Large N Gauge Theory from Lattice

or 2,3,4,...∞

Saumen Datta

Asian School on Lattice Field Theory 2011, TIFR

Outline

•Introduction to SU(N_c) at large N_c
•Properties of the $N_c \to \infty$ theory; phase structure

Narayanan and Neuberger

•Measurement of various observables and estimate of N_c dependence

Teper and collaborators; Lucini, et al.; Bali, et al; ...

•Deconfinement transition of SU(N_c) gauge theory

Teper and Coll.; Panero; Datta and Gupta

Outline

•Introduction to SU(N_c) at large N_c
•Measurement of various observables and estimate of N_c dependence
Teper and collaborators; Lucini, et al.; Bali, et al; ...
•Properties of the $N_c \to \infty$ theory; phase structure
Narayanan and Neuberger

•Deconfinement transition of SU(N_c) gauge theory

Introduction

The theory of Strong interactions: SU(3) gauge theory
Quark fields ψ_i : triplet of SU(3), $i = 1, 2, 3$ color

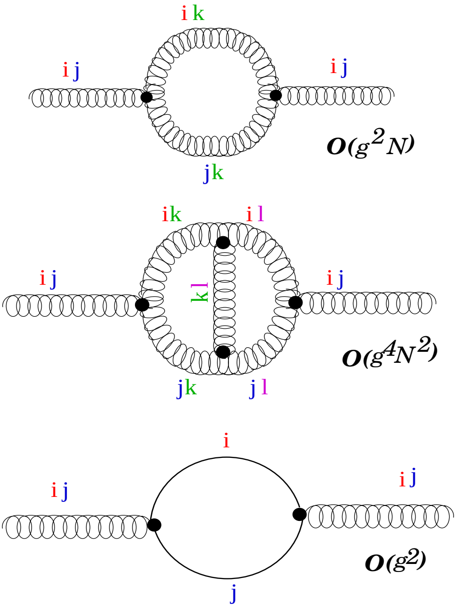
Gluon fields: traceless Hermitian matrices A^{μ}_{ij}

$$\alpha_S(Q^2) = \frac{g^2(Q^2)}{4\pi} = \frac{4\pi}{(11 - \frac{2N_f}{3}) \ln Q^2/\Lambda^2} \quad \textit{in leading order}$$

The interactions become strong at hadron physics scales:
no natural small expansion parameter

G. 't Hooft 1974 Consider SU(N_c), use $1/N_c$ as expansion parameter
Meson spectra of QCD with $N_c \to \infty$ colors in two dimensions

Large N



$g \sim 1/\sqrt{N}$ for nontrivial tractable large N limit

$\lambda_{tH} = g^2 N$

Quark loops ignored in leading order

unless $N_f \sim N_c$

$$\lambda_{tH}(Q^2) \sim \frac{48\pi^2}{11 \ln Q^2/\Lambda^2}$$

see, e.g., S. Coleman, Aspects of Symmetry

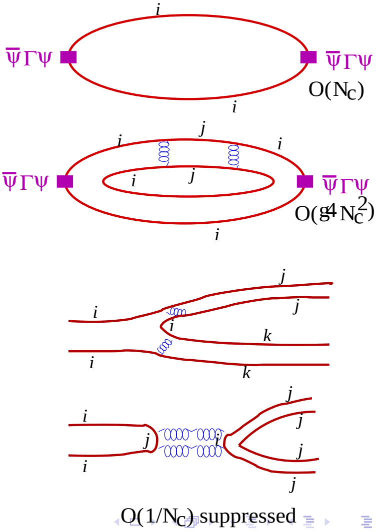
Qualitative hadron physics

Some qualitative understandings of hadron physics in four dimensions, *assuming SU(N) theory is confined.*

- Mesons are stable e.g., $\frac{\Gamma_\rho}{m_\rho} \sim 0.2$
  Mass $\sim O(N_c^0)$, decay width $\sim O(1/N_c)$
- Meson-meson scattering weak, amplitude $\sim O(1/N_c)$
- Meson states are $\bar{q}q$ (quark sea suppressed)
- Zweig's rule (quark line disconnected decays suppressed)

But $N_c = 3 = \mathcal{O}(N_f)$!

