

MSSM HIGGS BOSON PRODUCTION : GLUON FUSION

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I Introduction

II Higgs Production

III Conclusions

in collaboration with M. Mühlleitner and H. Rzehak

I INTRODUCTION

MSSM

- 2 Higgs doublets $\xrightarrow{\text{ESB}}$ 5 Higgs bosons: $h, H, A, H^\pm$

- LO: 2 input parameters: $M_A, \tan\beta = \frac{v_2}{v_1}$

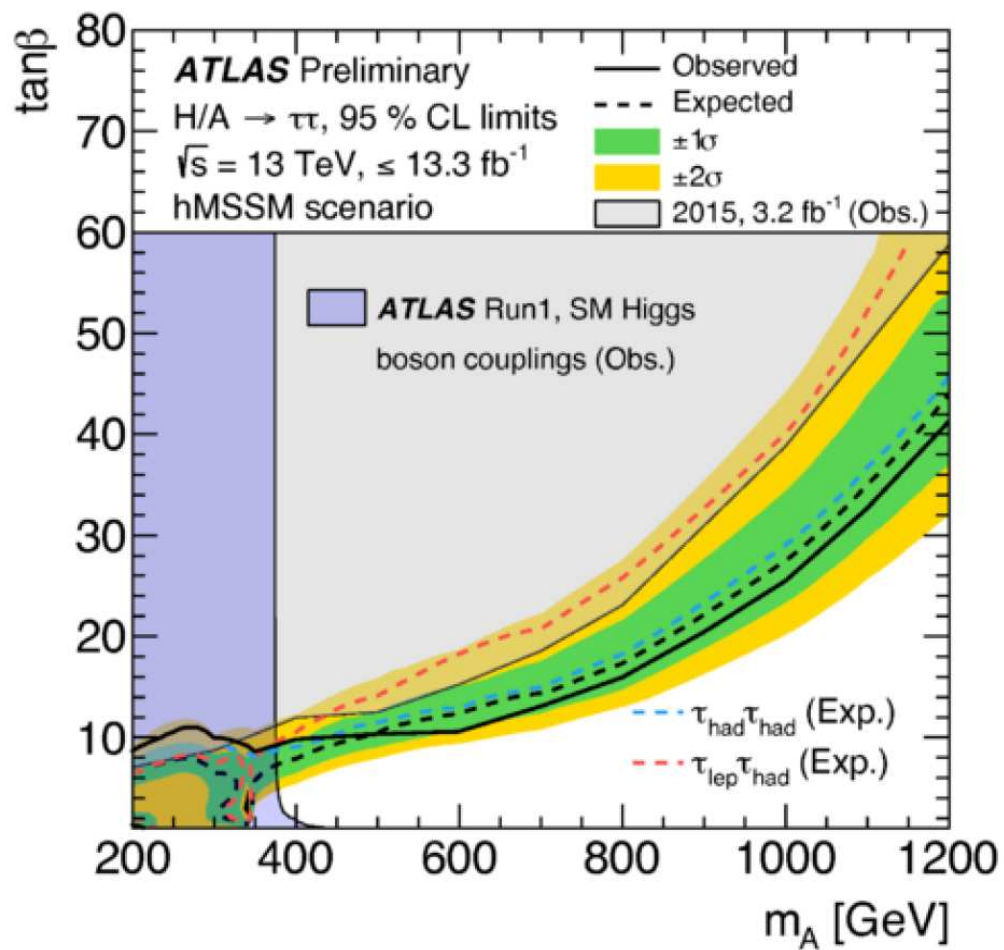
- radiative corrections $\propto m_t^4 \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \rightarrow \boxed{M_h \lesssim 135 \text{ GeV}}$

Haber
Carena, ...
Heinemeyer, ...
Zhang
Slavich, ...
...

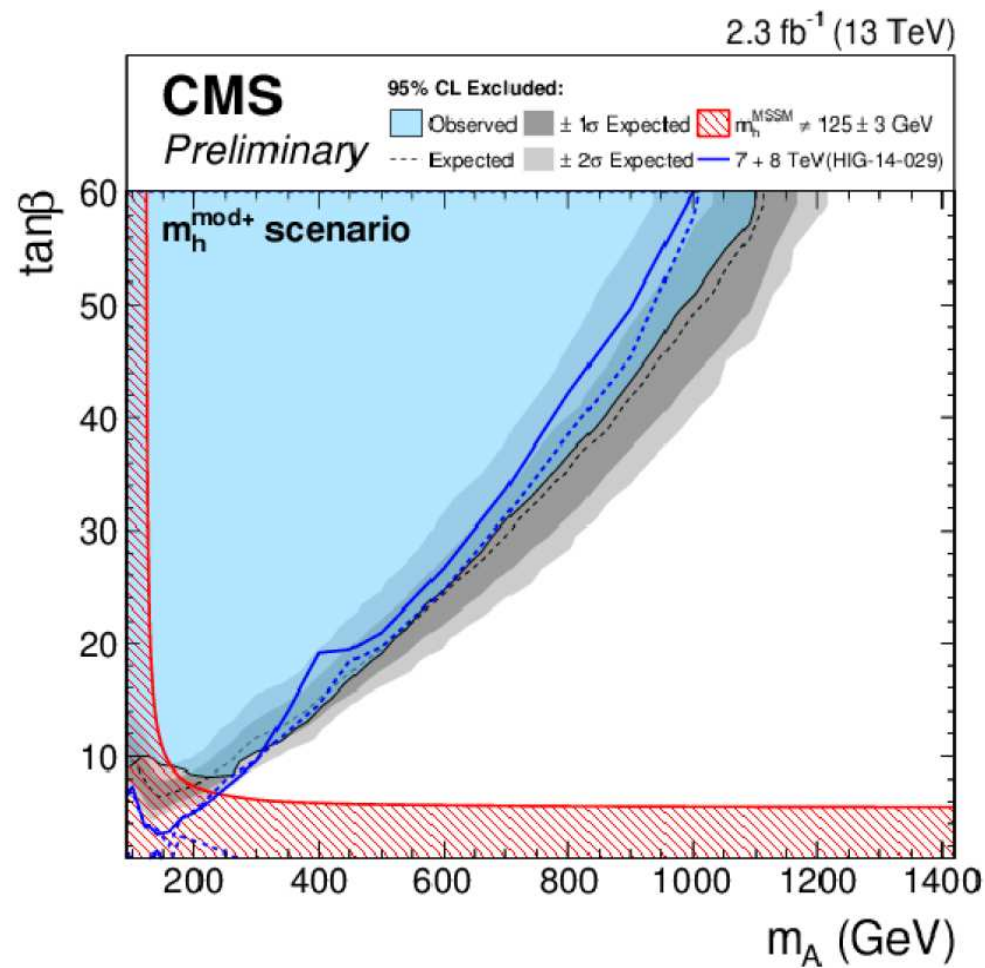
- Yukawa couplings: $\tan\beta \uparrow \Rightarrow g_u^\phi \downarrow \quad g_d^\phi \uparrow \quad g_V^\phi \downarrow$

- LHC: $gg \rightarrow \phi$ dominant for $\tan\beta \lesssim 10$
 $gg \rightarrow \phi b\bar{b}$ dominant for $\tan\beta \gtrsim 10$

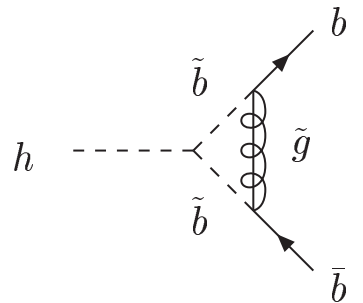
$$gg \rightarrow b\bar{b}\phi^0, \quad gg \rightarrow \phi^0$$



$$\phi^0 \rightarrow \tau^+\tau^-$$

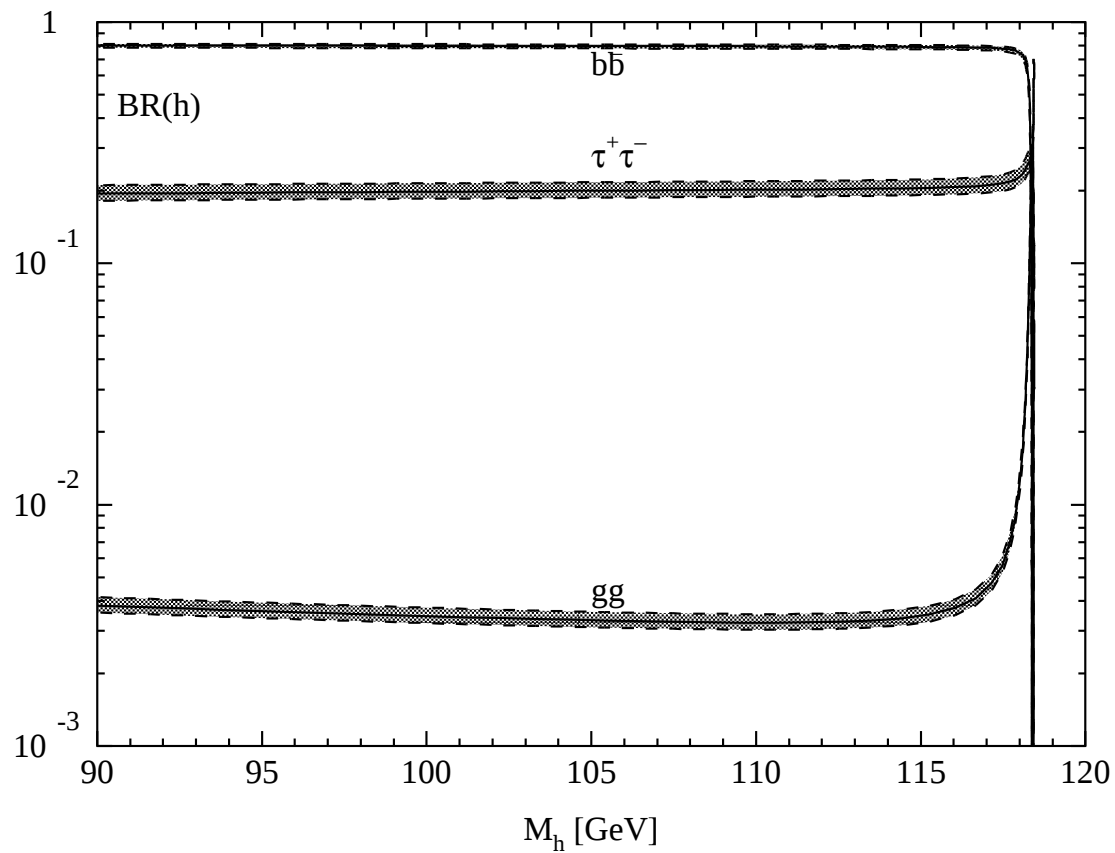


- large SUSY-QCD corrections to $\phi^0 \rightarrow b\bar{b}$



$$+ \dots \propto \frac{\alpha_s}{\pi} \frac{m_{\tilde{g}} \mu \tan\beta}{m_{\tilde{b}}^2}$$

Hall, ...
Carena, ...
Nierste, ...
Guasch, ...
etc.



Guasch, Häfliger, S.

