

Modified NRQCD and Quarkonium Production

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- Non-Relativistic QCD (NRQCD) (Bodwin, Braaten and Lepage, Phys. Rev. D **51**, 1125 (1995)), as an effective theory approach to the study of quarkonia, met with considerable success in predicting production cross-sections.
- NRQCD is derived from the QCD Lagrangian by neglecting all states of momenta much larger than the heavy quark mass, M_Q and to account for this exclusion by adding new interaction terms yielding the effective Lagrangian.

Colour-octet states

- The precursor to NRQCD was the colour-singlet model (Baier, Rückl, Z. Phys. C **19**, 251 (1983)) where from the short-distance amplitude of the production of the $Q\bar{Q}$ pair, a colour-singlet state with the correct $^{2S+1}L_J$ quantum numbers is projected out.
- A similar deployment of projection-operators in NRQCD yields both colour-singlet and colour-octet quarkonium states.
- The colour-singlet or colour-octet state then makes the appropriate non-perturbative transition to the physical quarkonium state.

NRQCD Factorisation

- In NRQCD, the following factorised form of the cross-section is obtained:

$$\sigma(J/\psi) = \sum_{n=\{\alpha,S,L,J\}} \frac{F_n}{M^{d_n-4}} \langle \mathcal{O}_n^{J/\psi}(^{2S+1}L_J) \rangle, \quad (1)$$

Where F_i 's refer to the short-distance co-efficients and $\mathcal{O}$ refers to the non-perturbative matrix elements and M is the mass of the J/ψ .

Fock decomposition

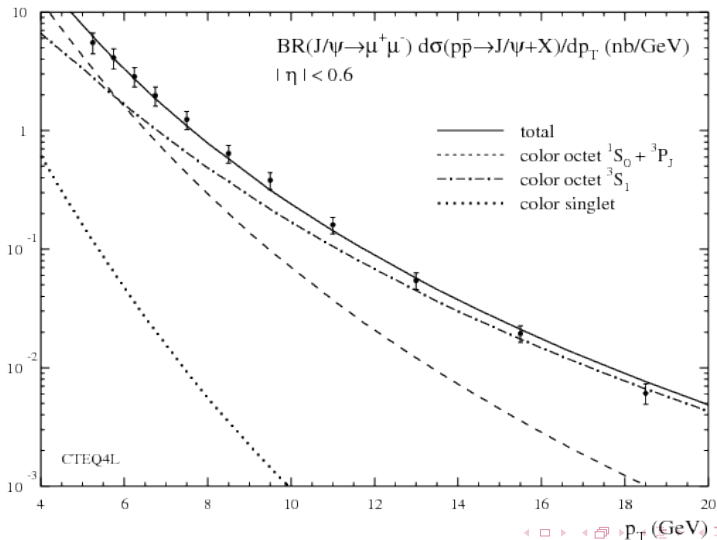
- The quarkonium state admits of a Fock-state expansion in orders of v .

$$\begin{aligned} |J/\psi\rangle = & \mathcal{O}(1) \left| Q\bar{Q}[{}^3S_1^{[1]}] \right\rangle + \mathcal{O}(v^2) \left| Q\bar{Q}[{}^3P_J^{[8]}] g \right\rangle + \\ & \mathcal{O}(v^4) \left| Q\bar{Q}[{}^1S_0^{[8]}] g \right\rangle + \mathcal{O}(v^4) \left| Q\bar{Q}[{}^3S_1^{[8]}] gg \right\rangle + \cdots \end{aligned} \quad (2)$$

The J/ψ cross section

$$\begin{aligned}\sigma_{J/\psi} &= \hat{F}_{3S_1^{[1]}} \times \langle \mathcal{O}(^3S_1)^{[1]} \rangle \\ &+ \hat{F}_{3S_1^{[8]}} \times \langle \mathcal{O}(^3S_1)^{[8]} \rangle \\ &+ \hat{F}_{1S_0^{[8]}} \times \langle \mathcal{O}(^1S_0)^{[8]} \rangle \\ &+ \left[\hat{F}_{3P_J^{[8]}} \times \langle \mathcal{O}(^3P_J)^{[8]} \rangle \right] / M^2\end{aligned}\tag{3}$$

J/ψ at the Tevatron



The problems

- The predictions that exploit the heavy-quark symmetry of the NRQCD Lagrangian viz. polarization of the quarkonium state produced at high- p_T and the η_c cross-section, are both in serious conflict with the experimental results.
- This suggests that a modification of NRQCD is called for.

