[A universal texture zero for flavour]

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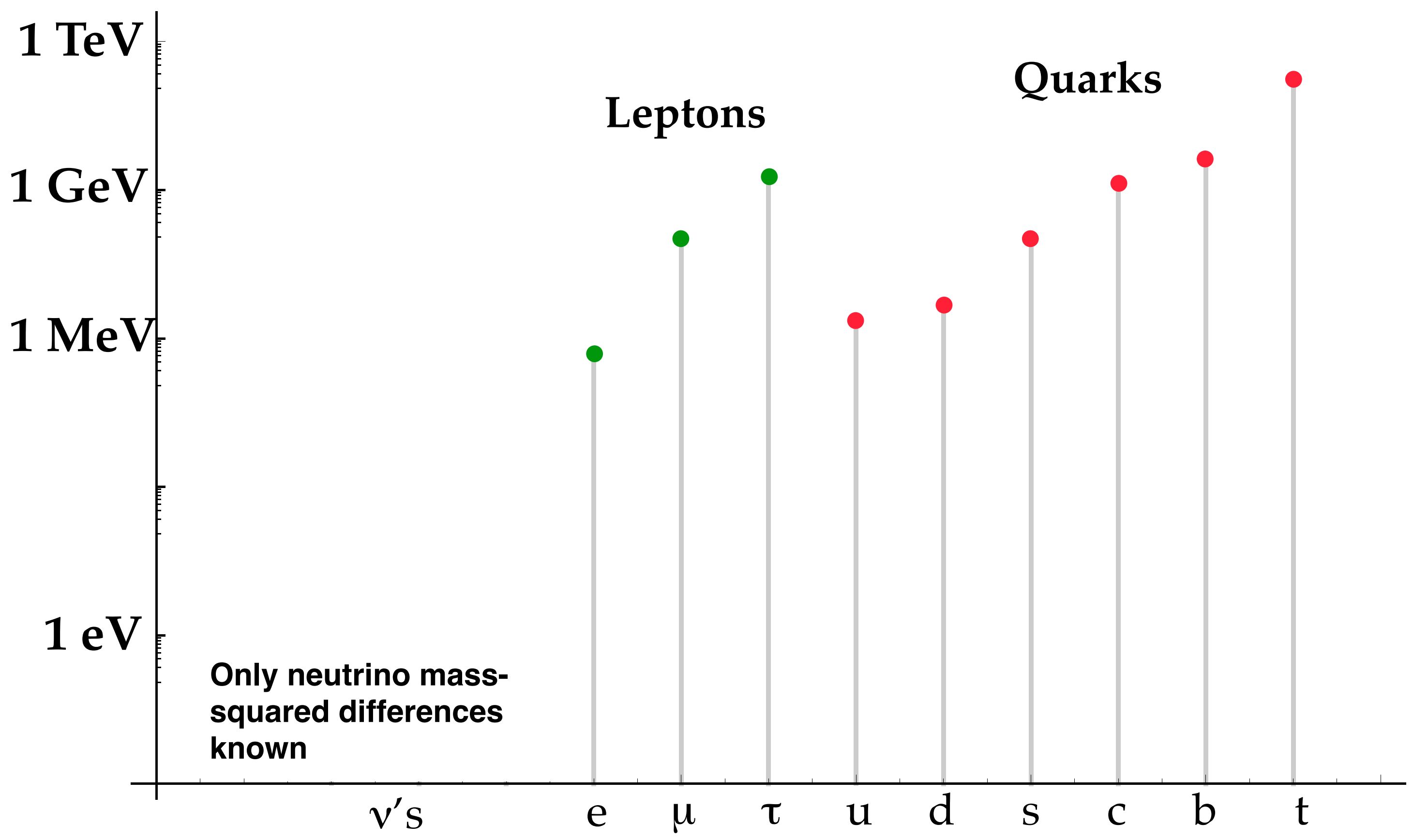
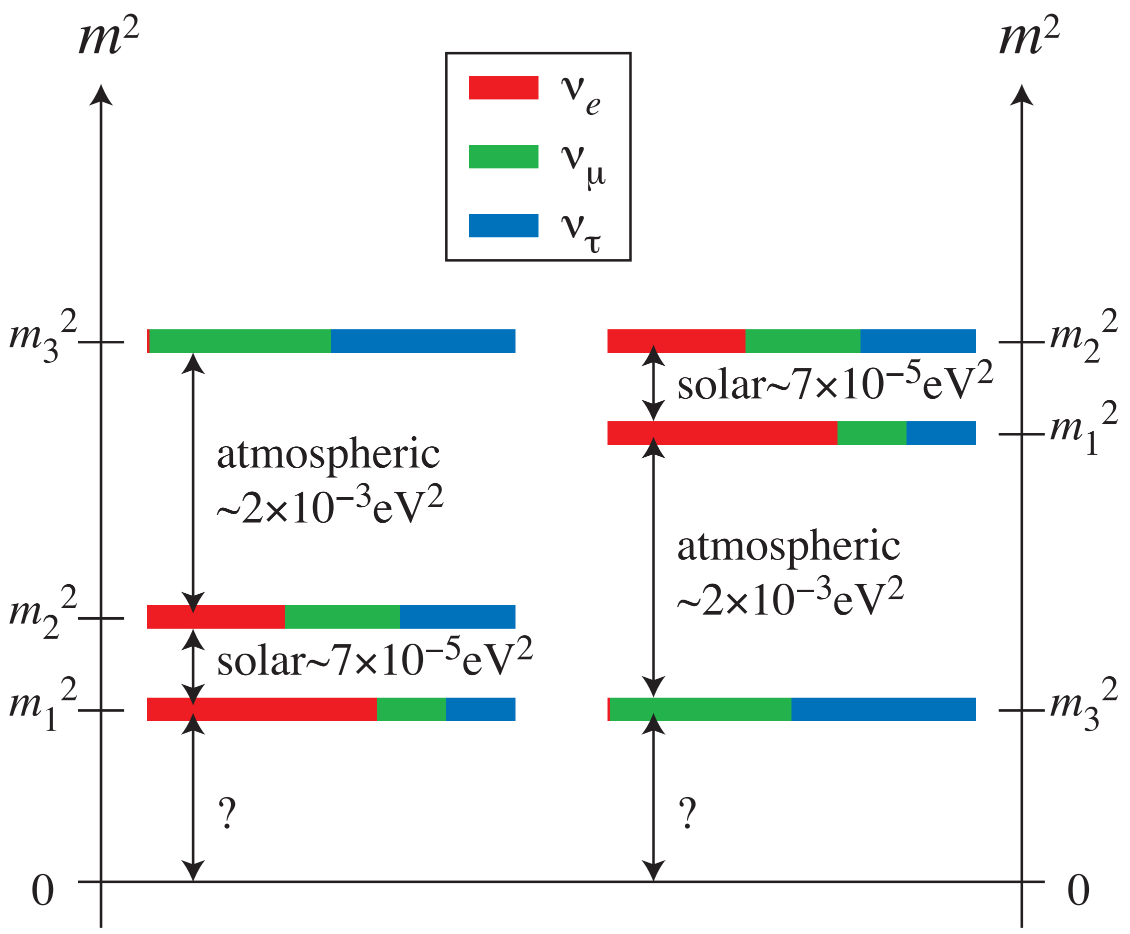
hep-ph/1710.01741