Witten: $e \sim \frac{1}{3}$!



Lattice study of SU(N_c) Gauge Theory

Can we use lattice to understand why large N intuitions hold?
Teper: study the theory with 3,4,5,... colors and estimate coefficient of leading correction term.

Body of work; see arXiv:0812.0085 (plenary, Lattice 2008) and references therein. Also works of Lucini, Bali, etc.

- SU(N) gauge theory confines for all N; measurement of string tension.
- Glueball spectra measured
- Computational cost $\sim O(N_c^3)$
- Meson spectra measured in the quenched theory
- Computational cost $\sim O(N_c^2)$ (actually less)

Narayanan & Neuberger: Study theory with $N_c \gg 3$
Use large N_c simplifications

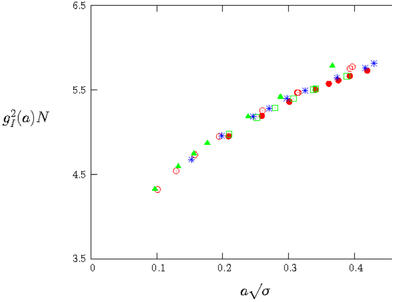


Figure 3: The (mean-field improved) bare 't Hooft coupling as a function ot the sc of the calculated string tension, for $N = 2$ ($\triangle$), $N = 3$ ($\circ$), $N = 4$ ($*$), $N = 6$ ($\square$), ($\bullet$).

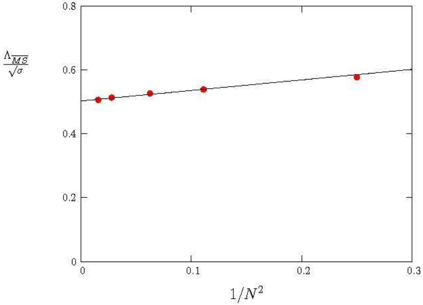


Figure 7: Calculated values of $\Lambda_{\overline{MS}}/\sqrt{\sigma}$ versus $1/N^2$ with a linear extrapolation to $N = \infty$ shown.

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.503(2)(40) + \frac{0.33(3)(3)}{N_c^2}$$

24

28

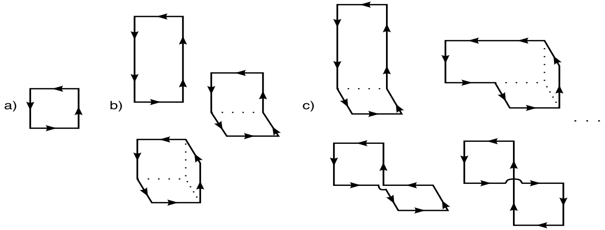
Allton, Trivini, Teper, JHEP 0807 ('08) 021

Glueball spectrum: Operators

Detailed measurement of glueball spectrum in all the channels at $a = 1/6T_c$

J	A_1	A_2	E	T_1	T_2
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1

	$++$	$-+$	$+-$	$--$
A_1	8	2	1	3
A_2	3	1	3	3
E	22	7	7	14
T_1	19	24	48	27
T_2	44	33	33	29



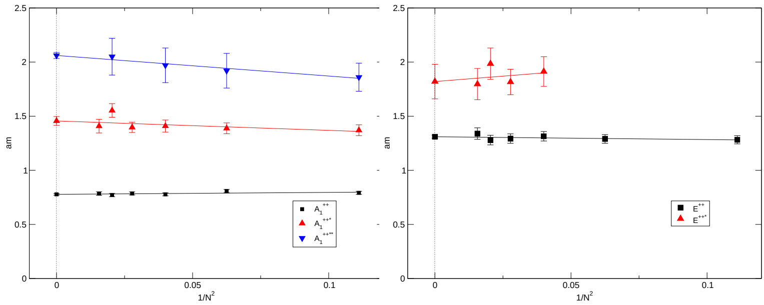
SU(N_c) Glueball spectra

Variational technique to get ground and excited states.

