nu_{L\beta}$$

  if the mass marix M_ν satisfies the invariance equation

$$G^T M_\nu G = M_\nu.$$

- Furthermore, since M_ν is a complex symmetric, it can be diagonalized by an unitary matrix U as

$$U^T M_\nu U = M^d_\nu = \text{diag}(m_1, m_2, m_3).$$

[1] C.S. Lam, Symmetry of Lepton Mixing, Phys. Lett. B 656 (2007) 193

Residual Symmetry (contd.)

- It's simple to see that if U diagonalize M_ν so does $U' = GU$.
- If m_1, m_2 and m_3 are non-degenerate, then G has eigenvalues ± 1, and is diagonalized by U i.e.,

$$GU = Ud, \quad \text{with} \quad d_{lm} = \pm\delta_{lm}. \tag{1}$$

- From Eq.(1)

$$\det(G) = \det(d) = \pm 1.$$

- Without any loss of generality we can confine to $\det G = +1$ which corresponds to

$$d_1 = \mathrm{diag}\,(1, -1, -1) \longrightarrow G_1,$$
$$d_2 = \mathrm{diag}\,(-1, 1, -1) \longrightarrow G_2,$$
$$d_3 = \mathrm{diag}\,(-1, -1, 1) \longrightarrow G_3.$$

Residual Symmetry (contd.)

- All three G matrices, G_1, G_2 and G_3, out of which only two are independent on account of a relation

$$G_a G_b = G_b G_a = G_c \quad \text{with} \quad a \neq b \neq c.$$

- Since $G^2 = I$, these two independent G_a matrices define a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry.

- Given a mass matrix M_ν in flavour space, one can obtain the PMNS matrix U, consistent with the symmetries of M_ν.

- From U, G_a's can be obtained as

$$G_a = U d_a U^\dagger \text{ with } a = 1, 2, 3.$$

- The explicit form of the $\mathbb{Z}_2$ generators will depend on the specific flavour structure of M_ν.

Strong Scaling Ansatz (SSA)

- We explore the *Strong Scaling Ansatz*[2] (SSA) which proposes a column-wise scaling relations in the elements of M_ν:

$$\frac{(M_\nu^0)_{e\mu}}{(-M_\nu^0)_{e\tau}} = \frac{(M_\nu^0)_{\mu\mu}}{(-M_\nu^0)_{\mu\tau}} = \frac{(M_\nu^0)_{\tau\mu}}{(-M_\nu^0)_{\tau\tau}} = k.$$

- The structure of M_ν dictated by this ansatz is given by

$$M_\nu^0 = \begin{pmatrix} P & -Qk & Q \\ -Qk & Rk^2 & -Rk \\ Q & -Rk & R \end{pmatrix}$$

which is diagonalized by

$$U^0 = \begin{pmatrix} c_{12}^0 & s_{12}^0 e^{i\alpha} & 0 \\ -\frac{ks_{12}^0}{\sqrt{1+k^2}} & \frac{kc_{12}^0}{\sqrt{1+k^2}} e^{i\alpha/2} & \frac{1}{\sqrt{1+k^2}} e^{i\beta/2} \\ \frac{s_{12}^0}{\sqrt{1+k^2}} & -\frac{c_{12}^0}{\sqrt{1+k^2}} e^{i\alpha/2} & \frac{k}{\sqrt{1+k^2}} e^{i\beta/2} \end{pmatrix}$$

[2] R.N Mohapatra and W. Rodejohann, Phys. Lett. B **644**, 59 (2007).

Strong Scaling Ansatz (Contd.)

- Explicit form of the $\mathbb{Z}_2$ generators G_a can be calculated as

$$G_a^{(k)} = U^0 d_a U^{0\dagger}.$$

$$G_1^{(k)} = \begin{pmatrix} \cos 2\theta_{12}^0 & -k(1+k^2)^{-1/2}\sin 2\theta_{12}^0 & -(1+k^2)^{-1/2}\sin 2\theta_{12}^0 \\ -k(1+k^2)^{-1/2}\sin 2\theta_{12}^0 & -(1+k^2)^{-1}(k^2\cos 2\theta_{12}^0+1) & -k(1+k^2)^{-1}(1-\cos 2\theta_{12}^0) \\ -(1+k^2)^{-1/2}\sin 2\theta_{12}^0 & -k(1+k^2)^{-1}(1-\cos 2\theta_{12}^0) & -(1+k^2)^{-1}(k^2+\cos 2\theta_{12}^0) \end{pmatrix}$$