[The Yukawa sector: mass spectrum]





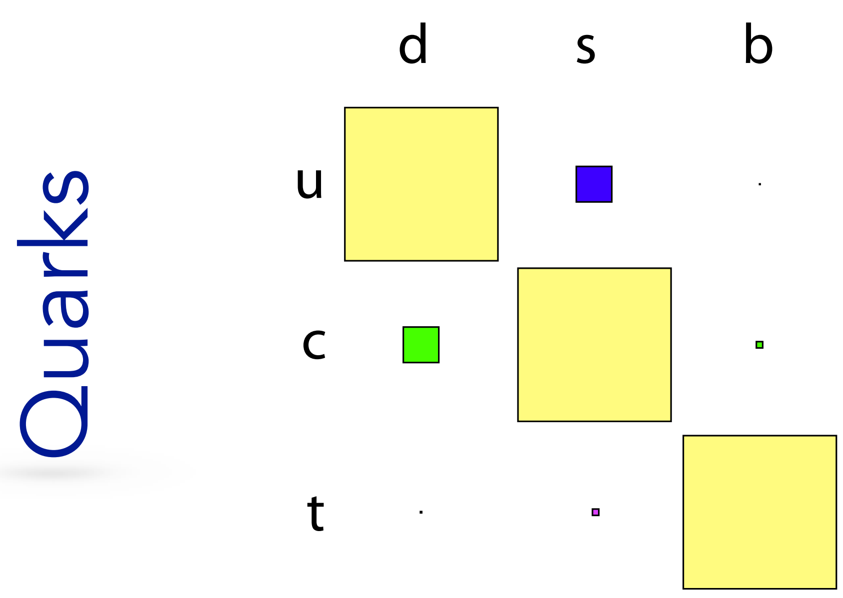
hep-ph/1301.1340

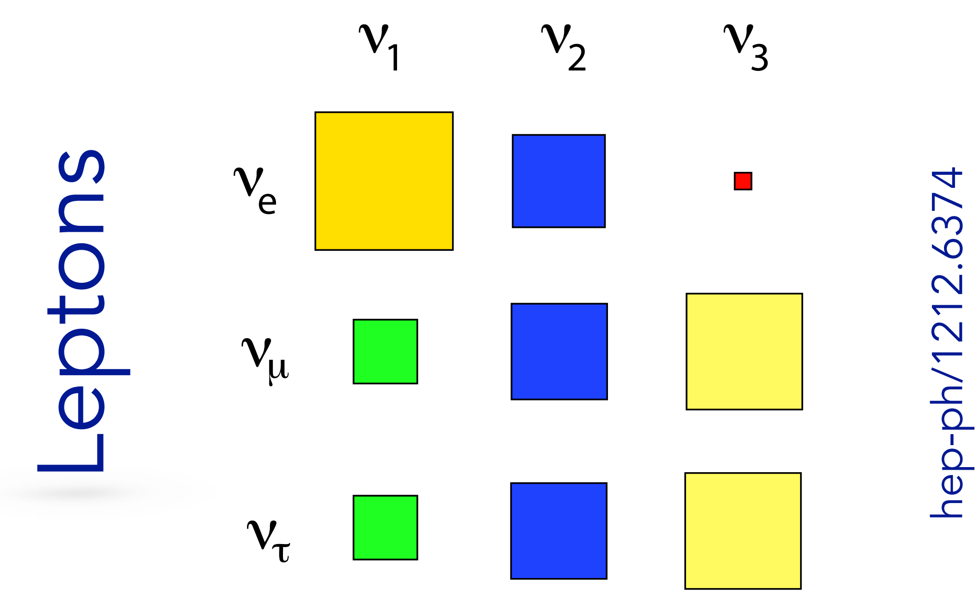
hep-ph/1405.5495

- Quark masses generically hierarchical
- Charged lepton masses generically hierarchical
- Neutrino mass generation not known…high-energy see-saw?
- Absolute neutrino mass not yet known, only mass-squared differences up to a sign

[The Yukawa sector: mixing matrices]

Mixing matrices have 3 mixing angles and a CP violating phase (+2 for Maj. neutrino)





$$|U_{\rm CKM}| = \begin{pmatrix} \begin{pmatrix}0.97441\\0.97413\end{pmatrix} & \begin{pmatrix}0.22597\\0.22475\end{pmatrix} & \begin{pmatrix}0.00370\\0.00340\end{pmatrix} \\ \begin{pmatrix}0.22583\\0.22461\end{pmatrix} & \begin{pmatrix}0.97358\\0.97328\end{pmatrix} & \begin{pmatrix}0.0426\\0.0402\end{pmatrix} \\ \begin{pmatrix}0.00919\\0.00854\end{pmatrix} & \begin{pmatrix}0.0416\\0.0393\end{pmatrix} & \begin{pmatrix}0.99919\\0.99909\end{pmatrix} \end{pmatrix}$$

PDG

$$|U_{\rm PMNS}| \begin{pmatrix} \begin{pmatrix}0.845\\0.791\end{pmatrix} & \begin{pmatrix}0.592\\0.512\end{pmatrix} & \begin{pmatrix}0.172\\0.133\end{pmatrix} \\ \begin{pmatrix}0.521\\0.254\end{pmatrix} & \begin{pmatrix}0.698\\0.455\end{pmatrix} & \begin{pmatrix}0.782\\0.604\end{pmatrix} \\ \begin{pmatrix}0.521\\0.254\end{pmatrix} & \begin{pmatrix}0.698\\0.455\end{pmatrix} & \begin{pmatrix}0.782\\0.604\end{pmatrix} \end{pmatrix}$$

Nu-fit

- CKM mixing small and hierarchical.
- PMNS mixing large and varied. Special patterns emerging? E.g.TBM, GR, BM…
- CP violation is large in the quark sector, and still unknown in the leptonic sector!

