Quark mixing in an S_3-symmetric 2HDM

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Outline

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Discrete symmetries in particle physics

- In SM, fermions (both leptons and quarks) come in three generations
- There are inter-generational differences, in contrast to their uniformity in gauge interactions
- Two types of hierarchies in the flavor sector:
  - Large hierarchy within the charged fermion sector and enormous hierarchy between charged fermion and neutrino masses
  - Mixing information in quark and lepton sector
- Finite discrete symmetry groups (e.g., S_4, D_4, A_4 etc.) provide an effective way of explaining some of these flavor issues
- We will consider S_3 symmetry

Basics of S_3

- S_3 is the permutation group of three objects
- The order of S_3 is $3! = 6$
- The six elements correspond to the following transformations

$$
\begin{aligned}
e &: (x_1, x_2, x_3) \to (x_1, x_2, x_3), \\
a_1 &: (x_1, x_2, x_3) \to (x_2, x_1, x_3), \\
a_2 &: (x_1, x_2, x_3) \to (x_3, x_2, x_1), \\
a_3 &: (x_1, x_2, x_3) \to (x_1, x_3, x_2), \\
a_4 &: (x_1, x_2, x_3) \to (x_3, x_1, x_2), \\
a_5 &: (x_1, x_2, x_3) \to (x_2, x_3, x_1).
\end{aligned}
$$

- S_3 can also be thought of as the symmetry of an equilateral triangle with a_1 and $a_1 a_2$ being the reflection and the $2\pi/3$ rotation respectively

contd.

- S_3 has three irreducible representation $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{2}$
- We pick a basis such that the generators in the $\mathbf{2}$ representation are given by

$$a = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}, \qquad b = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}.$$

- In this basis the quark fields transform under S_3 as:

$$\begin{aligned} \mathbf{2} &: \quad \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}, \begin{bmatrix} u_{1R} \\ u_{2R} \end{bmatrix}, \begin{bmatrix} d_{1R} \\ d_{2R} \end{bmatrix}, \\ \mathbf{1} &: \quad Q_3,\ u_{3R},\ d_{3R}, \end{aligned}$$

☆ Note that the square brackets denote the doublet representation of S_3, and has nothing to do with the representation of the enclosed fields under $\mathrm{SU}(2)$

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Scalar potential

- The scalar sector consists of two $\mathrm{SU}(2)$ doublets ϕ_i $(i = 1, 2)$, and their transformation under the S_3 symmetry is as follows:

$$\mathbf{2} \quad : \quad \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \equiv \Phi$$

- Each doublet can be represented as

$$\phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + i\zeta_i) \end{pmatrix}$$

- The scalar potential:

$$V(\Phi) = V_2(\Phi) + V_4(\Phi)\,,$$
$$V_2(\Phi) = \mu_1^2(\phi_1^\dagger\phi_1) + \mu_2^2(\phi_2^\dagger\phi_2) - \left(\mu_{12}^2\phi_1^\dagger\phi_2 + \text{h.c.}\right)\,,$$
$$V_4(\Phi) = \lambda_1(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2)^2 + \lambda_2(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1)^2 + \lambda_3\left\{(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)^2 + (\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2)^2\right\}.$$

Physical scalars

- Charged scalar:

$$\begin{pmatrix} w^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}, \quad \text{with} \quad M_{H^\pm}^2 = \frac{2\mu_{12}^2}{\sin 2\beta} - 2\lambda_3 v^2$$

where $\beta = \tan^{-1}(v_2/v_1)$

- Pseudoscalar:

$$\begin{pmatrix} z \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix}, \quad \text{with} \quad M_A^2 = \frac{2\mu_{12}^2}{\sin 2\beta} - 2(\lambda_2 + \lambda_3)v^2$$

- Other neutral scalar:

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad \text{with}$$

$$m_H^2 = \frac{2\mu_{12}^2}{\sin 2\beta}, \quad m_h^2 = 2(\lambda_1 + \lambda_3)v^2$$

contd.

