Heavy quark spin polarization in QCD medium

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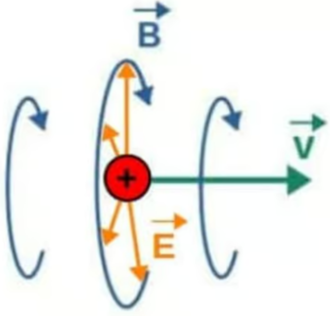
[S. Dey and A. Jaiswal, arXiv:2502.20352]

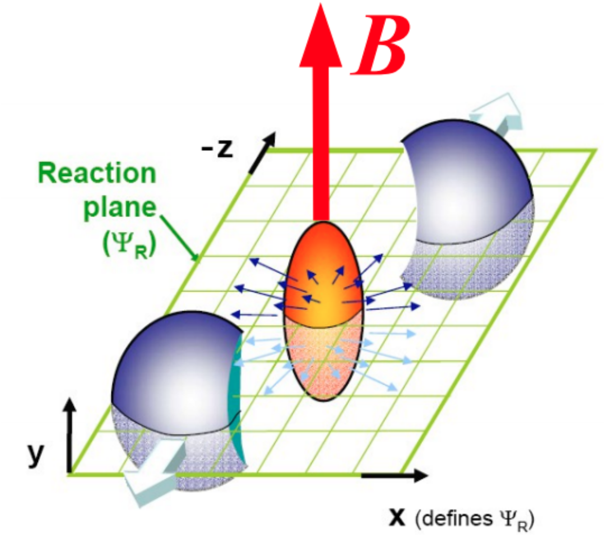
Moving charges create a magnetic field



A moving charged particle generates magnetic field due to its motion.

$$\text{Biot} - \text{Savart law :} \qquad \mathbf{B}(t, \mathbf{r}) = \frac{\mu_0}{4\pi} \frac{q\,\mathbf{v} \times \hat{\mathbf{R}}}{\mathbf{R}^2}, \qquad \mathbf{R} \equiv \mathbf{r} - \mathbf{r}_0$$

Generation of magnetic field in heavy ion collisions



[Adapted from D. Kharzeev @ CPOD 2013.]

Moving nuclei with relativistic velocities

Using Lienard-Wiechert potentials [K. Tuchin, AHEP (2013) 490495]:

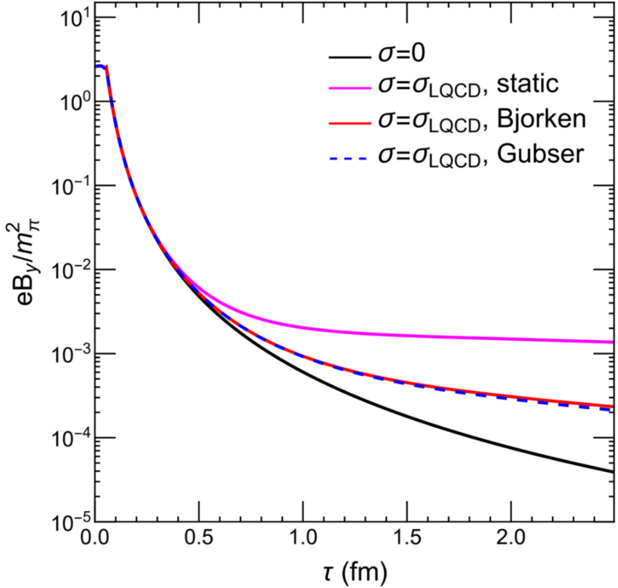
$$e\mathbf{E}(t,\mathbf{r}) = \alpha_{\mathrm{em}} \sum_a \frac{\left(1 - v_a^2\right)\mathbf{R}_a}{R_a^3\left[1 - \left(\mathbf{R}_a \times \mathbf{v}_a\right)^2 / R_a^2\right]^{3/2}},$$

$$e\mathbf{B}(t,\mathbf{r}) = \alpha_{\mathrm{em}} \sum_a \frac{\left(1 - v_a^2\right)\left(\mathbf{v}_a \times \mathbf{R}_a\right)}{R_a^3\left[1 - \left(\mathbf{R}_a \times \mathbf{v}_a\right)^2 / R_a^2\right]^{3/2}},$$