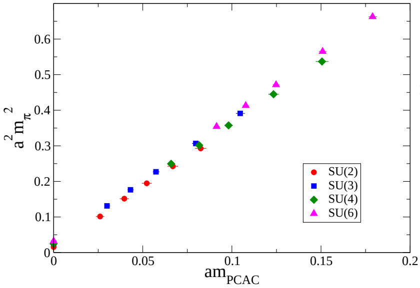
$$am_G(N_c \to \infty) = am_G(N_c) + \frac{c}{N_c^2}$$

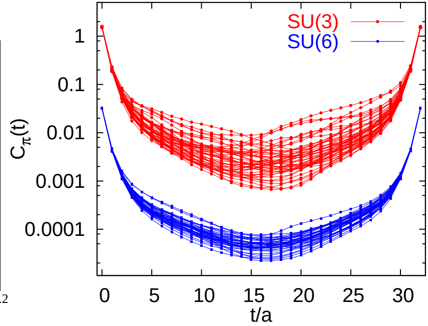
good fit for N_c = 3-7, $c \leq 1$

Lucini et al., arXiv:1007.3879



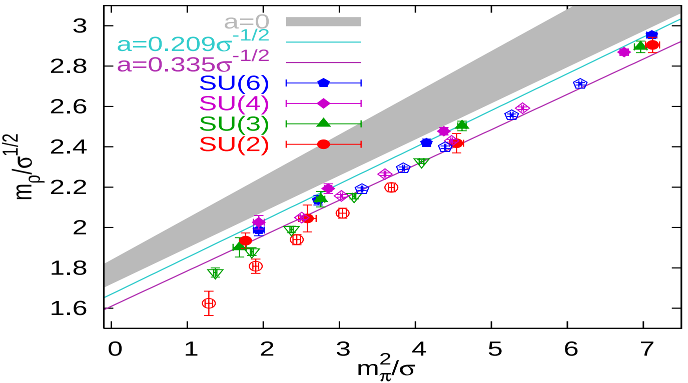
Meson spectra in quenched approximation





Del Debbio, et al., JHEP03('08)062; Bali & Bursa, JHEP09('08)110

ρ meson in SU(N_c)



$$\frac{m_\rho}{\sqrt{\sigma}} = 1.670(24) - \frac{0.22(23)}{N_c^2} + \left(0.182(5) - \frac{0.01(5)}{N_c^2}\right)\frac{m_\pi^2}{\sigma}$$

Gives $m_\rho = 794(23)$ MeV for the $N_c \to \infty$ theory!

Narayanan-Neuberger Approach

Narayanan and Neuberger follow a different approach of using simplifications arising at large N_c.

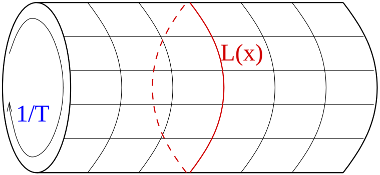
see, e.g, Narayanan & Neuberger, arXiv:0710.0098 (Lattice 2007 plenary)

The $N_c \to \infty$ theory is volume independent so long as the system is in confined phase.
Used this to study very large $N_c \sim 20$ theories at very small volume.
They found an increasing slope of m_ρ $vs. m_\pi^2$ with increasing N_c.