- Natural emergence of *alignment limit*
- h can be identified with the 125 GeV SM Higgs with all couplings to be SM-like
- A few comment about the quadratic part of the potential
  - If $\mu_1^2 = \mu_2^2$ and $\mu_{12}^2 = 0$, $V_2(\Phi)$ is completely S_3-symmetric $\Rightarrow$ after EWSB this results in a massless scalar (*Not desirable*)
  - If $\mu_1^2 \neq \mu_2^2$ and $\mu_{12}^2 = 0$, the potential is not S_3 symmetric, but still have a massless boson (*Not desirable*)
  - If $\mu_1^2 = \mu_2^2$ and $\mu_{12}^2 \neq 0$, $\nexists$ massless scalar, but $\tan\beta = 1$ or $v_1 = v_2$ because $\exists$ symmetry $\phi_1 \leftrightarrow \phi_2$ (*Not good [see later]*)
  - If $\mu_1^2 \neq \mu_2^2$ and $\mu_{12}^2 \neq 0$, $\nexists$ massless scalar and also $\tan\beta$ can be arbitrary (*Useful case*)

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Quark Yukawas

The most general S_3 symmetric Yukawa couplings for u-type quarks

$$\mathscr{L}_Y^{(u)} = -A_u\left(\bar{Q}_1\tilde{\phi}_1 + \bar{Q}_2\tilde{\phi}_2\right)u_{3R} - B_u\left\{\left(\bar{Q}_1\tilde{\phi}_2 + \bar{Q}_2\tilde{\phi}_1\right)u_{1R} + \left(\bar{Q}_1\tilde{\phi}_1 - \bar{Q}_2\tilde{\phi}_2\right)u_{2R}\right\} - C_u\bar{Q}_3\left(\tilde{\phi}_1 u_{1R} + \tilde{\phi}_2 u_{2R}\right) + \text{h.c.}$$

where $\tilde{\phi}_i = i\sigma_2\phi_i^*$

- For d-type quarks replace: $u_{AR} \to d_{AR}$, $\{A, B, C\}_u \to \{A, B, C\}_d$, and $\tilde{\phi}_i \to \phi_i$

Mass matrix after EWSB

$$\mathcal{M}_q = \frac{v}{\sqrt{2}}\begin{pmatrix} B_q \sin\beta & B_q\cos\beta & A_q\cos\beta \\ B_q\cos\beta & -B_q\sin\beta & A_q\sin\beta \\ C_q\cos\beta & C_q\sin\beta & 0 \end{pmatrix}, \quad (q = u, d)$$

Diagonalizing the mass matrix

- For the up-sector one can find two unitary matrices U_u and V_u such that $U_u \mathcal{M}_u V_u^\dagger$ is diagonal
- The CKM matrix is then given by $U_u U_d^\dagger$
- The matrices U_u and U_d are the unitary matrices which diagonalize, through similarity transformations, the hermitian matrices $\mathcal{M}_u \mathcal{M}_u^\dagger$ and $\mathcal{M}_d \mathcal{M}_d^\dagger$ respectively

$$\mathcal{M}_q \mathcal{M}_q^\dagger = \frac{1}{2} v^2 \begin{pmatrix} a_q^2 \cos^2\beta + b_q^2 & \frac{1}{2} a_q^2 \sin 2\beta & B_q C_q^* \sin 2\beta \\ \frac{1}{2} a_q^2 \sin 2\beta & a_q^2 \sin^2\beta + b_q^2 & B_q C_q^* \cos 2\beta \\ B_q^* C_q \sin 2\beta & B_q^* C_q \cos 2\beta & c_q^2 \end{pmatrix}$$

where $a_q = |A_q|$ etc.

- the three eigenvalues of $\mathcal{M}_u \mathcal{M}_u^\dagger$ would be the mass squared of the three up sector quarks, namely m_u^2, m_c^2 and m_t^2 etc.

contd.

The characteristic equation has the following form:

$$x^3 - (a^2 + 2b^2 + c^2)x^2 + (a^2 + b^2)(b^2 + c^2)x - a^2 b^2 c^2 \sin^2 3\beta = 0$$

where $x = (2m^2/v^2)$

Observations:

- If the x independent term vanish one eigenvalue will be vanishing
- Then the mass eigenvalues will be $\{0,\ \frac{1}{2}v^2(b^2 + c^2),\ \frac{1}{2}v^2(a^2 + b^2)\}$

Towards diagonalizing $MM^\dagger$, as a first step, diagonalize only the terms proportional to a_q^2; this is done, e.g., by a matrix

$$\mathcal{U} = \begin{pmatrix} 0 & 0 & 1 \\ \sin\beta & -\cos\beta & 0 \\ \cos\beta & \sin\beta & 0 \end{pmatrix}$$

contd.

