ATLAS Searches for Vh and hh Resonances

$m_{\mathrm{JJ}} = 3023$ GeV

Higgs Candidate:

$m = 102$ GeV

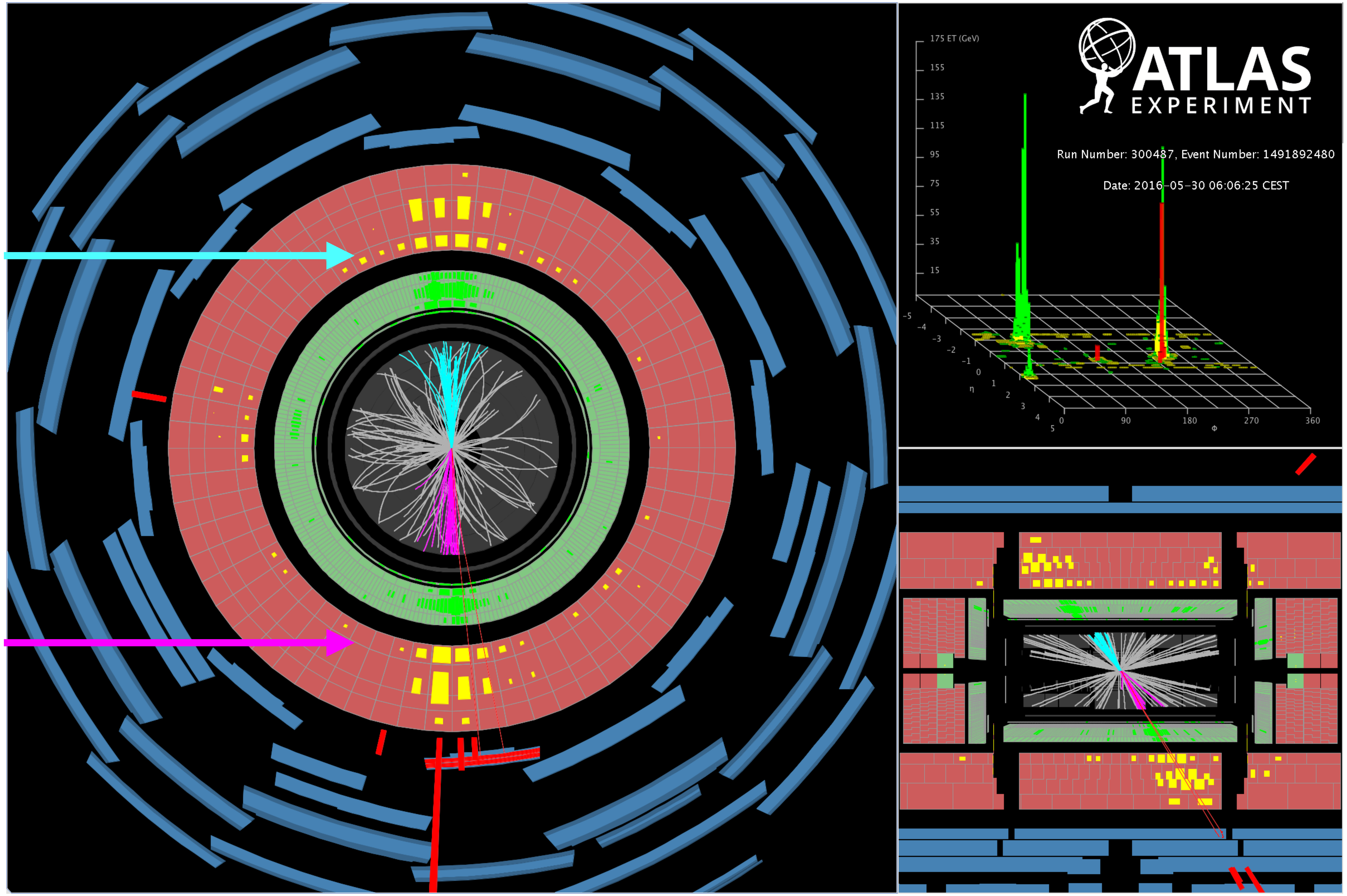
$p_{\mathrm{T}} = 1220$ GeV

b-tags $= 2$

Vector Boson Candidate:

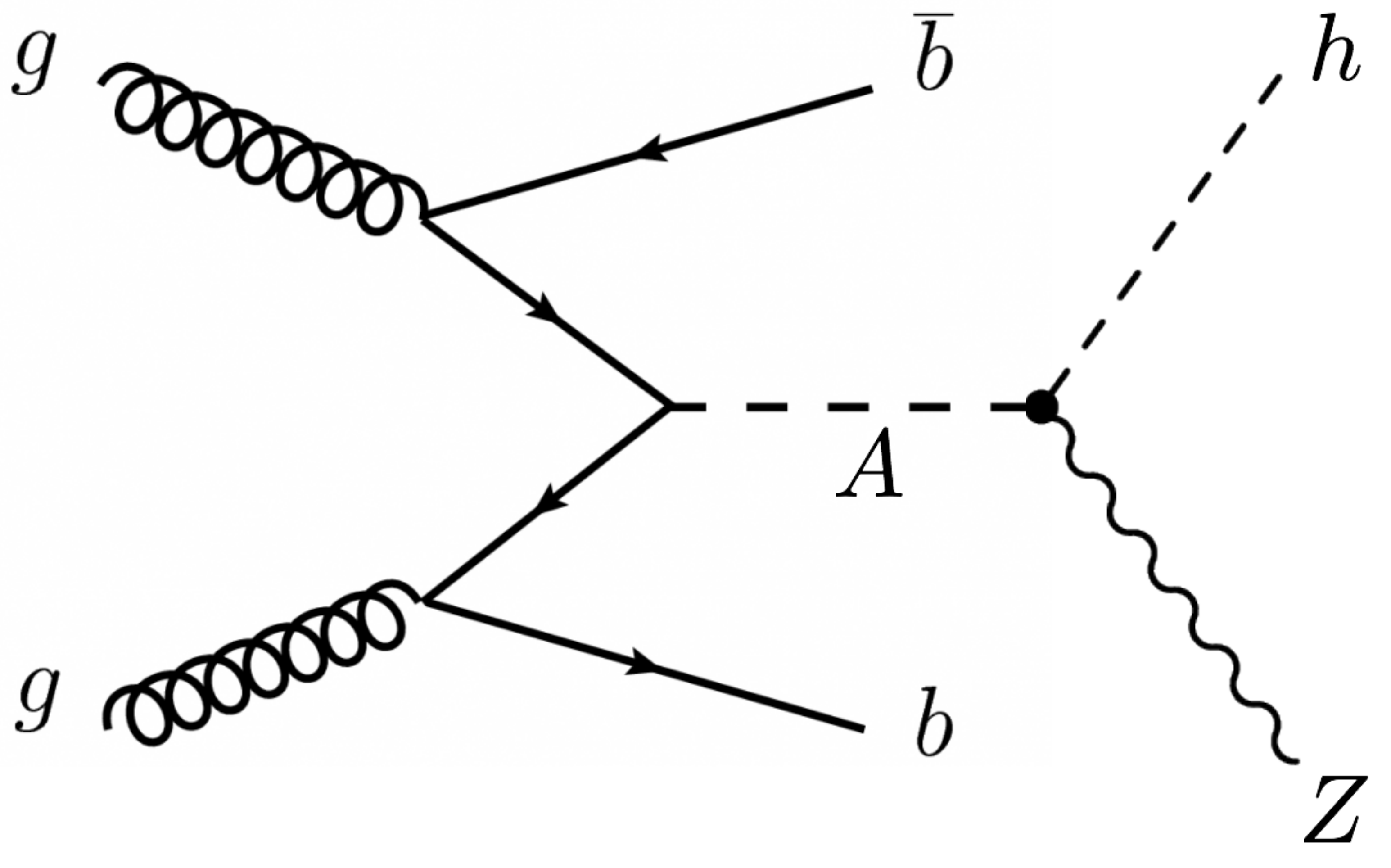
$m = 94$ GeV

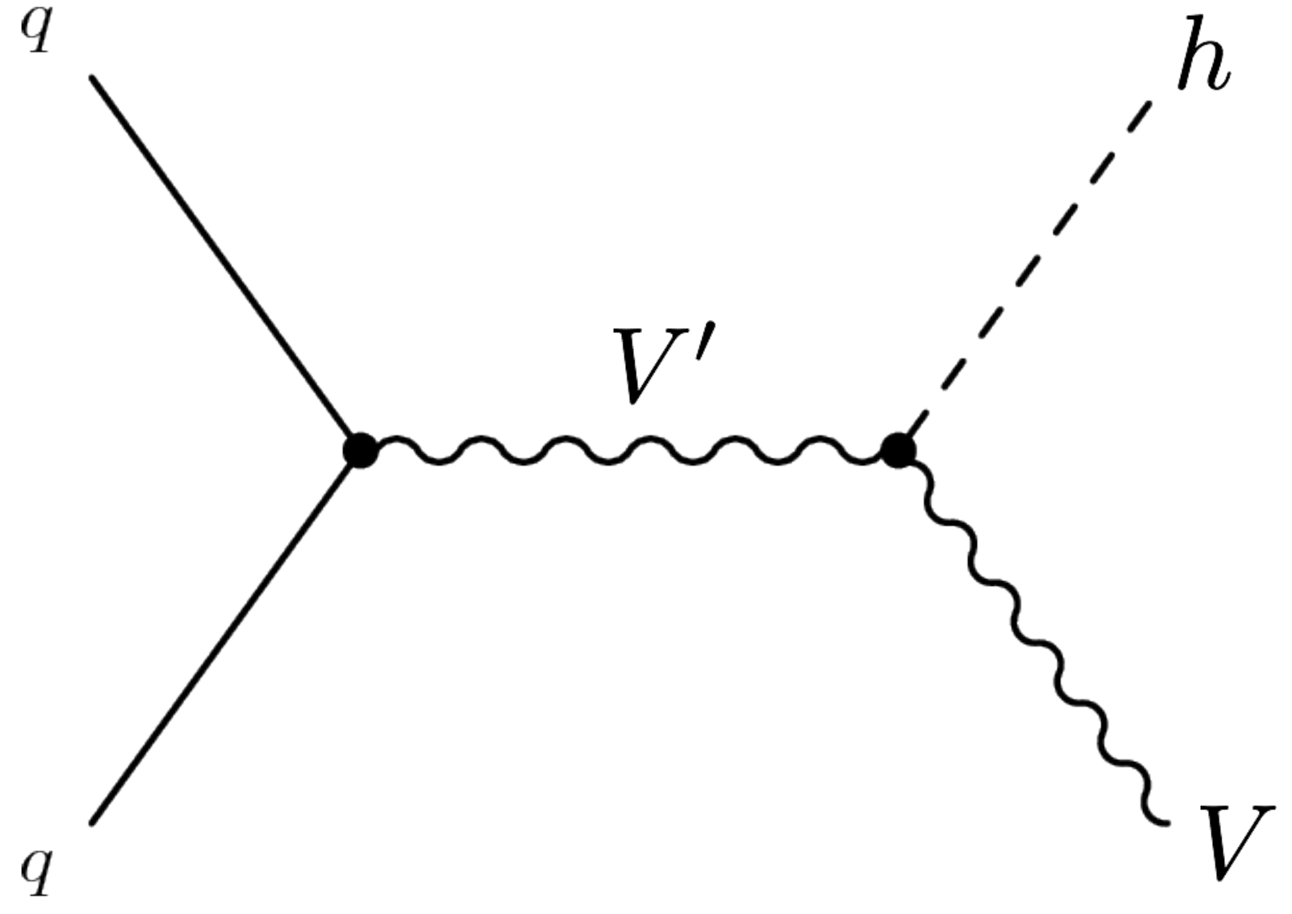
$p_{\mathrm{T}} = 1263$ GeV

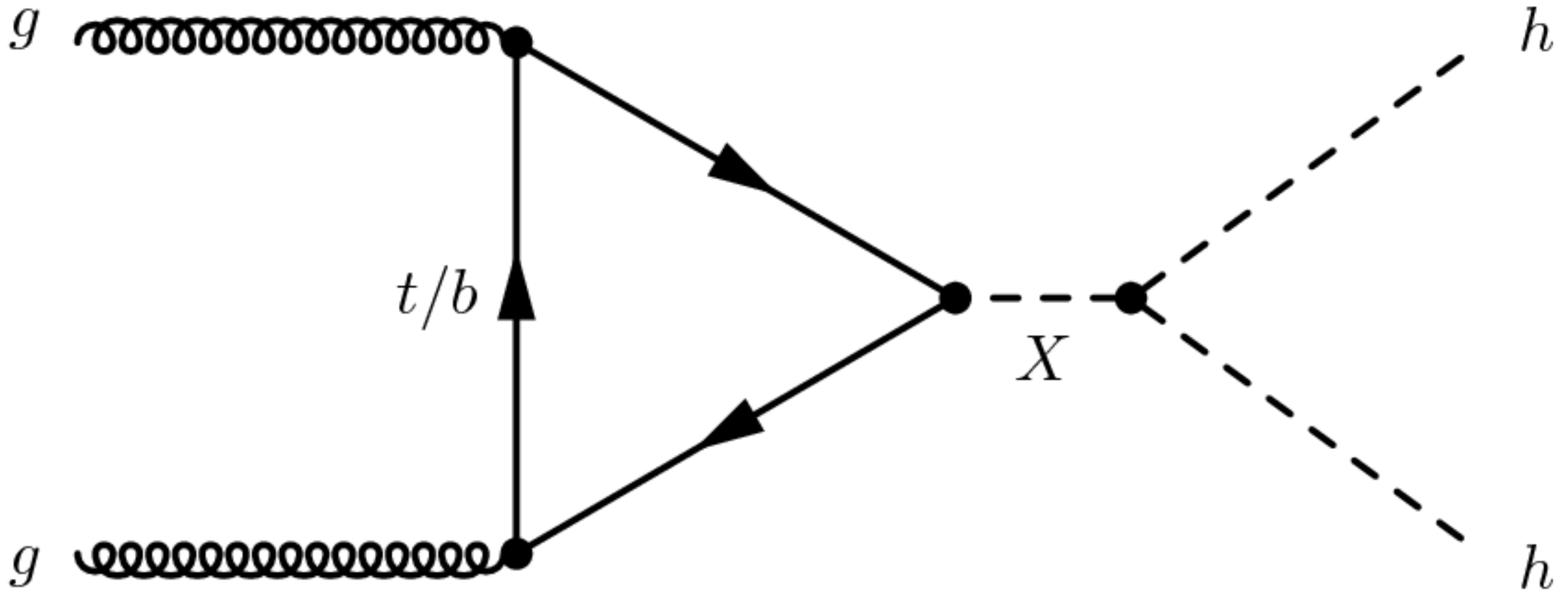


Outline

- Motivation
- Vh ($h \to b\bar{b}$)
  - Leptonic V decays
  - Hadronic V decays
- hh
  - $b\bar{b}\gamma\gamma$
  - $WW^*\gamma\gamma$
  - $b\bar{b}b\bar{b}$
- Conclusion







Motivation

- Higgs is new physics!
  - Coupled to nearly everything in SM, why not BSM degrees of freedom?
    - Extended gauge groups (HVT framework) and/or additional scalars/tensors
- Modifications to scalar sector offer explanations for:
  - Fine tuning of fundamental(?) scalar mass (SUSY, 2HDM, compositeness)
  - Baryogenesis via CP violation in extended scalar sector
  - Dark Matter
- Modifications to GR generally have additional scalar/tensor degrees of freedom:
  - Randall-Sundrum/Kaluza-Klein models with Radions, massive gravitons

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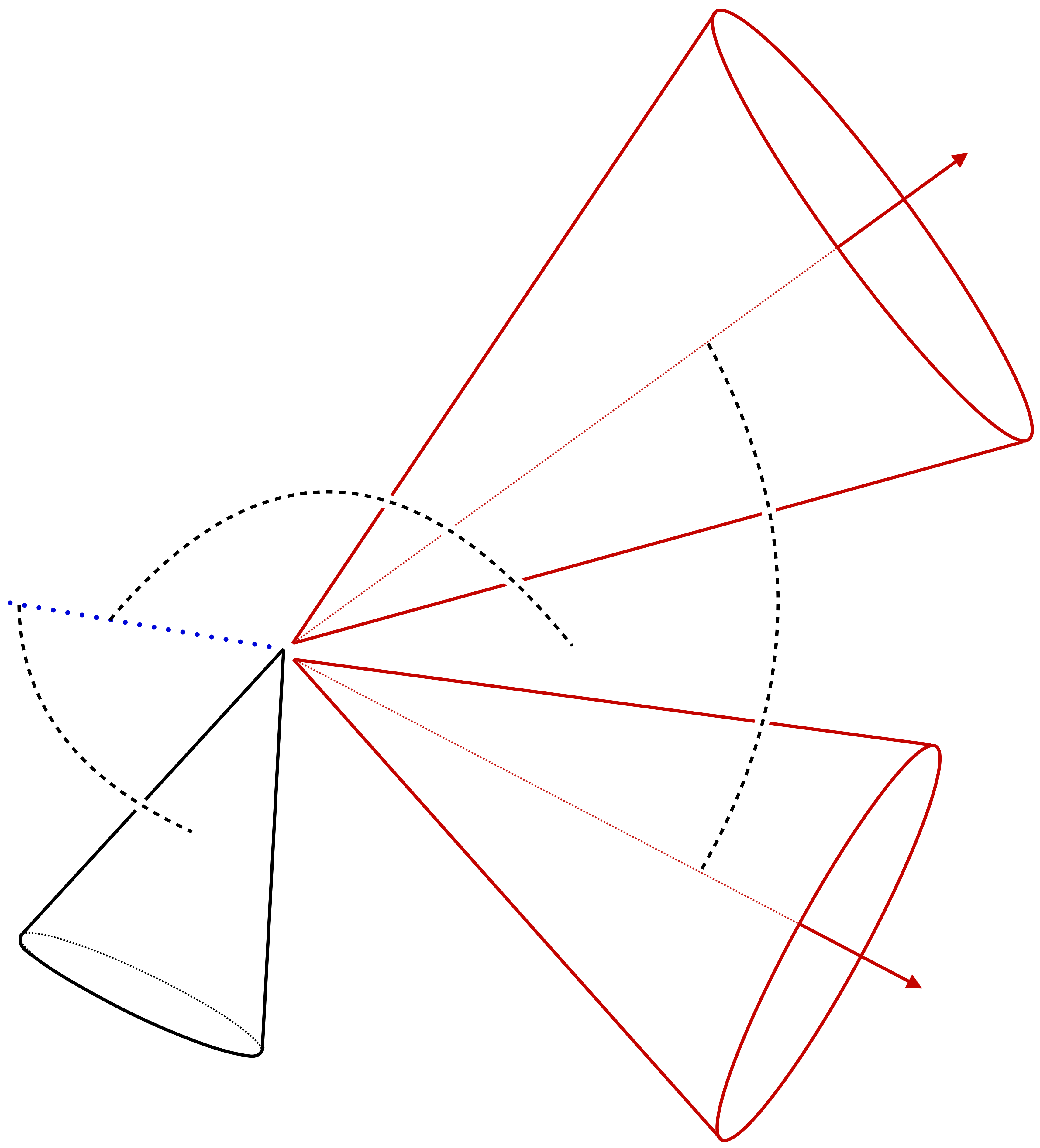
$Zh \to \nu\bar{\nu}b\bar{b}$

$E_{\mathrm{T}}^{\mathrm{miss}} > 150$



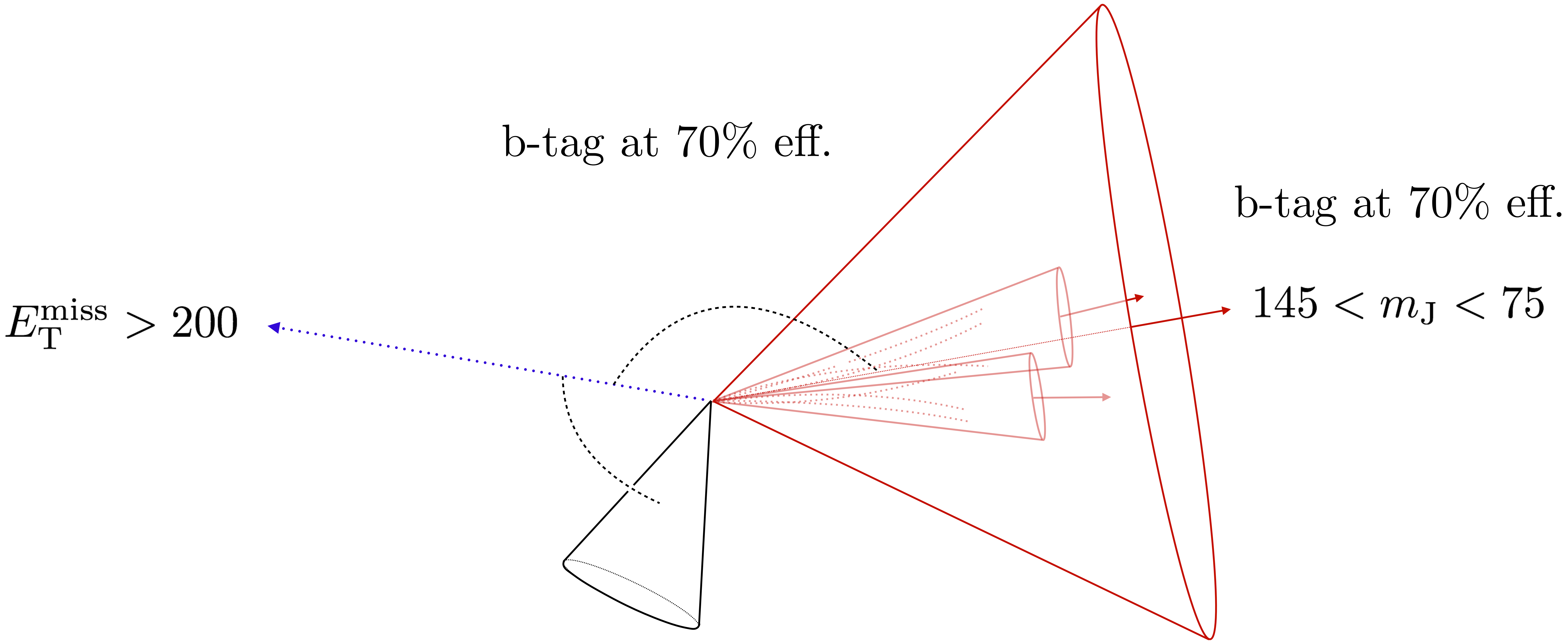
b-tag at 70% eff.

$140 < m_{\mathrm{jj}} < 110$

$$m_{\mathrm{T},Vh} = \sqrt{(E_{\mathrm{T}}^{h} + E_{\mathrm{T}}^{\mathrm{miss}})^2 - (\vec{p}_{\mathrm{T}}^{\,h} + \vec{E}_{\mathrm{T}}^{\mathrm{miss}})^2}$$

details

$Zh \to \nu\bar{\nu}b\bar{b}$



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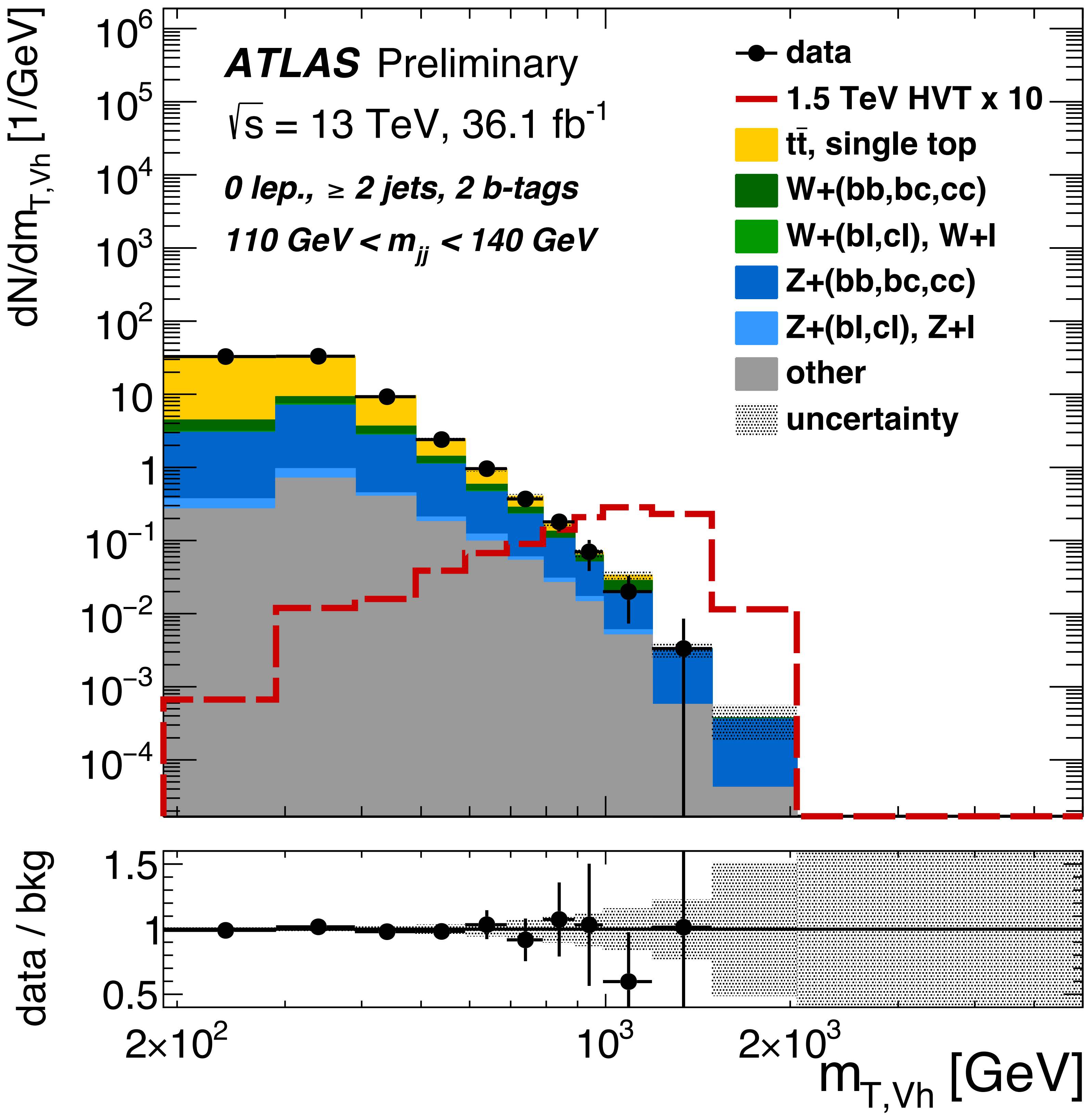
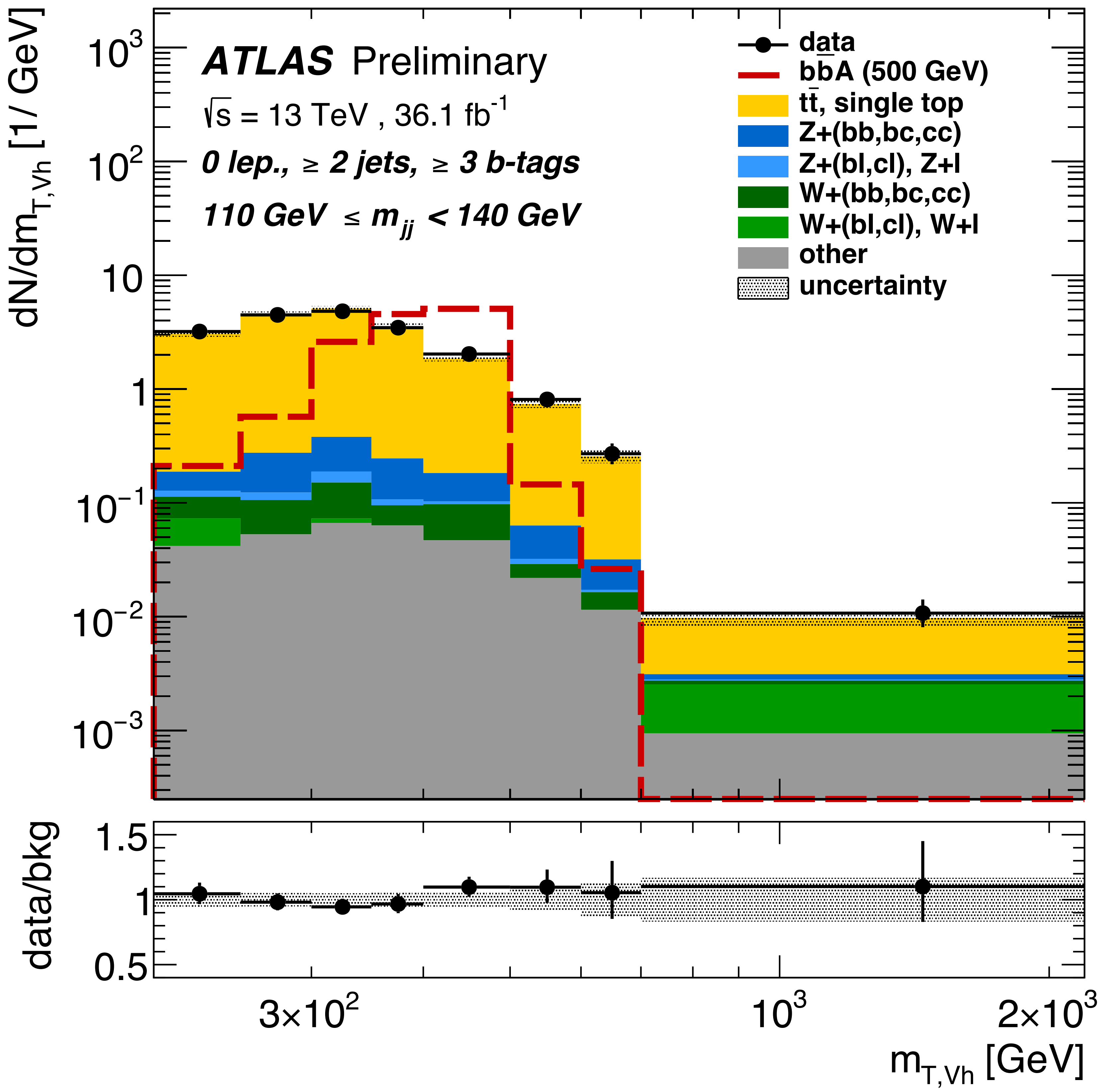
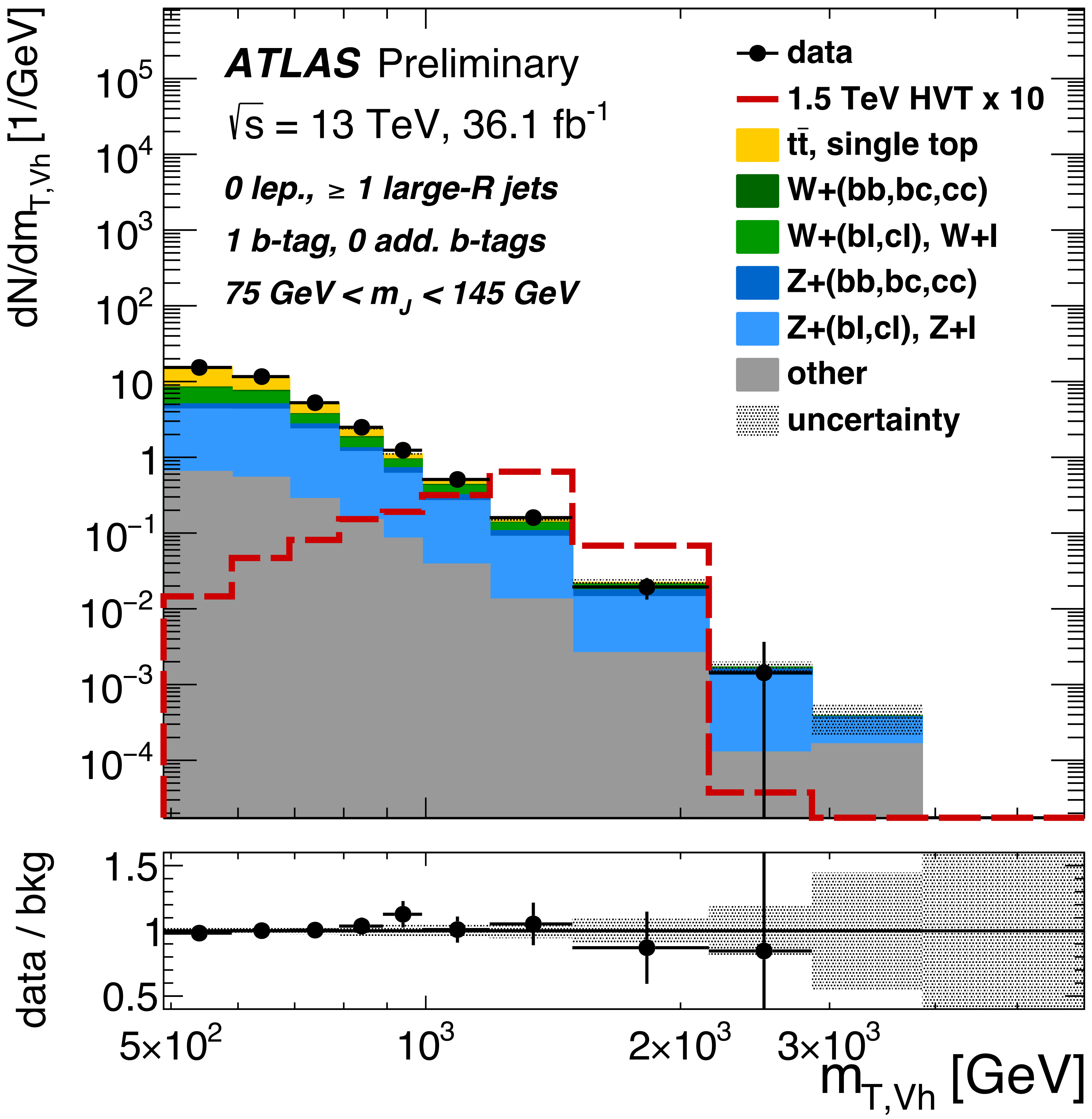
$Zh \to \nu\bar{\nu}b\bar{b}$

