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CHARMONIUM-NUCLEON INTERACTION FROM LATTICE QCD

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Why $c\bar{c}$-nucleon interaction ?

This is Ideal system to study the effect of the multi-gluon exchange.

✦Flavor singlet interaction

1) No quark interchange

2) Multiple gluon exchange plays essential role

→ Interaction is described by color van der Waals interaction, which is weakly attractive in principle. (e.g. $-1/r^7$ behavior given by color dipoles)

H. Fujii and D. Kharzeev PRD60, 114039 (1999)

✦Scattering length $a^{J/\psi\text{-}N}$

Note that "a>0" means attractive

	Method	$a^{J/\psi\text{-}N}$
Brodsky et al. PLB412 (1997) 125	pQCD	0.24
Hayashigaki, PTP. 101 (1999) 923	QCD sum rules	0.12
Yokokawa et al., PRD74 (2006) 034504	lattice	0.71(48)
Liuming et al., arXiv:0810.5412.	lattice	0.24(35)

Why cc^{bar}-nucleon interaction ?

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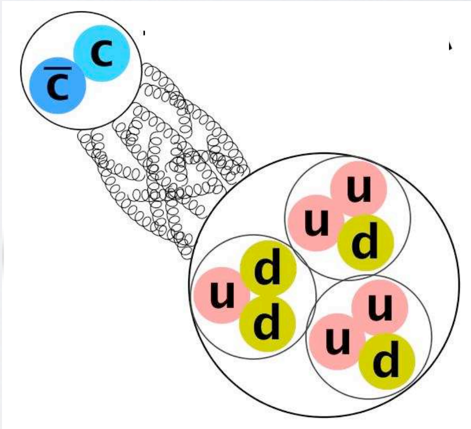
H. Fujii and D. Kharzeev PRD60, 114039 (1999)

✦If such an attraction is strong enough, charmonium may be bound to the nucleon or to the large nuclei.



Why $c\bar{c}$-nucleon interaction ?

The $c\bar{c}$–nucleon couples little to other hadronic channels.

$m_{\eta c}$=2.98GeV $m_{J/\psi}$=3.097GeV m_D≈1.87GeV $m_{\Lambda c}$=2.29GeV

1)**The J/ψ(η_c) mass is below the $D\bar{D}$ mass by more than 600(700) MeV.**

$J/\psi(c\bar{c}) \to D^+ + D^-$ ×forbidden $\phi(s\bar{s}) \to K^+ + K^-$ ○allowed

3.097GeV 3.74GeV 1.019GeV 0.987GeV

$\Upsilon(b\bar{b}) \to B^+ + B^-$ ×forbidden

9.460GeV 10.56GeV →Width: $\Gamma(J/\psi)$=0.093MeV, $\Gamma(\Upsilon)$=0.054MeV

2)**The J/ψ-N(η_c) mass is below the Λ_c-$\bar{D}$ mass by above 120(220)MeV.**

Mass(J/ψ–N) = 4.0352 ± 0.0001 GeV Mass(Λ_c–$\bar{D}$) = 4.1561 ± 0.0002 GeV

All other OZI-allowed hadronic states with the same quantum number of $c\bar{c}$ + N have higher masses.

Model study of nuclear-bound charmonium

✦ A semi-quantitative study of the charmonium-nucleus bound state was given by Brodsky et al. Brodsky, Schmidt, de Teramond, PRL 64 (1990) 1011

1. A simple Yukawa-type potential is assumed for the cc^{bar} -N system.

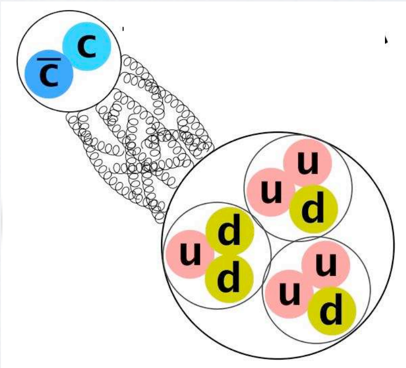
$$V(r) = -\gamma \frac{\exp(-\alpha r)}{r}$$

γ=0.4-0.6 α=600 MeV
fixed by Pomeron exchange model

2. The cc^{bar}-Nucleus potential

$$V_{c\bar{c}\text{-}A}(r) \sim A \times V_{c\bar{c}\text{-}N}(r)$$

They predicted a formation of nuclear-bound charmonium when A ≧ 3.



Model study of nuclear-bound charmonium

- Several calculation about charmonium-nuclear bound state are carried out. All results indicate that formation of the bound state is started from A=3

	Method	cc^{bar}–^{3}He Binding energy [MeV]
Brodsky et al PRL 64 (1990) 1011	Variational calculation $V_{c\bar{c}\text{-}A}(r) = -\gamma \frac{\exp(-\alpha r)}{r}$	−19
D. A. Wasson PRL 67 (1991) 2237	Folding Model $V_{c\bar{c}\text{-}A}(r) = \int d^3\vec{r}'\rho(\vec{r}')V_{c\bar{c}\text{-}N}(\vec{r}-\vec{r}')$	−0.8
V. B Belyaev et al NPA 780 (2006) 100	4–body calculation	−12.6

Note: these calculations assumed a simple Yukawa form for the $c\bar{c}$-N interaction, which parameters are barely fixed by a phenomenological Pomeron exchange model

- Precise information of the cc^{bar}-N potential $V_{c\bar{c}\text{-}N}(r)$ is indispensable for exploring nuclear-bound charmonium state.

Our strategy

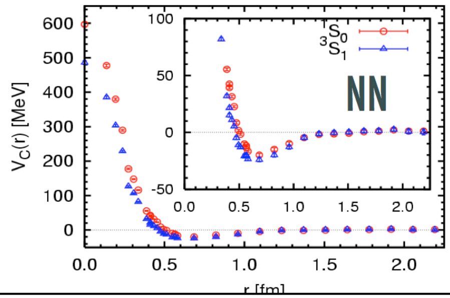
Lattice calculation

Fermilab approach for heavy quark: RHQ action[1,2,3]

Aoki-Hatsuda-Ishii approach[4,5]
for hadron-hadron potential

Lüscher formula with
twisted boundary conditions[6]

$c\bar{c}$-N potential



Scattering length a_0
Effective range r_0

Precise information of the $c\bar{c}$-N interaction

Quenched QCD calculations

Nf2+1 full QCD calculations

Theoretical input

Exact few body calculation[7], EFT, ...

[1] A. X. El-Khadra et al, PRD55 (1997) 3933
[2] S. Aoki et al., PTP109 (2003) 383
[3] N. H. Christ et al., PRD76 (2007) 074505
[4] N. Ishii et al., PRL90, 022001 (2007)
[5] S. Aoki et al., PTP123 (2010) 89
[6] P.F. Bedaque, PLB593 (04) 82
[7] E. Hiyama et al., PPNP51 (2003) 223

CC^{bar}-nucleon potential

Relativistic heavy quark action

✦Heavy quark mass introduces discretization errors of **O((ma)^n)**

✓ At charm quark mass, it becomes severe: **m_c ~ 1.5 GeV** and **1/a ~ 2 GeV**, then **$m_c a$ ~O(1)**.