Hietanen, Narayanan, et al., PLB 674 ('09) 80

Equilibrium Thermodynamics on Lattice

$$Z(T) = \int dU \ \exp(-\beta \int_{0,pbc}^{1/T} d\tau \int d^3x \mathcal{L}(U))$$



As lattice spacing $a \to 0$,
$\beta\mathcal{L} \to \mathrm{Tr} F_{\mu\nu}^2$ with
$\beta = 2N/g^2 = 2N^2/\lambda_{tH}$

- $A_\mu(\vec{x}, 0) = A_\mu(\vec{x}, 1/T)$
  Z_N group of aperiodic gauge transformation
  $U_\mu(\vec{x}, 1/T) = e^{2\pi i/N} U_\mu(\vec{x}, 0)$
- Order parameter for deconfinement transition:

$$L \;=\; \frac{1}{N} \mathrm{Tr} \prod_{x_0=1}^{N_\tau} U_{(x_0,\mathbf{x}),\hat{0}} \to 1/N \, \mathrm{Tr} \, \mathrm{P} \, \mathrm{e}^{\int_0^{1/\mathrm{T}} \mathrm{d}\tau \, \mathrm{A}_0(\tilde{\mathrm{x}},\tau)}$$

$$Z_N \quad : \quad L = \frac{1}{V} \sum_x L(\vec{x}) \to e^{2\pi i/N} L$$

Z_N Symmetry and Deconfinement

- $|<L>| \sim e^{-F}$, F free energy of a static quark source
- $<L> \neq 0 \;\; \rightarrow \;\; Z_N$ broken and deconfinement
- SU(2): 2nd order transition in Z_2 universality class

Engels et al. 1982-90

- For $N > 2$: Z_N: 1st order transition expected
- The QCD transition $N_f = 2 + 1$ is a *crossover*.
  Expected to be first order for all finite mass at SU(∞)
- Chiral limit may be more involved.
- Similarly, complicated phase structures have been predicted for $\mu_B \sim O(N_c)$.

McLerran & Pisarski '07

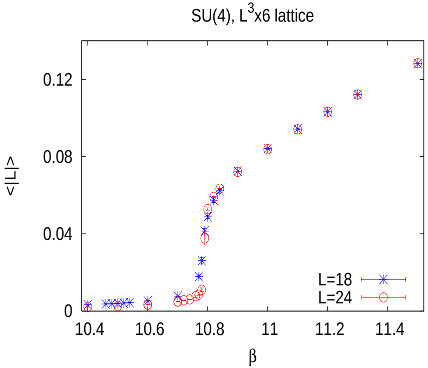
Study of SU(N) theory at finite T

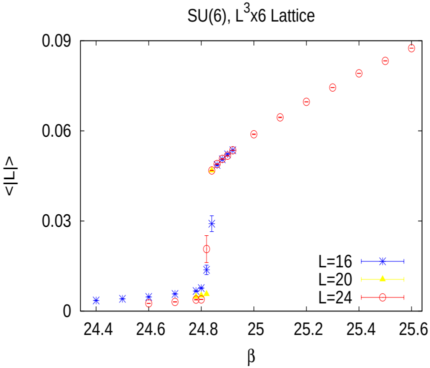
Interesting predictions at finite temperature, which may help our understanding of SU(3)

- Symmetry of Polyakov loop $\rightarrow$ strong 1st order transition?
- How does $T_c/\Lambda_{\overline{MS}}$ scale?
- For $N > 3$ the latent heat $\sim N^2$
- Does one reach the asymptotic state soon after the transition?
- Deviation from conformality in the plasma phase?

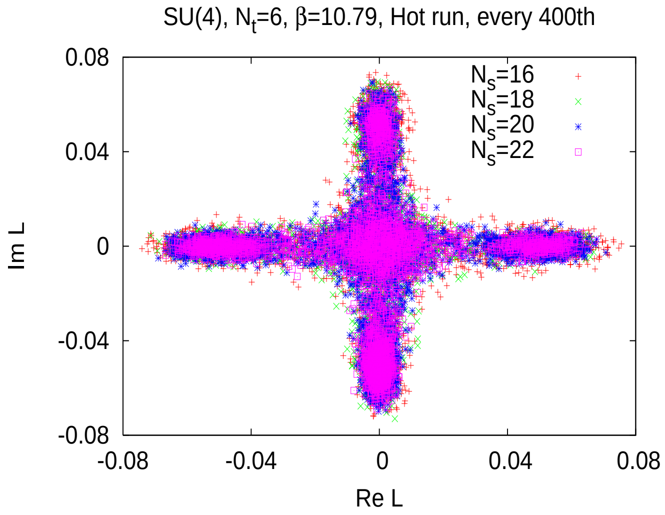
Gavai; Ohta & Wingate; 2001
Lucini & Teper; Lucini, Teper & Wenger; Bringholtz & Teper 2003–
Panero 2009; Datta & Gupta 2009-10

