The hadron spectrum on the lattice

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Topics in lattice hadron spectroscopy

- Introduction and motivation
- A small review
- Spin on the lattice

A constituent picture of hadrons

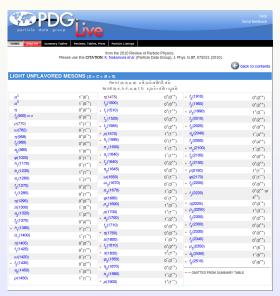
- QCD has quarks (in six flavours) and gluons
- The confinement conjecture: fields of the QCD lagrangian must be combined into colourless combinations: the mesons and baryons

A constituent model

constituents			quark model label
3 ⊗ 3	=	1 ⊕ 8	meson
3 ⊗ 3 ⊗ 3	=	1 ⊕ 8 ⊕ 8 ⊕ 10	baryon
8 & 8	=	$1 \oplus 8 \oplus 8 \oplus 10 \oplus 10$	glueball
3 ⊗ 8 ⊗ 3	=	1 ⊕ 8 ⊕ 8 ⊕ 8 ⊕ 10 ⊕ 10	hybrid
		:	:

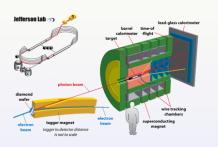
 QCD does not always respect this constituent labelling! There can be strong mixing.

The PDG view



What are these states? $\bar{q}q$ mesons?

The GlueX experiment at JLab



- 12 GeV upgrade to CEBAF ring
- New experimental hall: Hall D
- New experiment: GlueX
- Aim: photoproduce mesons, in particular the hybrid meson (with intrinsic gluonic excitations
- Expected to start taking data 2014

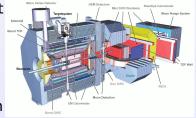
Panda@FAIR, GSI



- Extensive new construction at GSI Darmstadt
- Expected to start operation 2014

PANDA: Anti-<u>P</u>roton <u>AN</u>nihilation at <u>DA</u>rmstadt

- Anti-proton beam from FAIR on fixed-target.
- Physics goals include searches for hybrids and glueballs (as well as charm and baryon spectroscopy).



A renaissance in spectroscopy

- Early in the noughties, new narrow structures were seen by Belle and BaBar above the open-charm threshold.
- This led to substantial renewed interest in spectroscopy. Were these more quark-anti-quark states, or something more?
 - X(3872): very close to $D\bar{D}$ threshold a molecule?
 - Y(4260): a 1⁻⁻ hybrid?
 - $Z^{\pm}(4430)$: charged, can't be $\bar{c}c$.
- Very little is known and no clear picture seems to be emerging...

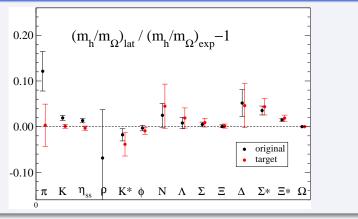
Lattice Hadron Spectroscopy

- Significant experimental effort hoping to understand light hadron and charm spectroscopy
 - Are there resonances that don't fit in the quark model?
 - Are there gluonic excitations in this spectrum?
 - What structure does confinement lead to?
 - How do resonances decay?
- To use LQCD to address these questions means:
 - identifying continuum properties of states
 - computing scattering and resonance widths
- To acheive this we need
 - Techniques that give statistical precision
 - Spin identification
 - Control over extrapolations $(m_a \to 0, V \to \infty, a \to 0.$

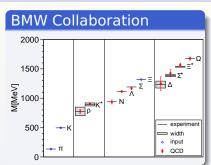
$N_f = 2 + 1$ simulations at the physical point

- First $N_f = 2 + 1$ simulations at physical quark mass parameters.
- PACS-CS computer, U Tsukuba. 14.3 Tflops peak
- Lattice spacing: a = 0.08995(40)fm (from m_{Ω}).

