Baryon Interactions on the Lattice

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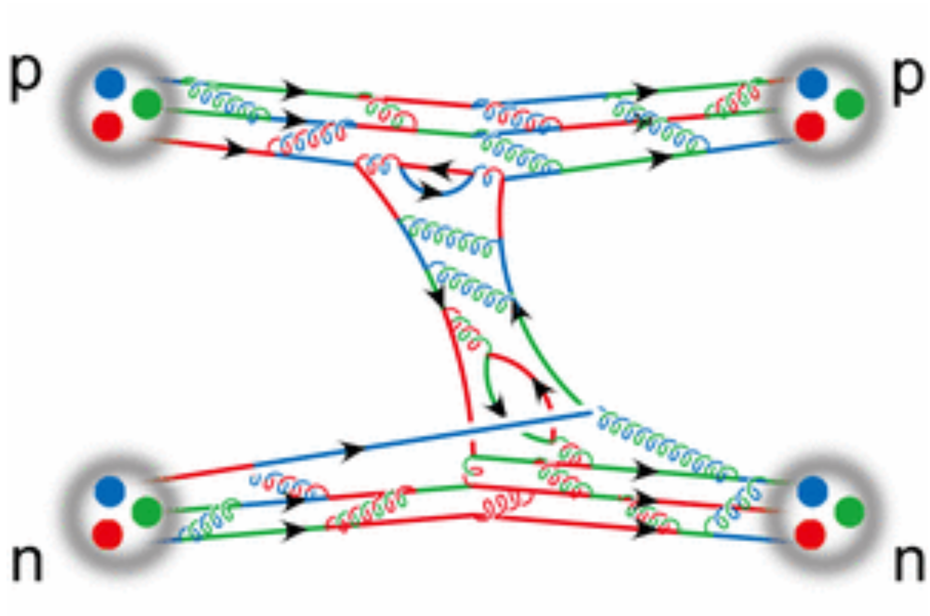
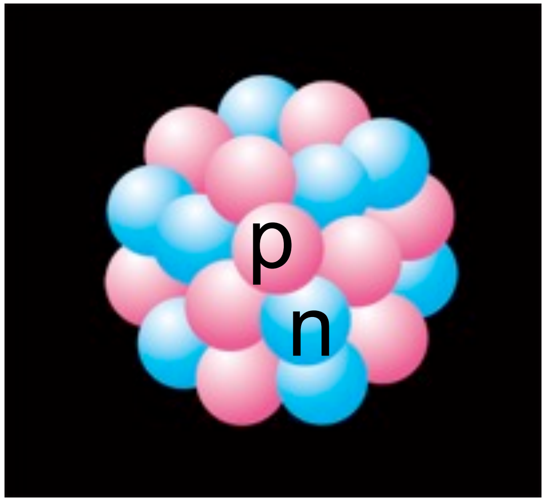


"Asian School on Lattice Field Theory"
March 14-25, 2011
Tata Institute of Fundamental Research, Mumbai, India

I would like to thank you for your support and encouragement to peoples in Japan from all over the world.
We, Japanese, should and will fight against difficult situations now to overcome this tragedy, together with your help.

1. Introduction

What binds protons and neutrons inside a nuclei ?