✦The Fermilab group proposed relativistic heavy quark action (RHQ) approach where all O((ma)^n) errors are removed by the appropriate choice of **m_0, ξ, r_s, C_B, C_E** .

A. X. El-Khadra et al, PRD55 (1997) 3933
S. Aokki et al., PTP109 (2003) 383
N. H. Christ et al., PRD76 (2007) 074505

$$S_{\rm lat} = \sum_{n,n'} \bar\psi_{n'}(\gamma^0 D^0 + \zeta \vec\gamma \cdot \vec D + m_0 a - \frac{r_t}{2} a (D^0)^2 - \frac{r_s}{2} a (\vec D)^2 + \sum_{i,j} \frac{i}{4} c_B a \sigma_{ij} F_{ij} + \sum_i \frac{i}{2} c_E a \sigma_{0i} F_{0i})_{n',n} \psi_n$$

We take the Tsukuba procedure in our study.

Y. Kayaba et al. [CP-PACS Collaboration], JHEP 0702, 019 (2007).

Hadron-hadron potential

✦ Equal-time Bethe-Salpeter (BS) amplitude

M. Lüscher, Nucl. Phys. B 354, 531 (1991)

$$\begin{aligned} F_{\eta_c\text{-}N}(\vec{x},\vec{y},t;t_0) &= \langle 0|N(\vec{x},t)\eta_c(\vec{y},t)J_{\eta_c\text{-}N}|0\rangle \\ &= \sum_n A_n \langle 0|N(\vec{x},t)\eta_c(\vec{y},t)|n\rangle e^{-W_n(t-t_0)t} \\ &\rightarrow A_0 \underbrace{\phi_0(\vec{r})}_{\text{BS wave function}} e^{-W_0(t-t_0)t} \qquad t \gg t_0, \quad \vec{r} = \vec{x}-\vec{y} \end{aligned}$$

Interpolating operators

$$\begin{aligned} N(\vec{x}) &= \epsilon_{abc}(u_a^t C\gamma_5 d_b)d_c(\vec{x}) \\ \eta_c(\vec{y}) &= \bar{c}\gamma_5 c_a(\vec{y}) \end{aligned}$$

✦ Schrödinger type equation for general cases.

$$E\phi(\vec{r}) + \frac{1}{2m_{red}}\nabla^2\phi(\vec{r}) = \int d^3r' U(\vec{r},\vec{r}')\phi(\vec{r}')$$

For cc^{bar}-N scattering at low energy

$$U(\vec{r},\vec{r}') = V_{\eta_c-N}(\vec{r})\delta(\vec{r}-\vec{r}')$$

Reduced mass $m_{red} = m_{\eta c} m_N / (m_{\eta c} + m_N)$

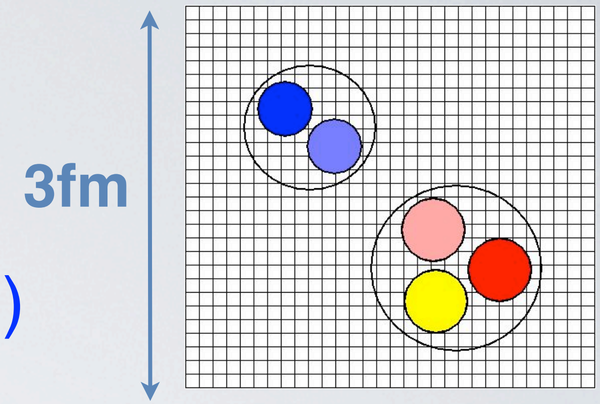
Velocity expansion

$$\begin{aligned} U(\vec{r},\vec{r}') &= V(\vec{r},\vec{v})\delta(\vec{r}-\vec{r}') \\ V(\vec{r},\vec{v}) &= V_0(r) + \frac{1}{2}\{V_{v^2}(r),\vec{v}^2\} + V_{l^2}(r)\vec{L}^2 + \cdots \end{aligned}$$

Quenched Lattice

Lattice set up

✦ Quenched QCD simulation

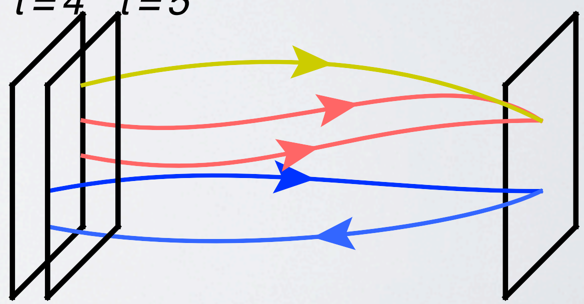


✦Lattice size:
$L^3 \times T$ = **32^3 x 48**,16^3 x 48 (La ≈ **3.0**,1.5 fm)

✦plaquette action (gauge) β=6.0 (a=0.093 fm or a^{-1}=2.1GeV)
+ non-perturbative O(a) improvement action (up & down)
+ **RHQ** action with one-loop PT coefficients (charm)

