
Basic Python

ML4HEP 2026 Pre-school Lectures

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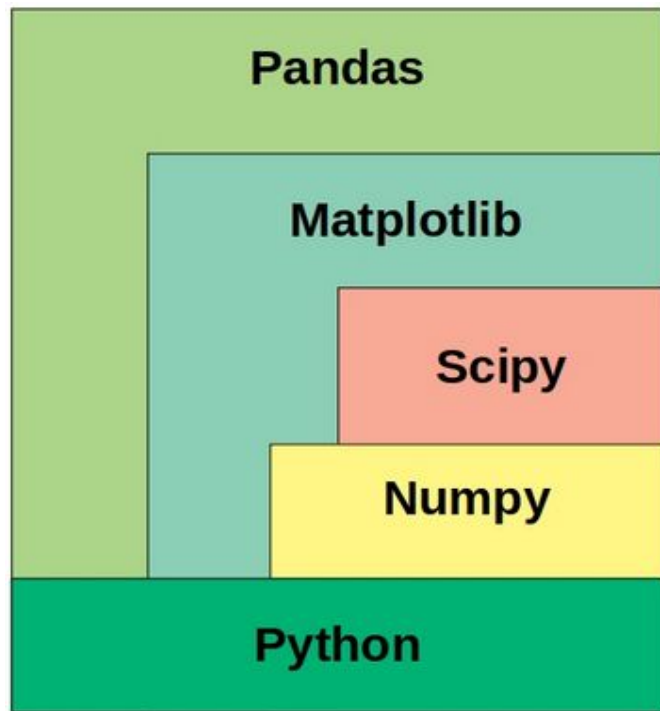
Lecture 7, 14/05/2026

What is Numpy

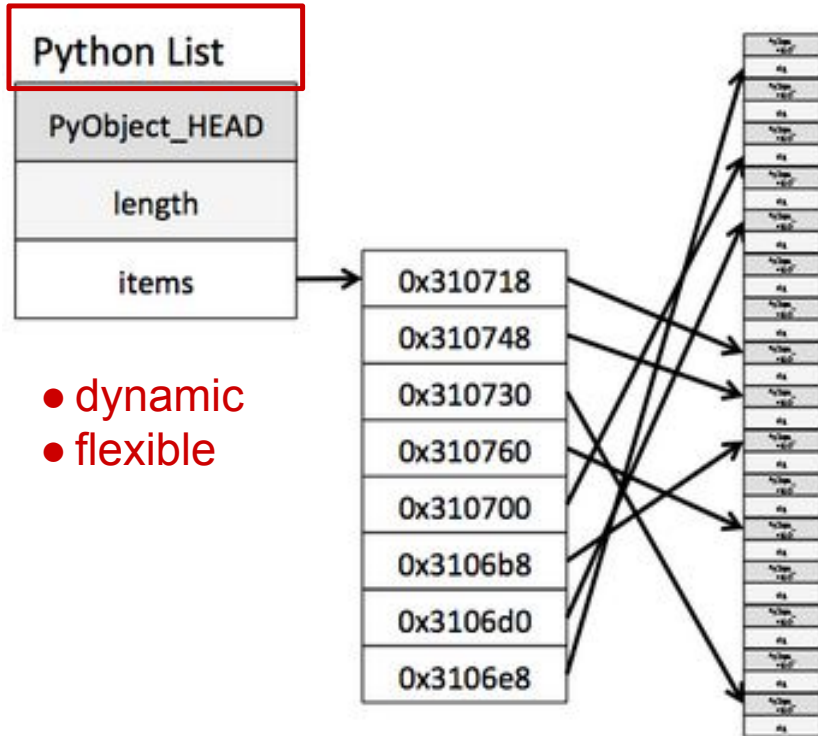
- Numpy is a C extension module for array oriented computing
 - adds powerful data structures like multi-dimensional arrays
 - fixed size arrays holding data of the same underlying type for memory and speed optimization
 - precompiled mathematical and numerical functions for performance
 - performance similar to C with flexibility of python

What is Numpy

- Numpy is a C extension module for array oriented computing
 - adds powerful data structures like multi-dimensional arrays
 - fixed size arrays holding data of the same underlying type for memory and speed optimization
 - precompiled mathematical and numerical functions for performance
 - performance similar to C with flexibility of python
- Numpy forms the basis of scientific computing in Python
 - **Linear Algebra**
 - **Image and signal processing**
 - **Machine learning**