Resolved ≥ 2 b-tags





Merged 1-tag



- tt, Z+(bb,bc,cc), Z+(bl,cl) backgrounds determined in fit to SR
  - For bbA interpretation in ≥ 3b SR, Z+(bb,bc,cc) norm. is fit independently

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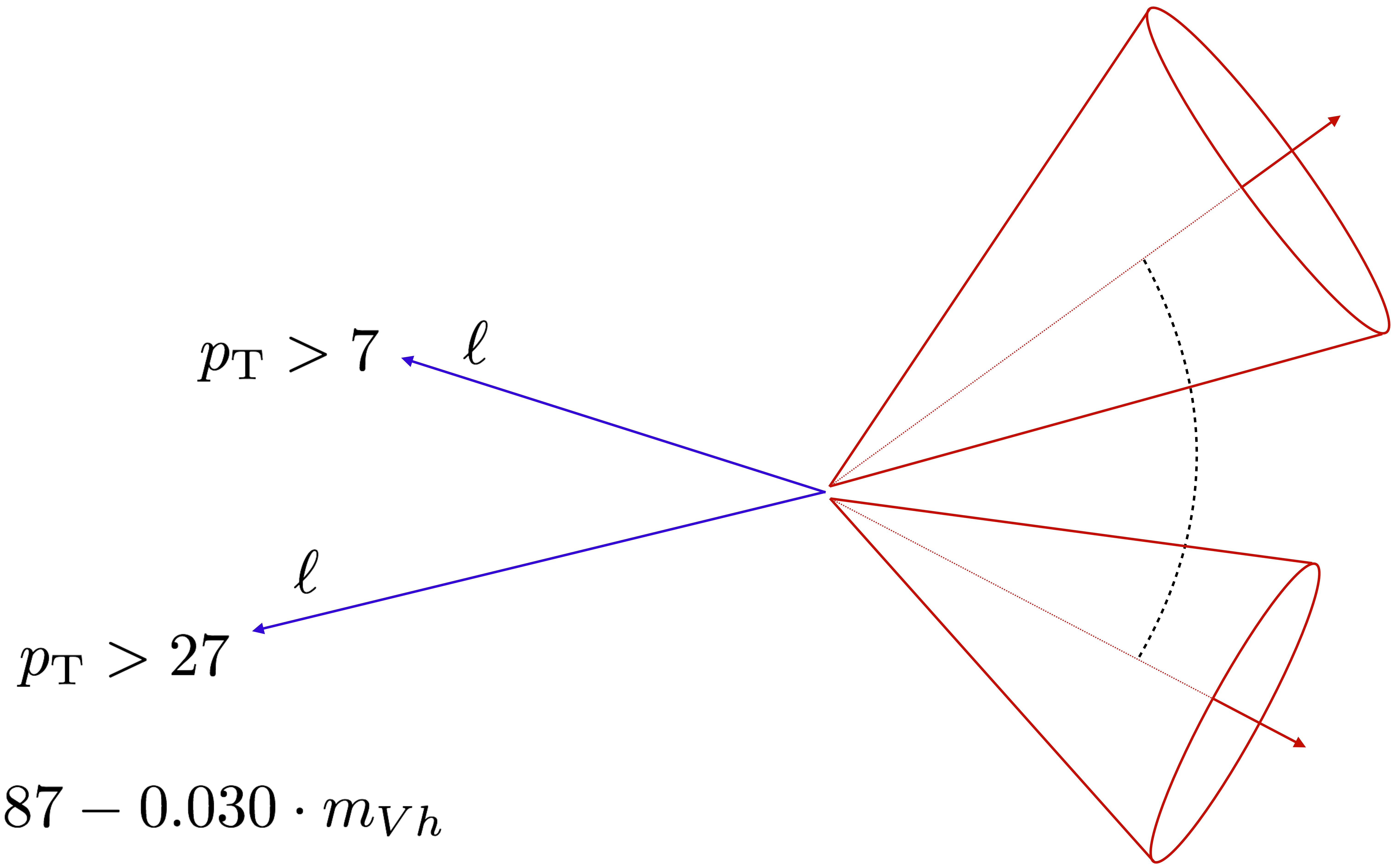
$Zh \to \ell^+\ell^- b\bar{b}$



b-tag at 70% eff.

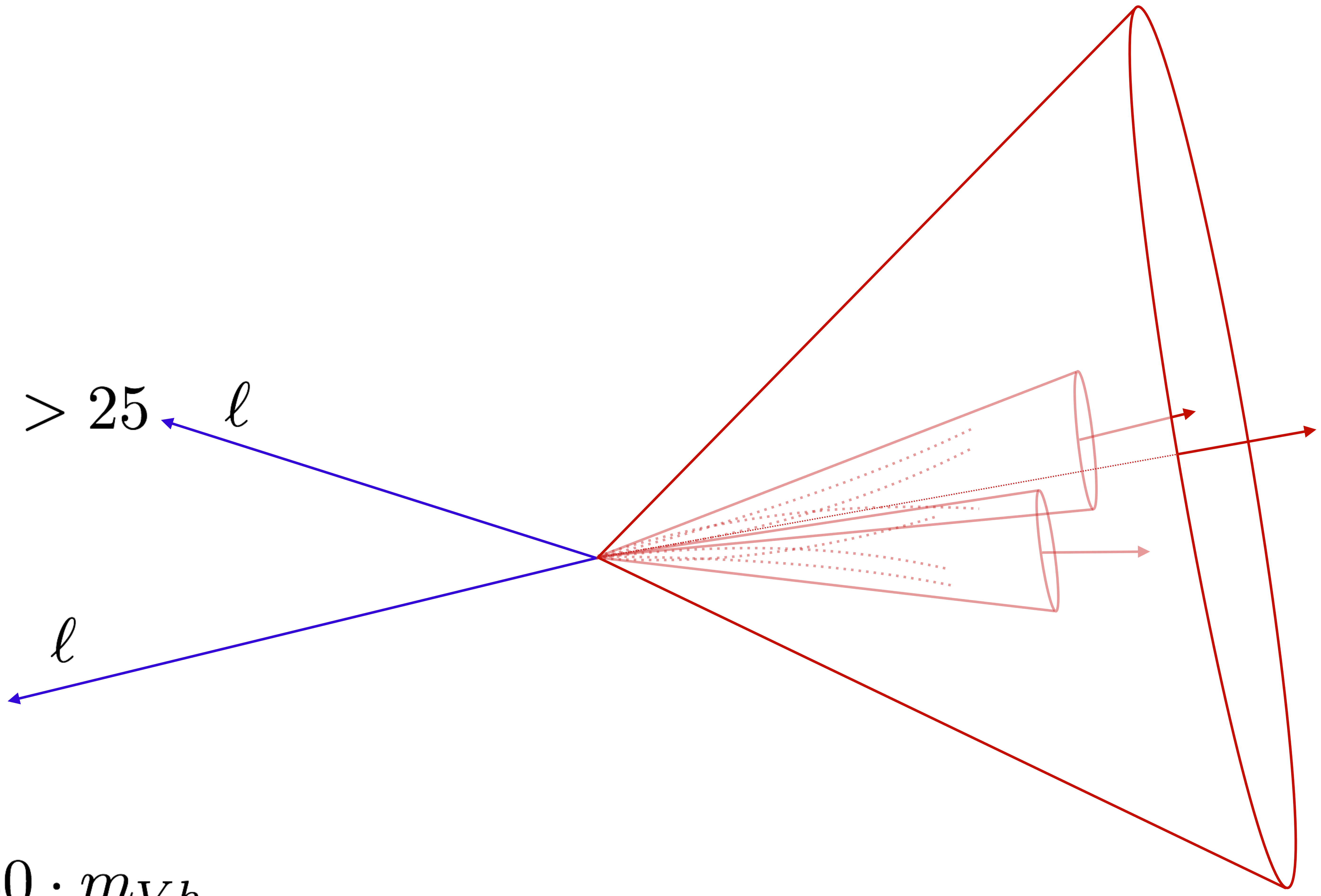
$145 < m_{jj} < 100$

$$m_{\ell\ell} \begin{cases} > 87 - 0.030 \cdot m_{Vh} \\ < 97 + 0.013 \cdot m_{Vh} \end{cases}$$

$$E_T^{miss}/\sqrt{H_T} < 1.15 + 8 \cdot 10^{-3} m_{Vh}$$

details

$Zh \to \ell^+\ell^- b\bar{b}$



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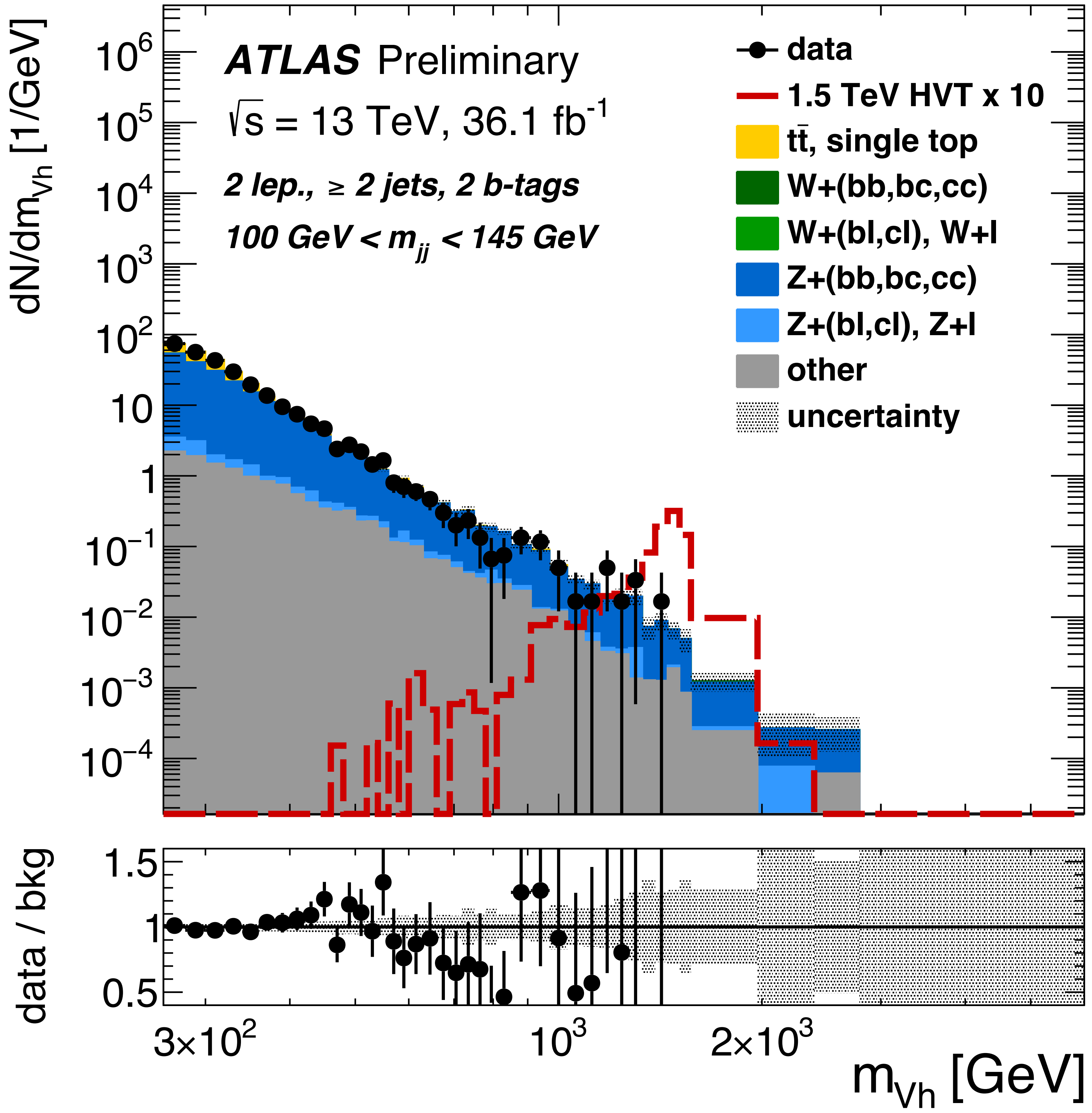
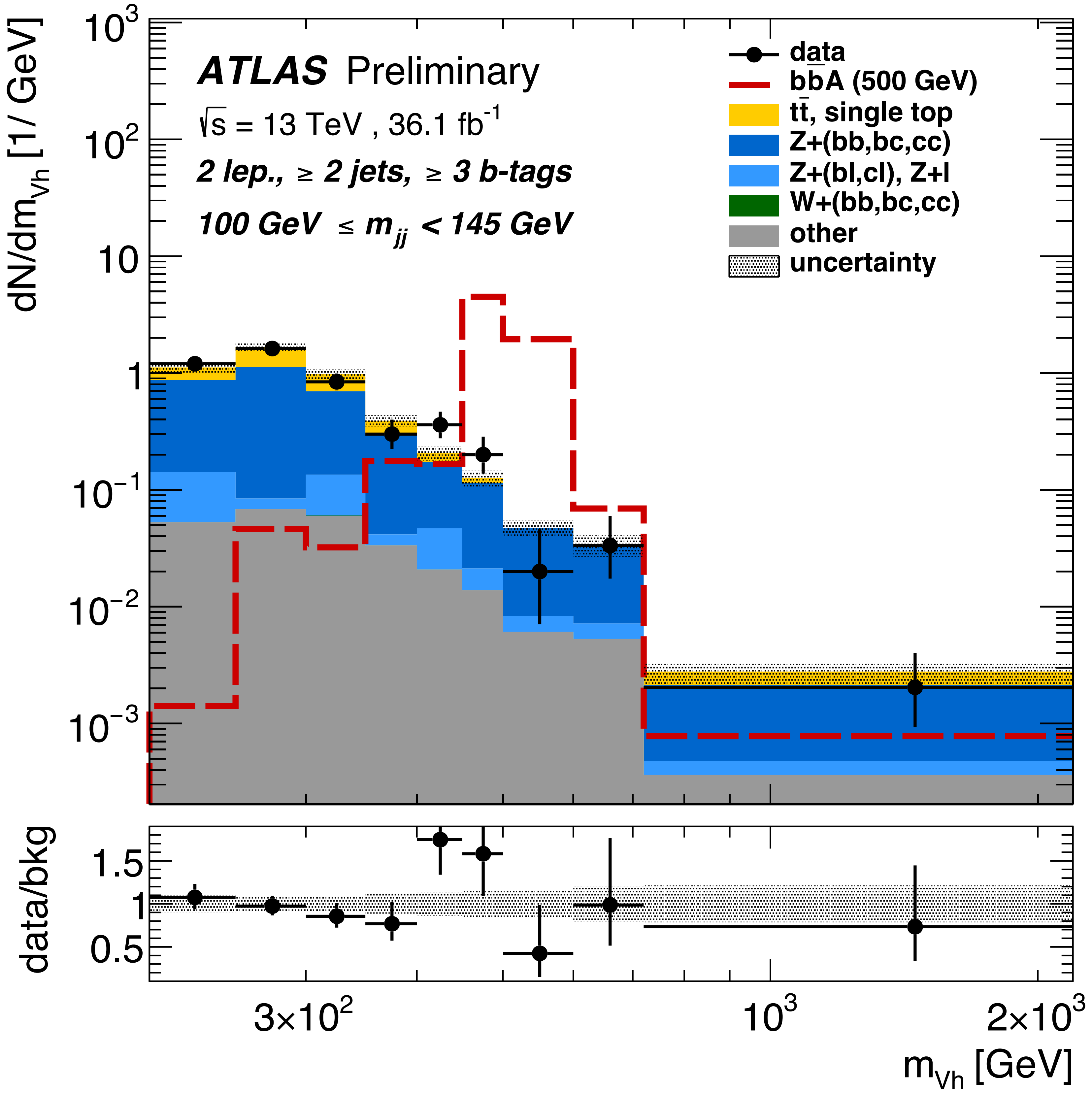
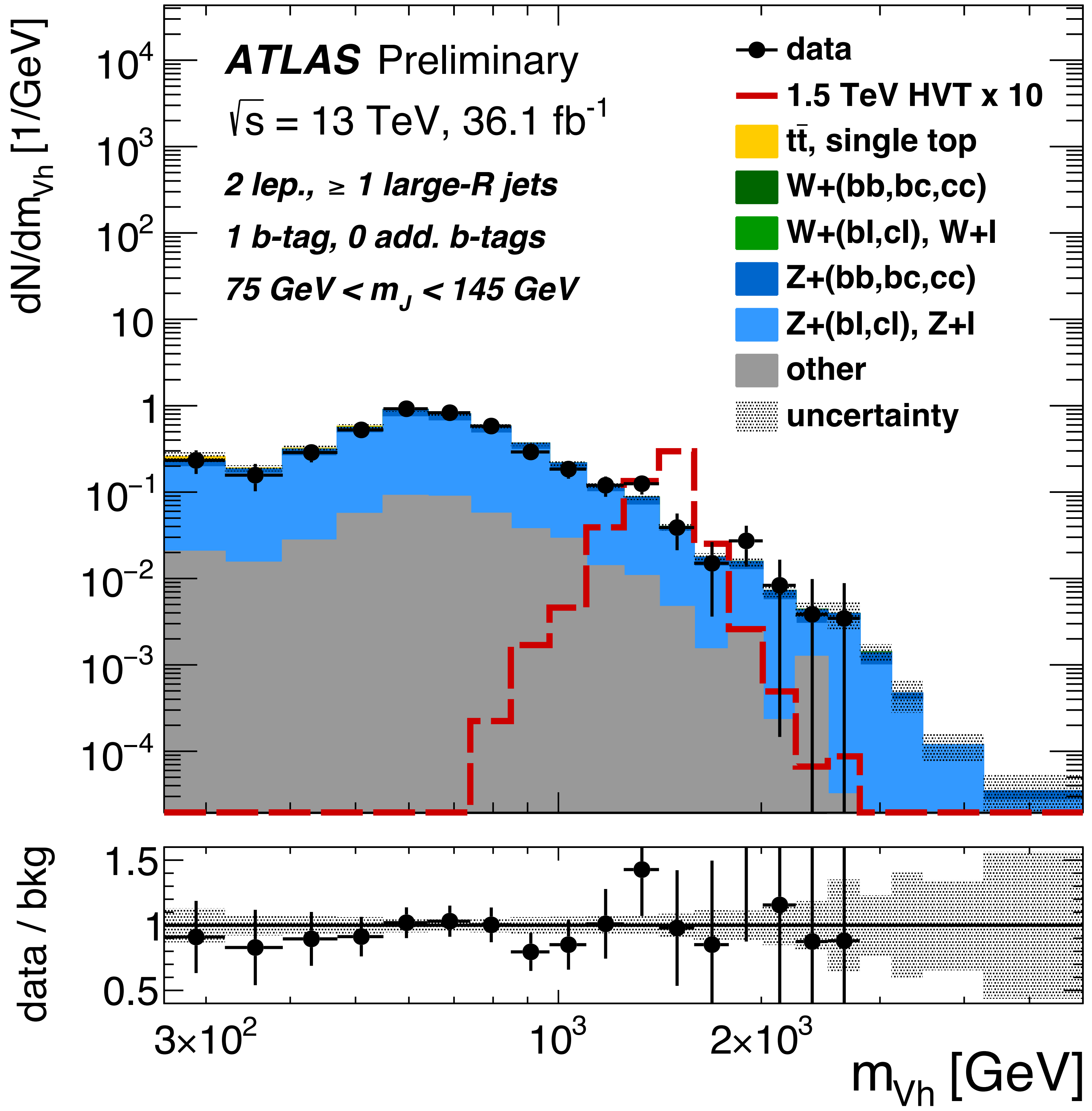




Merged 1-tag



Z+jets and $t\bar{t}$ normalizations fit in data with dedicated $t\bar{t}$ control region with $e\mu$ selection and no $E_{\mathrm{T}}^{\mathrm{miss}}/\sqrt{H_{\mathrm{T}}}$ requirement

$A \to Zh \to (\nu\nu, \ell^+\ell^-)b\bar{b}$

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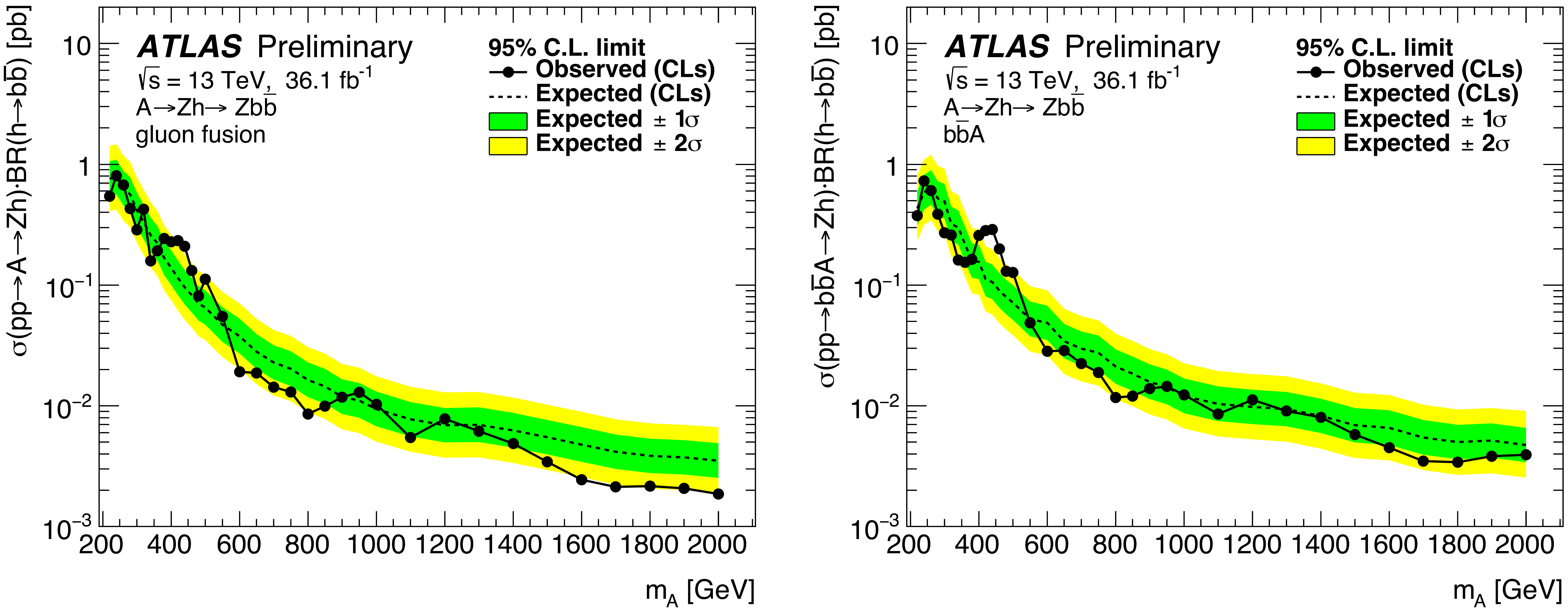