Y. Kayaba et al. [CP-PACS Collaboration], JHEP 0702, 019 (2007).

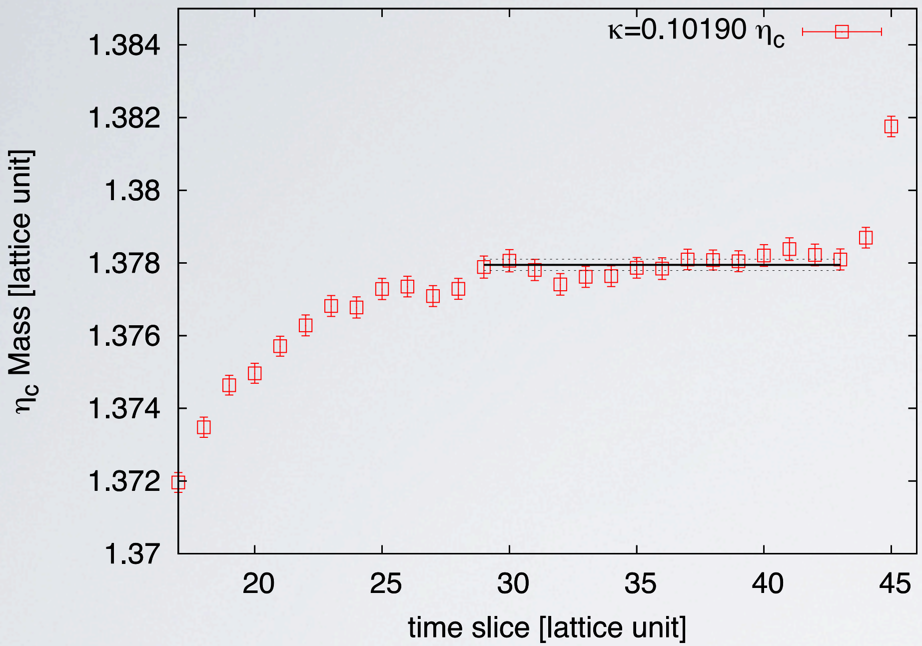
✦Statistics : **602 configs**

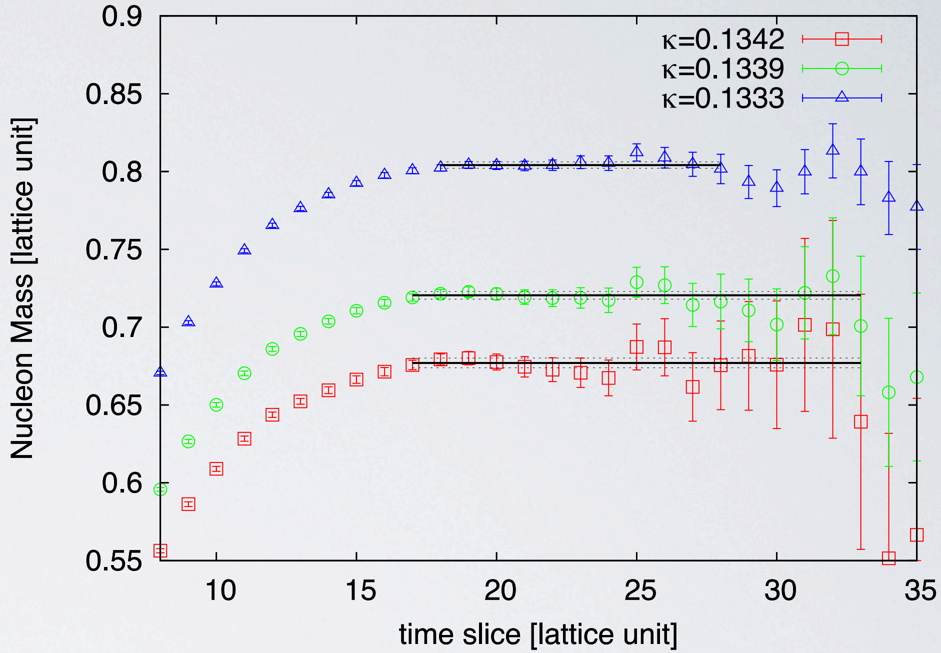


✦ Quark mass
- charm κ_Q = 0.10190 $m_{\eta c}$= 2.92 GeV
- Light

κ	0.1342	0.1339	0.1333
m_π [GeV]	0.64	0.73	0.87
m_N [GeV]	1.43	1.52	1.70

masses of η_c and nucleon





η_c

$m_{\eta c}$ = 2.91761(16) GeV

Nucleon

m_N = 1.4339(66), 1.5258(53), 1.7030(44) GeV

κ	am_{η_c}	$[t_{\min}, t_{\max}]$		
0.10190	1.37753(16)	[25,39]		
κ	am_π	$[t_{\min}, t_{\max}]$	am_N	$[t_{\min}, t_{\max}]$
0.1342	0.3014(3)	[15,34]	0.6770(31)	[17,33]
0.1339	0.3414(3)	[15.34]	0.7204(25)	[17,33]
0.1333	0.4115(2)	[15,34]	0.8041(21)	[18:28]

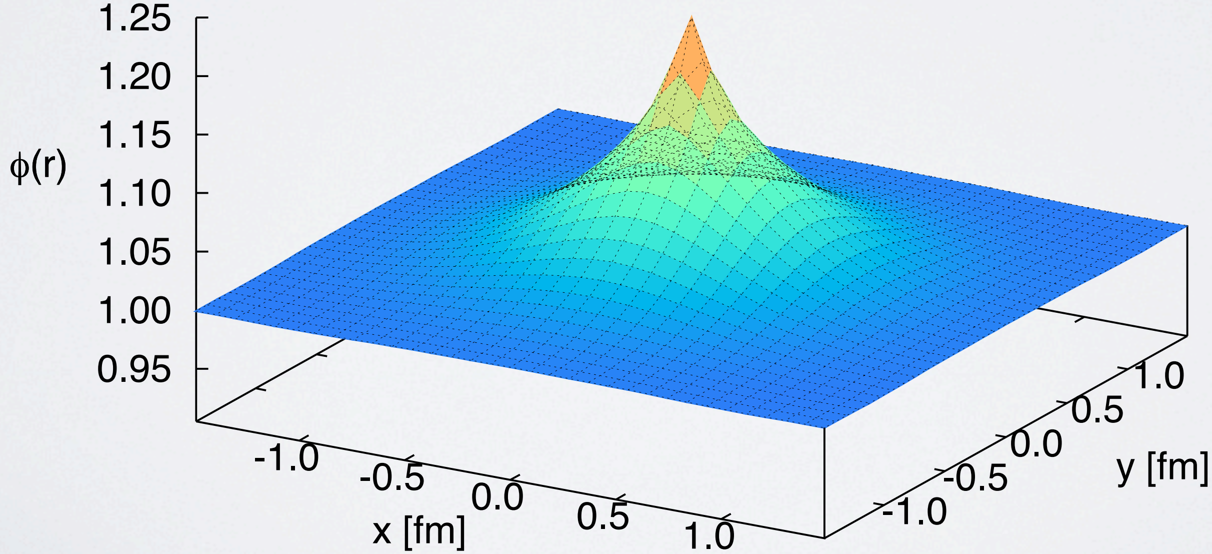
η_c-N wave function

★"S-wave" BS wave fucntion can be projected out as

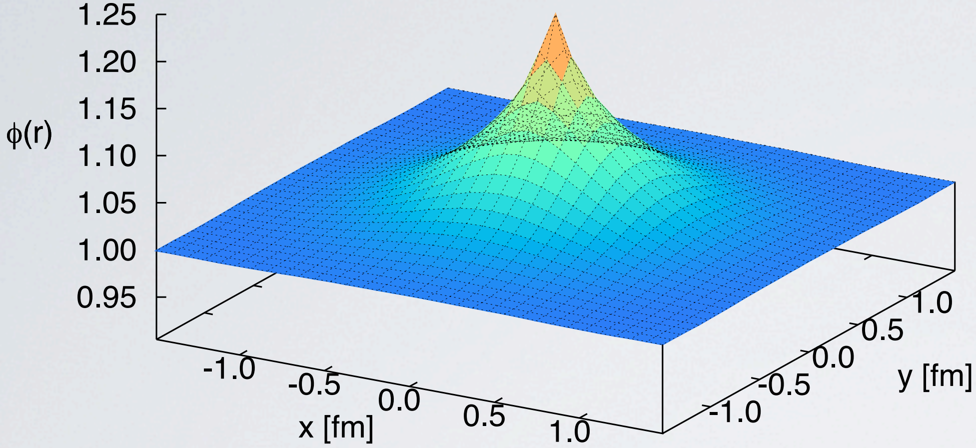
$$\phi(\vec{r}) = \frac{1}{24}\sum_{R\in O}\frac{1}{L^3}\sum_{\vec{x}}\langle 0|N(R[\vec{r}]+\vec{x})\eta_c(\vec{x})|N\eta_c\rangle$$

R represents an element of cubic group. The summation over R and x projects out the A_1^+ sector of cubic group and zero total momentum.

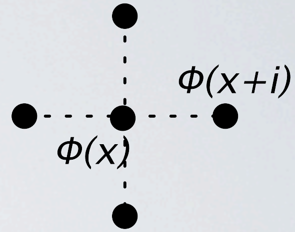
★The "S-wave" η_c-N wave function.