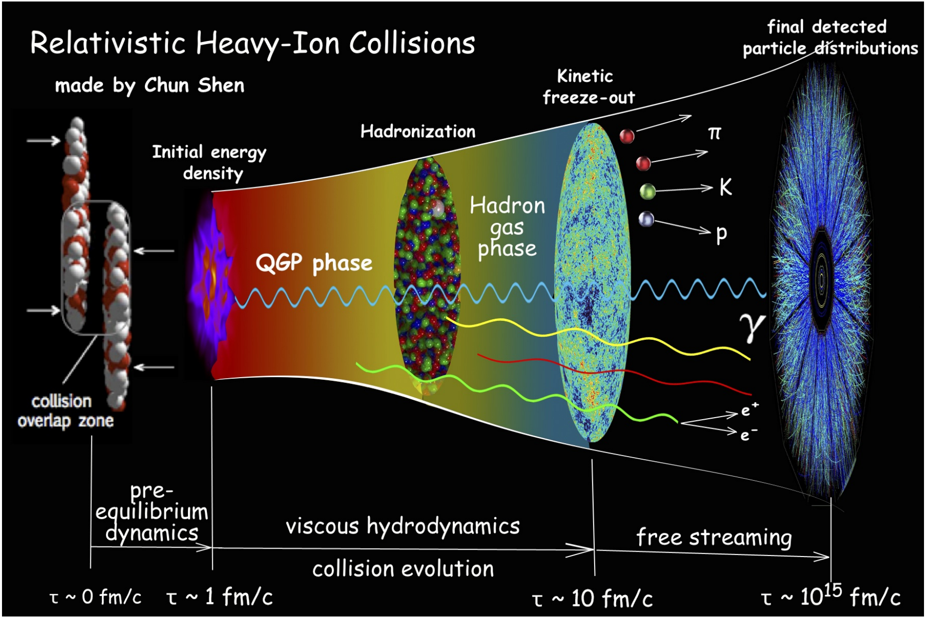
where $\mathbf{R}_a \equiv \mathbf{r} - \mathbf{r}_a$.

- Magnetic field due to motion of charges relative to an observer.
- Interplay between $\mathbf{E}$ and $\mathbf{B}$ in different reference frames.
- $\mathbf{B} \sim 10^{14}$ Tesla produced in relativistic heavy-ion collisions.

Magnetic field time evolution



[A. Huang, D. She, S. Shi, M. Huang and J. Liao, Phys. Rev. C 107, 034901 (2023).]



Relativistic Heavy-Ion Collisions
made by Chun Shen
final detected particle distributions
Kinetic freeze-out
Hadronization
Initial energy density
Hadron gas phase
QGP phase
π
K
p
γ
e+
e-
collision overlap zone
pre-equilibrium dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c

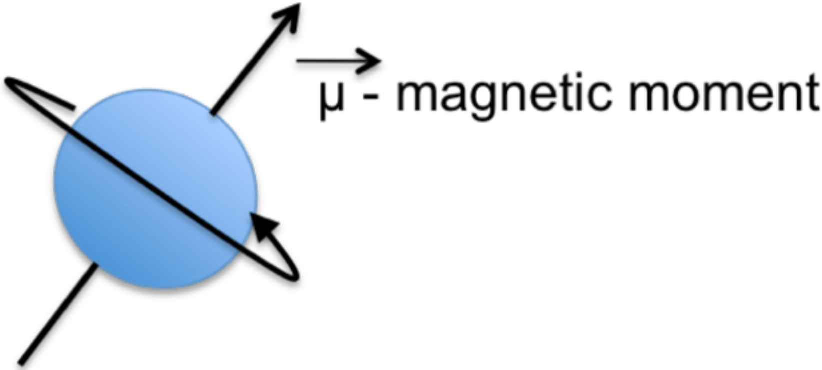
Heavy quarks in relativistic heavy-ion collisions