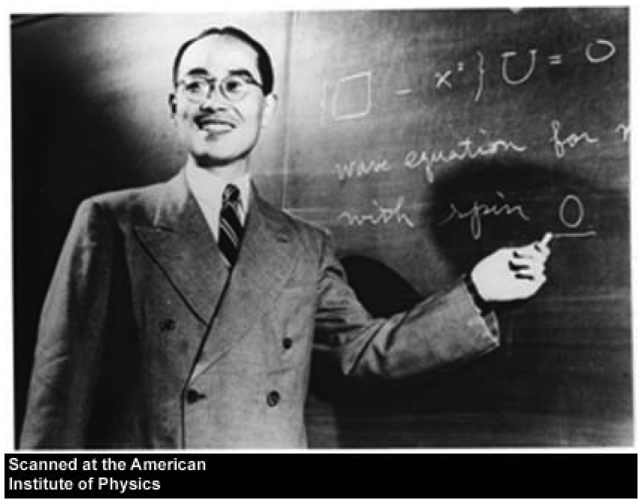
gravity: too weak

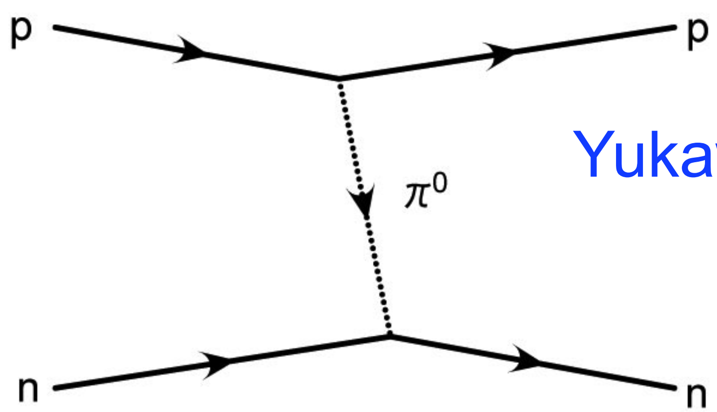
Coulomb: repulsive between pp
no force between nn, np

New force (nuclear force) ?

1935 H. Yukawa

introduced virtual particles (mesons) to explain the nuclear force





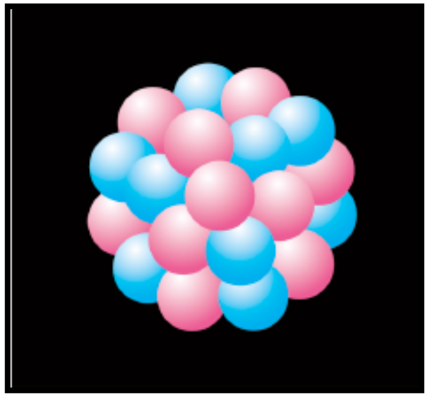
Yukawa potential

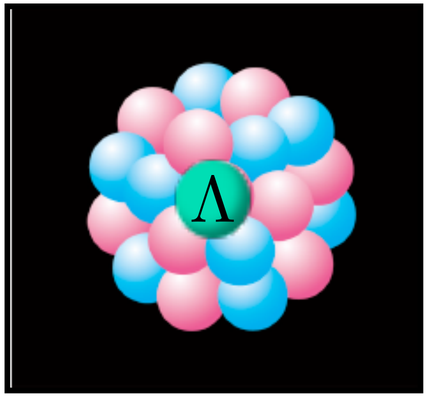
$$V(r) = \frac{g^2}{4\pi} \frac{e^{-m_\pi r}}{r}$$

1949 Nobel prize

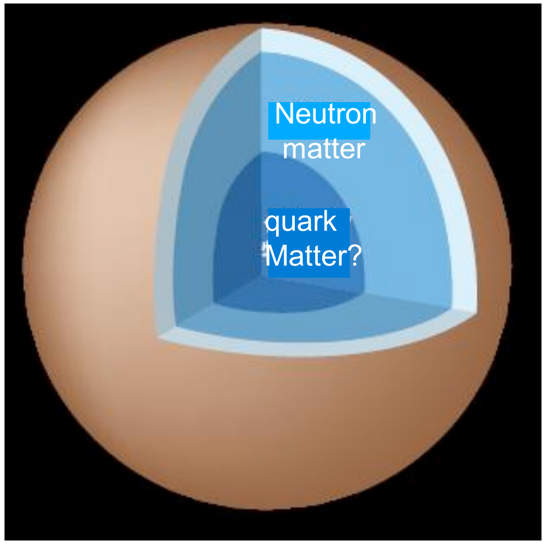
Nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei





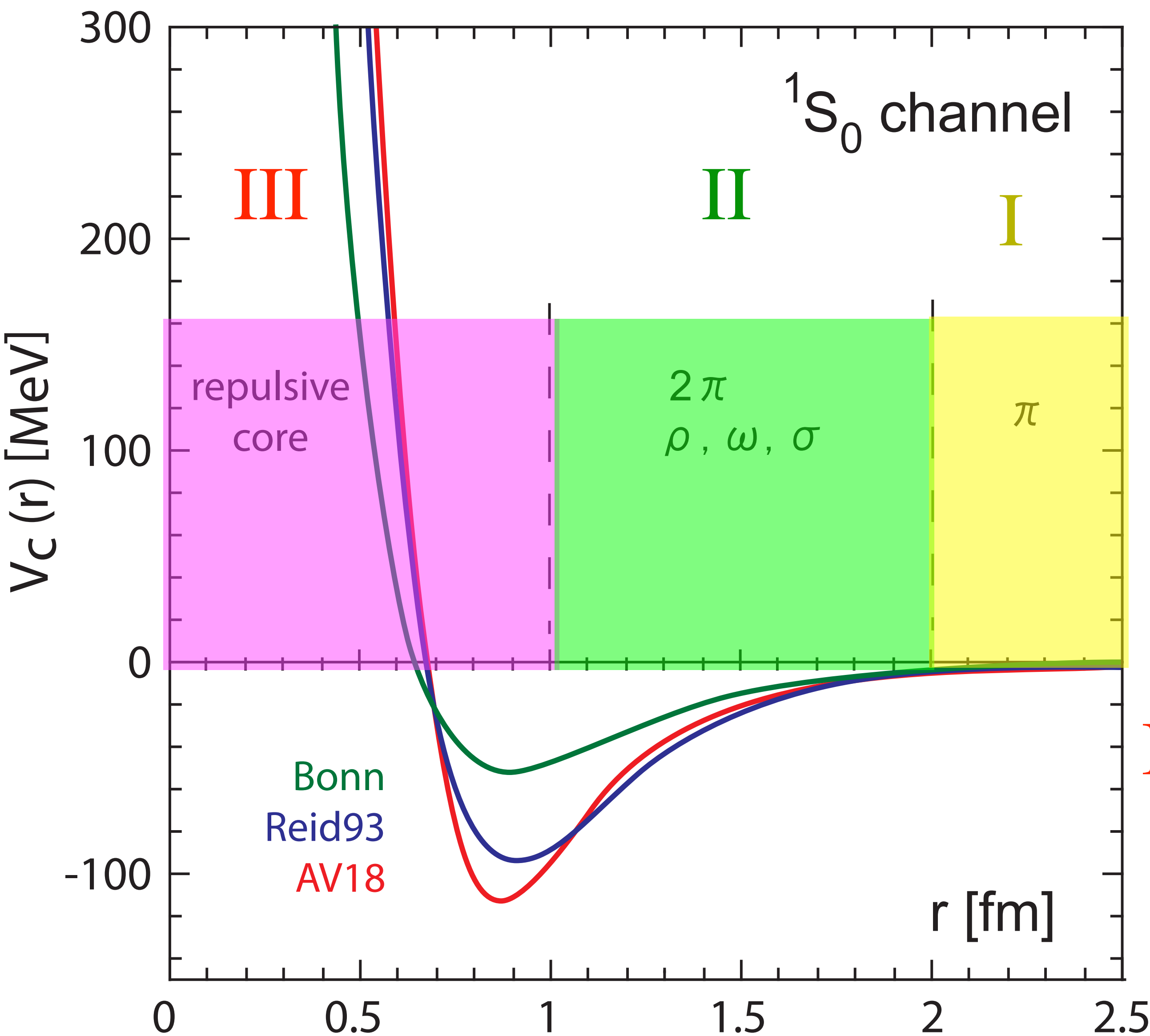
- Structure of neutron star



- Ignition of Type II SuperNova

Phenomenological NN potential

(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

Yiukawa(1935)

II Multi-pions

Taketani et al.(1951)

III Repulsive core

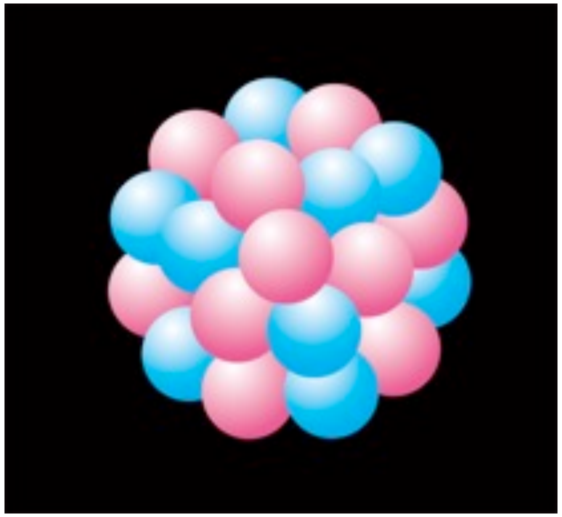
Jastrow(1951)