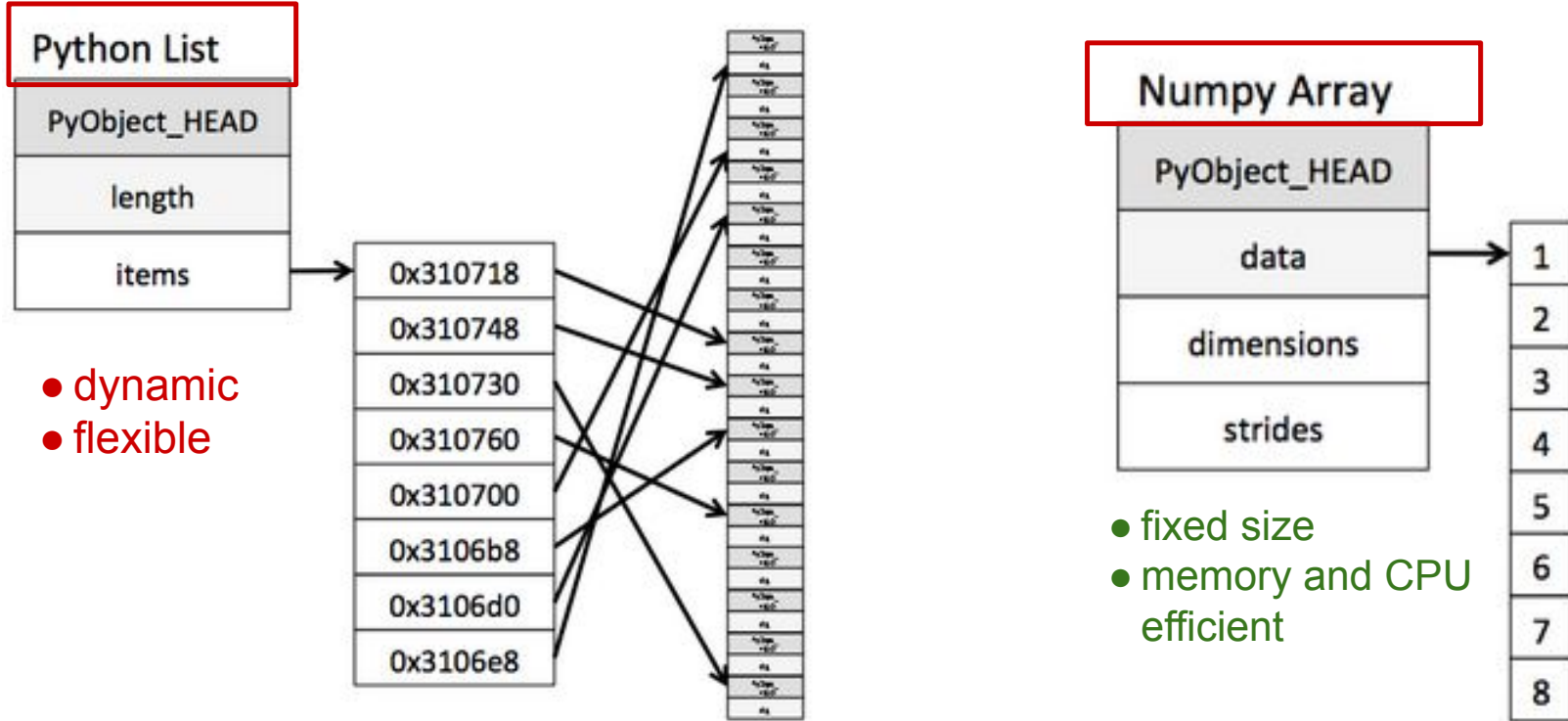


List vs Numpy Array



<https://jakevdp.github.io/PythonDataScienceHandbook/02.01-understanding-data-types.html>

List vs Numpy Array



- dynamic
- flexible

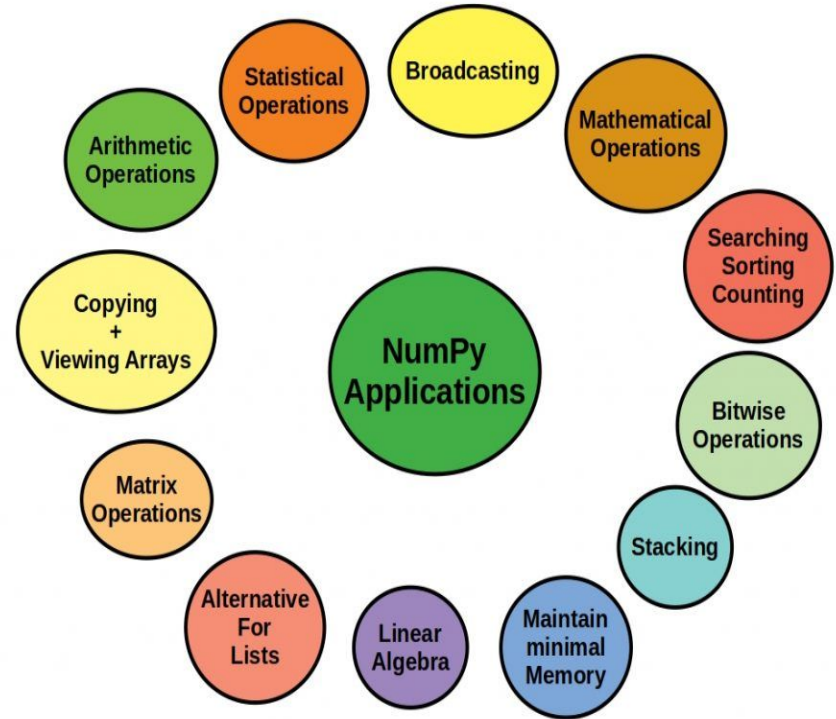
- fixed size
- memory and CPU efficient

<https://jakevdp.github.io/PythonDataScienceHandbook/02.01-understanding-data-types.html>

- The *strides* of an array tell us how many bytes we have to skip in memory to move to the next position along a certain axis.

