3. QCD transition I
deconfinement transition

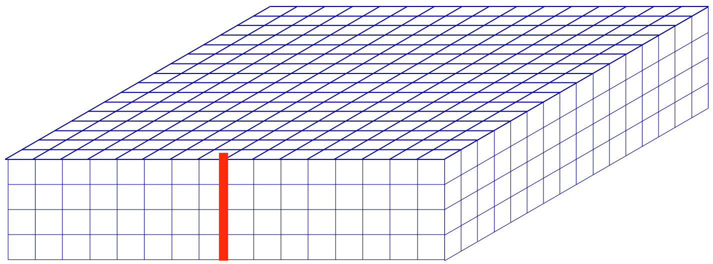
pure gauge YM theory

Let us first study the deconfinement transition of QCD. This transition is most cleanly studied in **pure gauge theories**.

Order parameter for the deconfinement transition

Polyakov loop:

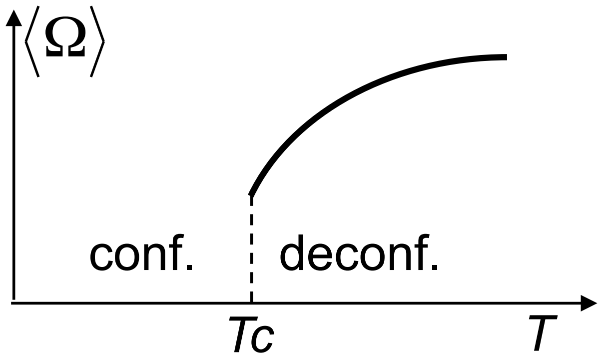
$$\Omega_{\mathbf{x}} = \frac{1}{N_c} Tr\left(\prod_{t=1}^{N_t} U_{\mathbf{x},t;4} \right)$$



$\propto$ current of a static quark at $\mathbf{x}$ (static charge in the fund. repr.)

$$\langle \Omega \rangle \propto e^{-F_q/T}, \quad F_q : \text{static quark free energy}$$

$$= \begin{cases} = 0 & \text{confined} \quad (F_q = \infty) \\ > 0 & \text{deconfined} \quad (F_q < \infty) \end{cases}$$



What is the symmetries behind??

pure gauge YM theory

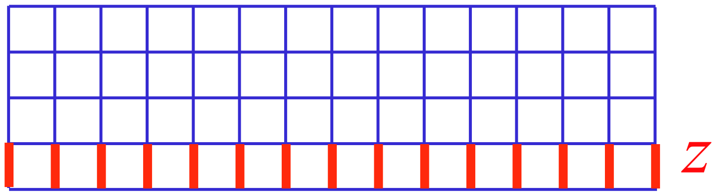
Center symmetry

center = $\{z | z \in G, \text{ s.t. } \forall g \in G, \ [z, g] = 0\}$: a subgroup of G

For $SU(N_c)$ YM, the center is $Z(N_c) = \{\exp(2\pi i\, n/N_c)\ ;\ n = 0, 1, \ldots, N_c\}$.

$$U_{x;\mu} \to \begin{cases} zU_{x;\mu} & \text{if } t = 0 \text{ and } \mu = 4, \\ U_{x;\mu} & \text{otherwise} \end{cases} \quad z \in Z(N_C) \subset SU(N_C)$$

$$S \to S, \quad \Omega_{\mathbf{x}} \to z\Omega_{\mathbf{x}}$$



t=0 is not crucial. It can be any globally space-like surface by a gauge transformation.

This is a global symmetry of the system.

But the Polyakov loop is not invariant.

=> Polyakov loop detects SSB of the center sym.

i.e. **deconf. trans. of QCD ≈ SSB of the center $Z(N_c)$ sym.**

effective G-L for deconf. trans.

$Z(N_c)$ effective spin model

<= center part of the Polyakov loop

\+ effective interaction between Polyakov loops

- high T expansion, strong coupl. expansion

  Svetisky, Yaffe, NP B210('82)423; PR D26('82)395, Polonyi, Szlachanyi, PL 110B('82)395

- MC confirmation of short-range effective interactions Okawa, PRL 60('88)1805

➡ $N_c = 2$ YM $\approx$ 3-d Ising => 2nd order

➡ $N_c = 3$ YM $\approx$ 3-d $Z(3)$ Potts => 1st order

MC study of SU(3) gauge theory

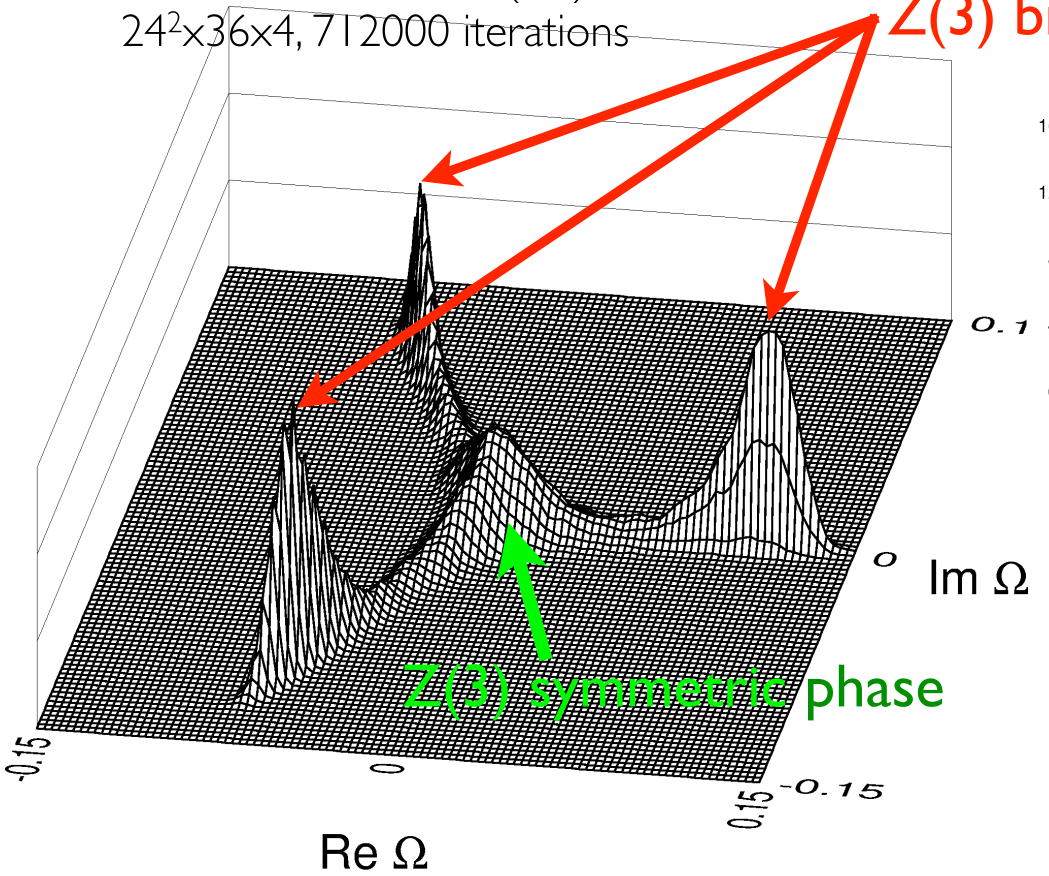
The expected 1st order transition turned out to be quite weak.