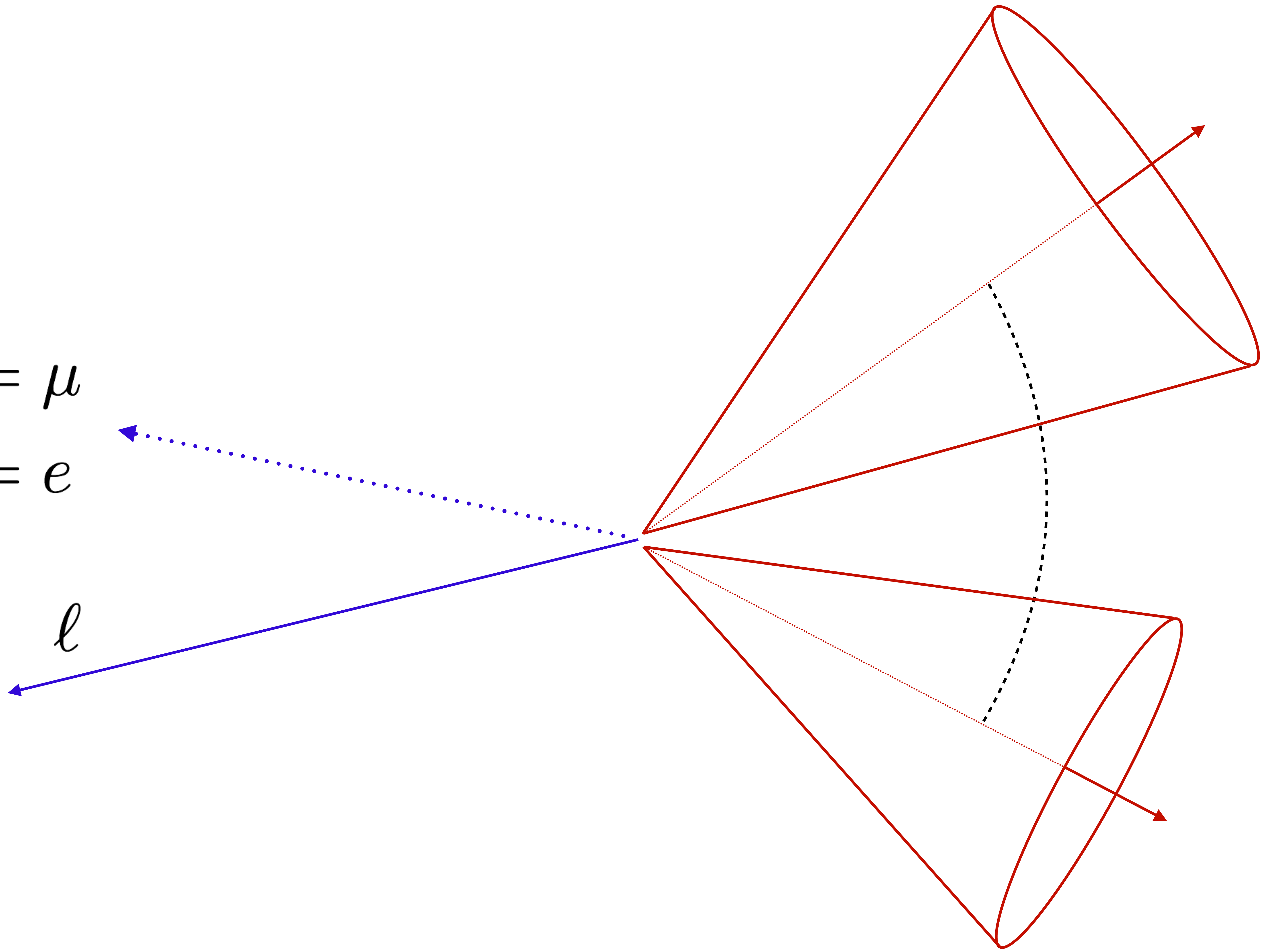
(a) Pure gluon fusion production

(b) Pure b-quark associated production

Excess at 440 GeV mostly from dimuon 3+ b-tag SR. Global significance only 2.4σ

Also interpreted as limits on 2HDM models

$Wh \to \ell^{\pm}\nu b\bar{b}$



m_{Vh} constructed by setting p_z^{ν} such that $\text{mass}(E_{\mathrm{T}}^{\mathrm{miss}}, p_{\mathrm{T}}^{\ell}) = m_W$

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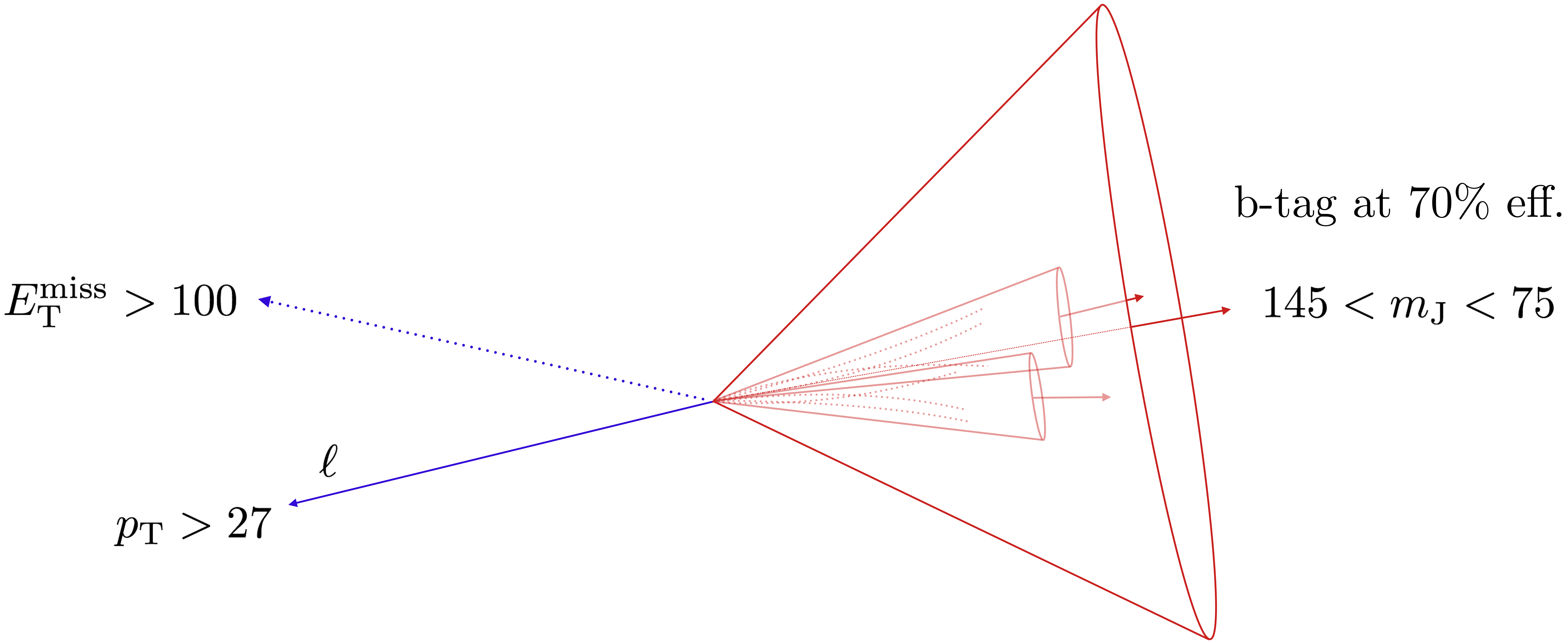
$Wh \to \ell^{\pm} \nu b\bar{b}$



m_{Vh} constructed by setting p_z^{ν} such that mass$(E_{\mathrm{T}}^{\mathrm{miss}}, p_{\mathrm{T}}^{\ell}) = m_W$

details

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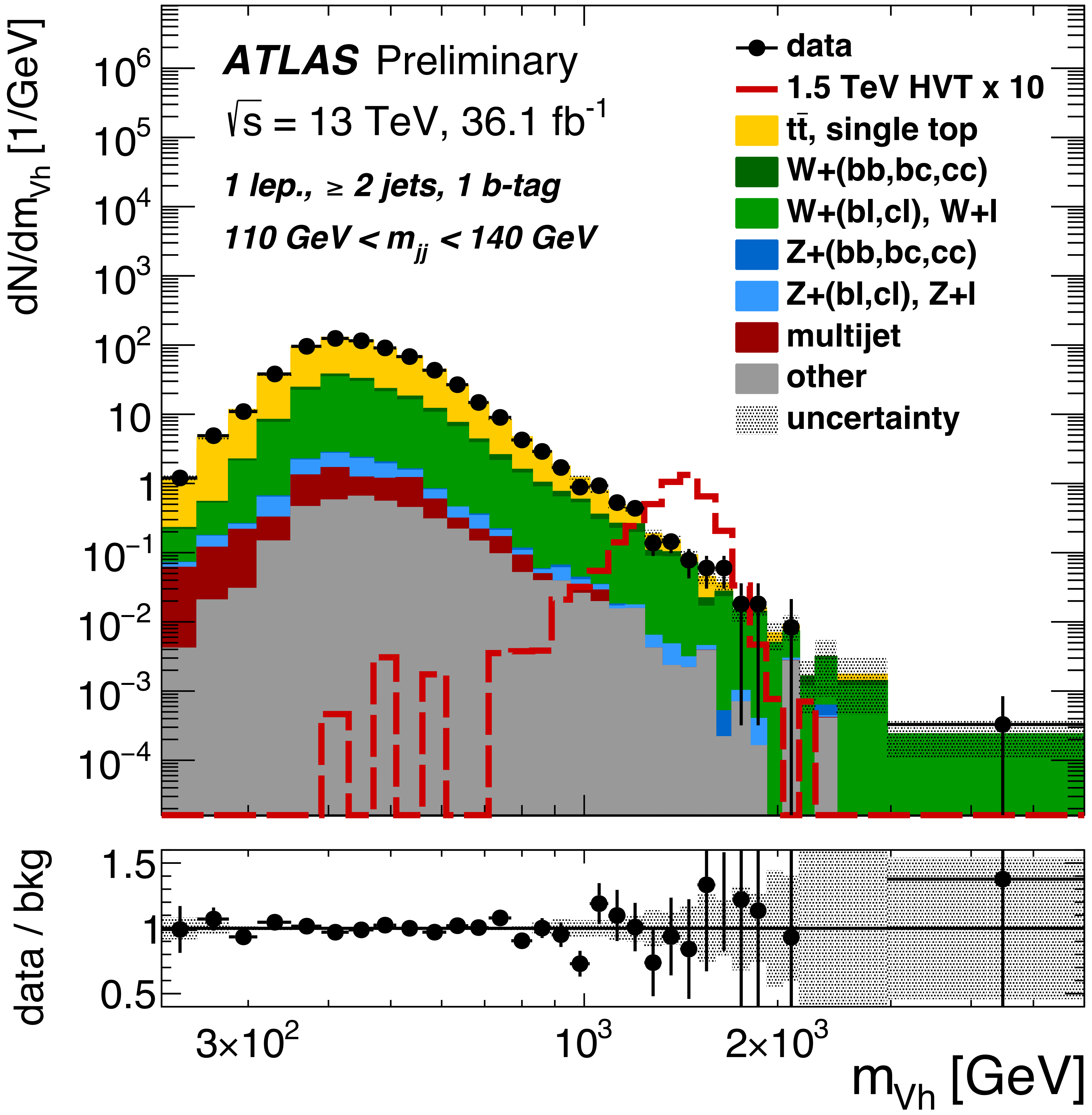
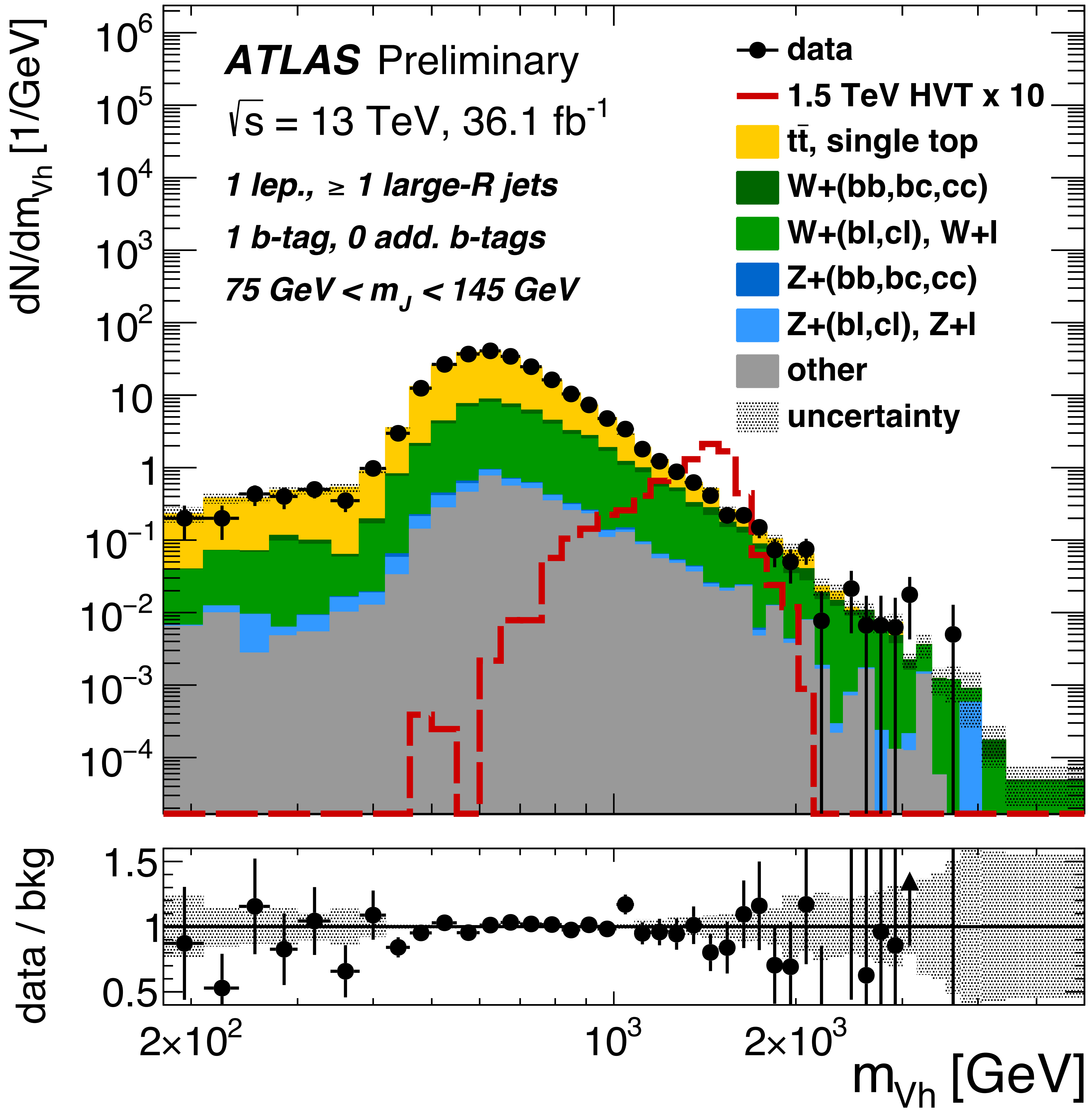
$Wh \to \ell^{\pm}\nu b\bar{b}$





- m_{bb} sideband mostly tt and W+jets
  - used in fit to constrain dominant backgrounds
- Multijet template taken from data with inverted lepton isolation

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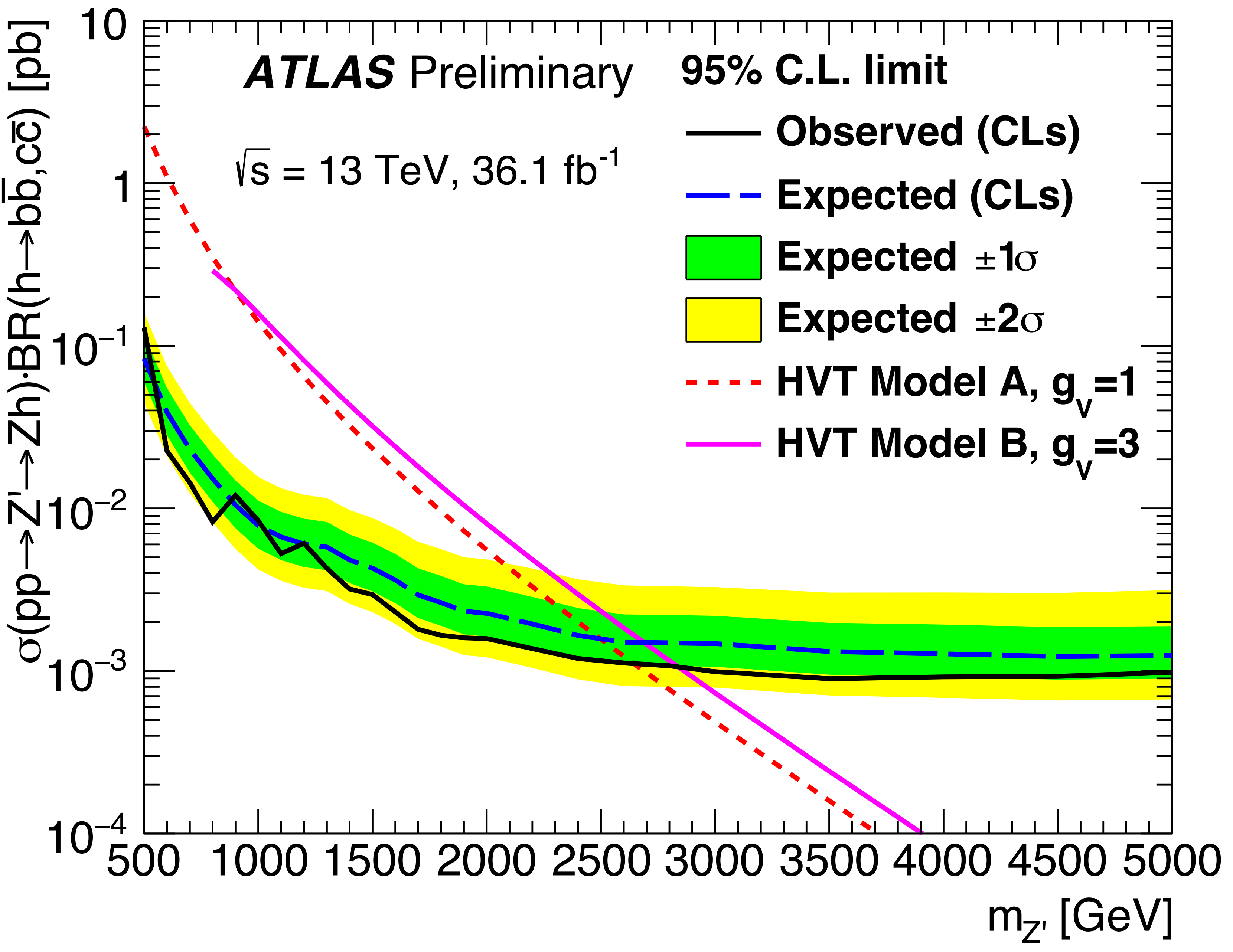
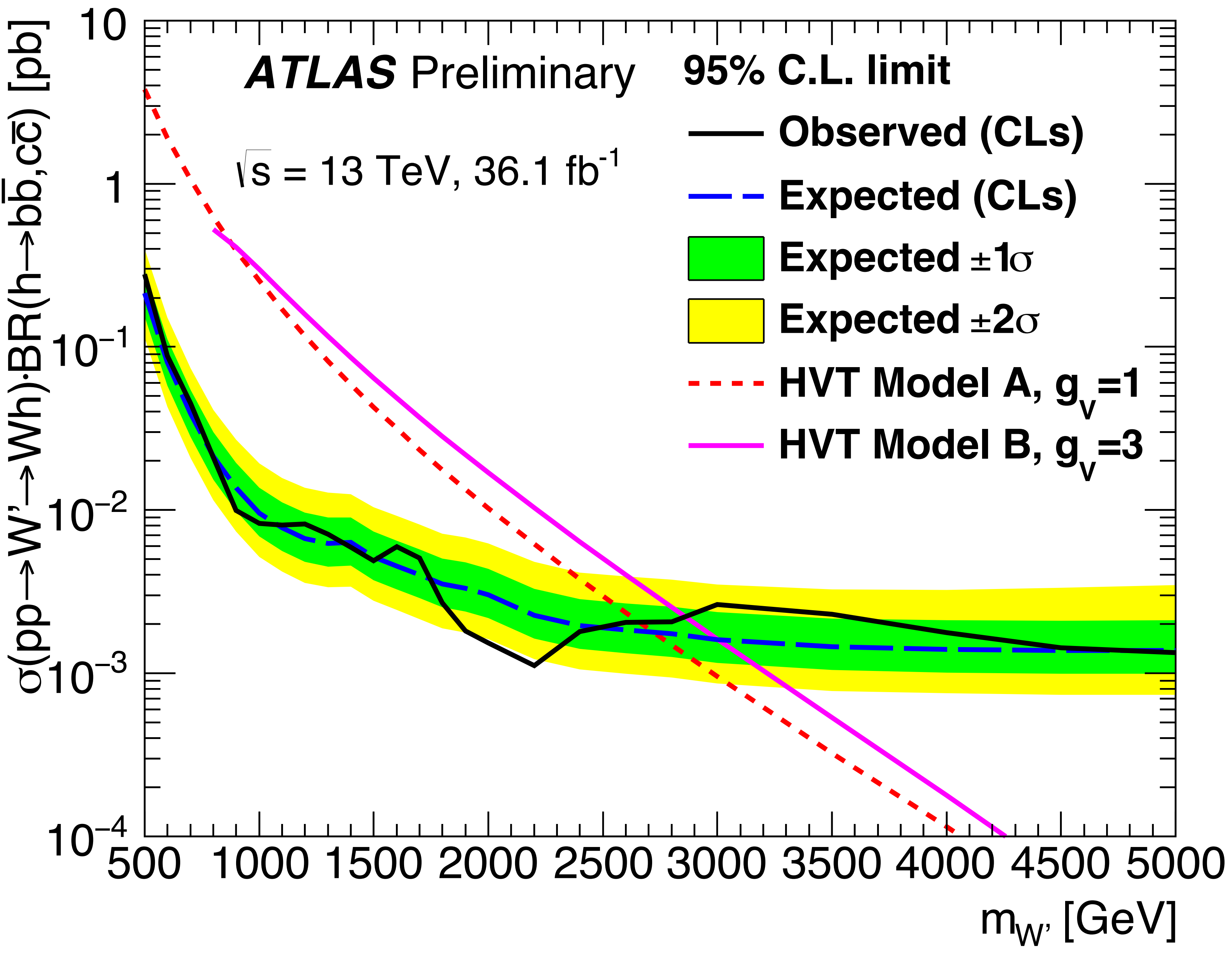
$X \to Vh \to (\nu\nu, \ell^+\ell^-, \ell^\pm\nu)b\bar{b}$

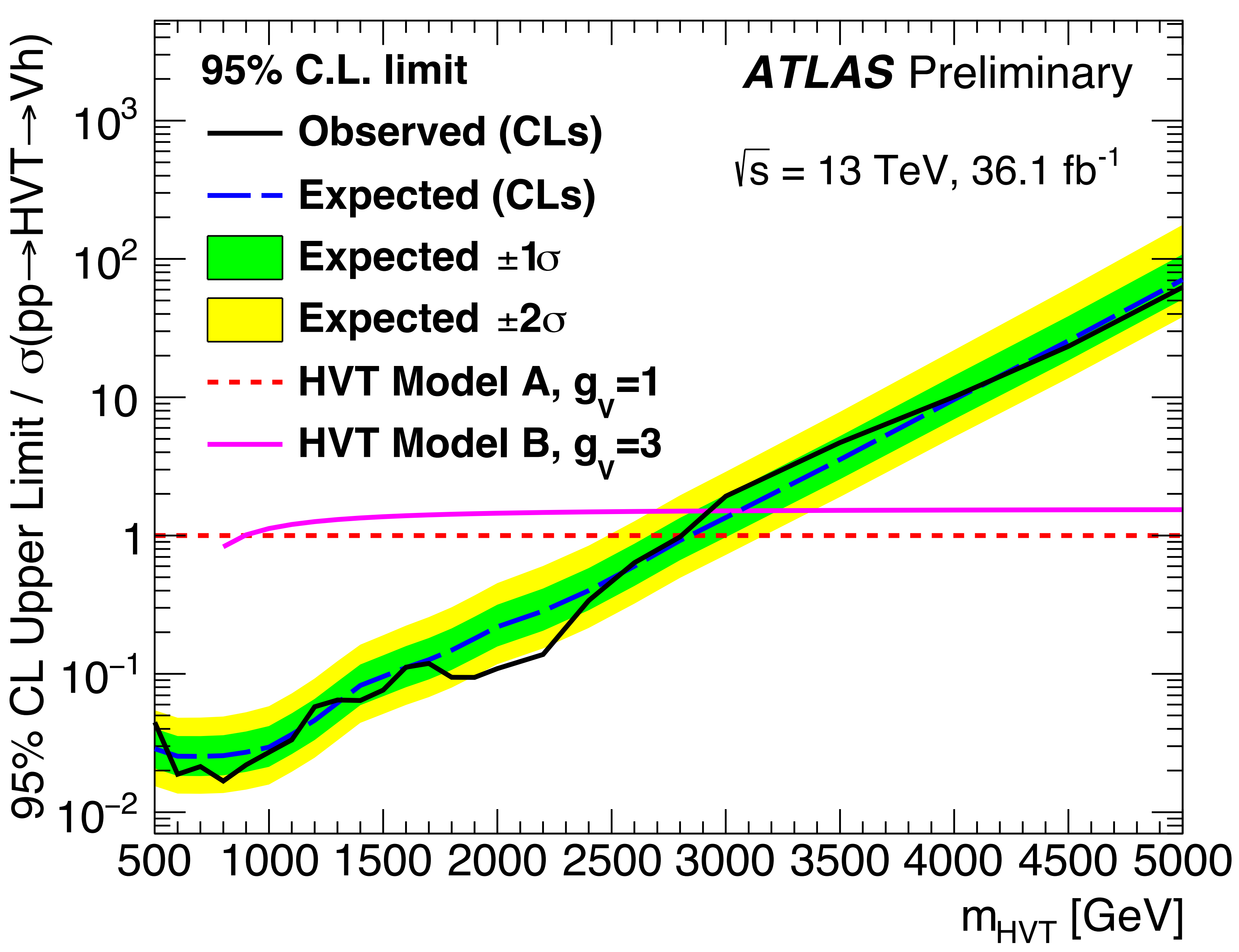
Z’



W’



HVT ($m_{Z'} \approx m_{W'}$)

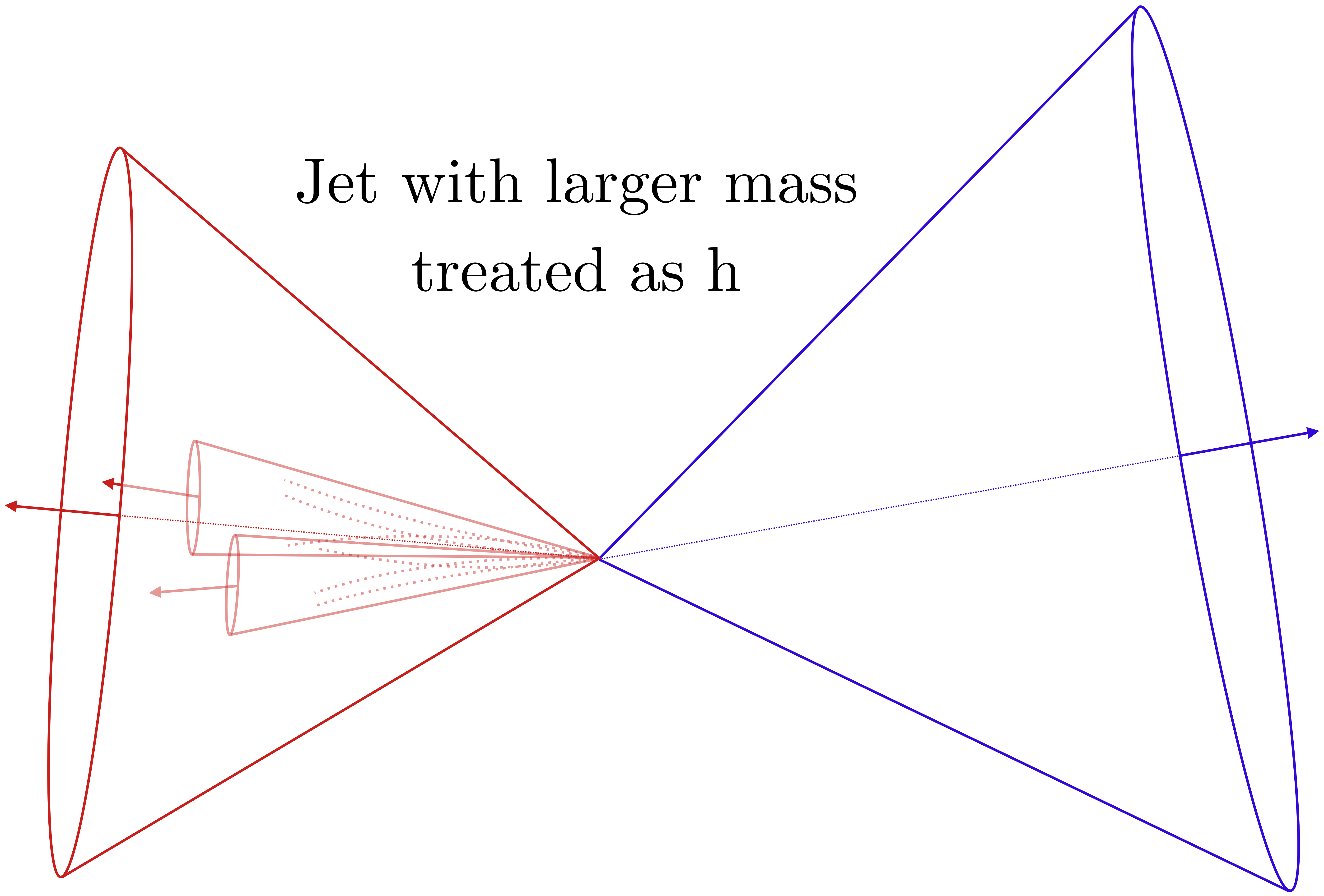


Model A has comparable BR to fermions and bosons.