Numpy Features

- Smaller memory footprint
- CPU efficiency
- Natural look and feel (e.g algebraic operation)
 - vectorization
 - broadcasting
 - operator overloading



<https://starship-knowledge.com/awesome-python-data-science-libraries>

Vectorization

- Modern CPUs have "vector" or Single Instruction Multiple Data (SIMD) capabilities which apply the same operation simultaneously to two, four, or more pieces of data
- An over-simplified view of Vectorization
 - process of rewriting a loop to process 4 elements of the array of size N simultaneously N/4 times instead of doing one element at a time

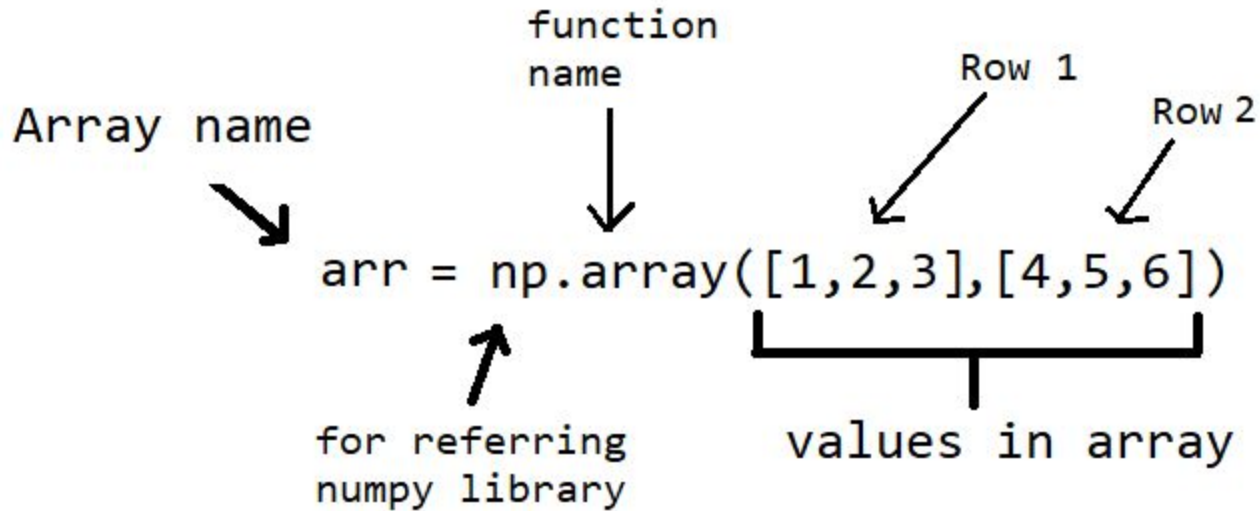
client code:

```
const int N = 16;
int A[N], B[N], C[N];
for (int i = 0; i < N; ++i)
    C[i] = A[i] + B[i];
```

will be unrolled as, assuming 4 independent SIMD instructions

```
for (int i = 0; i < N; i += 4) {
    C[i]    = A[i]    + B[i];
    C[i+1] = A[i+1] + B[i+1];
    C[i+2] = A[i+2] + B[i+2];
    C[i+3] = A[i+3] + B[i+3];
}
```

Arrays from Python lists



numpy.array

```
In [1]: import numpy as np
```

```
In [6]: np.array([1, 2, 3]) # one can also use a tuple
```

```
Out[6]: array([1, 2, 3])
```

```
In [3]: np.array([[1, 2], [3, 4]])
```

```
Out[3]: array([[1, 2],  
               [3, 4]])
```

```
In [4]: np.array([1, 2, 3], ndmin=2)
```

```
Out[4]: array([[1, 2, 3]])
```

ndarray (alias array)

ndarray Attributes	Description
<code>ndarray.ndim</code>	number of axes (dimensions) of the array
<code>ndarray.shape</code>	dimensions of the array
<code>ndarray.size</code>	total number of elements of the array
<code>ndarray.dtype</code>	type of the elements in the array
<code>ndarray.itemsize</code>	size in bytes of each element of the array
<code>ndarray.data</code>	buffer containing the actual elements of the array

```
>>> import numpy as np
>>> a = np.array([[0,1,2,3,4],[5,6,7,8,9],[10,11,12,13,14]])

>>> a
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14]])

>>> a.ndim
2

>>> a.shape
(3, 5)

>>> a.dtype.name
'int64'

>>> a.itemsize
8

>>> a.size
15

>>> type(a)
<class 'numpy.ndarray'>
```