SUSY-QCD Corrections to $b\bar{b}\phi^0$

$[\Delta \lesssim 1\%]$

$$\mathcal{L}_{eff} = -\lambda_b \bar{b}_R \left[\phi_1^0 + \frac{\Delta_b}{\text{tg}\beta} \phi_2^{0*} \right] b_L + h.c. \quad \text{valid to all orders in } \Delta_b$$

$$= -m_b \bar{b} \left[1 + i\gamma_5 \frac{G^0}{v} \right] b - \frac{m_b/v}{1 + \Delta_b} \bar{b} \left[g_b^h \left(1 - \frac{\Delta_b}{\text{tg}\alpha \text{tg}\beta} \right) h \right. \\ \left. + g_b^H \left(1 + \Delta_b \frac{\text{tg}\alpha}{\text{tg}\beta} \right) H - g_b^A \left(1 - \frac{\Delta_b}{\text{tg}^2\beta} \right) i\gamma_5 A \right] b$$

$$\Delta_b = \Delta_b^{QCD(1)} + \Delta_b^{elw(1)}$$

$$\Delta_b^{QCD(1)} = \frac{2}{3} \frac{\alpha_s(\mu_R)}{\pi} M_{\tilde{g}} \mu \text{tg}\beta I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, M_{\tilde{g}}^2)$$

$$\Delta_b^{elw(1)} = \frac{\lambda_t^2(\mu_R)}{(4\pi)^2} \mu A_t \text{tg}\beta I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2)$$

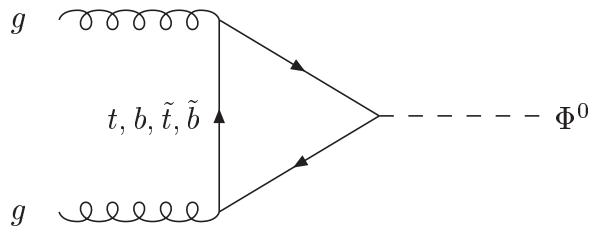
$$I(a, b, c) = -\frac{ab \log \frac{a}{b} + bc \log \frac{b}{c} + ca \log \frac{c}{a}}{(a-b)(b-c)(c-a)}$$

$\Rightarrow$ resummed Yukawa couplings

Carena, Garcia, Nierste, Wagner
Guasch, Häfliger, S.

II HIGGS PRODUCTION

Gluon fusion: $pp \rightarrow gg \rightarrow h/H/A$



Georgi, Glashow, Machacek, Nanopoulos

- third generation dominant
- NLO QCD corrections: $\sim 10 \dots 100\%$
- elw. corrections: $\sim 5\%$
- NNLO QCD for $m_t \gg M_\phi \Rightarrow + 20\text{--}30\%$
[mass effects small]

S., Djouadi, Graudenz, Zerwas
Dawson, Kauffman

Aglietti, . . .
Degrassi, Maltoni
Actis, Passarino, Sturm, Uccirati

Harlander, Kilgore
Anastasiou, Melnikov
Ravindran, Smith, van Neerven
Marzani, Ball, Del Duca, Forte, Vicini
Harlander, Ozeren
Pak, Rogal, Steinhauser

- N³LO for $m_t \gg M_\phi \Rightarrow$ scale stabilization

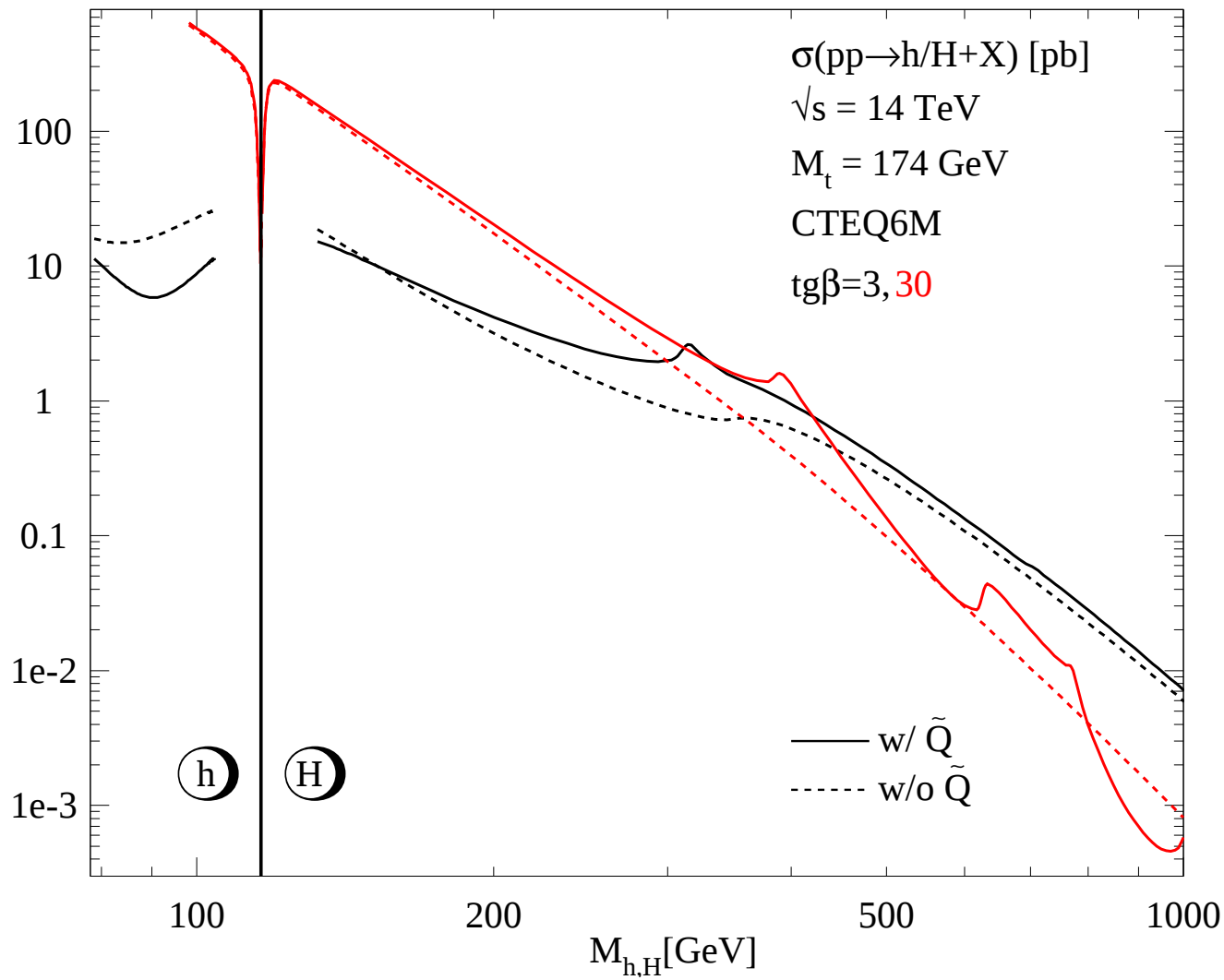
Moch, Vogt
Ravindran

de Florian, Mazzitelli, Moch, Vogt
Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger
Ball, Bonvini, Forte, Marzani, Ridolfi
Anastasiou, Duhr, Dulat, Herzog, Mistlberger
Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger

- N³LL soft gluon resummation [$m_t \gg M_\phi$]: $\lesssim 5\%$

Catani, de Florian, Grazzini, Nason
Ravindran

Ahrens, Becher, Neubert, Yang
Ball, Bonvini, Forte, Marzani, Ridolfi
Schmidt, S.

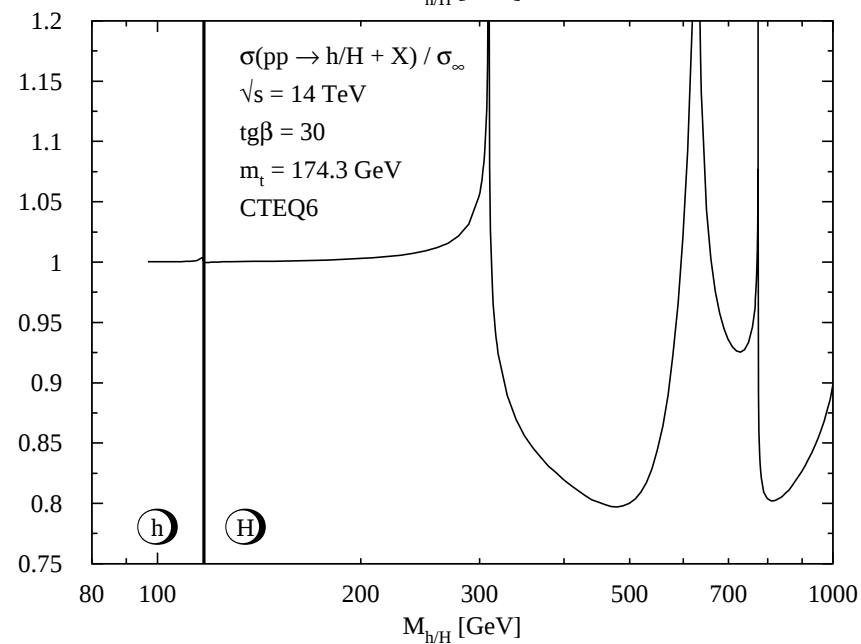
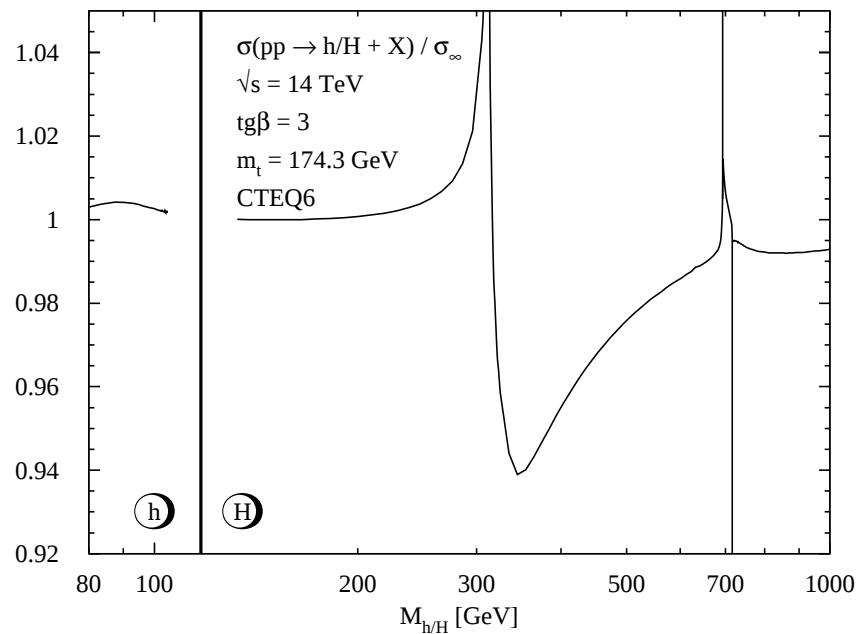
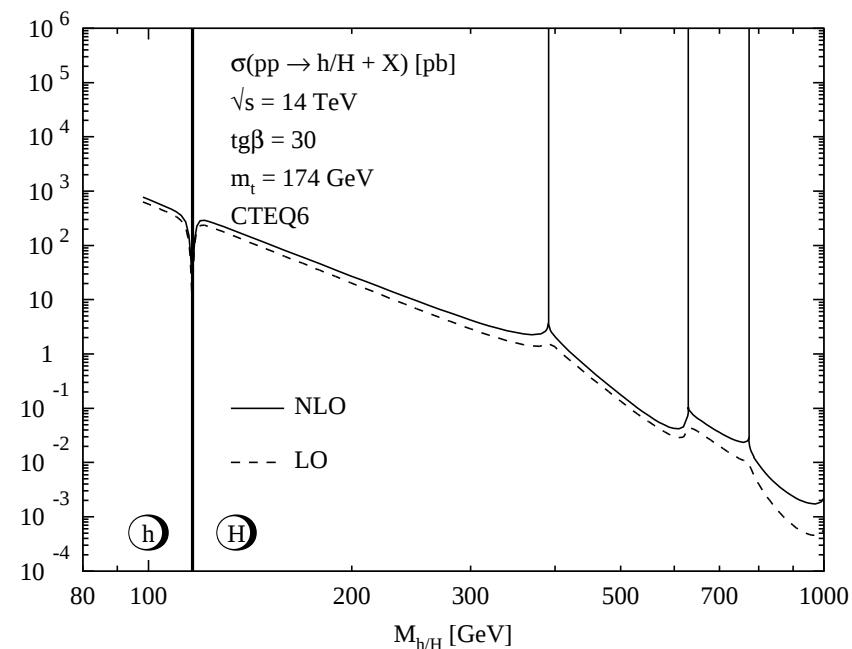
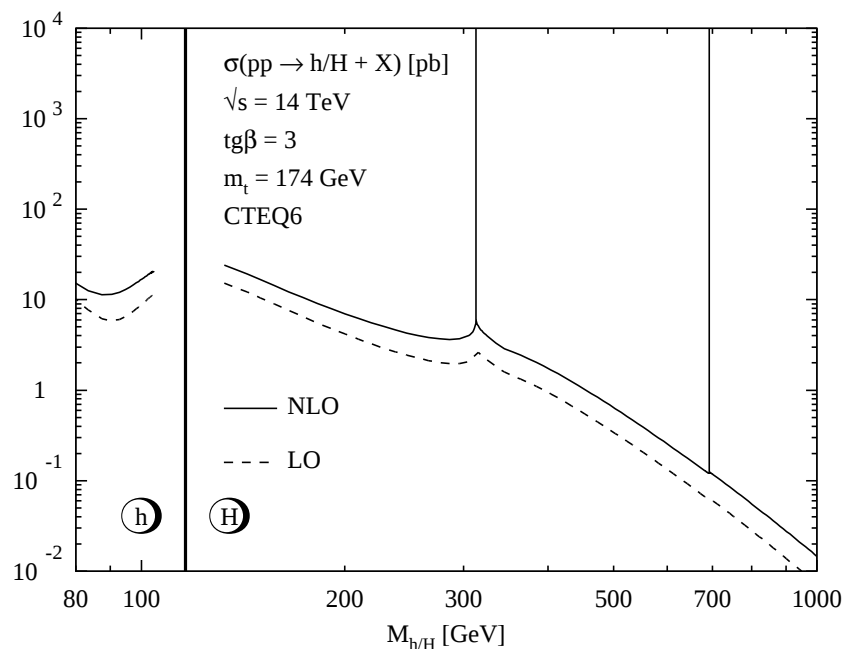