$$G_2^{(k)} = \begin{pmatrix} -\cos 2\theta_{12}^0 & k(1+k^2)^{-1/2}\sin 2\theta_{12}^0 & -(1+k^2)^{-1/2}\sin 2\theta_{12}^0 \\ k(1+k^2)^{-1/2}\sin 2\theta_{12}^0 & (1+k^2)^{-1}(k^2\cos 2\theta_{12}^0-1) & -k(1+k^2)^{-1}(1+\cos 2\theta_{12}^0) \\ -(1+k^2)^{-1/2}\sin 2\theta_{12}^0 & -k(1+k^2)^{-1}(1+\cos 2\theta_{12}^0) & -(1+k^2)^{-1}(k^2-\cos 2\theta_{12}^0) \end{pmatrix}$$

$$G_3^{(k)} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & (1-k^2)(1+k^2)^{-1} & 2k(1+k^2)^{-1} \\ 0 & 2k(1+k^2)^{-1} & -(1-k^2)(1+k^2)^{-1} \end{pmatrix}.$$

Modification of SSA with nonstandard CP

- The SSA predicts $\theta_{13} = 0$ (ruled out at more than 5.2σ) and therefore, no measurable Dirac CP-violation.

- *We have to sacrifice scaling symmetry*.

- A possible modification to SSA, was proposed[3],[4] by combining the $\mathbb{Z}_2 \times \mathbb{Z}_2$ residual symmetry with the non-standard CP transformation on the neutrino fields

$$\nu_{L\alpha} \to i(G_a^{(k)})_{\alpha\beta}\gamma^0 \nu_{L\beta}^C.$$

- This extends the real invariance of M_ν in (3) to its complex counterpart, i.e.

$$(G_a^{(k)})^T M_\nu G_a^{(k)} = M_\nu^*.$$

- Since only two of the three $G_a^{(k)}$'s ($a = 1, 2, 3$) are independent, there are 3 ways in which generalized CP symmetry can be implemented: $G_{1,2}$, $G_{2,3}$ and $G_{1,3}$.

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[3] R. Samanta, P. Roy and A. Ghosal, Eur. Phys. J. C **76**, no. 12, 662 (2016)

[4] W. Grimus and L. Lavoura, Phys. Lett. B **579** (2004) 113

The complex invariance relations of M_ν related to $G_{2,3}^{(k)}$ is now written as

$$(G_{2,3}^{(k)})^T M_\nu G_{2,3}^{(k)} = M_\nu^*,$$

which in turn implies

$$(G_1^{(k)})^T M_\nu G_1^{(k)} = M_\nu.$$

The most general form of Majorana neutrino mass matrix becomes

$$M_{\nu_1} = \begin{pmatrix} p & -q_1 k + i q_2 k^{-1} & q_1 + i q_2 \\ -q_1 k + i q_2 k^{-1} & r - s k^{-1}(k^2 - 1) + i \frac{2 q_2 \kappa_+}{\sqrt{1+k^2}} & s + i \frac{q_2 \kappa_+ (k^2-1)}{k\sqrt{1+k^2}} \\ q_1 + i q_2 & s + i \frac{q_2 \kappa_+ (k^2-1)}{k\sqrt{1+k^2}} & r - i \frac{2 q_2 \kappa_+}{\sqrt{1+k^2}} \end{pmatrix}$$

It has already been shown that $(G_3^{(k)})^T M_\nu G_3^{(k)} = M_\nu^*$ leads to the results

$$\tan\theta_{23} = k^{-1},$$

$$\sin\alpha = \sin\beta = \cos\delta = 0.$$

The overall real G_1 invariance of M_ν fixes the first column of U_{PMNS} to the first column of U_0. Therefore,

$$\sin^2\theta_{12} = 1 - \cos^2\theta_{12}^0 (1 + \tan^2\theta_{13}).$$

Case II: Complex extension of $G^{(k)}_{1,3}$ Invariance

In this case, the complex invariance relations of M_ν due to $G^{(k)}_{1,3}$ can be written as