Array Access

```
In [1]: import numpy as np
```

```
In [2]: a = np.array([1, 2, 3])
```

```
In [6]: print(a[1])
```

2

```
In [5]: print(a[0:2])
```

[1 2]

```
In [8]: print(a[1:])
```

[2 3]

```
In [9]: print(a[-2])
```

2

```
In [11]: m = np.array([[1, 2], [3, 4], [5, 6]])
```

```
In [12]: print(m)
```

```
[[1 2]
 [3 4]
 [5 6]]
```

```
In [14]: print(m[0])
```

[1 2]

```
In [15]: print(m[0,1])
```

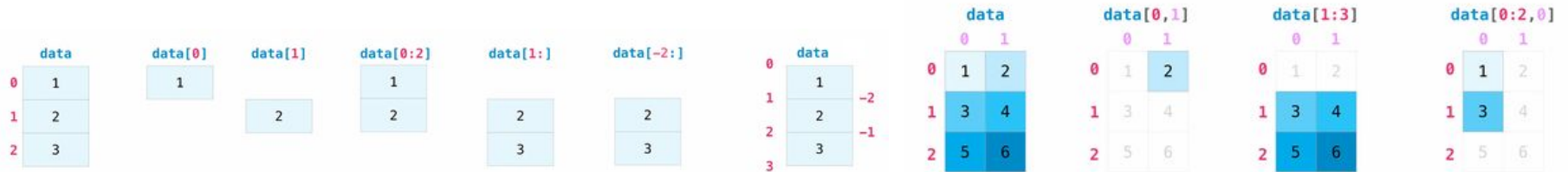
2

```
In [16]: print(m[1:3])
```

```
[[3 4]
 [5 6]]
```

```
In [17]: print(m[0:2,0])
```

[1 3]



Array Elements w/ conditions

```
In [1]: import numpy as np
```

```
In [8]: a = np.arange(1,13, dtype=np.int64).reshape(3,4)
```

```
In [14]: print(a[a < 9])
```

```
[1 2 3 4 5 6 7 8]
```

```
In [15]: even_only = a%2==0  
print(a[even_only])
```

```
[ 2  4  6  8 10 12]
```

```
In [16]: print(a[(a > 3) & (a < 10)]) # note the convention
```

```
[4 5 6 7 8 9]
```

```
In [17]: print(np.nonzero(a < 6))
```

```
(array([0, 0, 0, 0, 1]), array([0, 1, 2, 3, 0]))
```

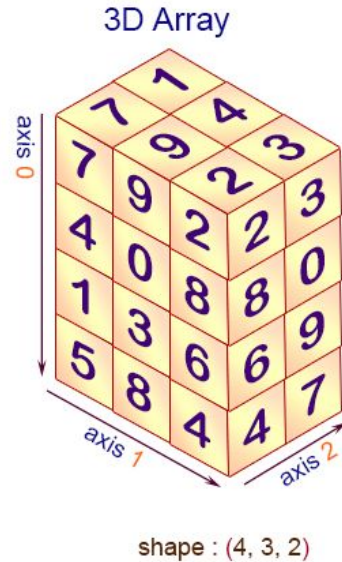
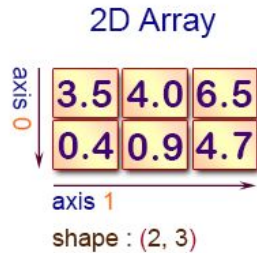
```
In [18]: print(np.count_nonzero(a < 6))
```

```
5
```

also see: `numpy.flatnonzero(array)`, `numpy.where(condition, ...)`, `numpy.argwhere(array)`

array axes

<https://www.w3resource.com/numpy/array-creation/index.php>



© w3resource.com

Arrays from scratch

numpy.arange

```
numpy.arange([start, ]stop, [step, ]dtype=None, *, like=None)
```

```
In [79]: np.arange(15, dtype=np.float64)
```

```
Out[79]: array([ 0.,  1.,  2.,  3.,  4.,  5.,  6.,  7.,  8.,  9., 10., 11., 12.,  
              13., 14.])
```

```
In [70]: np.arange(0, 20, 2)
```

```
Out[70]: array([ 0,  2,  4,  6,  8, 10, 12, 14, 16, 18])
```

```
In [76]: np.arange(1, 2, 0.1)
```

```
Out[76]: array([1. , 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9])
```

```
In [77]: np.arange(10, 1, -1)
```

```
Out[77]: array([10,  9,  8,  7,  6,  5,  4,  3,  2])
```

```
In [78]: np.arange(10, 0, -3)
```

```
Out[78]: array([10,  7,  4,  1])
```

Python List vs Numpy Array Memory Footprint

```
In [8]: import numpy as np
        from sys import getsizeof as sizeof

        print(sizeof(999)) # size of an integer
```

28

```
In [9]: # Empty List
        a = []
        print(sizeof(a))
```

56

```
In [10]: # list with N elements
        b = list(range(1000))
        print(sizeof(b)) # empty list + size of the pointers to 1000 integer objects
```

