Superconformal Chern-Simons-matter theories coupled to superconformal gravity

Motivation:

- ➔ Three dimensions
  - – Fun
  - – M2 branes
  - – Condensed matter

Outline:

- extended superspace approach
  - – on-shell equations
  - – classification of gauge groups
  - – supersymmetry enhancement
  - – gauge groups in presence of supergravity

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3d susy algebra

SO(N) R-symmetry

SO(2,1) Lorentz

$$\{D^I_\alpha, D^J_\beta\} = 2\mathrm{i}\delta^{IJ}(\gamma^a)_{\alpha\beta}\partial_a$$

Sl(2,R)=spin(2,1) Lorentz

Supercovariant derivatives:

$$D^I_\alpha = \partial^I_\alpha + \mathrm{i}\theta^{I\beta}(\gamma^a)_{\alpha\beta}\partial_a$$

Extended gauge multiplet

$$\{\mathscr{D}_\alpha^I, \mathscr{D}_\beta^J\} = 2\mathrm{i}\delta^{IJ}\mathscr{D}_{\alpha\beta} + F_{\alpha\beta}^{IJ}$$

$$[\mathscr{D}_\alpha^I, \mathscr{D}_b] = F_{\alpha b}^I$$

$$[\mathscr{D}_a, \mathscr{D}_b] = F_{ab}$$

Constraint: $\boxed{F_{\alpha\beta}^{IJ} = \mathrm{i}\varepsilon_{\alpha\beta}W^{IJ}}$ „Gauge multiplet“

Jacobi identities

$$F_{\alpha b}^I \sim (\gamma_b)_\alpha{}^\beta \mathscr{D}_\beta^J W^I{}_J$$

$$F_{ab} \sim (\gamma_{ab})^{\alpha\beta} \mathscr{D}_\alpha^I \mathscr{D}_\beta^J W_{IJ}$$