Mühlleitner, S.

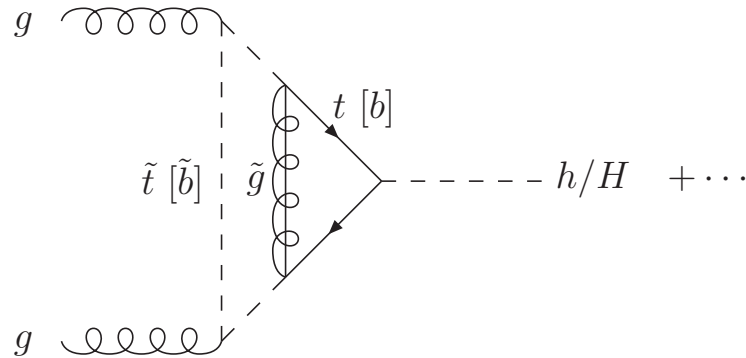
$\tan\beta = 3$: $m_{\tilde{t}_1} = 156$ GeV, $m_{\tilde{t}_2} = 516$ GeV, $m_{\tilde{b}_1} = 346$ GeV, $m_{\tilde{b}_2} = 358$ GeV
 $\tan\beta = 30$: $m_{\tilde{t}_1} = 195$ GeV, $m_{\tilde{t}_2} = 502$ GeV, $m_{\tilde{b}_1} = 315$ GeV, $m_{\tilde{b}_2} = 387$ GeV

• QCD corrections to squark loops:

Mühlleitner, S.



genuine SUSY-QCD corrections:



Harlander, Steinhauser, Hofmann
 Deggrasi, Slavich
 Anastasiou, Beerli, Daleo
 Mühlleitner, Rzehak, S.

$$\sigma_{LO}(pp \rightarrow \phi^0) = \sigma_0^\phi \tau_\phi \frac{d\mathcal{L}^{gg}}{d\tau_\phi}$$

$$\sigma_0^{h/H} = \frac{G_F \alpha_s^2}{288 \sqrt{2} \pi} \left| \sum_Q g_Q^{h/H} A_Q^{h/H}(\tau_Q) + \sum_{\tilde{Q}} g_{\tilde{Q}}^{h/H} A_{\tilde{Q}}^{h/H}(\tau_{\tilde{Q}}) \right|^2 \quad \sigma_0^A = \frac{G_F \alpha_s^2}{128 \sqrt{2} \pi} \left| \sum_Q g_Q^A A_Q^A(\tau_Q) \right|^2$$

- numerical analysis: $A_Q^{h/H}(\tau_Q) \rightarrow A_Q^{h/H}(\tau_Q) \left[1 + C_{SUSY}^Q \frac{\alpha_s}{\pi} \right]$

- $m_{Q/\tilde{Q}}^2 \rightarrow m_{Q/\tilde{Q}}^2(1 - i\epsilon)$

- 5-dimensional Feynman integrals $\rightarrow$ endpoint subtractions:

$$\int_0^1 dx \frac{f(x)}{x(1-x)} \rightarrow \int_0^1 dx \left\{ \frac{f(x)}{x(1-x)} - \frac{f(0)}{x} - \frac{f(1)}{1-x} \right\}$$

$\Rightarrow$ isolation of singularities

- thresholds for $M_H > 2m_Q \rightarrow$ numerical instabilities $\rightarrow$ integration by parts:

$$\int_0^1 dz \frac{f(z)}{(a+bz)^2} = -\frac{f(z)}{b(a+bz)} \Big|_0^1 + \int_0^1 dz \frac{f'(z)}{b(a+bz)}$$

$$\int_0^1 dz \frac{f(z)}{a+bz} = \frac{f(z)}{b} \log(a+bz) \Big|_0^1 - \int_0^1 dz \frac{f'(z)}{b} \log(a+bz)$$

$\Rightarrow$ thresholds in reduced powers of denominators or in arguments of logs $\Rightarrow$ stabilization

[more involved for quadratic polynomials]

- α_s : $\overline{\text{MS}}$ scheme [5 flavours]
- $m_Q, m_{\tilde{Q}}$: on-shell
- A_t, A_b : $\overline{\text{MS}}$
- A_t, A_b : anomalous SUSY-restoring counter terms
- θ_Q : anti-Hermitean:

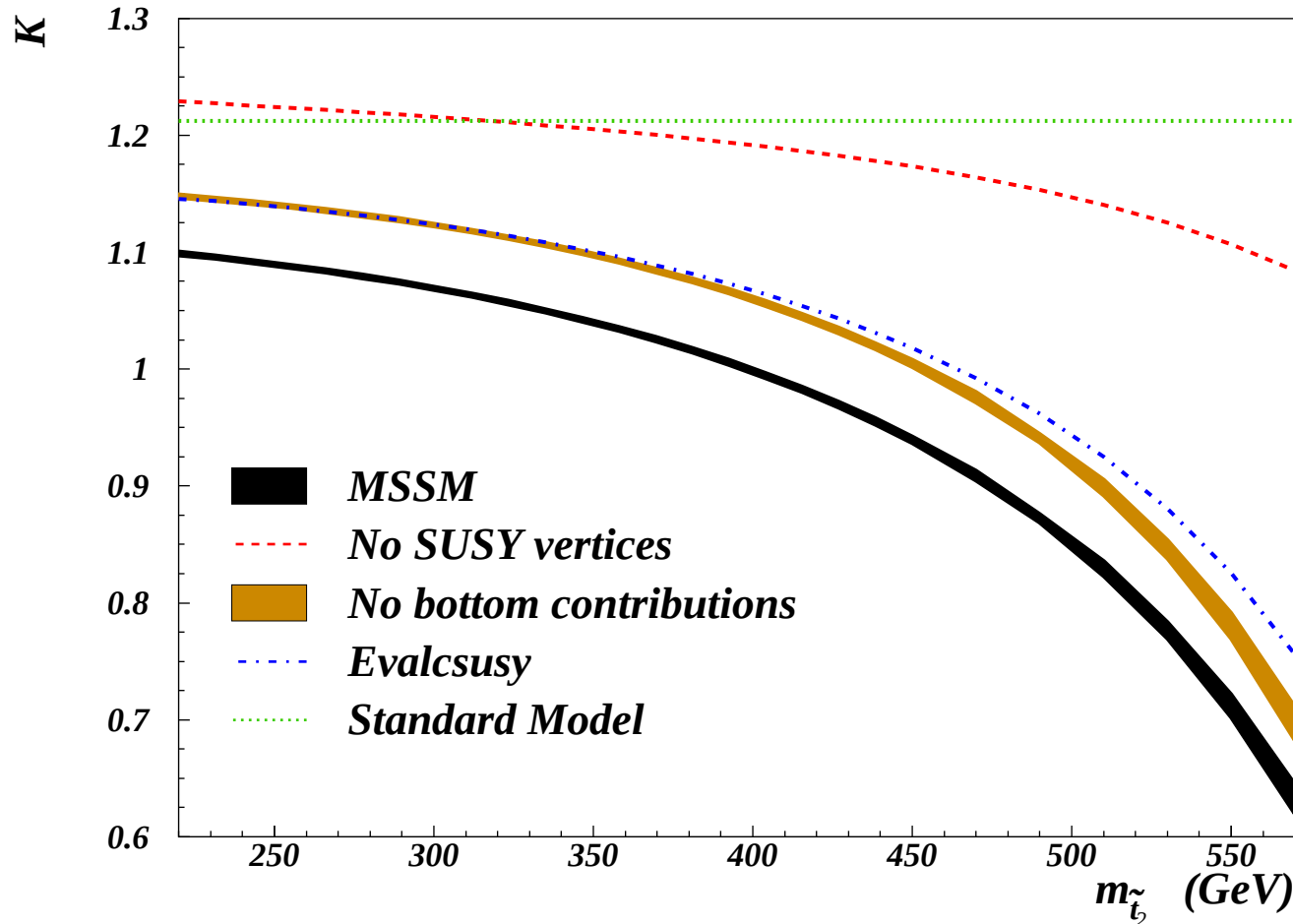
$$\delta\theta_t = \frac{1}{2} \Re e \frac{\Sigma_{12}(m_{\tilde{t}_1}^2) + \Sigma_{12}(m_{\tilde{t}_2}^2)}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}$$

- bottom Yukawa: resummation of Δ_b contributions

SUSY-QCD corrections:

$\alpha = 3^\circ, \tan\beta = 20, \mu = 300 \text{ GeV}, M_{\tilde{g}} = 500 \text{ GeV}, m_{\tilde{t}_1} = 150 \text{ GeV}$