$$(G^{(k)}_{1,3})^T M_\nu G^{(k)}_{1,3} = M_\nu^*,$$

which leads to

$$(G^{(k)}_{2})^T M_\nu G^{(k)}_{2} = M_\nu.$$

Most general Majorana neutrino mass matrix

$$M_{\nu_2} = \begin{pmatrix} p & -q_1 k + i q_2 k^{-1} & q_1 + i q_2 \\ -q_1 k + i q_2 k^{-1} & r - s k^{-1}(k^2-1) + i\frac{2q_2\kappa_-}{\sqrt{1+k^2}} & s + i\frac{q_2\kappa_-(k^2-1)}{k\sqrt{1+k^2}} \\ q_1 + i q_2 & s + i\frac{q_2\kappa_-(k^2-1)}{k\sqrt{1+k^2}} & r - i\frac{2q_2\kappa_-}{\sqrt{1+k^2}} \end{pmatrix}$$

with

$$\kappa_- = -\frac{1}{\kappa_+}.$$

Now the overall real G_2 (10) invariance of M_ν fixes the second column of U_{PMNS} to the second column of U_0 which gives

$$\sin^2\theta_{12} = \sin^2\theta^0_{12}(1+\tan^2\theta_{13}).$$

Case III: Complex extension of $G_{1,2}^{(k)}$ Invariance

- The complex invariance of M_ν related to $G_{1,2}^{(k)}$

$$(G_{1,2}^{(k)*})^T M_\nu G_{1,2}^{(k)} = M_\nu^*,$$

which implies

$$(G_3^{(k)})^T M_\nu G_3^{(k)} = M_\nu.$$

Since $G_3^{(k)}$ fixes the third column of U_{PMNS} to the third column of U_0. But that leads to a vanishing value of θ_{13} and therefore, must be discarded.

Modified Scaling with type-I seesaw

- To discss leptogenesis, we implement the non-standard CP in the context of type-I seesaw.
- We define two separate 'G' matrices G_L and G_R:

$$\nu_{L\alpha} \rightarrow i(G_L)_{\alpha\beta}\gamma^0\nu^C_{L\beta}, \qquad N_{R\alpha} \rightarrow i(G_R)_{\alpha\beta}\gamma^0 N^C_{R\beta}. \qquad (2)$$

- The Lagrangian of type-I seesaw in a weak basis can be written as

$$-\mathcal{L}_{mass} = \bar{N}_{iR}(m_D)_{i\alpha} l_{L\alpha} + \frac{1}{2}\bar{N}_{iR}(M_R)_i \delta_{ij} N^C_{jR} + \mathrm{h.c.}$$

- The invariance of the Lagrangian $\mathcal{L}_{mass}$ under Eq.(2) lead to

$$G_R^\dagger m_D G_L = m_D^*, \quad G_R^\dagger M_R G_R^* = M_R^*.$$

- With $M_R = \mathrm{diag}\,(\mathrm{M}_1, \mathrm{M}_2, \mathrm{M}_3)$, we obtain

$$(G_R)_{lm} = \pm\delta_{lm} \Rightarrow m_D G_3 = -m_D^*, \text{ and } m_D G_{1,2} = m_D^*. \qquad (3)$$

Modified Scaling with type-I seesaw

- For both both case-I and II, the most general form of m_Ds that satisfy the above constraints can be parameterized as

$$m_D = \begin{pmatrix} a & b_1 + ib_2 & -b_1/k + ib_2 k \\ e & c_1 + ic_2 & -c_1/k + ic_2 k \\ f & d_1 + id_2 & -d_1/k + id_2 k \end{pmatrix}$$

- In terms of the parameters of $(m_D)_{1,2}$, and M_R, the parameters of effective $M_\nu = -m_D^T M_R^{-1} m_D$ are given by

$$\boxed{\begin{array}{c} p = -(\frac{a^2}{M_1} + \frac{e^2}{M_2} + \frac{f^2}{M_3}) \\ q_1 = -\frac{\kappa_\pm p}{\sqrt{1+k^2}} \\ q_2 = -k(\frac{ab_2}{M_1} + \frac{ec_2}{M_2} + \frac{fd_2}{M_3}) \\ s = -\frac{\kappa_\pm^2 pk}{1+k^2} + k((\frac{b_2^2}{M_1} + \frac{c_2^2}{M_2} + \frac{d_2^2}{M_3})) \\ r = (sk + p) - 2q_1\sqrt{1+k^2}(\kappa_\pm - \frac{1}{\kappa_\pm}) \end{array}}$$

Numerical Analysis

- We utilize the (3σ) values of globally fitted neutrino oscillation data.

Table : Predictions on the light neutrino masses and $\sum_i m_i$.