η_c at the LHCb

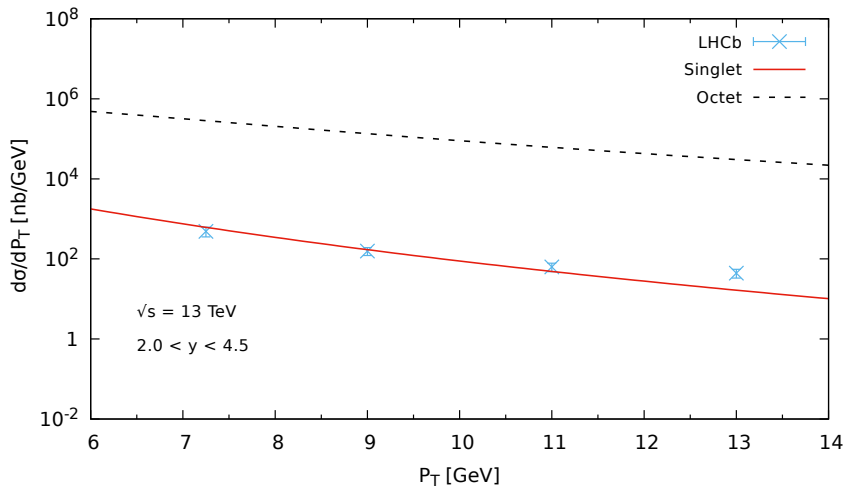


Figure: *The LHCb η_c anomaly*

Perturbative soft gluons

- The colour-octet $c\bar{c}$ state can radiate several soft *perturbative* gluons.
- In the multiple emissions that the colour-octet state can make before it makes the final NRQCD transition to a quarkonium state, the angular momentum and spin assignments of the $c\bar{c}$ state changes constantly.
- We make the assumption that the soft gluon transitions mix only the S and P states. We neglect the higher angular momentum states.
- We also assume that all the transition probabilities are equal.
- We also note that in the process of making these transitions an octet state can sometimes also transition to a singlet state.

The modified formula

- The formula for the production cross-section of a quarkonium state is then given by

$$\begin{aligned}\sigma_{J/\psi} &= \left[\hat{F}_{3S_1^{[1]}} \times \langle \mathcal{O}(^3S_1)^{[1]} \rangle \right] \\ &+ \left[\hat{F}_{3S_1^{[8]}} + \hat{F}_{1P_1^{[8]}} + \hat{F}_{1S_0^{[8]}} + (\hat{F}_{3P_J^{[8]}}) \right] \times \left(\frac{\langle \mathcal{O}(^3S_1)^{[1]} \rangle}{8} \right) \\ &+ \left[\hat{F}_{3S_1^{[8]}} + \hat{F}_{1P_1^{[8]}} + \hat{F}_{1S_0^{[8]}} + (\hat{F}_{3P_J^{[8]}}) \right] \times \langle \mathcal{O} \rangle, \quad (4)\end{aligned}$$

- Armed with the above formula, we will try to first check what predictions result for the cross-sections of the J/ψ , ψ' and χ_c states.