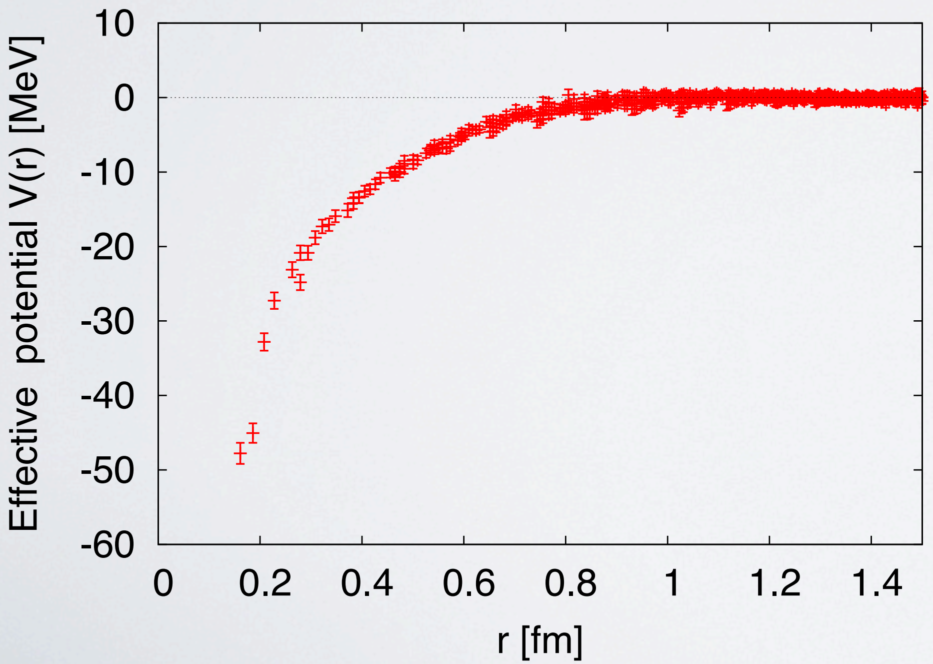
η_c-N potential



- The reduced mass is estimated from 2pt correlation functions
- ∇^2 denotes the discrete Laplacian