$m_{\tilde{t}_2} = 350 \text{ GeV}, m_{\tilde{b}_1} = 350 \text{ GeV}, m_{\tilde{b}_2} = 370 \text{ GeV}, \theta_{\tilde{t}/\tilde{b}} = 40^\circ$

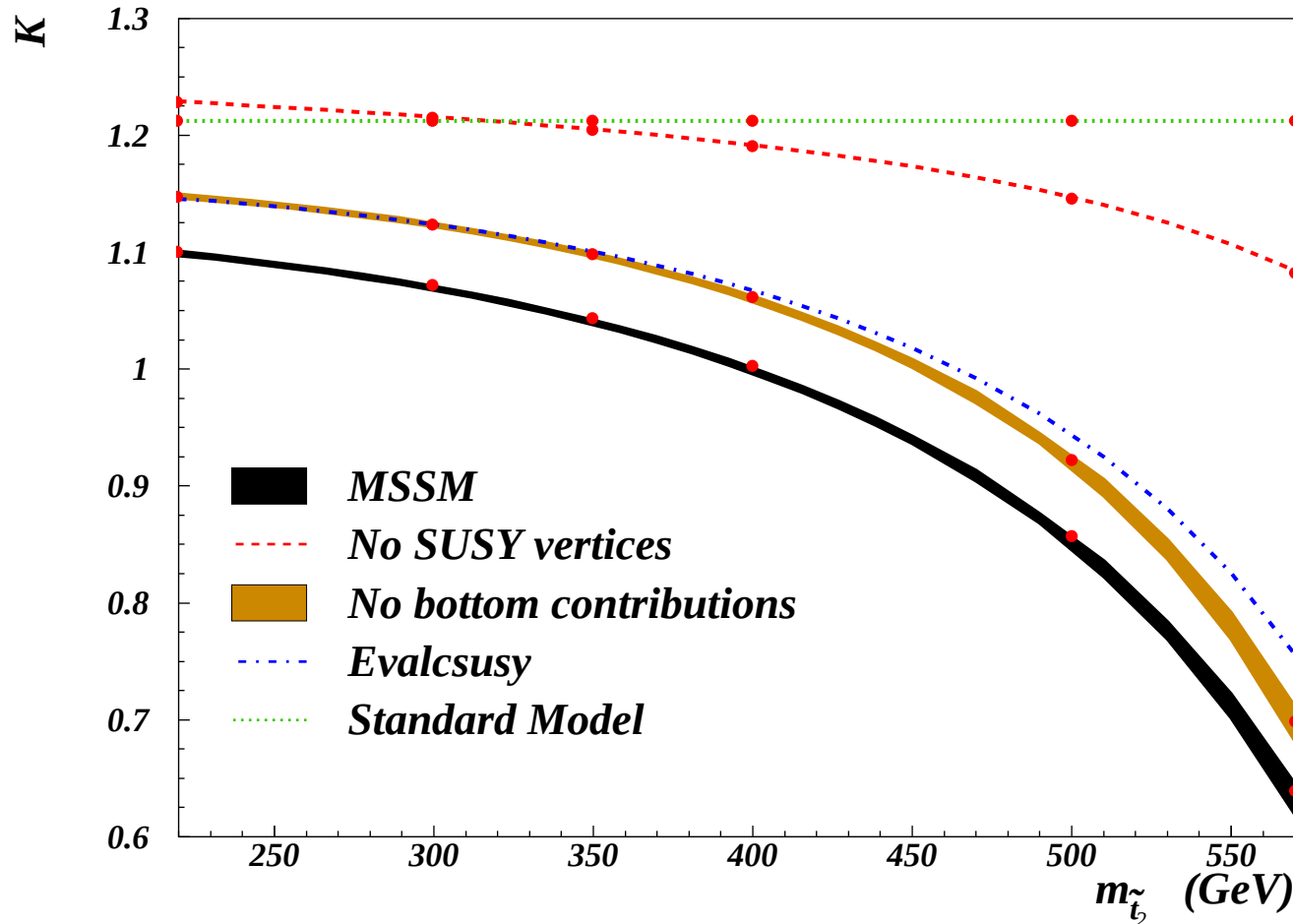


Anastasiou, Beerli, Daleo

SUSY-QCD corrections:

$\alpha = 3^\circ, \tan\beta = 20, \mu = 300 \text{ GeV}, M_{\tilde{g}} = 500 \text{ GeV}, m_{\tilde{t}_1} = 150 \text{ GeV}$

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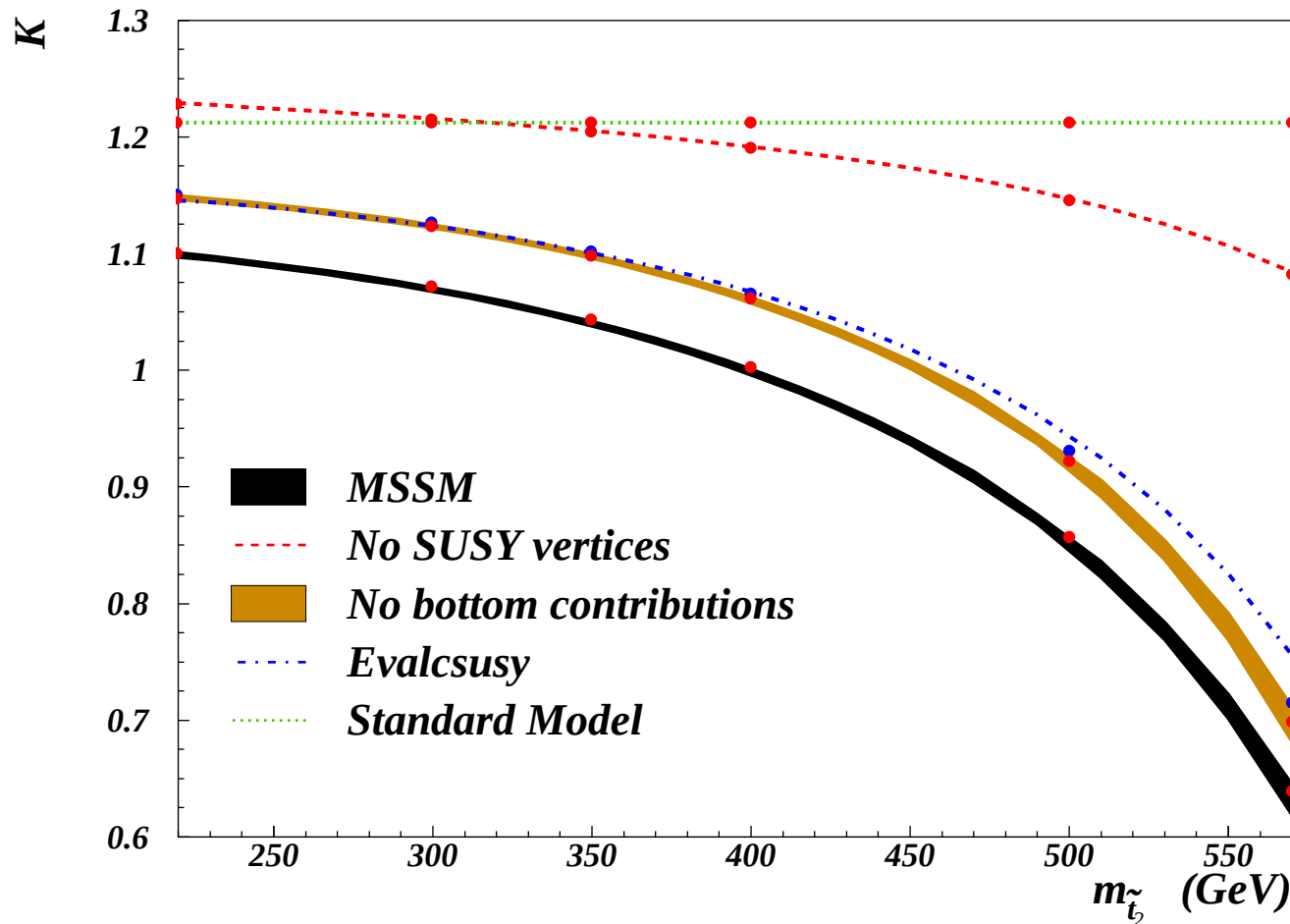
PRELIMINARY

Anastasiou, Beerli, Daleo
Mühlleitner, Rzehak, S.

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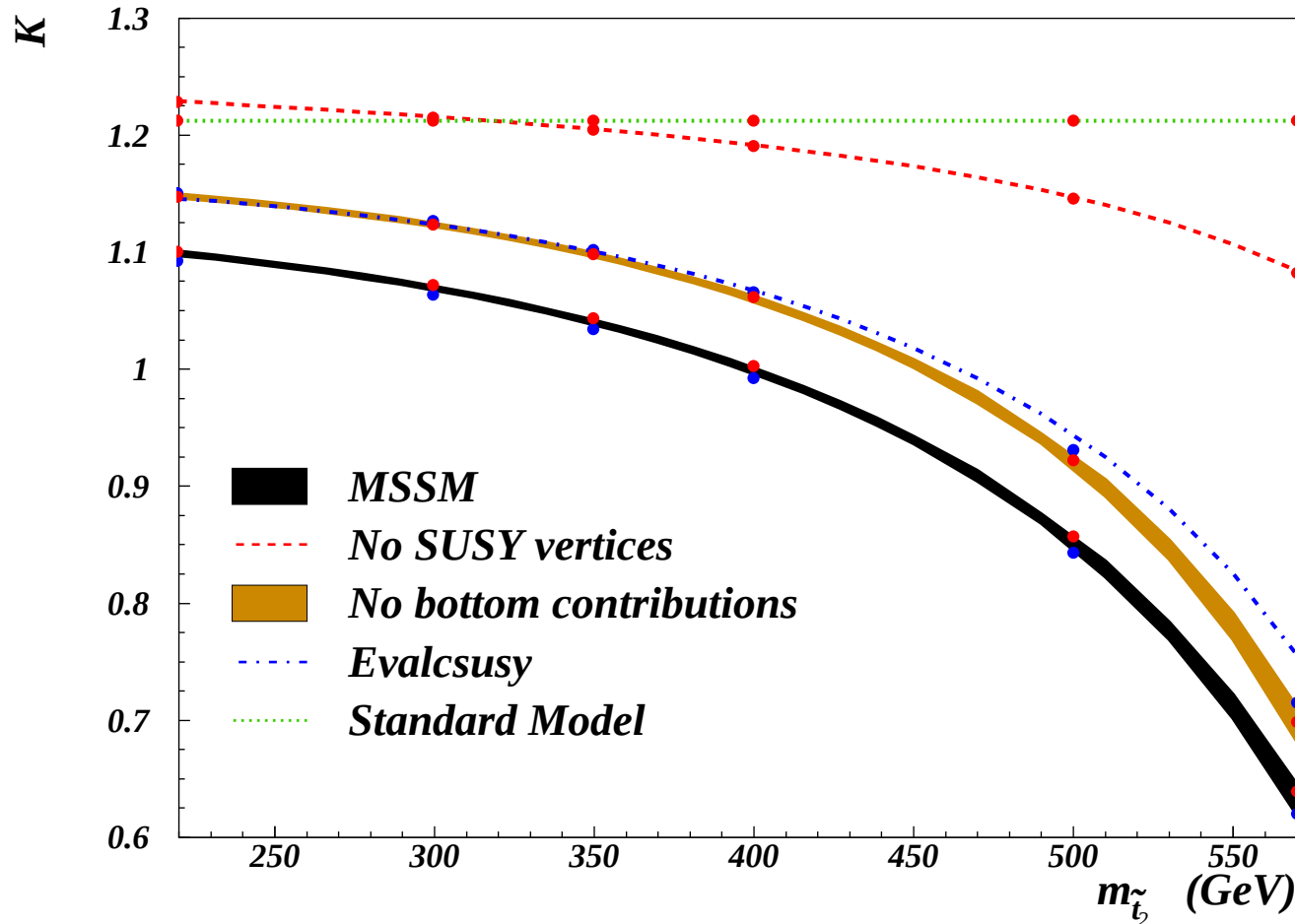
PRELIMINARY