- Heavy quarks (charm and bottom) has long been recognized as an excellent probe of transport properties of QCD medium. [D. Banerjee, S. Datta, R. Gavai, and P. Majumdar, PRD 85 (2012) 014510; S. K. Das, S. Plumari, S. Chatterjee, et. al. PLB 768 (2017) 260-264; $\cdots$]
- Heavy quarks are primarily generated in the initial hard scatterings of partons.
- Clean probe of the early-stage properties of heavy-ion collisions.
- Strong transient magnetic fields produced which are significant only during the early stages of the collision.
- Heavy quarks: ideal for observable signals of initial magnetic field.
- Our proposal:
  - Strong magnetic fields induce spin polarization of heavy quarks.
  - These induced spin polarization of heavy quarks can be observed in the polarization of open heavy-flavor hadrons.
  - Transverse momentum dependence of open heavy-flavor hadron polarization: distinctive signal for initial strong magnetic field.

Spin polarization of hadrons in heavy-ion collisions

- Spin polarization is a relatively new topic in heavy ion collisions.
- Attributed to angular momentum deposited in off-central coll.
- Provides unique opportunity to probe QGP properties.
- Several measurements of spin polarization of hadrons.
- In baryon sector:
  - Λ (spin 1/2): STAR, Nature, 548, 62–65 (2017); HADES; ALICE.
  - Ω (spin 3/2): STAR, Phys. Rev. Lett. 126, 162301 (2021).
  - Ξ (spin 1/2): STAR, Phys. Rev. Lett. 126, 162301 (2021).
- In meson sector:
  - K^{*0} : ALICE, PRL 125, 012301 (2020); STAR, Nature, 614, 244-248 (2023).
  - ϕ : ALICE, PRL 125, 012301 (2020); STAR, Nature, 614, 244-248 (2023).
  - Heavy quarkonium, J/ψ and Υ(1S) : ALICE, PLB 815, 136146 (2021).
- Polarization measurements for open heavy flavor underway.

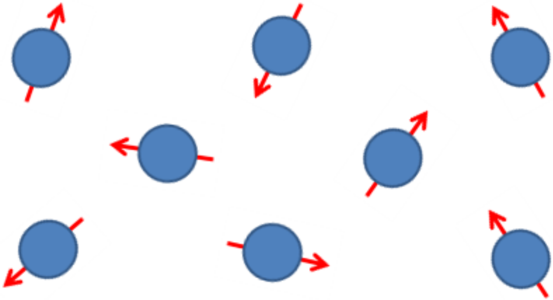
Heavy quarks: charged, spin-1/2 particles



Interaction with magnetic field: $\mathcal{H} = -\vec{\mu} \cdot \vec{B}$

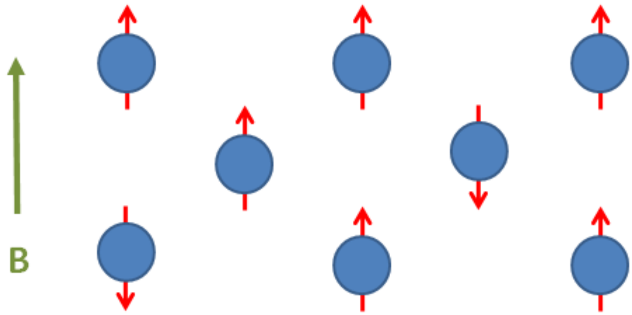
Magnetic moment and spin: $\vec{\mu} = \gamma \vec{s}$

No external magnetic field



Un-aligned spins of heavy quarks.

