Heavy Flavors in Finite Temperature

Probes of Quark-gluon plasma

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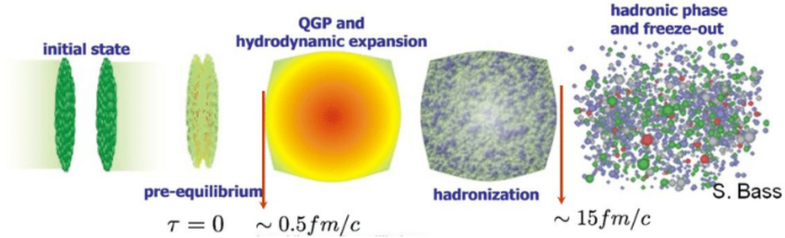
Heavy quark systems as probe of quark-gluon plasma

- In high energy heavy ion collisions in RHIC, LHC, ... deconfined quark gluon plasma is formed.
- The medium lasts for $\mathcal{O}(10\text{ fm})$, is strongly interacting. Need good probes to study its property.



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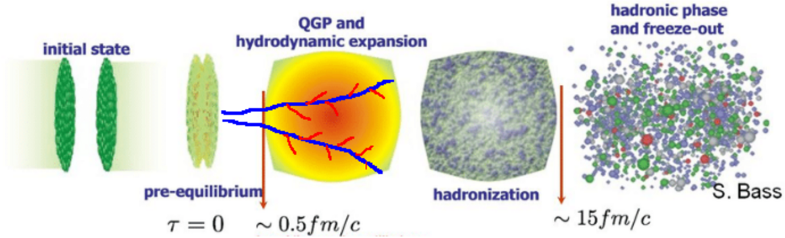

- Heavy quarks generated early in the collision, $\tau \sim \frac{1}{2M_Q}$. Interaction reveals properties of the medium.

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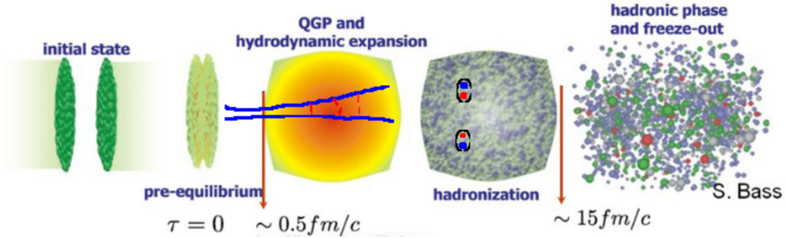
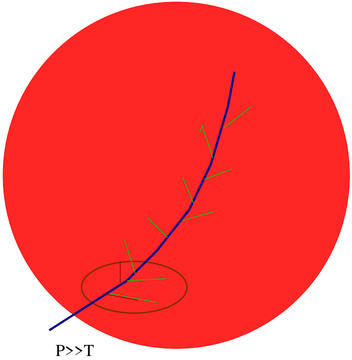


- Heavy quarks generated early in the collision, $\tau \sim \frac{1}{2M_Q}$. Interaction reveals properties of the medium.
- Suppression of quarkonia yield. Suggested as an indicator of deconfinement.

Matsui and Satz (1987)

Heavy flavor in QGP

- Energy loss of heavy quarks: simpler than the corresponding light quark jets.
- Large p_t: PQCD, simpler.
- Low p_t also tractable.



Δt s.t.

$1 << N_{sc} << \frac{M}{T}$

Heavy-light mesons in deconfined plasma

- For moderate momenta heavy quarks in a plasma, with $M \gg T$, a Langevin framework can be used.

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D(p) p_i, \qquad \langle \xi_i(t)\, \xi_j(t') \rangle = \kappa_{ij}\, \delta(t - t')$$

Svetitsky '88; Mustafa et al., '97; Moore & Teaney '05; Rapp & van Hees '05

- Leads to the Fokker-Planck equation

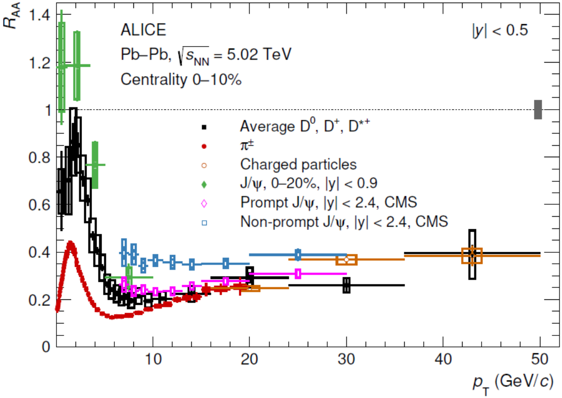
$$\frac{\partial f_Q(p,t)}{\partial t} = -\frac{\partial}{\partial p_i} [p_i\, \eta_D(p)\, f_Q(p,t)] + \frac{\partial^2}{\partial p_i\, \partial p_j} \left[\kappa_{ij}(p)\, f_Q(p,t)\right]$$

- For low momenta, just one coefficient κ.
  Standard nonrelativistic relations:

$$\eta_D = \frac{\kappa}{2\, M\, T}, \qquad \langle x^2(t) \rangle = 6 D_s t, \qquad D_s = \frac{2T^2}{\kappa}$$

- Thermalization of heavy quark requires relaxation time $\sim 1/\eta_D$ small.