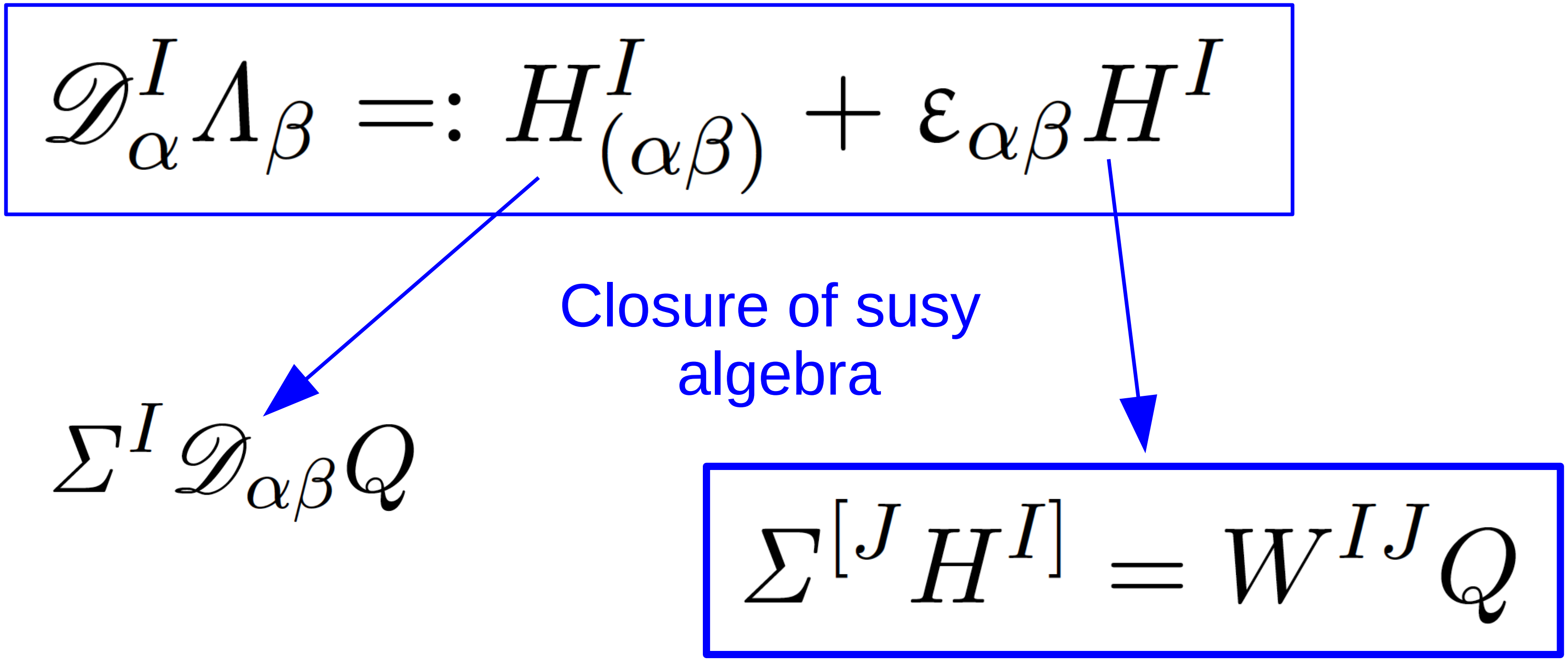
Matter: Scalar multiplet

(on-shell) condition:

$$\boxed{\mathscr{D}^I_\alpha Q_i = \mathrm{i}(\Sigma^I \Lambda_\alpha)_i}$$

spin(N)

Parametrisation:



$$\boxed{\mathscr{D}^I_\alpha \Lambda_\beta =: H^I_{(\alpha\beta)} + \varepsilon_{\alpha\beta} H^I}$$

Closure of susy algebra

$$\Sigma^I \mathscr{D}_{\alpha\beta} Q$$

$$\boxed{\Sigma^{[J} H^{I]} = W^{IJ} Q}$$

Only task: To solve this for H^I

idea from Gran, Greitz, Howe, Nilsson 2012

What is H^I ?

$$\boxed{\Sigma^{[J}H^{I]} = W^{IJ}Q}$$

What is W^{IJ} in terms of matter? → dimension one

$$\boxed{W^{IJ}Q = a\,\underbrace{\left(Q\Sigma^{IJ}\tau_A\bar{Q}\right)}_{W^{IJ}_A}\,\tau^A Q}$$

Gauge group generator

$$F_{ab} \sim (\gamma_{ab})^{\alpha\beta}\mathscr{D}^I_\alpha\mathscr{D}^J_\beta W_{IJ} \sim \varepsilon_{abc}\left(\overline{Q}\mathscr{D}^cQ - Q\overline{\mathscr{D}^cQ}\right)$$

indeed

Chern-Simons equation of motion: $F_{ab} = \varepsilon_{abc}J^c$

group factor	$(\tau^A)_{ij}(\tau_A)_{kl}$	$\boxed{W^{IJ}Q = (Q\Sigma^{IJ}\tau^A\bar{Q})(\tau_A Q)_k}$
SO(N)	$2\delta_{k[i}\delta_{j]l}$	$2(Q^{[k}\Sigma^{IJ}\bar{Q}^{l]})Q_l$
Sp(N)	$2\Omega_{k(i}\Omega_{j)l}$	$2(Q^{(k}\Sigma^{IJ}\bar{Q}^{l)})Q_l$
U(1)	$-q^2\delta_i{}^j\delta_k{}^l$	$-q^2(Q^l\Sigma^{IJ}\bar{Q}_l)Q_k$
SU(N)	$\frac{1}{N}\delta_i^j\delta_k^l - \delta_i^l\delta_k^j$	$\frac{1}{N}(Q^l\Sigma^{IJ}\bar{Q}_l)Q_k - (Q_k\Sigma^{IJ}\bar{Q}^l)Q_l$
U(N)	$-\delta_i^l\delta_k^j$	$-(Q_k\Sigma^{IJ}\bar{Q}^l)Q_l$

bifundamental representations: $\boxed{\Sigma^{[J}H^{I]} = W^{IJ}Q + QV^{IJ}}$

right-acting	$(Q_w{}^{\bar{v}}\Sigma^{IJ}\bar{Q}_{\bar{v}}{}^{v})Q_v{}^{\bar{w}}$	$(Q_v{}^{\bar{v}}\Sigma^{IJ}\bar{Q}_{\bar{v}}{}^{v})Q_w{}^{\bar{w}}$
left-acting	$Q_w{}^{\bar{v}}(\bar{Q}_{\bar{v}}{}^{v}\Sigma^{IJ}Q_v{}^{\bar{w}})$	$Q_w{}^{\bar{w}}(\bar{Q}_{\bar{v}}{}^{v}\Sigma^{IJ}Q_v{}^{\bar{v}})$
examples:	U(N)	U(1)

What is H^I ?