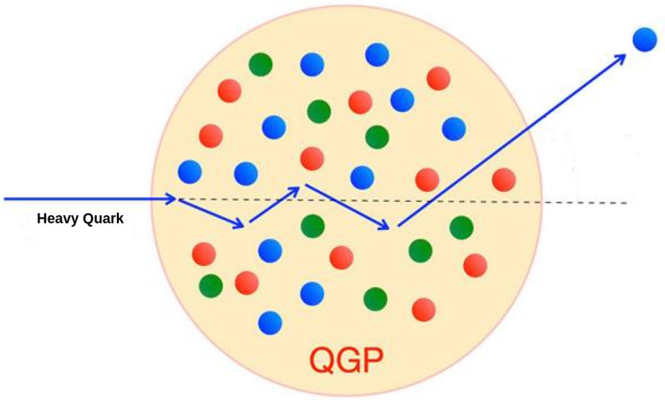
Heavy quarks in magnetic field



Aligned spins in presence of magnetic field.

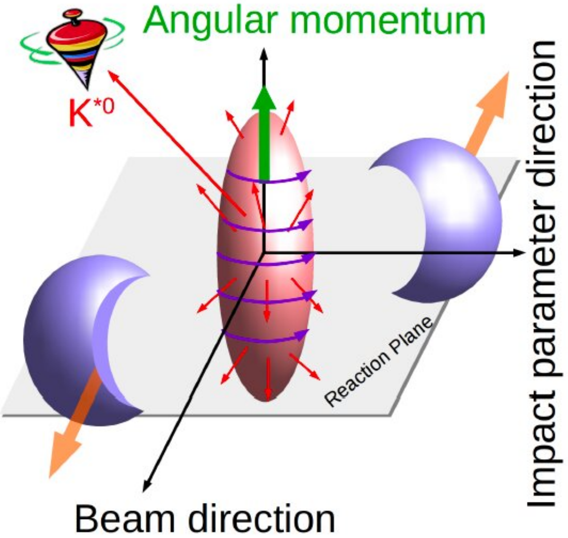
Spin-polarization of heavy quarks.

Heavy quarks in QGP



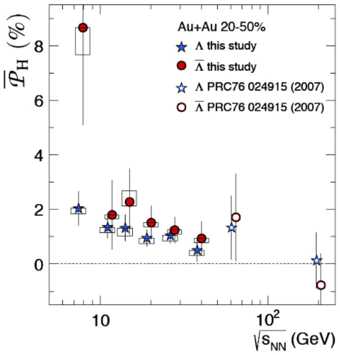
Polarized heavy quarks propagates through QGP.

Polarized QGP due to angular momentum deposition



[B. Mohanty, ICTS News 6, 18-20 (2020).]

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017



First evidence of a quantum effect in (relativistic) hydrodynamics

Adapted from F. Becattini
'Subatomic Vortices'

Heavy Quark Rotational Brownian Motion.

Rotational Brownian motion

- Random rotational motion (orientation and angular velocity) of a microscopic particle due to thermal fluctuations caused by collisions with surrounding medium particles.
- Rotational Brownian motion problem: first considered by Debye.
- For classical spins, the Langevin equation corresponds to the stochastic Landau–Lifshitz-Gilbert equation

$$\frac{d\mathbf{s}}{dt} = \mathbf{s} \times \left[\tilde{\mathbf{B}} + \boldsymbol{\xi}(t)\right] - \lambda\, \mathbf{s} \times \left(\mathbf{s} \times \tilde{\mathbf{B}}\right)$$

- Here $\tilde{\mathbf{B}} \equiv \gamma \mathbf{B} = -\frac{\partial \mathcal{H}}{\partial \mathbf{s}}$ and γ is the gyromagnetic ratio $\boldsymbol{\mu} = \gamma\, \mathbf{s}$.
- $\mathbf{s} \times \tilde{\mathbf{B}}$ represents precession dynamics of the system.
- $\boldsymbol{\xi}(t)$ is the random torque on the particle by the medium.
- λ is the damping coefficient.