PACS-CS Collaboration [arXiv:0911.2561]



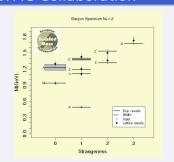
Convergence through universality





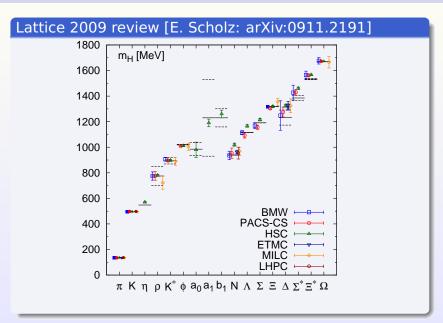


ETMC Collaboration



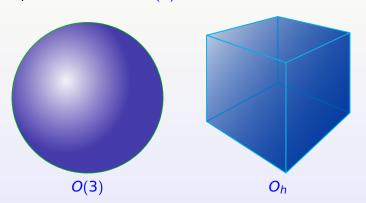
- BMW: SW-Wilson
- [Science 322:1224-1227,2008.]
- ETMC: Twisted Mass [arXiv:0910.2419,0803.3190]
- MILC: Staggered [arXiv:0903.3598]

Convergence through universality



A tale of two symmetries

 Continuum: states classified by J^P irreducible representations of O(3).



- Lattice regulator breaks O(3) → O_h
- Lattice: states classified by R^P "quantum letter" labelling irrep of O_h

Irreps of O_h

- O has 5 conjugacy classes (so O_h has 10)
- Number of conjugacy classes = number of irreps
- Schur: $24 = 1^2 + 1^2 + 2^2 + 3^2 + 3^2$
- These irreps are labelled A₁, A₂, E, T₁, T₂

		8 <i>C</i> ₃		6C ₄	
$\overline{A_1}$	1	1	1	1	1
A_2	1	1	-1	-1	1
E	2	-1	0	0	2
T_1	3	0	-1	1 -1 0 1	-1
<i>T</i> ₂	3	0	1	-1	-1

Spin on the lattice

- O_h has 10 irreps: $\{A_1^{g,u}, A_2^{g,u}, E^{g,u}, T_1^{g,u}, T_2^{g,u}, \}$, where $\{g, u\}$ label even/odd parity.
- Link to continuum: subduce representations of O(3) into O_b

	<i>A</i> ₁	A_2	E	<i>T</i> ₁	<i>T</i> ₂
J = 0	1				
J=1				1	
J = 2			1		1
J=3		1		1	1
<i>J</i> = 4	1		1	1	1
:	:	:	:	:	:

Baryons and double-cover irreps

- For fermions, need to consider irreps of rotation group with double cover.
- This has three more conjugacy classes, so three more irreps.
- $2^2 + 2^2 + 4^2 = 24$. Labelled G_1 , G_2 and H.

J	G_1	G_2	Н
$\frac{1}{2}$	1	0	0
3 2	0	0	1
<u>5</u>	0	1	1
$\frac{7}{2}$	1	1	1
1232527292	1	0	2
:	:	÷	:

Example: $J^{PC} = 2^{++}$ meson creation operator

Need more information to discriminate spins.
 Consider continuum operator that creates a 2⁺⁺ meson:

$$\Phi_{ij} = \bar{\psi} \left(\gamma_i D_j + \gamma_j D_i - \frac{2}{3} \delta_{ij} \gamma \cdot D \right) \psi$$

- Lattice: Substitute gauge-covariant lattice finite-difference D_{latt} for D
- A reducible representation:

$$\begin{split} & \Phi^{T_2} = \{\Phi_{12}, \Phi_{23}, \Phi_{31}\} \\ & \Phi^E = \left\{ \frac{1}{\sqrt{2}} (\Phi_{11} - \Phi_{22}), \frac{1}{\sqrt{6}} (\Phi_{11} + \Phi_{22} - 2\Phi_{33}) \right\} \end{split}$$

Look for signature of continuum symmetry:

$$\langle 0|\Phi^{(T_2)}|2^{++(T_2)}\rangle = \langle 0|\Phi^{(E)}|2^{++(E)}\rangle$$

