SUSY 2017 Dec. 11 2017

# On the Validity of Effective Potential and the Precision of Higgs Self Coupling

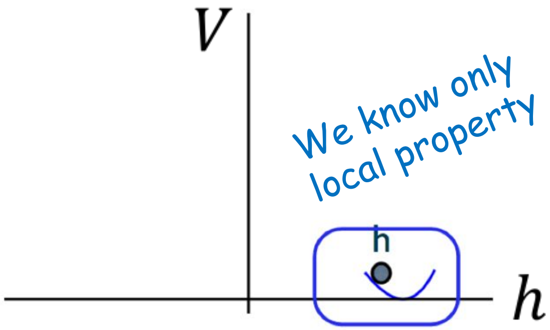
Bithika Jain

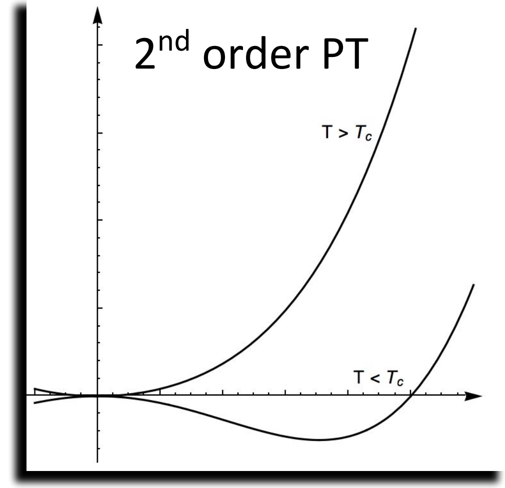
IFT-UNESP & ICTP-SAIFR
Sao Paulo, Brazil

On behalf of Minho Son

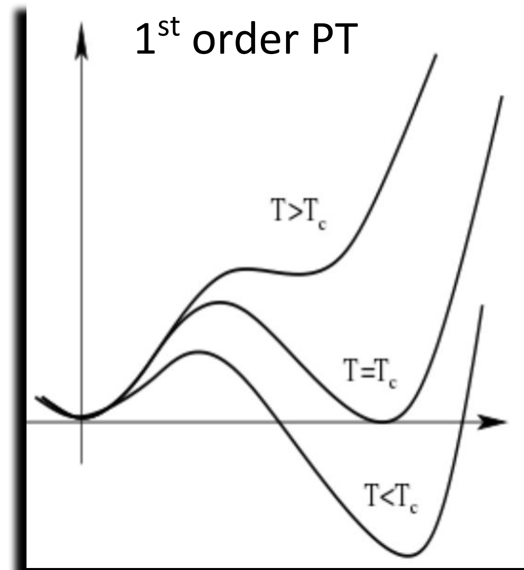
Based on work with S. Lee, M. Son 1709.03232

Various global structure/thermal histories
are still plausible




$$V_h = \frac{1}{2} m_h^2 h^2 + c_3 \frac{1}{6} \left( \frac{3\, m_h^2}{v} \right) h^3 + c_4 \frac{1}{24} \left( \frac{3\, m_h^2}{v^2} \right) h^4$$




An interesting physics still allowed
is baryogenesis based on
strong 1st order EW-phase transition

# Strong 1$^{st}$ order Electroweak Phase Transition

An interesting physics that can be related to the

**large** deviation of the Higgs self coupling

# The effective potential

is a main tool to examine the thermal history of the Higgs potential

$$V_{eff} = V_{tree} + V_{CW}[m_i^2(h) + \Pi_i] + V_T[m_i^2(h) + \Pi_i]$$

$$V_T = \sum_{i=\mathrm{B,F}} (-1)^{F_i} \frac{g_i T^4}{2\pi^2} \int_0^\infty dx\, x^2 \log\left[1 \mp \exp\left(-\sqrt{x^2 + (m_i^2(h) + \Pi_i)/T^2}\right)\right]$$

Obtaining the exact thermal mass is very non-trivial

What most people do is using

Truncated Full Dressing (TFD):

We call this Prescription A

: thermal mass $\Pi_i$ is still obtained in the high-T approximation

At Leading order in Temperature, $\Pi_i$ is mass-independent
→ non-decoupling issue

: the related uncertainty has not been well understood in BSM scenarios

Curtin, Meade, Ramani 16' for a recent discussion

# The effective potential

We call this **Prescription B**

In the High-T approximation

$$V_{eff} = V_{tree} + V_{CW} + V_T + V_{ring}$$

$$\begin{cases} V_{CW} = \sum_{i=t,W,Z,h,G,\cdots} (-1)^{F_i} \frac{g_i}{64\pi^2}\left[m_i^4(h)\left(\log\frac{m_i^2(h)}{m_i^2(v)} - \frac{3}{2}\right) + 2m_i^2(h)m_i^2(v)\right] \\ V_T = \sum_{i=\mathrm{B,F}} (-1)^{F_i} \frac{g_i T^4}{2\pi^2} J_{B/F}\left(\frac{m_i^2(h)}{T^2}\right) \\ V_{ring} = \sum_{i=\mathrm{bosons}} \frac{\bar{g}_i T}{12\pi}\left[m_i^3(h) - \left(m_i^2(h) + \Pi_i(T)\right)^{\frac{3}{2}}\right] \end{cases}$$

$$x^2 = \frac{m^2}{T^2} \ll 1$$

$$J_B(x^2) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}x^2 - \frac{\pi}{6}x^3 - \frac{1}{32}x^4 \log\left(\frac{x^2}{c_b}\right)$$

$$J_F(x^2) \approx \frac{7\pi^4}{360} - \frac{\pi^2}{24}x^2 - \frac{1}{32}x^4 \log\left(\frac{x^2}{c_f}\right)$$

$1^{st}$ order PT via thermal effect

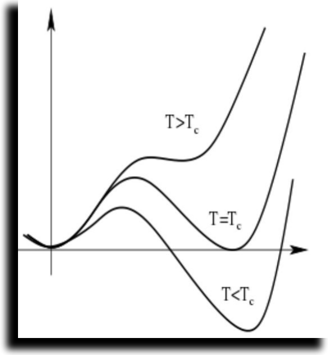
✓ Validity of this approx. carefully has to be checked

# On the criteria for strong 1st order phase transition

: There has been an ambiguity on how to quantify the strong first order phase transition and this ambiguity can cause $\mathcal{O}(1)$ fluctuation on the precision of Higgs self coupling

List of sources that can cause $\mathcal{O}(1)$ uncertainty

$$\frac{m_i^2(v_c)}{T_c^2} = \frac{m^2}{T_c^2} + \text{coupling}\times\frac{v_c^2}{T_c^2} \gtrsim 1$$




For $v_c \gtrsim T_c$ and $\gtrsim \mathcal{O}(1)$ coupling,
integral needs to be exactly evaluated

1. The High-temperature Approximation
2. A Large coupling
3. $\frac{v_c}{T_c}$ vs $\frac{v_N}{T_N}$ , and confusion on $\frac{v_c}{T_c} \gtrsim 0.6 - 1.4$

: checks if the potential develops degenerate vacua

: checks if the transition actually happens

The impact on the precision of the Higgs self-coupling of these issues has not been well studied in most literature in the context of BSM physics

To illustrate the issues we take

# Most commonly considered frameworks

$$V_{eff} = \sum_{i=t,W,Z,h,G,\boxed{\mathrm{BSM}}} V_i$$

No mixing with Higgs
: less constrained

Higgs portal $Z_2$-symmetry, e.g. $\langle S \rangle = 0$:

new scalar S

## Effective Field Theory:

higher-dimensional operators

$\mathcal{O}_H = (\partial |H|^2)^2$ vs $\mathcal{O}_6 = |H|^6$

Higgs decay
: strongly constrained

Higgs self coupling
: poorly constrained

E.g. Strongly coupled theory

PGB vs non-pGB

Higgs Portal

$\mathcal{O}_H \ll \mathcal{O}_6$ possible

Azatov, Contino, Panico, Son 15’

Strong 1st order
Electroweak Phase Transition

# Higgs Portal
# with a singlet scalar

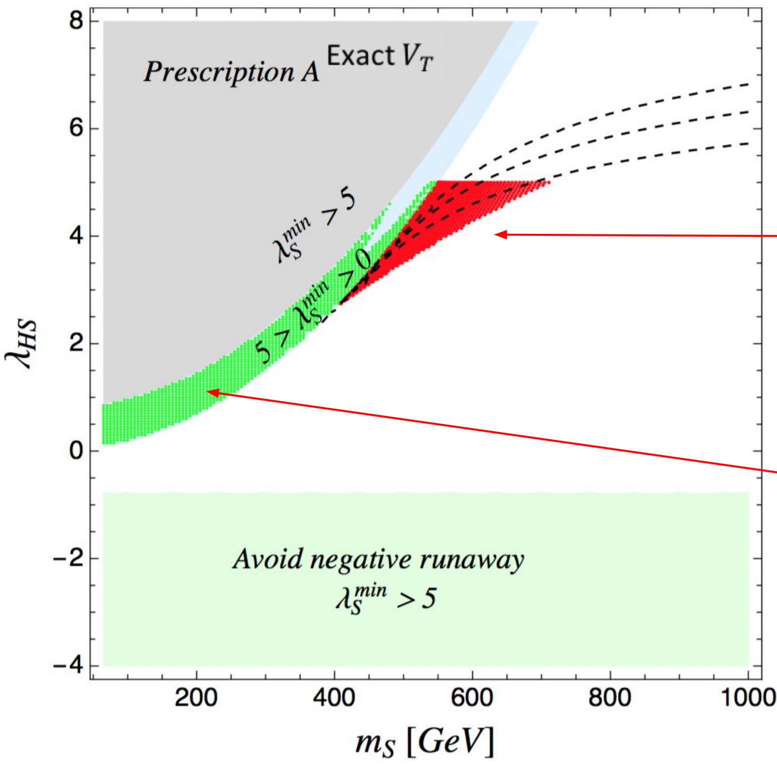
Noble, Perelstein 08’
Katz, Perelstein 14’
Curtin, Meade, Yu 14’
Kurup, Perelstein 17’
Barger, Langacker, McCaskey,Ramsey-Musolf, Shaughness 03’
Espinosa, Konstandin, Riva 11’
Cline, Kaiulainen 12’
Alanne, Tuominen, Vaskonen 14’
Many others ….