All possible terms cubic in Q and ranks-1

$$H^I = \{(\Sigma^I Q)\bar{Q}Q\} + \ldots + \ldots + \{(Q\Sigma^I\bar{Q})Q\} + \ldots$$

{…} all possible gauge index combinations of rank-1

$\Sigma^{[J}H^{I]} \overset{!}{=}$ → only rank-2 bilinears are allowed

Result (comes out for all N):

$$\boxed{\Sigma^{[J}H^{I]} = \{(Q\Sigma^{IJ}\bar{Q})Q - Q(\bar{Q}\Sigma^{IJ}Q)\}}$$

right- and left-acting terms with same gauge index structure

What is H^I ?

* N=4,5,6

$$\boxed{\Sigma^{[J}H^{I]} = \{(Q\Sigma^{IJ}\bar{Q})Q - Q(\bar{Q}\Sigma^{IJ}Q)\}}$$

Which groups fulfil this? → bifundamental matrix products

U(M)XU(N) $(Q_w{}^{\bar{v}}\Sigma^{IJ}\bar{Q}_{\bar{v}}{}^{v})Q_v{}^{\bar{w}} - Q_w{}^{\bar{v}}(\bar{Q}_{\bar{v}}{}^{v}\Sigma^{IJ}Q_v{}^{\bar{w}})$

SU(N)XSU(N), b/c of cancelling „U(1) terms“

$$\ldots - \frac{1}{N}(Q_v{}^{\bar{v}}\Sigma^{IJ}\bar{Q}_{\bar{v}}{}^{v})Q_w{}^{\bar{w}} + \frac{1}{N}Q_w{}^{\bar{w}}(\bar{Q}_{\bar{v}}{}^{v}\Sigma^{IJ}Q_v{}^{\bar{v}})$$

SU(N)XSU(M)XU(1)°, with compensating U(1)° factor

SO(N)XSp(M), only for $\mathcal{N}$=4,5 (with reality condition), for $\mathcal{N}$=6 if N=2

fundamental SU(N)XU(1)° and Sp(N)XU(1)° (special limits of above)

These groups allow a solution for H^I and closure of the algebra

(agrees with Gaiotto,Witten; Hosomichi, 3xLee, Park; Schnabl,Tachikawa 2008)

* N=7,8

What is H^I ?

$$\Sigma^{[J}H^{I]} = \{(Q\Sigma^{IJ}\bar{Q})Q - Q(\bar{Q}\Sigma^{IJ}Q)\}$$

For N=7,8 one gets $(\Sigma^{IJ})_{[ij]}Q_i\bar{Q}_kQ_j$

only possible with real SU(2)XSU(2)

where $\bar{Q}_{\bar{v}}{}^{v} = \varepsilon^{vw}Q_w{}^{\bar{w}}\varepsilon_{\bar{w}\bar{v}}$

there it holds that

$$Q_{[i}\bar{Q}_{|k|}Q_{j]} = -\frac{1}{2}Q_{[i}\bar{Q}_{j]}Q_k - \frac{1}{2}Q_k\bar{Q}_{[i}Q_{j]}$$

representation:	fundamental	bifundamental
$\boldsymbol{\mathcal{N}=4}$	$\mathrm{SU}(N)_a \times \mathrm{U}(1)_{a-a/N}$ $\mathrm{Sp}(N)_a \times \mathrm{U}(1)_{-a}$	$\mathrm{U}(M)_a \times \mathrm{U}(N)_{-a}$ $\mathrm{SU}(N)_a \times \mathrm{SU}(N)_{-a}$ $\mathrm{Sp}(M)_a \times \mathrm{SO}(N)_{-a}$
$\boldsymbol{\mathcal{N}=5}$	$\mathrm{SU}(N)_a \times \mathrm{U}(1)_{a-a/N}$ $\mathrm{Sp}(N)_a \times \mathrm{U}(1)_{-a}$	$\mathrm{U}(M)_a \times \mathrm{U}(N)_{-a}$ $\mathrm{SU}(N)_a \times \mathrm{SU}(N)_{-a}$ $\mathrm{Sp}(M)_a \times \mathrm{SO}(N)_{-a}$
$\boldsymbol{\mathcal{N}=6}$	$\mathrm{SU}(N)_a \times \mathrm{U}(1)_{a-a/N}$ $\mathrm{Sp}(N)_a \times \mathrm{U}(1)_{-a}$	$\mathrm{U}(M)_a \times \mathrm{U}(N)_a$ $\mathrm{SU}(N)_a \times \mathrm{SU}(N)_{-a}$ $\mathrm{Sp}(M)_a \times \mathrm{SO}(2)_a$
$\boldsymbol{\mathcal{N}=7}$		$\mathrm{SU}(2)_a \times \mathrm{SU}(2)_a$
$\boldsymbol{\mathcal{N}=8}$		$\mathrm{SU}(2)_a \times \mathrm{SU}(2)_a$