R_{AA} and its flavor dependence

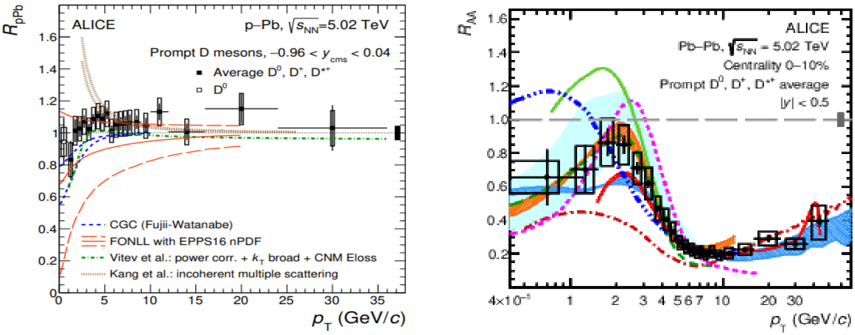


ALICE, JHEP 01 (2022) 174

Similar results also from CMS.

CMS, Phys. Lett. B 782 (2018) 474; B 850 (2024) 138389

R_{AA} for prompt D in pPb and Pb-Pb



ALICE, JHEP 01 (2022) 174

Indicate thermalization of low p_T charm in PbPb: small relaxation time.

Need reliable determination of κ.

Dong, Lee & Rapp, Ann. Rev. Nucl. Part. Sc. 69 (2019) 417

Calculation of κ

- A field theoretic definition of κ can be given:

$$3\kappa \;=\; \frac{1}{\chi}\lim_{\omega\to 0}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\;\int d^3x \left\langle \frac{1}{2}\left\{F_i(t,x)\,,\; F_i(0,0)\right\}\right\rangle$$

- Expanding the force term in a series in $1/M$:

$$F^i \;=\; M\frac{dJ^i}{dt} \;=\; \phi^\dagger \left\{ -gE^i \;+\; \frac{\left[D^i, D^2 \;+\; c_b g\sigma\cdot B\right]}{2M} \;+\; .... \right\} \phi$$

- In static $(M_Q \to \infty)$ limit one gets only the gE force.

$$\kappa \;=\; \frac{1}{3\chi}\lim_{\omega\to 0}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\;\langle \mathrm{Tr}\, W(t,-\infty)^\dagger\, gE_i(t)\, W(t,0)\, gE_i(0)\, W(0,-\infty)\rangle$$

J. Casalderrey-Solana & D. Teaney, PRD 74 (2006) 085012;
S. Caron-Huot, M. Laine & G. Moore, JHEP 04 (2009) 53;

- At $\frac{1}{M_Q}$ the magnetic field force needs to be included.

$$\kappa_Q \;\approx\; \kappa_E \;+\; \frac{2}{3}\langle v^2\rangle\,\kappa_B\,, \qquad \langle\gamma v^2\rangle \;=\; \frac{3T}{M_{\rm kin}}$$

A. Bouttefeux & M. Laine, JHEP 12 (2020) 150

Calculation of κ_Q

- κ_Q cannot be calculated in perturbation theory at temperatures of interest to ALICE or CMS.
- Nonperturbative evaluation: calculate the *EE* Matsubara correlator on the lattice.
- $$G_{EE}(\tau) \; = \; \int_0^\infty \frac{d\omega}{\pi} \, \rho_{EE}(\omega) \, \frac{\cosh \, \omega(\tau - 1/2T)}{\sinh \omega/2T}$$

  κ_E can be extracted from the infrared behavior of $\rho_{EE}(\omega)$:

  $$\rho_{IR} \underset{\omega \to 0}{\approx} \frac{\kappa_E \, \omega}{2T}$$

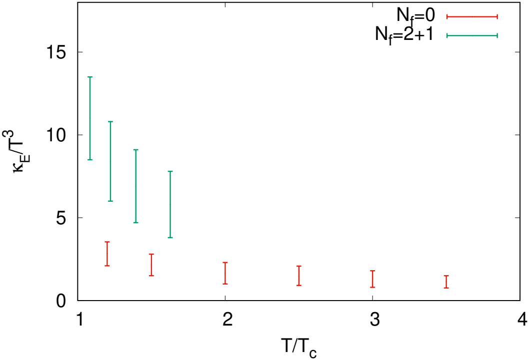
- Calculation for gluonic plasma by several groups. Results agree.