8056

```
In [11]: # Numpy array
        c = np.arange(1000)
        print(c.size * c.itemsize) + numpy.array overhead
```

8000

Python List vs Numpy Array CPU Performance

```
In [1]: import numpy as np
```

```
# size of arrays and lists
```

```
size = 1000000
```

```
# lists
```

```
lsta = range(size)
```

```
lstb = range(size)
```

```
# numpy arrays
```

```
arra = np.arange(size)
```

```
arrb = np.arange(size)
```

```
# multiply the lists and store the result in another list
```

```
%timeit lstc = [(a * b) for a, b in zip(lsta, lstb)]
```

79.8 ms \pm 4.07 ms per loop (mean \pm std. dev. of 7 runs, 10 loops each)

```
In [2]: # multiply the Numpy arrays and store the result in another Numpy array
```

```
%timeit arrc = arra * arrb
```

1.89 ms \pm 470 μ s per loop (mean \pm std. dev. of 7 runs, 100 loops each)

reshape, resize

```
In [135]: a = np.floor(10 * np.random.random((3, 4)))
```

```
In [136]: print(a)
```

```
[[6. 3. 0. 7.]  
 [0. 8. 4. 9.]  
 [2. 0. 9. 2.]]
```

```
In [137]: a.ravel() # view
```

```
Out[137]: array([6., 3., 0., 7., 0., 8., 4., 9., 2., 0., 9., 2.])
```

```
In [144]: print(a.reshape(4, 3)) # view
```

```
[[6. 3. 0.]  
 [7. 0. 8.]  
 [4. 9. 2.]  
 [0. 9. 2.]]
```

```
In [145]: print(a.transpose()) # (a.T) view
```

```
[[6. 0. 2.]  
 [3. 8. 0.]  
 [0. 4. 9.]  
 [7. 9. 2.]]
```

```
In [148]: a.resize((3, 4)) # changes a
```

```
In [149]: print(a)
```

```
[[6. 3. 0. 7.]  
 [0. 8. 4. 9.]  
 [2. 0. 9. 2.]]
```

zeros, ones, full, empty, identity

```
In [10]: import numpy as np  
         from math import pi
```

```
In [9]: np.zeros(10, dtype=float)
```

```
Out[9]: array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

```
In [11]: np.ones((3, 5), dtype=np.float64)
```

```
Out[11]: array([[1., 1., 1., 1., 1.],  
               [1., 1., 1., 1., 1.],  
               [1., 1., 1., 1., 1.]])
```

```
In [7]: np.full((3, 5), pi)
```

```
Out[7]: array([[3.14159265, 3.14159265, 3.14159265, 3.14159265, 3.14159265],  
               [3.14159265, 3.14159265, 3.14159265, 3.14159265, 3.14159265],  
               [3.14159265, 3.14159265, 3.14159265, 3.14159265, 3.14159265]])
```

```
In [6]: np.empty(5)
```

```
Out[6]: array([1.39758760e-316, 0.00000000e+000, 7.38191927e+271, 2.25563599e-153,  
               1.41818073e+195])
```

```
In [7]: np.empty((2, 3))
```

```
Out[7]: array([[1.39824135e-316, 0.00000000e+000, 0.00000000e+000],  
               [0.00000000e+000, 0.00000000e+000, 0.00000000e+000]])
```

```
In [11]: np.identity(4)
```

```
Out[11]: array([[1., 0., 0., 0.],  
               [0., 1., 0., 0.],  
               [0., 0., 1., 0.],  
               [0., 0., 0., 1.]])
```

Random numbers

```
In [7]: np.random.random((3, 3))
```

```
Out[7]: array([[0.20469884, 0.69742749, 0.45218705],  
              [0.11663043, 0.29524859, 0.74615482],  
              [0.46118038, 0.31752885, 0.88003084]])
```

```
In [8]: np.random.normal(0, 1, (3, 3))
```

```
Out[8]: array([[-1.07596625, -1.04521417, 0.94400503],  
              [-0.11817752, -0.42738321, -0.23172611],  
              [-1.15899737, 1.38676666, -1.64432531]])
```

```
In [3]: np.random.randint(0, 100, (3, 3))
```

```
Out[3]: array([[ 7, 68, 23],  
              [ 3, 85, 53],  
              [ 0, 11, 75]])
```

```
In [14]: np.random.uniform(5,10,size = 4)
```

```
Out[14]: array([8.5184692 , 8.72702707, 6.82632451, 5.36786343])
```

```
In [15]: np.random.uniform(size = (2,3))
```

```
Out[15]: array([[0.83883722, 0.94502378, 0.88101416],  
              [0.00827861, 0.58941644, 0.52890729]])
```

linspace, logspace, geomspace

```
In [39]: ## numpy.linspace(start, stop, num=50, endpoint=True, retstep=False, dtype=None, axis=0)  
np.linspace(0, 1, num=20) # Generate evenly spaced numbers
```