Relation of different N

→ via their SO(N) spin matrices

SO(N)/spin(N) algebra: $[\mathcal{N}^{IJ}, \mathcal{N}^{KL}] = 4\delta^{[K[I}\mathcal{N}^{J]L]}$

Clifford algebra: $\gamma^I\gamma^J = \delta^{IJ} + \gamma^{IJ}$

→ then $\mathcal{N}^{IJ} = -\frac{1}{2}\gamma^{IJ}$ generate spin(N)

Chiral representation:

$$\gamma^1 = \sigma_1 \otimes \mathbb{1} \otimes \ldots \otimes \mathbb{1} \qquad \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

$$\gamma^{2,\ldots,2m} = \mathrm{i}\sigma_2 \otimes \mathrm{i}\tilde{\gamma}^{1,\ldots,\mathcal{N}-1} \qquad \begin{pmatrix} 0 & \mathrm{i}\tilde{\gamma} \\ -\mathrm{i}\tilde{\gamma} & 0 \end{pmatrix}$$

$$\gamma^* = \gamma^{2m+1} = -\mathrm{i}^m\gamma^1 \cdot \ldots \cdot \gamma^{2m} = \sigma_3 \otimes \mathbb{1} \otimes \ldots \otimes \mathbb{1} \qquad \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$$

For even N: $\gamma^{IJ} = \begin{pmatrix} \Sigma^{IJ} & 0 \\ 0 & \bar{\Sigma}^{IJ} \end{pmatrix}$ $\quad P_{\mathrm{L/R}} = \frac{1}{2}(\mathbb{1} \pm \gamma^*)$ → reducible

$$\Sigma^I\bar{\Sigma}^J = \delta^{IJ} + \Sigma^{IJ}$$

$$\bar{\Sigma}^I\Sigma^J = \delta^{IJ} + \bar{\Sigma}^{IJ}$$

call them *Clifford* representations
and *chiral* representations

Relation of different N

$$\begin{tikzcd}
\textbf{N=4 chiral} \arrow[r, "?"] & \textbf{N=4 Clifford} \arrow[d, "\text{inclusion of } \gamma^* = \gamma^5"'] & \\
& \textbf{N=5} \arrow[d, "\text{inclusion of } \Sigma^1 = \bar{\Sigma}^1 = \mathbb{1}"'] & \\
& \textbf{N=6 chiral} \arrow[r, "?"] & \textbf{N=6 Clifford} \arrow[d, "\text{inclusion of } \gamma^* = \gamma^7"'] \\
& & \textbf{N=7} \arrow[d, "\text{inclusion of } \Sigma^1 = \bar{\Sigma}^1 = \mathbb{1}"'] \\
& & \textbf{N=8 chiral}
\end{tikzcd}$$

This line has to be explored regarding the form of H^I

and the calculation of $\Sigma^{[J} H^{I]}$

Suppose the situation: $(\Sigma^I)(\Sigma_I) \sim \delta, \varepsilon$ metric tensors

Then generally: $H^I = \{(\Sigma^I Q)\bar{Q}Q\} + \ldots + \{(Q\Sigma^I\bar{Q})Q\} + \ldots$ and no $(\Sigma^{IK})(\Sigma_K)$

in order to get rank-2 bilinears: $(\Sigma^{[J})(\Sigma^{I]}) \overset{!}{\sim} (\delta)(\Sigma^{IJ})$