Banerjee, Datta, Gavai, Majumdar (2012, 2023)
Francis et al. (2015); Altenkort et al. (2021); Brambilla et al. (2020, 2023)

κ_E for QCD with $N_f = 2 + 1$

- Calculated recently for $N_f = 2 + 1$ (with $m_\pi \approx 320$ MeV)

HotQCD (Altenkort, et al.), PRL 130 (2023) 231902



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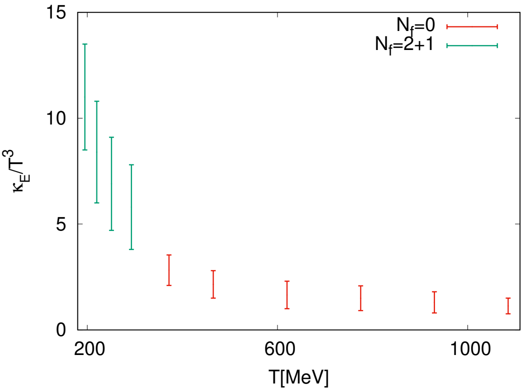
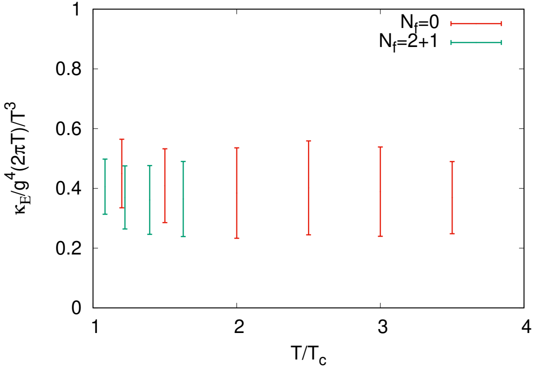


- The difference is largely due to the different T_c in the two theories.

Parametrization of κ_E

An interesting observation: (Altenkort et al.,PRD 109(2024) 114505) results for κ_E for T near T_c can be parametrized as $c\, g^4(2\pi T)\; T^3$, with similar c for both the gluonic and the 2+1 flavor theory.

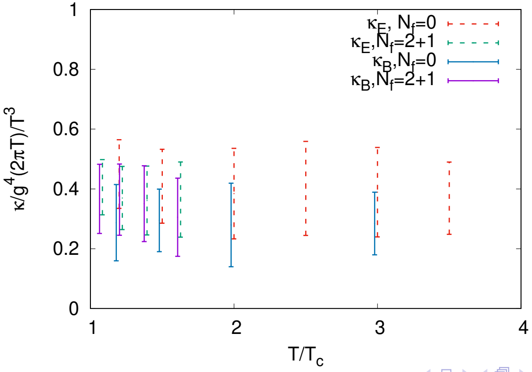


- This is not perturbative: coefficient N_f dependent in PT.
- Also in this temperature range g^5 term dominates in NLO PT.

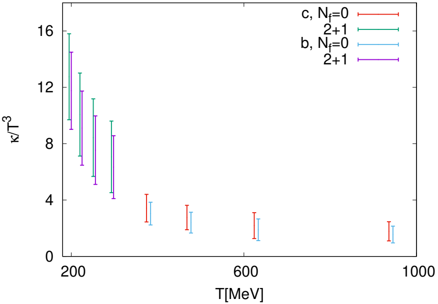
κ_B from lattice

- κ_B can similarly be extracted from BB correlators.
- Some additional complications due to renormalization property of the BB correlator.
- Calculated for both $N_f = 0$ and $2+1$ theories.

Banerjee, Datta & Laine, JHEP 08 (2022) 128; Brambilla et al., PRD107(2023)074503. Altenkort et al. (HotQCD), PRL 132 (2024) 051902



κ_c, κ_b

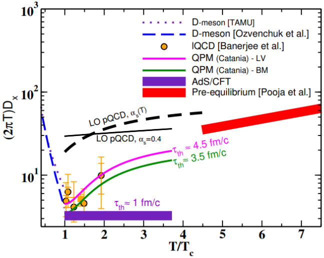


- Correction to the static limit is $\lesssim 10\%$ for bottom near T_c
- Even for charm the $1/M$ correction is $\lesssim 25\%$ near T_c
- $\eta_D \approx \frac{\kappa \langle v^2 \rangle}{6\, T^2}$ smaller for bottom by $\frac{1}{M_Q}$.

Open heavy flavors in QGP

- Extraction of mass dependent κ from experiment will be interesting.
- At this level of description, one can hope to have a controlled theory, with nonperturbative estimates of diffusion coefficient.
- Effect of pre-equilibrium state: much more difficult, and necessarily model dependent. Some attempts to include it.

Pooja, Das, Greco, Ruggieri, EPJ Web Conf. 316(2025)02002; Das, PoSHardProbes2023(2024)011



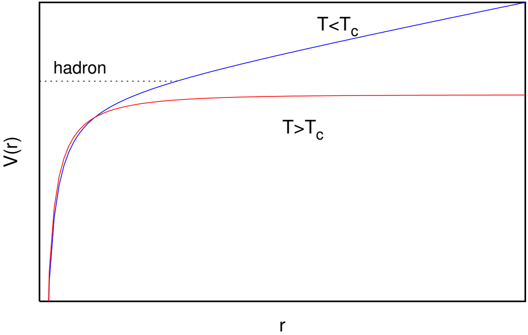
- Other interesting observables: polarization and directed flow of $D, \bar{D}, B, \bar{B}$.