Anastasiou, Beerli, Daleo
Mühlleitner, Rzehak, S.
Harlander, Steinhauser
Degrassi, Slavich

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$m_{\tilde{t}_2} = 350 \text{ GeV}, m_{\tilde{b}_1} = 350 \text{ GeV}, m_{\tilde{b}_2} = 370 \text{ GeV}, \theta_{\tilde{t}/\tilde{b}} = 40^\circ$

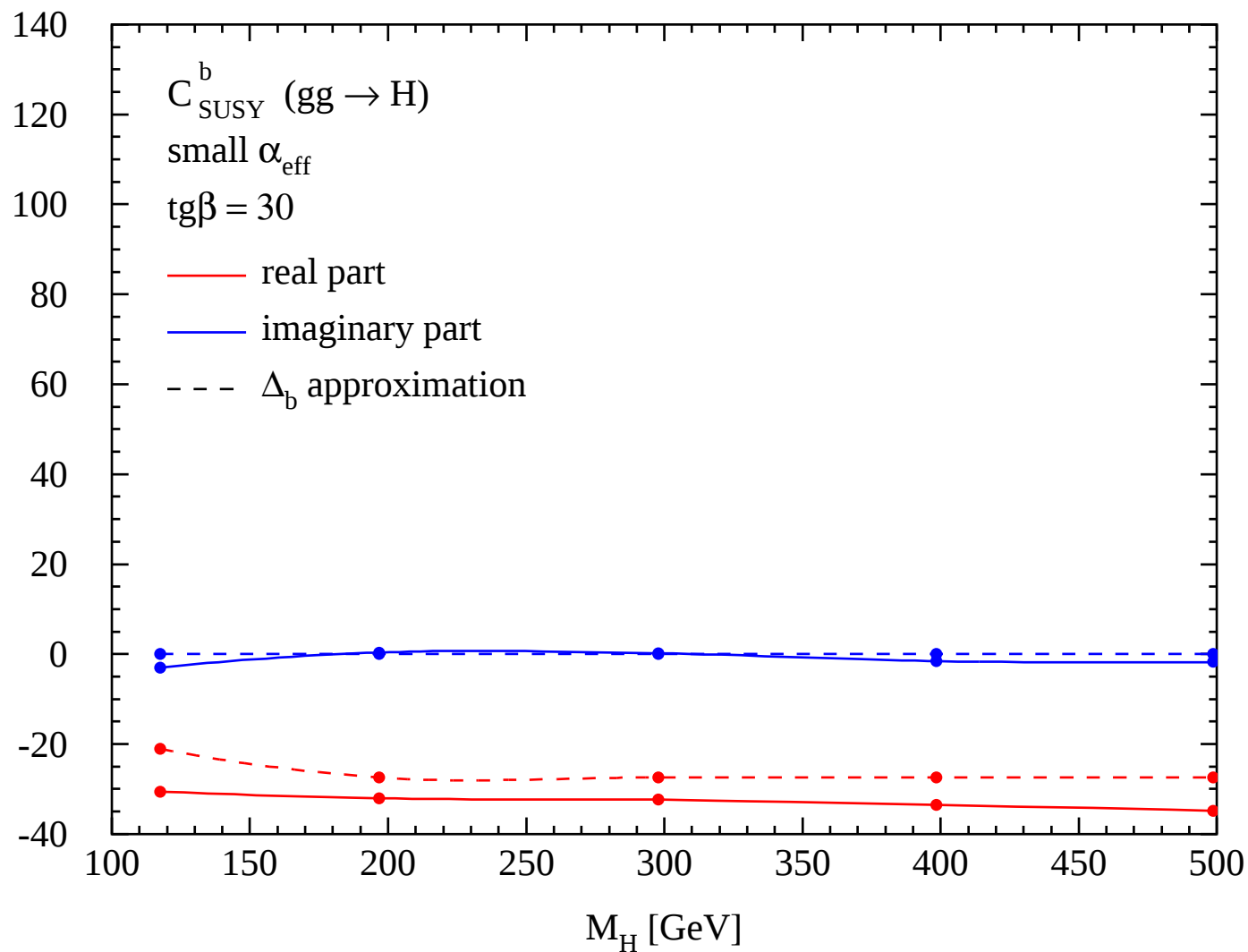


PRELIMINARY

Anastasiou, Beerli, Daleo
Mühlleitner, Rzehak, S.
Harlander, Steinhauser
Degrassi, Slavich

• 2009:

PRELIMINARY



$$\Delta \sim -5 \frac{\alpha_s}{\pi} \sim -15\%$$

Mühlleitner, Rzehak, S.

τ -phobic scenario

[scale = 1 TeV]