[Symmetries as solutions]

- **9** charged fermion masses + **3** active neutrino masses
- **6** mixing angles and **2 - 4** CP violating phases

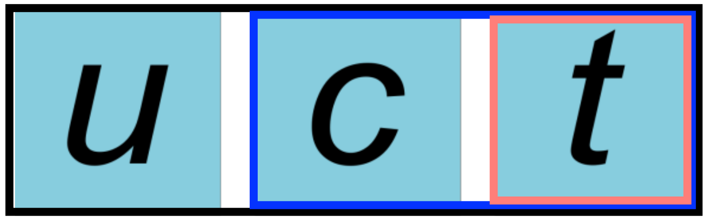
20-22 free and unexplained parameters exist in the SM Yukawa sector

- Can we address this by appending the SM with a new symmetry?

$$BSM\ theory \quad \sim \quad \mathcal{G}_{BSM} \times SM$$

- Such a symmetry would presumedly relate fermions in a given family— i.e. a `horizontal' or `family' or `flavour' symmetry

$$\mathcal{R}\left(\mathcal{G}_{BSM}\right) \sim 3, \bar{3}, 2, 1, \ldots$$



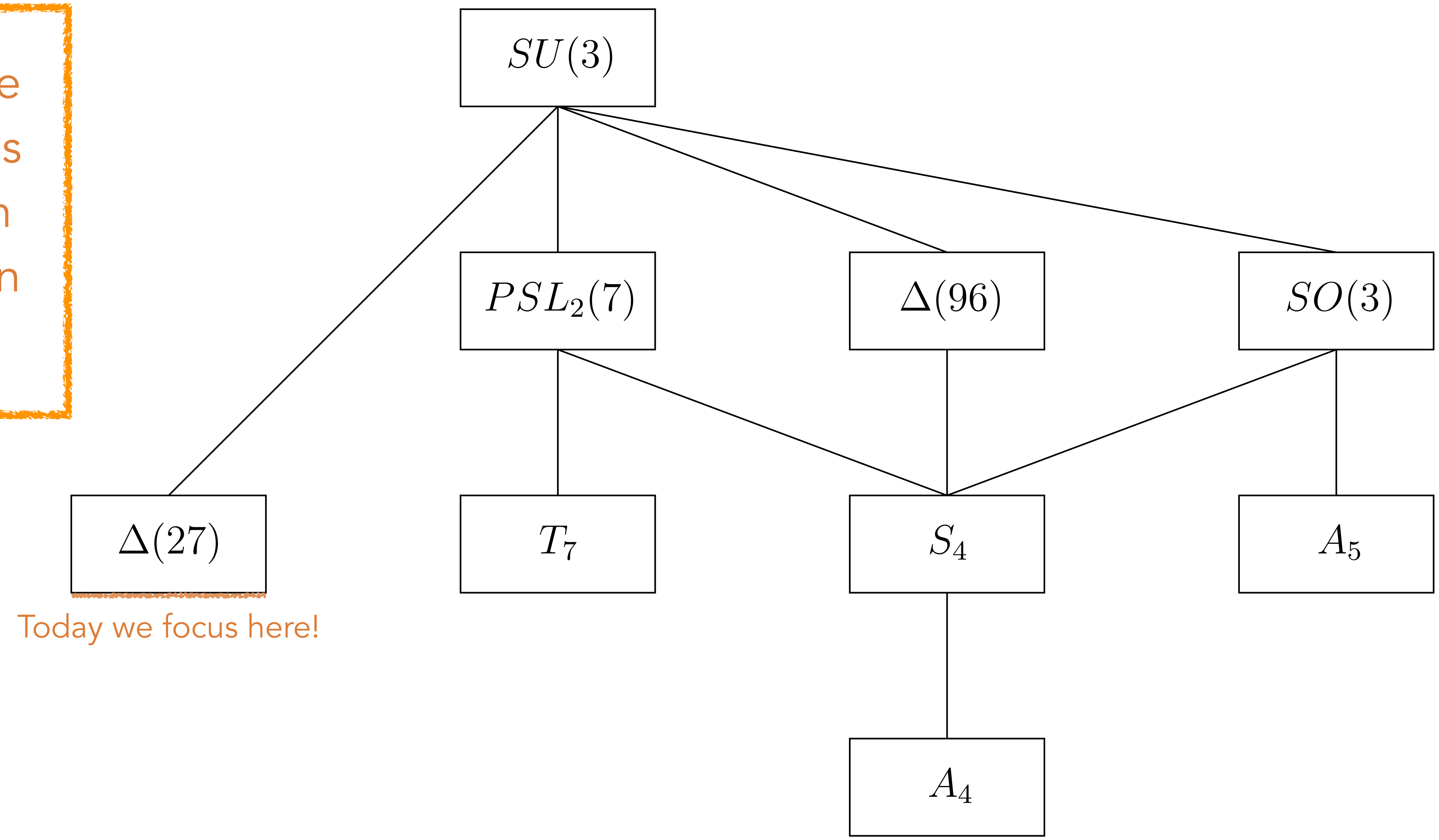
- New dynamical scalar sector to realize its breaking patterns?

- Also, what are the mathematical properties of the required symmetry?

[The discrete approach]

Reviews: King, Luhn: hep-ph/1301.1340, Grimus, Ludl: hep-ph/1110.6376, Altarelli, Feruglio: hep-ph/1002.0211
Encyclopedia: Ishimori et al.: hep-ph/1003.3552

All of these symmetries have been explored in models…



- $U(1)_{FN}$ symmetries difficult to reconcile with large neutrino mixing -> non-Abelian groups
- Discrete symmetries avoid Goldstone modes that could spoil phenomenology, easily embedded in SUSY GUTs, extra dimensional theories — naturally pumped out of orbifold compactifications!
- Easier facilitation of vacuum alignment than with continuous symmetries
- Huge literature: Pakvasa, Sugawara (1977) use S3 for Cabibbo angle. Resurgence(?) in early 90s (Kaplan, Schmaltz; Frampton, Kephart), TBM and GUT models established early-mid 00s (Ma, Rajasekaran; Altarelli, Feruglio, de M. Varzielas, King, Ross +), new flood in 2012/13 after non-zero reactor angle…also see talk by Kumar Rey

[SUSY and discrete flavour models]

Discrete flavour models generally do not rely on SUSY, but it (can) make life simpler:

- Holomorphicity constrains the form of the Yukawa sector (superpotential).
- SUSY helps to align flavoured vacua via "F-Type" or "D-Type" mechanisms:

$$-F^*_{X_i} = \frac{W_{\text{flavon}}}{X_i} = 0 \qquad V_{\text{flavon}} = \sum_i \left|\frac{W_{\text{flavon}}}{X_i}\right|^2 + \left|\frac{W_{\text{flavon}}}{\phi_i}\right|^2 + m^2_{X_i}|X_i|^2 + m^2_{\phi_i}|\phi_i|^2 + \cdots$$

or

$$V = -m^2 \sum_i \phi^{i\dagger}\phi^i + \lambda \left(\sum_i \phi^{i\dagger}\phi^i\right)^2 + \Delta V \qquad \Delta V = \kappa \sum_i \phi^{i\dagger}\phi^i\phi^{i\dagger}\phi^i$$

$$\Delta V = \kappa(\phi^1\phi^2\phi^3 + H.c.)$$

hep-ph/1301.1340

We utilize this type of mechanism!