Talks of Amaresh Jaiswal, Sandeep Chatterjee

Quarkonia in deconfined QGP

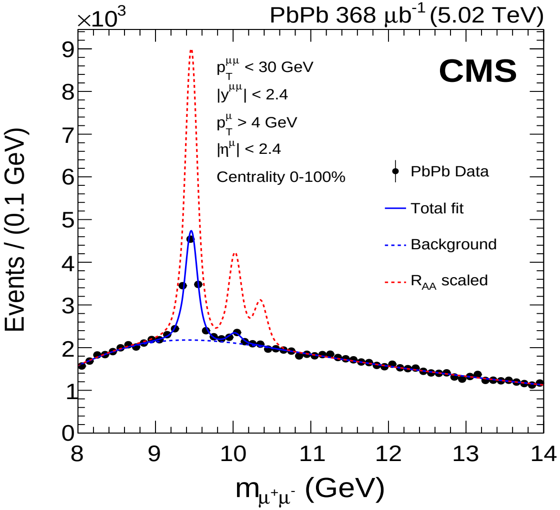
- Screening of the $\bar{Q}Q$ interaction in QGP: suppression of quarkonia peak in dilepton spectra at sufficiently high temperatures.

Matsui & Satz (1986)



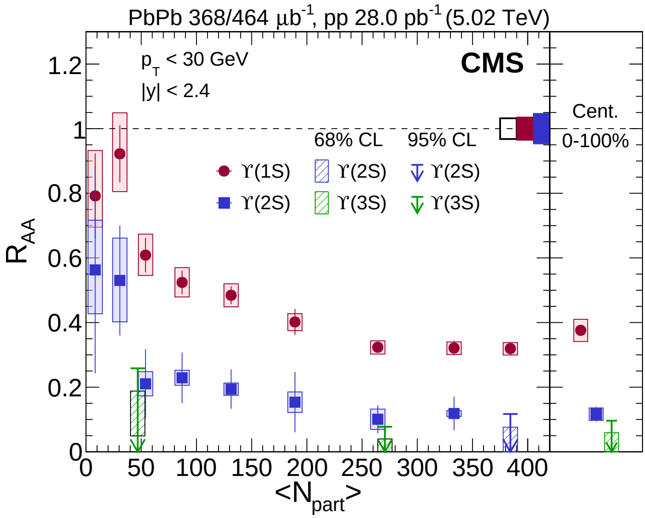
- Yield of various quarkonia: thermometer of the medium?

Υ in LHC $Pb - Pb$



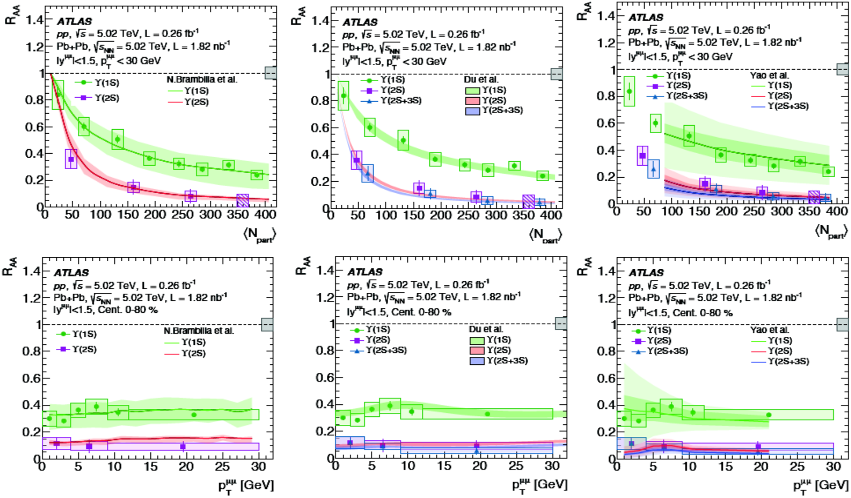
CMS, PLB (2019)

Υ in LHC $Pb-Pb$



CMS, PLB (2019)

Different theoretical models



N.Brambilla et al., PRD 104 (2021) 094049

M.H.X. Du and R. Rapp, PRC 96 (2017) 054901

X. Yao et al., JHEP 2021 (2021) 46

ATLAS, PR C107(2023)054912; Z. Citron, Hard Probes 2023

Theory of $\bar{Q}Q$ in QGP

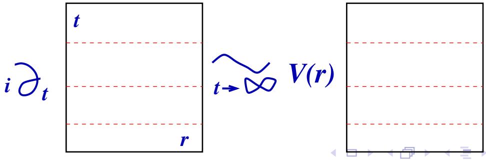
- Different theoretical approaches can explain the data, by tuning of the parameters and the evolution of the system.
- In particular, open quantum system calculations (Brambilla et al.) and rate equation based calculations (Du and Rapp) fit the data, even though they are not equivalent.

Akamatsu 2015; Sharma and Tiwari, PRD 101 (2020) 074004

- $\bar{Q}Q$ evolution Hamiltonian:

$$H_{\bar{Q}Q} = \left(\frac{P_Q^2}{2M_Q} + gA_0^a(\vec{x}_Q)\, t_Q^a\right) + \left(\frac{P_{Q_c}^2}{2M_Q} - gA_0^a(\vec{x}_{Q_c})\, t_{Q_c}^{\star\,a}\right) + \mathcal{O}\left(\frac{1}{M_Q}\right)$$

- The leading order interaction leads to an effective thermal potential, which can be defined nonperturbatively.