N=4 chiral $\xrightarrow{q_a \doteq (Q_i, Q^{\bar{i}})^{\mathrm{T}}}$ **N=4 Clifford**

$$(\Sigma^I)_{i\bar{i}}(\bar{\Sigma}_I)^{\bar{j}j} = 2\delta_i^j\delta_{\bar{i}}^{\bar{j}}$$

$$(\Sigma^{[I})_{i\bar{j}}(\bar{\Sigma}^{J]})^{\bar{l}k} = \ldots$$

SU(2) indices

$$(\gamma^I)_a{}^b \doteq \begin{pmatrix} 0 & (\Sigma^I)_{i\bar{j}} \\ (\bar{\Sigma}^I)^{\bar{i}j} & 0 \end{pmatrix}$$

$$C^{ab} \doteq \begin{pmatrix} \varepsilon^{ij} & 0 \\ 0 & \varepsilon_{\bar{i}\bar{j}} \end{pmatrix}$$

realises USp(4)=spin(5)

N=5

↕

N=6 chiral

SU(4) $(\Sigma^I)_{ij}, (\bar{\Sigma}^I)^{ij}$

$$(\gamma^{[I})_{ij}(\gamma^{J]})^{kl} = 2\delta_{[i}^{[k}(\gamma^{IJ})_{j]}{}^{l]}$$

follows from $\varepsilon^{ijkl}(\gamma^I)_{kl} = -2(\gamma^I)^{ij}$

or from SU(2) properties

$$H^{I,k} = -a(\gamma^I)^{kl}\{(q_l\bar{q}^i q_i - q_i\bar{q}^i q_l) + 2(\gamma^I)^{ij}q_i\bar{q}^k q_j\}$$

$$(\Sigma^I)_{ij}(\bar{\Sigma}_I)^{kl} = -4\delta^k_{[i}\delta^l_{j]}$$
$$(\bar{\Sigma}^I)^{ij}(\bar{\Sigma}_I)^{kl} = 2\varepsilon^{ijkl}$$

→

$$(\gamma^I)(\gamma_I)\not\sim\delta\delta$$
$$(\gamma^{[J})(\gamma^{I]})\not\sim(\delta)(\gamma^{IJ})$$

fails b/c of complexness of spin(6)=SU(4)

N=6 chiral → **N=6 Clifford**

N=6 chiral → **N=6 Majorana** $q_a = \begin{pmatrix} Q_i \\ \bar{Q}^i \end{pmatrix}$

realises spin(7)

in a real basis:

$$(\gamma^I)_{i(j}(\gamma^I)_{l)k} = \delta_{ik}\delta_{jl} - \delta_{i(j}\delta_{l)k}$$

Fierz lemma:

$$8\delta_{ac}\delta_{bd} = \delta_{ad}\delta_{bc} + \gamma^I_{ad}\gamma^I_{bc} - \tfrac{1}{2}\gamma^{IJ}_{ad}\gamma^{IJ}_{bc} - \tfrac{1}{6}\gamma^{IJK}_{ad}\gamma^{IJK}_{bc}$$

→

$$\begin{aligned} \gamma^{I[K}_{ab}\gamma^{L]I}_{cd} &= 4\delta_{[a[c}\gamma^{KL}_{d]b]} - \gamma^{[K}_{ab}\gamma^{L]}_{cd} \\ -8\delta_{(c[a}\gamma^K_{b]d)} &= \delta_{cd}\gamma^K_{ab} - \tfrac{1}{2}\gamma^{KIJ}_{cd}\gamma^{IJ}_{ab} \\ -8\delta_{[c[a}\gamma^K_{b]d]} &= -\gamma^{KI}_{cd}\gamma^I_{ab} + \gamma^{KI}_{ab}\gamma^I_{cd} \end{aligned}$$

N=7

↕

N=8 chiral

$(\Sigma^I)_{i\bar{i}}, (\bar{\Sigma}^I)_{\bar{i}i}$

N=6 M
$$H^I_k = \gamma^I_{kl}\,[q_l\bar{q}_i q_i - q_i\bar{q}_l q_i] + \gamma^I_{ij}q_i\bar{q}_j q_k - \gamma^{IK}_{ij}\gamma^K_{kl}q_i\bar{q}_j q_l - \gamma^{I*}_{ij}\gamma^*_{kl}q_i\bar{q}_j q_l$$