- In the current study we also find that we need SUSY (or some other BSM spectrum) to obtain successful predictions in the UV, where our model holds (radiative corrections to mass and mixing)
- Also, interesting phenomenology depending on relative SUSY and flavour breaking scales (see hep-ph/1607.06827 and hep-ph/1710.02593 for recent thoughts…)

Most models assume a very high scale for flavour, often relating it to GUTs. This also typically avoids having to deal with other phenomenological effects…

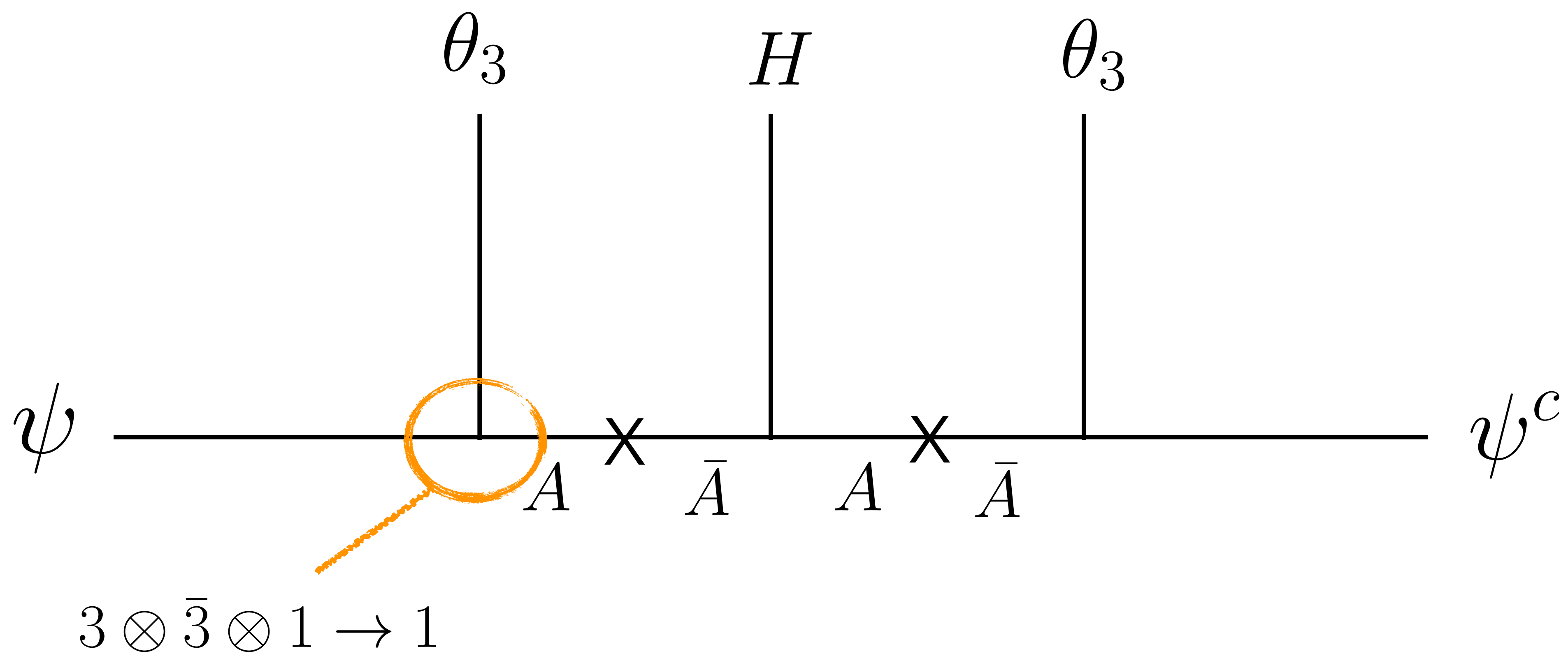
however…

$$\Lambda_F \leq \Lambda_{SUSY}$$

tree level flavour violating effects possible -> interesting hints for flavour from SUSY signals…

[Effective operators]

- In the IR, we can build effective mass matrices with higher dimensional operators:



- In the UV, each vertex is part of the full Lagrangian (messengers A integrated out):

$$\mathcal{L}_{UV} \sim \psi\, \theta_3\, A + \bar{A}\, H\, A + \ldots \qquad \mathcal{L}_{IR} \sim \psi\, \theta_3\, H\, \theta_3\, \psi^c$$

- Hence by assigning the messengers to trivial singlets, one can form family symmetry invariants:

$$\mathcal{L} \sim \psi_i\, \theta_3^i\, \theta_3^j\, \psi_j^c\, H \sim \mathbf{1}$$

[Mass matrices from flavons]

- Flavons acquire vacuum expectation values along specific directions in flavour space:

$$\langle \theta_3 \rangle = v_3 \cdot (0, 0, 1)$$

Alignment is generally non-trivial to achieve, but facilitated with the NADS

- Mass matrices then follow from the form of the effective operator:

$$\mathcal{L}_Y \left(\psi, \psi^c, H, \theta_i\right) \Leftrightarrow \mathcal{M}\left(\psi, \psi^c, \langle H, \theta_i \rangle\right)$$

$$\mathcal{L} \sim \psi_i \, \theta_3^i \, \theta_3^j \, \psi_j^c \, H \quad \Rightarrow \quad \mathcal{M} \propto v_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Family symmetries shape the Yukawa sector, align VEVs, and thereby control fermionic mass and mixing matrices

- Let's use phenomenologically successful mass patterns to guide the construction of our model…

See e.g Roberts, Romanino, Ross, Velasco-Sevilla: hep-ph/0104088

[A universal texture zero (UTZ) for fermions]

- A (1,1) texture zero can accurately reproduce the phenomenology of the charged fermions.

$$M_a^D \approx m_3 \begin{pmatrix} 0 & \varepsilon_a^3 & \varepsilon_a^3 \\ \varepsilon_a^3 & r_a\varepsilon_a^2 & r_a\varepsilon_a^2 \\ \varepsilon_a^3 & r_a\varepsilon_a^3 & 1 \end{pmatrix}, \quad r_{u,d} = 1, \quad r_l = -3$$

$$\epsilon_u \simeq 0.05, \qquad \epsilon_{d,l} \simeq 0.15$$

- It implements the well known **Georgi-Jarlskog (PLB 86 1979)** mass relation and also the successful **Gatto-Sartori-Tonin (PLB 28 1968)** relation:

$$\sin\theta_c = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}} \right|$$

$$m_b \approx 3\, m_\tau$$

$$m_s \approx 3 \times \frac{1}{3} m_\mu$$

$$m_d \approx 3 \times 3\, m_e$$

Red indicates RGE

IDEA: Can this successful texture be extended to the neutrino sector?

- If so, can we realize this phenomenology in a concrete model that also addresses the leptons?