```
Out[39]: array([0.          , 0.05263, 0.10526, 0.15789, 0.21053, 0.26316, 0.31579,  
               0.36842, 0.42105, 0.47368, 0.52632, 0.57895, 0.63158, 0.68421,  
               0.73684, 0.78947, 0.84211, 0.89474, 0.94737, 1.          ])
```

```
In [40]: ## numpy.logspace(start, stop, num=50, endpoint=True, base=10.0, dtype=None, axis=0)  
np.logspace(2.0, 3.0, num=4)
```

```
Out[40]: array([ 100.          , 215.44347, 464.15888, 1000.          ])
```

```
In [42]: y = np.linspace(2.0, 3.0, num=4)  
(10**y).astype(np.float64)
```

```
Out[42]: array([ 100.          , 215.44347, 464.15888, 1000.          ])
```

```
In [43]: ## numpy.geomspace(start, stop, num=50, endpoint=True, dtype=None, axis=0)  
np.geomspace(1, 1000, num=4) # Geometric progression
```

```
Out[43]: array([ 1., 10., 100., 1000.])
```

```
In [44]: np.geomspace(1, 256, num=9)
```

```
Out[44]: array([ 1., 2., 4., 8., 16., 32., 64., 128., 256.])
```

```
In [45]: np.geomspace(1000, 1, num=4)
```

```
Out[45]: array([1000., 100., 10., 1.])
```

numpy.eye

```
In [17]: print(np.eye(4,dtype=np.int8))
```

```
[[1 0 0 0]
 [0 1 0 0]
 [0 0 1 0]
 [0 0 0 1]]
```

```
In [18]: print(np.eye(4,k=1,dtype=np.int8))
```

```
[[0 1 0 0]
 [0 0 1 0]
 [0 0 0 1]
 [0 0 0 0]]
```

```
In [19]: print(np.eye(4,k=-1,dtype=np.int8))
```

```
[[0 0 0 0]
 [1 0 0 0]
 [0 1 0 0]
 [0 0 1 0]]
```

save/load data

```
In [1]: import numpy as np
```

```
In [2]: a = np.arange(15).reshape(3,5)  
np.save("matrix_data.npy", a)
```

```
In [3]: b = np.load('matrix_data.npy')  
print(b)
```

```
[[ 0  1  2  3  4]  
 [ 5  6  7  8  9]  
 [10 11 12 13 14]]
```

```
In [4]: np.savetxt('matrix_data.txt', a)
```

```
In [5]: c = np.loadtxt('matrix_data.txt')  
print(c)
```

```
[[ 0.  1.  2.  3.  4.]  
 [ 5.  6.  7.  8.  9.]  
 [10. 11. 12. 13. 14.]]
```

Arrays operations

Vectorization and Broadcasting

- Vectorization
 - Array element-by-element computation, i.e explicit loops, indices pushed to “behind the scenes” in optimized, pre-compiled C code
 - Python code
 - clean, concise, easy to follow
 - resembles standard mathematical notation
- Broadcasting
 - describes implicit element-by-element behavior of operations on arrays of different shapes

The diagram shows the following sequence of operations:

1
2

 * **1.6** =

1
2

 *

1.6
1.6

 =

1.6
3.2

https://numpy.org/doc/stable/user/absolute_beginners.html

Array Operations

```
In [81]: a = np.random.random((3, 2))  
print(a)
```

```
[[0.07644 0.15268]  
 [0.40377 0.33181]  
 [0.54763 0.92259]]
```

```
In [65]: print(a.sum(), a.sum(axis=0), a.sum(axis=1))
```

```
2.240318562104763 [1.4351  0.80522] [0.84843 0.87412 0.51778]
```

```
In [82]: print(a.min(), a.max())
```

```
0.07644475727068445 0.9225939469162023
```

```
In [67]: print(a.min(axis=0), a.max(axis=0), a.min(axis=1), a.max(axis=1))
```

```
[0.39084 0.12694] [0.5808  0.38497] [0.38497 0.29331 0.12694] [0.46346 0.5808  0.39084]
```

```
In [68]: print(a.cumsum(axis=0))
```

```
[[0.46346 0.38497]  
 [1.04426 0.67829]  
 [1.4351  0.80522]]
```

```
In [69]: print(a.cumsum(axis=1))
```

```
[[0.46346 0.84843]  
 [0.5808  0.87412]  
 [0.39084 0.51778]]
```

sort, concatenate

```
In [13]: import numpy as np
```

```
In [14]: a = np.array([2, 1, 5, 3, 7, 4, 6, 8])
```

```
In [15]: print(np.sort(a))
```

```
[1 2 3 4 5 6 7 8]
```

```
In [16]: print(np.argsort(a))
```

```
[1 0 3 5 2 6 4 7]
```

```
In [17]: a.sort() # in-place sort  
print(a)
```

```
[1 2 3 4 5 6 7 8]
```

```
In [19]: a = np.array([1, 2, 3, 4])  
b = np.array([5, 6, 7, 8])  
print(np.concatenate((a, b)))
```