How to calculate potential nonperturbatively

- $V(r)$ has been calculated in weak coupling perturbation theory, in various hierarchy of scales.

$$V(r) \;=\; V_{\rm re}(r;T)(r) \;-\; i\,V_{\rm im}(r;T)(r)$$

M. Laine, O. Philipsen, P. Romatschke & M. Tassler, JHEP 03 (2007) 054
N. Brambilla, J. Ghiglieri, A. Vairo & P. Petreczky, PRD 78 (2008) 014017.

- Nonperturbative study: Euclidean Wilson loop $W(r, \tau; T)$.
- Need an analytical continuation $\Rrightarrow$ spectral function.

$$W(R,\tau) \;=\; \mathcal{N} \int d\omega \; e^{-\omega\tau} \; \rho(R,\omega)$$

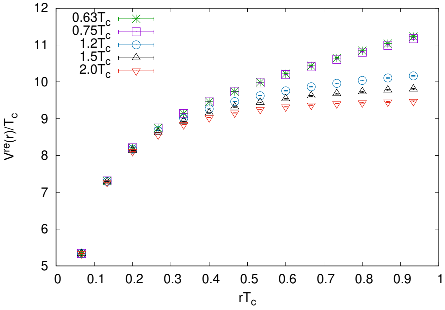
A. Rothkopf T. Hatsuda & S. Sasaki, PRL 108 (2012) 162001

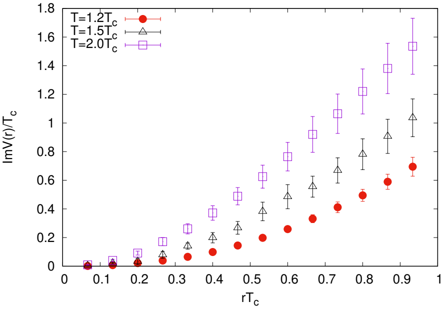
- A direct extraction of $\rho(R, \omega)$ from lattice very difficult.

See, e.g., Y. Burnier, O. Kaczmarek & A. Rothkopf, PRL 114 (2015) 082001.

Finite temperature potential for quarkonia

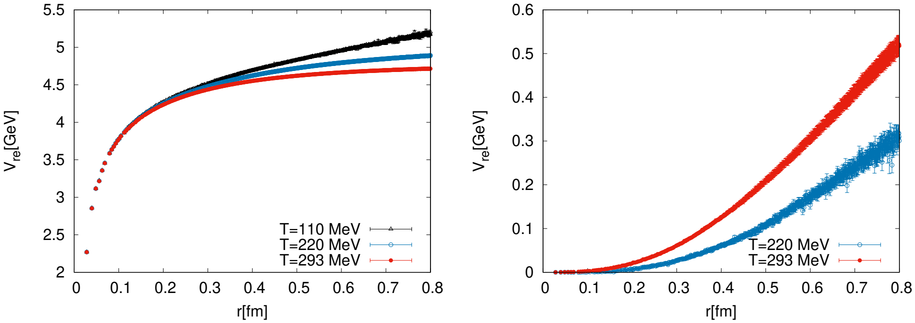
Demanding a low ω structure that leads to a potential, an effective thermal potential can be obtained which gives an excellent description of the Wilson loop data.





D. Bala & S. Datta, PRD 101 (2020) 034507.

Results for full QCD



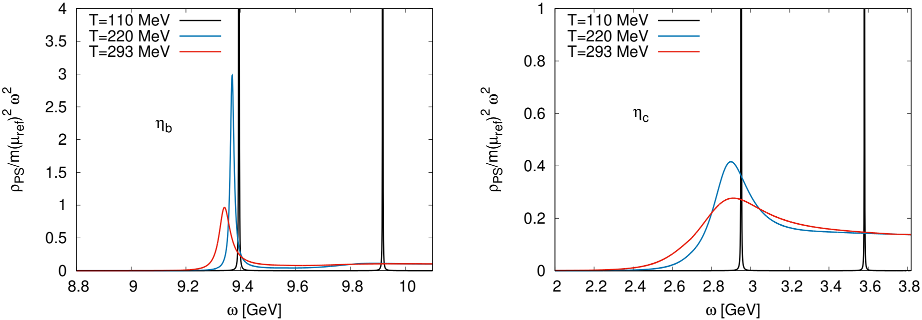
HotQCD: D. Bala, S. Ali, O. Kaczmarek, Pavan, arXiv:2412.17570

- Gives excellent description of the quarkonium correlators.
- Physically motivated input for peak structure crucial. Other kind of peak structures can lead to ambiguous results.

HotQCD: A. Bazavov et al., PRD 109 (2024) 074504

Spectral function for $\bar{Q}\gamma_i Q$ current

Vector quarkonia peak using the thermal potential:



Sharp $\Upsilon(1S)$ peak above 1.6 T_c. J/ψ peak broad, but well-defined at 1.2 T_c, survives till 1.6 T_c.

D. Bala, S. Ali, O. Kaczmarek, Pavan, arXiv:2412.17570

$\bar{Q}, Q$ in octet color configuration

- $V_o(r, T)$, the interaction potential between $\bar{Q}, Q$ in a color octet configuration?
- No direct way of defining V_o nonperturbatively.