Repulsive core is important

stability of nuclei

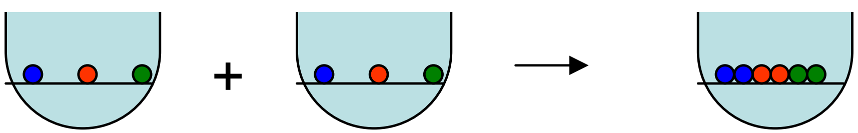
maximum mass of neutron star

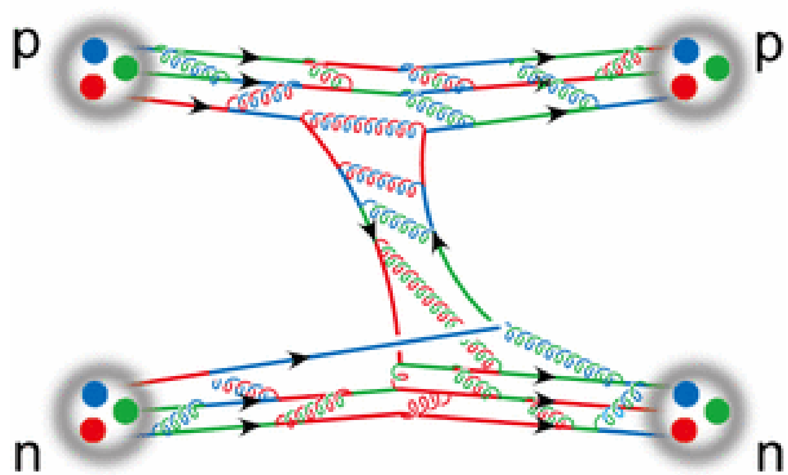
explosion of type II supernova



Origin of RC: “The most fundamental problem in Nuclear physics.”

Note: Pauli principle is not essential for the “RC”.

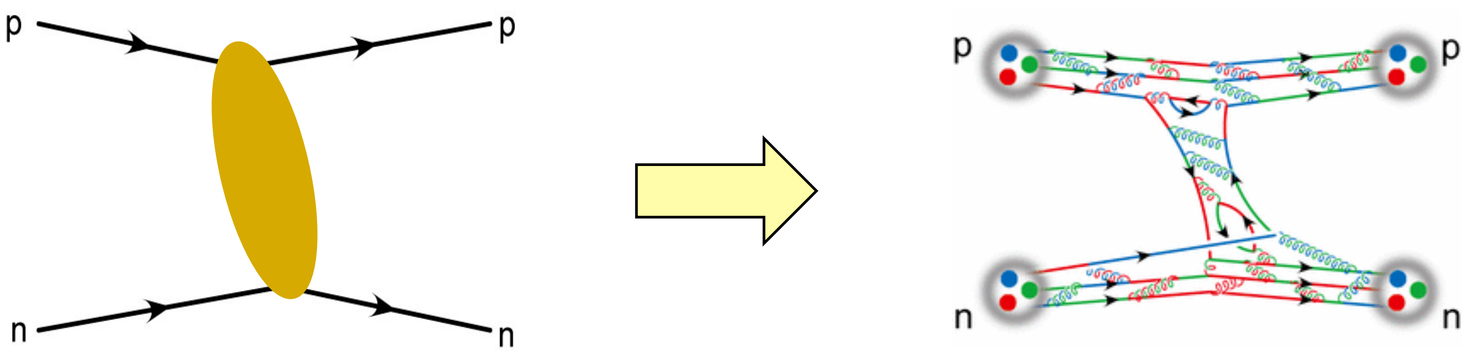




QCD based explanation is needed

Lattice QCD can explain ?

2. Strategy to extract "potential" in QCD



Y. Nambu, "Quarks : Frontiers in Elementary Particle Physics", World Scientific (1985)

"Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task."