See de Medeiros Varzielas, Ross, JT: hep-ph/1710.01741

[The UTZ model]

$$\mathcal{G}_F = \Delta(27) \times \mathbb{Z}_N$$

$$SO(10) \longrightarrow SU(4) \times SU(2)_L \times SU(2)_R \longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$$

Fields	$\psi_{q,e,\nu}$	$\psi^c_{q,e,\nu}$	H_5	Σ	S	θ_3	θ_{23}	θ_{123}	θ	θ_X
$\Delta(27)$	3	3	1_{00}	1_{00}	1_{00}	$\bar{3}$	$\bar{3}$	$\bar{3}$	$\bar{3}$	3
Z_N	0	0	0	2	-1	0	-1	2	0	x

$$\mathcal{L}^D_a = \psi_i \left(\frac{1}{M^2_{3,a}} \theta^i_3 \theta^j_3 + \frac{1}{M^3_{23,a}} \theta^i_{23} \theta^j_{23} \Sigma + \frac{1}{M^3_{123,a}} \left(\theta^i_{123} \theta^j_{23} + \theta^i_{23} \theta^j_{123} \right) S \right) \psi^c_j H_5$$

$a \in \{u, d, l, \nu\}$

Messenger masses distinguish fermion species

VEV provides contact with GUTs —- implements Georgi-Jarlskog

Needed for shaping symmetry

$$\mathcal{L}^M_\nu = \psi^c_i \left(\frac{1}{M} \theta^i \theta^j + \frac{1}{M^4} \left[\alpha \theta^i_{23} \theta^j_{23} \left(\theta^a \theta^a \theta^a_{123} \right) + \beta \left(\theta^i_{23} \theta^j_{123} + \theta^i_{123} \theta^j_{23} \right) \left(\theta^a \theta^a \theta^a_{23} \right) \right] \right) \psi^c_j$$

Additional flavon allows for LNV

- A Type-I See-saw generates active light neutrinos:

$$M^D_\nu \cdot M^{M,-1}_\nu \cdot M^{D,T}_\nu \Longrightarrow M_\nu$$

[Discrete anomaly freedom]

Krauss, Wilczek : PRL 62 (1989)
Ibanez, Ross : PLB 260 (1991)
Banks, Dine : PRD 45 (1992)
Araki et al. : NPB 805 (2008)

- A longstanding argument of Krauss and Wilczek suggests that discrete symmetries must be gauged in the UV. This means anomaly cancellation must be enforced.

$$D-G-G, \quad D-g-g, \quad Z-G-G, \quad Z-g-g$$

$Z/D-G-G:$
$$\sum_{\mathbf{r}^{(f)},\mathbf{d}^{(f)}} tr\left[\tau(\mathbf{d}^{(f)})\right]\cdot l(\mathbf{r}^{(f)}) \stackrel{!}{=} 0 \bmod \frac{N}{2}$$
$$tr\left[\tau(\mathbf{d}^{(f)})\right] = N\frac{\ln\det U(\mathbf{d}^{(f)})}{2\pi i}$$

$D-g-g:$
$$\sum_{\mathbf{d}^{(f)}} tr\left[\tau(\mathbf{d}^{(f)})\right] \stackrel{!}{=} 0 \bmod \frac{N}{2}$$

$Z-g-g:$
$$\sum_f q^{(f)} = \sum_m q^{(m)}\cdot \dim \mathbf{R}^{(m)} \stackrel{!}{=} 0 \bmod \frac{N}{2}$$

Fields	$\psi_{q,e,\nu}$	$\psi^c_{q,e,\nu}$	H_5	Σ	S	θ_3	θ_{23}	θ_{123}	θ	θ_X
$\Delta(27)$	3	3	1_{00}	1_{00}	1_{00}	$\bar{3}$	$\bar{3}$	$\bar{3}$	$\bar{3}$	3
Z_N	0	0	0	2	-1	0	-1	2	0	x

$\Delta(3N^2)$	$\mathbf{1}_{\mathbf{k},\mathbf{l}}$	$\mathbf{3}_{[\mathbf{k}][\mathbf{l}]}$
$\det(b)$	ω^k	1
$\det(a)$	ω^l	1
$\det(a')$	ω^l	1

- **D-G-G and D-g-g anomalies trivially satisfied**
- **Z-G-G and Z-g-g only involve massive state contributions**

[UTZ mass matrices]

- Generic vacuum alignment vectors depend on complex phases:

$$\langle\theta_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \mathrm{v}_3, \quad \langle\theta_{123}\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\beta} \\ e^{i\alpha} \\ -1 \end{pmatrix} \mathrm{v}_{123}, \quad \langle\theta_{23}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ e^{i\alpha} \\ 1 \end{pmatrix} \mathrm{v}_{23}$$

- The UTZ Lagrangian then generates Dirac matrices of the following form:

$$\mathcal{M}_i^D \equiv \frac{M_i^D}{c} \simeq \begin{pmatrix} 0 & a\, e^{i(\alpha+\beta+\gamma)} & a\, e^{i(\beta+\gamma)} \\ a\, e^{i(\alpha+\beta+\gamma)} & (b\, e^{-i\gamma} + 2a\, e^{-i\delta})\, e^{i(2\alpha+\gamma+\delta)} & b\, e^{i(\alpha+\delta)} \\ a\, e^{i(\beta+\gamma)} & b\, e^{i(\alpha+\delta)} & 1 - 2a\, e^{i\gamma} + b\, e^{i\delta} \end{pmatrix}$$

- Heavy Majorana singlet mass matrix is of the same form, and active light neutrinos generated with a Type-I See-saw.

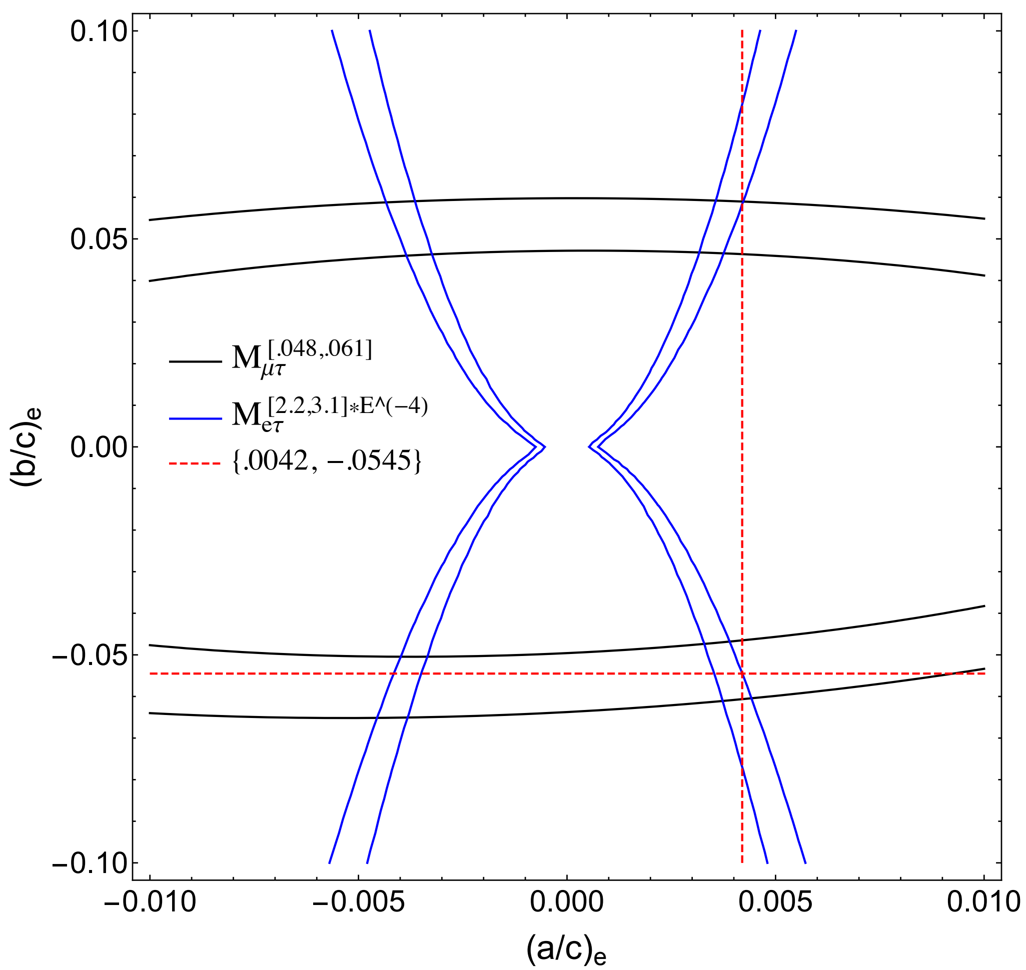
$$\mathcal{M}_i^D := \mathcal{M}_i^D\,(a_i, b_i, \gamma_i, \delta_i)$$
$$\mathcal{M}_{\nu_R}^M := \mathcal{M}_{\nu_R}^M\,(x, y, \rho, \phi) \underbrace{\Longrightarrow}_{SS} \mathcal{M}_\nu^M\,(a_\nu, b_\nu, \gamma_\nu, \delta_\nu, x, y, \rho, \phi)$$