Model B has suppressed fermionic couplings.

Phys. Let. B

$Vh \to q\bar{q}b\bar{b}$



<u>h Jet:</u>

$145 > m_{\mathrm{J}} < 75$

b-tag at 77% eff.

$p_{\mathrm{T}}^{lead} > 450$

<u>V Jet:</u>

mass <u>window</u> 68%

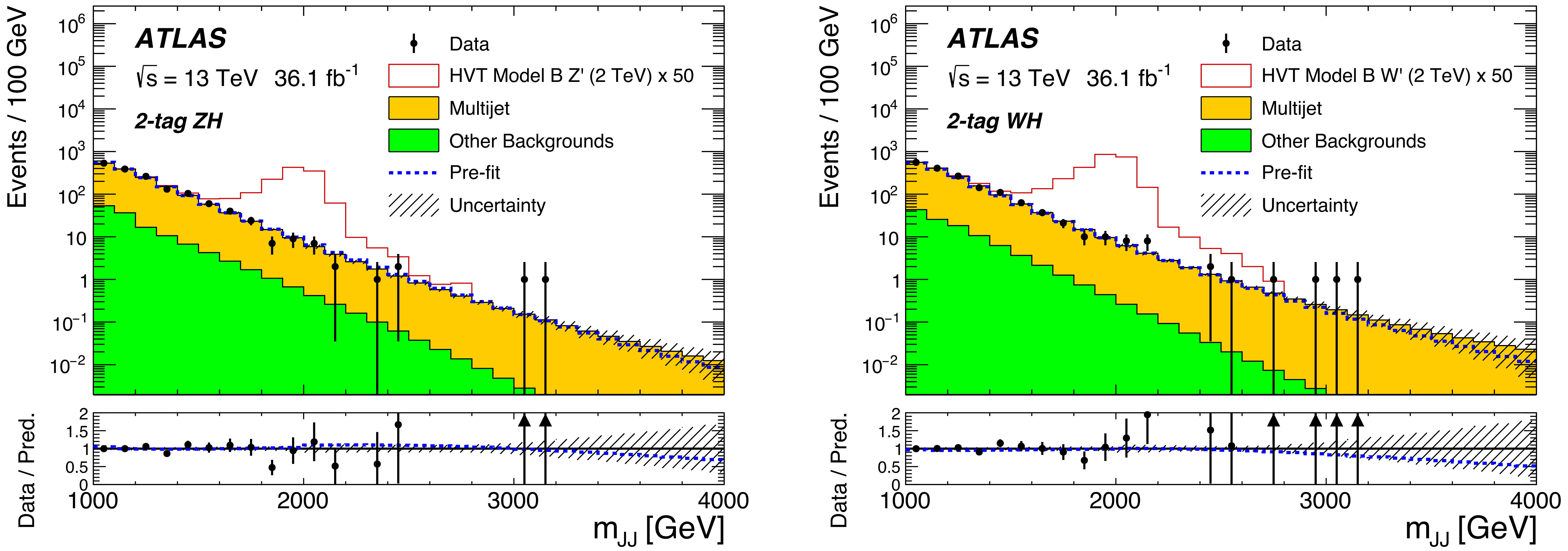
D_2 <u>cut</u> to 50% eff.

Flat eff. in p_{T}

<u>ATL-PHYS-PUB-2015-033</u>

$Vh \to q\bar{q}b\bar{b}$

Phys. Let. B



- Multijet background from 0-tag data
  - Normalized in high h mass sideband
  - Kinematic reweighting for 2-tag model

Phys. Let. B

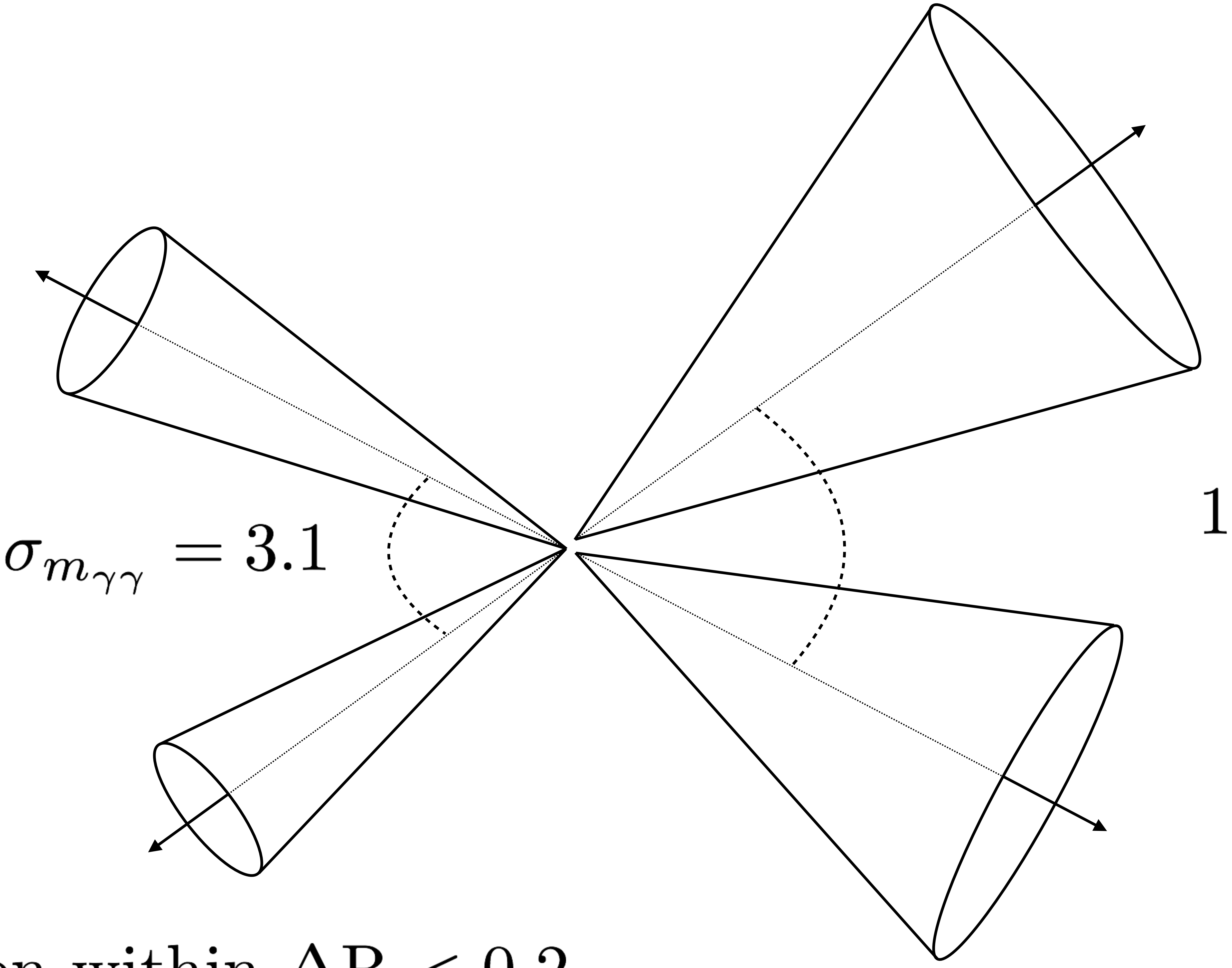
$Vh \to q\bar{q}b\bar{b}$

Observed Limit from leptonic channels

ATLAS-CONF-2016-004

$hh \to b\bar{b}\gamma\gamma$

b-tag at 85% eff.



$|m_{\gamma\gamma} - m_h| < 2\sigma_{m_{\gamma\gamma}} = 3.1$

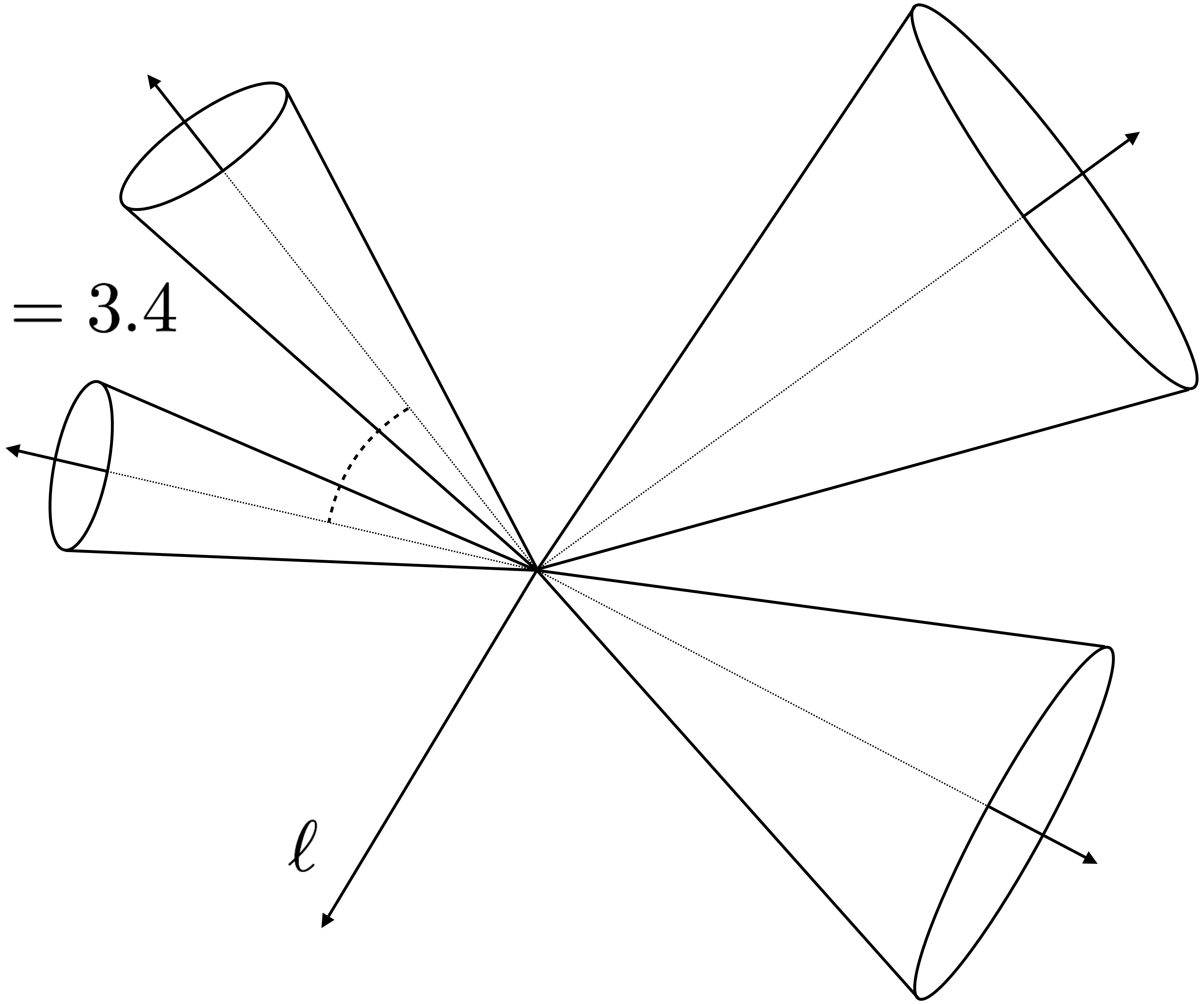
$135 > m_{b\bar{b}} < 95$

Track isolation within ΔR < 0.2
(excluding $\gamma \to e^+e^-$)

ATLAS-CONF-2016-071

$hh \to WW^* \gamma\gamma$

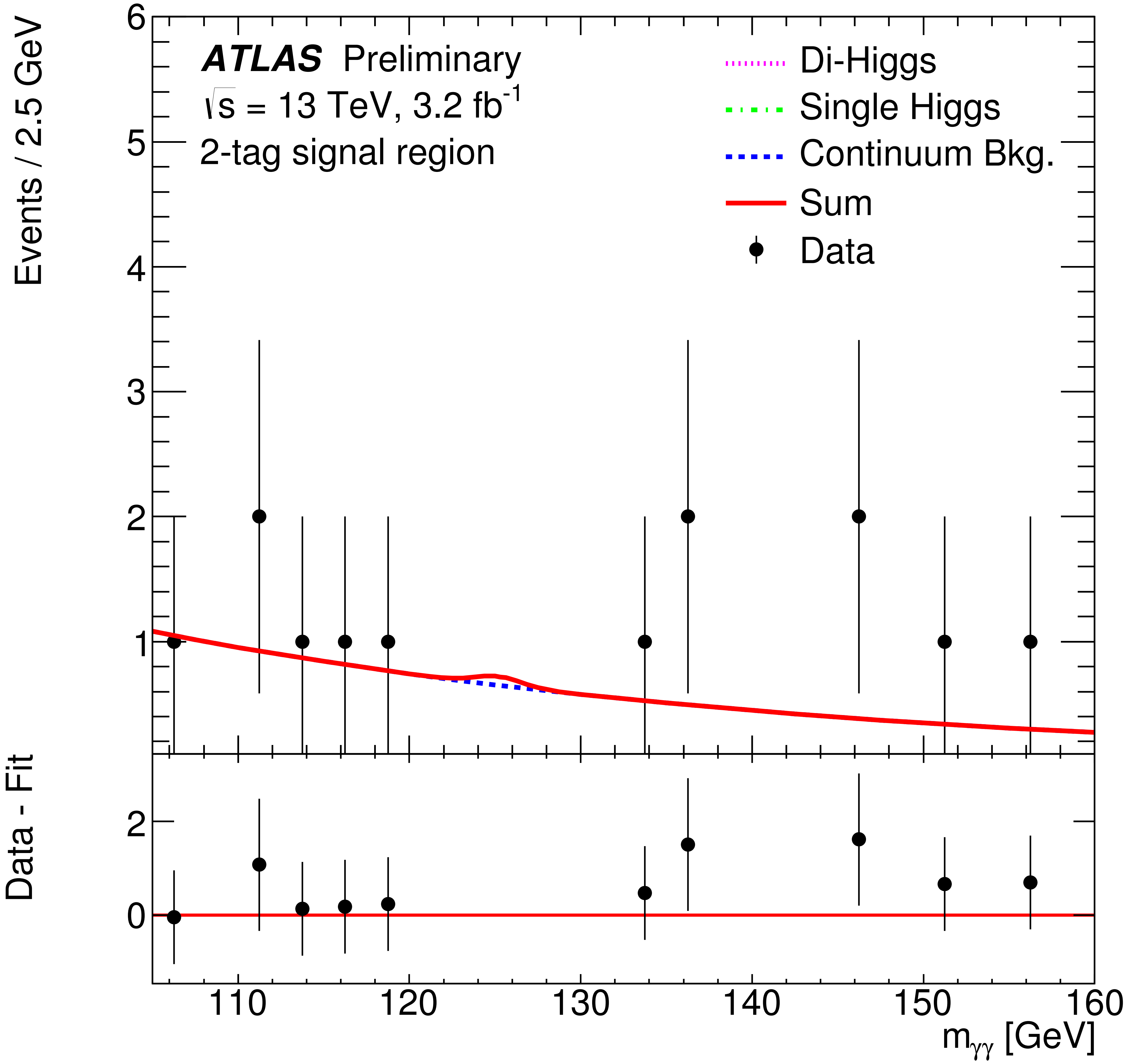
$$|m_{\gamma\gamma} - 125.09| < 2\sigma_{m_{\gamma\gamma}} = 3.4$$



Veto events with b-jets
(Tagged at 70% eff.
≈8% c-jet mis-tag)