- Applying a similarity transformation with this matrix on $\mathcal{M}\mathcal{M}^\dagger$

$$M^2 = \mathcal{U}\mathcal{M}\mathcal{M}^\dagger\mathcal{U}^\dagger = \frac{1}{2}v^2 \begin{pmatrix} c^2 & -bc\cos 3\beta & bc\sin 3\beta \\ -bc\cos 3\beta & b^2 & 0 \\ bc\sin 3\beta & 0 & a^2+b^2 \end{pmatrix}$$

- Now near-masslessness of first generation of quarks can be obtained in two ways:

(i) some Yukawa couplings vanish [*bad way!*]

(ii) $\sin 3\beta = 0$ [*good way!*]

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Construction of the CKM matrix

- Zero eigenvalue can be ensured with,

$$\sin 3\beta = 0 \Rightarrow \tan\beta = \sqrt{3} \Rightarrow v_2 = \sqrt{3}v_1 = \frac{\sqrt{3}v}{2}$$

- This value of β also makes the matrix M^2 block-diagonal,

$$M^2 = \mathcal{U}\mathcal{M}\mathcal{M}^\dagger\mathcal{U}^\dagger = \frac{1}{2}v^2 \begin{pmatrix} c^2 & bc & 0 \\ bc & b^2 & 0 \\ 0 & 0 & a^2+b^2 \end{pmatrix}$$

- Third generation has been singled out, and therefore $v\sqrt{(a^2+b^2)/2}$ $\Rightarrow$ mass of the third generation quark
- Since it is heavier than the quarks in the first two generations, we need $a^2 \gg b^2, c^2$ in both up and down sectors

contd.

- Complete diagonalization of M^2 would require a further similarity transformation affecting the upper 2×2 block
- This will involve the values of the Yukawa couplings

$$U_u = \mathcal{O}_u \mathcal{U}\,, \qquad U_d = \mathcal{O}_d \mathcal{U}\,,$$

where

$$\mathcal{O}_q = \begin{pmatrix} \cos\theta_q & -\sin\theta_q & 0 \\ \sin\theta_q & \cos\theta_q & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ with } \tan\theta_q = \frac{c_q}{b_q}$$

contd.

The CKM matrix can be written as

$$\begin{aligned} V_{\mathrm{CKM}} &= U_u U_d^\dagger = \mathcal{O}_u \mathcal{O}_d^\dagger \\ &= \begin{pmatrix} \cos(\theta_u - \theta_d) & -\sin(\theta_u - \theta_d) & 0 \\ \sin(\theta_u - \theta_d) & \cos(\theta_u - \theta_d) & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

- The difference $(\theta_u - \theta_d)$ can be identified with the Cabibbo angle, θ_C
- Smallness of first generation quarks $\Leftrightarrow$ block diagonal CKM

Possible issues

As a leading order effect we connect smallness of first generation quarks and the block diagonal nature of CKM matrix

1. Can the CKM be *exactly* reproduced?
2. FCNC

Exact reproduction of CKM

- Note that the VEV relation $v_2 = \sqrt{3} v_1$ i.e., $\sin 3\beta = 0$ is not protected by any symmetry, so let's perturb it by a small amount such that $\sin 3\beta = \delta$
- After a little algebra and a few reasonable approximations,

$$m_u^2 \approx \frac{1}{4} m_c^2 \delta^2 \sin^2 2\theta_u$$

$$m_d^2 \approx \frac{1}{4} m_s^2 \delta^2 \sin^2 2\theta_d$$

- Since $(\theta_u - \theta_d) = \theta_C$, one can solve for δ and θ_u or θ_d
- Taking all the uncertainties into account we have found $\delta > 0.2$ which is inconsistent with our assumption of small δ
- Thus this minimal framework is not sufficient to reproduce the *exact* observed masses of the first generation quarks ☹

FCNC

- It can be shown that the FCNCs are uniquely determined by θ_u or θ_d
- A trivial but viable solution to the FCNC problem would be to make all the scalars except h sufficiently heavy
- The bounds from the electroweak T-parameter can also be evaded if the non-standard scalars, H, A and $H^{\pm}$ are nearly degenerate

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Summary and Conclusion

- We connect two apparently disjoint experimental observations namely, the tiny masses of first generation of quarks and the near block-diagonal structure of the CKM matrix in a simple set-up of 2HDM with an S_3 symmetry
- These two features of the quark sector can be attributed to a particular value of tan β
- An added bonus of our model is the existence of a light scalar, which can be identified with the 125 GeV Higgs observed at the LHC, in view of a naturally emerging alignment limit
- Admittedly, the exact CKM matrix and correct non-zero masses for the first generation of quarks could not be reproduced in this minimalistic scenario
- Perhaps our set-up can be taken as a constituent towards a more elaborate framework which can address the full quark structure . . .

Thank You