Very sorry if I missed your paper

# Higgs Portal SM + a singlet scalar with Z2

$$V_{tree} = -\frac{1}{2}\mu^2 h^2 + \frac{1}{4}\lambda h^4 + \frac{1}{2}\lambda_{HS} h^2 S^2 + \frac{1}{2} m_{0\,s}^2 S^2 + \frac{1}{4}\lambda_S S^4$$

$(\langle h \rangle, \langle s \rangle) = (v, 0)$ is a global minimum




Based on naïve criterion, existence of degenerate vacua, with $v_c/T_c > 1$ * Note cutoff $\lambda_{HS} < 5$ by hand

1. One-step strong 1st phase transition (**RED**)

$$V(0,0) \rightarrow V(v,0) \quad , \langle S \rangle = 0$$

2. Two-step strong 1st phase transition (**GREEN**)

$$V(0,0) \rightarrow V(0,v_s) \rightarrow V(v,0)$$

$$V(0,v_s) > V(v,0) \rightarrow$$

$$\lambda_S > \lambda_S^{\min} \equiv \lambda \frac{m_{0\,s}^4}{\mu^4} = \frac{2(m_S^2 - v^2\lambda_{HS})^2}{m_h^2 v^2}$$

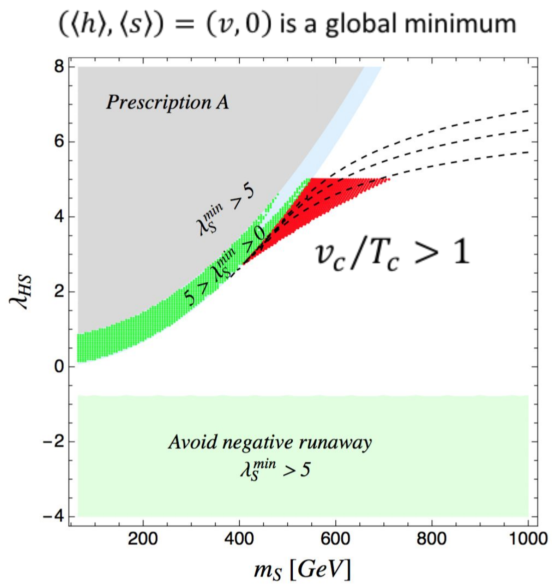
parametrize $\lambda_s = \lambda_s^{min} + \delta_s$

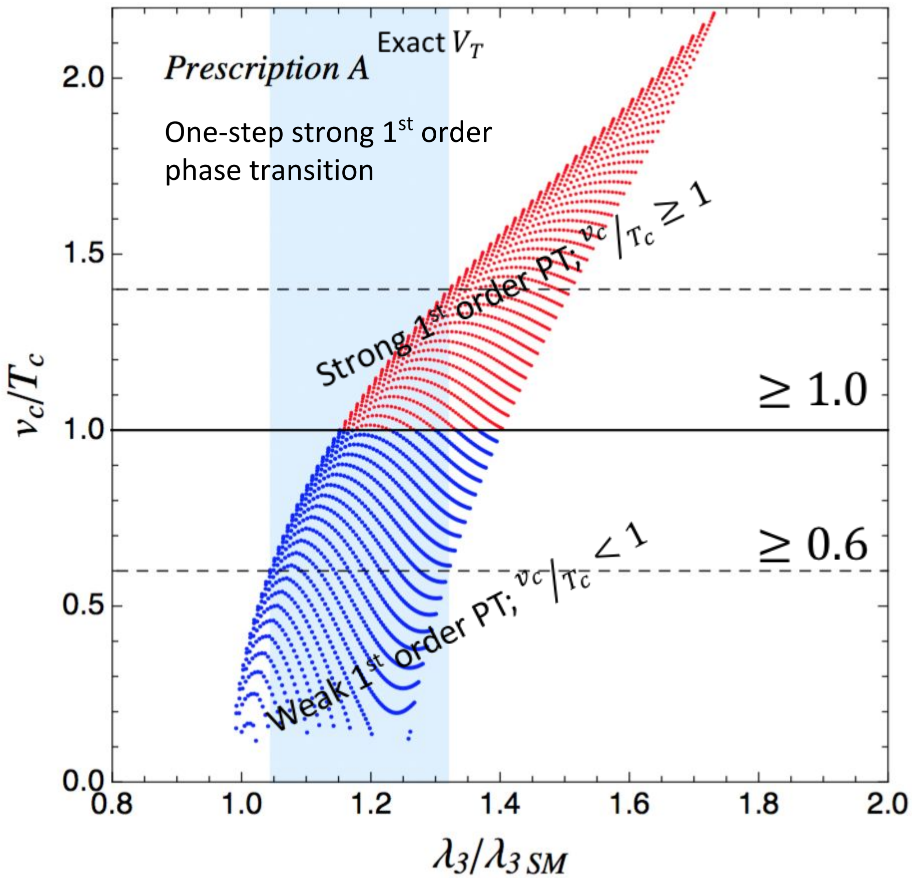
Scan:

$m_s = [100, 800]$ GeV in steps of 10 GeV

$\lambda_{HS} = [1, 5]$ in steps of 0.2

Similar plot in Curtin, Meade, Ramani 14'




Future collider plan can sensitively depend on the exact criteria

E.g. only 100 TeV pp
vs various colliders 100 TeV, ILC?

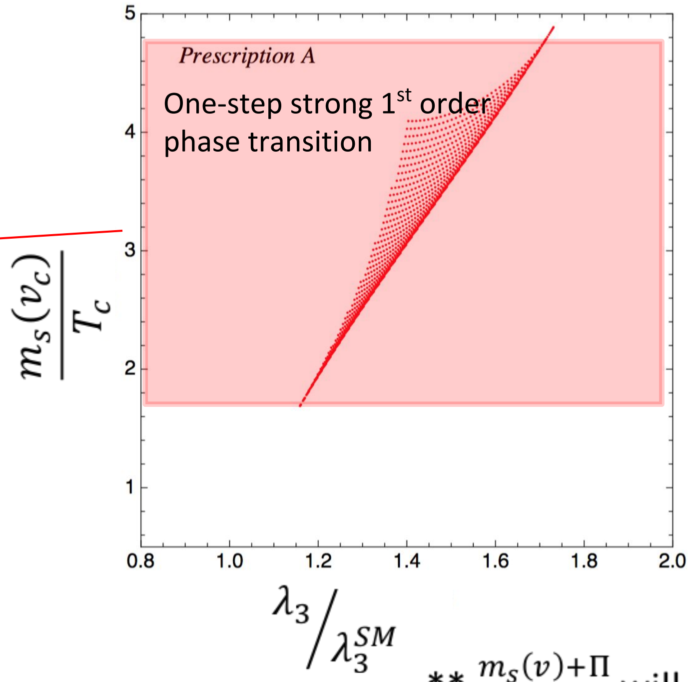
~ 5% ~ 32%

✓ fluctuates by $\mathcal{O}(1)$ amount

# Q. Is there a preferred $v_c/T_c$?

Jain, SON in progress

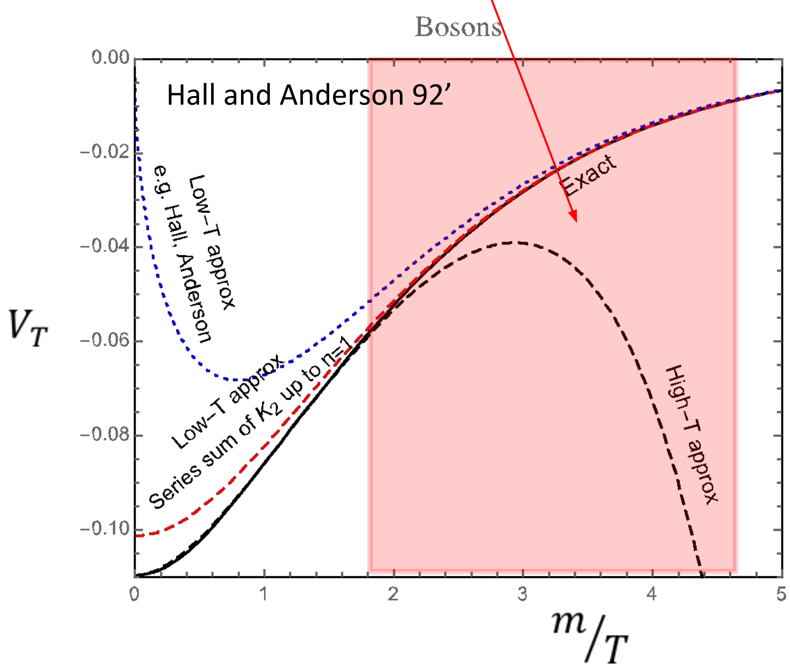
# Validity of High-T approximation




One-step PT

High-T approx. fails

(The issue is more pronounced in two step PT)