$$m_t = 173.2 \text{ GeV}$$

$$\text{tg}\beta = 30$$

$$M_{\tilde{Q}} = 1.5 \text{ TeV}$$

$$M_{\tilde{g}} = 1.5 \text{ TeV}$$

$$M_2 = 200 \text{ GeV}$$

$$A_b = A_t = 4.417 \text{ TeV} \quad [X_t = 2.9 M_{\tilde{Q}}]$$

$$\mu = 2 \text{ TeV}$$

$$M_{\tilde{\ell}_3} = 500 \text{ GeV}$$

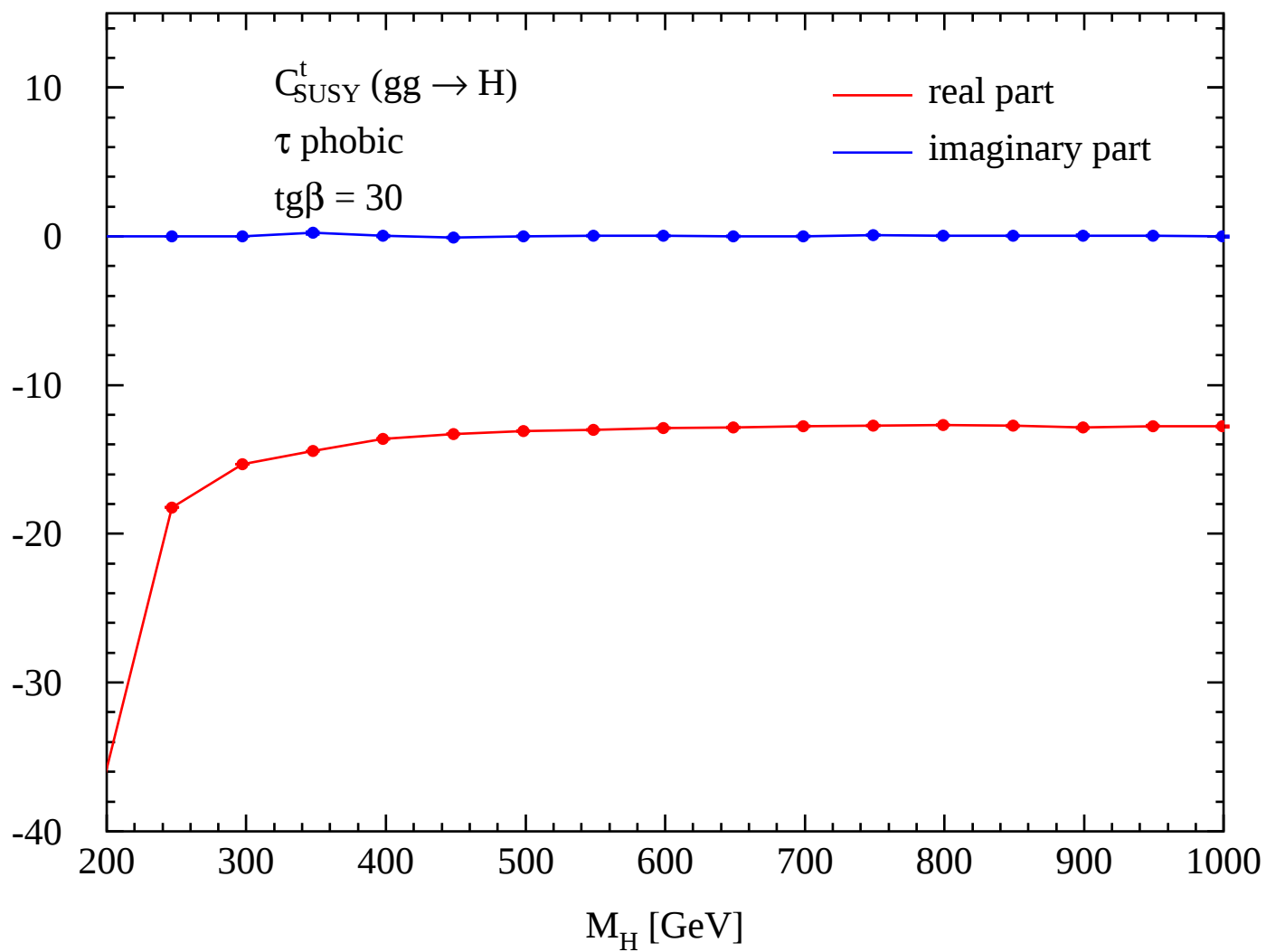
$$m_{\tilde{t}_1} = 1.318 \text{ TeV}$$

$$m_{\tilde{t}_2} = 1.726 \text{ TeV}$$

$$m_{\tilde{b}_1} = 1.501 \text{ TeV}$$

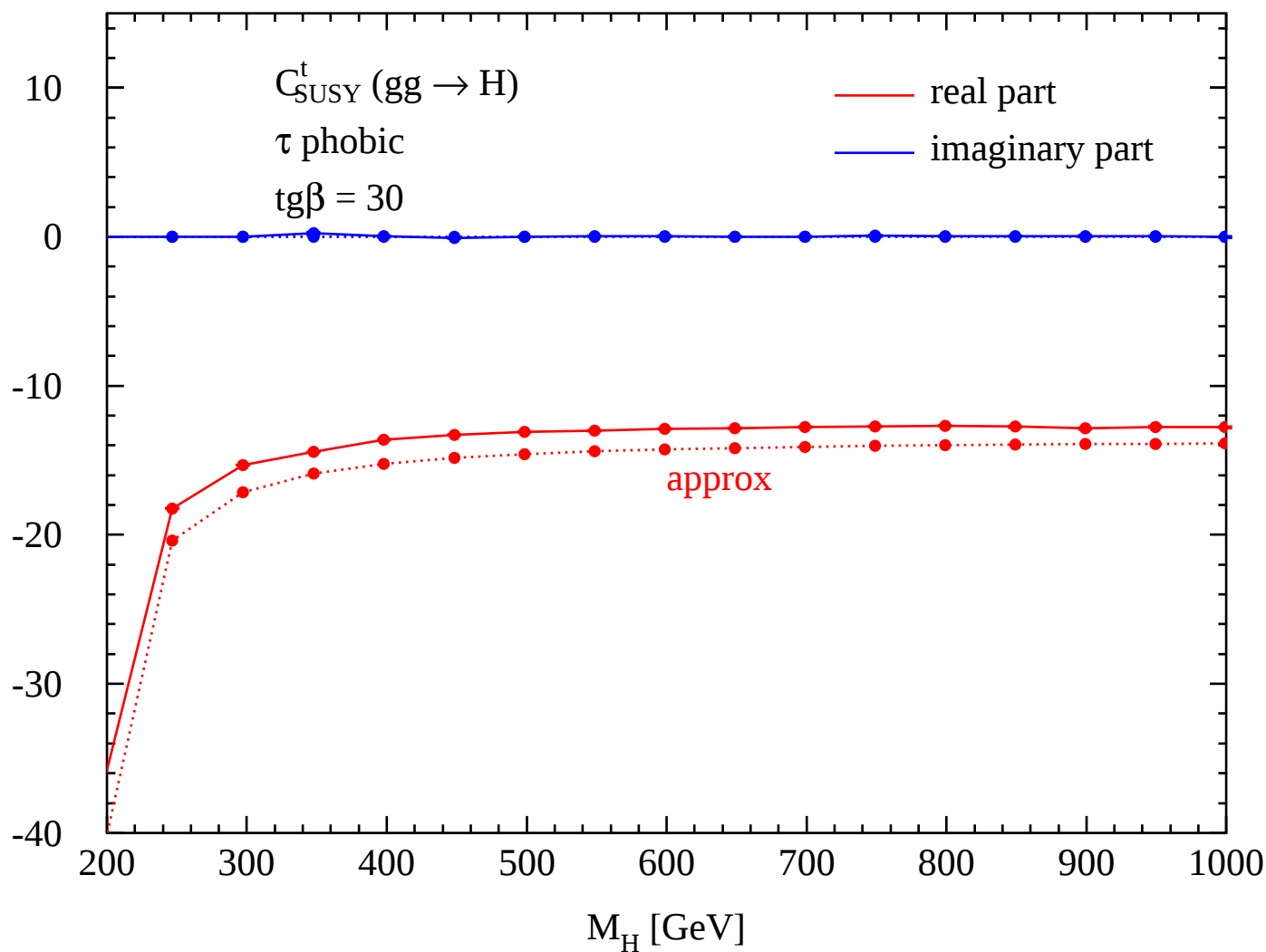
$$m_{\tilde{b}_2} = 1.565 \text{ TeV}$$

PRELIMINARY



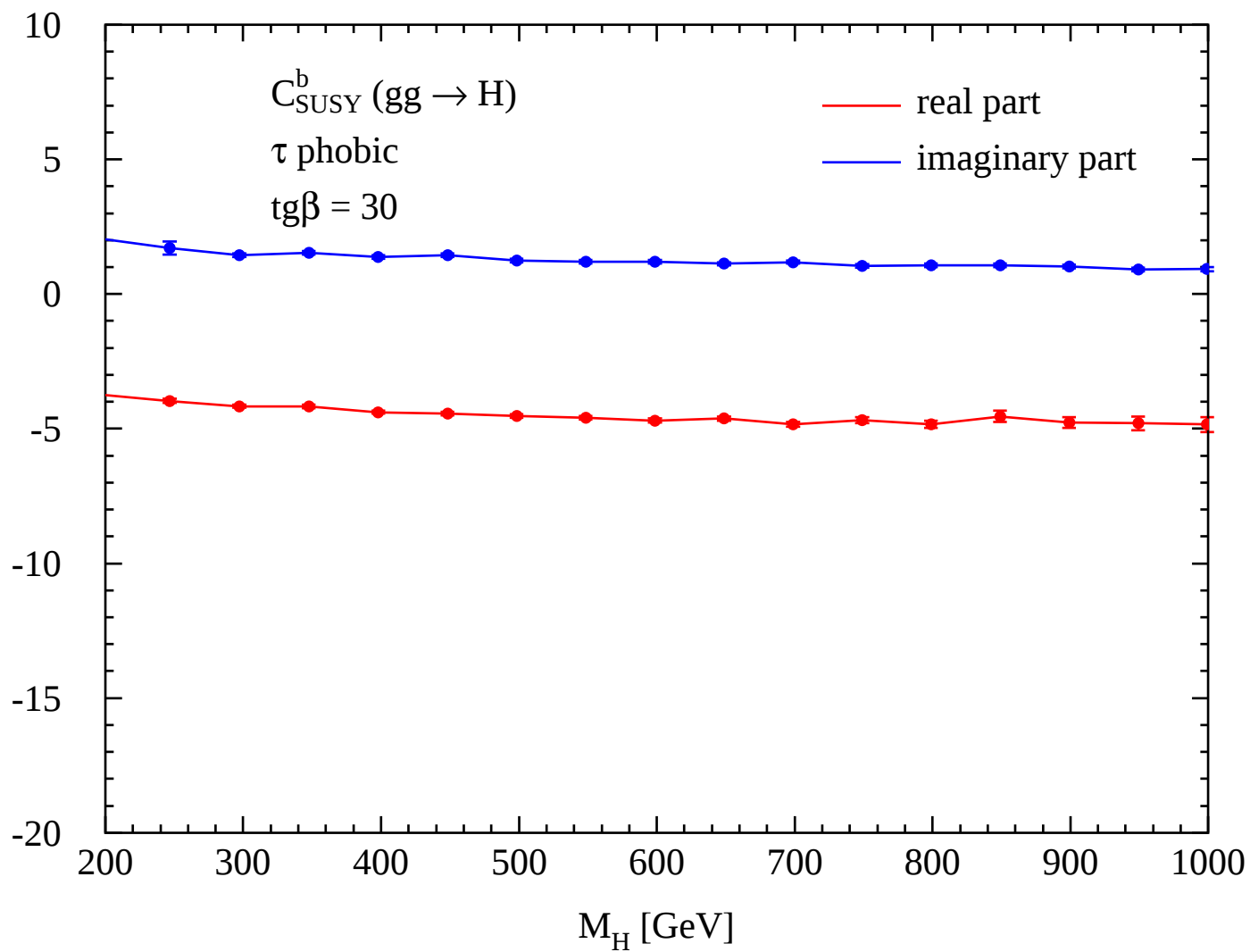
Mühlleitner, Rzehak, S.

PRELIMINARY



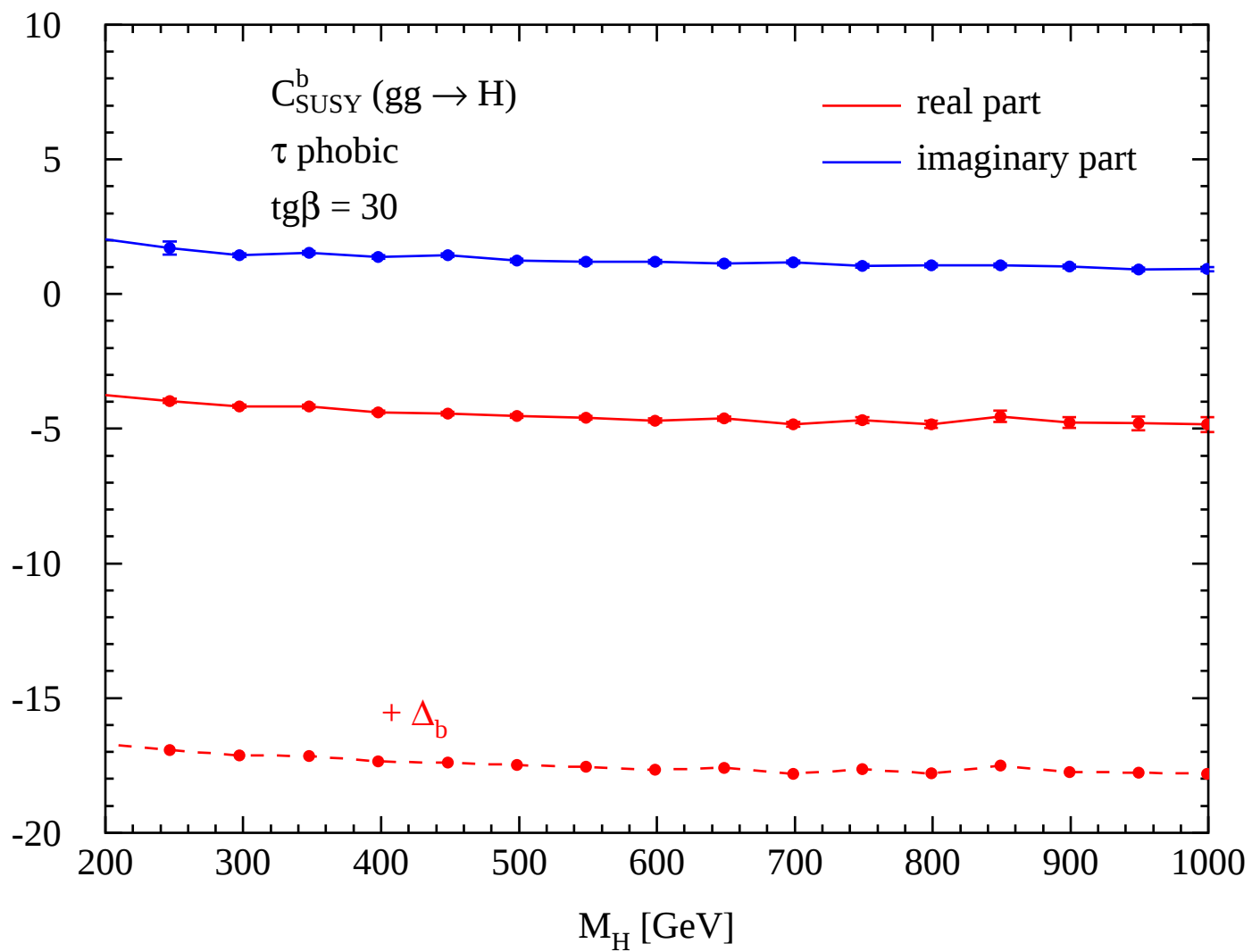
Mühlleitner, Rzehak, S.
Harlander, Steinhauser
Degrassi, Slavich