2-1. Nambu-Bethe-Salpeter (NBS) wave function

$$\varphi^W(\mathbf{x})e^{-Wt} = \langle 0|T\{N(\mathbf{r}+\mathbf{x},t)N(\mathbf{r},t)\}|2N, W, s_1 s_2\rangle \qquad W = 2\sqrt{\mathbf{k}^2+m_N^2}$$

QCD eigen-state for 2 nucleons with energy W

local nucleon operator $N_\alpha(x) \equiv \begin{pmatrix} p_\alpha(x) \\ n_\alpha(x) \end{pmatrix} = \varepsilon^{abc}\left(u_a(x)C\gamma_5 d_b(x)\right)q_\alpha(x), \quad q(x) = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}$

Below pion-production threshold $W < 2m_N + m_\pi$ $\qquad S = e^{2i\delta}$ S-matrix

$r \longrightarrow \infty$

$$\varphi^W(\mathbf{r})_{S=0} \simeq \sum_{l,l_z} Z^{l,l_z}(S=0)Y_{ll_z}(\Omega_{\mathbf{r}})\frac{\sin(kr - l\pi/2 + \delta_{l0}(k))}{kr}e^{i\delta_{l0}(k)}$$

spin-singlet

$$\varphi^W(\mathbf{r})_{S=1} \propto \sum Y_{ll_z}(\Omega_{\mathbf{r}})\frac{\sin(kr - l\pi/2 + \delta_{l1}(k))}{kr}e^{i\delta_{l1}(k)}.$$

spin-triplet

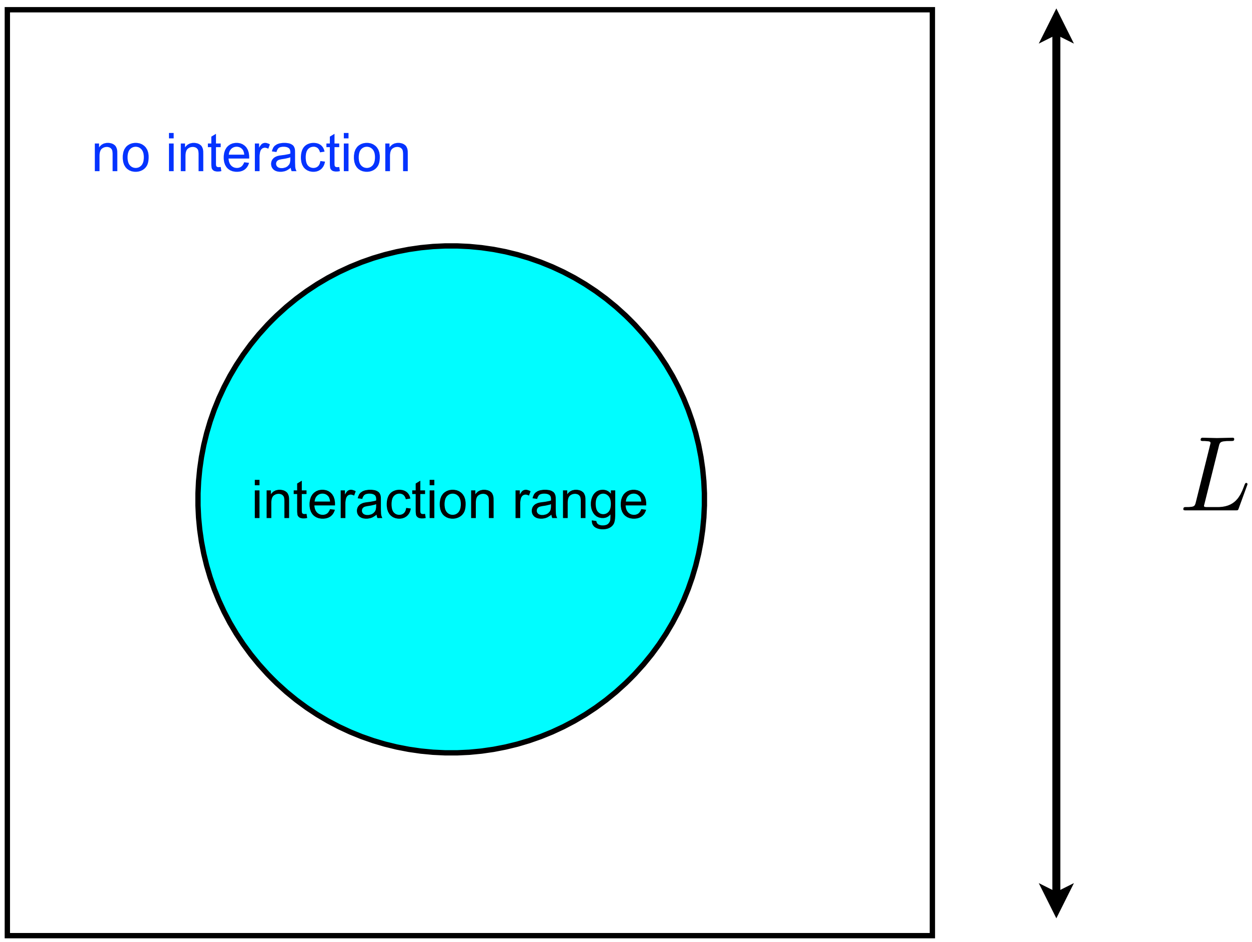
$$1/2 \otimes 1/2 = 0 \oplus 1$$

$$\left[\frac{k^2}{2\mu} - H_0\right]\varphi^W(\mathbf{r}) \simeq 0, \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