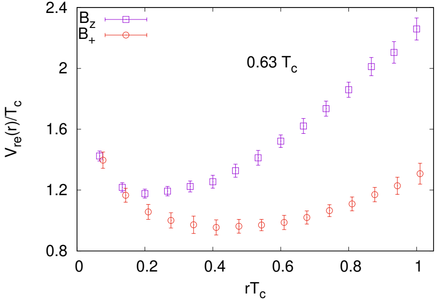
Philipsen & Wagner, PR D89 (2014) 014509

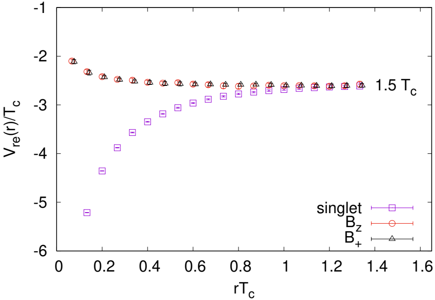
- One can study instead potential for hybrid $\bar{Q}Q + G$ states, and infer results for the octet potential.
- In perturbation theory: ladder sum independent of G^a and in lowest orders, gives octet potential

$$V_o(\vec{r}) = -\frac{1}{N_c^2 - 1} V_s(r) \; + \; \mathcal{O}(\alpha^3)$$

Potential for B_z and $B_x \pm iB_y$

Operators B_z and $B_x \pm iB_y$: Σ_u^- and Π_u states respectively





Above T_c: we do not see dependence on G^a

D. Bala and S. Datta, PRD 103 (2021) 014512

This is for gluon plasma. A full QCD calculation is very important.

$\bar{Q}Q$ density matrix

- For a (near) static medium, the thermal potential can describe the temperature dependence of the $\bar{Q}Q$ system.
- The proper framework to study quarkonia evolving with the plasma is the open quantum system framework.
  Y. Akamatsu, 2013-2020; Review, R. Sharma, 2101.04268
- The total system, including the heavy quark system and the "medium" of light quarks and gluons, can be described by the density matrix $\rho_{\rm tot}$:

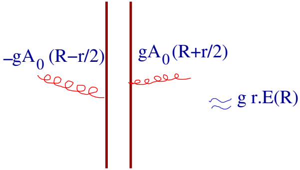
$$\rho_{\rm tot}(t) \;=\; \sum_i \omega_i \left|\psi_i(t)\right\rangle \left\langle\psi_i(t)\right| , \qquad i\frac{d}{dt}\rho_{\rm tot}(t) = [H_{\rm tot}, \rho_{\rm tot}(t)]$$

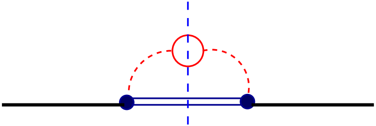
  The HQ "system" is described by $\rho(t) = {\rm Tr}_M \; \rho_{\rm tot}(t)$
- Assuming $\rho_{\rm tot}(0) = \rho(0) \otimes \rho_M(0)$ and that the system relaxation time is much larger than the medium relaxation time, one can write an Evolution equation for the $\bar{Q}Q$ density matrix ρ:

$$\begin{aligned}\frac{d\rho(t)}{dt} &= -i[H_0 + \Delta H, \rho] + \sum_n \left(C_n \,\rho C_n^\dagger - \frac{1}{2}\left\{ C_n^\dagger \, C_n \, , \rho \right\} \right) \\ &= -i\left(H_{\rm eff}\,\rho - \rho \, H_{\rm eff}^\dagger \right) + \sum_n C_n \,\rho C_n^\dagger , \qquad H_{\rm eff} = H_0 + \Delta H + \frac{i}{2}\sum_n C_n^\dagger C_n\end{aligned}$$

Dipole approximation: pNRQCD

- For $r \ll 1/T$, e.g., for Υ, interaction of $\bar{Q}Q$ with medium: color dipole.





- Let us expand in r and further, write the $\bar{Q}Q$ in $|S\rangle$, $|O\rangle$ basis (pNRQCD).

$$
\begin{aligned}
\mathcal{L}_{pNR} &= \mathrm{Tr} \quad \left[S^\dagger \left(i\partial_0 - V_s(r) \right) S \;+\; O^\dagger \left(iD_0 - V_o(r) \right) O \right] \\
&+ \mathrm{Tr} \quad \left[S^\dagger \vec{r}.g\vec{E} O \;+\; h.c. + \frac{1}{2}\, O^\dagger \{ \vec{r}.g\vec{E}\,,\, O \} \right] + \mathcal{O}\left(\frac{1}{M}\right)
\end{aligned}
$$

Pineda & Soto (1998); Brambilla, et al., RMP 77 (2005) 1423

- Thermal correction depends on

$G(t) \;=\; \frac{1}{3} \sum_i \langle T\, E^{a,i}(t,\vec{0})\, W^{ab}(t,0)\, E^{b,i}(0,\vec{0}) \rangle.$

ρ in pNRQCD

- ρ, H_{eff} can be split into singlet and octet subspaces,

$$\rho \;=\; \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}, \; H_{\text{eff}} \;=\; \begin{pmatrix} H_s(T) & 0 \\ 0 & H_o(T) \end{pmatrix},$$

  and $C_\pm, C_o$ control singlet $\longleftrightarrow$ octet and octet $\longleftrightarrow$ octet transitions, respectively.