PRELIMINARY



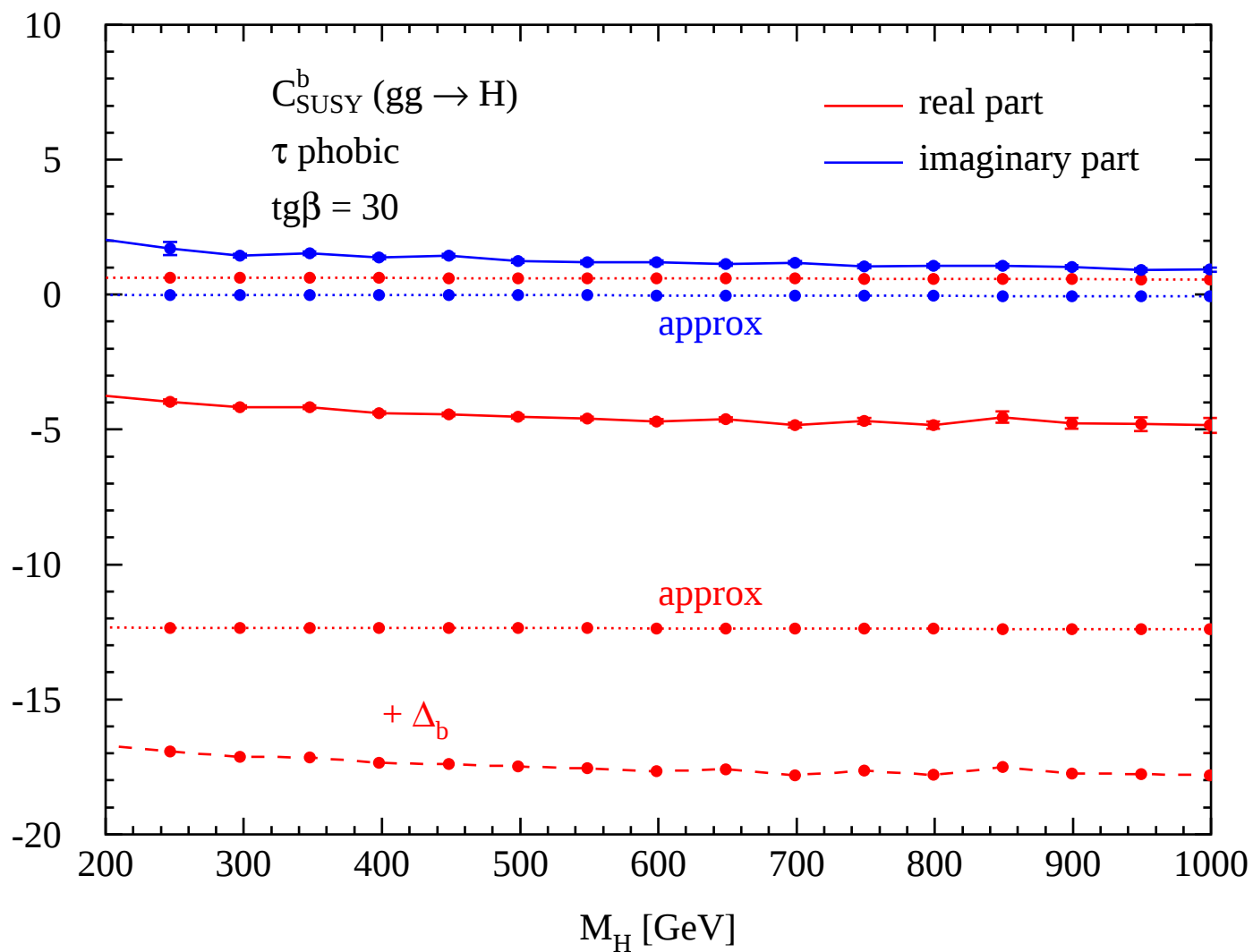
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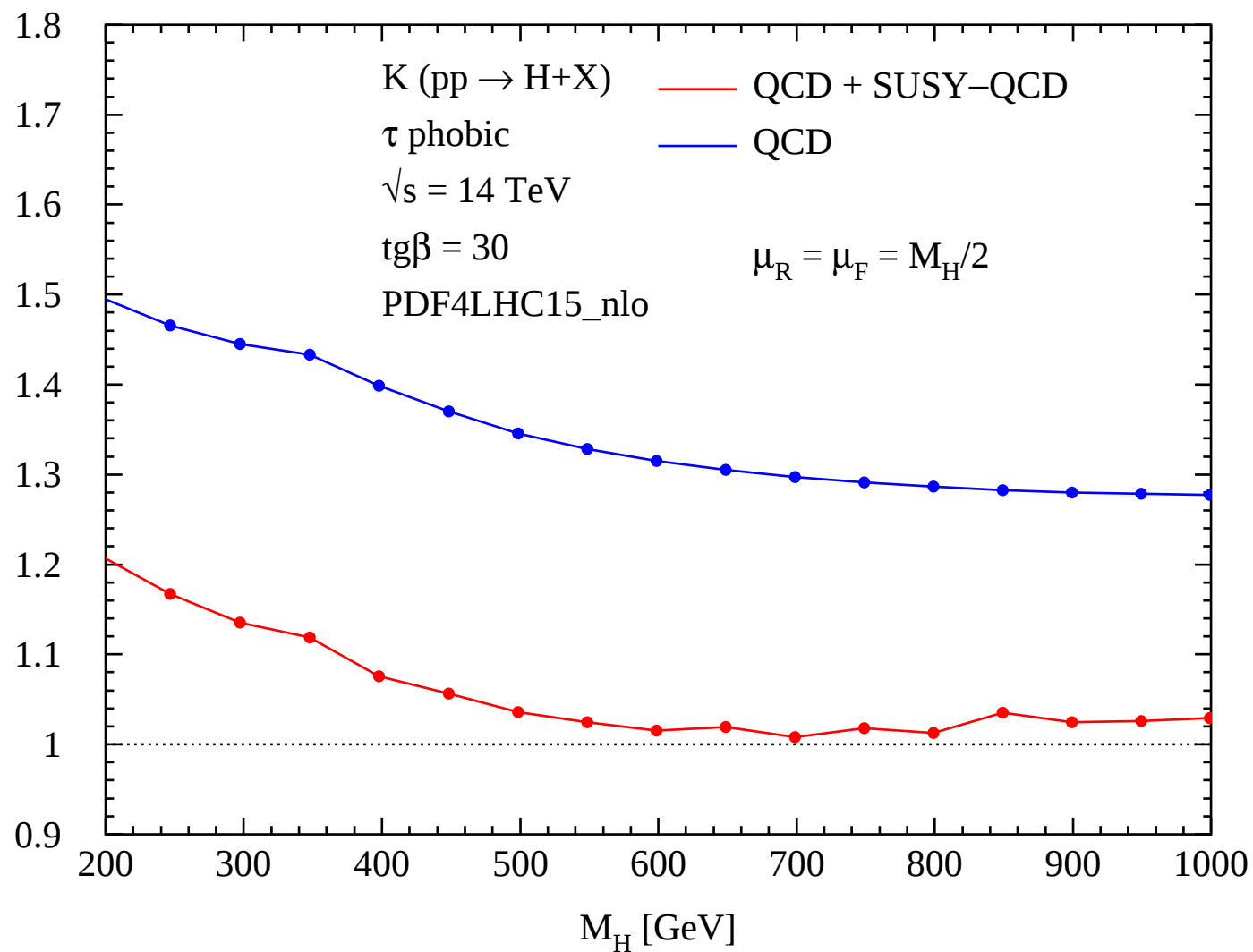
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PRELIMINARY



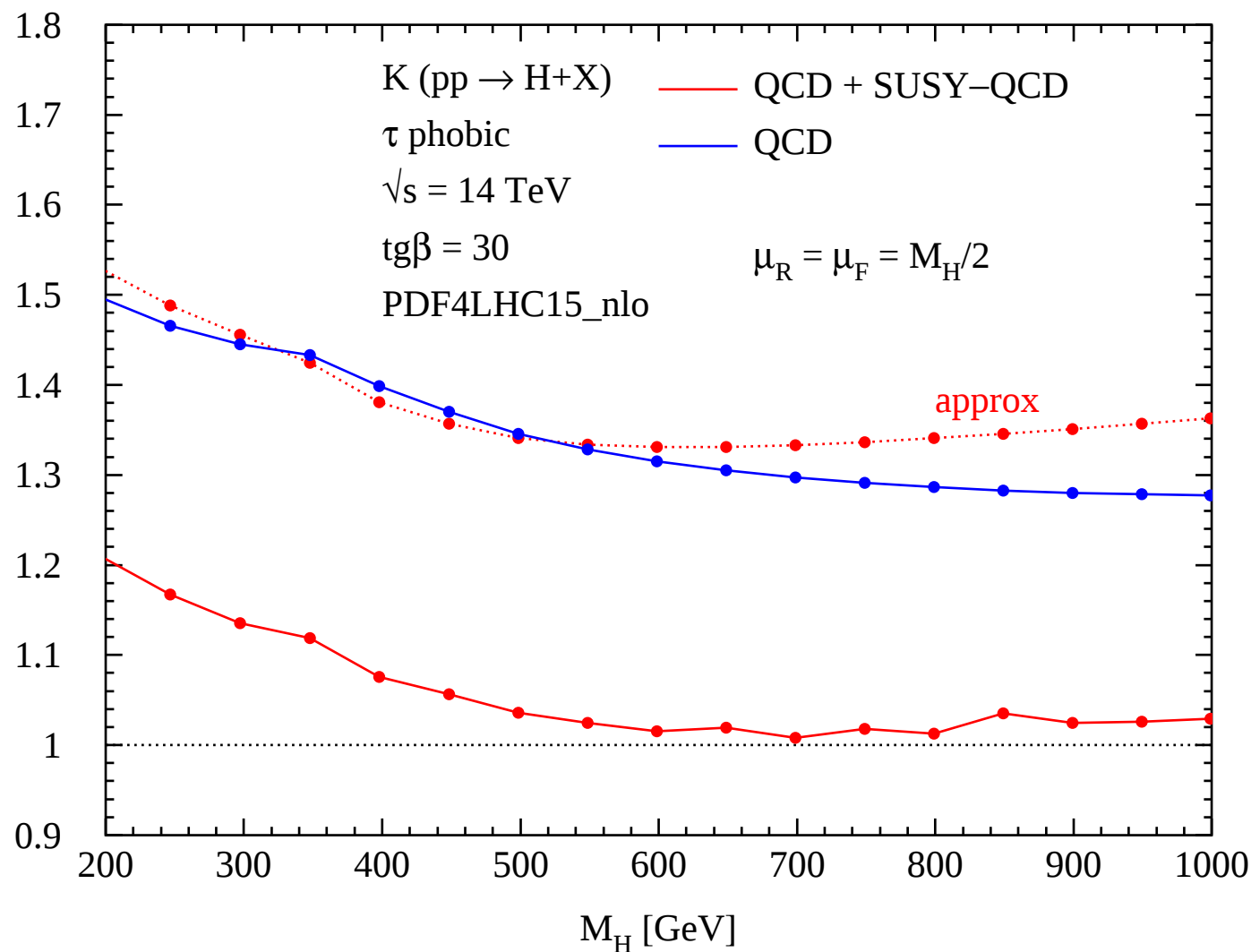
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Degrassi, Slavich

PRELIMINARY



Mühlleitner, Rzehak, S.

PRELIMINARY



Mühlleitner, Rzehak, S.
Harlander, Steinhauser
Degrassi, Slavich

III CONCLUSIONS

- $gg \rightarrow h/H$: genuine SUSY–QCD corrections large
- sizeable corrections beyond Δ_b approximation
⇒ sizeable differences to approximate calculations
- outlook: other scenarios
pseudoscalar A
application to HH @ NLO
[results soon...]