J/ψ and ψ' at the Tevatron

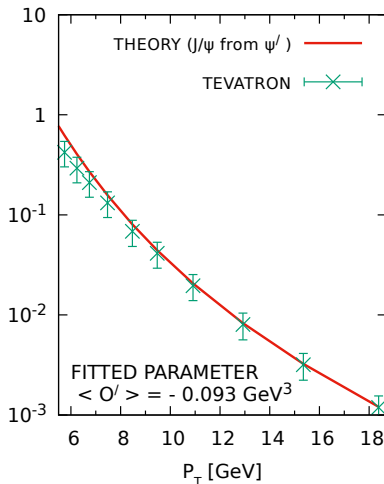
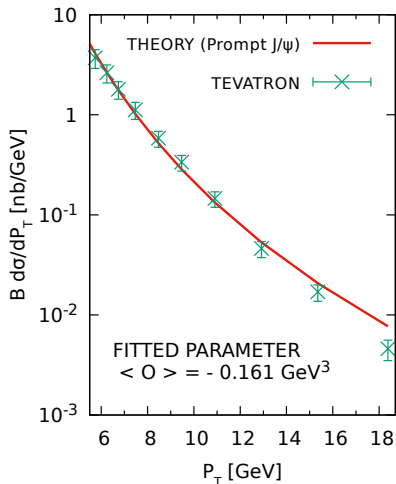
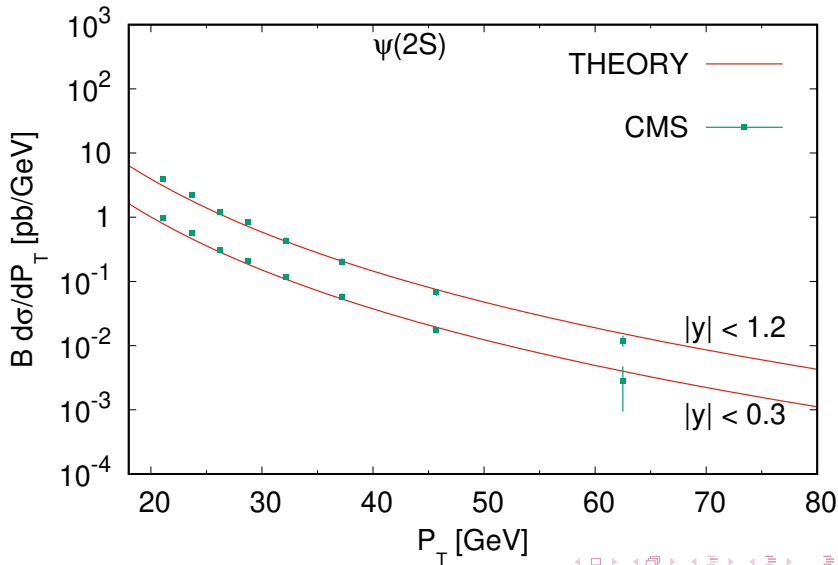


Figure: Fit to the J/ψ and ψ' cross-sections at the Tevatron - CDF.

ψ' at the LHC



χ_c at the Tevatron

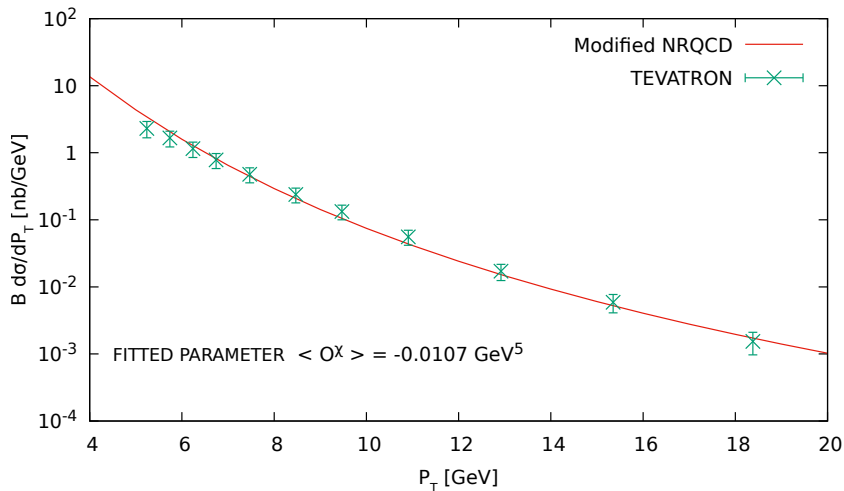


Figure: Fit to the χ_c cross-section at the Tevatron - CDF.