** $\frac{m_s(v)+\Pi}{T_c}$ will be even bigger

$$x^2 = \frac{m^2}{T^2} \ll 1: \text{High T approximation}$$

$$J_B(x^2) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}x^2 - \frac{\pi}{6}x^3 - \frac{1}{32}x^4 \log\left(\frac{x^2}{c_b}\right)$$

$$J_F(x^2) \approx \frac{7\pi^4}{360} - \frac{\pi^2}{24}x^2 - \frac{1}{32}x^4 \log\left(\frac{x^2}{c_f}\right)$$

$$x^2 = \frac{m^2}{T^2} \gg 1: \text{Low T approximation}$$

$$J_B(x^2; n) = -\sum_{k=1}^{n} \frac{1}{k^2} x^2 K_2(x\,k)$$

$$J_F(x^2; n) = -\sum_{k=1}^{n} \frac{(-1)^k}{k^2} x^2 K_2(x\,k)$$

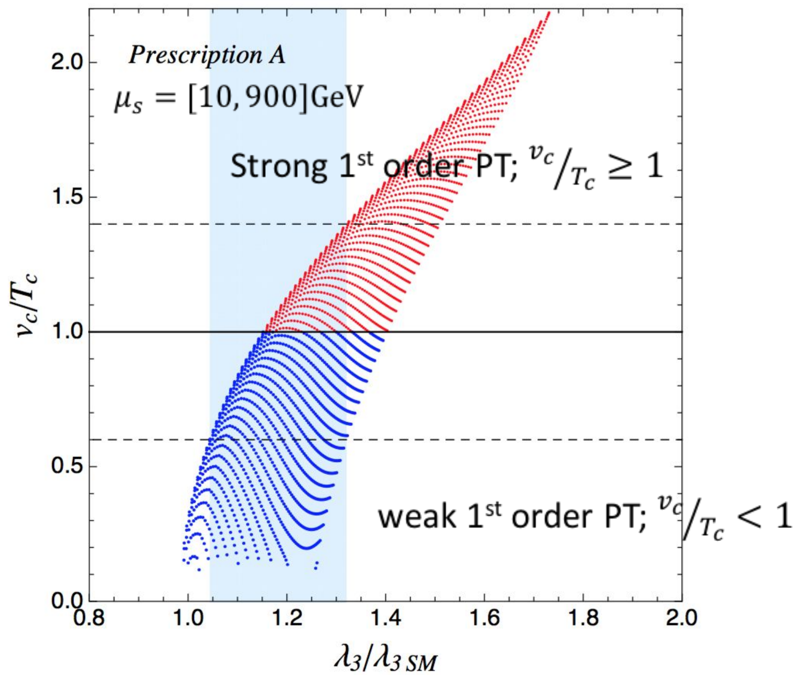
Low-T approx with $n = \mathcal{O}(10)$
= Exact evaluation, e.g. chose $n = 50$

# $\mathcal{O}(1)$ fluctuation in Higgs self-coupling

Prescription A

## Exact $V_T$

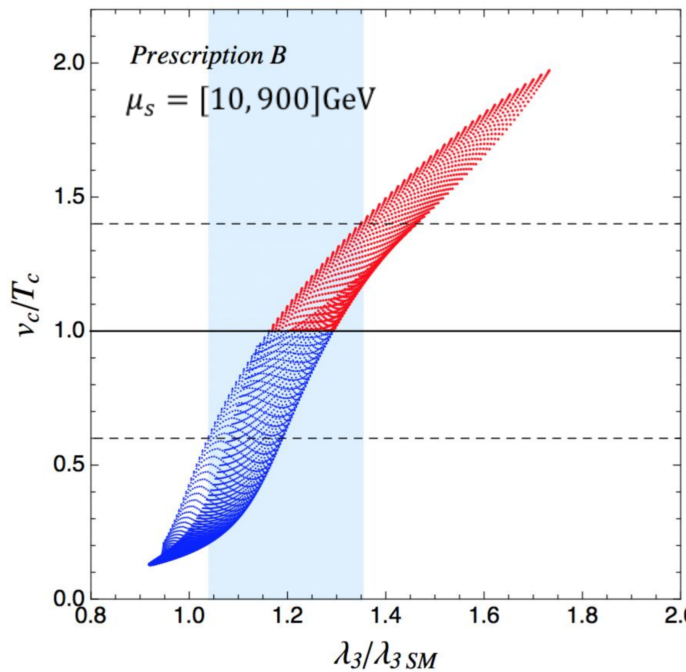
One-step strong 1st order phase transition




Prescription B

## High-T approx $V_T$

One-step strong 1st order phase transition



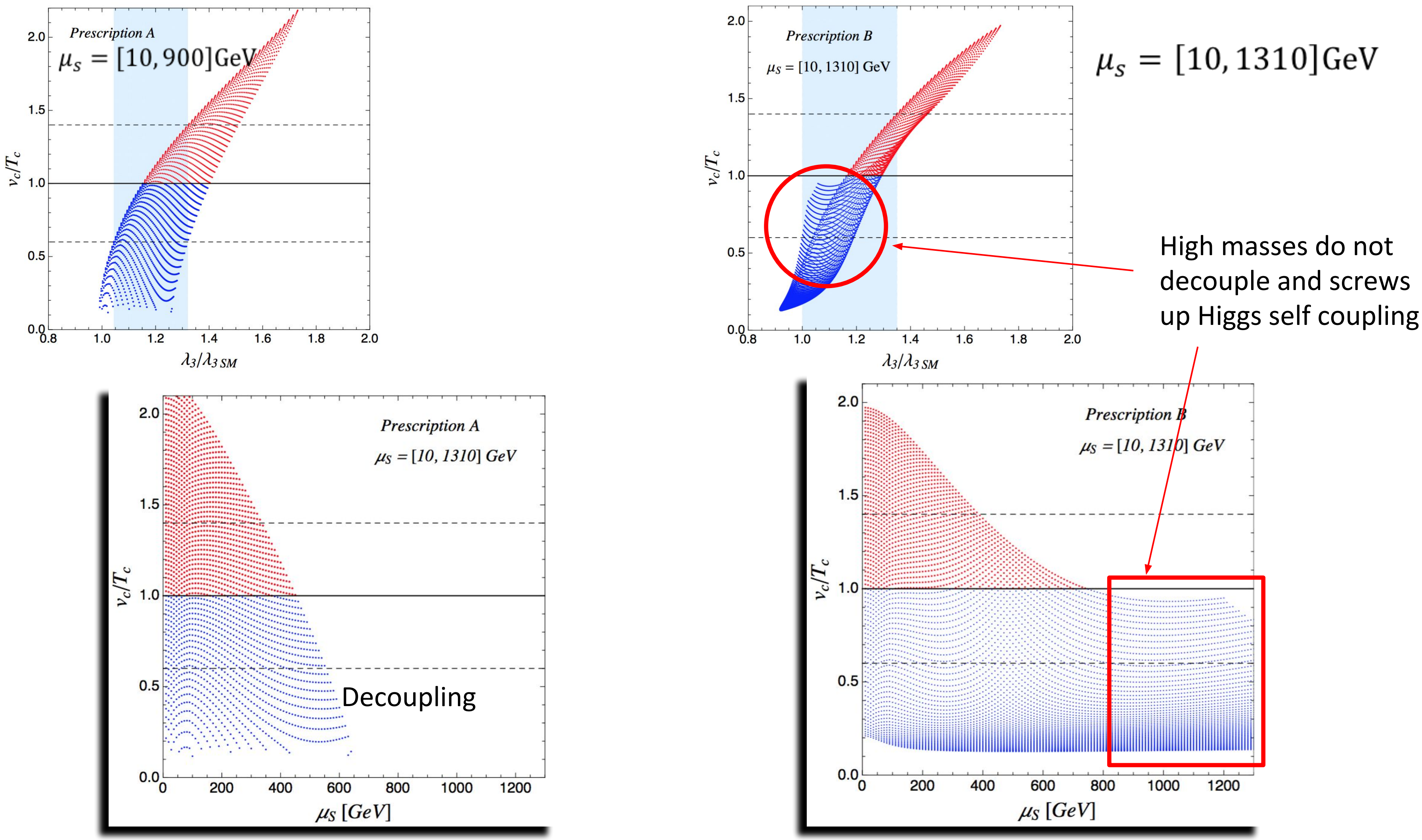

Similarly for both, the smallest deviation, $\delta\left(\lambda_3/\lambda_3^{SM}\right) \sim [5, 32]\%$ for $v_c/T_c \geq [0.6, 1.4]$

The patterns of parameter space in two cases look very different! What's going on ?

# $\mathcal{O}(1)$ fluctuation in Higgs self-coupling

Exact $V_T$

High-T approx. $V_T$




High-T approx. does not seem to be appropriate for Higgs portal.

Strong 1st order
Electroweak Phase Transition

# EFT approach

Grojean, Gervant, Well 04'
Noble, Perelstein 06'
Delaunay, Grojean, Wells, 08'
Huang, Joglekar, Li, Wagner 15'
Huang, Gu, Yin, Yu, Zhang 16'
Chung, Long, Wang 16'
Gan, Long, Wang 17'
Reichert, Eichhorn, Gies, Pawlowski, Plehn, Scherer 17'
Many others….