Case-I					
Normal Ordering			Inverted Ordering		
$m_1/10^{-3}$ (eV)	$m_2/10^{-3}$ (eV)	$m_3/10^{-3}$ (eV)	$m_1/10^{-3}$ (eV)	$m_2/10^{-3}$ (eV)	$m_3/10^{-3}$ (eV)
$4.0-8.5$	$9.28-12.0$	$49-52$	$47-61$	$49-62$	$9-36$
$\sum_i m_i < 0.08$ eV			$\sum_i m_i < 0.16$ eV		

Case-II					
Normal Ordering			Inverted Ordering		
$m_1/10^{-3}$ (eV)	$m_2/10^{-3}$ (eV)	$m_3/10^{-3}$ (eV)	$m_1/10^{-3}$ (eV)	$m_2/10^{-3}$ (eV)	$m_3/10^{-3}$ (eV)
$4.1-8.8$	$9.23-13.1$	$48-52$	$47-60$	$49-61$	$10-38$
$\sum_i m_i < 0.08$ eV			$\sum_i m_i < 0.16$ eV		

Neutrinoless double beta decay (Case-I)

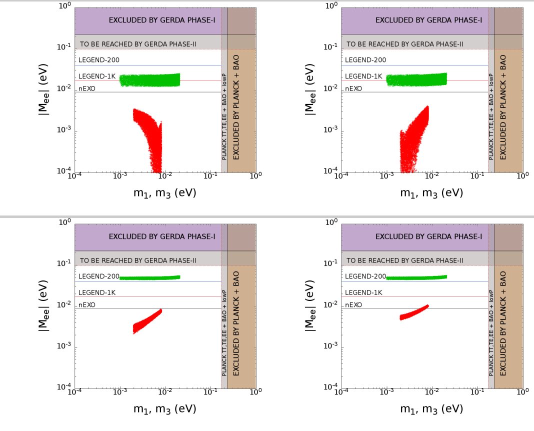

Figure : Plot of $|M_{ee}|$ vs. the lightest neutrino mass: the top two figures represent Case A: $\alpha = \pi$, $\beta = 0$ (left) and Case B: $\alpha = \pi$, $\beta = \pi$ (right) while the figures in the lower panel represent Case C: $\alpha = 0$, $\beta = 0$ (left) and Case D: $\alpha = 0$, $\beta = \pi$ (right).

Baryogenesis via Leptogenesis

- Bayogenesis via leptogenesis through the decays of the RH neutrinos, is governed by the Lagrangian

$$-\mathcal{L} = \lambda_{i\alpha} \bar{N}_{Ri} \tilde{\phi}^\dagger l_{L\alpha} + \frac{1}{2} \bar{N}_{iR} (M_R)_i \delta_{ij} N^C_{jR} + \mathrm{h.c.}$$

- We assume a hierarchical scenario, e.g., $M_1 \ll M_2 \ll M_3$.
- The decays of N_1 matter for the creation of lepton asymmetry.
- The standard expression for CP asymmetry parameter due to the decay of N_i is given by

$$\epsilon_i^\alpha = \frac{1}{4\pi v^2 h_{ii}} \sum_{j \neq i} \left\{ \mathrm{Im}[h_{ij} (m_D)_{i\alpha} (m_D^*)_{j\alpha}] g(x_{ij}) + \frac{\mathrm{Im}[h_{ji} (m_D)_{i\alpha} (m_D^*)_{j\alpha}]}{1 - x_{ij}} \right\}$$

where $h \equiv m_D m_D^\dagger,\ m_D = v\lambda/\sqrt{2},\ x_{ij} = M_j^2/M_i^2$.

Leptogenesis in three temperature regimes

- I) $T \sim M_1 > 10^{12}$ **GeV** In this case, all the flavors are indistinguishable, and the total CP asymmetry is a sum over individual flavors $\varepsilon_i = \sum_\alpha \varepsilon_i^\alpha$.

  In our model, $\varepsilon_i = 0 \Rightarrow$ unflavoured leptogenesis does not occur.

- II) $T \sim M_1 < 10^9$ **GeV** when all the flavors (e, μ, τ) are in equilibrium and distinguishable.