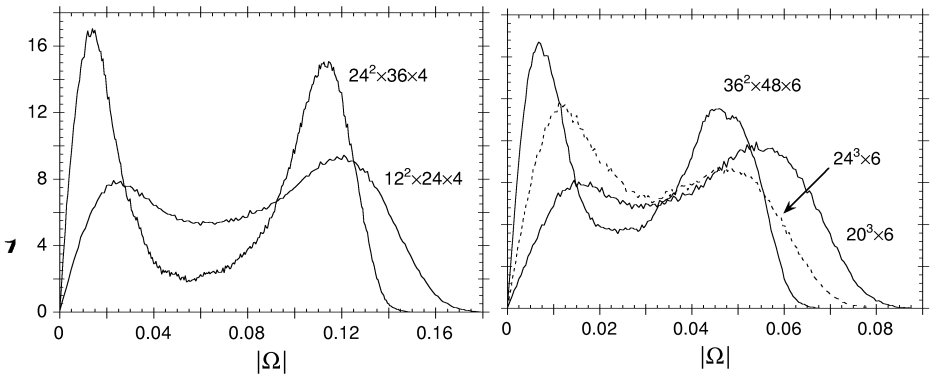
=> Large lattice with high statistics required.

Polyakov loop histogram at T_c

Iwasaki et al., PR D46('92)4657

24^2x36x4, 712000 iterations





Z(3) nature clearly seen.

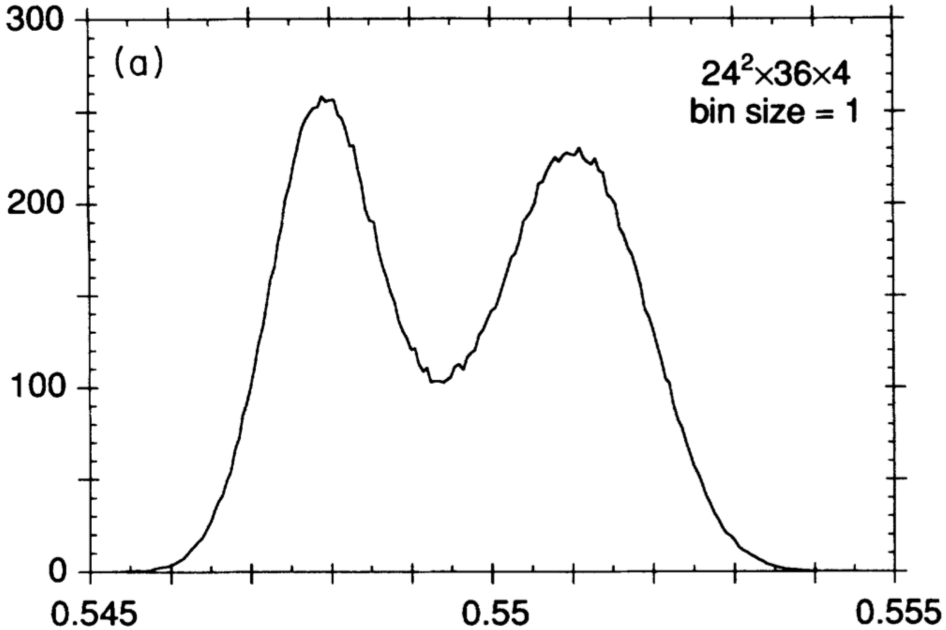
2 phase coexistence

=> **1st order** suggested

MC study of SU(3) gauge theory

Plaquette histogram at T_c

Iwasaki et al., PR D46('92)4657



$$P = \frac{1}{6N_{\mathrm{site}}} \sum_{n,\mu<\nu} \frac{1}{3} \mathrm{Re\ tr} \left[U_{n,\mu} U_{n+\hat{\mu},\nu} U^{\dagger}_{n+\hat{\nu},\mu} U^{\dagger}_{n,\nu} \right]$$

~ energy density

1st order visible with other observables too, when the volume is sufficiently large.

MC study of SU(3) gauge theory

To confirm the nature of the transition,

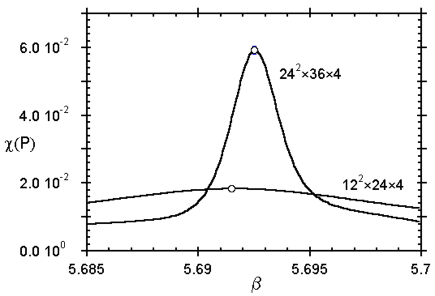
Finite size scaling test

Fukugita et al., NP B337('90)181, Iwasaki et al, PR D46('92)4667

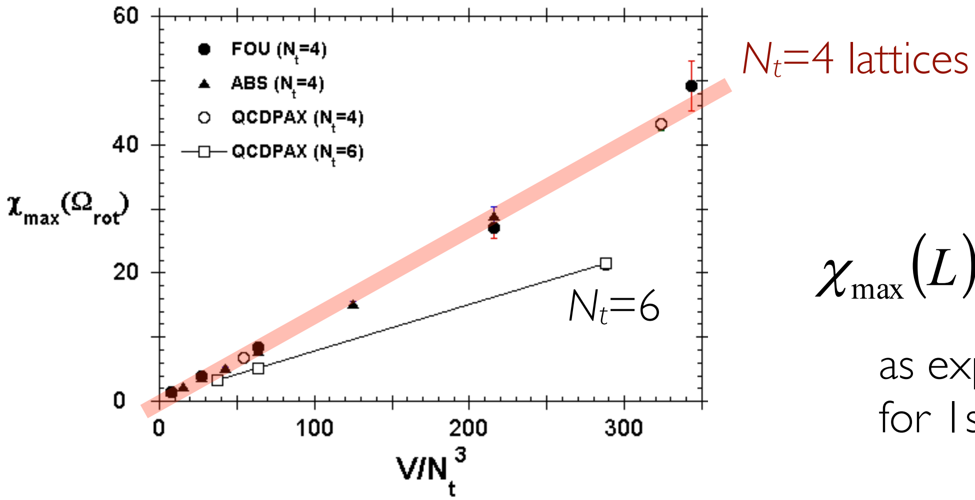
Polyakov loop susceptibility

$$\chi = V\left(\langle\Omega_{\rm rot}^2\rangle - \langle\Omega_{\rm rot}\rangle^2\right)$$

=> peak height



<= *Reweighting method*



$$\chi_{\max}(L) \propto V$$

as expected for 1st order.

FSS test with SU(2) gauge theory

Engels et al., NP B332('90)737

$$\chi_{\max}(L) \propto V^{\rho}, \quad \rho = \gamma/3\nu$$

$\rho = 0.655(2)$ 3d Ising

$= 0.64(1)$ SU(2)

	Ising	SU(2)
β/ν	0.516(5)	0.543(30)
γ/ν	1.965(5)	1.93(3)
ν	0.630(3)	0.65(4)

Scaling argument works well.