  Brambilla, Vairo, et al., 2017-2023

- $C_\pm \propto \sqrt{\kappa}\, r$, where the transport coefficient κ incorporates system information, and is related to $\tilde{G}(\omega \to 0)$.
- It is possible to analytically continue $G(t)$ to the Euclidean correlator $G(\tau)$.

  B. Scheihing-Hitschfeld & X. Yao, PRD108 (2023) 054024

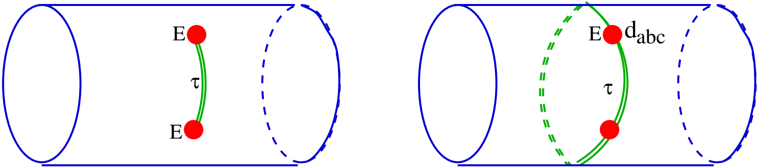
- $G(\tau)$ related to κ through analytic continuation.

$$G(\tau) \;=\; \int_{-\infty}^{\infty} d\omega \; \rho(\omega) \; \frac{e^{\omega(1/2T \,-\, \tau)}}{2 \sinh \frac{\omega}{2T}}$$

$$\rho(\omega) \;=\; \rho_{\text{odd}}(\omega) \;+\; \rho_{\text{even}}(\omega)\,; \qquad \kappa \;=\; \lim_{\omega \to 0} \frac{T}{\omega}\; \rho_{\text{odd}}(\omega)$$

$G(\tau)$ and $G_{\rm oct}(\tau)$

- The $O^\dagger \{r.gE\,,\,O\}$ term in L_{pNR} leads to a correction to the octet potential, which can be obtained from the correlator $G_{\rm oct}(\tau)$.
- $G(\tau)$ and $G_{\rm oct}(\tau)$ look similar, and also similar to the $G_{EE}(\tau)$ involved in calculation of heavy quark diffusion coefficient κ_E.

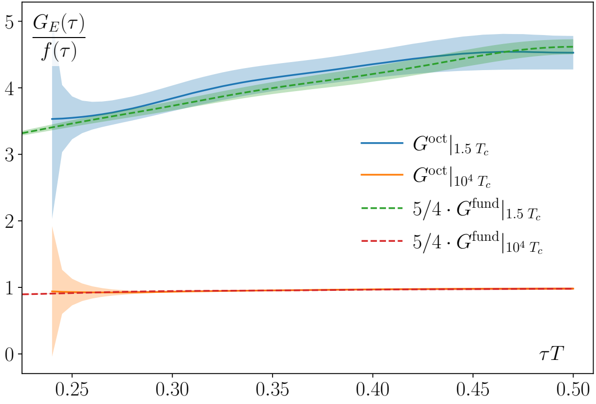


- They agree in LO, modulo color factor, but not in NLO.

$$\left.\begin{matrix} G(\tau)|_{LO} \\ G_{\rm oct}(\tau)(\tau)|_{LO} \\ G_{\rm fund}(\tau)|_{LO} \end{matrix}\right\} = g^2 \left\{\begin{matrix} N^2-1 \\ (N^2-4)/N \\ C_f \end{matrix}\right\} f(\tau), \qquad f(\tau) \;=\; \frac{\pi^2 T^4}{\sin^2 \pi t T}\left(\cot^2 \pi t T \;+\; \frac{1}{3}\right)$$

Octet-octet correlator

Structure of $G_{\text{oct}}(\tau)$, which calculates a correction of the thermal octet potential, is simple: it has a similar structure as $G_{\text{fund}}(\tau)$, and shows a color scaling, $\Rightarrow \kappa_{\text{oct}} \approx \frac{5}{4}\kappa_{\text{fund}}$

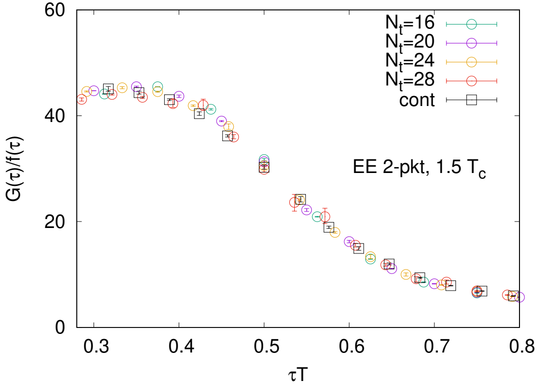


N. Brambilla, S. Datta, M. Jener, V. Leino, J. Mayer-Steudte, P. Petreczky, A. Vairo, in preparation

Structure of $G(\tau)$

Structure of $G(\tau)$ is very different. The renormalization is also involved.

$$G(\tau; a) \;=\; Z_E^2(a)\, e^{\delta m(a)\,\tau}\, Z_{\rm bare}(\tau; a); \qquad a \;=\; \frac{1}{N_t\, T}$$