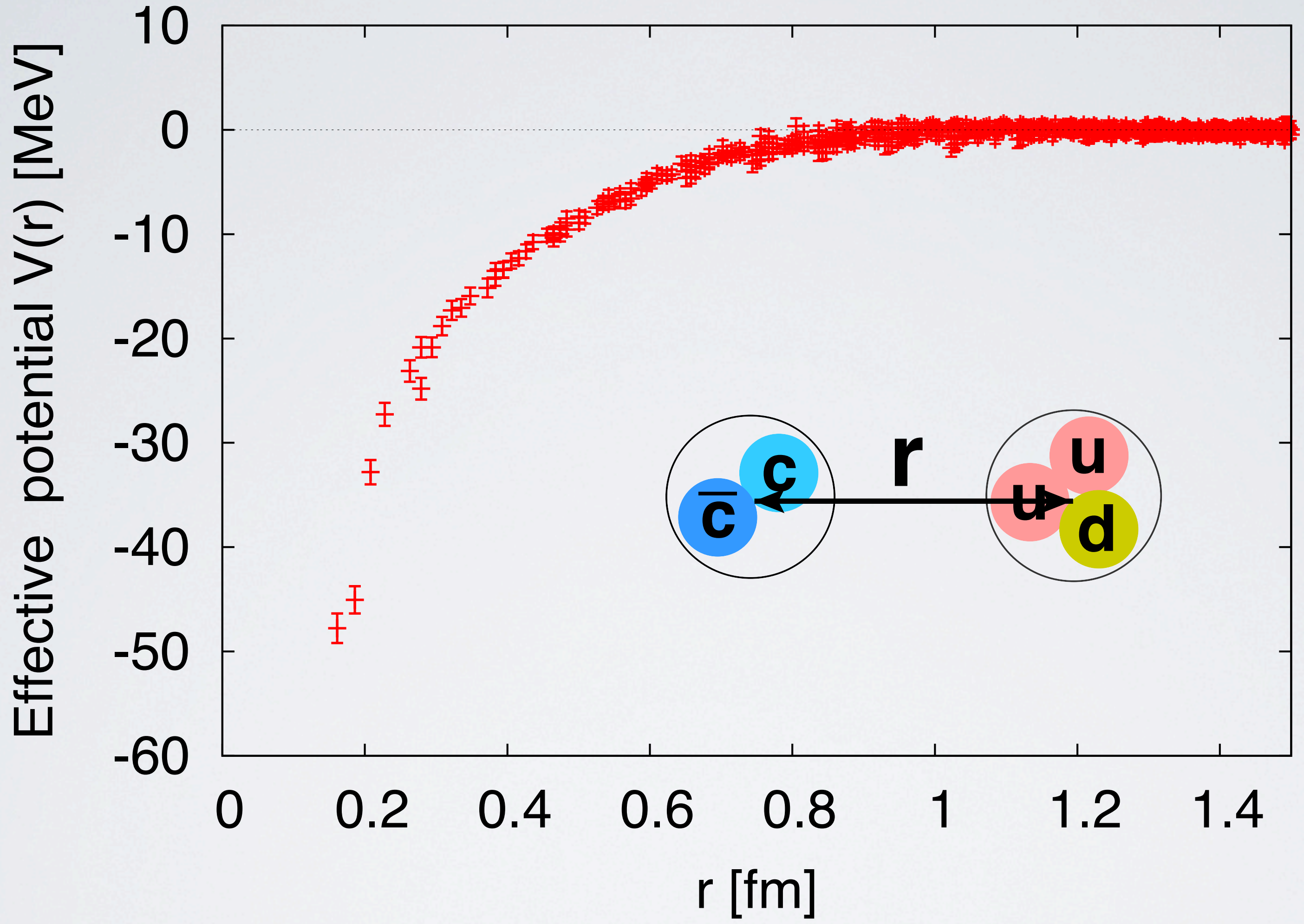


$$\frac{1}{2m_{\mathrm{red}}}\frac{\nabla^2\phi(r)}{\phi(r)} = V(r) - E$$



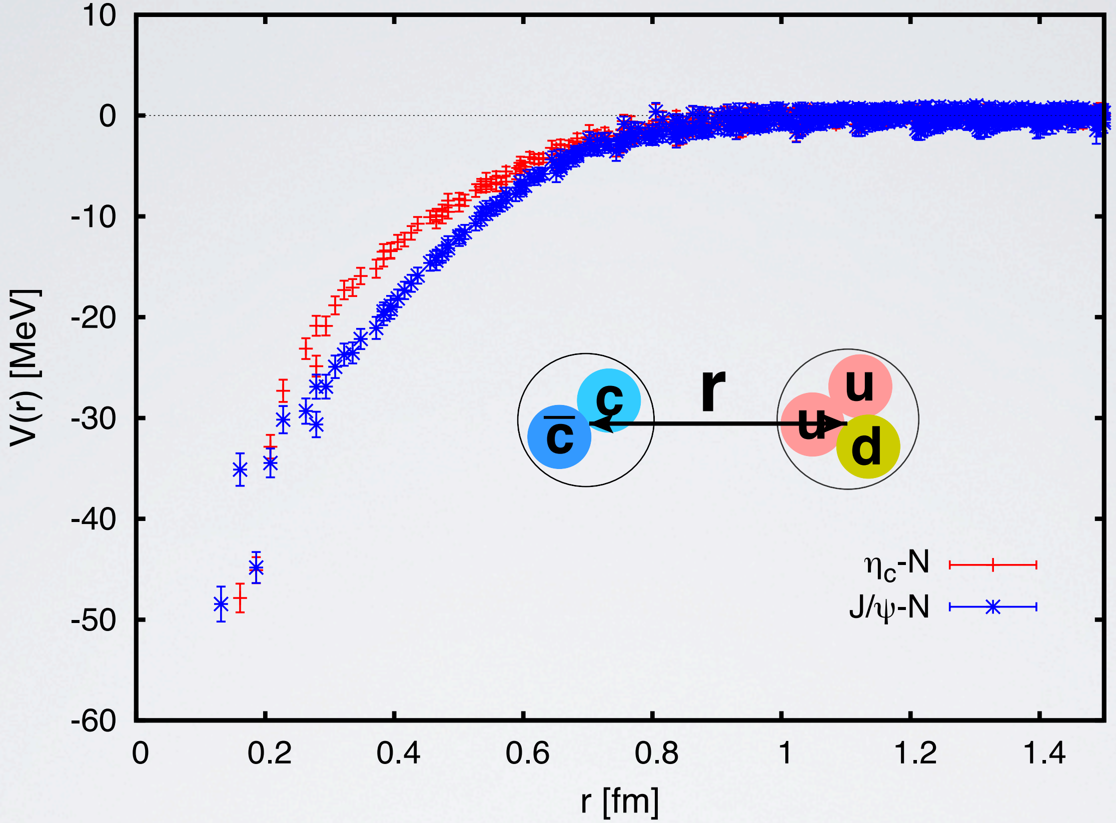
- V(r) is adjusted as V(r)=0 for r>1fm
  → Energy shift E

η_c-N potential



- The η_c-N potential exhibits entire attraction without any repulsion.
- The interaction is exponentially screened in long distance region.

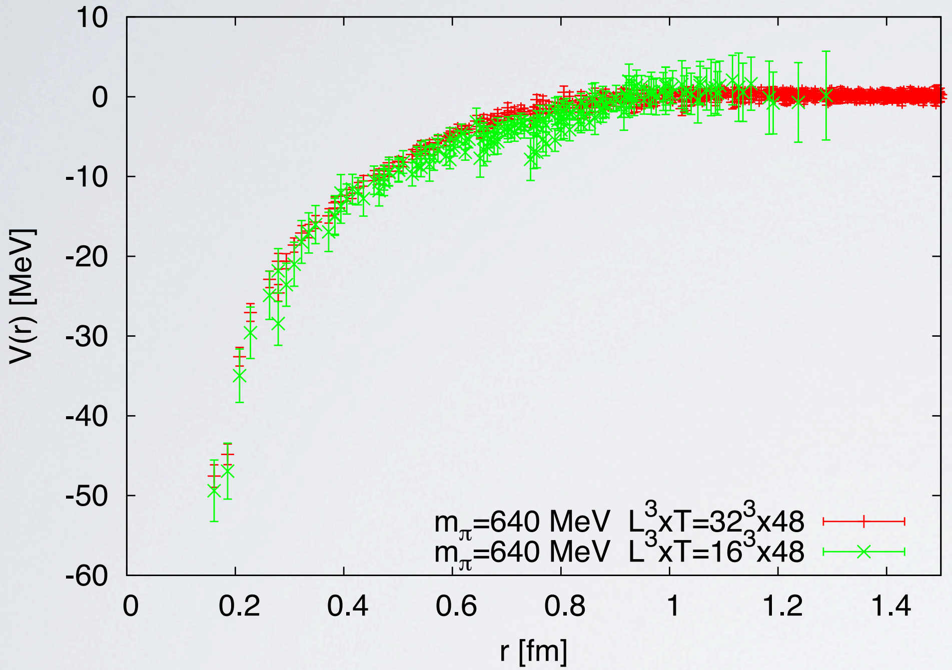
J/ψ-N potential

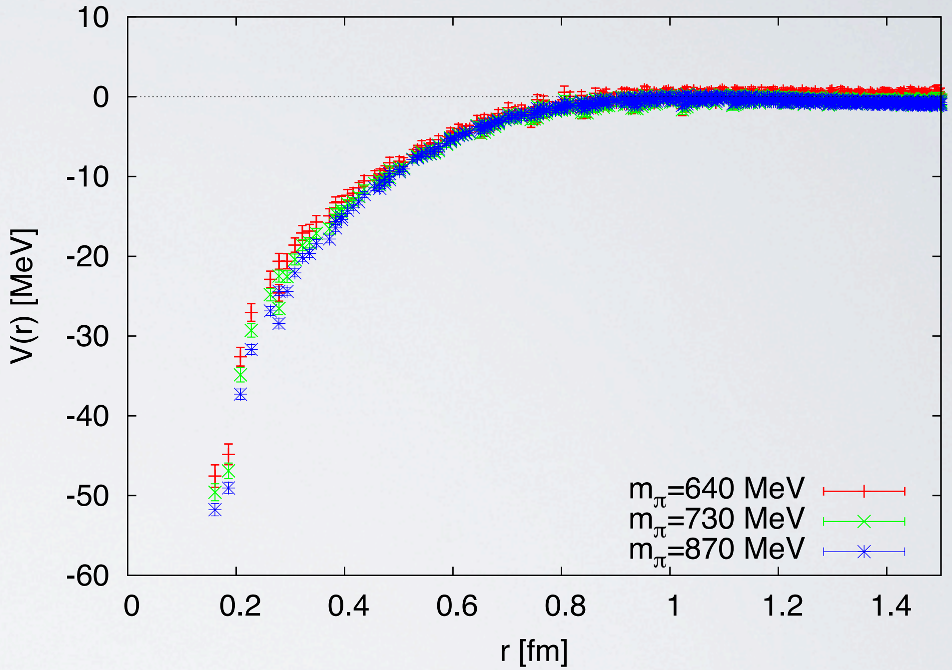


- The attraction of J/ψ-N potential is slightly stronger than that of η_c-N.
  →Ordinary van der Waals interaction is sensitive to the size of charge distribution.