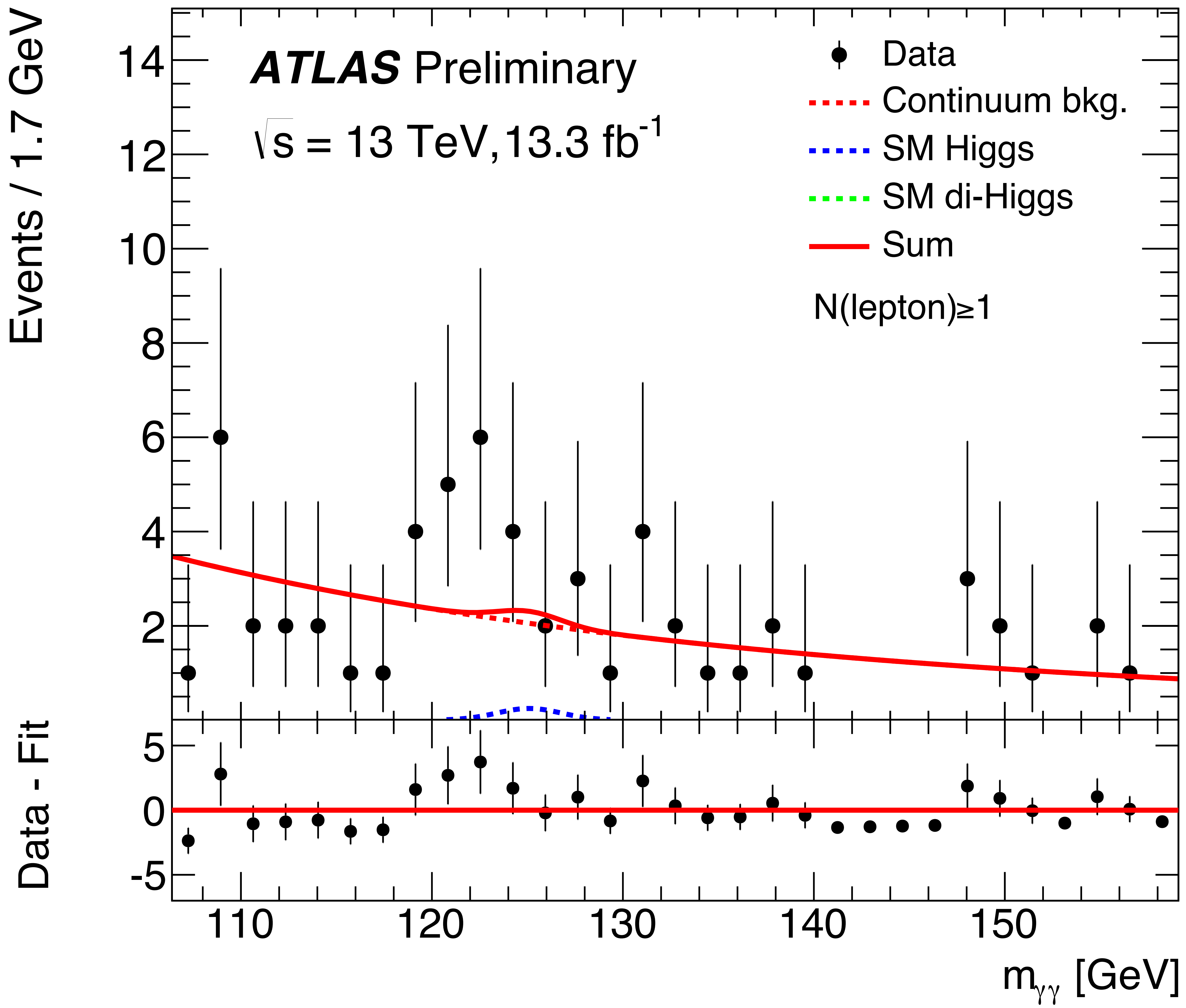
details

$hh \to b\bar{b}\gamma\gamma$



No events in m_h window

$hh \to WW^*\gamma\gamma$

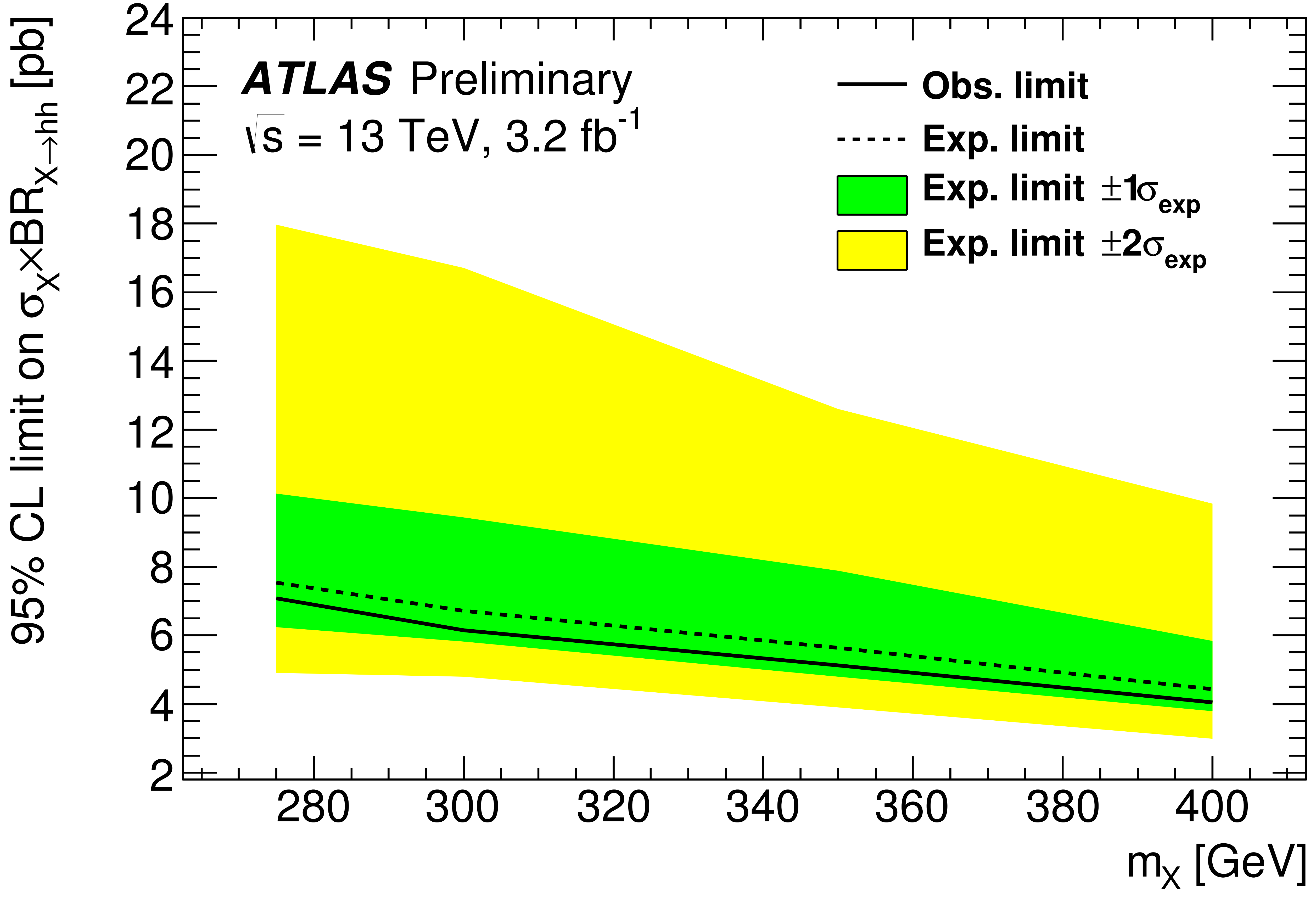


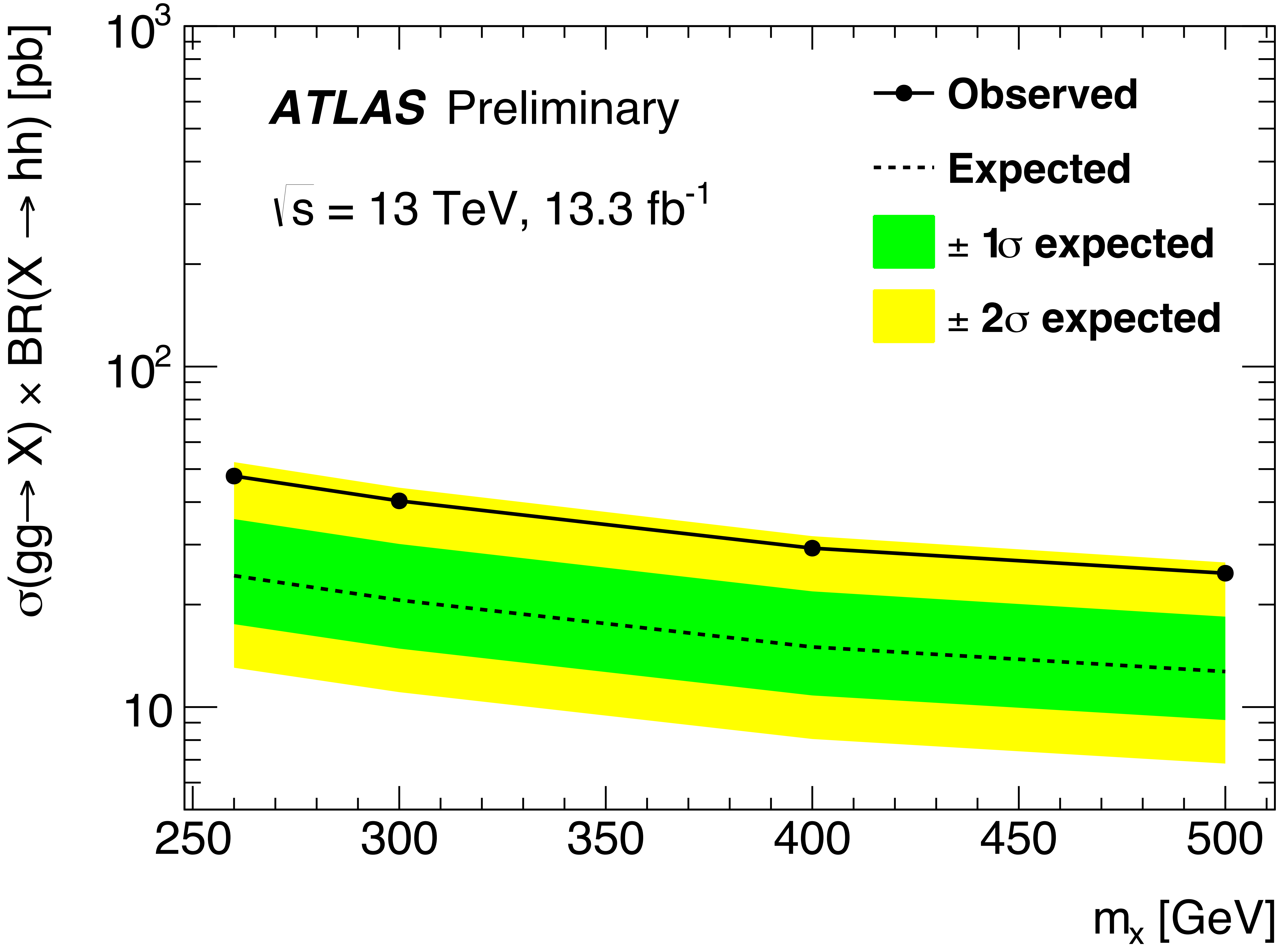
ATLAS-CONF-2016-004

ATLAS-CONF-2016-071

$$hh \to b\bar{b}\gamma\gamma$$

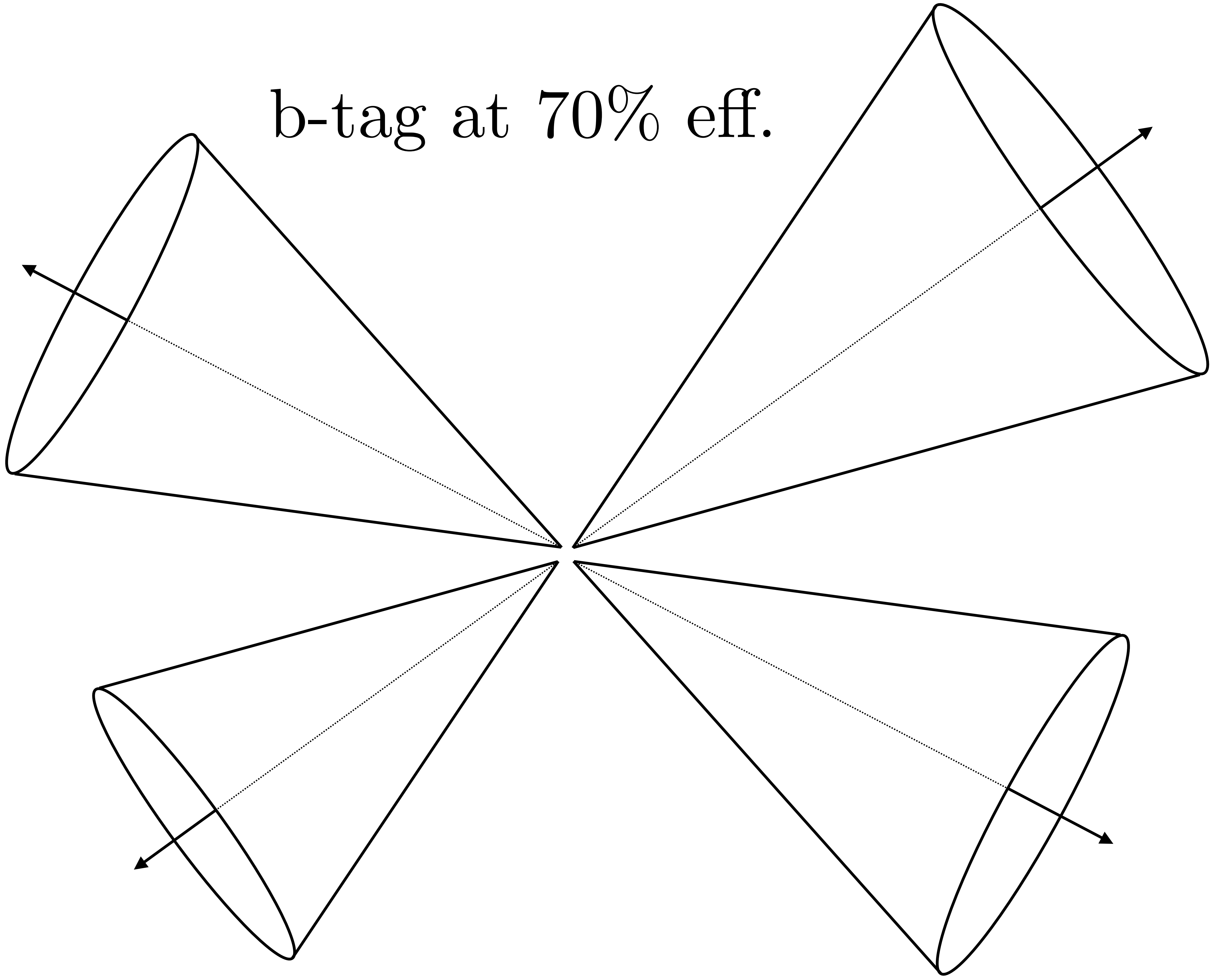
$$hh \to WW^*\gamma\gamma$$





ATLAS-CONF-2016-049

$hh \to b\bar{b}b\bar{b}$



≥ 4 b-jets

Consider pairings in sliding ΔR_{jj} windows as function of m_{4j}

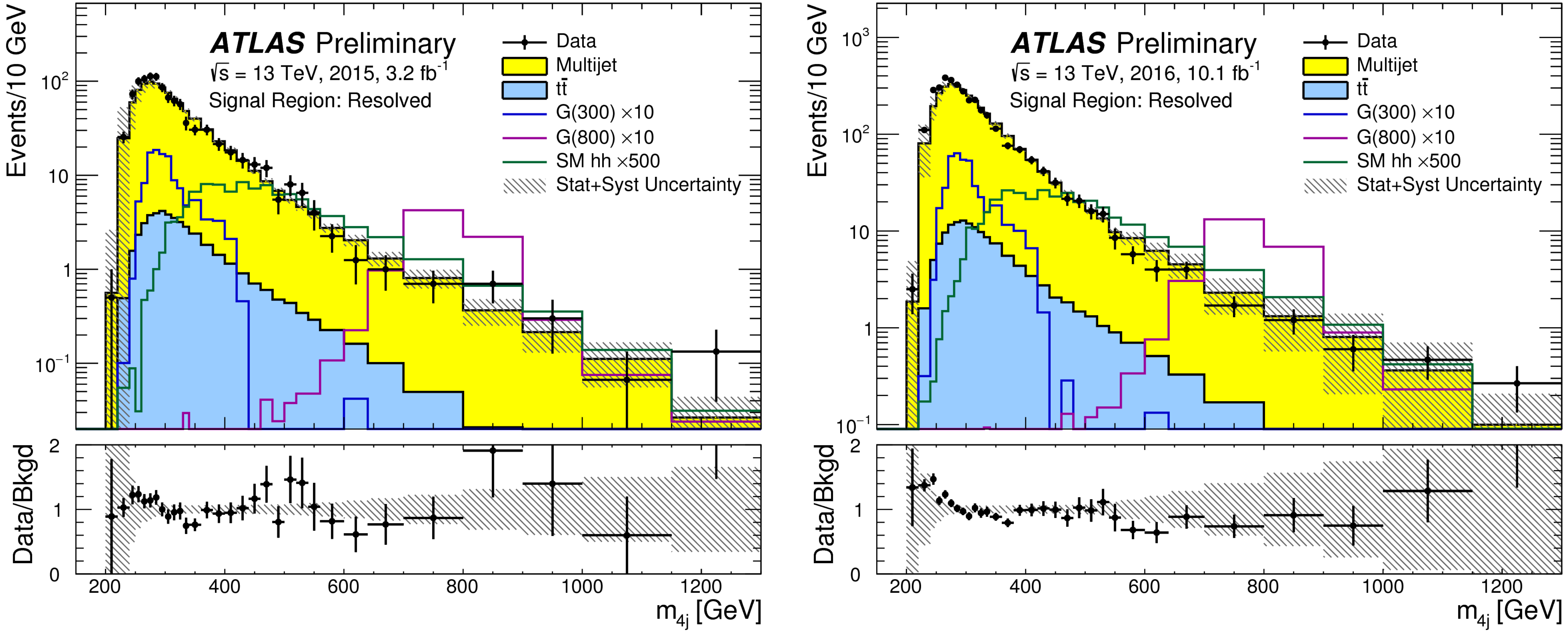
Construct di-jets with pairing that minimizes di-jet mass difference

$$X_{hh} = \sqrt{\left(\frac{m_{h_1} - 120}{0.1 m_{h_1}}\right)^2 \left(\frac{m_{h_2} - 115}{0.1 m_{h_2}}\right)^2} < 1.6$$

details

ATLAS-CONF-2016-049

$hh \to b\bar{b}b\bar{b}$



2015/2016 data used slightly different online b-triggers.

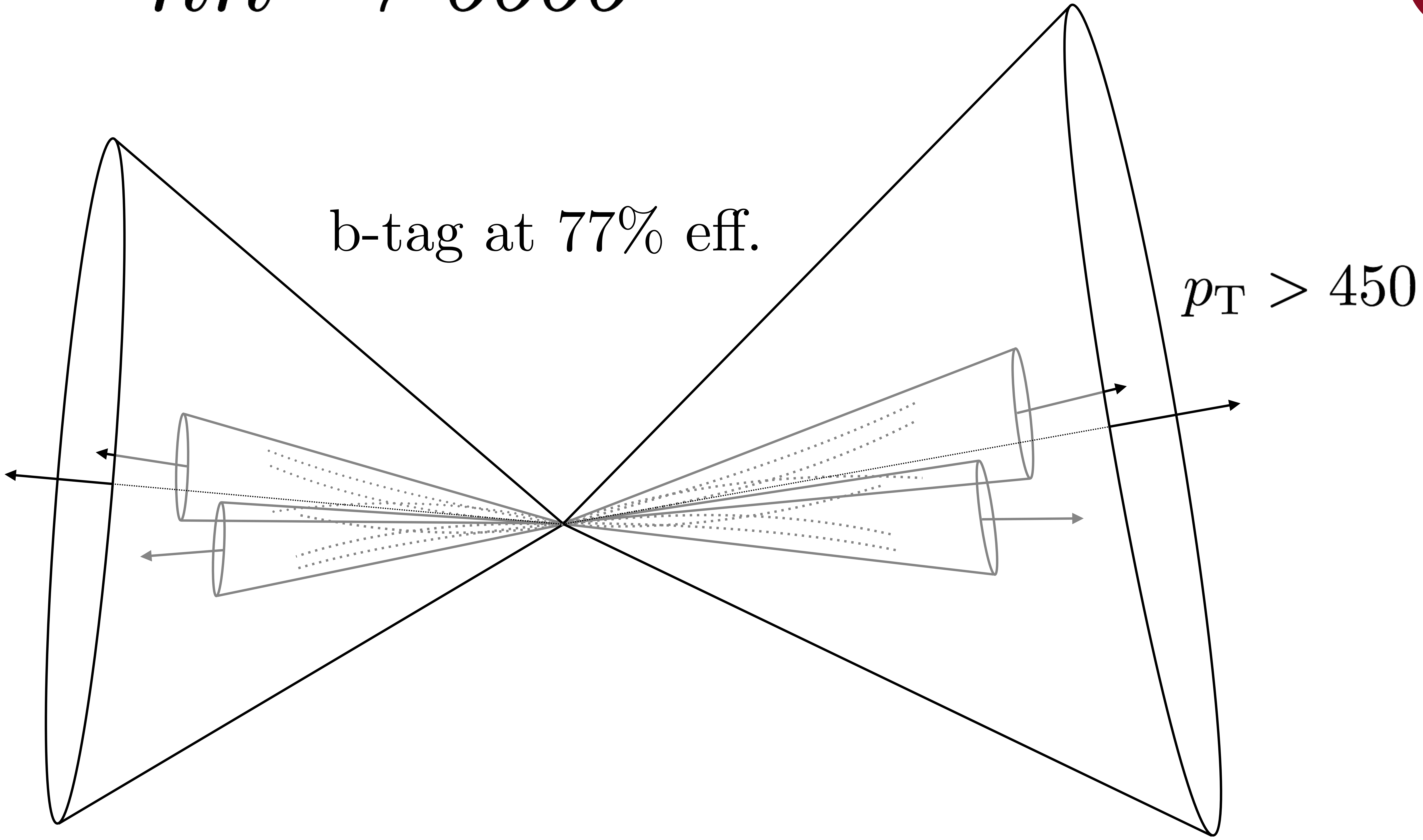
Modeled separately and combined in final fit.

ATLAS-CONF-2016-049

$hh \to b\bar{b}b\bar{b}$

≥ 1 b-tagged track jet on each side

Split into 2b, 3b, 4b signal regions



$$X_{hh} = \sqrt{\left(\frac{m_{h_1} - 120}{0.1 m_{h_1}}\right)^2 \left(\frac{m_{h_2} - 115}{0.1 m_{h_2}}\right)^2} < 1.6$$

ATLAS-CONF-2016-049

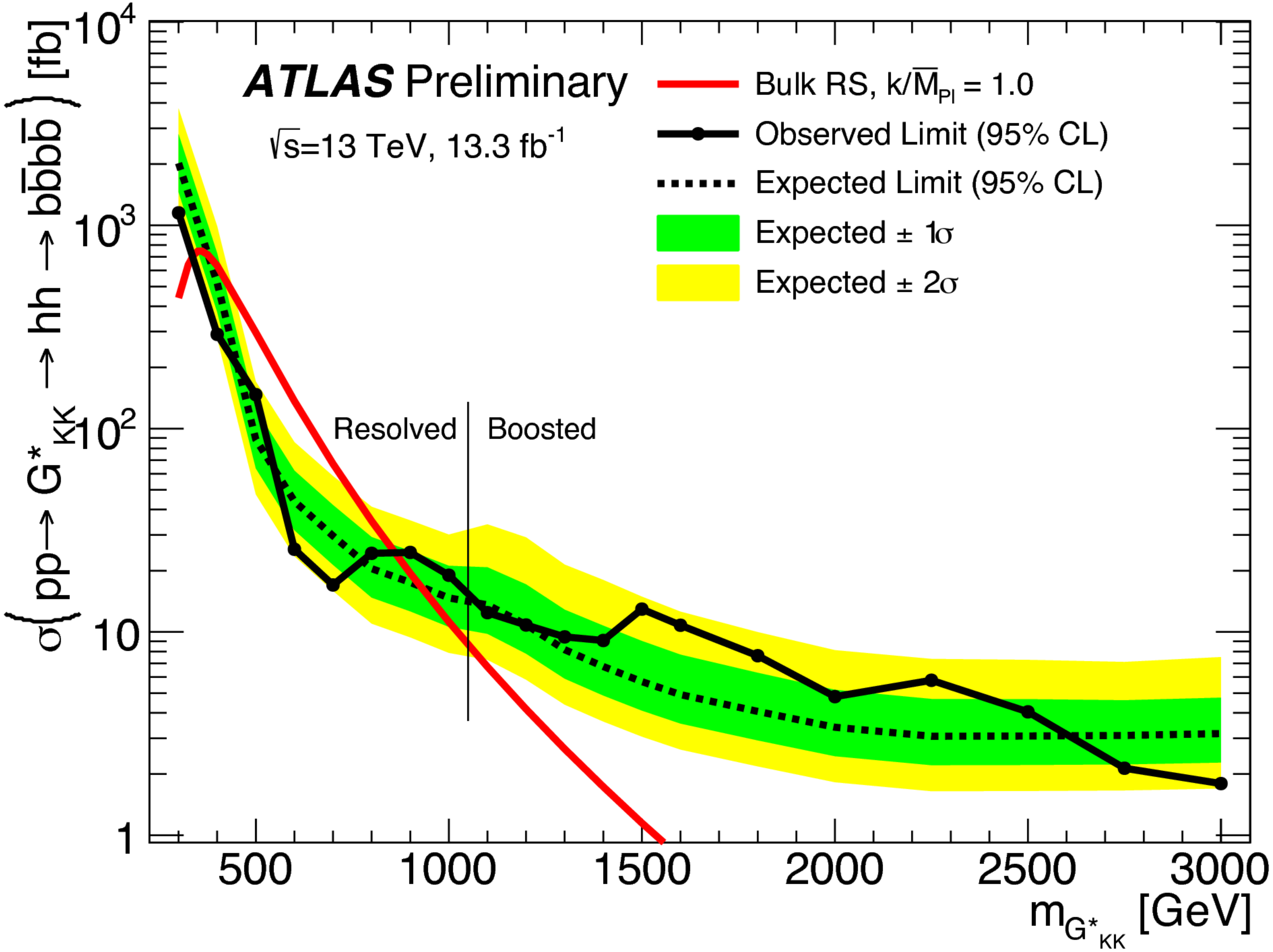
$hh \to b\bar{b}b\bar{b}$

4b, 3b and 2b SRs combined in final fit.

ATLAS-CONF-2016-049

$hh \to b\bar{b}b\bar{b}$

- Switch to boosted analysis based on crossing of expected limit ≈ 1050 GeV



Conclusions

- Huge number of final states!
  - Great for combinations and cross-checks of fluctuations
- No evidence yet of BSM physics
  - Modest excesses in bbA and all-hadronic Vh look like fluctuations
- Many exciting avenues forward. Two that I am especially excited about:
  - Increased use of machine learning techniques
  - Constrain background/signal systematics with SM measurements (eg. $ZZ \to b\bar{b}b\bar{b}$)

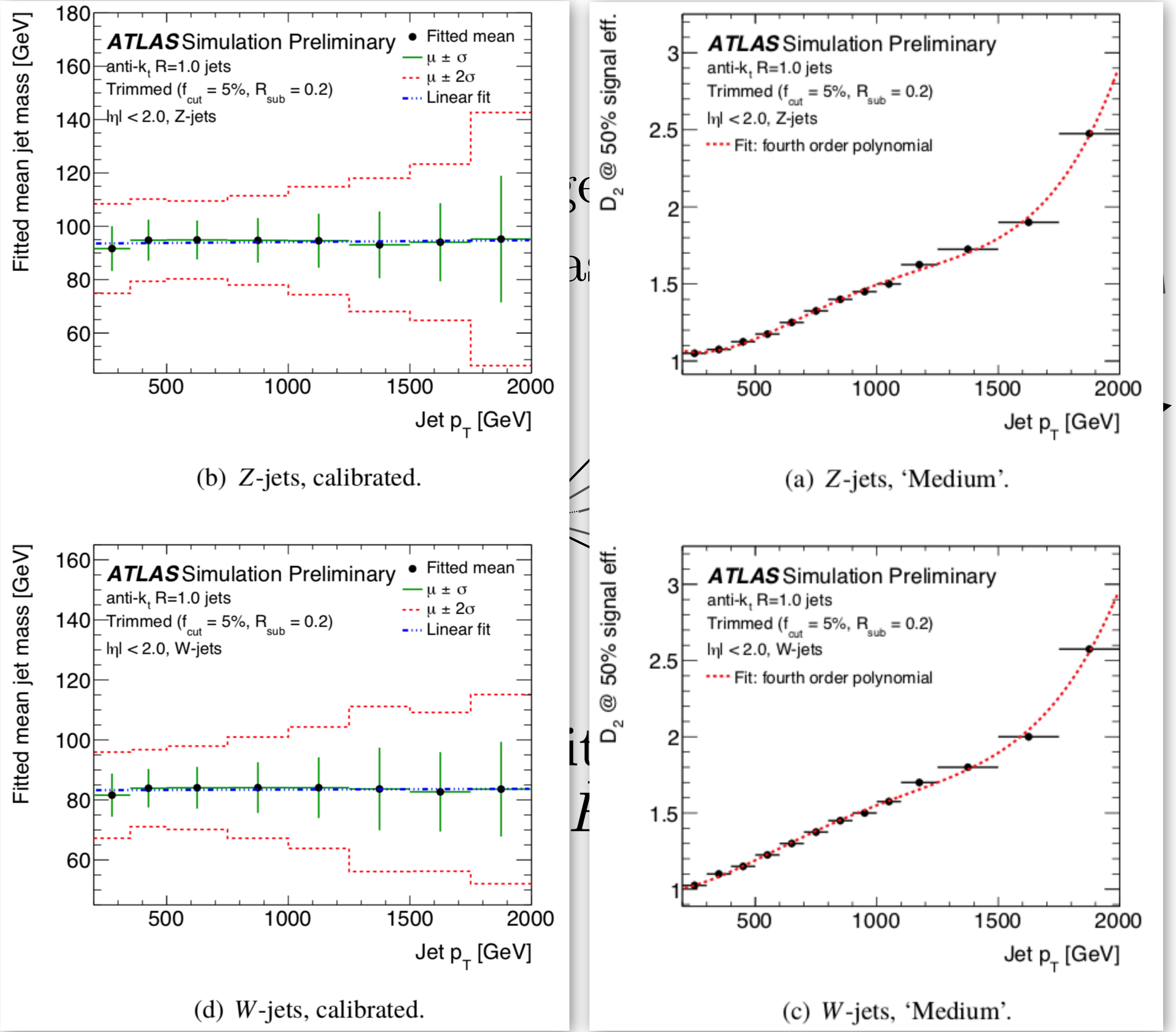
Backup

Phys. Let. B

$Vh \to q\bar{q}b\bar{b}$

$p_{\mathrm{T}}^{sublead} > 250$

$p_{\mathrm{T}}^{lead} > 450$



(b) Z-jets, calibrated.

(a) Z-jets, 'Medium'.

(d) W-jets, calibrated.

(c) W-jets, 'Medium'.

h Jet:

≥ 1 track jet
with $p_{\mathrm{T}} \geq 10$
b-tag at 77%

$145 > m_{\mathrm{J}} < 75$

V Jet:

mass window 68%

D_2 cut to 50% eff.

Flat eff. in p_{T}

ATL-PHYS-PUB-2015-033

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$A \to Zh \to (\nu\nu, \ell^+\ell^-)b\bar{b}$