$\delta_l(k)$ is the scattering phase shift

Finite volume → allowed value: k_n^2

→ Lueshcer's formula $\delta_l(k_n)$



2-2. Non-local potential from the NBS wave function

Define $$[E_k - H_0]\,\varphi^W_{\alpha\beta}(\mathbf{x}) = \int U_{\alpha\beta;\gamma\delta}(\mathbf{x},\mathbf{y})\varphi^W_{\gamma\delta}(\mathbf{y})d^3y, \qquad E_k = \frac{k^2}{2\mu}.$$

4x4 $\quad \alpha,\beta,\gamma,\delta = 1,2$

1. U is E-independent.

$$N(W_1,W_2) = \int \varphi^{W_1}(\mathbf{r})^\dagger \varphi^{W_2}(\mathbf{r})\,d^3r,$$

$$P^{W_{\rm th}}(\mathbf{x},\mathbf{y}) = \int_{W_{1,2}\le W_{\rm th}} \rho(W_1)dW_1\,\rho(W_2)dW_2\,\varphi^{W_1}(\mathbf{x})N^{-1}(W_1,W_2)(\varphi^{W_2})^*(\mathbf{y}) \equiv \int_{W_1\le W_{\rm th}} \rho(W_1)dW_1\,P(W_1;\mathbf{x},\mathbf{y})$$

$$U^{W_{\rm th}}(\mathbf{x},\mathbf{y}) = \int_{W_{1,2}\le W_{\rm th}} \rho(W_1)dW_1\,\rho(W_2)dW_2\,[E_k-H_0]\,\varphi^{W_1}(\mathbf{x})N^{-1}(W_1,W_2)(\varphi^{W_2})^*(\mathbf{y})$$
$$= \int_{W_1\le W_{\rm th}} \rho(W_1)dW_1\,[E_k-H_0]\,P(W_1;\mathbf{x},\mathbf{y}).$$

$$\int U(\mathbf{x},\mathbf{y})^{W_{\rm th}}\varphi^W(\mathbf{y})\,d^3y = \int_{W_1\le W_{\rm th}} \rho(W_1)dW_1\,[E_k-H_0]\,\varphi^{W_1}(\mathbf{x})\frac{1}{\rho(W)}\delta(W_1-W) = \theta(W_{\rm th}-W)\,[E_k-H_0]\,\varphi^W(\mathbf{x})$$

2. U is not unique. We can freely add $$\int_{W>W_{\rm th}} \rho(W)\,dW\,f_W(\mathbf{x})P(W;\mathbf{x},\mathbf{y})$$

For example
$$U^\infty(\mathbf{x},\mathbf{y}) = \int_0^\infty \rho(W)dW\,[E_k-H_0]\,P(W;\mathbf{x},\mathbf{y})$$

2-3. Velocity expansion of the non-local potential

Velocity expansion $$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

$$\mathbf{L} = \mathbf{r}\times\mathbf{p} \qquad \mathbf{S} = (\vec{\sigma}_1+\vec{\sigma}_2)/2$$

At the lowest few orders

$$V(\mathbf{r},\nabla) = \underbrace{V_0(r) + V_\sigma(r)\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T(r)S_{12}}_{\mathrm{LO}} + \underbrace{V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S}}_{\mathrm{NLO}} + O(\nabla^2)$$

tensor operator $$S_{12} = 3\frac{(\mathbf{r}\cdot\vec{\sigma}_1)(\mathbf{r}\cdot\vec{\sigma}_2)}{r^2} - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

Isospin decomposition

$$V_X(r) = V_X^0(r) + V_X^\tau(r)\vec{\tau}_1\cdot\vec{\tau}_2, \qquad X = 0,\sigma,T,\mathrm{LS},\cdots,$$