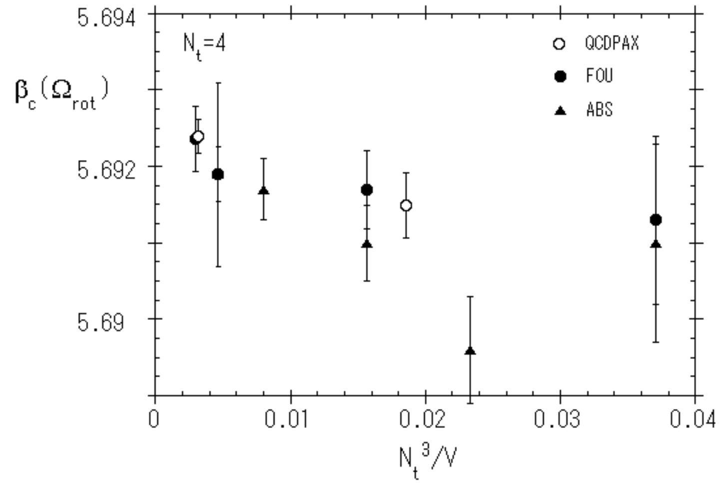
MC study of SU(3) gauge theory

Critical temperature

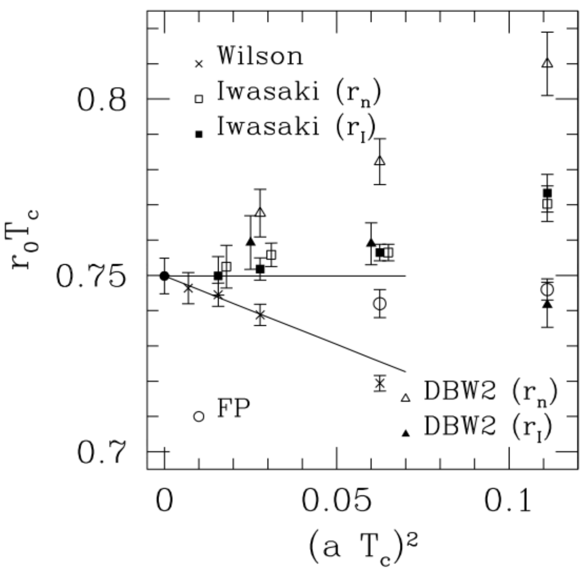
step 1) compute $\beta_C(N_t, V=\infty)$
<= extrapolation using FSS

step 2) determine the scale at β_c => $T_c(N_t)$
<= physical quantities at $T=0$
string tension, Sommer scale, ...

step 3) continuum extrapolation: $N_t \rightarrow \infty$



Iwasaki et al., PR D46('92)4657



$T_C\, r_0 = 0.7498(50)$

$r_0 \sim 0.5$ fm => $T_C \sim 295$ MeV

Necco, NP B683('04)137

influence of heavy quarks

Quarks violates the $Z(N_c)$ symmetry explicitly.

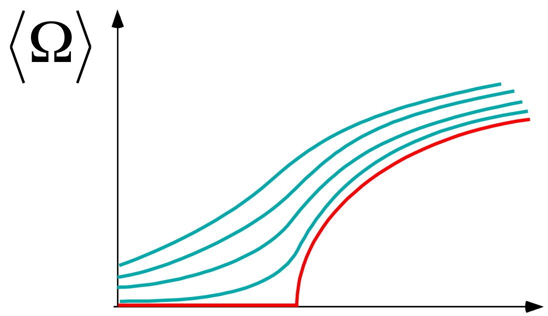
Quark action not invariant under the $Z(N_c)$ transformation.

Light quarks : chiral symmetry => to be discussed in the next section.
Heavy quarks : perturbation to the pure gauge system

Polyakov-loop effective theory + hopping param. expansion

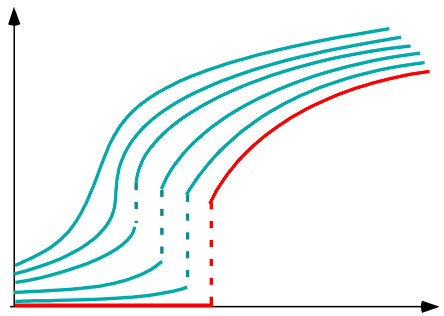
➡ heavy quarks act as **external magnetic field** to the $Z(N_c)$ spins

2nd order case [SU(2)]



Quickly turns into an analytic crossover.

1st order case [SU(3)]



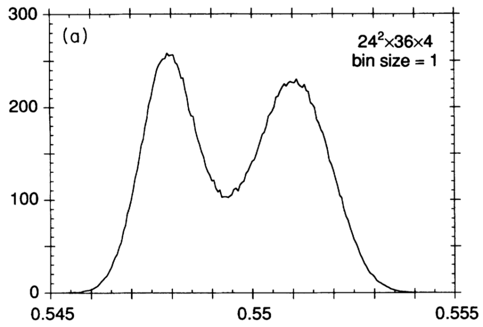
Remains to be 1st order for a while,
and then turns into a crossover.

$\langle \Omega \rangle$ is no more an order parameter, but is useful as an **indicator** of the transition/crossover.

influence of heavy quarks

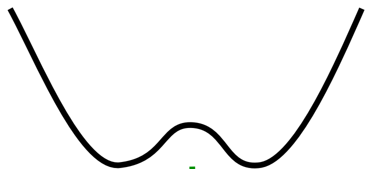
Effective potential of plaquette

H. Saito et al. (WHOT-QCD Collab.) Lattice 2010,; paper in preparation

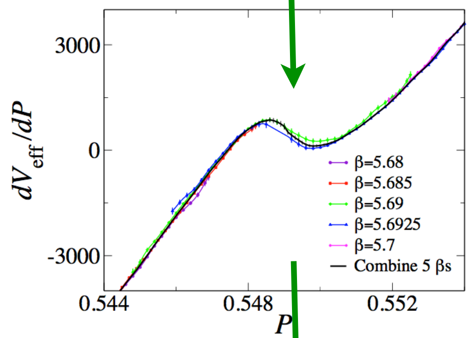


$$V_{\rm eff}(P,\beta,\kappa) = -\ln w(P,\beta,\kappa)$$

double-well effective potential

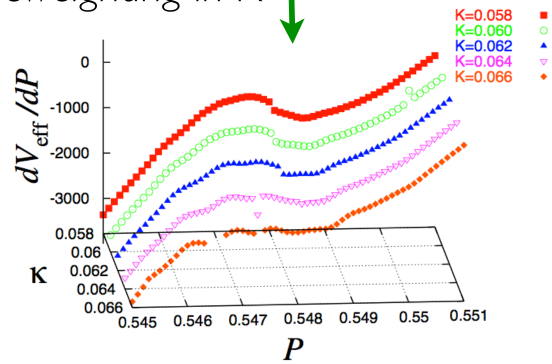


d / dP + reweighting in β

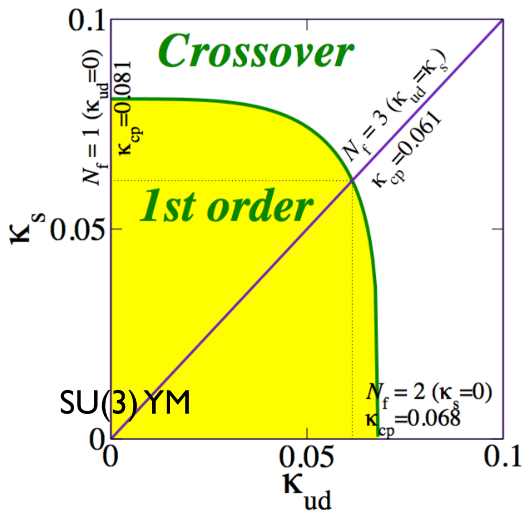


- plaquette gauge + standard Wilson, Nt=4
- hopping parameter expansion

reweighting in K



phase diagram



4. QCD transition II
chiral transition

effects of light quarks

Quark pair-creation => $\langle \Omega \rangle \neq 0$ even with confinement. => $\langle \Omega \rangle$ not quite sensitive.