Langevin and Fokker-Planck equations

- Generalized multivariate Langevin equation

$$\frac{dy_i}{dt} = A_i(y,t) + C_{ik}(y,t)\,\xi_k(t).$$

- Reduces to Landau–Lifshitz-Gilbert equation with $s_i = y_i$ and

$$A_i = \epsilon_{ijk} s_j \tilde{B}_k + \lambda\,(s^2\delta_{ik} - s_i s_k)\tilde{B}_k, \qquad C_{ik} = \epsilon_{ijk} s_j.$$

- Assume statistical properties of white noise for the random torque

$$\langle \xi_k(t) \rangle = 0, \qquad \langle \xi_k(t_1)\,\xi_l(t_2) \rangle = 2\,D\,\delta_{kl}\,\delta(t_1 - t_2).$$

- Using the Kramers–Moyal expansion, one arrives at the Fokker–Planck equation

$$\frac{\partial \mathcal{P}}{\partial t} = -\frac{\partial}{\partial y_i}\left[A_i(y,t) + D C_{jk}(y,t)\frac{\partial C_{ik}(y,t)}{\partial y_j}\right]\mathcal{P} + D\frac{\partial^2}{\partial y_i \partial y_j}[C_{ik}(y,t)C_{jk}(y,t)\mathcal{P}]$$

Fokker-Planck equation for spin

- Fokker–Planck equation corresponding to the stochastic Landau–Lifshitz-Gilbert equation

$$\frac{\partial \mathcal{P}}{\partial t} = -\frac{\partial}{\partial s_i}\Big[\epsilon_{ijk}\, s_j \tilde{B}_k + \lambda(s^2\delta_{ik} - s_i\, s_k)\tilde{B}_k - 2Ds_i\Big]\mathcal{P} + D\,\frac{\partial^2}{\partial s_i\,\partial s_j}\big[s^2\delta_{ij} - s_i s_j\big]\mathcal{P}$$

- Assuming that the field $\tilde{\mathbf{B}}$ is independent of particle spin $\mathbf{s}$

$$\frac{\partial \mathcal{P}}{\partial t} = \lambda\,\frac{\partial}{\partial \mathbf{s}} \cdot \left[\mathbf{s} \times \left(\mathbf{s} \times \left(\tilde{\mathbf{B}} - T\frac{\partial}{\partial \mathbf{s}}\right)\right)\right] \mathcal{P}$$

- To find: Probability of a spin-polarized particle having an instantaneous orientation in the direction (θ, ϕ).
- Consider a sphere in spin-space of fixed radius s, i.e., $\mathbf{s} = (s, \theta, \phi)$: each point on the sphere represents a different spin orientation.
- Choose z-axis to be along $\tilde{\mathbf{B}}$.

Fokker-Planck equation for spin contd...

- Since we have an axially symmetric Hamiltonian

$$2\tau_s \frac{\partial \mathcal{P}}{\partial t} = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\left(\beta\frac{\partial \mathcal{H}}{\partial\theta}\mathcal{P} + \frac{\partial \mathcal{P}}{\partial\theta}\right)\right], \qquad \tau_s \equiv \frac{1}{2D} = \frac{1}{2\lambda T}$$

- Here, $\beta \equiv 1/T$ and $\mathcal{H} = -\boldsymbol{\mu}\cdot\mathbf{B}$.
- For spatially homogeneous and time varying magnetic field $\mathbf{B}$,

$$2\tau_s \frac{\partial \mathcal{P}}{\partial t} = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\left(\frac{\partial}{\partial\theta} + \beta\mu B(t)\sin\theta\right)\right]\mathcal{P}\,.$$

- This equation can be written in shorthand notation as

$$2\tau_s\,\partial_t\mathcal{P}(\theta,t) = \mathcal{L}_\theta(t)\,\mathcal{P}(\theta,t) \quad \Longrightarrow \quad 2\tau_s\,\partial_t\,|\mathcal{P},t\rangle = \hat{\mathcal{L}}(t)\,|P,t\rangle$$

- The generic solution has the structure

$$|\mathcal{P},t\rangle = \exp\left[\frac{1}{2\tau_s}\int_0^t dt'\,\hat{\mathcal{L}}(t')\right]|\mathcal{P},0\rangle$$