Spin average $$G_{\mathrm{ave}}^{J/\psi\text{-}N} = \frac{1}{3}G^{1/2} + \frac{2}{3}G^{3/2}$$

volume & quark mass dependence





- No volume dependence
- No quark mass dependence

Nf2+1 Lattice

New technique for Aoki-Hatsuda-Ishii approach

T. Inoue et al. (HAL QCD Collaboration), arxiv:1012.5928

✦ Saturation time we can take may possibly be too short to obtain the BS wave function for grand state of hadron-hadron system.

low energy excitation $E_1 - E_0 \sim \frac{1}{2m_{\rm red}}\left(\frac{2\pi}{L}\right)^2 \sim 90\mathrm{MeV}$

Time dependent BS wave function

$$\phi_n(\vec{r},t) = \phi_n(\vec{r})e^{-E_n t} = \phi_n(\vec{r})e^{-(W_n - 2M)t}$$

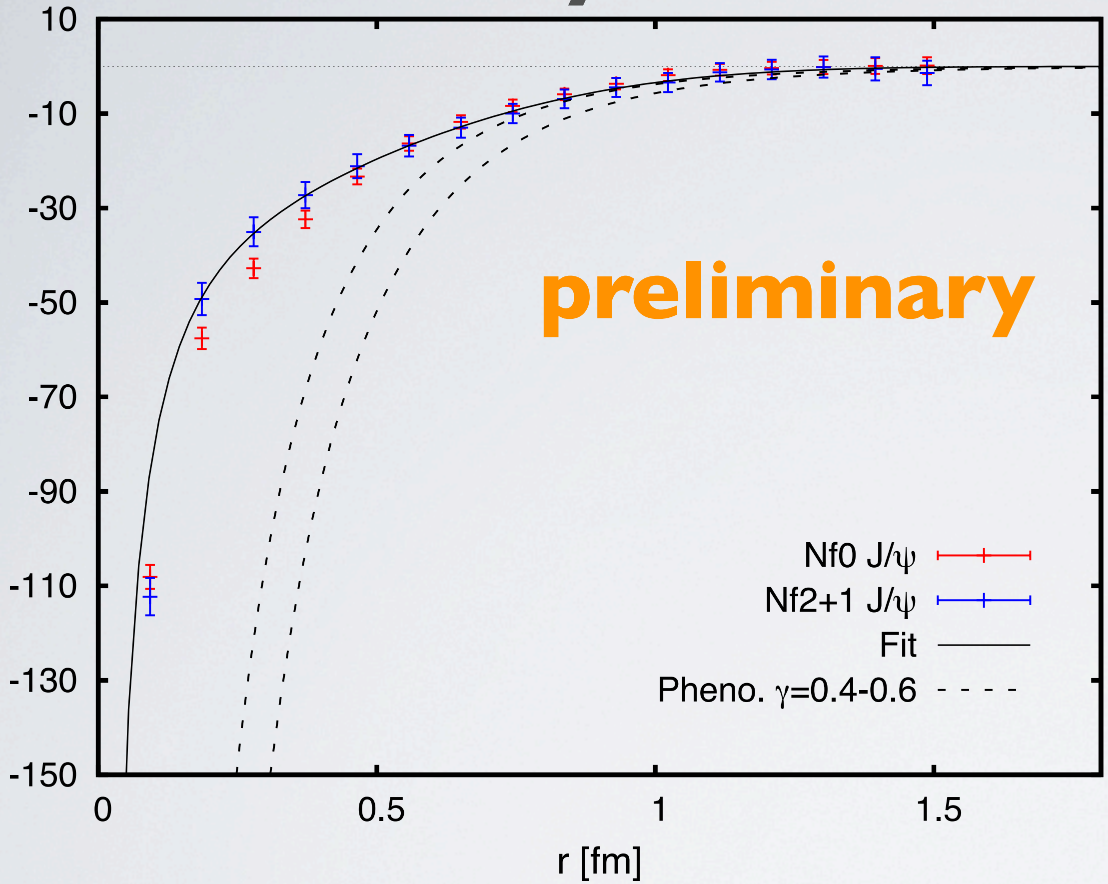
The two-body potential in low energy QCD dictates all the elastic scattering states simultaneously through the Schrodinger equation.

$$-\frac{\nabla^2}{2m_{\rm red}}\phi_n(\vec{r},t) + \int d^3r' U(\vec{r},\vec{r}')\phi_n(\vec{r}',t) = -\frac{\partial}{\partial t}\phi_n(\vec{r},t)$$

$$\phi(\vec{r},t) = \sum_n A_n \phi_n(\vec{r},t) = G_{J/\psi\text{-}N}(\vec{r},t)/e^{-2Mt}$$

Central potential $V_C(r) = \frac{\left(\frac{\nabla^2}{2m_{\rm red}} - \frac{\partial}{\partial t}\phi(\vec{r},t)\right)}{\phi(\vec{r},t)}$

$c\bar{c}$-N potential using PACS-CS 2+1 flavor dynamical configuration



S.Aoki et al., PRD 79, 034503 (2009)

- **Quark mass;**
  κ_{ud}=0.13754 ,κ_s=013640
  (m_π=0.41GeV, m_N=1.2GeV)
  κ_c =0.106787 ($m_{\eta c}$ = 3GeV)
- **Lattice size;**
  $L^3 \times T = 32^3 \times 64$ (La ≈ 3.0 fm)
- **# of statistics** ; 250

- There is neither qualitative nor quantitative differences between the quenched QCD result and the 2+1 flavor QCD result.
- The $c c^{bar}$-N potential may become more attractive in the vicinity of the physical point, where nucleon become more larger.

Physical quantity calculated by potential

preliminary

✦By using the potentials which fit the lattice data, we can calculate observables such as the scattering length and effective range.

Scattering length $a_{J/\psi\text{-}N} \sim 0.3$ fm
effective range $r_{0\ J/\psi\text{-}N} \sim 2$ fm

✦ Finally, using the potential calculated from BS wave function, we solve the 2-body Schrodinger equation for J/ψ - A in the infinite volume with physical nucleon mass. we can observe the J/ψ-A bound state for A ≧ 7 with folding model.

Folding model $$V_{c\bar{c}A}(\vec{r}) = \int d^3r' V_{c\bar{c}N}(\vec{r} - \vec{r}')\rho_A(\vec{r}')$$

D. A. Wasson PRL 67 (1991) 2237

e.g. ^{7}Li B ~ 150 keV
^{12}C B ~ 2 MeV

Lüscher formula with twisted boundary conditions

Twisted boundary condition

P.F. Bedaque, PLB593 (04) 82

- Generalized spatial boundary condition (b.c.)

$$\psi(x+L) = e^{i\phi}\psi(x)$$

$\phi = 0$: periodic boundary condition (PBC)
$\phi = \pi$: anti-periodic boundary condition (APBC)