(a) 2HDM Type-I (b) 2HDM Type-II (c) 2HDM Lepton-specific (d) 2HDM Flipped

ATLAS-CONF-2017-055

$A \to Zh \to (\nu\nu, \ell^+\ell^-)b\bar{b}$

(a) 2HDM Type-I

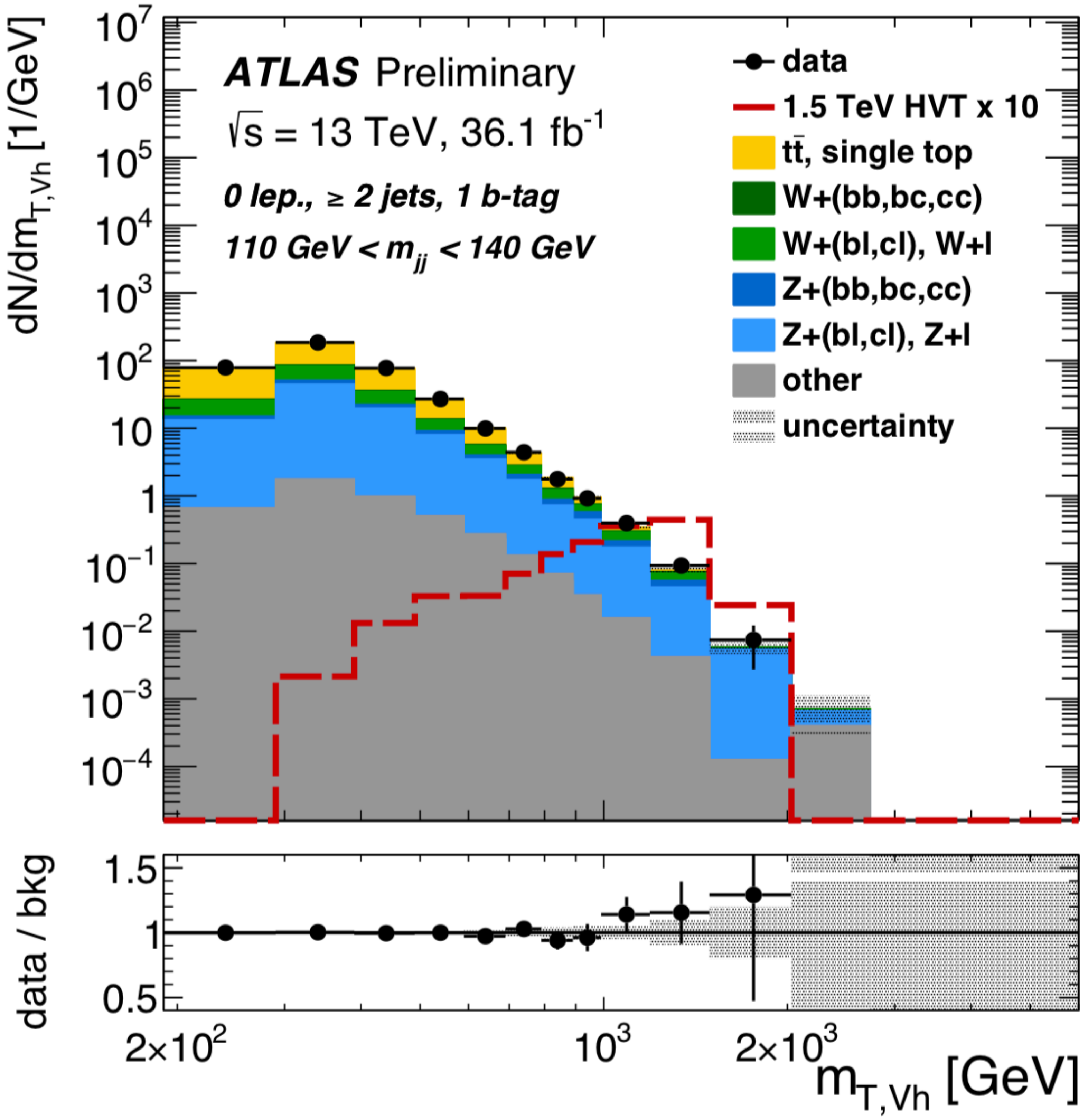
(b) 2HDM Type-II

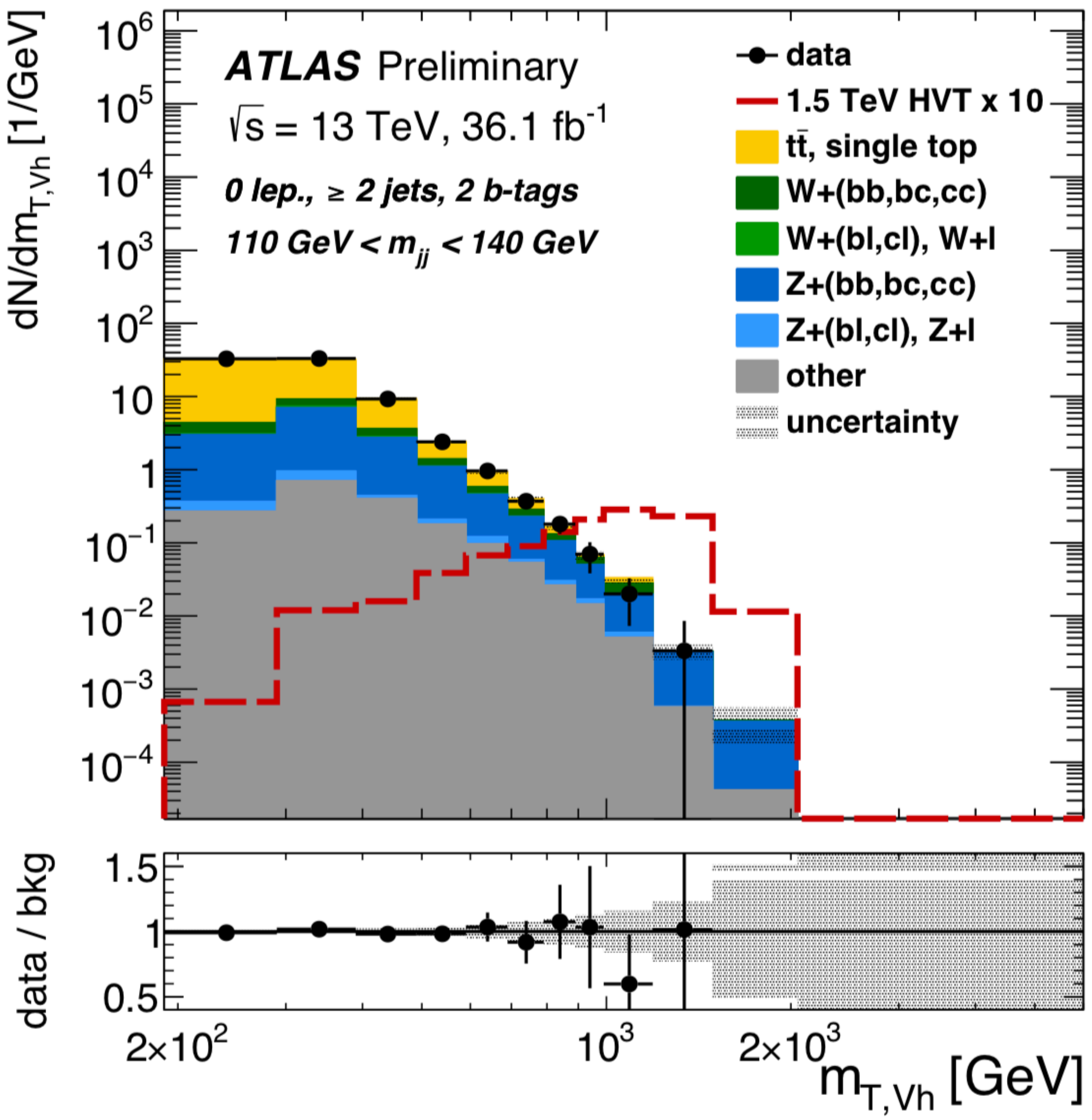
(c) 2HDM Lepton-specific

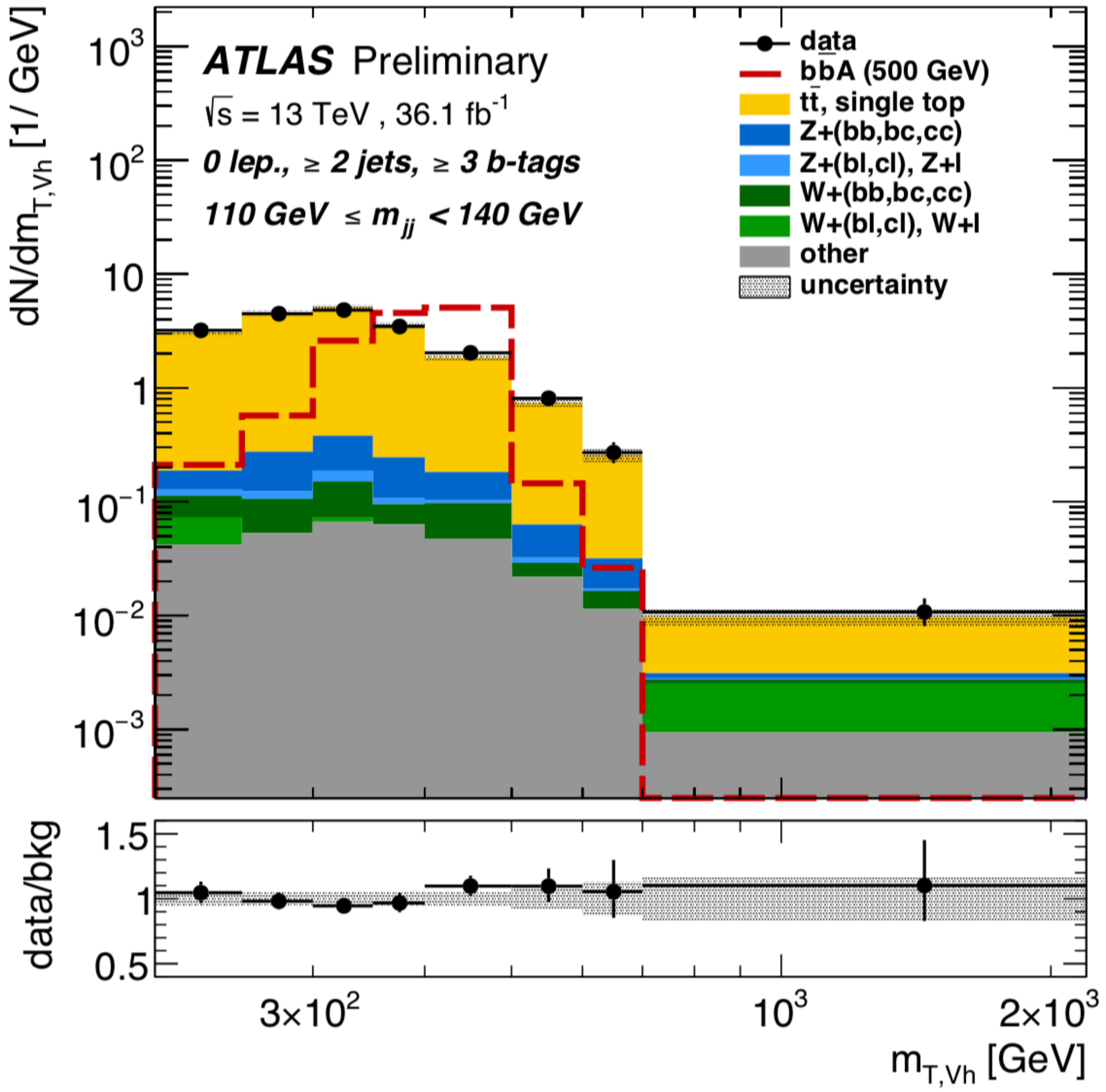
(d) 2HDM Flipped

ATLAS-CONF-2017-055

$Zh \to \nu\bar{\nu}b\bar{b}$







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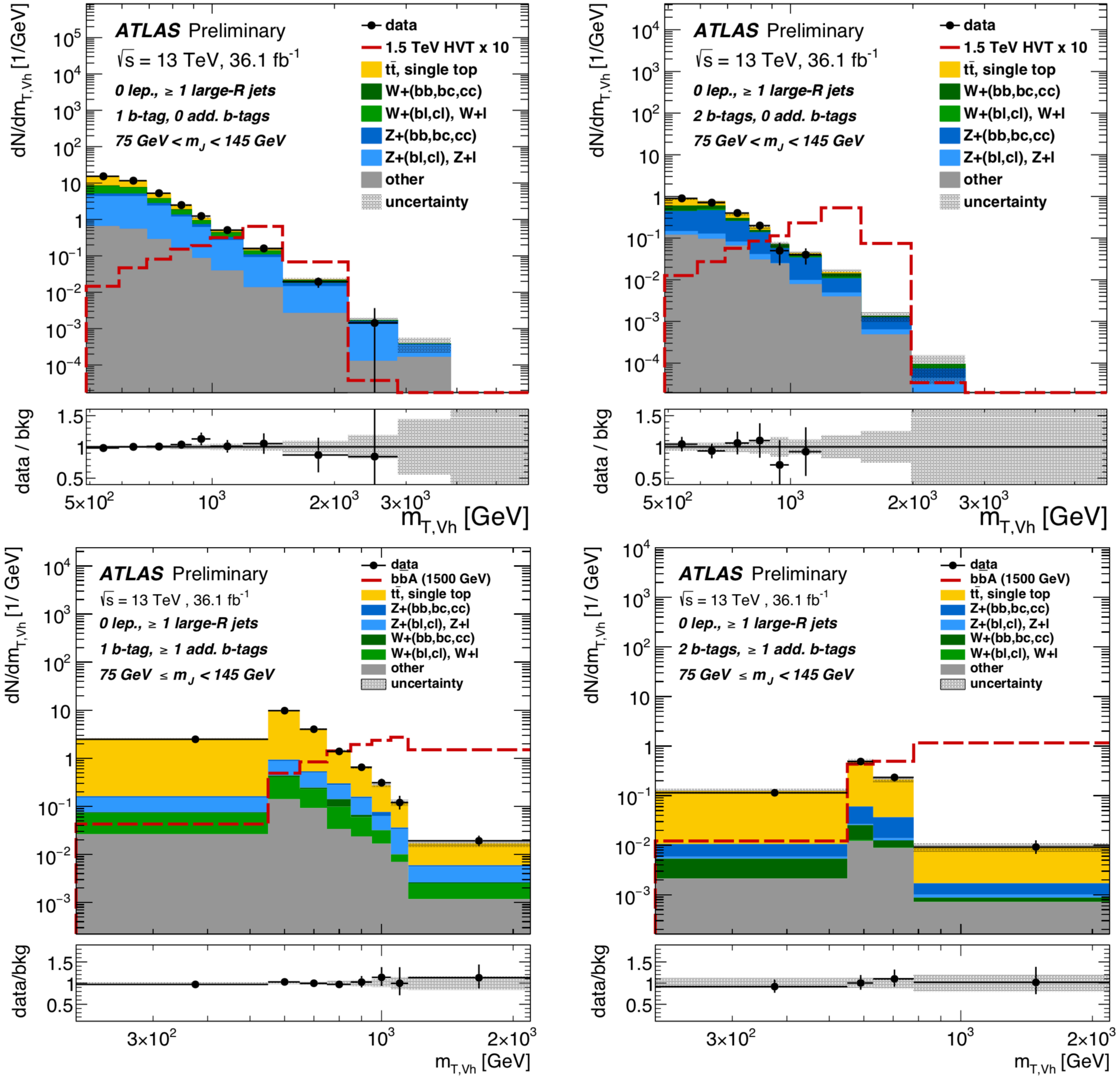
$Zh \to \nu\bar{\nu}b\bar{b}$



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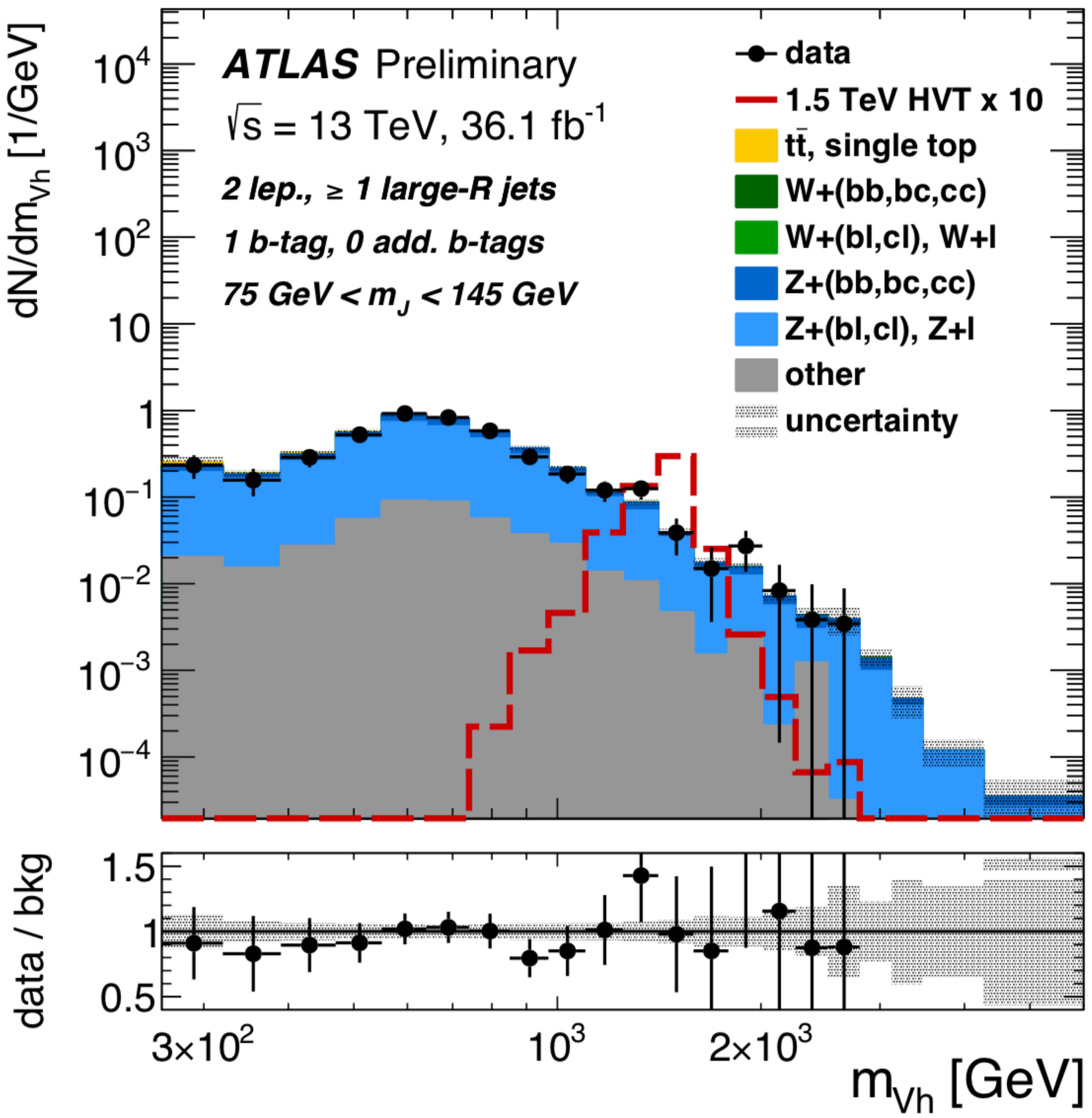
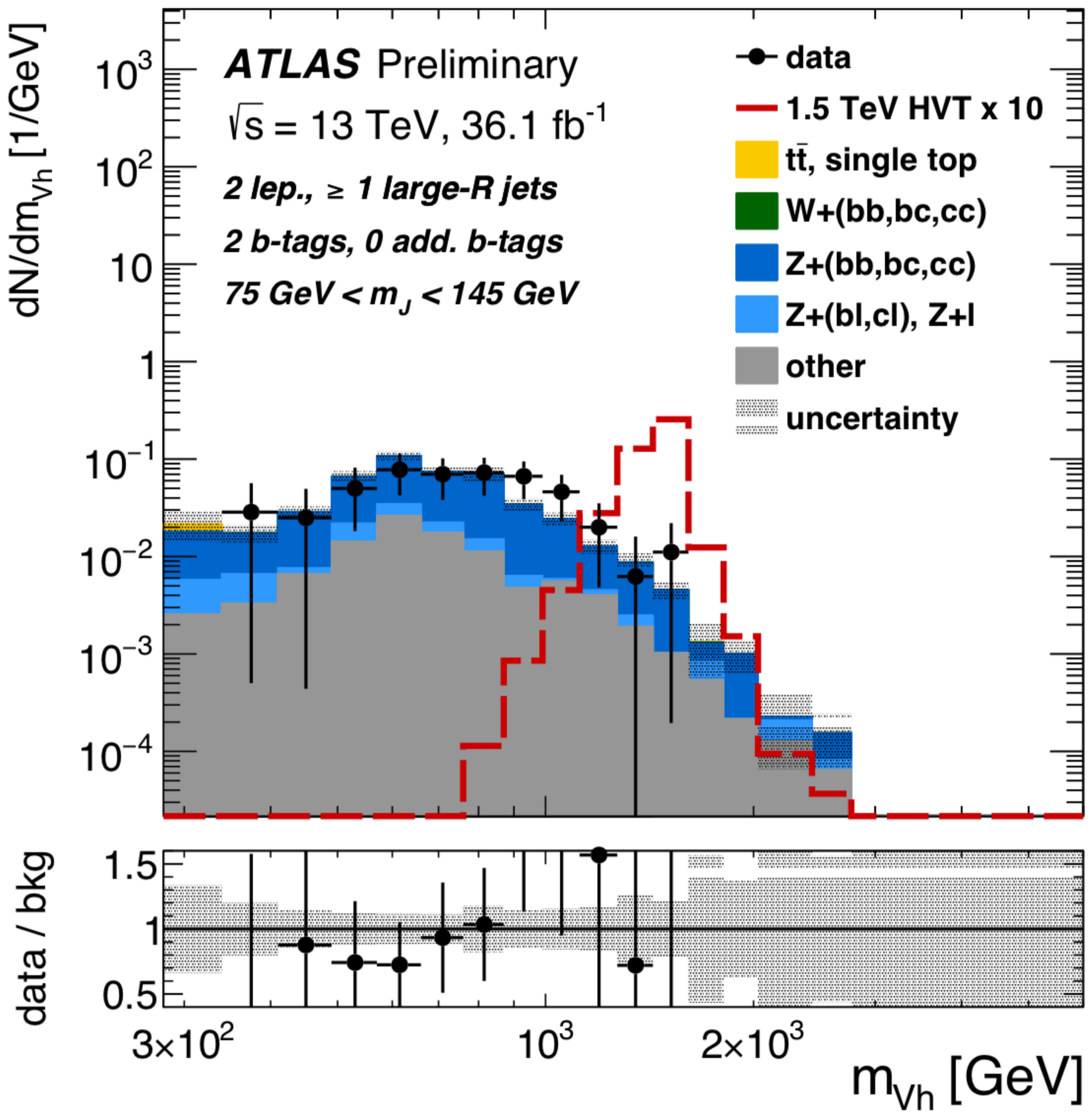
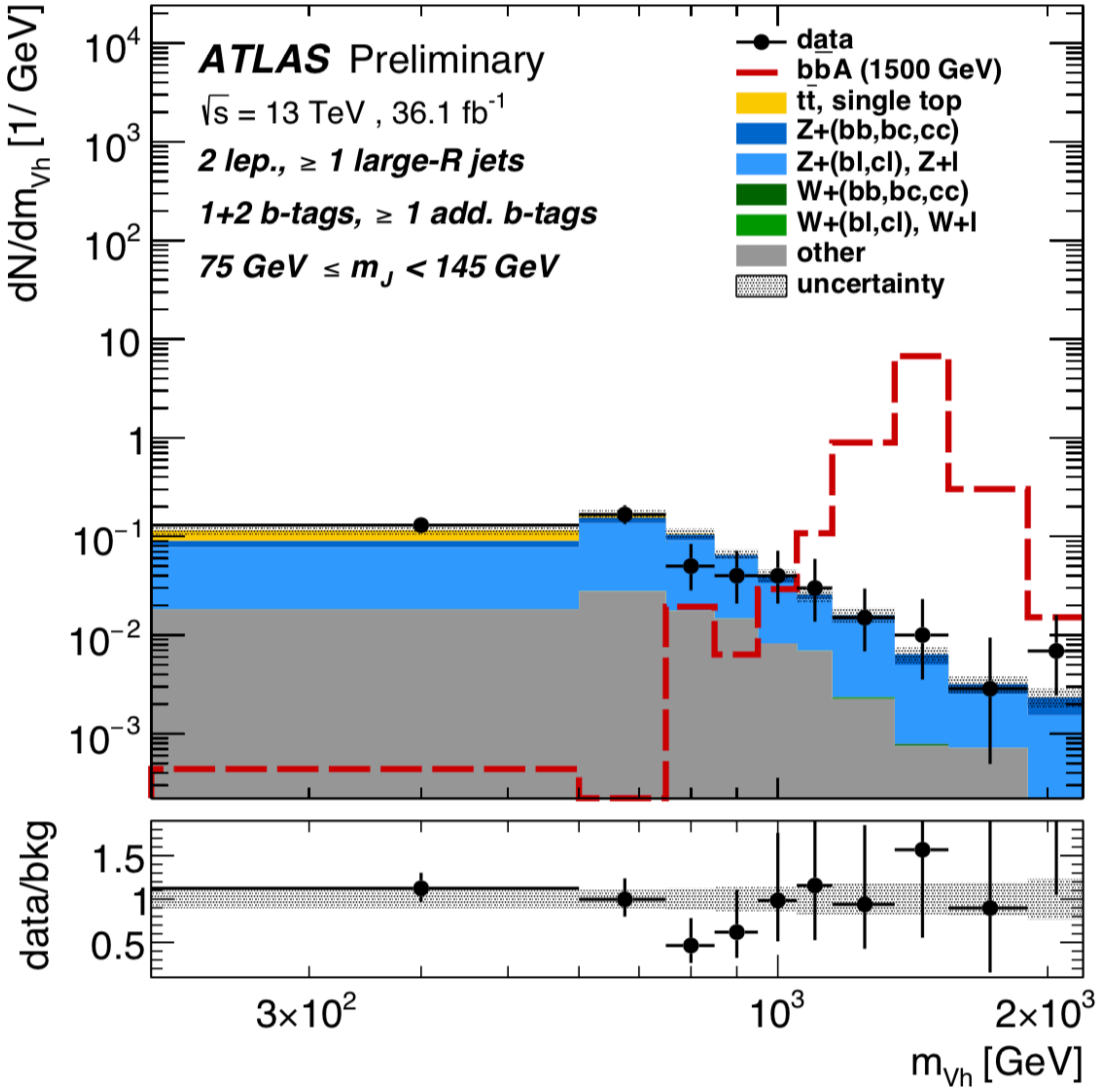
$Zh \to \ell^+\ell^-b\bar{b}$

ATLAS-CONF-2017-055

$Zh \to \ell^+\ell^- b\bar{b}$







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Vh→(leptons)+bb

variable	Resolved	Merged
	Common selections	
number of jets	$\geq$2 small-R jets (==2 or 3 1-lep.)	$\geq$1 large-R jet
leading jet p_{T} [GeV]	> 45	> 250
m_{jj}, m_{J} [GeV]	110–140 (0,1-lep.), 100–145 (2-lep.)	75–145
	0-lepton selection	
$E_{\mathrm{T}}^{\mathrm{miss}}$ [GeV]	> 150	> 200
$\sum p_T^{\mathrm{jet}_i}$ [GeV]	> 150 $(120^{(*)})$	–
$\Delta\phi(\mathrm{j},\mathrm{j})$	$< 7\pi/9$	–
$p_{\mathrm{T}}^{\mathrm{miss}}$ [GeV]	> 30	
$\Delta\phi(\vec{E}_{\mathrm{T}}^{\mathrm{miss}}, \vec{p}_{\mathrm{T}}^{\mathrm{\,miss}})$	$< \pi/2$	
$\Delta\phi(\vec{E}_{\mathrm{T}}^{\mathrm{miss}}, h)$	$> 2\pi/3$	
$\min[\Delta\phi(\vec{E}_{\mathrm{T}}^{\mathrm{miss}}, \text{small-}R\text{ jet})]$	$> \pi/9$ (2 or 3 jets) , $> \pi/6$ ($\geq$ 4 jets)	
$\mathrm{N}_{\tau_{\mathrm{had}}}$	0 **	
	1-lepton selection	
leading lepton p_{T} [GeV]	> 27	> 27
$E_{\mathrm{T}}^{\mathrm{miss}}$ [GeV]	$> 40(80^{(\ddagger)})$	> 100
p_{T}^{W} [GeV]	$> \max[150,\ 710 - 3.3\cdot 10^5\ GeV/m_{Vh}]$	$> \max[150,\ 394\cdot\ln(m_{Vh}/1\ GeV) - 2350]$
$m_T(W)$ [GeV]	¡300	
	2-lepton selection	
leading lepton p_{T} [GeV]	> 27	> 27
sub-leading lepton p_{T} [GeV]	> 7	> 25
$E_{\mathrm{T}}^{\mathrm{miss}}/\sqrt{H_{\mathrm{T}}}$ [$\sqrt{\mathrm{GeV}}$]	$< 1.15\ + 8\cdot 10^{-3}\ m_{Vh}/1\ GeV$	
$p_{T,\ell\ell}$ [GeV]	$> 20 + 9\cdot\sqrt{m_{Vh}/1GeV - 320}^{\ddagger\ddagger}$	
$m_{\ell\ell}$ [GeV]	[87 - $0.030\cdot m_{Vh}/1\ GeV$, $97 + 0.013\cdot m_{Vh}/1\ GeV$]	

ATLAS-CONF-2017-055

Vh→(leptons)+bb

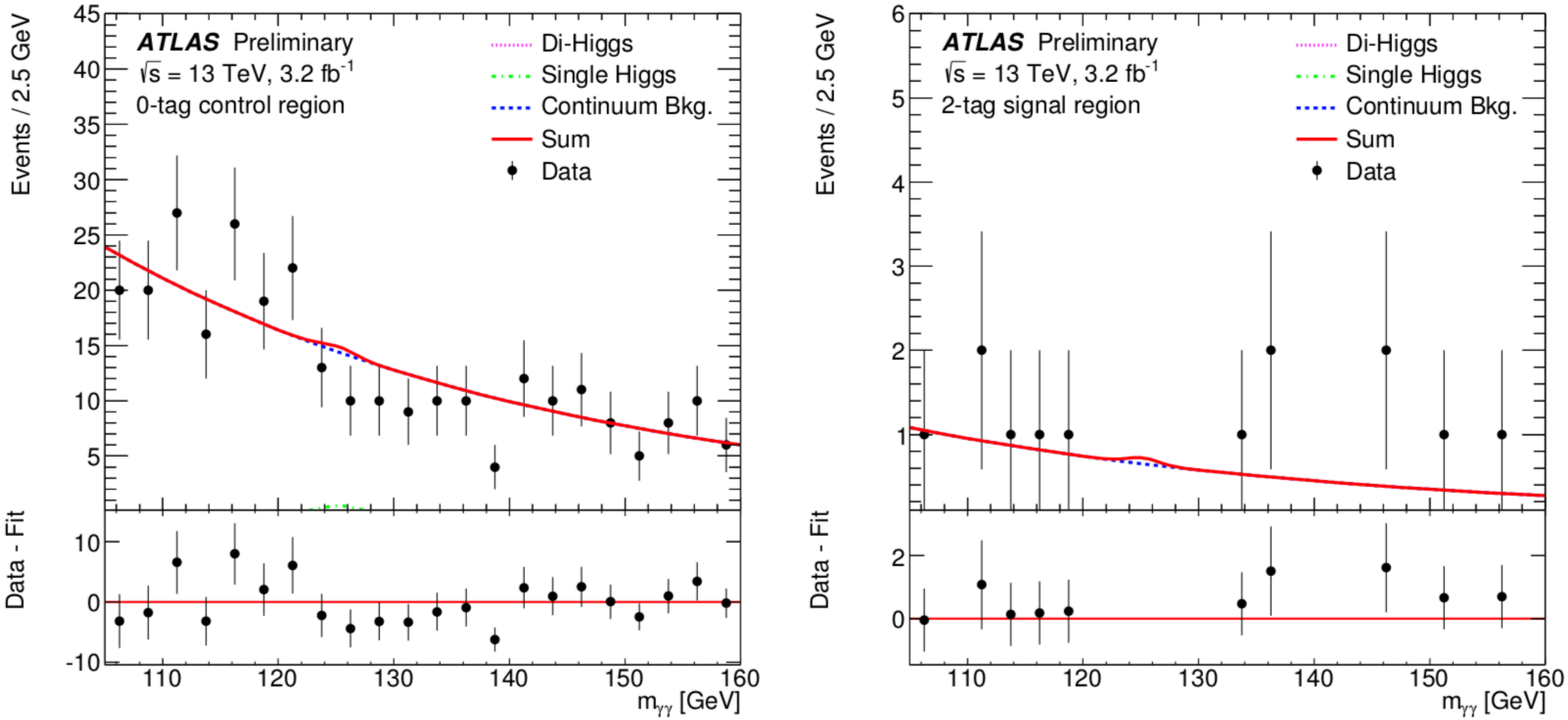
Systematics

Process	quantity/source	value
signal	acceptance	3-7%
SM Vh, $t\bar{t}V$, $t\bar{t}h$	norm.	50%
diboson	norm.	11%
multijet (1-lep.)	norm.	50%
	template method	S
single top	norm.	19%
	resolved / merged	24%
	m_{jj} SR / m_{jj} CR (1-lep.)	7%
$t\bar{t}$	resolved / merged	15-45%
	m_{jj} SR / m_{jj} CR (1-lep.)	7%
	SR / $e\mu$ CR (2-lep.)	2%
	PS, ISR/FSR, ME	S
	p_{T} reweight	S
$Z+(bb, bc, cc)$	resolved / merged	19%
	0-lep. / 2-lep.	15%
	generator, PDF, scale	S
$Z+(bl, cl)$	resolved / merged	28%
	0-lep. / 2-lep.	12%
	generator, PDF, scale	S

Process	quantity/source	value
$Z+l$	norm.	19%
	resolved / merged	23%
	0-lep. / 2-lep.	8%
	generator, PDF, scale	S
$W+(bb, bc, cc)$	norm. (A,Z')	26%
	resolved / merged	18-43%
	m_{jj} SR / m_{jj} CR (1-lep.)	6%
	0-lep. / 1-lep.	26%
	generator, PDF, scale	S
$W+(bl, cl)$	norm. (A,Z')	23%
	resolved / merged	15-35%
	m_{jj} SR / m_{jj} CR (1-lep.)	5%
	0-lep. / 1-lep.	22%
	generator, PDF, scale	S
$W+l$	norm.	20-30%
	resolved / merged	16-20%
	m_{jj} SR / m_{jj} CR (1-lep.)	2%
	0-lep. / 1-lep.	19%
	generator, PDF, scale	S