Heavy quark polarization

- Considering the time dependence of the magnetic field to be of the form $B(t) = B_0\,\phi(t)$

$$\hat{\mathcal{L}}(t) = \hat{\mathcal{L}}^0 + \alpha\,\hat{\mathcal{L}}'(t), \quad \hat{\mathcal{L}}^0 = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right), \quad \mathcal{L}'(t) = \frac{\phi(t)}{\sin\theta}\frac{\partial}{\partial\theta}\,\sin^2\theta\,.$$

- Here $\alpha \equiv \mu B_0/T$. Solve perturbatively assuming $\alpha \ll 1$.

$$\alpha \equiv \frac{\mu\,B_0}{T} = \frac{\gamma\,s\,B_0}{T} = \frac{\mathrm{g}\,q\,s\,B_0}{2\,m_Q\,T} = \frac{f\,(e\,B_0)}{2\,m_Q T}$$

- $\mathrm{g} \approx 2$ is the g-factor, the charge $q = f\,e$, where f is $2/3$ and $-1/3$ for charm and bottom quarks, respectively.

- For $eB_0 = 10m_\pi^2$ and the spin $s = \hbar/2$, we obtain $\alpha = 0.171$ for charm quarks and $\alpha = -0.021$ for bottom quarks.

Heavy quark polarization contd...

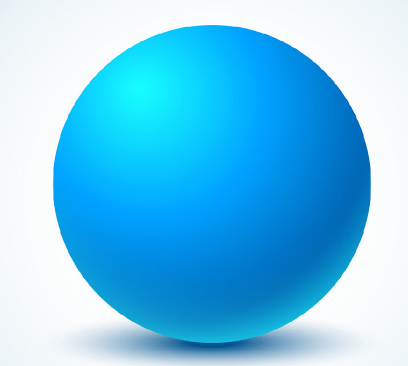
- Assume all heavy quarks are initially spin polarized along $\theta = \theta_0$ direction, i.e., for the initial condition $\mathcal{P}(\theta, 0) = \delta(\cos\theta - \cos\theta_0)$.
- Considering $\phi(t) = e^{-t/\tau_B}$, vector polarization (baryons) is

$$\langle\cos\theta\rangle = \cos\theta_0\, e^{-t/\tau_s} + \frac{\alpha\tau_B}{3} e^{-t/\tau_s}\left[\frac{1-\exp\left(-\frac{(\tau_s-\tau_B)t}{\tau_s\tau_B}\right)}{\tau_s - \tau_B} - \frac{1-\exp\left(-\frac{(2\tau_B+\tau_s)t}{\tau_s\tau_B}\right)}{2\tau_B + \tau_s}\right]$$

- Tensor polarization (vector mesons) is

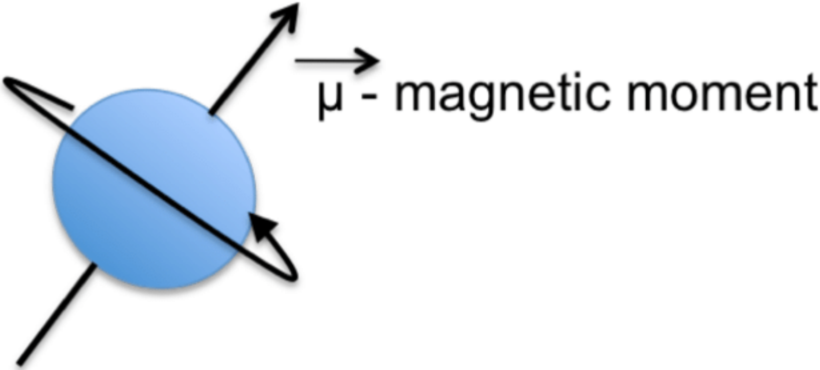
$$\langle\cos^2\theta\rangle = \frac{1}{3} + \frac{2}{3}\, e^{-3t/\tau_s} + \alpha\tau_B\left[\frac{4\, e^{-4t/\tau_s}}{15(\tau_s - \tau_B)}\left(1 - \exp\left[-\frac{(\tau_s - \tau_B)}{\tau_s\tau_B}t\right]\right)\right.$$
$$- \frac{4\, P_3(\cos\theta_0) e^{-9t/\tau_s}}{3(\tau_s - 3\tau_B)}\left(1 - \exp\left[-\frac{(\tau_s - 3\tau_B)}{\tau_s\tau_B}t\right]\right)$$
$$\left. + \frac{2}{3\tau_s}\cos\theta_0 e^{-t/\tau_s}\left(1 - e^{-t/\tau_B}\right)\right].$$

Decay of scalar particles



No anisotropy in the rest frame: isotropic decay products.