In the massless limit, we instead have the **chiral symmetry**.

- **massless N_F -flavor QCD (continuum theory)**:

$$SU(N_F)_L \times SU(N_F)_R \times U(1)_V \times \cancel{U(1)_A}$$

SSB ↓ ↑ high T anomaly

$$SU(N_F)_V \times U(1)_V$$

effective 3d σ model (G-L model) Pisarski-Wilczek, PR D29('84)338, Wilczek, IJMP A7(92)3911, Rajagopal-Wilczek NP B399(93)395

order parameter $M_{ab} \sim \left\langle \bar{q}_a \frac{1+\gamma_5}{2} q_b \right\rangle$ $N_F \times N_F$ complex matrix

$$\mathbf{M} \rightarrow U^{+}\mathbf{M}V, \quad U \in U(N_F)_L, V \in U(N_F)_R$$

$$L = Tr\partial_i \mathbf{M}^{+}\partial_i \mathbf{M} + \mu^2 Tr\mathbf{M}^{+}\mathbf{M} + \lambda_1 Tr(\mathbf{M}^{+}\mathbf{M})^2 + \lambda_2 (Tr\mathbf{M}^{+}\mathbf{M})^2$$

$$+ c_{U(1)_A}\{\det \mathbf{M} + \det \mathbf{M}^{+}\} \qquad \Leftarrow U(1)_A \text{ anomaly}$$

chiral sigma model

- $N_F \geq 3$: **1st order**
- $N_F = 2$: more complicated
  anomaly term ~ M^2 ~ mass term: relevant !
  - when anomaly negligible around T_c => **1st order**
  - with anomaly
    model ≈ O(4) Heisenberg model

$$\mathbf{M}(\mathbf{x}) = (\text{real positive const.}) \times SU(2)\,\text{matrix} = \sigma(\mathbf{x}) + i\pi_a(\mathbf{x}) \cdot \tau^a, \quad \tau : \text{Pauli matrix}$$

$$\mathbf{M} \to U^+ \mathbf{M} V, \quad U, V : \text{same } U(1) \text{ phase}$$

This model is much simpler than the sigma model, and is well investigated.

=> **2nd order**

γ/ν	ν	β/ν
1.97(1)	0.73(2)	
1.9749(38)	0.7479(90)	0.5129(11)
1.964(13)	0.749(4)	

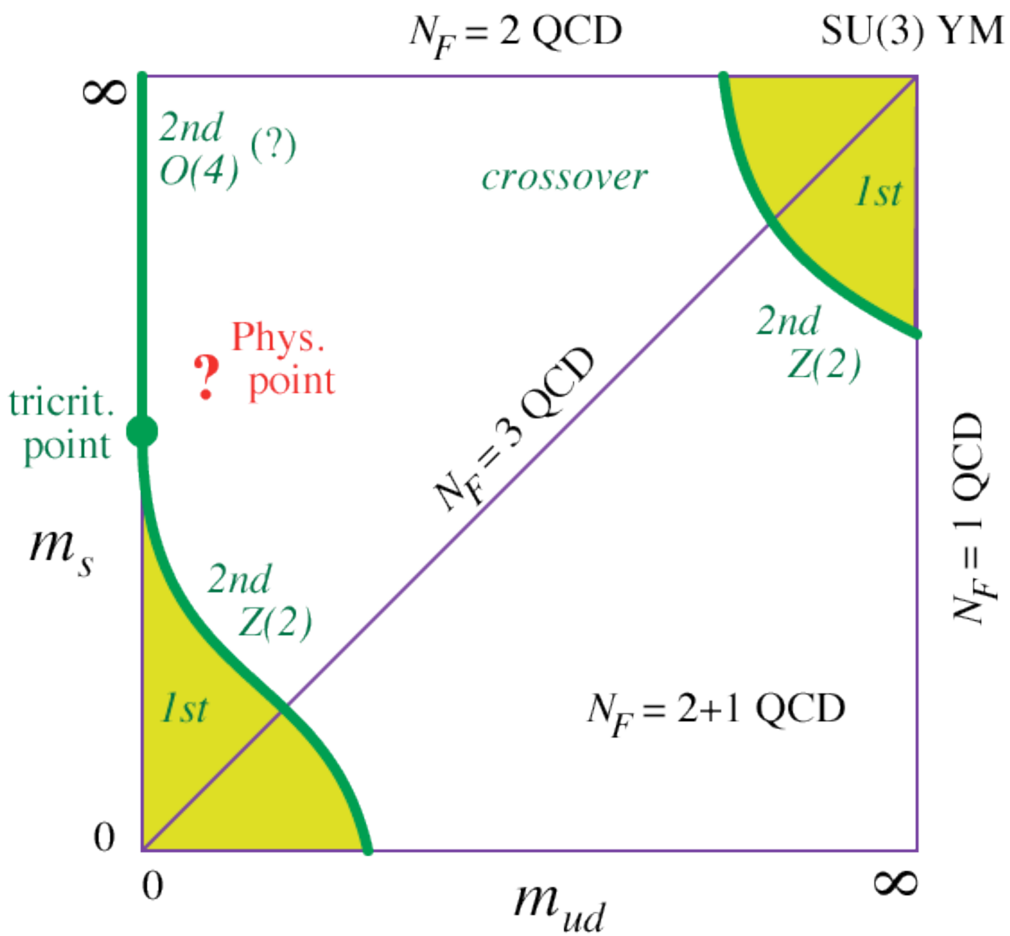
$$M/h^{1/\delta} = f(t/h^{1/\beta\delta})$$