Very sorry if I missed your paper

# $1^{st}$ order phase transition in EFT approach

I. Only dim-6 operator $\mathcal{O}_6 \sim |H|^6$

** Suitable for the case with $\mathcal{O}_H \ll \mathcal{O}_6$, nevertheless we will present results in SILH basis

& assume Higgs as pGB: NDA in SILH basis: $c_6 \sim \left(\frac{v}{f}\right)^2 \equiv \xi$

$$V_{EFT} = m^2|H|^2 + \lambda|H|^4 + \frac{c_6}{v^2}\frac{m_h^2}{2\,v^2}|H|^6$$

$$\to \frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4 + \frac{1}{8}\frac{c_6}{v^2}\frac{m_h^2}{2\,v^2}h^6$$

At "tree"-level

$$\lambda_3 / \lambda_3^{SM} = 1 + c_6$$

$$\lambda_4 / \lambda_4^{SM} = 1 + 6\,c_6$$

Relation holds only at the level of dimension-six

# $1^{st}$ order phase transition in EFT approach

## II. Resum all $\mathcal{O}_{4+2n} \sim |H|^{4+2n}$

$$V_{EFT} = m^2|H|^2 + \lambda|H|^4 + \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2\,v^2} |H|^{4+2n}$$

$$\rightarrow \frac{1}{2} m^2 h^2 + \frac{1}{4}\lambda h^4 + \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2\,v^2} \left(\frac{h^2}{2}\right)^{2+n}$$

NDA in SILH basis: $c_{4+2n} \sim \left(\frac{v}{f}\right)^{2n}$ → $c_{4+2n} \sim c\left(\frac{v}{f}\right)^{2n} \equiv c\xi^n$

:can be resumed up to infinite order in Higgs field and
``EFT can make sense as long as $E < \Lambda_{\text{cutoff}}$"

At "tree"-level

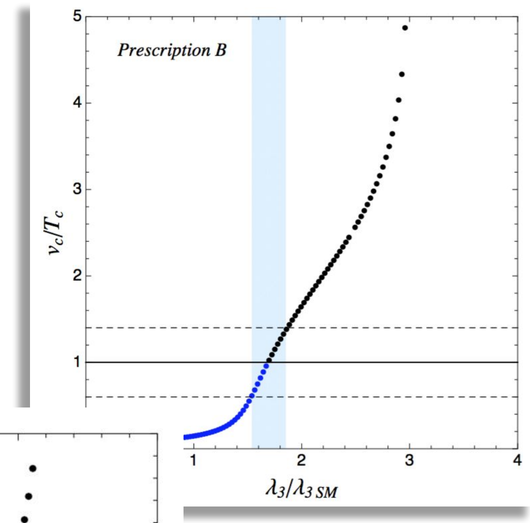
$$\lambda_3 / \lambda_3^{SM} = 1 + 16c \frac{\xi}{(2-\xi)^4}$$

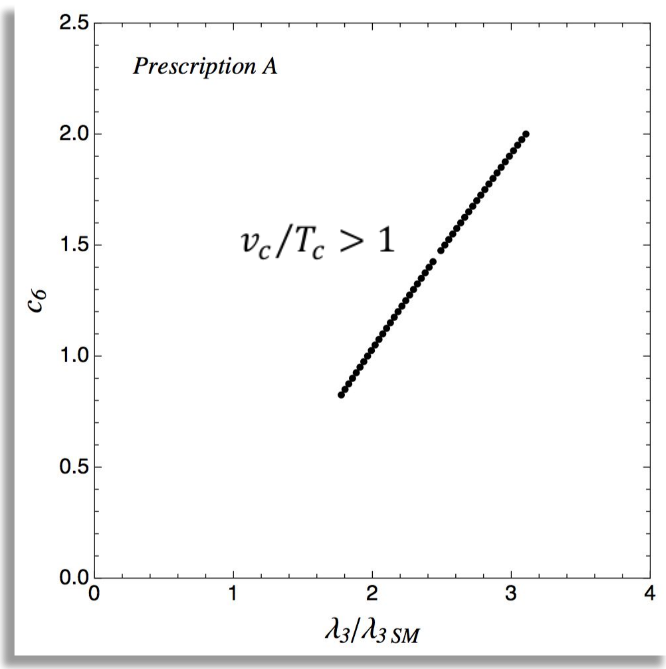
$$\lambda_4 / \lambda_4^{SM} = 1 + 32c \frac{(6+\xi)\xi}{(2-\xi)^5}$$

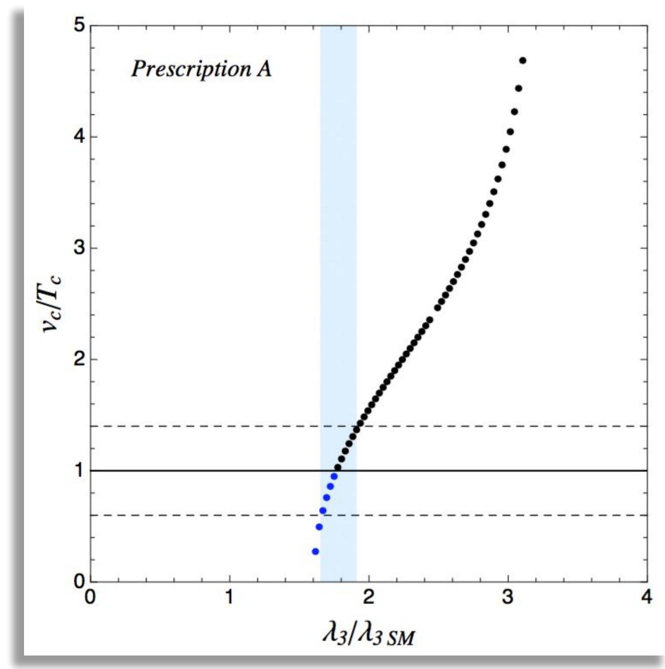
$$\frac{\lambda_4 / \lambda_4^{SM}}{\lambda_3 / \lambda_3^{SM}} = 2\frac{6+\xi}{2-\xi} \rightarrow \boxed{14} \text{ as } \xi \rightarrow 1 \text{ (or } f \rightarrow v)$$

# Only dim-6 operator

$$\Delta V_{EFT} = \frac{1}{8}\frac{c_6}{v^2}\frac{m_h^2}{2\,v^2}h^6$$



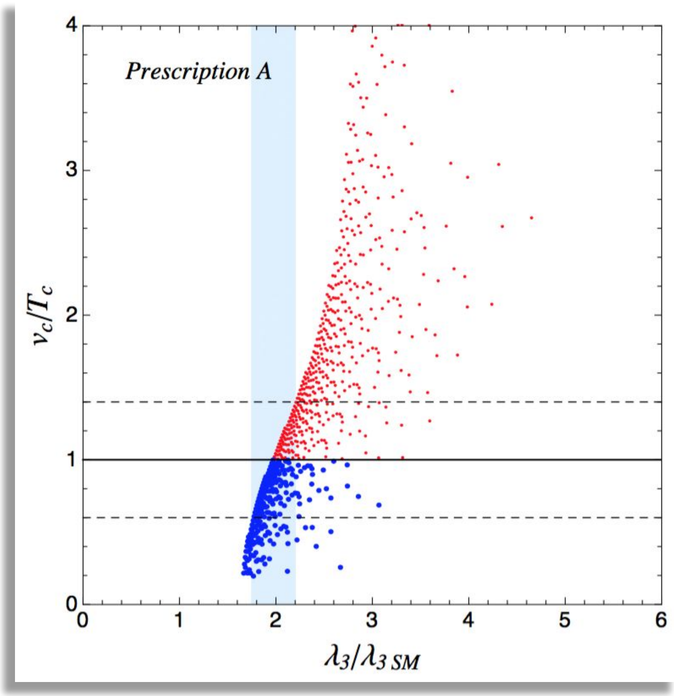

$c_6 \sim \mathcal{O}(1)$ : Validity of EFT

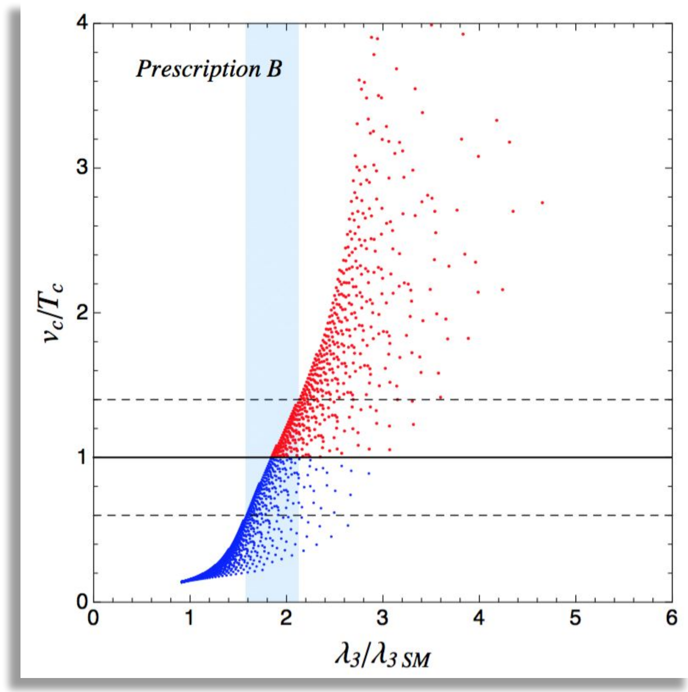
- ✓ High-T approximation seems to be ok, e.g. no large mass involved
- ✓ The uncertainty due to the finite $v_c/T_c$ is not pronounced