9 low energy parameters successfully describe 18 observables!

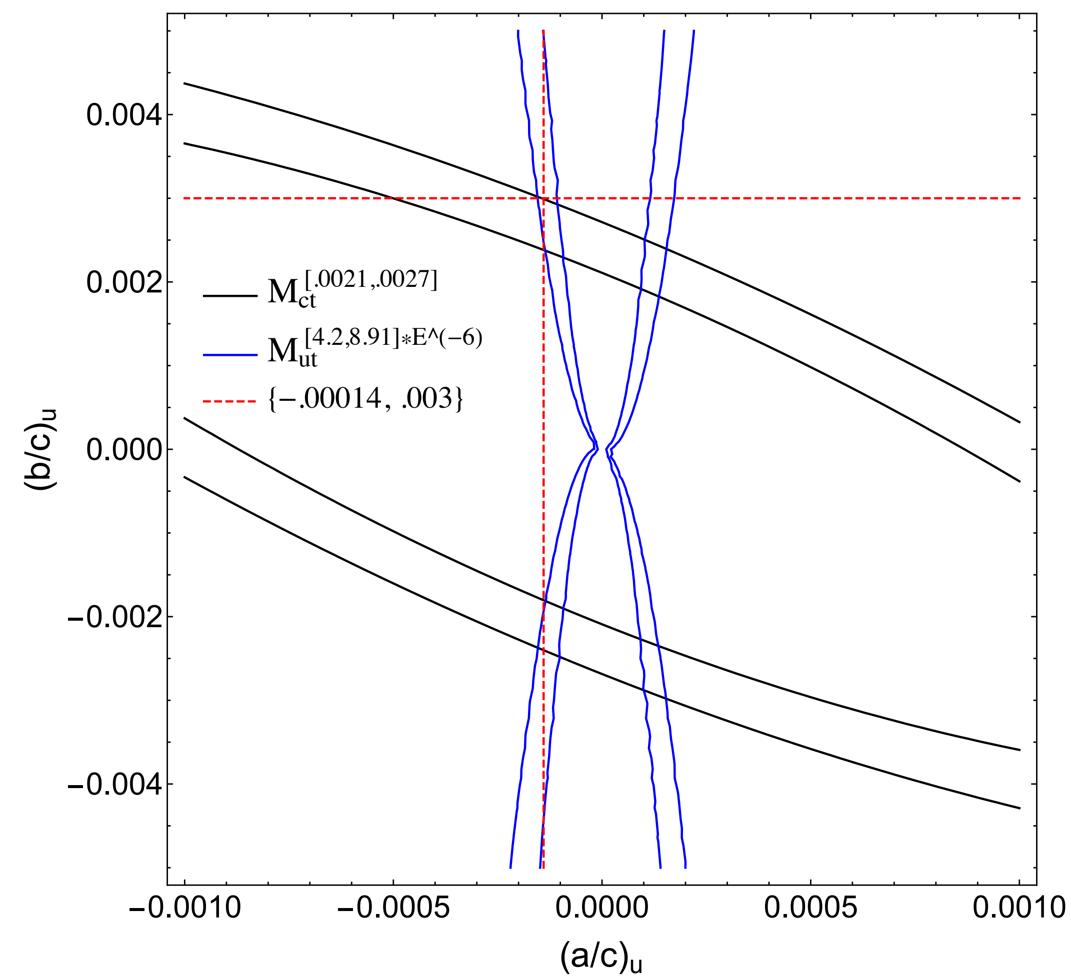
[A fit to masses]

Serna, Ross : PLB 664 (2008)
Antusch, Kersten, Lindner, Ratz : NPB 674 (2003)

Charged Leptons



Up Quarks



$(\mu = M_X)$	m_e/m_τ	m_μ/m_τ	m_u/m_t	m_c/m_t	m_d/m_b	m_s/m_b	$\Delta m^2_{sol}/\Delta m^2_{atm}$
Max	.00031	.061	8.91×10^{-6}	.0027	.0012	.021	.0336
Min	.00022	.048	4.2×10^{-6}	.0021	.00035	.008	.021
L.O.	.00031	.055	7.16×10^{-6}	.0027	.00090	.020	.0213
H.O.	.00026	.049	7.89×10^{-6}	.0025	.0010	.020	.0213

Data radiatively corrected to the GUT scale with an MSSM spectrum

[A fit to mixing]

Serna, Ross : PLB 664 (2008)
Antusch, Kersten, Lindner, Ratz : NPB 674 (2003)