LO local potential $$V^{\mathrm{LO}}(\mathbf{r}) = V_0(r) + V_\sigma(r)\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T(r)S_{12},$$

ex: spin-singlet $$V_c(r,S=0) \equiv V_c(r) - 3V_\sigma(r) = \frac{[E_k - H_0]\,\varphi^W(\mathbf{r})}{\varphi^W(\mathbf{r})}.$$

2-4. Remarks

[Q1] Scheme/Operator dependence of the potential

- the potential depends on the definition of the wave function, in particular, on the choice of the nucleon operator N(x). (Scheme-dependence)
  - local operator = convenient choice for reduction formula
- Moreover, the potential itself is NOT a physical observable. Therefore it is NOT unique and is naturally scheme-dependent.
  - Observables: scattering phase shift of NN, binding energy of deuteron

- Is the scheme-dependent potential useful ? Yes !
  - useful to understand/describe physics
  - a similar example: running coupling
    - Although the running coupling is scheme-dependent, it is useful to understand the deep inelastic scattering data (asymptotic freedom).
- "good" scheme ?
  - good convergence of the perturbative expansion for the running coupling.
  - good convergence of the derivative expansion for the potential ?
    - completely local and energy-independent one is the best and must be unique if exists. (Inverse scattering method)

[Q2] Energy dependence of the potential

Non-local, E-independent ⟺ Local, E-dependent

$$\left(E+\frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y})$$

$$V_E(\mathbf{x})\varphi_E(\mathbf{x}) = \left(E+\frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x})$$

non-locality can be determined order by order in velocity expansion (cf. ChPT)

$$V(\mathbf{x},\nabla) = V_C(r) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \{V_D(r),\nabla^2\} + \cdots$$

If the velocity expansion is good,

↓

“QCD” justifies the use of quantum mechanism with potential to describe the NN system.

Later explicitly consider this problem.

3. Lattice formulation

4-pt Correlation function

$$F(\mathbf{r}, t - t_0) = \langle 0|T\{N(\mathbf{x}+\mathbf{r}, t)N(\mathbf{x}, t)\}\underline{\overline{\mathcal{J}}(t_0)}|0\rangle$$

source for NN

complete set

$$\begin{aligned} F(\mathbf{r}, t - t_0) &= \langle 0|T\{N(\mathbf{x}+\mathbf{r}, t)N(\mathbf{x}, t)\} \sum_{n,s_1,s_2} \underline{|2N, W_n, s_1, s_2\rangle\langle 2N, W_n, s_1, s_2|}\overline{\mathcal{J}}(t_0)|0\rangle \\ &= \sum_{n,s_1,s_2} A_{n,s_1,s_2}\varphi^{W_n}(\mathbf{r})e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2|\overline{\mathcal{J}}(0)|0\rangle. \end{aligned}$$

Large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r}, t - t_0) = A_0\varphi^{W_0}(\mathbf{r})e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

It is important to find the large t region where this condition is approximately satisfied while the signal is still reasonably good.

3-1. Choice of source operator

Spacial symmetry in the hyper-cubic lattice $SO(3,\mathbf{R}) \Rightarrow SO(3,\mathbf{Z})$

Representation A_1, A_2, E, T_1, T_2

P: parity

Total spin J=L+S L S

$$R_1 \otimes R_2$$

	A_1	A_2	E	T_1	T_2
A_1	A_1	A_2	E	T_1	T_2
A_2	A_2	A_1	E	T_2	T_1
E	E	E	$A_1 \oplus A_2 \oplus E$	$T_1 \oplus T_2$	$T_1 \oplus T_2$
T_1	T_1	T_2	$T_1 \oplus T_2$	$A_1 \oplus E \oplus T_1 \oplus T_2$	$A_2 \oplus E \oplus T_1 \oplus T_2$
T_2	T_2	T_1	$T_1 \oplus T_2$	$A_2 \oplus E \oplus T_1 \oplus T_2$	$A_1 \oplus E \oplus T_1 \oplus T_2$

L	P	A_1	A_2	E	T_1	T_2
0 (S)	+	1	0	0	0	0
1 (P)	−	0	0	0	1	0
2 (D)	+	0	0	1	0	1
3 (F)	−	0	1	0	1	1
4 (G)	+	1	0	1	1	1
5 (H)	−	0	0	1	2	1
6 (I)	+	1	1	1	1	2