J/ψ at the LHC

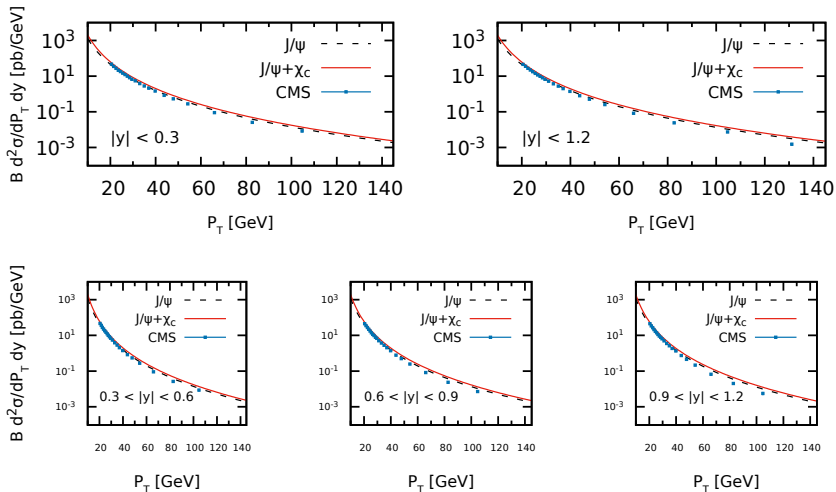


Figure: Prediction for the J/ψ cross-section at the LHC - CMS.

χ_c at the LHC

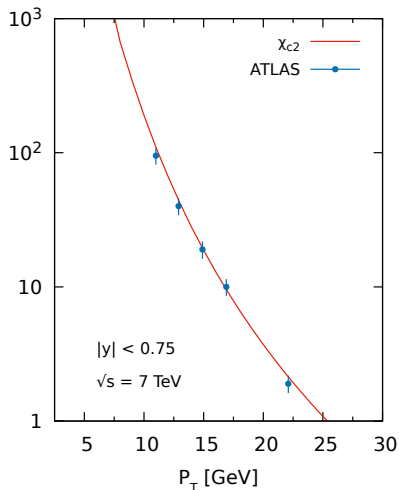
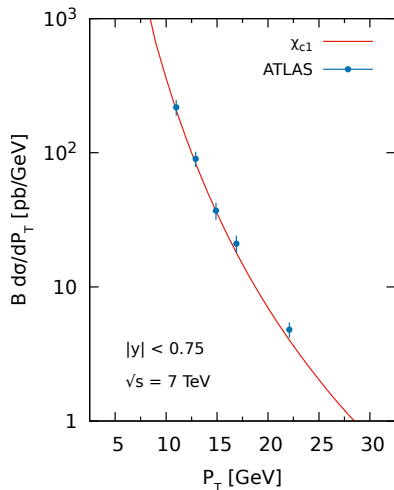


Figure: Prediction for the χ_c cross-section at the LHC - ATLAS.

η_c at the LHC - I

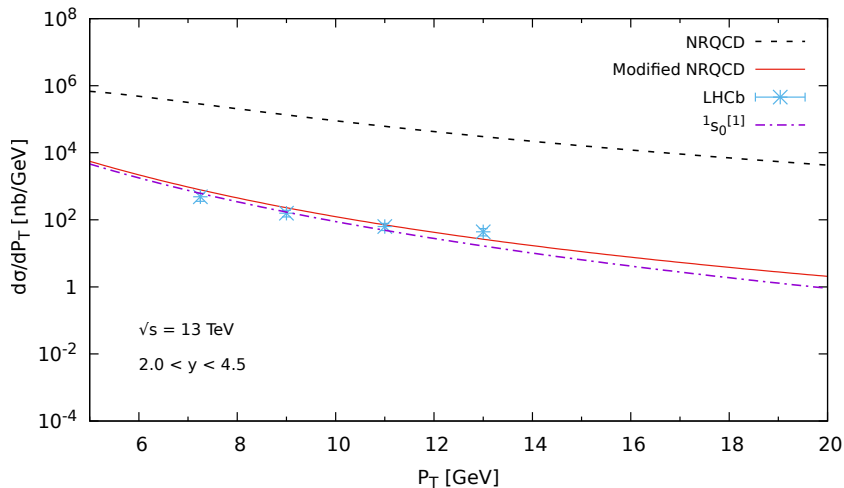


Figure: Prediction for the η_c cross-section at the LHC - LHCb 13 TeV.