Deconfinement Transition for SU(4) and SU(6)



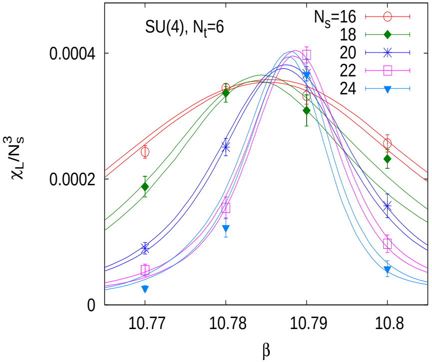


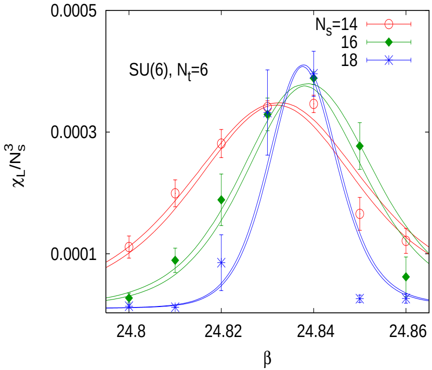
Degenerate Vacuua at T_c



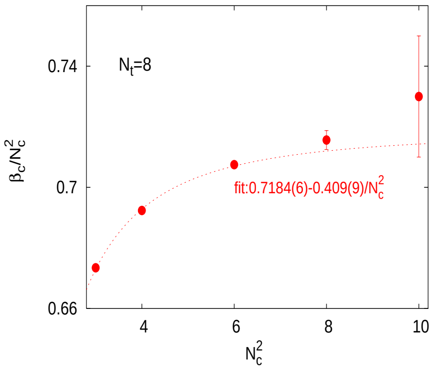
Finite Size Analysis

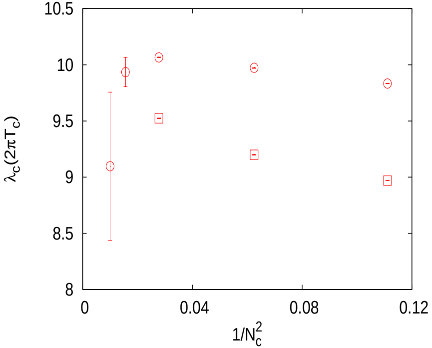
Susceptibility $\chi_L = V(\langle |L|^2 \rangle - \langle |L| \rangle^2) \quad \sim V$ for 1st order.





N_c dependence of transition point





We use for coupling $V_{Q,\bar{Q}}(q^2) = -\frac{N^2-1}{2N}\frac{g^2}{q^2}$ (V-scheme)

$T_c/\Lambda_{\overline{MS}}$

- Use 2-loop running to directly calculate $T_c/\Lambda_{\overline{MS}}$
- For each N_c, continuum limit of $g^2_{\rm Vscheme}$
- We also checked using other schemes.
  Scheme-dependence observed to be stronger than cutoff dependence.
- N_c-dependence using $c_0 + \frac{c_1}{N_c^2}$

$$\boxed{\frac{T_c}{\Lambda_{\overline{MS}}}\Big|_{N\to\infty} = 1.121(2) + \frac{0.51(5)}{N_c^2}}$$