# All orders of $|H|^{4+2n}$

$$\Delta V_{EFT} = \sum \frac{c_{4+2n}}{v^{2n}} \frac{m_h^2}{2\,v^2}\left(\frac{h^2}{2}\right)^{2+n}$$

where $c_{4+2n} \sim c\left(\frac{v}{f}\right)^{2n} \equiv c\xi^n$



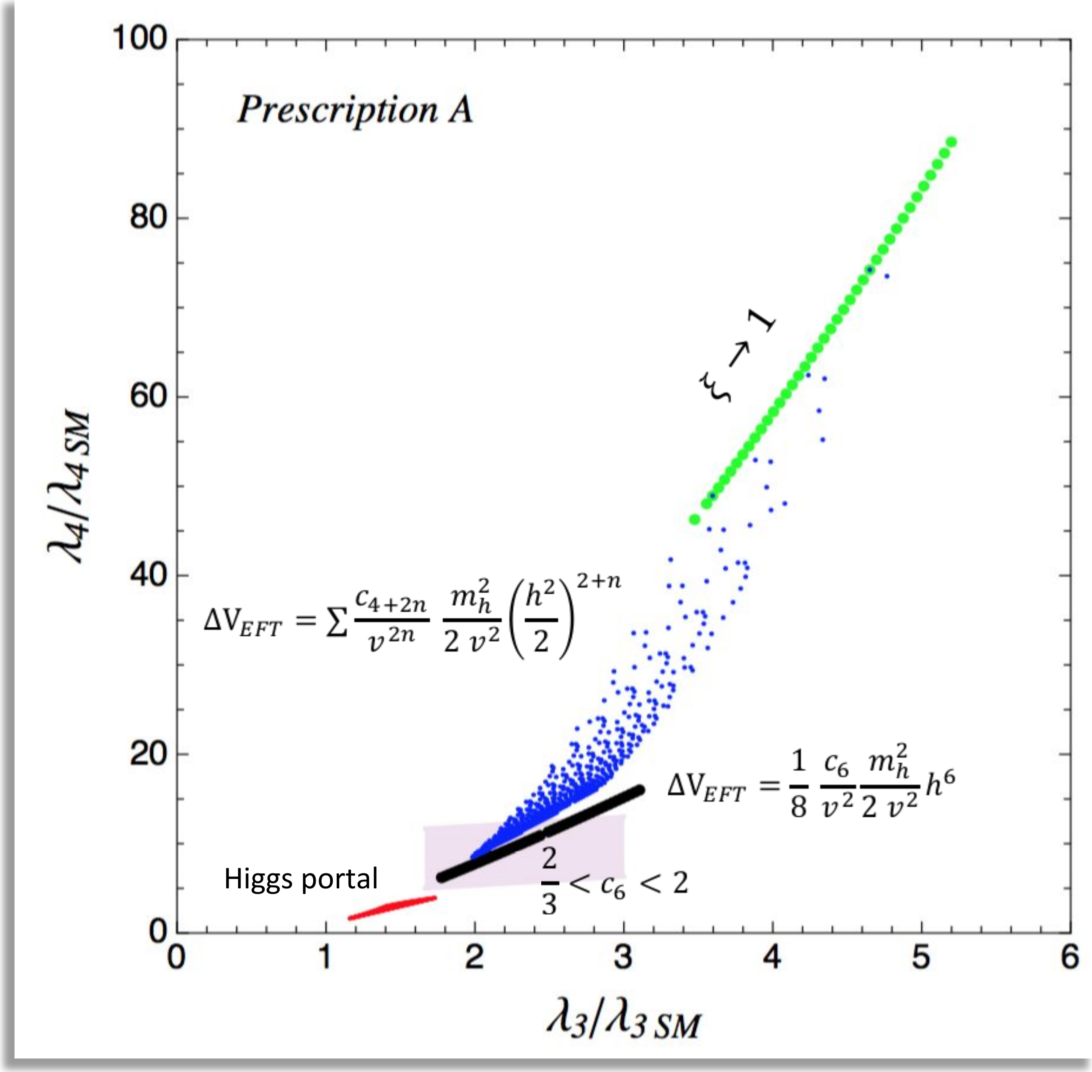




✓ High-T approximation seems to be ok, e.g. no large mass involved

✓ The uncertainty due to the finite $v_c/T_c$ is not pronounced

# Cubic vs. Quartic
# Higgs self-coupling

# Cubic vs. Quartic



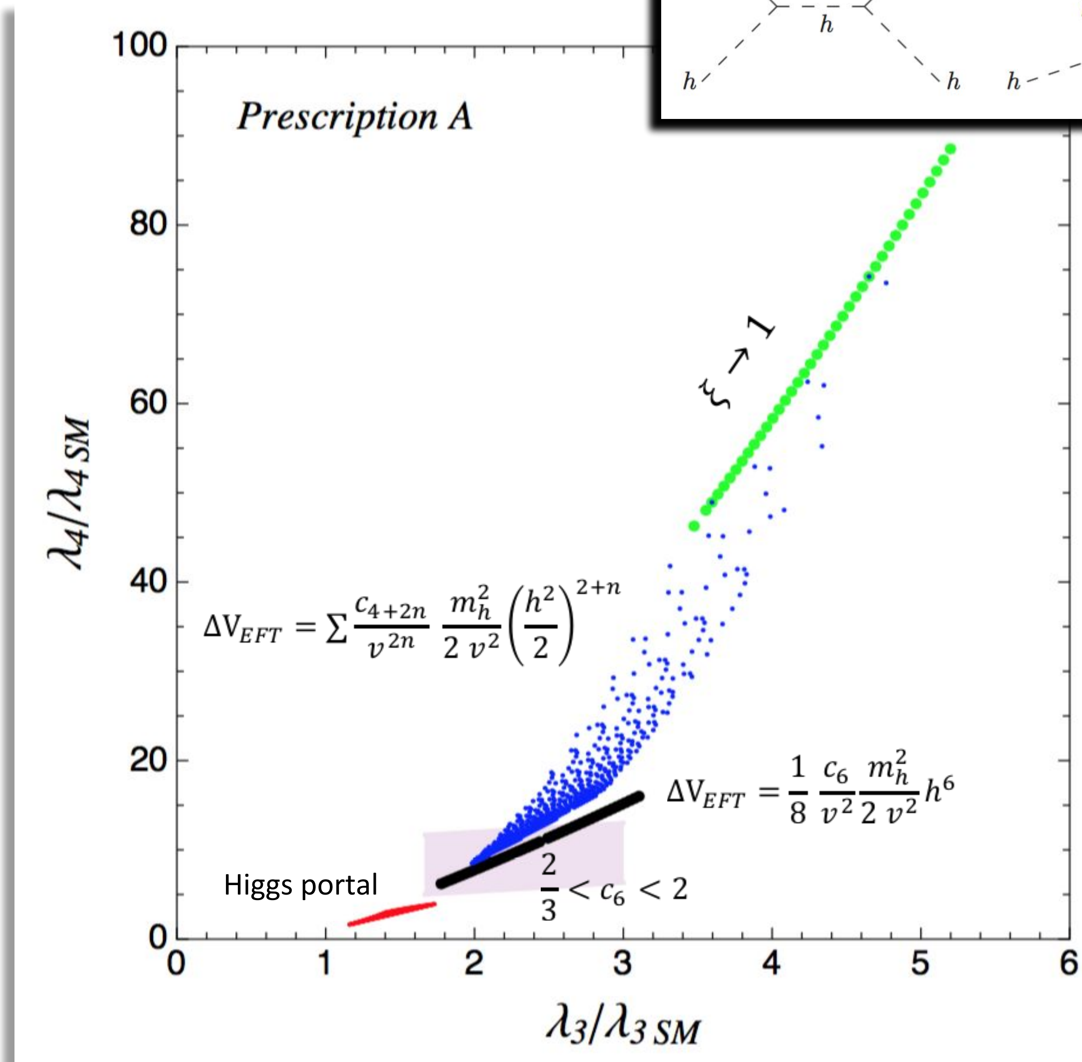

EFT prefers large Higgs self-couplings

Large quartic coupling might be able to be tested at future collider

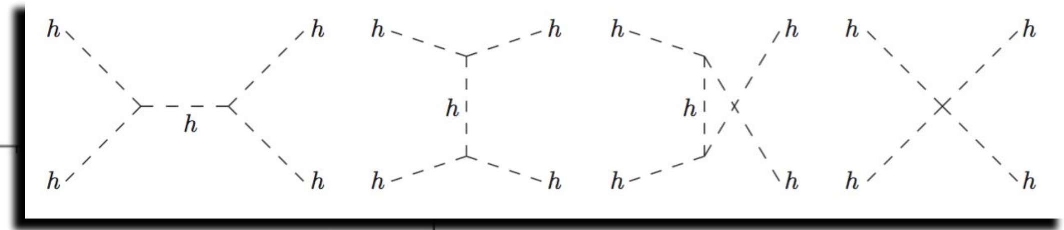
Unitarity bound

Electroweak Precision Test

# Cubic vs. Quartic



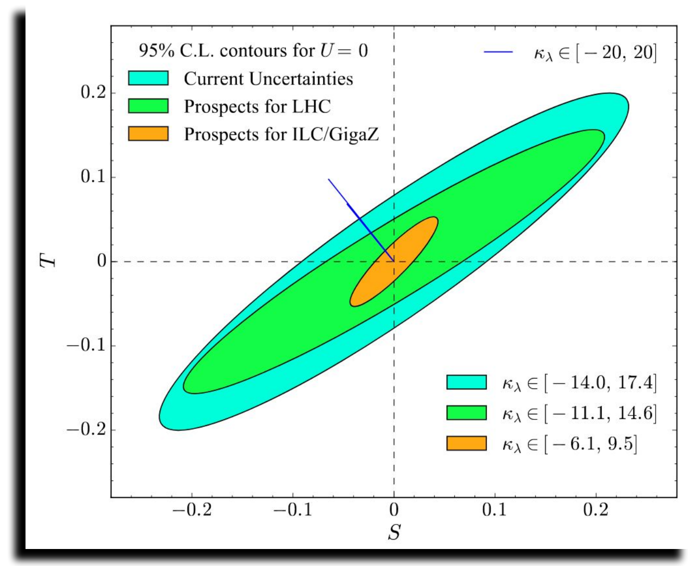

## Unitarity bound




$$\left|\lambda_3/\lambda_3^{SM}\right| \lesssim 6.5 \qquad \left|\lambda_4/\lambda_4^{SM}\right| \lesssim 65$$