η_c at the LHC - II

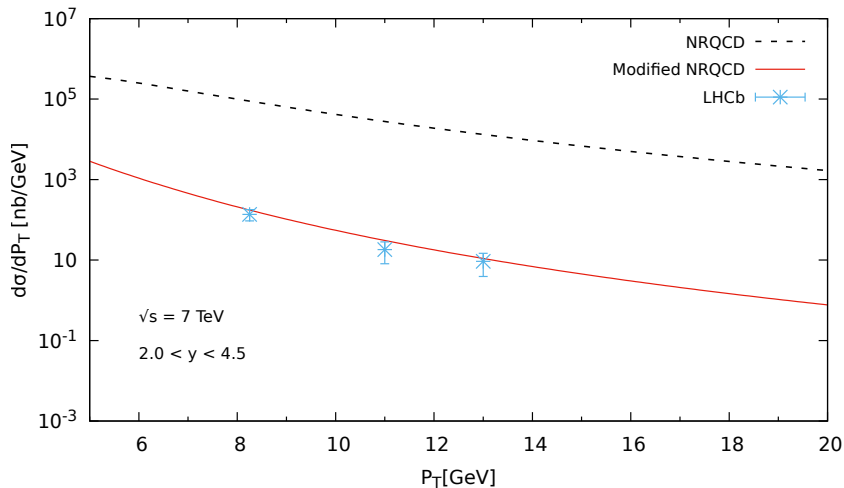


Figure: Prediction for the η_c cross-section at the LHC - LHCb 7 TeV.

η_c at the LHC - III

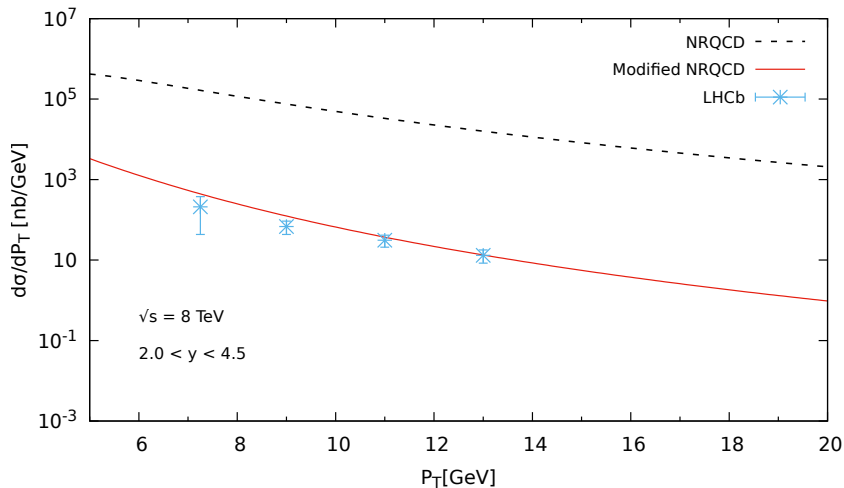


Figure: Prediction for the η_c cross-section at the LHC - LHCb 8 TeV.

η_c to J/ψ ratio at the LHC

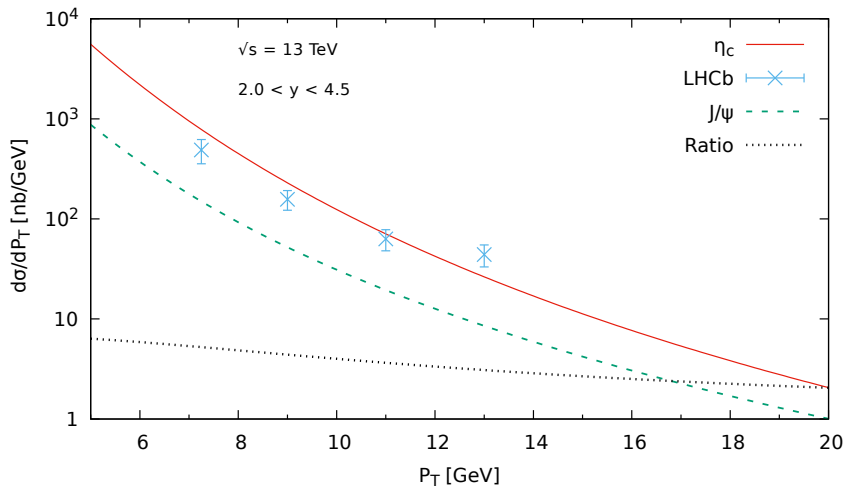


Figure: Prediction for the η_c to J/ψ cross-section ratio at the LHC - LHCb 13 TeV.