$1/\beta\delta = 0.537(7)$

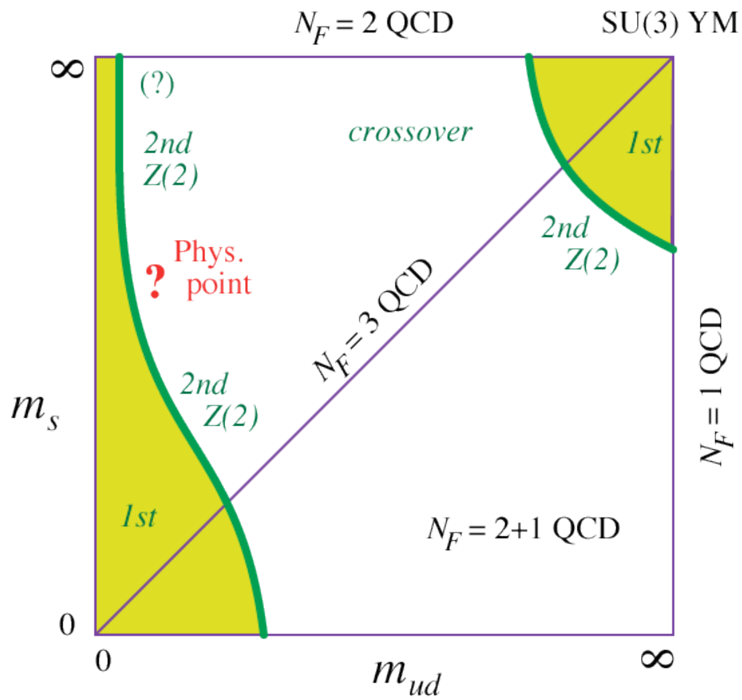
$1/\delta = 0.2061(9)$

(Note: RG flow enhances the anomaly towards the IR limit.)

order of the QCD transition



or, alternatively, when N_F=2 is 1st order



Staggered simulations

=> The physical point locates in the crossover region.

lattice quarks

caveat!!
Not easy to keep the chiral sym. of continuum QCD on the lattice.

Nielesen-Ninomiya No-Go theorem:

One flavor lattice fermion cannot be local and chiral simultaneously.

=> several options

- Wilson-type: keep the flavor sym., but violates chiral
- staggered-type: keep a combination of taste-chiral sym.,, but violates locality, ...
- domain-wall / overlap: (approximately) keep the chiral sym., but expensive.

Life is not easy yet.

LATTICE QUARKS

Staggered-type quarks

Most widely used.

Merits: ✓ A kind of "chiral symmetry" preserved at $a > 0$.
=> location of the chiral limit protected (no additive ren. to m_q)

✓ Light quark simulations less expensive. <= det$M$ positive definite

Problems:

❖ 4 copies of flavors ("*tastes*") => 4th root trick   det$M$ ⇒ $[\det M]^{1/4}$

❖ non-local => universality arguments fragile

If a continuum limit exists, and if it belongs to the universality class of QCD ?

Vital discussions: Sharpe@Lat06, Creutz@Lat07, Kronfeld@Lat07, ...
See also Rossi-Testa 1005.3672.

I *assume* that the staggered quarks have the continuum limit in the universality class of QCD with desired N_F.

Message: Continuum extrapolation must be done first.

Still a couple of worrisome issues.

Taste violation problem

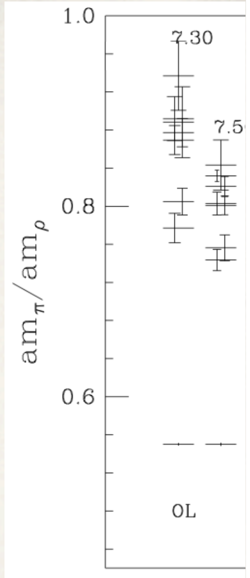
taste sym. violated at $a > 0$ => errors in flavor identifications

(e.g.) many π's in the taste space

Lighterst π usually chosen as physical.

Other π's do contribute to dynamical / loop effects

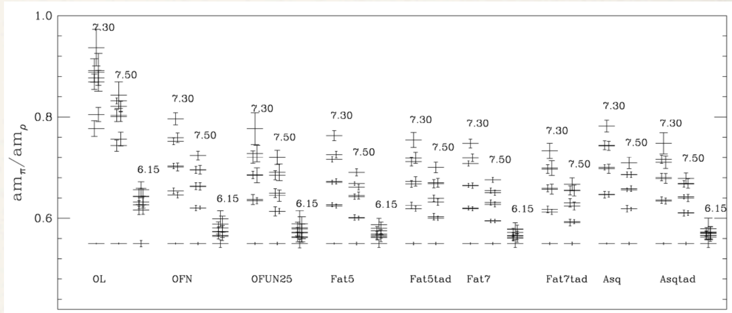
=> lattice artifacts.



Improved staggered quarks

Various actions proposed: asqtad / p4 / HYP / stout / HISQ / ...

The degree of improvement differs.



Orginos et al,
hep-lat/9909087
$m_\pi/m_\rho=0.55$,

It turned out from recent lessons of Tc-determination with improved staggered-type quarks that

it is important to have a good control on the taste-violation.

Still, the caveat with the non-locality remains,

in particular, when we want to apply universality arguments.

Wilson-type quarks

Most conservative.

Merits:
- ✓ Describe a single flavor.
- ✓ QCD continuum limit exists.

Problems:
- ❖ Explicit violation of the chiral symmetry at $a > 0$ => Additive m_q renormalization
- ❖ det$M$ not positive definite => light quarks more expensive.

Phase structure:

S. Aoki PRD30('84)

2nd order transition to explain massless π's without the chiral symmetry

Sharpe-Singleton PRD58('98)

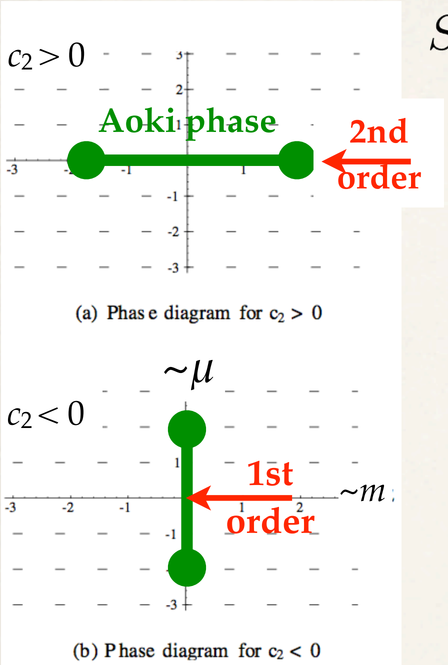
Wilson term -> effective int. c_2 the phase structure depends on the sign of c_2.

$$\mathcal{V}_\chi = -\frac{c_1}{4}\mathrm{Tr}(\Sigma + \Sigma^\dagger) + \frac{c_2}{16}\{\mathrm{Tr}(\Sigma + \Sigma^\dagger)\}^2$$

$c_2 > 0$: 2nd order

$c_2 < 0$: 1st order

Twisting



Sharpe-Wu PRD70('04)

$$S_q^{\rm TW} = a^4 \sum_x \bar{\psi}(x)[D_W + m_0 + i\gamma_5\tau_3\mu_0]\psi(x)$$

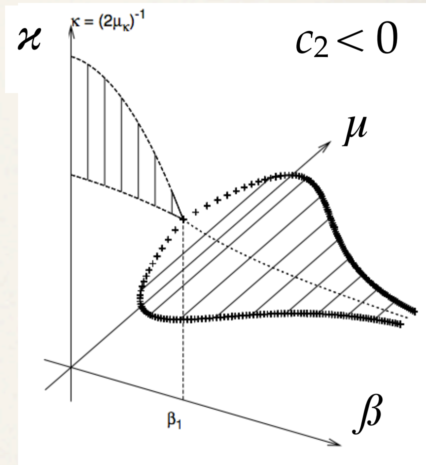
c_2 depends on the lattice action.

various glues + $S_q^{\rm TW}$ => 1st order

In this case, we have to avoid the 1st order region to approach the continuum limit.