N=7
$$H^I_k = \gamma^I_{kl}\,[q_l\bar{q}_i q_i - q_i\bar{q}_l q_i] + \gamma^I_{ij}q_i\bar{q}_j q_k - \gamma^{IK}_{ij}\gamma^K_{kl}q_i\bar{q}_j q_l$$

N=8 ch
$$H^I_{\bar{k}} = (\bar{\Sigma}^I)_{\bar{k}l}\,\left[Q_l\bar{Q}_i Q_i - Q_i\bar{Q}_l Q_i\right] - (\Sigma^{IK})_{ij}(\bar{\Sigma}^K)_{\bar{k}l}Q_i\bar{Q}_j Q_l$$

Supergravity

Extended curved superspace: local structure group Sl(2,R)XSO(N)

$$\mathscr{D}_A = E_A{}^M \partial_M + \tfrac{1}{2}\Omega_A{}^{mn}\mathscr{M}_{mn} + \tfrac{1}{2}\Phi_A{}^{PQ}\mathscr{N}_{PQ}$$

Superalgebra:

$$[\mathscr{D}_A, \mathscr{D}_B\} = -T_{AB}{}^C\mathscr{D}_C + \tfrac{1}{2}R_{AB}{}^{PQ}\mathscr{N}_{PQ} + \tfrac{1}{2}R_{AB}{}^{mn}\mathscr{M}_{mn}$$

Weyl invariant constraints:

$$T^{IJ,c}_{\alpha\beta} = -\mathrm{i}\delta^{IJ}(\gamma^c)_{\alpha\beta}$$

$$T_{a\beta,K}{}^{J,\gamma} = (\gamma_a)_\beta{}^\gamma K^{JK} + (\gamma^b)_\beta{}^\gamma L_{ab}{}^{JK}$$

$$T_{ab,K}{}^{\gamma} = \Psi_{ab,K}{}^{\gamma}$$

For anti de Sitter, only

$$K^{IJ} = \delta^{IJ}K \neq 0$$

super Jakobi identity implies:

$$\{\mathscr{D}^I_\alpha, \mathscr{D}^J_\beta\} = 2\mathrm{i}\delta^{IJ}\mathscr{D}_{\alpha\beta} + \mathrm{i}\varepsilon_{\alpha\beta}\left(\underset{\uparrow}{W}{}^{IJKL} + 4\delta^{K[I}K^{J]L}\right)\mathscr{N}_{KL} + 4\mathrm{i}K^{IJ}\mathscr{M}_{\alpha\beta}$$

super Cotton tensor

Supergravity-matter coupling

SO(N) field strength

$$F^{IJ}_{ab} \sim (\gamma_{ab})^{\alpha\beta} \mathscr{D}^K_\alpha \mathscr{D}^L_\beta W^{IJKL}$$

coupling to matter

$$W^{IJKL} = -\tfrac{\lambda}{16} \bar{Q} \Sigma^{IJKL} Q$$

$$\Sigma^{[J} H^{I]} = -\tfrac{1}{2} \left(W^{IJKL} \Sigma_{KL} + 4K \Sigma^{IJ} \right) Q$$

solvable with W^{IJKL} off shell for N=4,5,6

e.g. N=6:

special U(1)
R-symmetry factor

$$\{\mathscr{D}^I_\alpha, \mathscr{D}^J_\beta\} Q = 2\mathrm{i} (\gamma^a)_{\alpha\beta} \mathscr{D}_a Q - \tfrac{1}{2} \varepsilon_{\alpha\beta} W_{PQ} \Sigma^{IJPQ} Q + q \varepsilon_{\alpha\beta} W^{IJ} Q - 2\mathrm{i} \varepsilon_{\alpha\beta} K \Sigma^{IJ} Q$$

$$H^I_{\mathrm{SG}} = -\tfrac{\mathrm{i}}{2} W_{PQ} \bar{\Sigma}^{IPQ} Q + \mathrm{i} W^{IK} \bar{\Sigma}_K Q + 2K \bar{\Sigma}^I Q$$

$$W^{IJ} = \tfrac{\lambda}{16} \mathrm{i} |Q \bar{\Sigma}^{IJ} \bar{Q}|$$

$$H^{I,k}_{\mathrm{SG}} = -\tfrac{\lambda}{4} |Q_i \bar{Q}^k| (\bar{\Sigma}^I)^{il} Q_l + \tfrac{\lambda}{16} |Q_i \bar{Q}^i| (\bar{\Sigma}^I)^{kl} Q_l + 2K \bar{\Sigma}^I Q$$