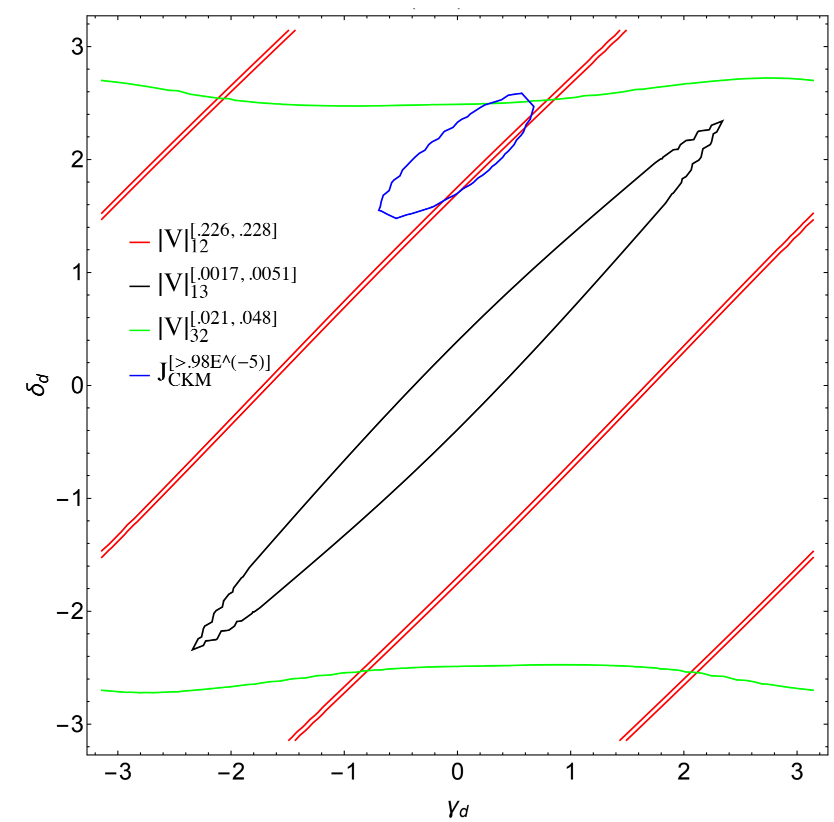
$$|V_{CKM}|^{LO} = \begin{pmatrix} .974 & .226 & .00420 \\ .226 & .974 & .0191 \\ .00248 & .0194 & .9998 \end{pmatrix}, \quad \mathcal{J}_{CKM}^{LO} = 9.898 \times 10^{-6}$$

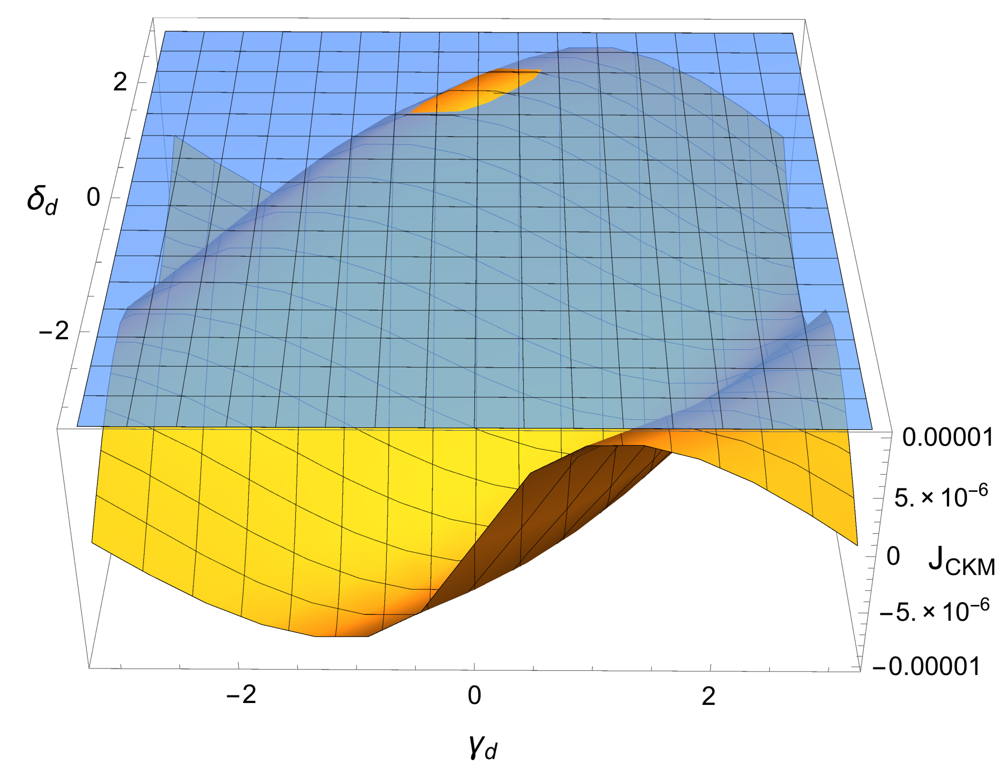
$$|V_{PMNS}|^{LO} = \begin{pmatrix} .823 & .547 & .152 \\ .400 & .499 & .769 \\ .404 & .672 & .621 \end{pmatrix}, \quad \mathcal{J}_{PMNS}^{LO} = -.0304$$

- Minor discrepancies in exterior off diagonal CKM elements @ LO
- Include **two additional parameters** from HO operators in OPE:

$$|V_{CKM}|^{HO} = \begin{pmatrix} .974 & .226 & .00307 \\ .226 & .974 & .0313 \\ .00574 & .0309 & .9995 \end{pmatrix}, \quad \mathcal{J}_{CKM}^{HO} = 1.665 \times 10^{-5}$$

$$|V_{PMNS}|^{HO} = \begin{pmatrix} .830 & .536 & .153 \\ .405 & .534 & .742 \\ .384 & .654 & .652 \end{pmatrix}, \quad \mathcal{J}_{PMNS}^{HO} = -.0311$$





This represents excellent agreement in the UV!

[Status of discrete flavour symmetries?]

Bad

- Symmetry landscape underdetermined: multiple symmetries can predict the same mixing mixing patterns, and the same symmetry can predict multiple patterns.
- Shaping symmetries still required to constrain the form of Lagrangians (Yukawa and alignment)
- Outside of `direct' models, making concrete predictions from the UV is difficult without additional input — guideposts from RGE, SUSY, anomaly constraints, higher dimensions?

Good

- NADS are well-motivated by data and can be easily incorporated into UV theories.
- They are also naturally pumped out of stringy compactifications.
- They are more powerful than conventional local symmetries at aligning flavoured vacua.
- We have shown that they can economically model both quarks and leptons, no easy task.

A UTZ successfully describes both quarks and leptons and is economically implemented with discrete symmetries!

[Thanks!]

- See Scalars 2017 for a complementary talk from I. de Medeiros Varzielas…

[Backup: vacuum alignment]

- We want to achieve the 3, 123, and 23 alignments.

$$\langle \theta_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \mathrm{v}_3, \quad \langle \theta_{123} \rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\beta} \\ e^{i\alpha} \\ -1 \end{pmatrix} \mathrm{v}_{123}, \quad \langle \theta_{23} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ e^{i\alpha} \\ 1 \end{pmatrix} \mathrm{v}_{23}, \quad \left\langle \theta_X^\dagger \right\rangle = \frac{1}{\sqrt{6}} \begin{pmatrix} 2e^{i\beta} \\ -e^{i\alpha} \\ 1 \end{pmatrix} \mathrm{v}_X$$

$$V_1(\theta_i) = m_i^2 |\theta_i|^2 \qquad V_2(\theta_i) = h_i (\theta_i)^2 \left(\theta^{\dagger i}\right)^2.$$

SU(3) invariant - insufficient to align as desired

Allowed by discrete symmetry!