Luzio, Grober, Spannowsky 17'

## Electroweak Precision Test

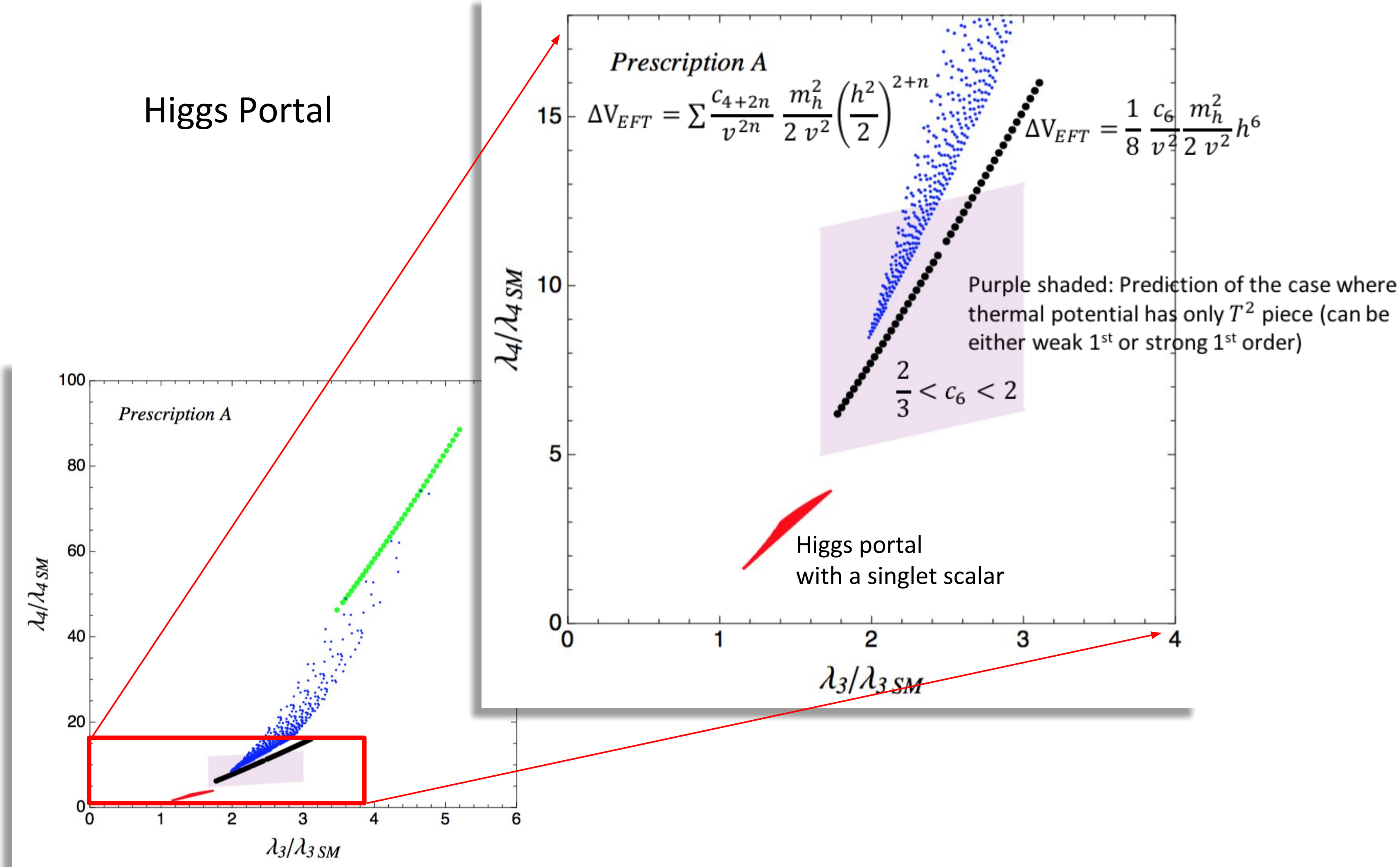



Kribs, Maier,Rzehak, Spannowsky, Waite 17'

# Cubic vs. Quartic

$$V(h) = \frac{1}{2} m_h^2 h^2 + c_3 \frac{1}{6}\left(\frac{3\, m_h^2}{v}\right) h^3 + c_4 \frac{1}{24}\left(\frac{3\, m_h^2}{v^2}\right) h^4$$