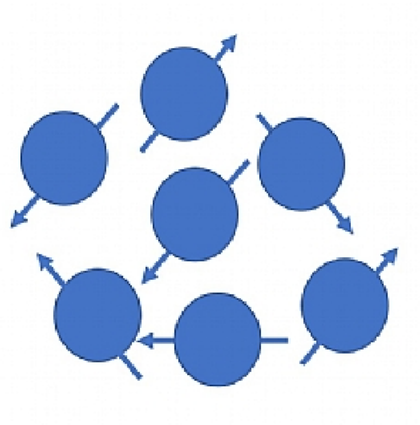
Decay of particles with spin



Preferred direction due to spin: anisotropic decay products

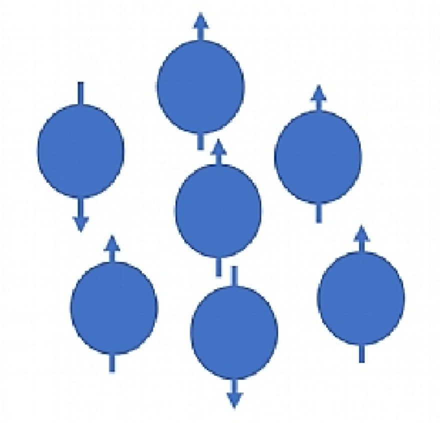
Basis for polarization observables.

Several random decays



Averaging over random decays should lead to isotropic decay products.

Decay of spin polarized particles



Averaging over decay of spin-polarized particles should lead to anisotropic decay products.

Heavy baryon and meson polarization

- For baryons, the angular distribution of one of the decay daughter

$$\frac{dN}{d\cos\theta} = \frac{1}{2}\Big(1 + \alpha_B |\vec{P}_B| \cos\theta\Big)$$

- α_B is decay parameter. Using this distribution, one gets

$$\langle\cos\theta\rangle = \int \cos\theta \frac{dN}{d\cos\theta} d\cos\theta \implies |\vec{P}_B| = \frac{3}{\alpha_B}\langle\cos\theta\rangle$$

- Similarly, for mesons, the angular distribution is

$$\frac{dN}{d\cos\theta} = \frac{3}{4}\Big[1 - \rho_{00} + \big(3\rho_{00} - 1\big)\cos^2\theta\Big]$$

- ρ_{00} is element of spin density matrix; unpolarized $\implies$ $\rho_{00} = 1/3$.
- Using this distribution, one gets

$$\langle\cos^2\theta\rangle = \int \cos^2\theta \frac{dN}{d\cos\theta} d\cos\theta \implies \Delta\rho_{00} = \frac{5}{2}\Big[\langle\cos^2\theta\rangle - \frac{1}{3}\Big]$$