- All momenta are quantized in finite volume as

$$p = \frac{2\pi}{L}\left(n + \frac{\phi}{2\pi}\right) \text{ with integer } n$$

accessible to any small momentum with the angle ϕ

Lüscher formula with twisted boundary conditions

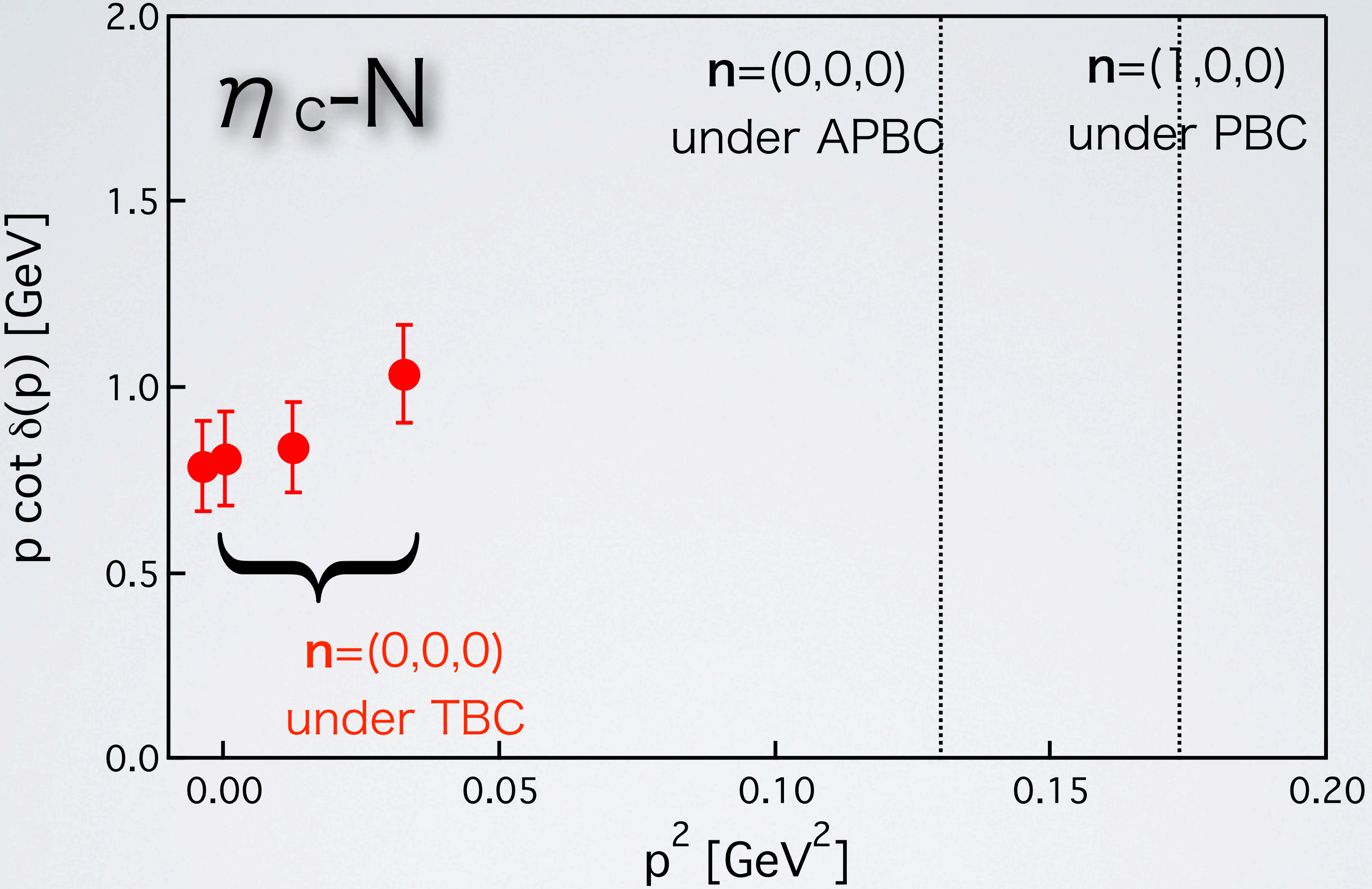
P.F. Bedaque, PLB593 (04) 82

$$p \cot \delta(p) = \frac{\mathcal{Z}^{\mathrm{d}}_{00}(1, q^2)}{L\pi}$$

where $\mathcal{Z}^{\mathbf{d}}_{00}(s, q^2) = \sum_{\mathbf{n} \in \mathbf{Z}^3} \frac{1}{((\mathbf{n} + \mathbf{d})^2 - q^2)^s}$

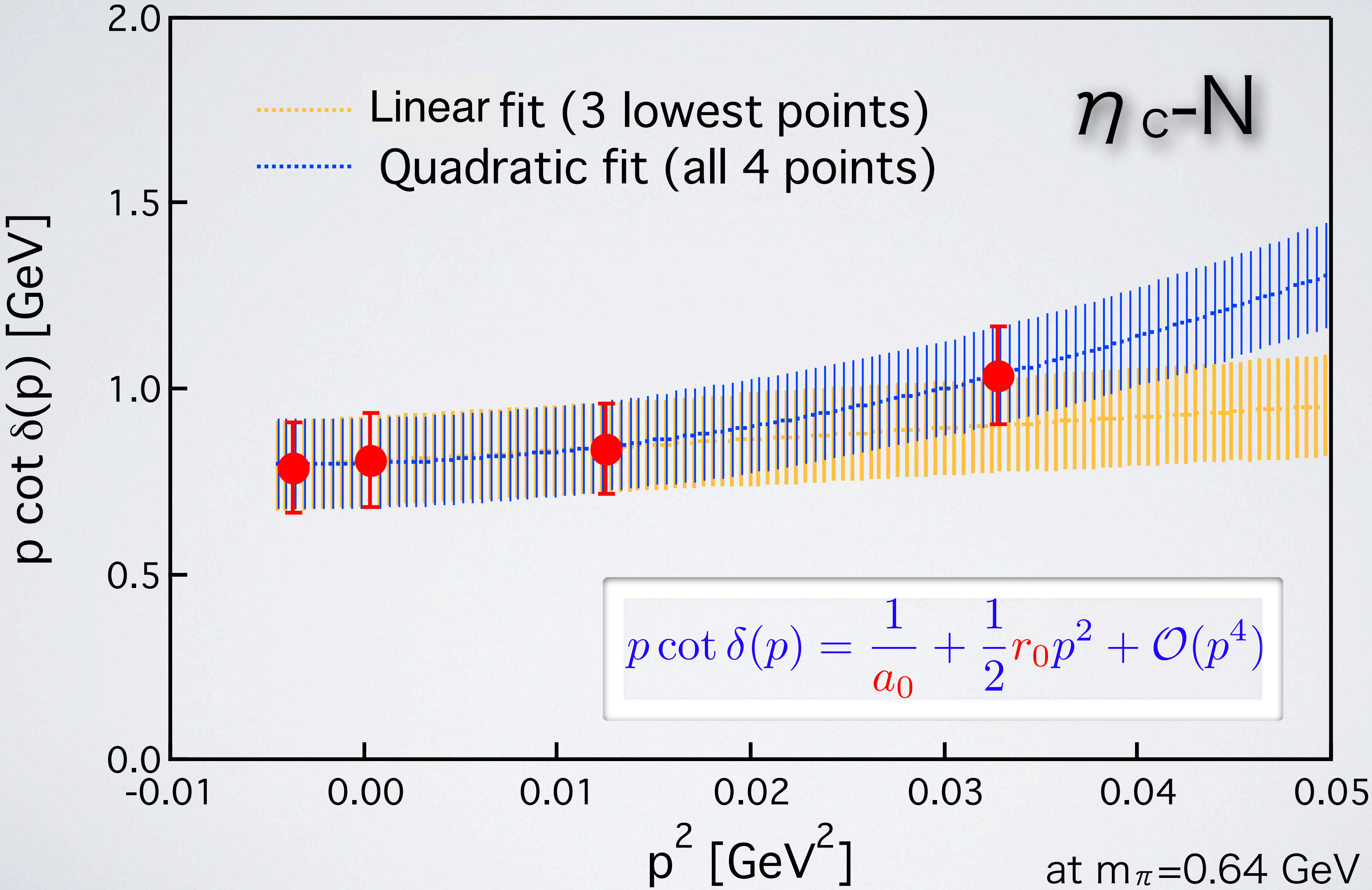
with $\mathbf{d} = \left(\frac{\phi_1}{2\pi}, \frac{\phi_2}{2\pi}, \frac{\phi_3}{2\pi} \right)$