```
[1 2 3 4 5 6 7 8]
```

```
In [20]: x = np.array([[1, 2], [3, 4]])  
y = np.array([[5, 6]])  
np.concatenate((x, y), axis=0)
```

```
Out[20]: array([[1, 2],  
               [3, 4],  
               [5, 6]])
```

reverse

```
In [1]: import numpy as np
```

```
In [2]: a = np.arange(15)  
print(a)
```

```
[ 0  1  2  3  4  5  6  7  8  9 10 11 12 13 14]
```

```
In [3]: print(np.flip(a)) # view, returns a copy
```

```
[14 13 12 11 10  9  8  7  6  5  4  3  2  1  0]
```

```
In [4]: b = np.arange(15).reshape(3,5)
```

```
In [5]: print(b)
```

```
[[ 0  1  2  3  4]  
 [ 5  6  7  8  9]  
 [10 11 12 13 14]]
```

```
In [6]: print(np.flip(b))
```

```
[[14 13 12 11 10]  
 [ 9  8  7  6  5]  
 [ 4  3  2  1  0]]
```

```
In [7]: print(np.flip(b, axis=0))
```

```
[[10 11 12 13 14]  
 [ 5  6  7  8  9]  
 [ 0  1  2  3  4]]
```

```
In [8]: print(np.flip(b, axis=1))
```

```
[[ 4  3  2  1  0]  
 [ 9  8  7  6  5]  
 [14 13 12 11 10]]
```

```
In [11]: b[1] = np.flip(b[1])
```

```
In [12]: print(b)
```

```
[[ 0  1  2  3  4]  
 [ 9  8  7  6  5]  
 [10 11 12 13 14]]
```

```
In [13]: b[:,1] = np.flip(b[:,1])
```

```
In [14]: print(b)
```

```
[[ 0 11  2  3  4]  
 [ 9  8  7  6  5]  
 [10  1 12 13 14]]
```

Finding Unique Elements

```
In [1]: import numpy as np
a = np.array([11, 11, 12, 13, 14, 15, 16, 17, 12, 13, 11, 14, 18, 19, 20])
```

```
In [3]: print(np.unique(a))

[11 12 13 14 15 16 17 18 19 20]
```

```
In [5]: values, indices = np.unique(a, return_index=True)
print(indices)

[ 0  2  3  4  5  6  7 12 13 14]
```

```
In [8]: values, count = np.unique(a, return_counts=True)
print(values, '\n', count)

[11 12 13 14 15 16 17 18 19 20]
[3 2 2 2 1 1 1 1 1 1]
```

```
In [9]: b = np.array([[1, 2, 3, 4], [5, 6, 7, 8], [9, 10, 11, 12], [1, 2, 3, 4]])
print(np.unique(b, axis=0))

[[ 1  2  3  4]
 [ 5  6  7  8]
 [ 9 10 11 12]]
```

Comparing Arrays

```
In [26]: a = np.array([5e5, 1e-7, 4.000004e6])  
b = np.array([5.00001e5, 1e-7, 4e6])  
print(a,b)
```

```
[5.000000e+05 1.000000e-07 4.000004e+06] [5.00001e+05 1.00000e-07 4.00000e+06]
```

```
In [27]: rtol = 1e-05  
atol = 1e-08  
np.abs(a - b) <= (atol + rtol * np.abs(b))
```

```
Out[27]: array([ True,  True,  True])
```

```
In [30]: ## numpy.isclose(a, b, rtol=1e-05, atol=1e-08, equal_nan=False)  
np.isclose(a,b,rtol=rtol,atol=atol)
```

```
Out[30]: array([ True,  True,  True])
```

```
In [31]: ## numpy.allclose(a, b, rtol=1e-05, atol=1e-08, equal_nan=False)  
np.allclose(a,b,rtol=rtol,atol=atol)
```

```
Out[31]: True
```

```
In [35]: np.equal(np.array([1,2,3,4]),np.array([0,2,3,4]))
```

```
Out[35]: array([False,  True,  True,  True])
```

Statistics

```
In [1]: a = np.random.normal(0,1,1000)
```

```
In [13]: print(np.mean(a))
```

```
-0.034478607630422944
```

```
In [14]: print(np.median(a))
```

```
-0.013434852528562493
```

```
In [15]: print(np.std(a))
```

```
1.0055207559124688
```

```
In [17]: print(np.var(a))
```

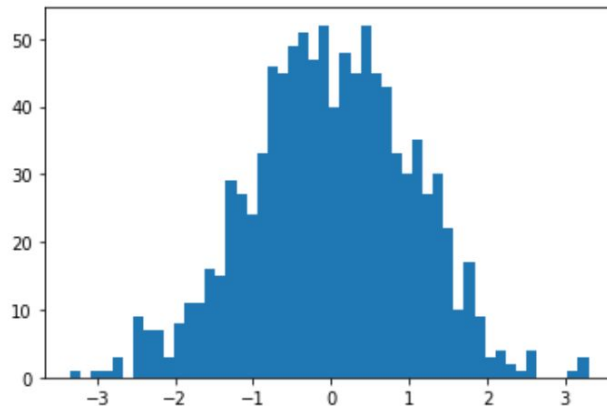
```
1.0110719905707826
```