- Here $\Delta\rho_{00} \equiv \rho_{00} - 1/3$

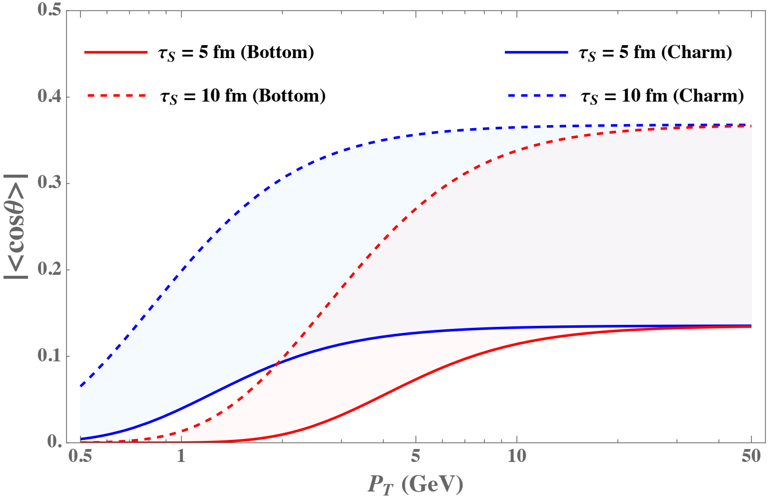
Predictions

- Since α is small, we find dominant contributions to be

$$\langle \cos\theta \rangle = \cos\theta_0 \, e^{-t/\tau_s}, \qquad \langle \cos^2\theta \rangle = \frac{1}{3} + \frac{2}{3} \, e^{-3t/\tau_s}.$$

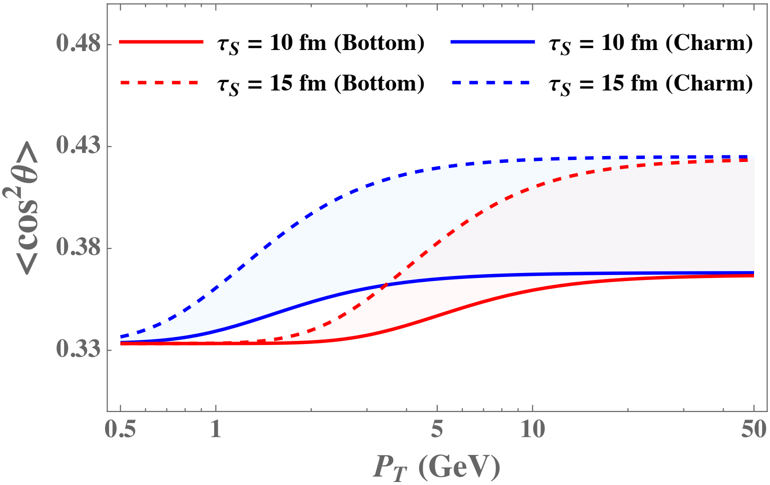
- Here τ_s is spin relaxation time. Supposedly large but not well constrained. Could be treated as a fit parameter.
- Initially $\theta_0 = 0, \pi$ depending on charge of the quark.
- Open heavy baryon and anti-baryon have opposite polarization.
- Open heavy vector mesons and its anti-particle, same sign.
- Quarkonium pol. small: heavy quark and anti-quark opp. pol.
- Assume static uniform fireball of constant temperature, $t = R/v_T$.
- In the mid-rapidity region, $v_T = p_T/\sqrt{m_Q^2 + p_T^2}$.

Open heavy baryon polarization



Heavy quark vector polarization along initial magnetic field direction.

Open heavy vector meson polarization



Heavy quark tensor polarization along initial magnetic field direction.

Summary

- Very strong magnetic field produced in early stages of relativistic heavy-ion collisions.
- Heavy quarks produced at early times in hard binary scatterings.
- Heavy quarks are charged fermions $\implies$ magnetic moment.
- Magnetic field aligns the magnetic moments $\rightarrow$ spin polarization.
- Interaction with QCD medium $\rightarrow$ de-polarization.
- De-polarization larger for larger time spent in the medium.
- Fast heavy quarks escape the medium quicker $\rightarrow$ larger spin polarization; slow stays longer $\rightarrow$ less spin polarization.
- Heavy quark polarization increasing with p_T is a signature of initial strong magnetic field.
- Further predictions for difference between polarization of open heavy vector mesons and baryons. Quarkonium polarization small.

Ongoing and future works in this direction

- Fireball assumed to be static with constant average temperature.
- More realistic space-time evolution of the fireball and external magnetic field necessary.
- Predictions at forward rapidities.
- Calculation of spin relaxation time τ_s for heavy quarks.
- Derivation of an Einstein-Stokes-like relation between the spin diffusion coefficient and the dissipative parameters in spin hydrodynamics.
- Derivation of rotational Fokker-Planck equation from Kinetic theory with non-local collision terms.

Thank you!