  The final baryon asymmetry in this regime is approximated with

$$Y_B \simeq -\frac{12}{37g^*}\left[\varepsilon_i^{(e)}\eta\left(\frac{151}{179}\tilde{m}_e\right) + \varepsilon_i^{(\mu)}\eta\left(\frac{344}{537}\tilde{m}_\mu\right) + \varepsilon_i^{(\tau)}\eta\left(\frac{344}{537}\tilde{m}_\tau\right)\right] \tag{4}$$

- In our model, $\varepsilon_1^e = 0$. Numerically, the maximum value of $|\varepsilon_1^{\mu,\tau}|$ is found to be $\sim 10^{-8}$. Y_B in the observed range cannot be generated with such a small CP asymmetry parameter.

  Fully flavored leptogenesis is numerically ruled out.

- III) 10^9 **GeV** $< T \sim M_1 < 10^{12}$ **GeV**, when only the τ flavor are in equilibrium: τ*-flavored* leptogenesis. In this regime there are two relevant CP asymmetry parameters; ε_i^τ and $\varepsilon_i^2 = \varepsilon_i^e + \varepsilon_i^\mu$.

- The final baryon asymmetry in this regime is approximated with

$$Y_B \simeq -\frac{12}{37g^*}\left[\varepsilon_i^{(2)}\eta\left(\frac{417}{589}\tilde{m}_2\right) + \varepsilon_i^{(\tau)}\eta\left(\frac{390}{589}\tilde{m}_\tau\right)\right]$$

  where $\tilde{m}_\alpha = \frac{|(m_D)_{1\alpha}|^2}{M_1}$ and $\eta(\tilde{m}_\alpha)$ is the efficiency factor that accounts for the inverse decay and the lepton number violating scattering process.

- It turns out that only this scenario is viable in our model.

Leptogenesis with normal mass ordering (Case-I and II)

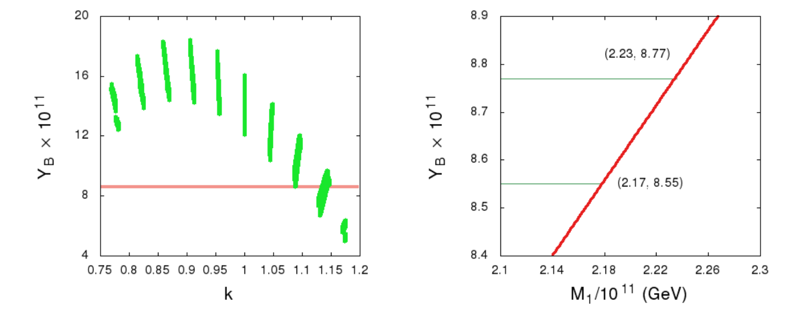

Figure : The plot on the LHS shows a variation of Y_B vs k. The red band in the same plot indicates the observed range of Y_B.

Leptogenesis with inverted mass ordering (Case-I and II)

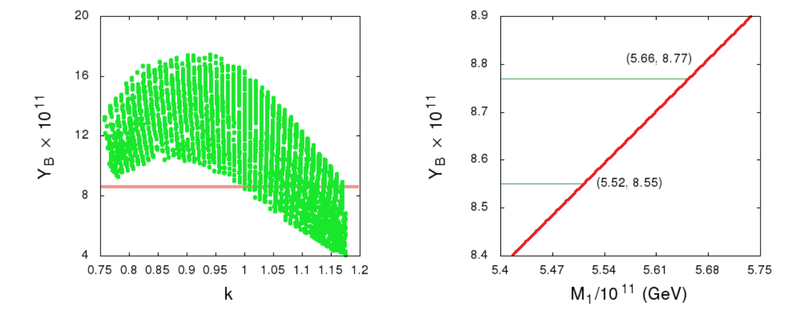

Figure : The plot on the LHS shows a variation of Y_B vs k. The red band in the same plot indicates the observed range of Y_B.

Summary

- Definite analytical relations between θ_{12} and θ_{13}.
- Predict maximal Dirac CP violation ($\cos\delta = 0$), vanishing Majorana phases ($\alpha, \beta = 0$).
- The model offers predictions for $\beta\beta_{0\nu}$ decay matrix element $|M_{ee}|$ and the light neutrino masses $m_{1,2,3}$ in the experiments such as nEXO, LEGEND, GERDA-II, T2K, NOνA, DUNE etc.
- With the assumption that the required CP asymmetry is created by the decay of the lightest (N_1) of the heavy Majorana neutrinos, only τ-flavored leptogenesis is found to be allowed in this model.
- For a normal ordering of light neutrino masses, θ_{23} is found be less than its maximal value, for the final baryon asymmetry Y_B to be in the observed range.
- An upper and a lower bound on the mass of N_1 have also been estimated.

Thank You