```
In [18]: print(np.average(a))
```

```
-0.034478607630422944
```

```
In [19]: print(np.ptp(a))
```

```
6.70344163147174
```



Histogramming

<code><u>histogram</u>(a[, bins, range, normed, weights, ...])</code>	Compute the histogram of a dataset.
<code><u>histogram2d</u>(x, y[, bins, range, normed, ...])</code>	Compute the bi-dimensional histogram of two data samples.
<code><u>histogramdd</u>(sample[, bins, range, normed, ...])</code>	Compute the multidimensional histogram of some data.
<code><u>bincount</u>(x, /[, weights, minlength])</code>	Count number of occurrences of each value in array of non-negative ints.
<code><u>histogram_bin_edges</u>(a[, bins, range, weights])</code>	Function to calculate only the edges of the bins used by the histogram function.
<code><u>digitize</u>(x, bins[, right])</code>	Return the indices of the bins to which each value in input array belongs.

<https://numpy.org/doc/stable/reference/routines.statistics.html>

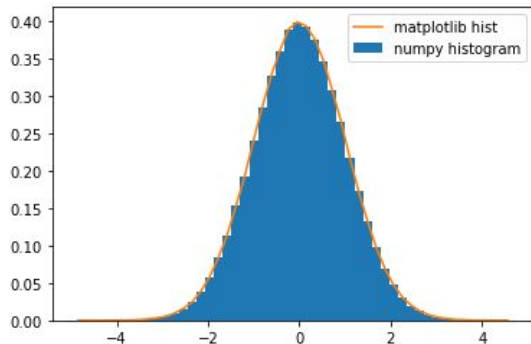
numpy.histogram

- `numpy.histogram(array, options)` generates data and returns
 - an array with histogram bin content and
 - an array containing the bin edges

```
In [14]: import numpy as np
import matplotlib.pyplot as plt
ndist = np.random.normal(0, 1.0, 1000000)
plt.hist(ndist, bins=50, density=True)
# n: bin content, bins: bin edges
(n, bins) = np.histogram(ndist, bins=50, density=True)
print(type(n))
print(type(bins))
plt.plot(.5 * (bins[1:] + bins[:-1]), n)
plt.legend(["matplotlib hist", "numpy histogram"], loc="upper right")
```

```
<class 'numpy.ndarray'>
<class 'numpy.ndarray'>
```

```
Out[14]: <matplotlib.legend.Legend at 0x7ff56c6b0910>
```



Numpy Universal Functions (ufuncs)

- Comparison: `<`, `<=`, `==`, `!=`, `>=` `>`
- Arithmetic: `+`, `-`, `*`, `/`, `reciprocal`, `square`
- Math
 - Exponential: `exp`, `log`, `log10`, `power`, `sqrt`
 - Trigonometric: `sin`, `cos`, `tan`, `arcsin`, `arccos`, `arctan`
 - Hyperbolic: `sinh`, `cosh`, `tanh`, `arcsinh`, `arccosh`, `arctanh`
- bitwise operations: `&`, `|`, `~`, `^`, `left_shift`, `right_shift`
- logical operations: `and`, `not`, `or`, `logical_xor`
- predicates: `isfinite`, `isinf`, `isnan`, `signbit`
- others: `abs`, `ceil`, `floor`, `mod`, `modf`, `round`, `sign`

numpy.newaxis

```
In [21]: a = np.array([1, 2, 3, 4, 5, 6])  
a.shape
```

```
Out[21]: (6,)
```

```
In [30]: print(a[np.newaxis, :])  
[[1 2 3 4 5 6]]
```

```
In [31]: print(a[np.newaxis, :].shape)  
(1, 6)
```

```
In [28]: print(a[:, np.newaxis])  
[[1]  
 [2]  
 [3]  
 [4]  
 [5]  
 [6]]
```

```
In [29]: print(a[:, np.newaxis].shape)  
(6, 1)
```

numpy.expand_dims

```
In [1]: import numpy as np
```

```
In [10]: a = np.arange([0,1,2,3,4,5,6,7,8,9], dtype=np.float64)
```

```
In [11]: print(a)
```

```
[0. 1. 2. 3. 4. 5. 6. 7. 8. 9.]
```

```
In [12]: print(np.expand_dims(a,axis=0)) # view
```

```
[[0. 1. 2. 3. 4. 5. 6. 7. 8. 9.]]
```

```
In [13]: print(np.expand_dims(a,axis=1)) # view
```

```
[[0.]  
 [1.]  
 [2.]  
 [3.]  
 [4.]  
 [5.]  
 [6.]  
 [7.]  
 [8.]  
 [9.]]
```