η_c at the LHCb

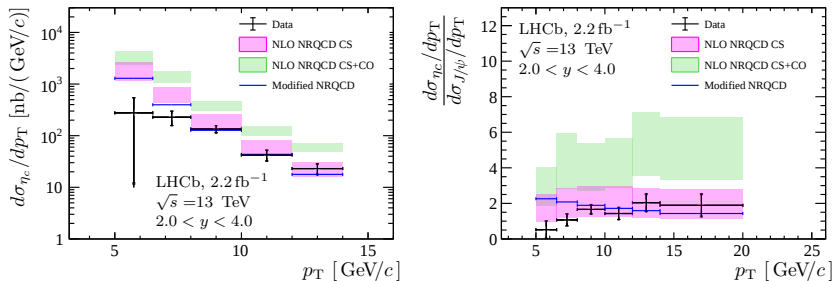


Figure: LHCb data for η_c compared to NRQCD and MNRQCD

$\Upsilon(1S)$ at the LHC

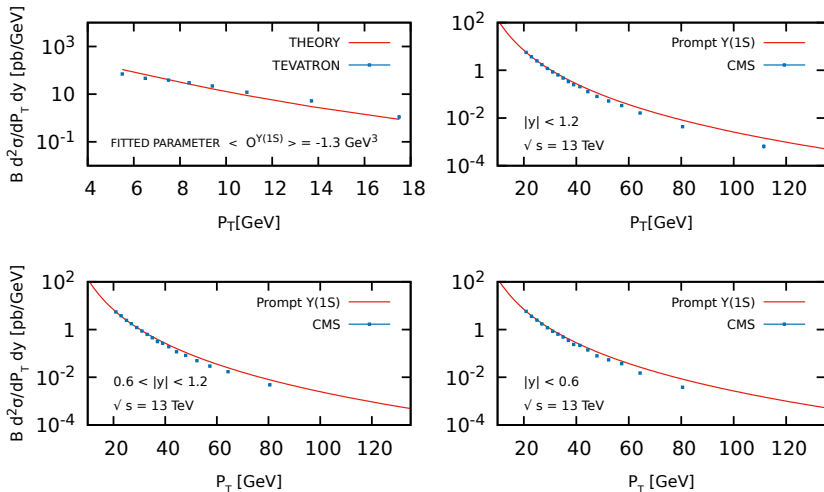


Figure: Prediction for $\Upsilon(1S)$ production at the LHC - CMS 13 TeV.

$\Upsilon(2S)$ at the LHC

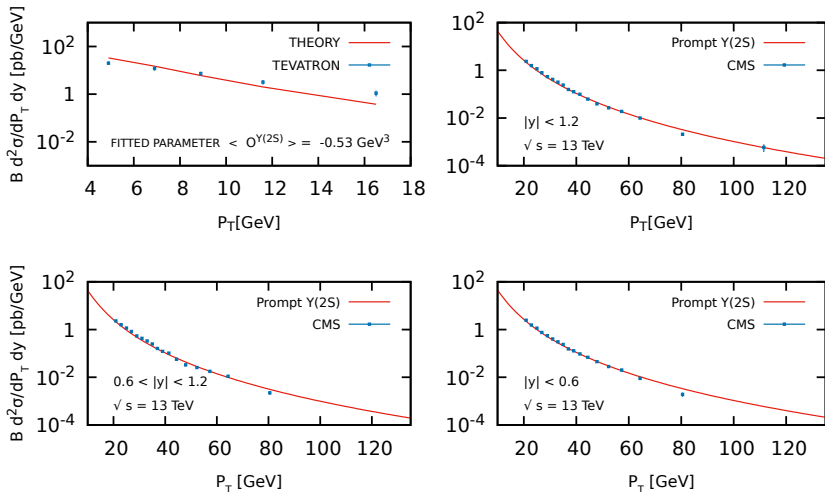


Figure: Prediction for $\Upsilon(2S)$ production at the LHC - CMS 13 TeV.