ATLAS-CONF-2016-004

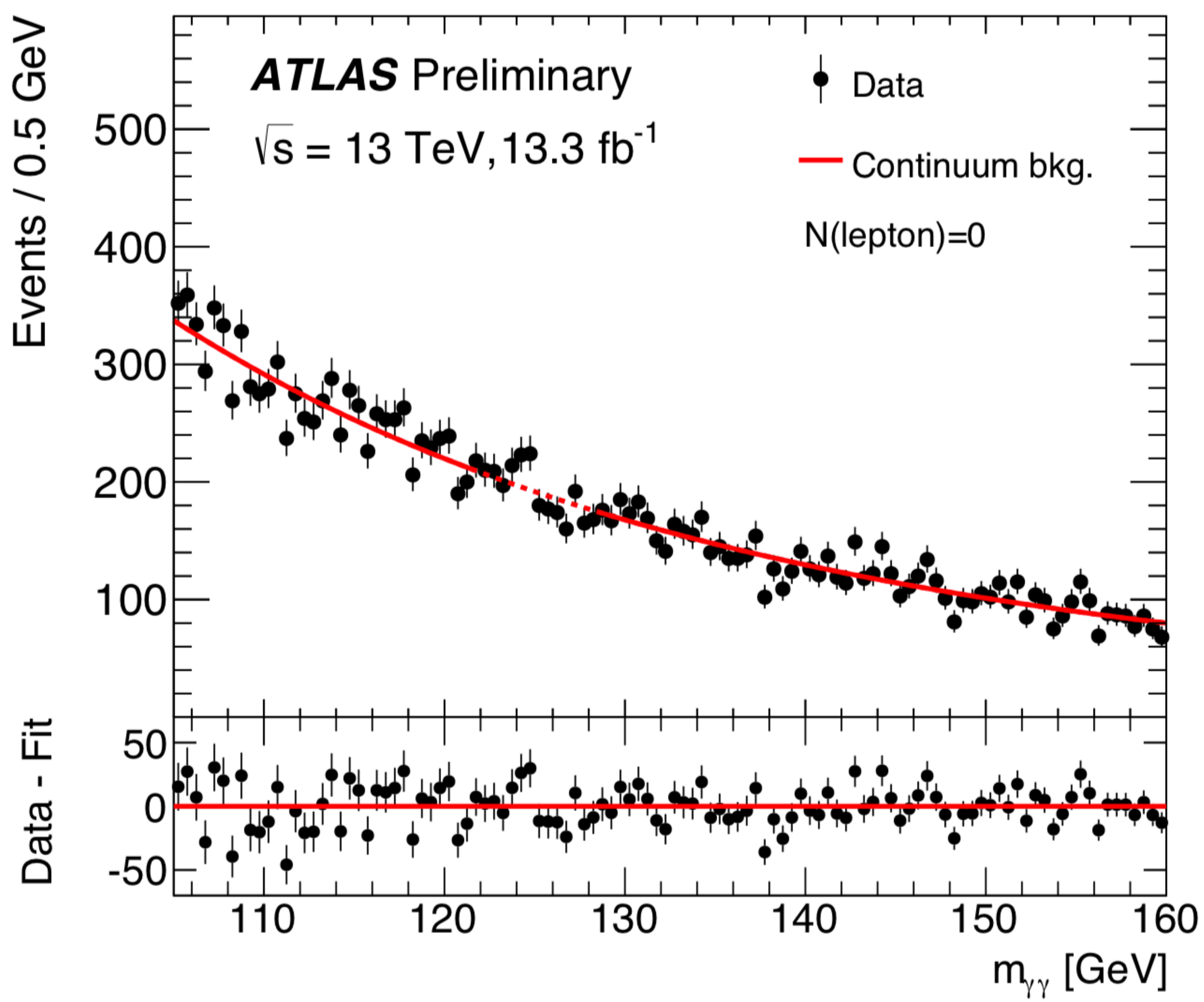
$hh \to b\bar{b}\gamma\gamma$

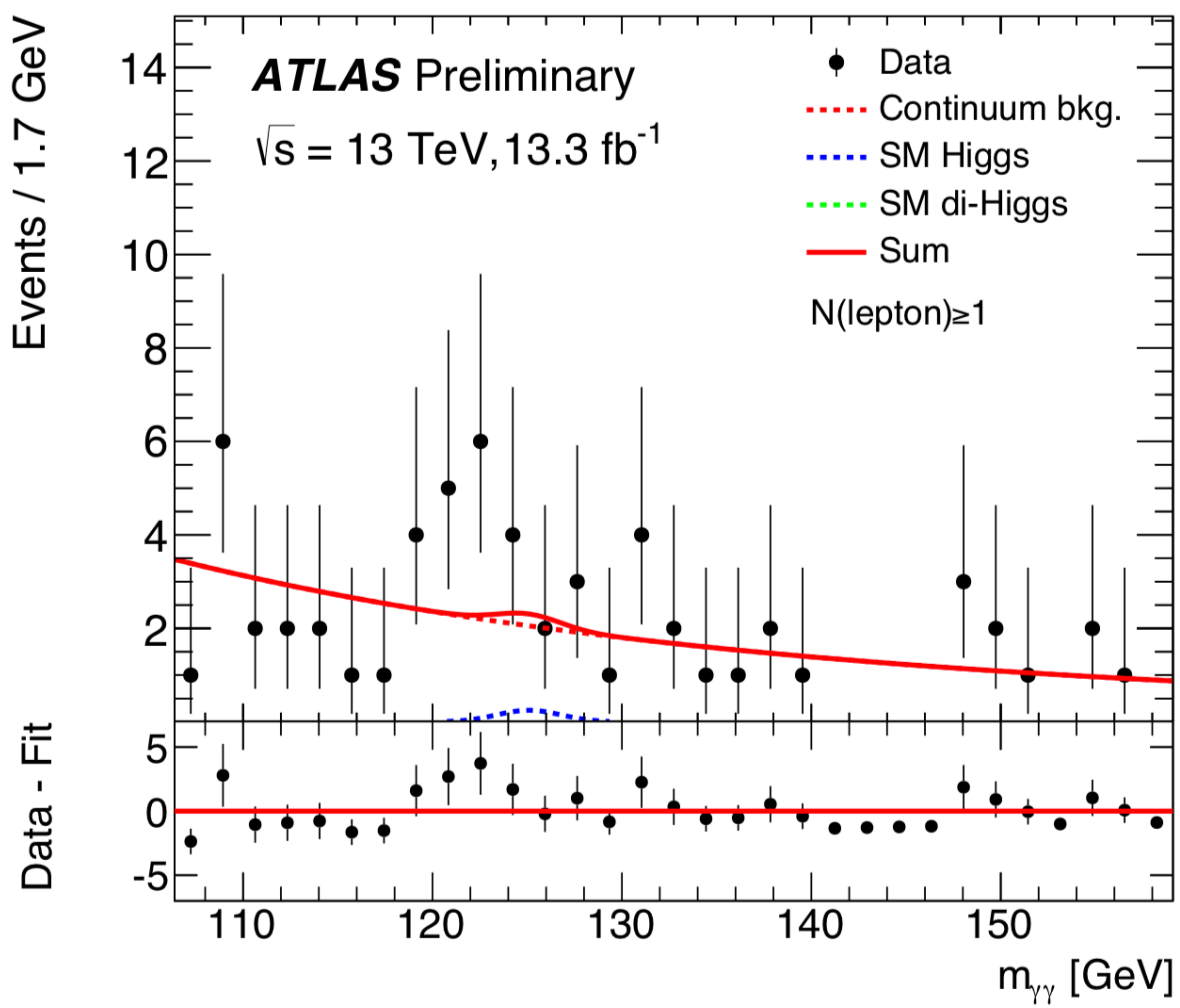


No events in m_h window

ATLAS-CONF-2016-071

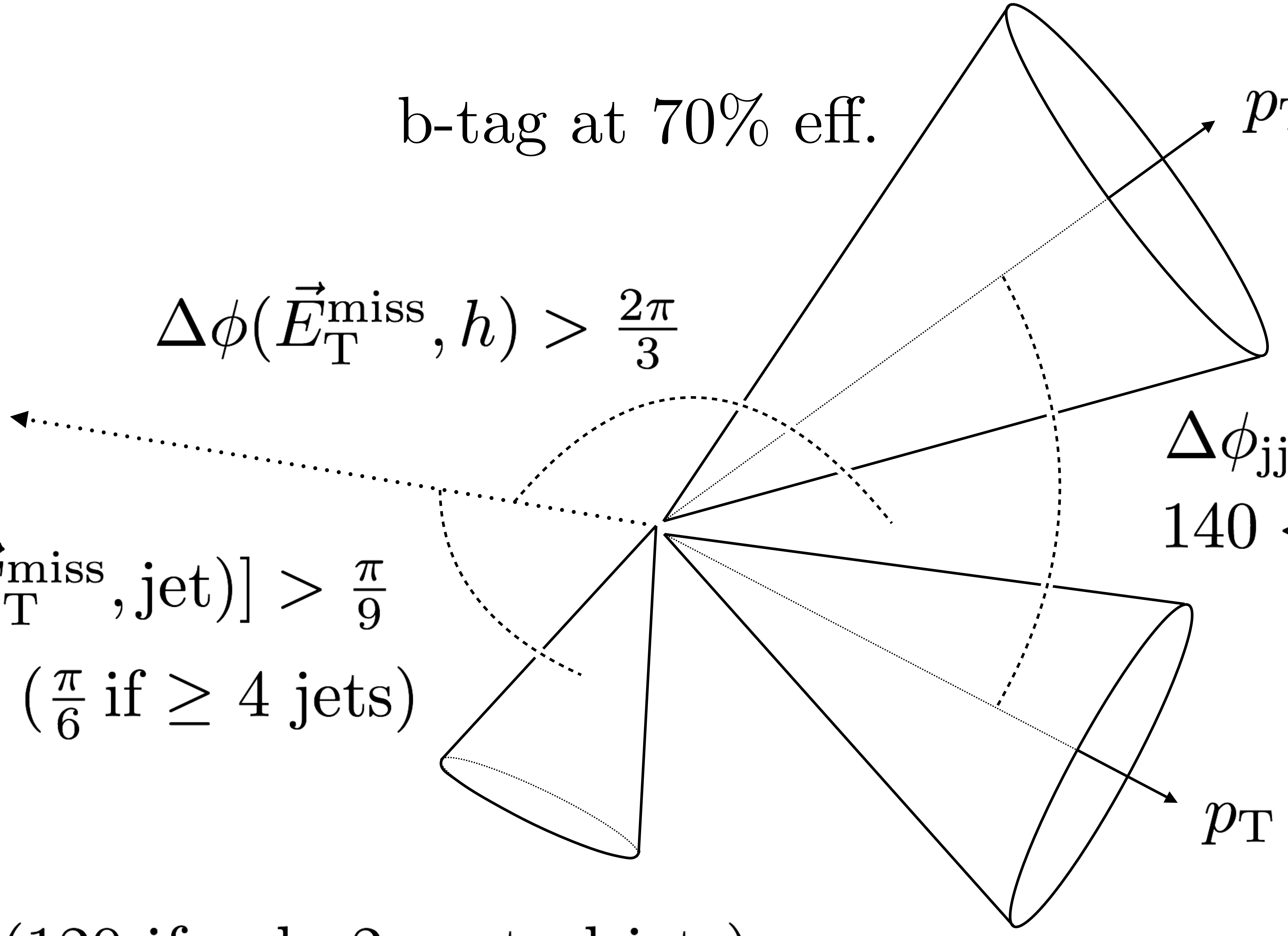
$hh \to WW^*\gamma\gamma$





ATLAS-CONF-2017-055

$Zh \to \nu\bar{\nu}b\bar{b}$

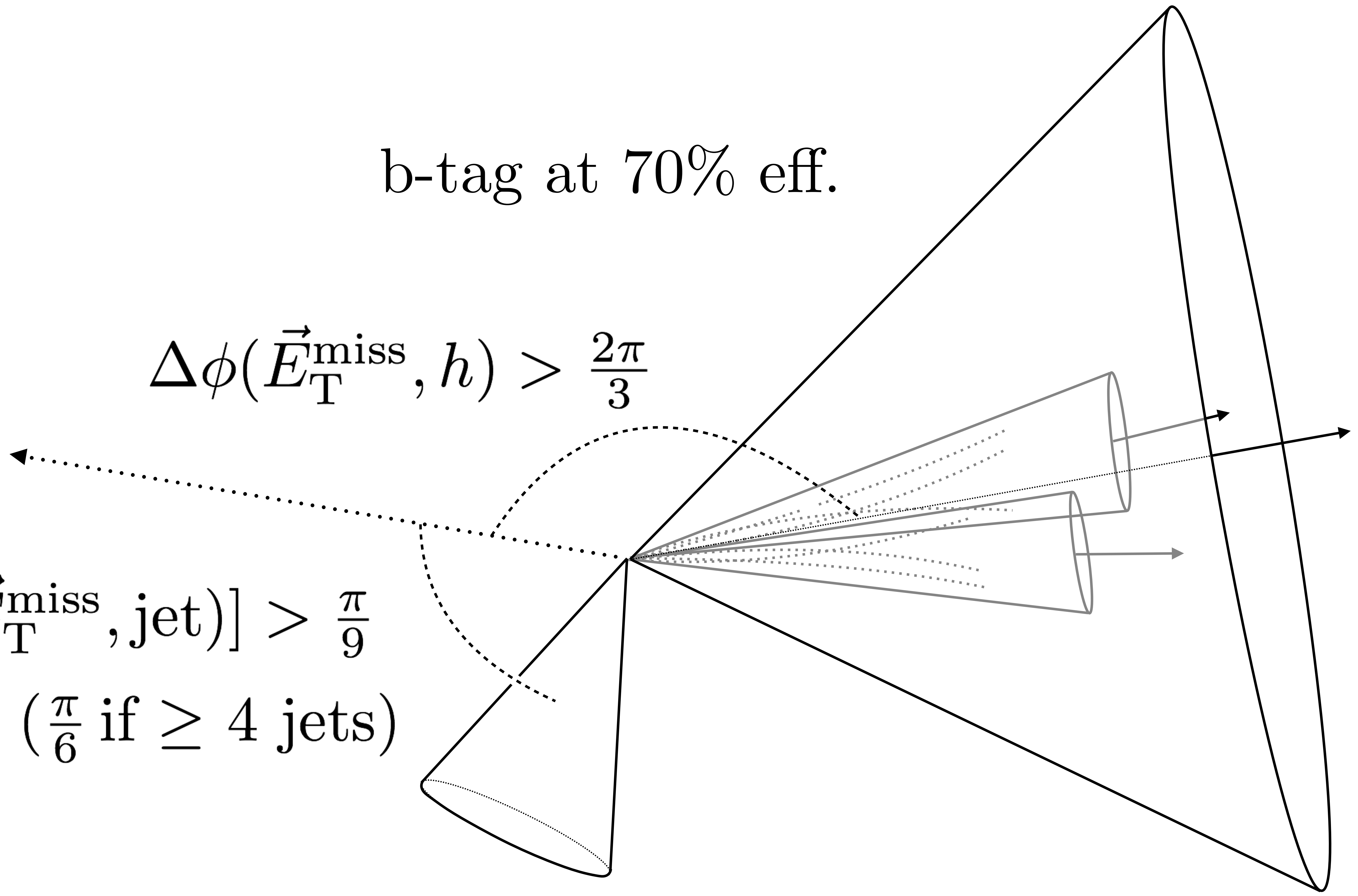


$\sum p_{\mathrm{T}}^{\mathrm{jet}_i} > 150\,(120$ if only 2 central jets$)$

$$m_{\mathrm{T},Vh} = \sqrt{(E_{\mathrm{T}}^{h} + E_{\mathrm{T}}^{\mathrm{miss}})^2 - (\vec{p}_{\mathrm{T}}^{\,h} + \vec{E}_{\mathrm{T}}^{\mathrm{miss}})^2}$$

ATLAS-CONF-2017-055

$Zh \to \nu\bar{\nu}b\bar{b}$



b-tag at 70% eff.

$\Delta\phi(\vec{E}_{\mathrm{T}}^{\mathrm{miss}}, h) > \frac{2\pi}{3}$

$E_{\mathrm{T}}^{\mathrm{miss}} > 200$

$\min[\Delta\phi(\vec{E}_{\mathrm{T}}^{\mathrm{miss}}, \mathrm{jet})] > \frac{\pi}{9}$

$(\frac{\pi}{6}$ if ≥ 4 jets)

$p_{\mathrm{T}} > 250$

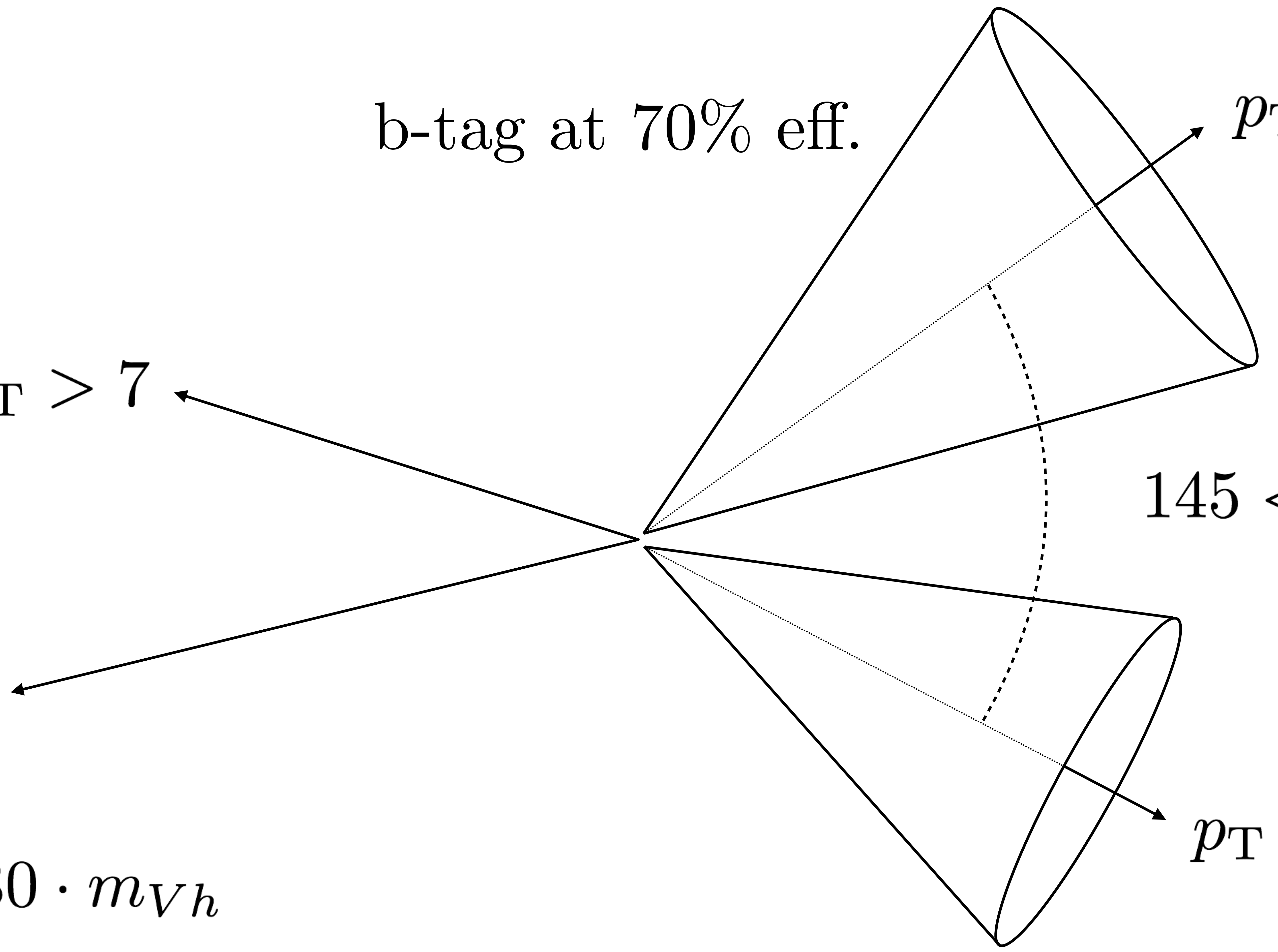
$145 < m_{\mathrm{J}} < 75$

≥ 1 track jet
with $p_{\mathrm{T}} \geq 10$

$$m_{\mathrm{T},Vh} = \sqrt{(E_{\mathrm{T}}^{h} + E_{\mathrm{T}}^{\mathrm{miss}})^2 - (\vec{p}_{\mathrm{T}}^{\,h} + \vec{E}_{\mathrm{T}}^{\mathrm{miss}})^2}$$

ATLAS-CONF-2017-055

$Zh \to \ell^+\ell^- b\bar{b}$



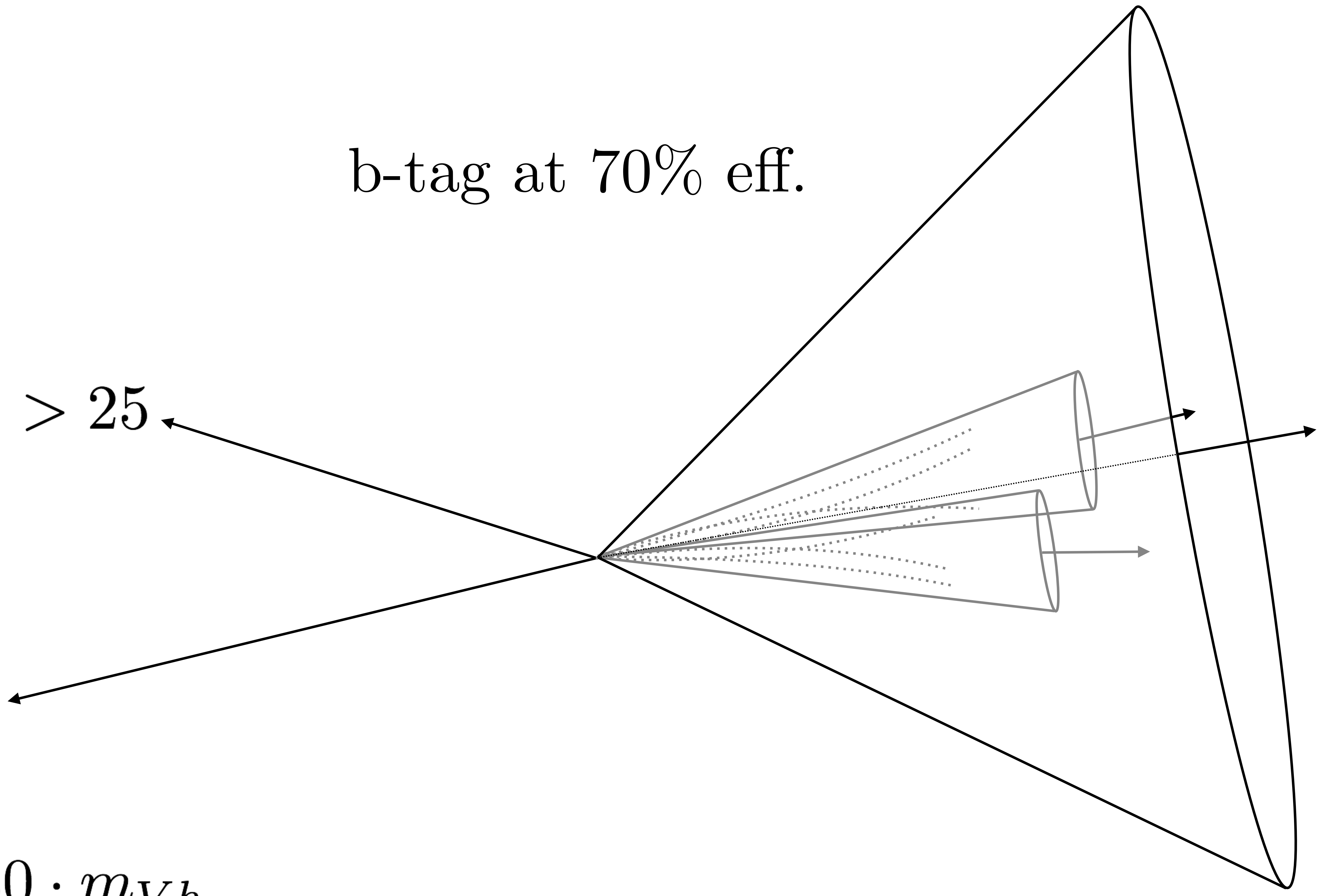
$$m_{\ell\ell} \begin{cases} > 87 - 0.030 \cdot m_{Vh} \\ < 97 + 0.013 \cdot m_{Vh} \end{cases}$$

$$p_{\mathrm{T},\ell\ell} > 20 + 9 \cdot \sqrt{m_{Vh} - 320} \qquad E_{\mathrm{T}}^{\mathrm{miss}}/\sqrt{H_{\mathrm{T}}} < 1.15 + 8 \cdot 10^{-3} m_{Vh}$$

ATLAS-CONF-2017-055

$Zh \to \ell^+\ell^- b\bar{b}$

b-tag at 70% eff.



$p_\mathrm{T} > 25$

$p_\mathrm{T} > 27$

$p_\mathrm{T} > 250$

$145 < m_\mathrm{J} < 75$

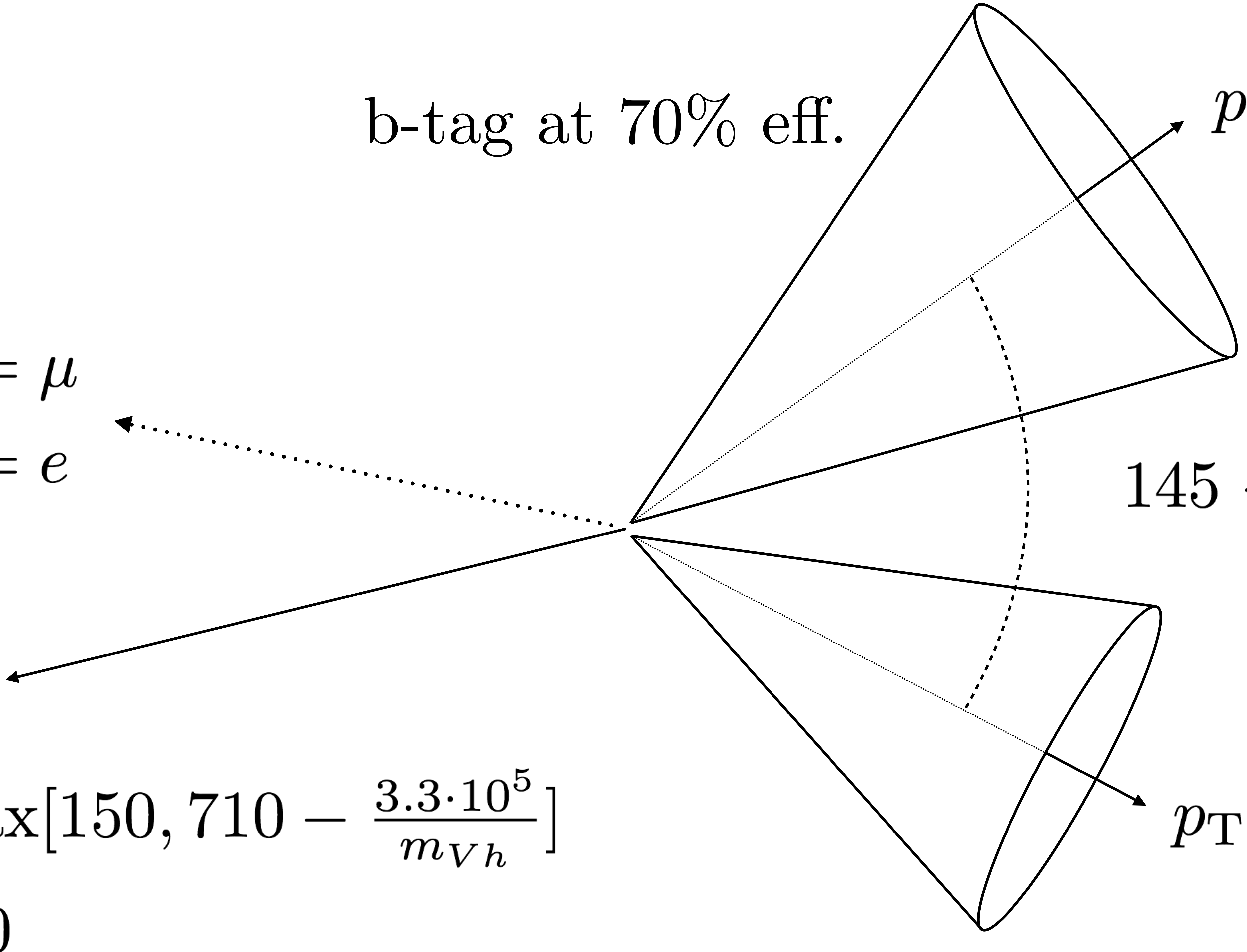
≥ 1 track jet with $p_\mathrm{T} \geq 10$

$$m_{\ell\ell} \begin{cases} > 87 - 0.030 \cdot m_{Vh} \\ < 97 + 0.013 \cdot m_{Vh} \end{cases}$$

$$p_{\mathrm{T},\ell\ell} > 20 + 9 \cdot \sqrt{m_{Vh} - 320} \qquad E_\mathrm{T}^\mathrm{miss}/\sqrt{H_\mathrm{T}} < 1.15 + 8 \cdot 10^{-3} m_{Vh}$$

ATLAS-CONF-2017-055

$Wh \to \ell^{\pm}\nu b\bar{b}$



$p_{\mathrm{T}}^{W} > \max[150, 710 - \frac{3.3 \cdot 10^5}{m_{Vh}}]$

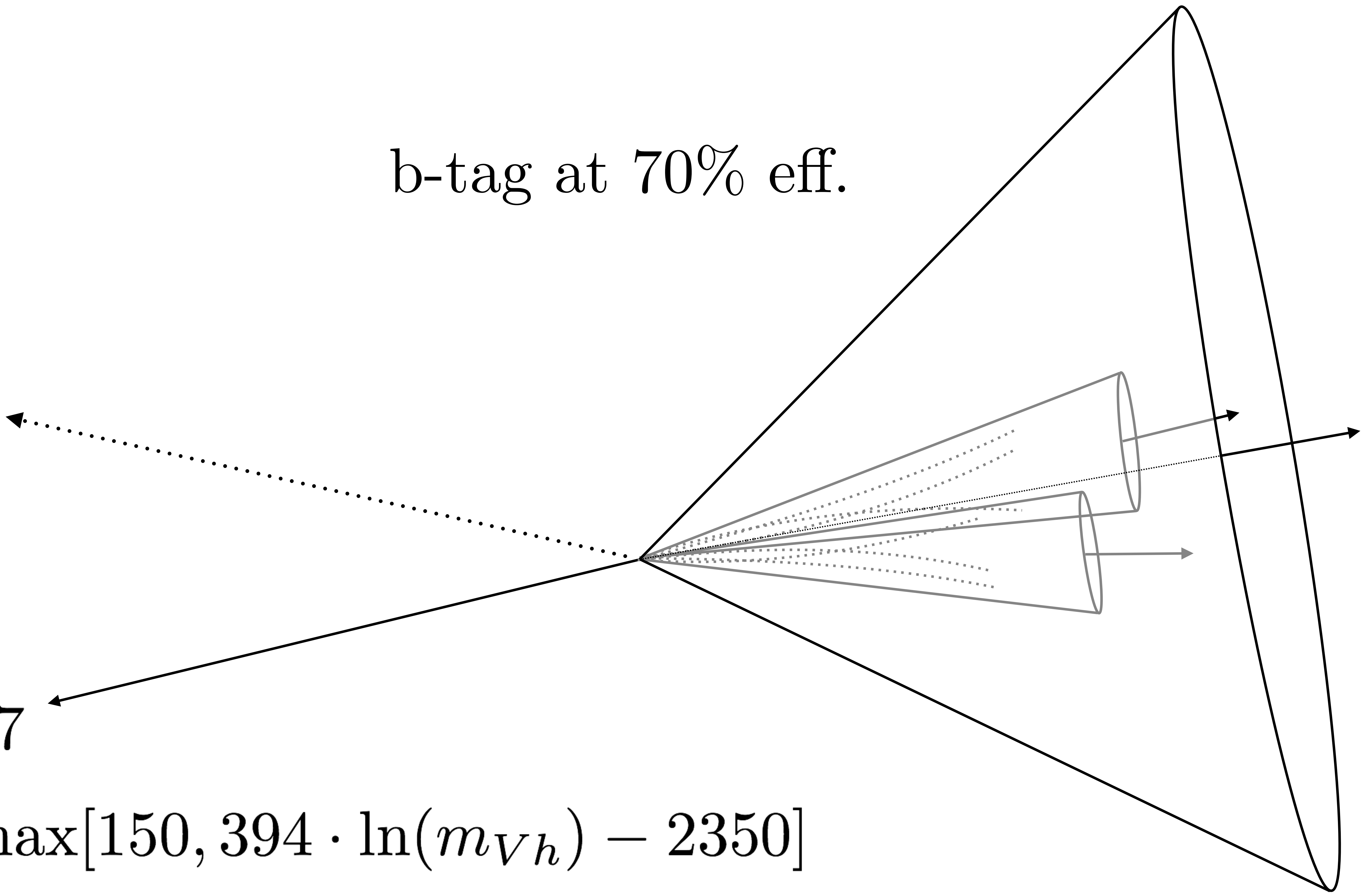
$m_{\mathrm{T}}^{W} < 300$

m_{Vh} constructed by setting p_z^{ν} such that $\mathrm{mass}(E_{\mathrm{T}}^{\mathrm{miss}}, p_{\mathrm{T}}^{\ell}) = m_W$

ATLAS-CONF-2017-055

$Wh \to \ell^{\pm}\nu b\bar{b}$

b-tag at 70% eff.