Very light quarks without twisting possible only at small a.

Farchioni et al. EPJ C42('05)



S. Aoki et al. (PACS-CS) PRD79('09); PRD81('01)

- Iwasaki gauge + Clover (C_{SW}^{NP})
- $N_F = 2+1$, 32^3x64, $a = 0.09$fm (π,K,Ω input), MPDDHMC algorithm

➡ m_{ud} could be reduced down to the phys. point w/o encountering a 1st order transition

i.e., either $c_2>0$, or harmless 1st order at smaller m_q.

Phase structures with tmClover not well clarified yet.

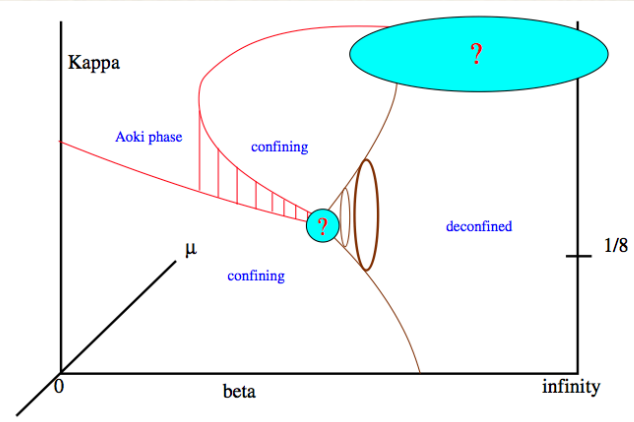
See also Becirevic et al. PRD74('06)
S.Aoki et al. PRD72('05)

$T > 0$

Creutz PRD76('07)

Cone-shaped deconfinement transition plane

Possible 1st order transition ($c_2 < 0$)
hidden in the "?" region.

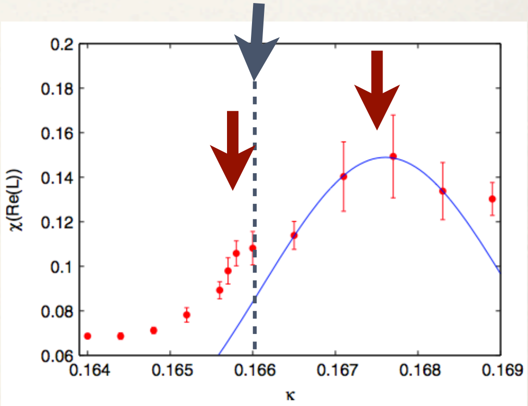


Illgenfritz et al. (tmfT) PRD80('09) Zeidlewicz (Mon)

- tree-level Symanzik gauge + tmWilson i.e. a $c_2 < 0$ case
- $N_F = 2$, $Nt = 8$

➡ consistent with cone-shaped deconfinement transition plane

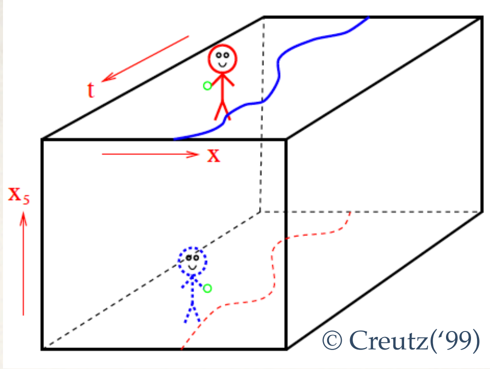
$\varkappa_c(T=0)$



$\mu_0 = 0.005$

Chiral quarks

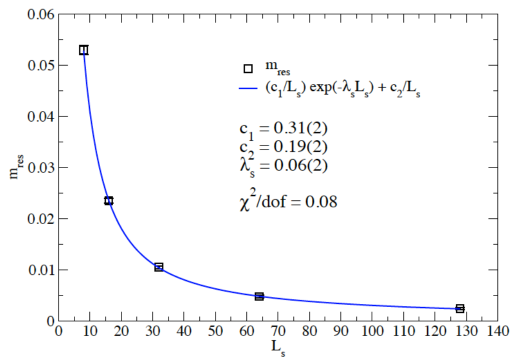
Most canonical.



✤ Domain-wall

Chirality realized in the limit Ls = ∞.   Ls: lattice size in the 5th direction.

m_{res}



Ls

Finite Ls => chiral violations => m_{res}

$m_q^{ren} = m_q^{bare} + m_{res}$ (*a la* Wilson quarks)

$m_{res} \sim 1/\mathrm{Ls}$ <= mobility edge

$T > 0$ simulations usually require coarse lattices
=> Control of chiral violations is a big issue.

First study: Chen et al. PRD64('01)

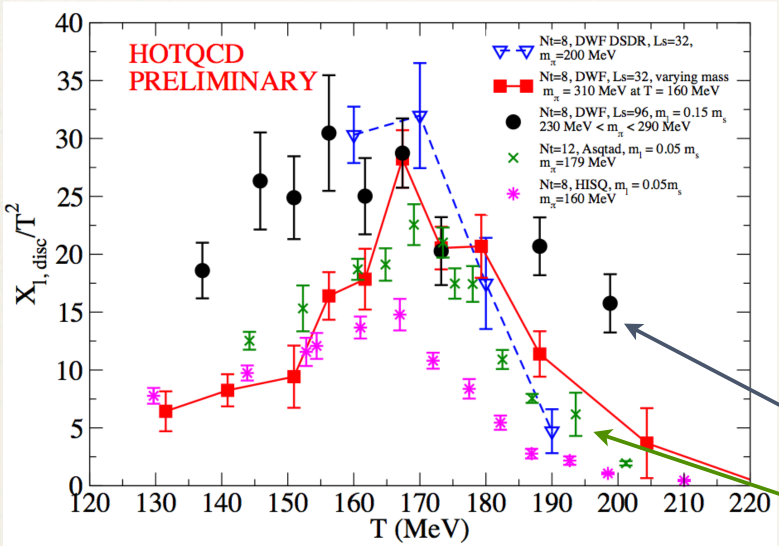
- plaquette gauge + DW
- $N_F = 2$, Nt = 4 (8^3x4), Ls = mainly 8

➡ Chiral modes confirmed, but large m_{res} effects.

New: Cheng et al. PRD81('10)

- Iwasaki gauge + DW
- $N_F = 2+1$, $Nt = 8$ (16^3x8), Ls = mainly 32
- a ~ 0.15fm, $m_\pi \sim 308$ MeV, $(m_l+m_{res})/(m_s+m_{res}) \sim 0.25$

Improved gauge & finer lattice & larger Ls
=> better control of chiral violations.



Qualitatively consistent with expectations.