$\Upsilon(3S)$ at the LHC

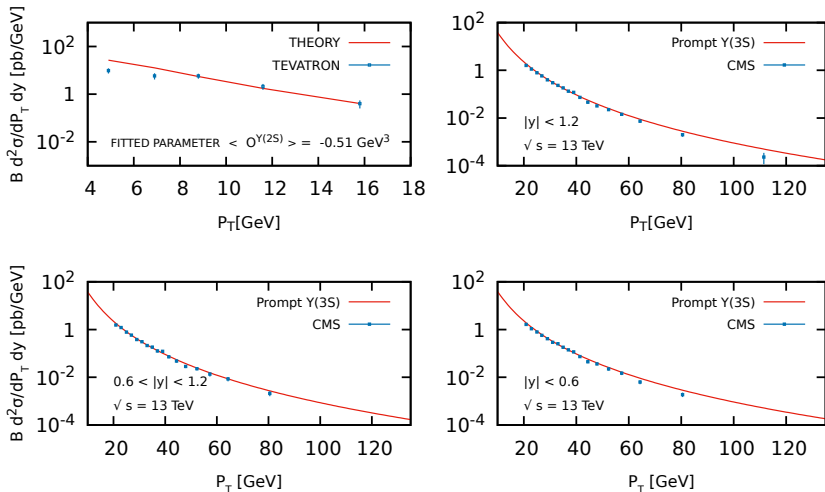


Figure: Prediction for $\Upsilon(3S)$ production at the LHC - CMS 13 TeV.

η_b at the LHC

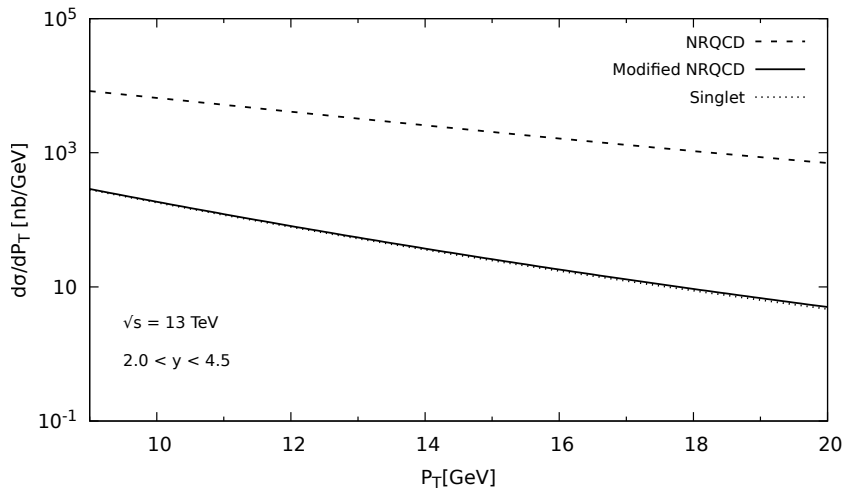


Figure: Prediction for η_b production at the LHC 13 TeV.

h_c at the LHC

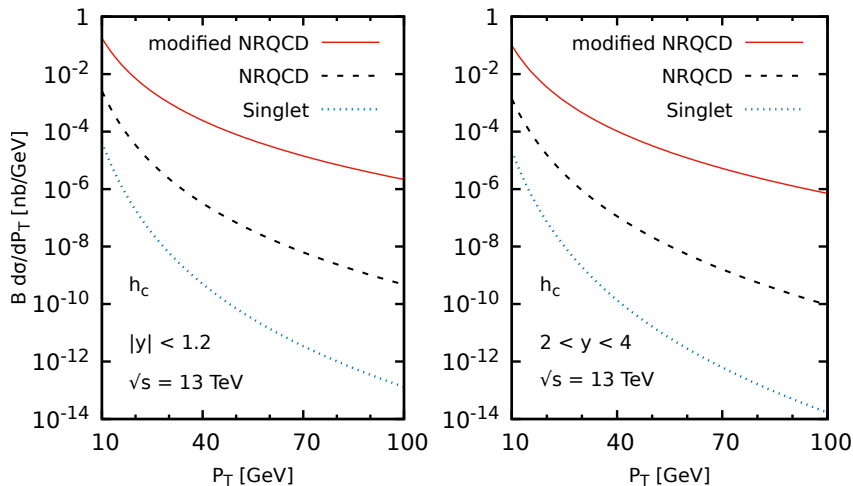


Figure: Prediction for h_c production at the LHC 13 TeV.

References

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