$E_{\mathrm{T}}^{\mathrm{miss}} > 100$

$p_{\mathrm{T}} > 250$

$145 < m_{\mathrm{J}} < 75$

≥ 1 track jet with $p_{\mathrm{T}} \geq 10$

$p_{\mathrm{T}} > 27$

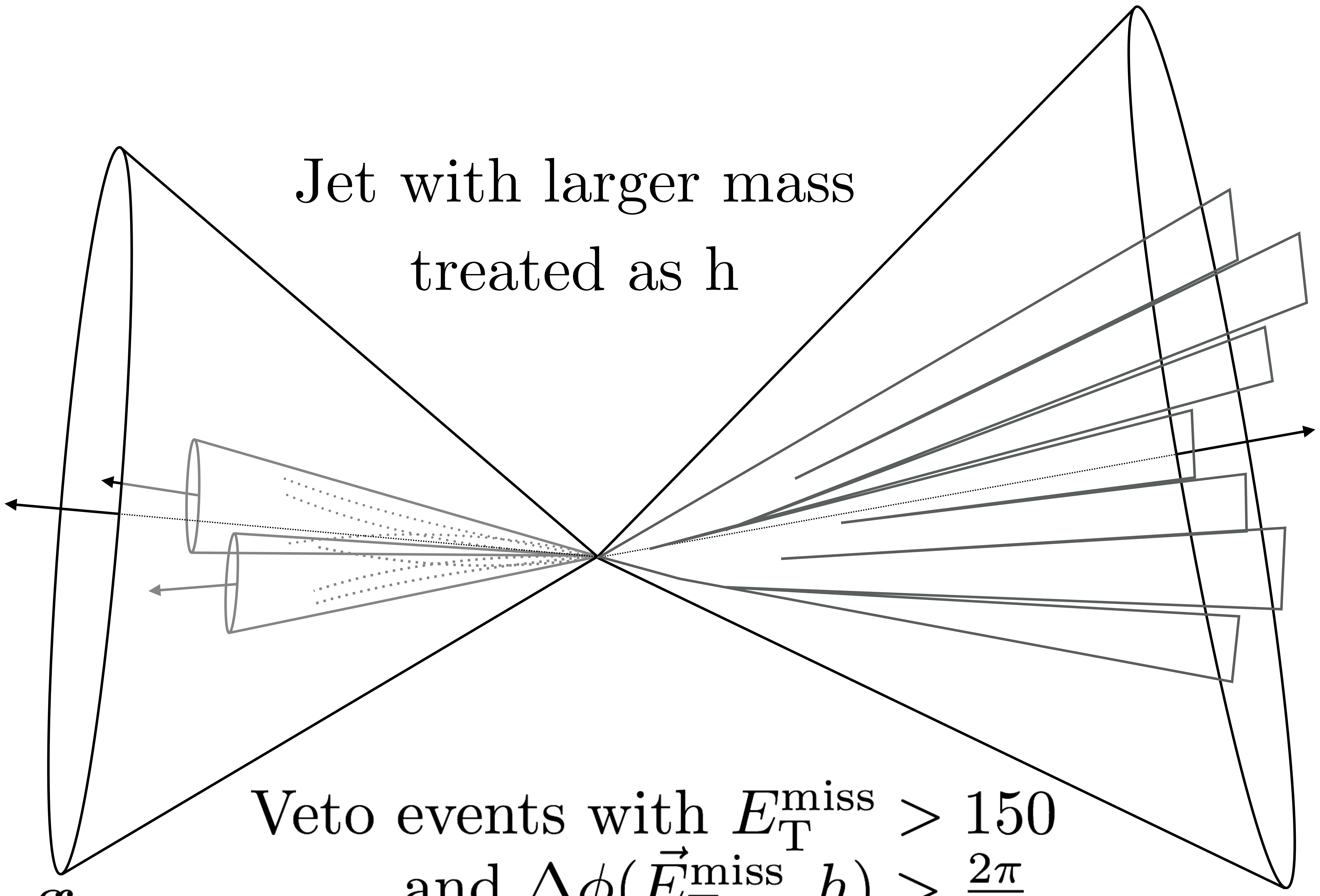
$p_{\mathrm{T}}^{W} > \max[150, 394 \cdot \ln(m_{Vh}) - 2350]$

$m_{\mathrm{T}}^{W} < 300$

m_{Vh} constructed by setting p_z^{ν} such that $\mathrm{mass}(E_{\mathrm{T}}^{\mathrm{miss}}, p_{\mathrm{T}}^{\ell}) = m_W$

[Phys. Let. B]

$Vh \to q\bar{q}b\bar{b}$



Jet with larger mass treated as h

$p_{\mathrm{T}}^{sublead} > 250$

$p_{\mathrm{T}}^{lead} > 450$

<u>h Jet:</u>

$\geq$ 1 track jet
with $p_{\mathrm{T}} \geq 10$
b-tag at 77% eff.

$145 > m_{\mathrm{J}} < 75$

Veto events with $E_{\mathrm{T}}^{\mathrm{miss}} > 150$
and $\Delta\phi(\vec{E}_{\mathrm{T}}^{\mathrm{miss}}, h) > \frac{2\pi}{3}$

<u>V Jet:</u>

mass <u>window</u> 68%

D2 <u>cut</u> to 50% eff.

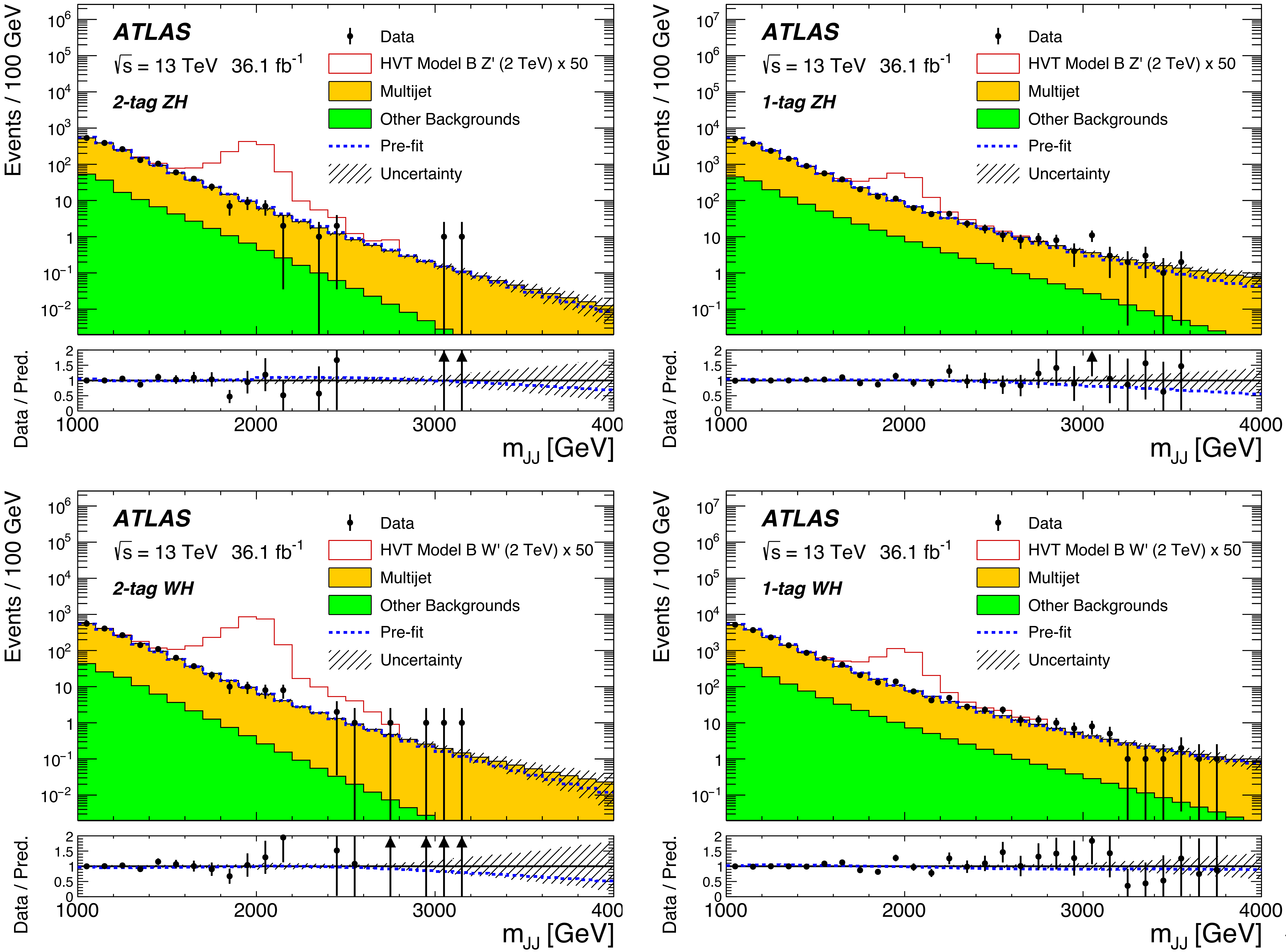
Flat eff. in p_{T}

<u>ATL-PHYS-PUB-2015-033</u>

Phys. Let. B

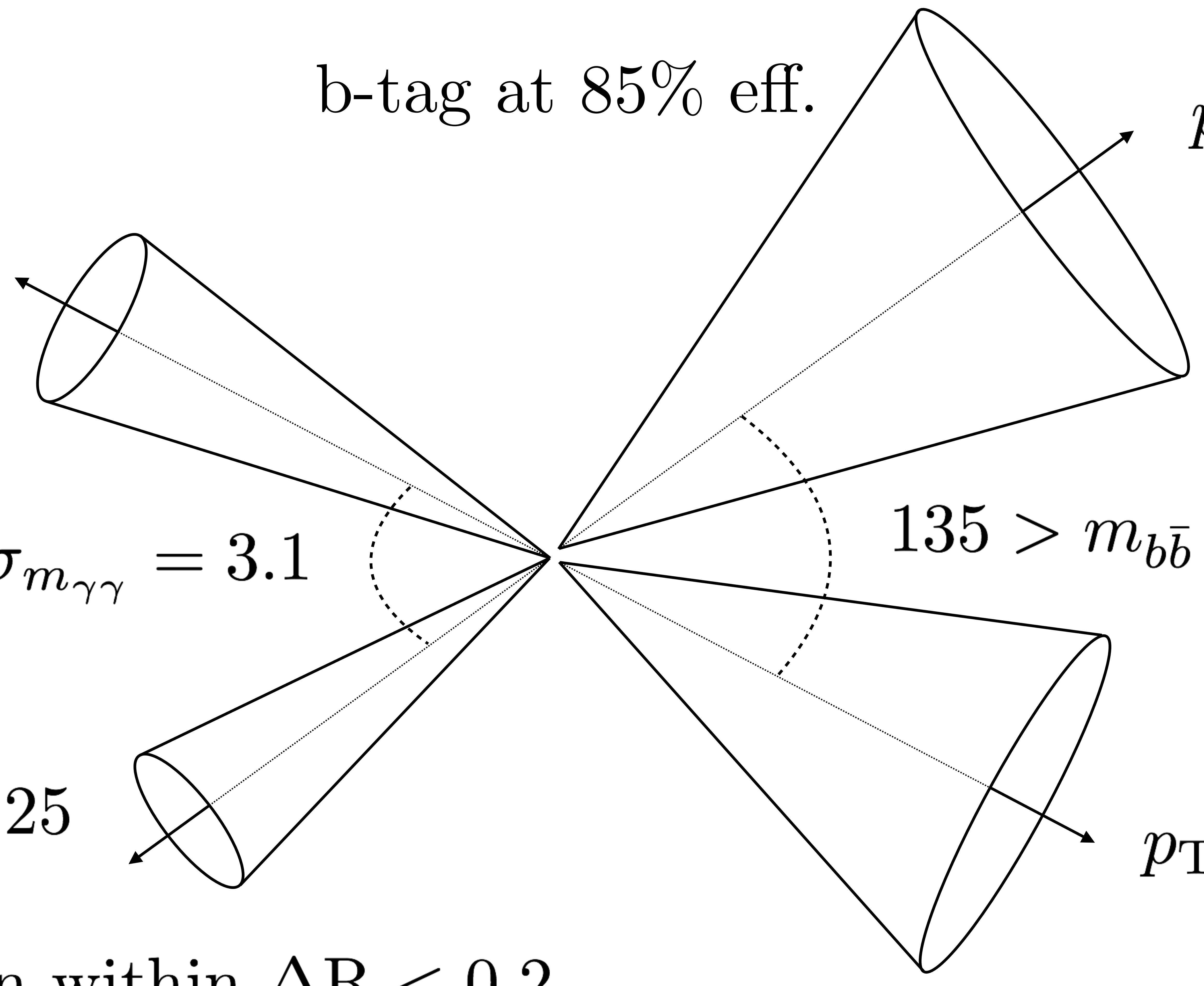
$Vh \to q\bar{q}b\bar{b}$

- Multijet background from 0-tag data
  - Normalized in high h mass sideband
  - Kinematic reweighting for 2-tag model



ATLAS-CONF-2016-004

$hh \to b\bar{b}\gamma\gamma$



ATLAS-CONF-2016-071

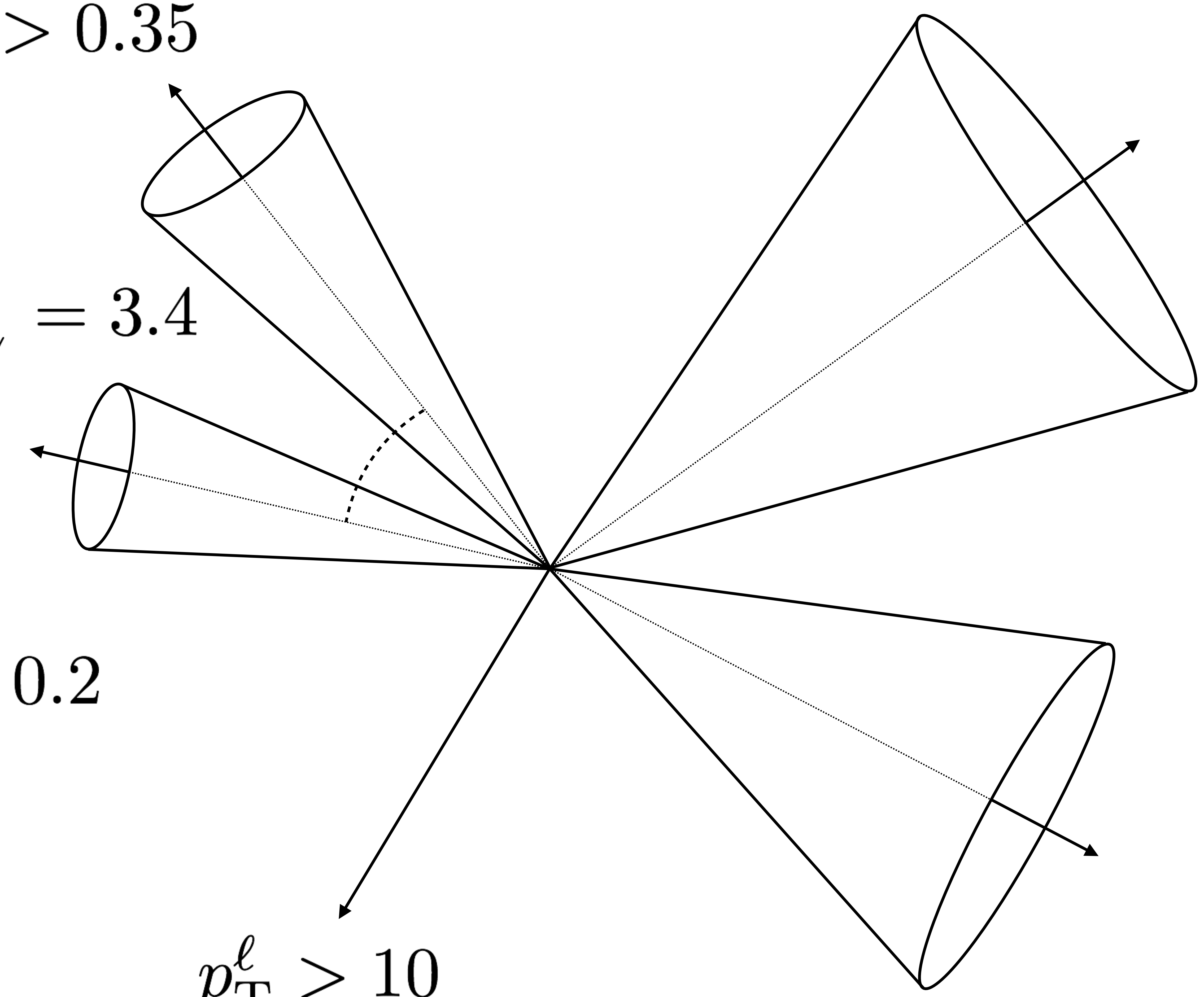
$hh \to WW^*\gamma\gamma$

$E_{\mathrm{T}}/m_{\gamma\gamma} > 0.35$

$|m_{\gamma\gamma} - 125.09| < 2\sigma_{m_{\gamma\gamma}} = 3.4$

$E_{\mathrm{T}}/m_{\gamma\gamma} > 0.25$

Track and calorimeter isolation within $\Delta\mathrm{R} < 0.2$



≥ 2 jets
$p_{\mathrm{T}} > 25$

Veto events with b-jets
(Tagged at 70% eff.
≈8% c-jet mis-tag)

$p_{\mathrm{T}}^{\ell} > 10$

$$|\eta| < \begin{cases} 2.47 \text{ (excluding } 1.37 < |\eta| < 1.52) \text{ if } \ell = e \\ 2.7 \text{ if } \ell = \mu \end{cases}$$

ATLAS-CONF-2016-049

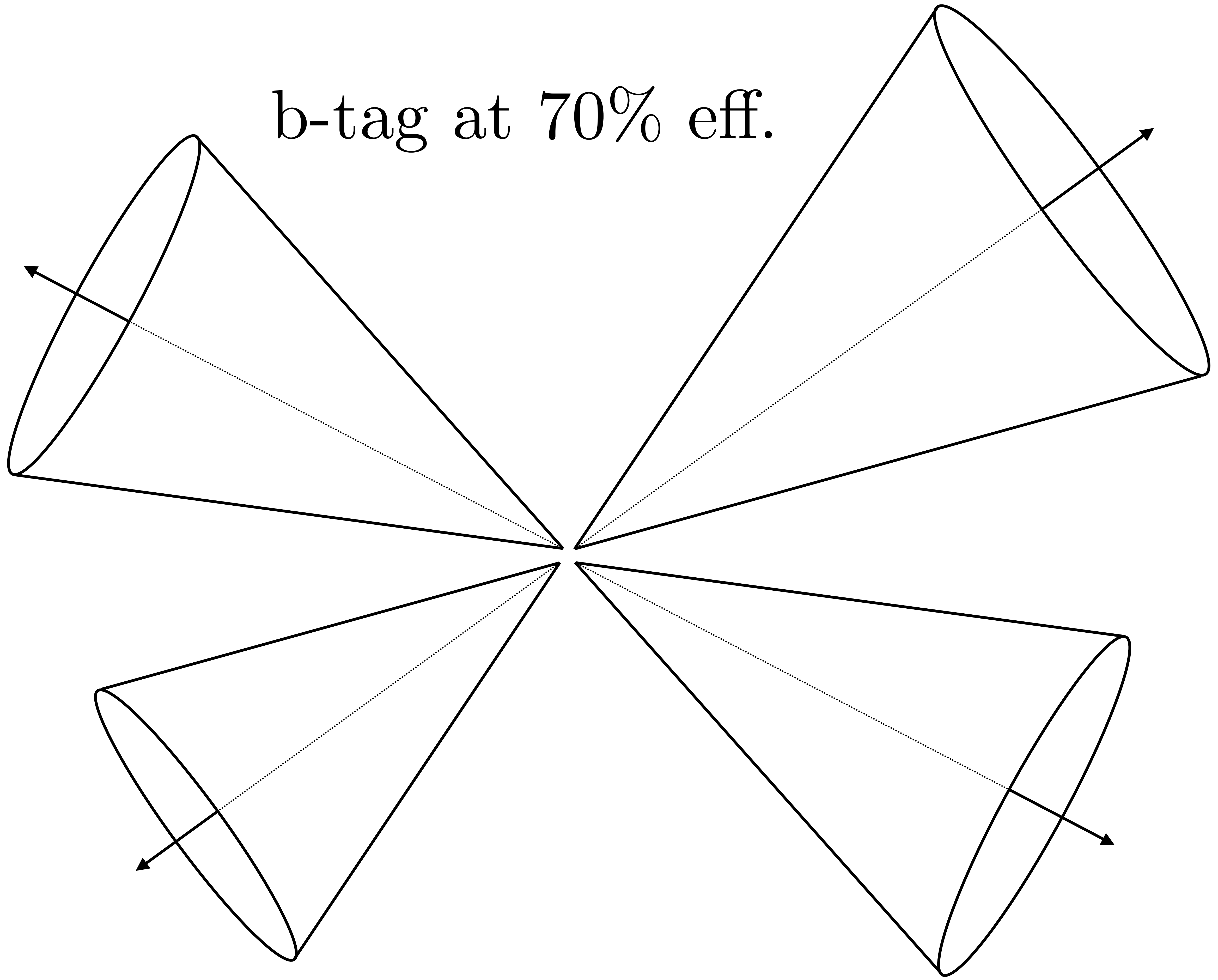
$hh \to b\bar{b}b\bar{b}$

≥ 4 b-jets, $p_{\mathrm{T}} > 30$

Consider pairings in sliding ΔR_{jj} windows as function of $m_{4\mathrm{j}}$

Construct di-jets with pairing that minimizes di-jet mass difference

$$p_{\mathrm{T}}^{\mathrm{h_1}} > 0.50 m_{4\mathrm{j}} - 90$$
$$p_{\mathrm{T}}^{\mathrm{h_2}} > 0.33 m_{4\mathrm{j}} - 70$$
$$|\Delta\eta_{hh}| < \max[1.1, 2 \times 10^{-3} m_{4\mathrm{j}} - 0.6]$$
$$\Delta R_{hh} > 1.5$$

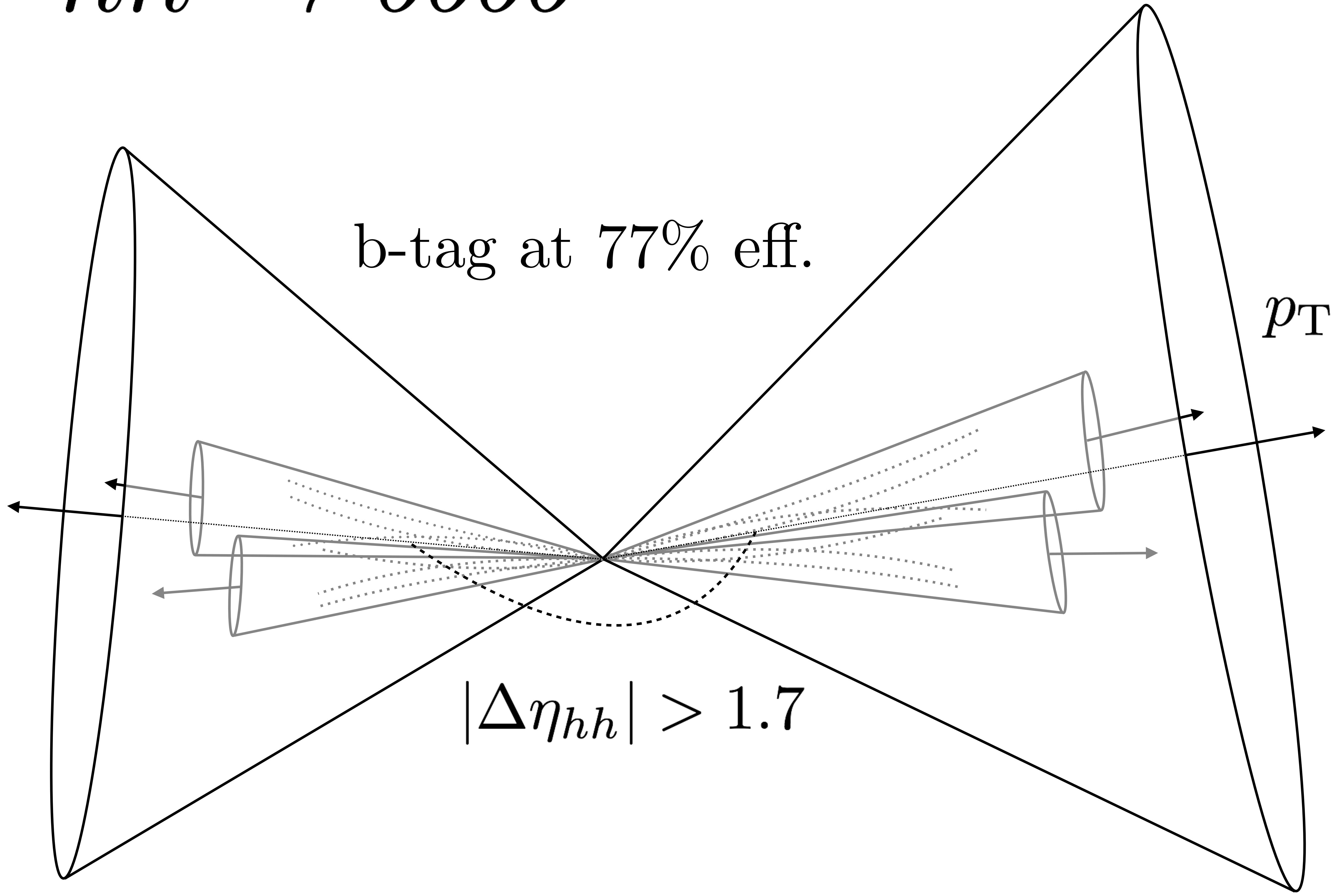


$$X_{hh} = \sqrt{\left(\frac{m_{h_1} - 120}{0.1 m_{h_1}}\right)^2 \left(\frac{m_{h_2} - 115}{0.1 m_{h_2}}\right)^2} < 1.6$$

ATLAS-CONF-2016-049

$hh \to b\bar{b}b\bar{b}$

$p_{\mathrm{T}} > 250$



≥ 1 b-tagged track jet
on each side

Split into 2b, 3b, 4b
signal regions

$$X_{hh} = \sqrt{\left(\frac{m_{h_1} - 120}{0.1 m_{h_1}}\right)^2 \left(\frac{m_{h_2} - 115}{0.1 m_{h_2}}\right)^2} < 1.6$$