$$V_3 = k_1 \theta_{X,i} \theta_{123}^{\dagger i} \theta_{123,j} \theta_X^{\dagger j}, \quad k_1 > 0.$$

Orthogonal to 123, but does not distinguish between (0,1,-1) and (2,-1,-1)

$$V_4 = k_2 m_0 \theta_X^1 \theta_X^2 \theta_X^3 \qquad V_5 = k_3 \theta_{23,i} \theta_X^i \theta_{23}^{\dagger j} \theta_X^{\dagger j} + k_4 \theta_{23,i} \theta_3^{\dagger i} \theta_{3,i} \theta_{23}^{\dagger i}, \quad \text{with } k_3 > 0 \text{ and } k_4 < 0$$

Achieves (0,1,1) alignment!

$$V = \sum_{i=3,123} \left(V_1(\theta_i) + V_2(\theta_i) \right) + V_3 + V_4 + V_5$$

- Alignment of LNV family discussed in paper…

[Backup: charged sector RGE]

Serna, Ross : PLB 664 (2008)

Parameters	Input SUSY Parameters					
$\tan\beta$	1.3	10	38	50	38	38
γ_b	0	0	0	0	-0.22	$+0.22$
γ_d	0	0	0	0	-0.21	$+0.21$
γ_t	0	0	0	0	0	-0.44
Parameters	Corresponding GUT-Scale Parameters with Propagated Uncertainty					
$y^t(M_X)$	6^{+1}_{-5}	0.48(2)	0.49(2)	0.51(3)	0.51(2)	0.51(2)
$y^b(M_X)$	$0.0113^{+0.0002}_{-0.01}$	0.051(2)	0.23(1)	0.37(2)	0.34(3)	0.34(3)
$y^\tau(M_X)$	0.0114(3)	0.070(3)	0.32(2)	0.51(4)	0.34(2)	0.34(2)
$(m_u/m_c)(M_X)$	0.0027(6)	0.0027(6)	0.0027(6)	0.0027(6)	0.0026(6)	0.0026(6)
$(m_d/m_s)(M_X)$	0.051(7)	0.051(7)	0.051(7)	0.051(7)	0.051(7)	0.051(7)
$(m_e/m_\mu)(M_X)$	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)
$(m_c/m_t)(M_X)$	$0.0009^{+0.001}_{-0.00006}$	0.0025(2)	0.0024(2)	0.0023(2)	0.0023(2)	0.0023(2)
$(m_s/m_b)(M_X)$	0.014(4)	0.019(2)	0.017(2)	0.016(2)	0.018(2)	0.010(2)
$(m_\mu/m_\tau)(M_X)$	0.059(2)	0.059(2)	0.054(2)	0.050(2)	0.054(2)	0.054(2)
$A(M_X)$	$0.56^{+0.34}_{-0.01}$	0.77(2)	0.75(2)	0.72(2)	0.73(3)	0.46(3)
$\lambda(M_X)$	0.227(1)	0.227(1)	0.227(1)	0.227(1)	0.227(1)	0.227(1)
$\bar{\rho}(M_X)$	0.22(6)	0.22(6)	0.22(6)	0.22(6)	0.22(6)	0.22(6)
$\bar{\eta}(M_X)$	0.33(4)	0.33(4)	0.33(4)	0.33(4)	0.33(4)	0.33(4)
$J(M_X)\times 10^{-5}$	$1.4^{+2.2}_{-0.2}$	2.6(4)	2.5(4)	2.3(4)	2.3(4)	1.0(2)
Parameters	Comparison with GUT Mass Ratios					
$(m_b/m_\tau)(M_X)$	$1.00^{+0.04}_{-0.4}$	0.73(3)	0.73(3)	0.73(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	$0.70^{+0.8}_{-0.05}$	0.69(8)	0.69(8)	0.69(8)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.82(7)	0.83(7)	0.83(7)	0.83(7)	1.05(8)	0.68(6)
$(\frac{\det Y^d}{\det Y^e})(M_X)$	$0.57^{+0.08}_{-0.26}$	0.42(7)	0.42(7)	0.42(7)	0.92(14)	0.39(7)

Table 2: The mass parameters continued to the GUT-scale M_X for various values of $\tan\beta$ and threshold corrections $\gamma_{t,b,d}$. These are calculated with the 2-loop gauge coupling and 2-loop Yukawa coupling RG equations assuming an effective SUSY scale $M_S = 500$ GeV.

[Backup: neutrino RGE]

Antusch, Kersten, Lindner, Ratz : NPB 674 (2003)

Generic enhancement and suppression factors

	$\dot{\theta}_{12}$	$\dot{\theta}_{13}$	$\dot{\theta}_{23}$	$\dot{\delta}$	$\dot{\varphi}_i$
n.h.	1	$\sqrt{\zeta}$	1	$\sqrt{\zeta}\,\theta_{13}^{-1}$	$\sqrt{\zeta}$
p.d.(n.)	$\frac{m_1^2}{\Delta m^2_{\text{sol}}}$	$\frac{m_1}{\sqrt{\Delta m^2_{\text{atm}}}}$	1	$\frac{m_1}{\sqrt{\Delta m^2_{\text{atm}}}}\theta_{13}^{-1} + \frac{m_1^2}{\Delta m^2_{\text{sol}}}$	$\frac{m_1^2}{\Delta m^2_{\text{sol}}}$
i.h.	ζ^{-1}	$\mathscr{O}(\theta_{13})$	1	ζ^{-1}	ζ^{-1}
p.d.(i.)	ζ^{-1}	$\frac{m_3}{\sqrt{\Delta m^2_{\text{atm}}}}$	1	$\frac{m_3}{\sqrt{\Delta m^2_{\text{atm}}}}\theta_{13}^{-1} + \zeta^{-1}$	ζ^{-1}
d.	$\frac{m^2}{\Delta m^2_{\text{sol}}}$	$\frac{m^2}{\Delta m^2_{\text{atm}}}$	$\frac{m^2}{\Delta m^2_{\text{atm}}}$	$\frac{m^2}{\Delta m^2_{\text{atm}}}\theta_{13}^{-1} + \frac{m^2}{\Delta m^2_{\text{sol}}}$	$\frac{m^2}{\Delta m^2_{\text{sol}}}$

$$\zeta := \frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}$$

- Degenerate neutrino masses, large ratios of SUSY higgs VEVs, and special convolutions of phases can drive large neutrino mixing.

$$\Delta_{\text{RG}} \sim 10^{-5}\left(1+\tan^2\beta\right)\Gamma_{\text{enh}}$$

- Further, we predict a severe hierarchy, and therefore expect minimal running of mixing angles and phases…

- On the other hand, neutrino masses are expected to run. For normal tan \beta, generations scale uniformly:

$$m_i(t) \approx \exp\left[\frac{1}{16\pi^2}\int_{t_0}^{t} \mathrm{d}\tau\,\alpha(\tau)\right] m_i(t_0) =: s(t,t_0)\,m_i(t_0)$$

Can be 1.1 - 1.2 over many decades of evolution

- For large tan \beta, splitting amongst generations expected —>