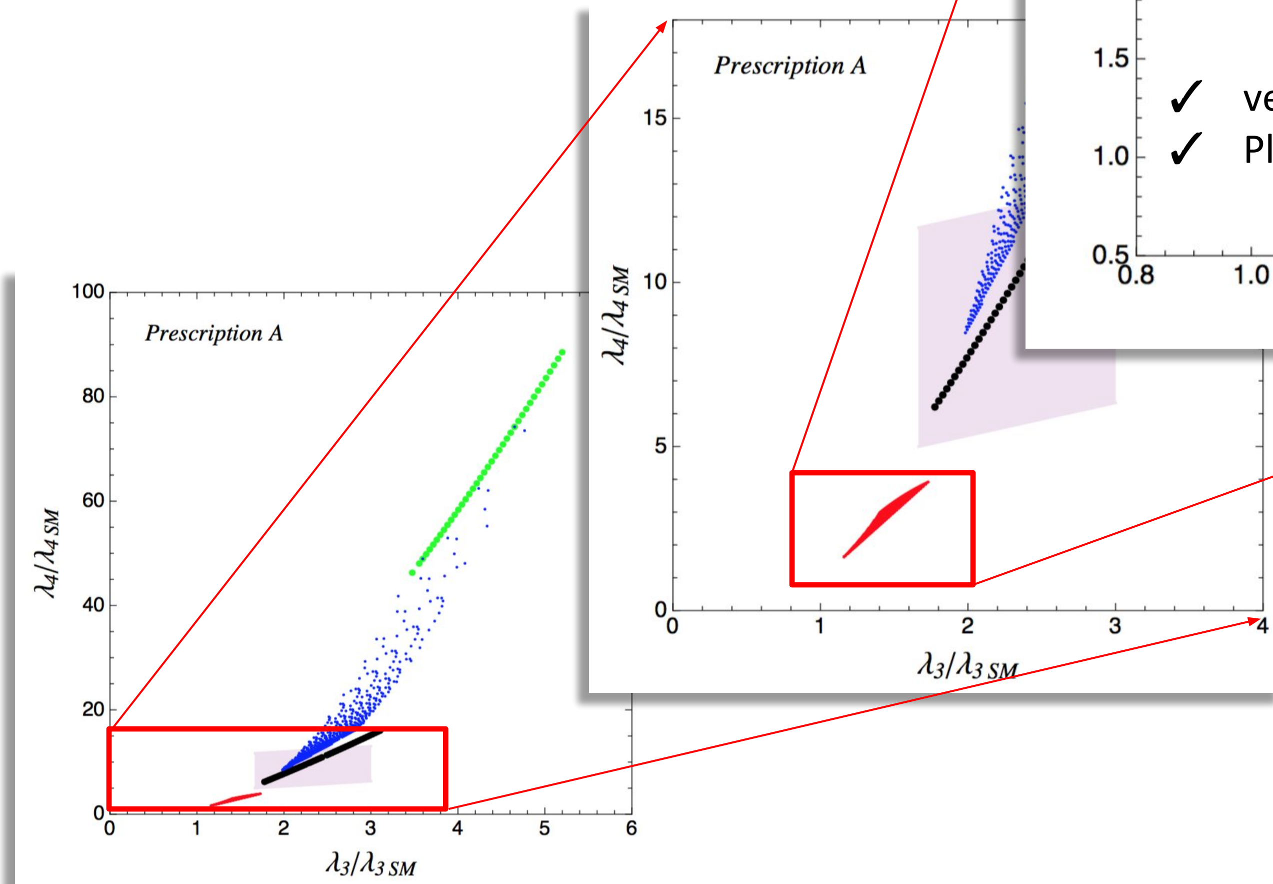
Higgs Portal

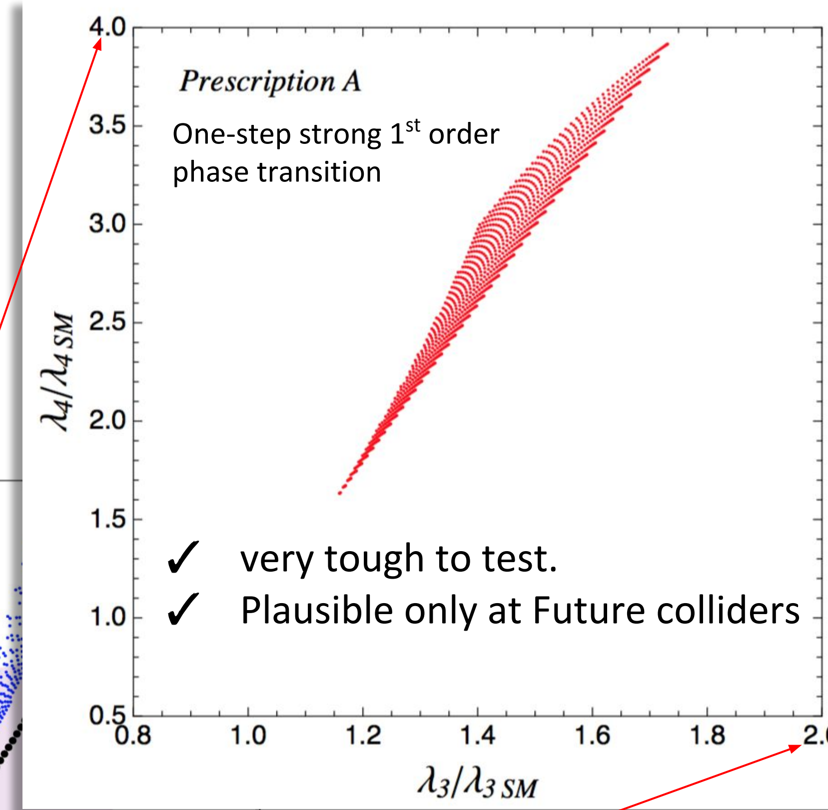


# Cubic vs. Quartic

$$V(h) = \frac{1}{2} m_h^2 h^2 + c_3 \frac{1}{6}\left(\frac{3\, m_h^2}{v}\right) h^3 + c_4 \frac{1}{24}\left(\frac{3\, m_h^2}{v^2}\right) h^4$$