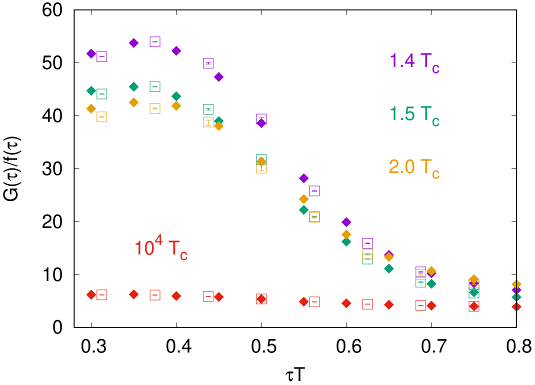


D. Banerjee, S. Datta, B. Singh, and N. Brambilla, M. Jener, V. Leino, J. Mayer-Steudte, P. Petreczky, A. Vairo, work in progress

Structure of $G(\tau)$

Structure of $G(\tau)$ is very different. The renormalization is also involved.

$$G(\tau; a) \;=\; Z_E^2(a)\, e^{\delta m(a)\,\tau}\, Z_{\rm bare}(\tau; a); \qquad a \;=\; \frac{1}{N_t\, T}$$

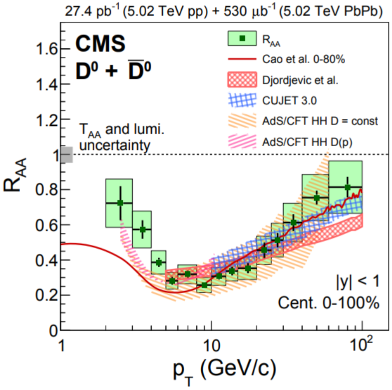


D. Banerjee, S. Datta, B. Singh, and N. Brambilla, M. Jener, V. Leino, J. Mayer-Steudte, P. Petreczky, A. Vairo, work in progress

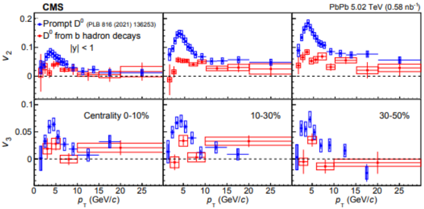
Summary

- Heavy flavor mesons, and quarkonia, provide very important probes of the medium created in relativistic heavy ion collisions, and can answer questions on the nature of the medium.
- The medium is complex, dynamic, and transient. To understand the whole system, it is important to try to get control over parts of it.
- For the evolution of low p_T heavy quarks in QGP, a Langevin description, with diffusion coefficient calculated nonperturbatively from QCD, should work.
- For the evolution of quarkonia in QCD, it is possible to have an open quantum system description, with parameters determined from lattice.
- Results for effective thermal potential available from lattice, work in progress for the jump operators.
- Huge scope for improvement in the lattice calculation.

Extra slide: R_{AA} and v_2 from CMS



CMS, PLB 782 (2018) 474



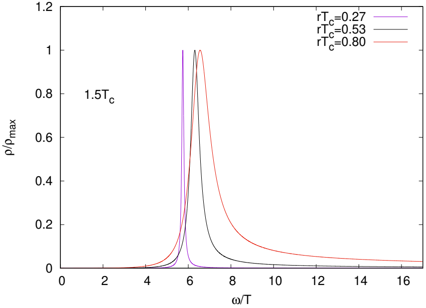
CMS, PLB 850 (2024) 138389

Low-ω peak

Demanding that the low ω structure leads to a potential in the long t limit, we are led to a structure which is Lorentzian-like near the maximum,

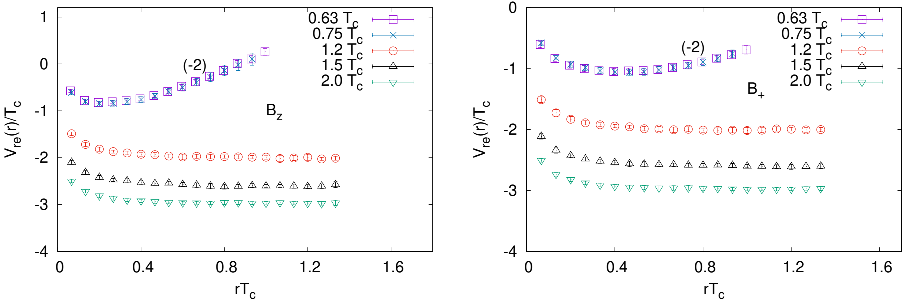
$$\rho(\omega)_{\rm low} \approx \sqrt{\frac{2}{\pi}} \frac{V_i}{(V_r - \omega)^2 + V_i^2}$$

but has a powerlike and an exponential fall-off, respectively, in the high and the low ω sides.



D. Bala & S. Datta, PRD 101 (2020) 034507.

Extra slide: $V^{o}_{\rm re}(r;T)$ above T_c



- $V^{o}_{\rm re}(r;T)$ screened and repulsive everywhere.
- $V^{o}_{\rm re}(r;T)$ for Σ_u^- and Π_u agree everywhere.
- Comes close to $V_{\rm re}(r;T)$ for $rT \gtrsim 2$.

D. Bala & S. Datta, PRD 103 (2021) 014512