Define thermodynamic quantities

$F(T,V) = T \ln Z(T,V)$ Z: grand canonical partition function

$\epsilon = -\frac{1}{V}\frac{\partial F(T,V)/T}{\partial(1/T)}$

$p = \frac{\partial F}{\partial V} \qquad = F/V$ for homogeneous

Trace of energy momentum tensor $\theta^{\mu\mu} = \Delta = \frac{\epsilon - 3p}{T^4}$

$$(\epsilon - 3p)/T^4 = 6N_\tau^4 \, a\frac{\partial \beta}{\partial a} \, (P(T) - P(T=0))$$

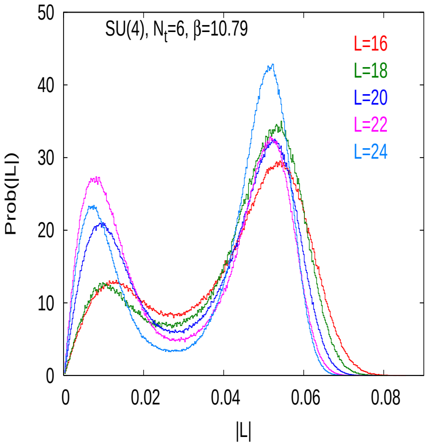
$$\boxed{a\frac{\partial\beta}{\partial a} \; = \; -\frac{\partial\beta}{\partial g_R^2(k/a)} \; \cdot \; 2g_R \; \beta(g_R)}$$

$$\boxed{\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{\beta_0}^{\beta} d\beta \, (P(\beta) - P(\beta_0))}$$

Latent Heat: Method

L_h/T_c^4 obtained from the discontinuity of $(\epsilon - 3p)/T^4$ at T_c

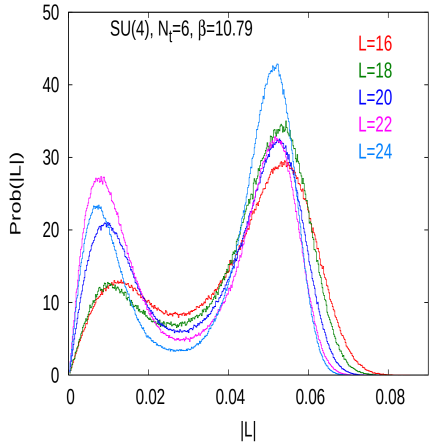
- Put $|L|$ cut to identify confined and deconfined phase at T_c



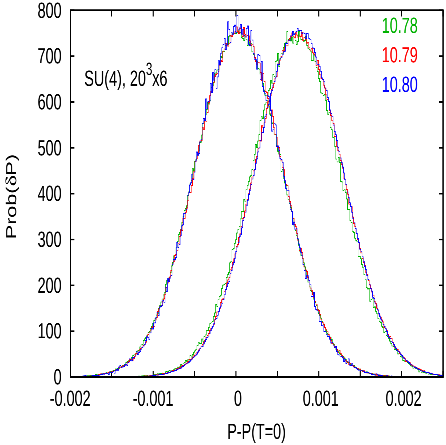
Latent Heat: Method

L_h/T_c^4 obtained from the discontinuity of $(\epsilon - 3p)/T^4$ at T_c

- Put $|L|$ cut to identify confined and deconfined phase at T_c



- Procedure stable in the metastability regime



Latent Heat: Results

- Results from $N_\tau = 8$ lattices:

N_c	β	L_h/T_c^4	$L_h/\Delta_{\max}$
3	6.0609	1.67(4)(4)	0.68(3)
4	11.08	4.32(6)(6)	0.82(2)
6	25.46	11.93(34)(5)	0.90(3)

$$\boxed{\frac{L_h}{T_A T_c^4} = 0.388(3) - \frac{1.61(4)}{N_c^2}}$$

(statistical error only)
correction considerable at $N_c = 3$

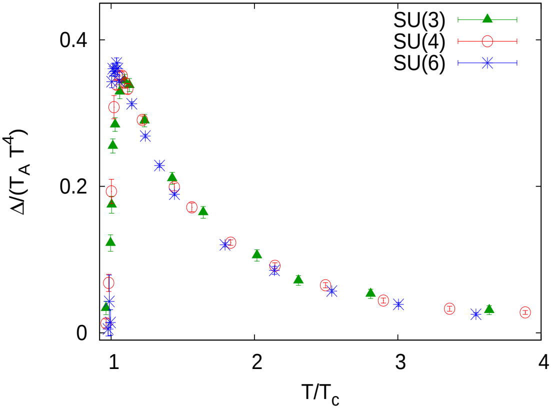
- In excellent agreement with the $N_t = 8$ results of Teper et al. (at smaller volume, lower statistics)