Chiral violations not small enough.

$$m_{res} \sim 0.008 \;>>\; m_l = 0.003$$

Next steps (HotQCD):

- Ls = 96
- improved action dedicated for DW
  ("dislocation suppressing determinant ratio")

Overlap (fixed Q)

Cossu @ Lat10

- Iwasaki gauge + Overlap + Fukaya-term to suppress topology flips
- $N_F = 2$, $Nt = 8$, Q=0 sector

O(4) scaling with Wilson-type quarks

Because the chiral symmetry is explicitly violated with Wilson-type quarks, an additive renormalization is required to define the chiral condensate etc.

Proper renormalization for Wilson-type quarks:

renormalize such that Ward-Takahashi identities from the chiral symmetry are satisfied.

Axial vector Ward identites

Bochicchio et al. NP B256('85)

$$\nabla_\mu A_\mu(x) = 2m_q P(x) + O(a), \quad \text{etc.}; \qquad A_\mu \equiv \overline{\psi}\gamma_5\gamma_\mu\psi, \quad P \equiv \overline{\psi}\gamma_5\psi$$

This in turn leads us to **a definition of quark mass** (AWI quark mass):

$$m_q^{AWI} \equiv \frac{-m_\pi Z_A \langle 0|A_4|\pi(\mathbf{p}=\mathbf{0})\rangle}{Z_P \langle 0|P|\pi(\mathbf{p}=\mathbf{0})\rangle}$$

Itoh et al., NP B274('86)33; Maiani, Martinelli, PL B178('86)

Similarly, we can define **a (properly subtracted) chiral condensate**:

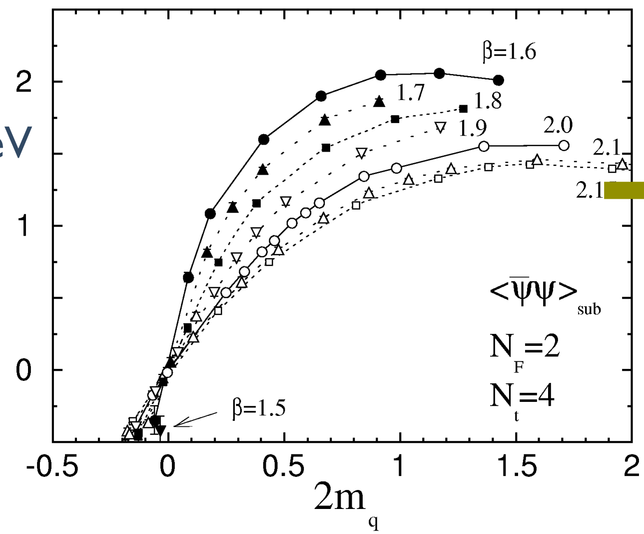
$$\langle \bar{\Psi}\Psi \rangle_{\rm sub} = 2m_q aZ \sum_x \langle \pi(x)\pi(0) \rangle$$

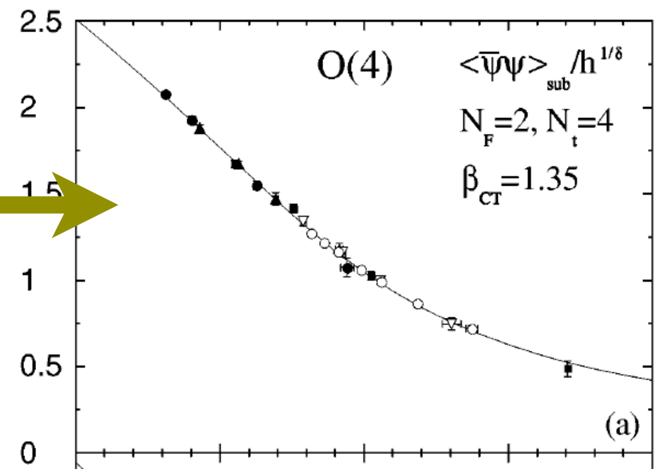
O(4) scaling with Wilson-type quarks

Test $M/h^{1/\delta} = f(t/h^{1/\beta\delta})$ with $M = \langle\bar{\Psi}\Psi\rangle_{\rm sub}, \quad h = 2m_q a, \quad t = \beta - \beta_{\rm ct}$

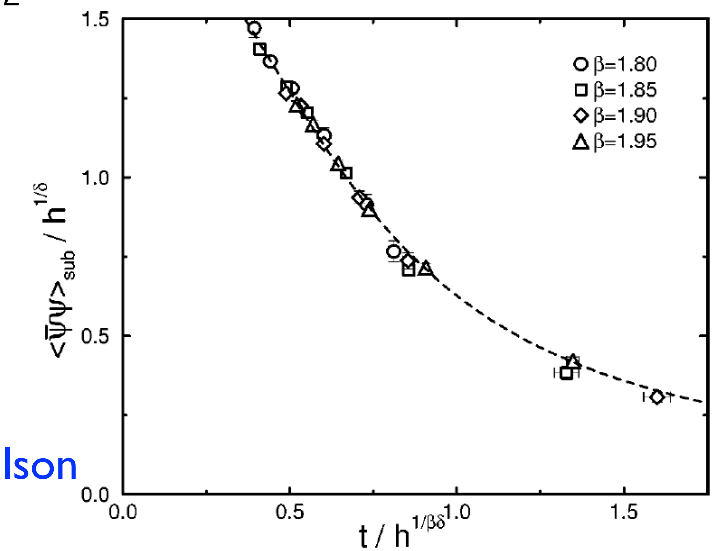
Iwasaki et al. PRL78('97)

- Iwasaki gauge + Wilson
- Nt=4, m_π ~ 600-900 MeV





QCD data well described by
O(4) critical exponents and
O(4) scaling function.



AliKhan et al.(CP-PACS) PRD63('01)

- Iwasaki gauge + clover-improved Wilson
- Nt=4, m_π ~ 600-1000 MeV

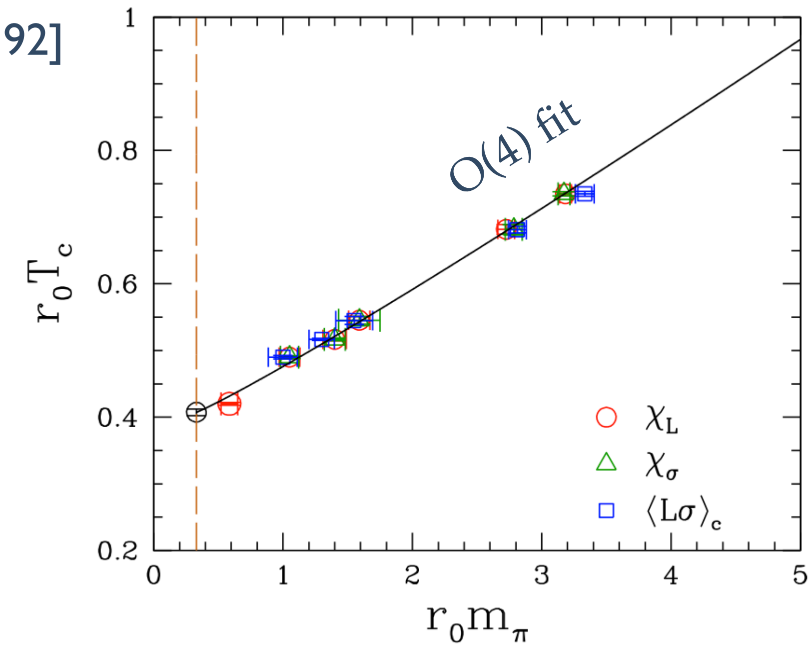
O(4) scaling with Wilson-type quarks

Bornyakov et al. (QCDSF-DIK Collab.) [arXiv:0910.2392]

- plaquette gauge + clover-improved Wilson (C_{SW}^{NP})
- $N_F = 2$, $Nt = 8,10,12$, $m_\pi \approx$ 420-1300 MeV

➡ "in accord with the predictions with the O(4) Heisenberg model"

➡ "a first order transition is very unlikely"



➡ Consistent with O(4) scaling, though quarks are heavy yet.