which is defined via analytic continuation from s>3/2 to s=1



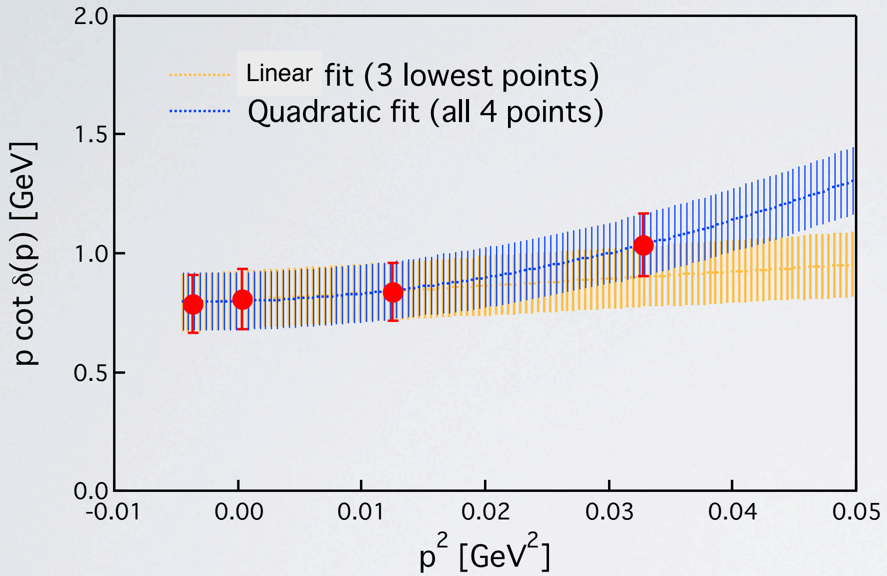
at m_π=0.64 GeV

Effective range expansion



at m_π=0.64 GeV

Effective range expansion

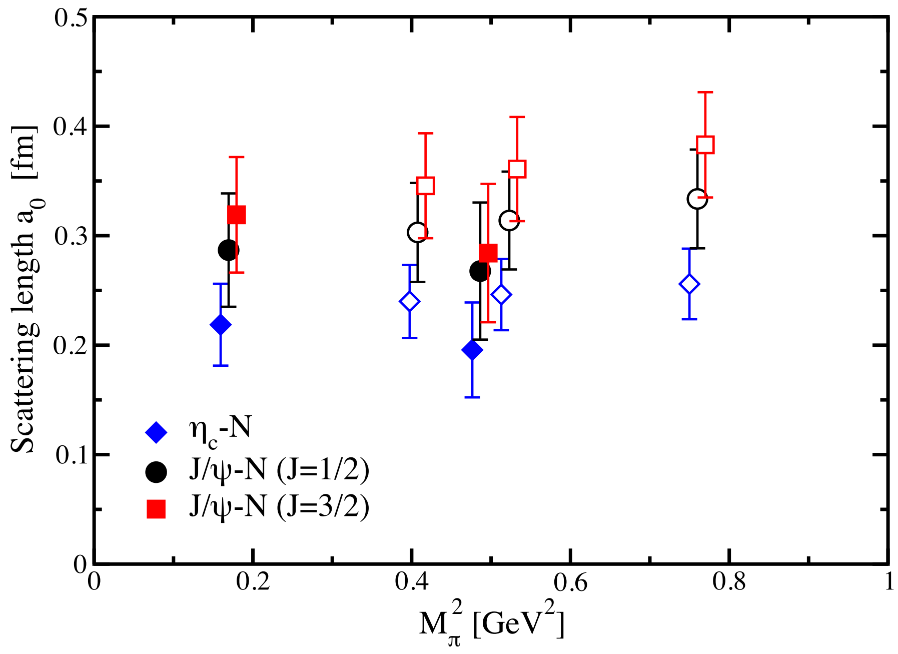


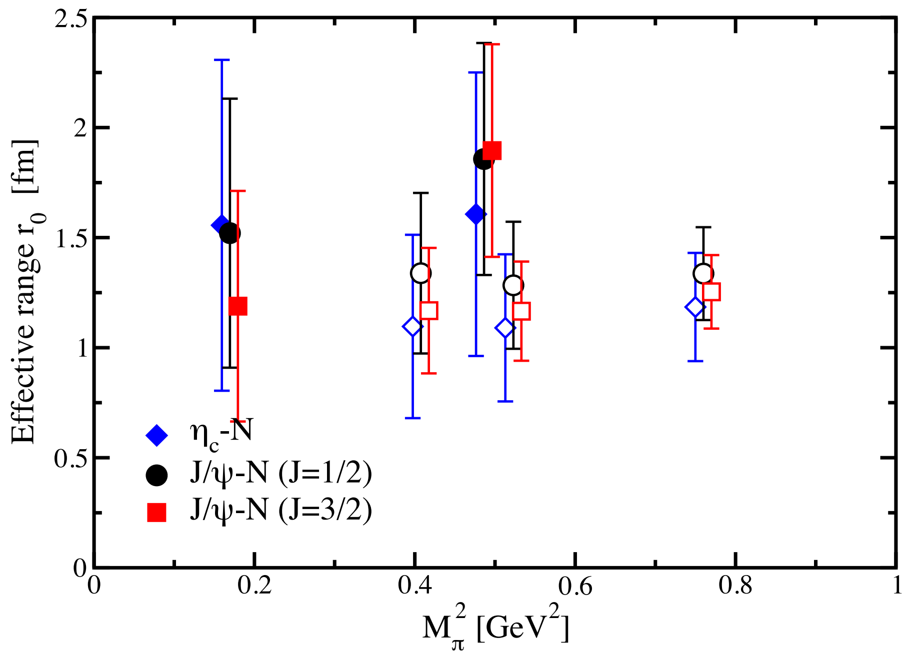
$$p \cot \delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \mathcal{O}(p^4)$$

fit	a_0 [fm]	r_0 [fm]	χ^2/ndf
Linear	0.245(37)	1.18(53)	0.003
Quadratic	0.247(37)	0.56(64)	0.012

at m_π=0.64 GeV

Scattering length and effective range





$a_{J/\psi\text{-}N} \approx 0.3$ fm $\quad r_{J/\psi\text{-}N} \approx 1.5$ fm

Summary

- We derived the $c\bar{c}$-nucleon potential with quenched QCD simulations (pilot study) and Nf2+1 full QCD simulations (preliminary)
  - ✓ The low energy $c\bar{c}$-N interaction is attractive in the whole range of r.
  - ✓ The Long-range part is likely suppressed exponentially.
    - No quark mass dependence up to $m_\pi \sim 400$MeV.
    - No drastic difference between quenched QCD and Full QCD.
- Using extended Lüscher formula with twisted boundary conditions, we derived the the low energy scattering parameters

$$a_{J/\psi\text{-}N} \approx 0.3 \text{ fm} \qquad r_{J/\psi\text{-}N} \approx 1.5 \text{ fm}$$

- Future perspective
  - ✓ We need to perform the simulation in lighter quark mass region.
  - ✓ Exploring nuclear-bound charmonium state with theoretical inputs.