Lucini, Teper, Wenger, JHEP02, 033 ('05)

- A larger value obtained for SU(4) by Gavai. Uses bare coupling.

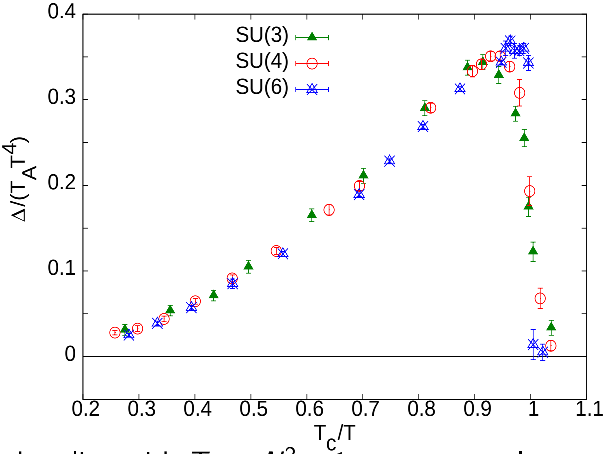
R. Gavai, Nucl. Phys. B 633, 127 ('02)

$(\epsilon - 3p)/T^4$



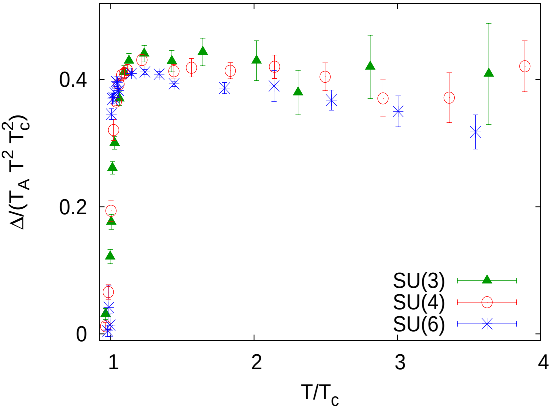
- Good scaling with $T_A = N_c^2 - 1$ except very close to T_c
- Peak moves towards T_c with increasing N_c, requires better accuracy to quantify the movement.

$(\epsilon - 3p)/T^4$



- Good scaling with $T_A = N_c^2 - 1$ except very close to T_c
- Peak moves towards T_c with increasing N_c, requires better accuracy to quantify the movement.
- Substantial conformal symmetry breaking at $2T_c$: $\Delta^{1/4} \sim T$

$\epsilon - 3p \sim T^2$?

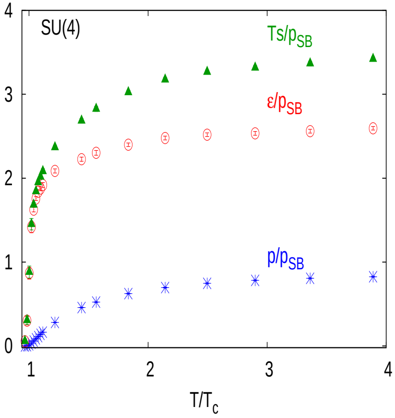


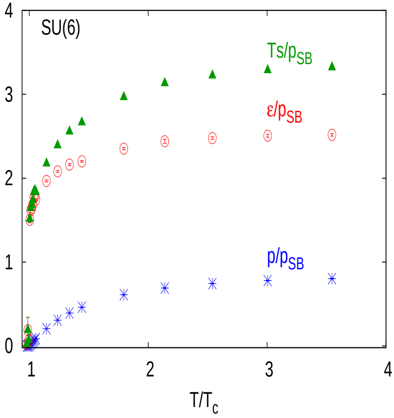
- $e - 3p \sim T^2$ observed for SU(3) over large T range.