O(2) scaling with staggered-type quarks

★ O(4) vs. O(2)

Realize desired N_F through the 4th root trick: $\mathbf{det}M \Rightarrow [\mathbf{det}M]^{1/4}$

Sym. of the system = sym. of M

= the "remnant" chiral symmetry of staggered quarks: $e^{i(\gamma_5 \otimes \gamma_5)\theta}$ in the taste-chiral space

= U(1) = O(2) for any $a > 0$ and any N_F

=> When the chiral transition is 2nd order on finite lattices, we expect

O(2) scaling for any N_F

(assuming here that the non-locality does not invalidate the universality arguments at $a > 0$ too).

O(4) may appear only when we take the continuum limit prior to the chiral fits.

In practice, however, it is not easy to discriminate between O(2) and O(4) numerically.

O(2) scaling with staggered-type quarks

★ Results of early efforts (N_F=2): puzzling

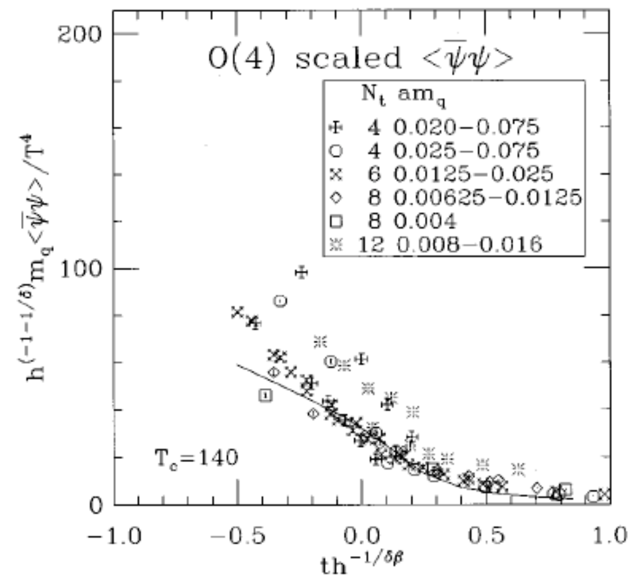
(all: *plaq. gauge* + *unimproved staggered*)

=> Transition looks continuous, but neither O(2) nor O(4)

Bielefeld ('94): $m_q a$=0.02-0.075 , Nt=4-8

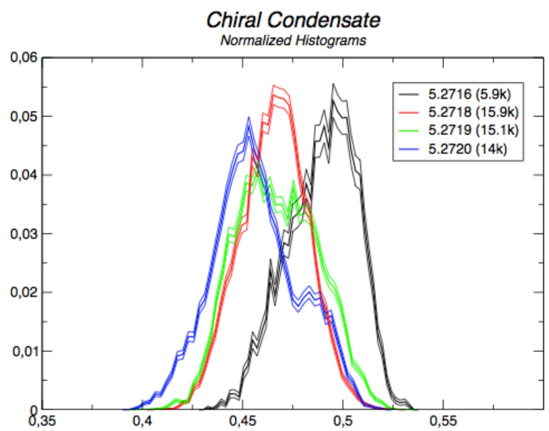
MILC ('94-96) :$m_q a$=0.008-0.075 , Nt=4-12 =>

JLQCD ('98): $m_q a$=0.01-0.075, Nt=4



=> 1st order?

Genova-Pisa ('05-08): $m_q a$=0.01335-, Nt=4



2 state signal?

O(2) scaling with staggered-type quarks

★ With improved staggered quarks (N_F=2+1):

Ejiri et al. (BNL-Bielefeld) PRD80('09)

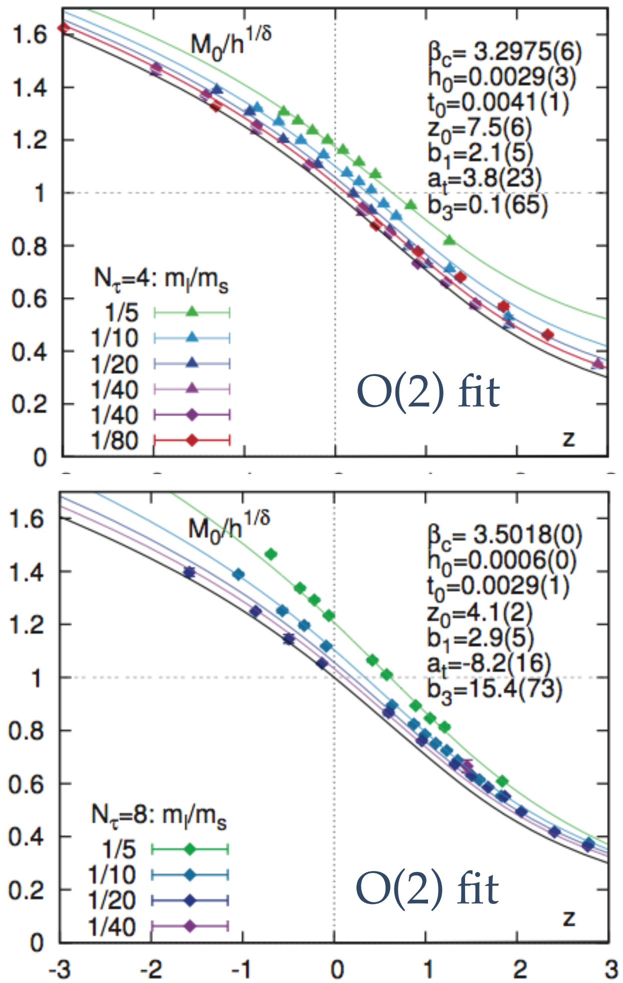
- tree-level improved Symanzik gauge + p4-improved staggered quarks
- $m_s \approx$ physical, m_l/m_s down to $1/80 - 1/20$ ($m_\pi^{\mathrm{pNG}} \approx 75 - 150$ MeV)
- $N_t = 4,\ \ 8$
- crossover region

Better control of the taste violation.

O(2) scaling with staggered-type quarks

★ With improved staggered quarks (N_F=2+1):

$$M_0 \;=\; m_s \langle \bar{\psi}\psi \rangle_l / T^4$$



➡ Consistent with O(2) scaling
[O(4) fit possible too, but it should be O(2).]

➡ Deviation for m_l/m_s > 1/20

➡ **Suggests a continuous transition in the chiral limit.**

➡ **Tricritical point may be lower than m_s^{phys}.**