BACKUP SLIDES

small α_{eff} scenario [modified]

$$\tan\beta = 30$$

$$M_{\tilde{Q}} = 800 \text{ GeV}$$

$$M_{\tilde{g}} = 1000 \text{ GeV} \quad \leftarrow$$

$$M_2 = 500 \text{ GeV}$$

$$A_b = A_t = -1.133 \text{ TeV}$$

$$\mu = 2 \text{ TeV}$$

$$m_{\tilde{t}_1} = 679 \text{ GeV} \quad m_{\tilde{t}_2} = 935 \text{ GeV}$$

$$m_{\tilde{b}_1} = 601 \text{ GeV} \quad m_{\tilde{b}_2} = 961 \text{ GeV}$$

$$\mathcal{L}_{eff} = -\lambda_b \bar{b}_R \left[\phi_1^0 + \frac{\Delta_b}{\tan\beta} \phi_2^{0*} \right] b_L + h.c. \quad \Rightarrow \quad \lambda_b = \frac{m_b}{v_1 (1 + \Delta_b)} = \frac{\hat{m}_b}{v_1}$$

• input squark mass matrix: $\hat{m}_b(Q_0) = \frac{\bar{m}_b(Q_0)}{1 + \Delta_b}$ $\hat{m}_t(Q_0) = \bar{m}_t(Q_0)$

$$\mathcal{M}_{\tilde{q}} = \begin{bmatrix} \tilde{M}_{\tilde{q}_L}^2(Q_0) + \hat{m}_q^2(Q_0) + \Delta_{11} & \hat{m}_q(Q_0)[A_q(Q_0) - \mu r_q] + \Delta_{12} \\ \hat{m}_q(Q_0)[A_q(Q_0) - \mu r_q] + \Delta_{21} & \tilde{M}_{\tilde{q}_R}^2(Q_0) + \hat{m}_q^2(Q_0) + \Delta_{22} \end{bmatrix}$$

$$m_{\tilde{q}_{1/2}}^2 = \hat{m}_q^2(Q_0) + \frac{1}{2} \left[\tilde{M}_{\tilde{q}_L}^2(Q_0) + \tilde{M}_{\tilde{q}_R}^2(Q_0) \mp \sqrt{[\tilde{M}_{\tilde{q}_L}^2(Q_0) - \tilde{M}_{\tilde{q}_R}^2(Q_0)]^2 + 4\hat{m}_q^2(Q_0)[\bar{A}_q(Q_0) - \mu r_q]^2} \right] + \Delta m_{\tilde{q}_{1/2}}^2$$

$$\tilde{q}_1 = \tilde{q}_L \cos \theta_q + \tilde{q}_R \sin \theta_q$$

$$\tilde{q}_2 = -\tilde{q}_L \sin \theta_q + \tilde{q}_R \cos \theta_q$$

• tree-level like mixing angle $\tilde{\theta}_q$ ($m_{\tilde{q}_i}$ pole masses):

$$\sin 2\tilde{\theta}_q = \frac{2\hat{m}_q(Q_0)[A_q(Q_0) - \mu r_q]}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} \quad \cos 2\tilde{\theta}_q = \frac{\tilde{M}_{\tilde{q}_L}^2(Q_0) - \tilde{M}_{\tilde{q}_R}^2(Q_0)}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2}$$

- SUSY–QCD corr. Δ_{ij} significant $\rightarrow$ new diagonalization (no iteration)
- θ_q in physical amplitudes: remove artificial sing. for $m_{\tilde{q}_1} \sim m_{\tilde{q}_2}$

$$\rightarrow \quad \delta\theta_q = \frac{1}{2} \frac{\Sigma_{12}(m_{\tilde{q}_1}) + \Sigma_{12}(m_{\tilde{q}_2})}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2}$$

- unique relation to tree-level like mixing angle $\tilde{\theta}_q \Rightarrow$ consistent scheme

$$\theta_q = \tilde{\theta}_q + \Delta\tilde{\theta}_q \quad \Delta\tilde{\theta}_q = \delta\tilde{\theta}_q - \delta\theta_q$$

$$\delta\tilde{\theta}_q = \frac{\text{tg } 2\tilde{\theta}_q}{2} \left\{ \frac{\delta\hat{m}_q}{\hat{m}_q(Q_0)} + \frac{\delta\bar{A}_q}{\bar{A}_q(Q_0) - \mu r_q} - \frac{\delta m_{\tilde{q}_1}^2 - \delta m_{\tilde{q}_2}^2}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} \right\}$$

- shift mass parameter: $M_{\tilde{q}_{L/R}}^2(Q_0) \rightarrow M_{\tilde{q}_{L/R}}^2(Q_0) + \Delta\bar{M}_{\tilde{q}_{L/R}}^2$

$$\tilde{M}_{\tilde{q}_L}^2(Q_0) = M_{\tilde{q}_L}^2(Q_0) + D_{\tilde{q}_L} = m_{\tilde{q}_1}^2 \cos^2 \tilde{\theta}_q + m_{\tilde{q}_2}^2 \sin^2 \tilde{\theta}_q - \hat{m}_q^2(Q_0)$$

$$\tilde{M}_{\tilde{q}_R}^2(Q_0) = M_{\tilde{q}_R}^2(Q_0) + D_{\tilde{q}_R} = m_{\tilde{q}_1}^2 \sin^2 \tilde{\theta}_q + m_{\tilde{q}_2}^2 \cos^2 \tilde{\theta}_q - \hat{m}_q^2(Q_0)$$

$$\begin{aligned}
\frac{\delta \hat{m}_q}{\hat{m}_q(Q)} &= -C_F \frac{\alpha_s}{\pi} \Gamma(1 + \epsilon) (4\pi)^\epsilon \frac{3}{4} \left\{ \frac{1}{\epsilon} + \log \frac{\bar{\mu}^2}{Q^2} + \delta_{SUSY} \right\} + \Delta_q \\
&- C_F \frac{\alpha_s}{4\pi} \left\{ B_1[\hat{m}_q^2(Q); M_{\tilde{g}}, m_{\tilde{q}_1}] + B_1[\hat{m}_q^2(Q); M_{\tilde{g}}, m_{\tilde{q}_2}] \right. \\
&\quad \left. + 2M_{\tilde{g}}(\bar{A}_q - \mu r_q) \frac{B_0[\hat{m}_q^2(Q); M_{\tilde{g}}, m_{\tilde{q}_1}] - B_0[\hat{m}_q^2(Q); M_{\tilde{g}}, m_{\tilde{q}_2}]}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} \right\}
\end{aligned}$$

$$\delta \bar{A}_q = C_F \frac{\alpha_s}{\pi} \Gamma(1 + \epsilon) (4\pi)^\epsilon M_{\tilde{g}} \left\{ \frac{1}{\epsilon} + \log \frac{\bar{\mu}^2}{\mu_R^2} \right\}$$

$$\begin{aligned}
g_{\tilde{f}_1 \tilde{f}_1}^{h,H} &= g_{\tilde{f}_L \tilde{f}_L}^{h,H} \cos^2 \theta_f + g_{\tilde{f}_R \tilde{f}_R}^{h,H} \sin^2 \theta_f + g_{\tilde{f}_L \tilde{f}_R}^{h,H} \sin 2\theta_f \\
g_{\tilde{f}_2 \tilde{f}_2}^{h,H} &= g_{\tilde{f}_L \tilde{f}_L}^{h,H} \sin^2 \theta_f + g_{\tilde{f}_R \tilde{f}_R}^{h,H} \cos^2 \theta_f - g_{\tilde{f}_L \tilde{f}_R}^{h,H} \sin 2\theta_f \\
g_{\tilde{f}_1 \tilde{f}_2}^{h,H} &= g_{\tilde{f}_2 \tilde{f}_1}^{h,H} = \frac{1}{2}(g_{\tilde{f}_R \tilde{f}_R}^{h,H} - g_{\tilde{f}_L \tilde{f}_L}^{h,H}) \sin 2\theta_f + g_{\tilde{f}_L \tilde{f}_R}^{h,H} \cos 2\theta_f \\
g_{\tilde{f}_1 \tilde{f}_1}^A &= g_{\tilde{f}_2 \tilde{f}_2}^A = 0 \\
g_{\tilde{f}_1 \tilde{f}_2}^A &= -g_{\tilde{f}_2 \tilde{f}_1}^A = g_{\tilde{f}_L \tilde{f}_R}^A
\end{aligned}$$

$$\begin{aligned}
g_{\tilde{q}_L \tilde{q}_L}^\Phi(\mu_R) &= \hat{m}_q^2(\mu_R) g_1^\Phi + M_Z^2 (I_{3q} - e_q \sin^2 \theta_W) g_2^\Phi \\
g_{\tilde{q}_R \tilde{q}_R}^\Phi(\mu_R) &= \hat{m}_q^2(\mu_R) g_1^\Phi + M_Z^2 e_q \sin^2 \theta_W g_2^\Phi \\
g_{\tilde{q}_L \tilde{q}_R}^\Phi(\mu_R) &= -\frac{\hat{m}_q(\mu_R)}{2} [\mu g_3^\Phi - \bar{A}_q(\mu_R) g_4^\Phi]
\end{aligned}$$

$$\hat{m}_q(\mu_R) = \hat{m}_q(m_q) \frac{c[\alpha_s(\mu_R)/\pi]}{c[\alpha_s(m_q)/\pi]}$$

$$c(x) = \left(\frac{23}{6}x\right)^{\frac{12}{23}} \left[1 + \frac{3731}{3174}x\right] \quad \text{for } q = b \quad (N_F = 5)$$

$$c(x) = \left(\frac{7}{2}x\right)^{\frac{4}{7}} \left[1 + \frac{137}{98}x\right] \quad \text{for } q = t \quad (N_F = 6)$$

$$\begin{aligned} \bar{A}_q(\mu_R) = \bar{A}_q(Q_0) + M_3(Q_0) & \left\{ -\frac{16}{9} \left[\frac{\alpha_{s,SUSY}(\mu_R)}{\alpha_{s,SUSY}(Q_0)} - 1 \right] \left[1 + \frac{1}{6} \frac{\alpha_{s,SUSY}(Q_0)}{\pi} \right] \right. \\ & \left. - \frac{16}{27} \frac{\alpha_{s,SUSY}(Q_0)}{\pi} \left[\frac{\alpha_{s,SUSY}^2(\mu_R)}{\alpha_{s,SUSY}^2(Q_0)} - 1 \right] \right\} \end{aligned}$$

$$\alpha_{s,SUSY}(\mu) = \frac{12\pi}{9 \log(\mu^2/\Lambda_{SUSY}^2)} \left\{ 1 - \frac{14}{9} \frac{\log \log(\mu^2/\Lambda_{SUSY}^2)}{\log(\mu^2/\Lambda_{SUSY}^2)} \right\}$$

$$\alpha_{s,SUSY}(Q_0) = \alpha_s(Q_0) \left\{ 1 + \frac{\alpha_s(Q_0)}{\pi} \left[\frac{1}{6} \log \frac{Q_0^2}{m_t^2} + \frac{1}{2} \log \frac{Q_0^2}{M_{\tilde{g}}^2} + \frac{1}{24} \sum_{\tilde{q}_i} \log \frac{Q_0^2}{m_{\tilde{q}_i}^2} \right] \right\}$$

$$\begin{aligned} M_3(Q_0) = M_{\tilde{g}} & \left\{ 1 - \frac{\alpha_s(M_{\tilde{g}})}{\pi} \left[C_A + \frac{3}{4} C_A \log \frac{Q_0^2}{M_{\tilde{g}}^2} \right. \right. \\ & + \frac{1}{4} \sum_{q,i} \left(B_1(M_{\tilde{g}}^2; m_q, m_{\tilde{q}_i}) + \frac{\Gamma(1+\epsilon)(4\pi)^\epsilon}{2\epsilon} + \frac{1}{2} \log \frac{\bar{\mu}^2}{Q_0^2} \right. \\ & \left. \left. \left. - (-1)^i \frac{m_q}{M_{\tilde{g}}} \sin 2\theta_q B_0(M_{\tilde{g}}^2; m_q, m_{\tilde{q}_i}) \right) \right] \right\} \end{aligned}$$