  Meisinger, Miller, Ogilvie '02; Pisarski '07

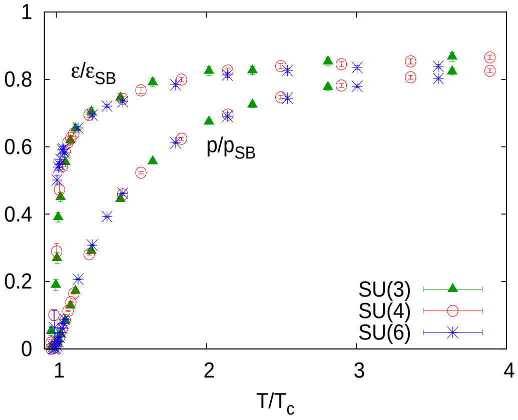
- Similar behavior observed for $N_c = 4{,}6$, subleading term also contribute at SU(6)

Bulk Thermodynamic Quantities

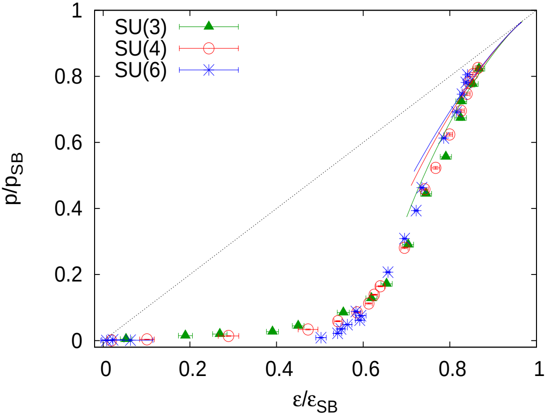




N_c scaling



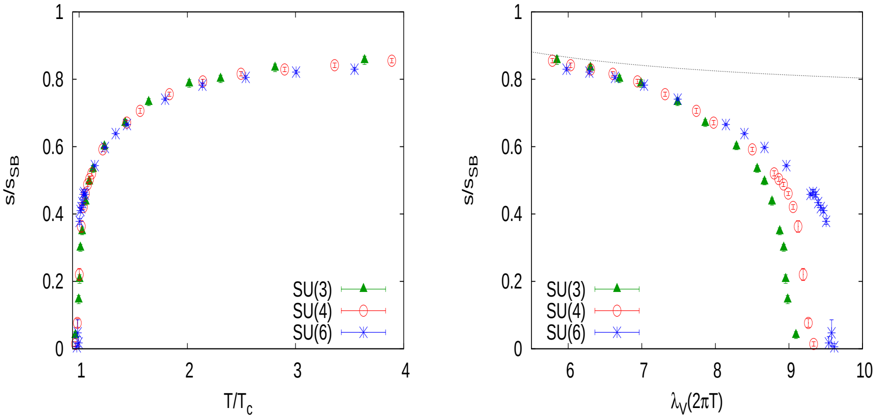
Approach to conformality



Closer to weak coupling theory (Laine & Schroeder, PR D 73('06)) than to conformal theory.

No evidence of a strongly coupled, near-conformal phase.

Entropy and 't Hooft scaling



Strong N_c scaling is better than scaling with the 't Hooft coupling, which holds only at the higher temperatures.
The line is the result for $\mathcal{N} = 4$ SYM (Klebanov et al. '02)

Summary

- Interesting physics insights from lattice study of SU(N_c) gauge theories with $N_c > 3$
- Helps in understanding qualitative success of large N_c intuitions in understanding features of hadron physics
- Quark loop effects, baryons in theory with large number of colors: not studied yet
- Insights about the deconfined gluonic plasma
- Here, quark loop effects will be even more interesting.