Modification of groups

Keep the ansatz general

$$H^I_{\mathrm{SG}} = XW_{PQ}\bar{\Sigma}^{IPQ}Q + YW^{IK}\bar{\Sigma}_K Q + 2K\bar{\Sigma}^I Q$$

solve it together with gauge groups ansatz

$$\begin{aligned}\Sigma^{[J}H^{I]}_{\mathrm{SG}} =& \tfrac{\lambda}{16}[2\mathrm{i}X|Q_i\bar{Q}^i|(\Sigma^{IJ}Q)_k - \mathrm{i}(6X+Y)(\Sigma^{IJ})_k{}^l|Q_l\bar{Q}^m|Q_m \\ &- \mathrm{i}(2X-Y)|Q_k\bar{Q}^j|(\Sigma^{IJ}Q)_j + \mathrm{i}(2X-Y)|Q\bar{\Sigma}^{IJ}\bar{Q}|Q_k] - 2K\Sigma^{IJ}Q\end{aligned}$$

$$\Sigma^{[J}H^{I]}_{\mathrm{CS}} = \ldots$$

reproduce the supergravity sector and potential field strengths

representation:	fundamental	bifundamental
$\boldsymbol{\mathcal{N}=4}$	$\mathrm{SU}(N)_a \times \mathrm{U}(1)_{a-a/N}$ $\mathrm{Sp}(N)_a \times \mathrm{U}(1)_{-a}$	$\mathrm{U}(M)_a \times \mathrm{U}(N)_{-a}$ $\mathrm{SU}(N)_a \times \mathrm{SU}(N)_{-a}$ $\mathrm{Sp}(M)_a \times \mathrm{SO}(N)_{-a}$
+SG		
$\boldsymbol{\mathcal{N}=5}$	$\mathrm{SU}(N)_a \times \mathrm{U}(1)_{a-a/N}$ $\mathrm{Sp}(N)_a \times \mathrm{U}(1)_{-a}$	$\mathrm{U}(M)_a \times \mathrm{U}(N)_{-a}$ $\mathrm{SU}(N)_a \times \mathrm{SU}(N)_{-a}$ $\mathrm{Sp}(M)_a \times \mathrm{SO}(N)_{-a}$
+SG		
$\boldsymbol{\mathcal{N}=6}$	$\mathrm{SU}(N)_a \times \mathrm{U}(1)_{a-a/N}$ $\mathrm{Sp}(N)_a \times \mathrm{U}(1)_{-a}$	$\mathrm{U}(M)_a \times \mathrm{U}(N)_a$ $\mathrm{SU}(N)_a \times \mathrm{SU}(N)_{-a}$ $\mathrm{Sp}(M)_a \times \mathrm{SO}(2)_a$
+SG	$\mathrm{SU}(N) \times \mathrm{U}(1)$	$\mathrm{SU}(N)_a \times \mathrm{SU}(M)_a$
$\boldsymbol{\mathcal{N}=7}$		$\mathrm{SU}(2)_a \times \mathrm{SU}(2)_a$
+SG	$\mathrm{SO}(N)_{-\lambda/16}$ $\mathrm{SU}(N)_{-\lambda/8} \times \mathrm{U}(1)_{(2-N)\lambda/16}$	$\mathrm{SU}(2)_a \times \mathrm{SU}(2)_{a-\lambda/8}$
$\boldsymbol{\mathcal{N}=8}$		$\mathrm{SU}(2)_a \times \mathrm{SU}(2)_a$
+SG	$\mathrm{SO}(N)_{-\lambda/8}$ $\mathrm{SU}(N)_{-\lambda/4} \times \mathrm{U}(1)_{(2-N)\lambda/8}$	$\mathrm{SU}(2)_a \times \mathrm{SU}(2)_{a-\lambda/4}$

Comments

- Things are for free in superspace
- On-shell approach is a quite universal treatment of different N
- Gravitationally coupled theories
  - Realise topologically massive gravity with fixed scale
  - Possible applications for M-theory or condensed matter
  - Relevant for adS/CFT with free boundary