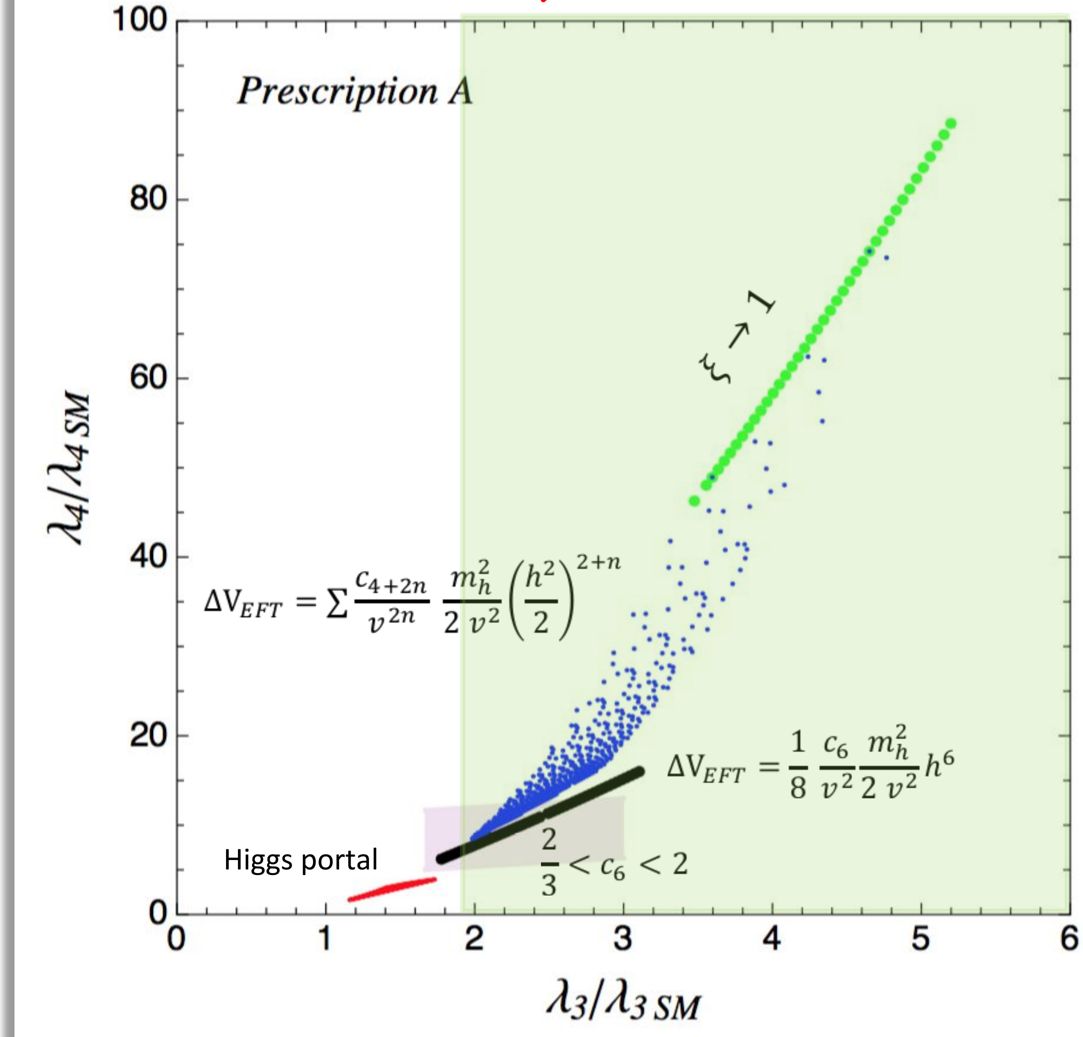
Higgs Portal

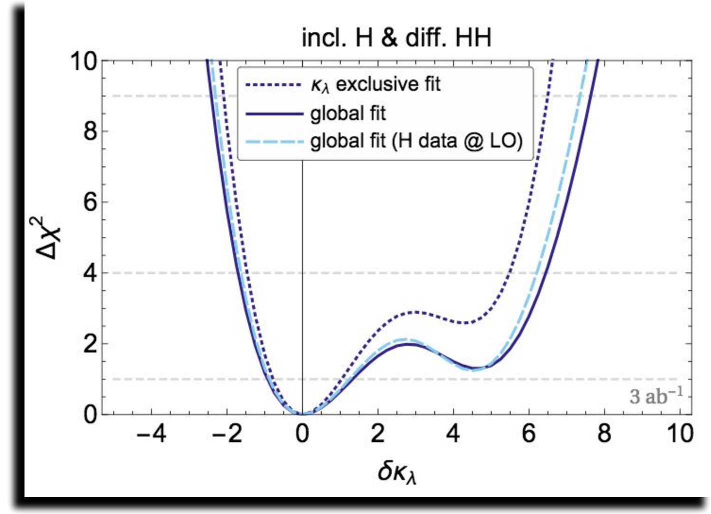


$\mathcal{O}(1)$ fraction of EFT parameter space can be tested at the HL LHC

Can be tested @ HL LHC with 68% CL

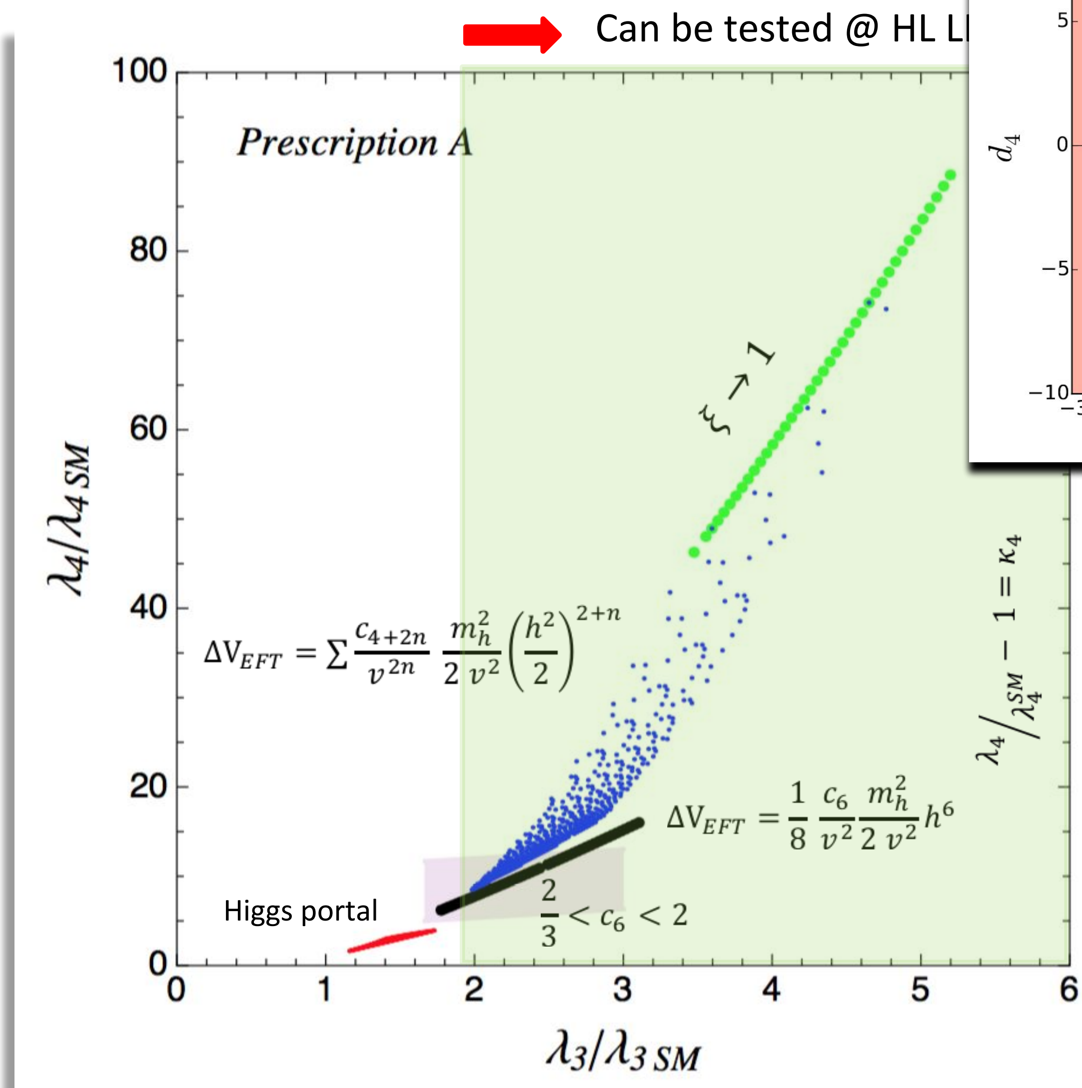


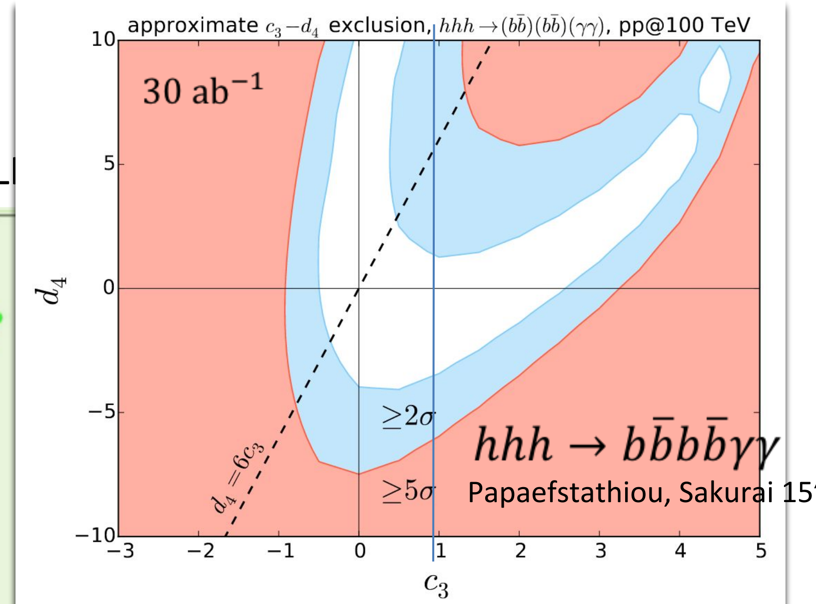




Vita, Grojean, Panico, Riembau, Vantalon 17'

$c_3 = [0.1, 2.3]$ @$1\sigma$

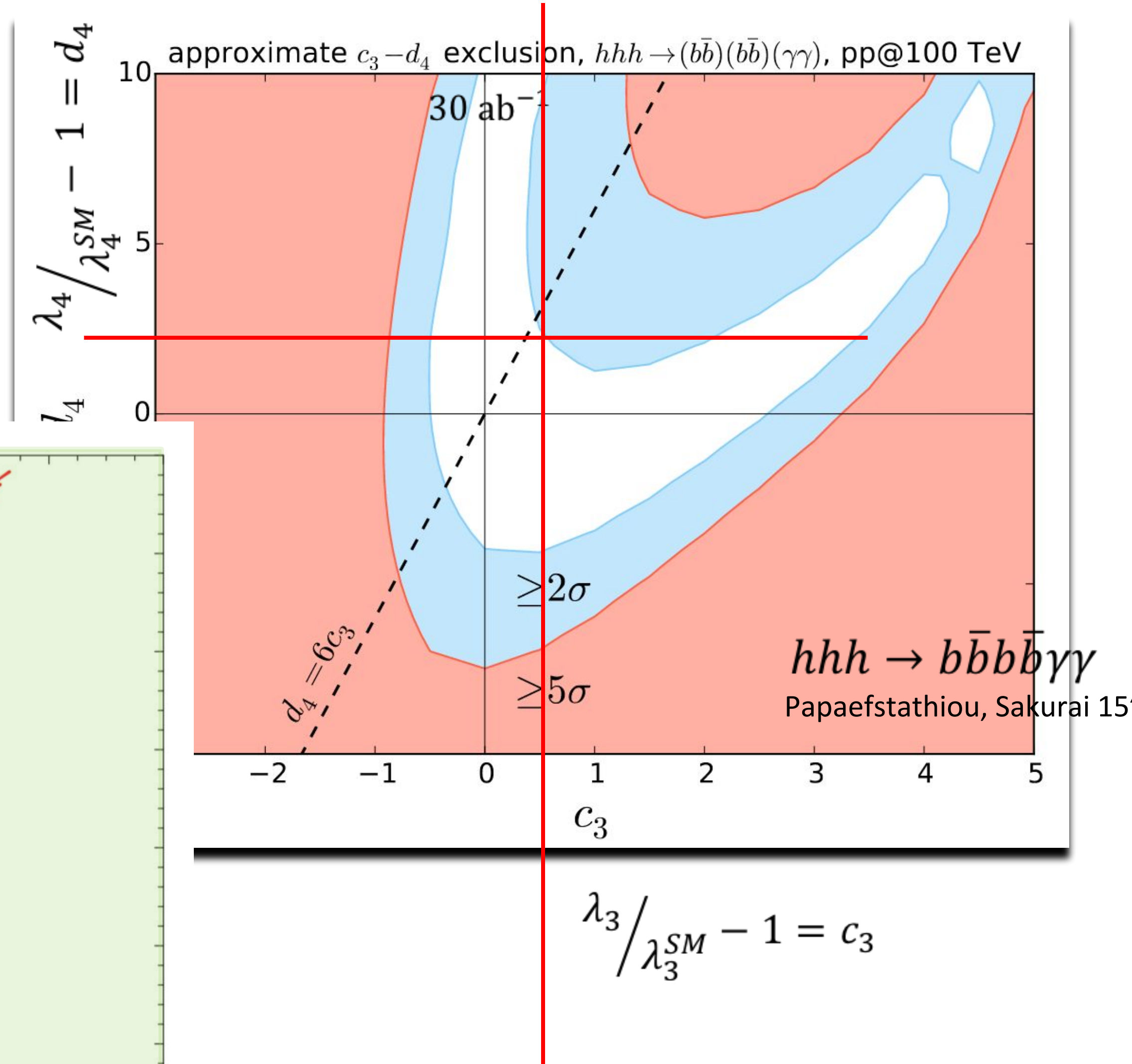
Quartic coupling IS useful to distinguish different EFTs!

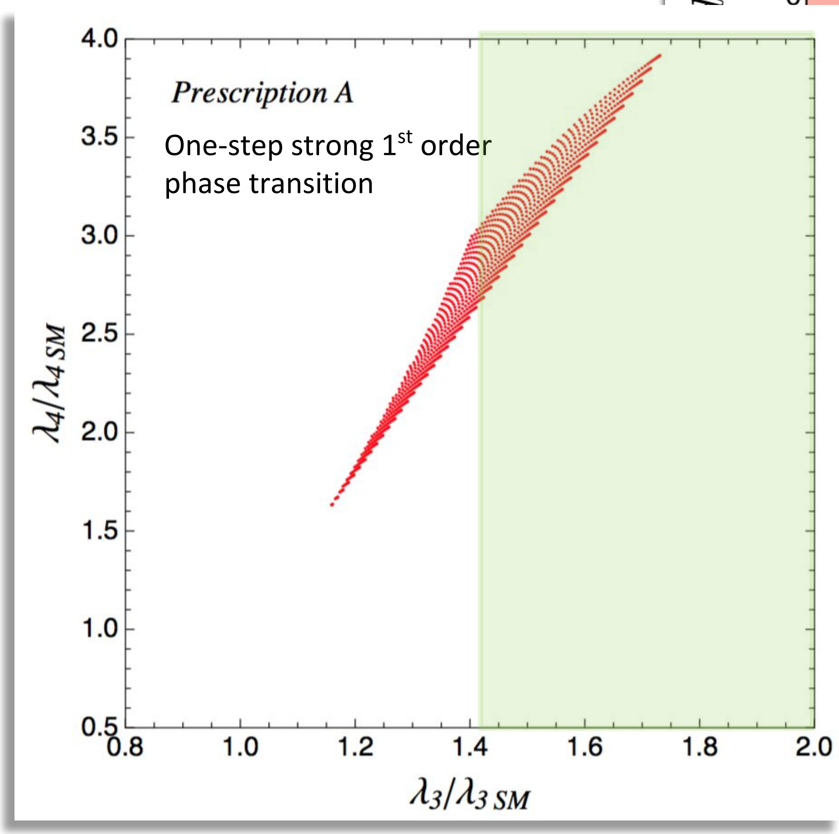



**Strong sensitivity on quartic when a large deviation of the cubic is observed

Quartic coupling IS also useful for Higgs portal



# Summary

- High-T approximation, finite v/T, A large coupling
  - ❑ Higgs Portal with a singlet scalar with Z2
    - ✓ Do not use high-T approximation
    - ✓ $\mathcal{O}(1)$ fluctuation on the precision of Higgs self-coupling due to v/T criteria
      → dramatic impact on future collider plan
  - ❑ Effective Field Theory Approach
    - ✓ Above issues become mild
    - ✓ Large deviation of coupling -> Validity of EFT -> Any reasonable EFT model?
- BSM maps in $(\lambda_3, \lambda_4)$
  - ✓ $\lambda_4$ can be used to differentiate different BSM scenarios in a situation when a large $\delta\lambda_3$ is observed
  - ✓ The quartic couplings in any BSM scenarios have a good sensitivity @ 100 TeV in that situation