A_1 spin-singlet(S=0) T_1 spin-triplet(S=1)

Wall source $\mathcal{J}^{\rm wall}(t_0)_{\alpha\beta,fg} = N^{\rm wall}_{\alpha,f}(t_0) N^{\rm wall}_{\beta,g}(t_0)$ $q^{\rm wall}(t_0) \equiv \sum_{\mathbf{x}} q(\mathbf{x},t_0)$

L=0 A_1^+

(J,I) conserved + fermion $(-1)^{S+1+I+1}P = -1$

S is also conserved.
But L is NOT.

$$t = t_0 \qquad \mathcal{J}(t_0; J^{P=+}, I) = P^{(S)}_{\beta\alpha} \mathcal{J}^{\rm wall}(t_0)_{\alpha\beta, fg} \qquad (S, I) = (0,1), (1,0)$$

spin projection

spin-singlet $(J^P, I) = (A_1^+, 1)$ spin-triplet $(J^P, I) = (T_1^+, 0)$

$$A_1^+ \otimes A_1 = A_1^+ \qquad A_1^+ \otimes T_1 = T_1^+$$

$$t > t_0 \qquad R_1 \otimes A_1 = A_1^+ \qquad R_1 \otimes T_1 = T_1^+$$

$$\Rightarrow R_1 = A_1^+ \qquad \Rightarrow R_1 = A_1^+, E^+, T_1^+, T_2^+$$

$$L = 0, 4, 6, \cdots \qquad L = 0, 2, 6, \cdots$$

	A_1	A_2	E	T_1	T_2
A_1	A_1	A_2	E	T_1	T_2
A_2	A_2	A_1	E	T_2	T_1
E	E	E	$A_1 \oplus A_2 \oplus E$	$T_1 \oplus T_2$	$T_1 \oplus T_2$
T_1	T_1	T_2	$T_1 \oplus T_2$	$A_1 \oplus E \oplus T_1 \oplus T_2$	$A_2 \oplus E \oplus T_1 \oplus T_2$
T_2	T_2	T_1	$T_1 \oplus T_2$	$A_2 \oplus E \oplus T_1 \oplus T_2$	$A_1 \oplus E \oplus T_1 \oplus T_2$

L	P	A_1	A_2	E	T_1	T_2
0 (S)	+	1	0	0	0	0
1 (P)	−	0	0	0	1	0
2 (D)	+	0	0	1	0	1
3 (F)	−	0	1	0	1	1
4 (G)	+	1	0	1	1	1
5 (H)	−	0	0	1	2	1
6 (I)	+	1	1	1	1	2

$t > t_0$

$$\varphi^W(\mathbf{r}; J^P, I, L, S) = P^{(L)} P^{(S)} \varphi^W(\mathbf{r}; J^P, I)$$

Projection

$$P^{(L)}\phi(\mathbf{r}) \equiv \frac{d_L}{24} \sum_{g \in SO(3,\mathbf{Z})} \chi^L(g) \phi(g^{-1} \cdot \mathbf{r})$$

character

$$L = A_1, A_2, E, T_1, T_2$$

$$F(\mathbf{r}, t - t_0; J^P, I) \simeq A(J^P, I) \varphi^W(\mathbf{r}; J^P, I) e^{-W(t-t_0)}$$

$$A(J^P, I) = \langle 2N, W | \bar{\mathcal{J}}(0; J^P, I) | 0 \rangle$$

3-2. Leading order potential: spin-singlet

LO potential $$V^{\rm LO}(\mathbf{r}) = V_0(r) + V_\sigma(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(r) S_{12}.$$

Spin-singlet potential $$(J^P, I) = (A_1^+, 1)$$

$$V_C(r)^{(S,I)=(0,1)} \equiv V_0^{I=1}(r) - 3V_\sigma^{I=1}(r) = \frac{[E_k - H_0]\,\varphi^W(\mathbf{r}; A_1^+, I=1, A_1, S=0)}{\varphi^W(\mathbf{r}; A_1^+, I=1, A_1, S=0)}$$

$$V_X^{I=1} = V_X^0 + V_X^\tau \qquad\qquad L = 0, 4, 6, \cdots$$

3-3. Leading order potential: spin-triplet

$$\left[H_0 + V_C(r)^{(S,I)=(1,0)} + V_T(r)S_{12}\right]\varphi^W(\mathbf{r}; J^P = T_1^+, I=0) = E_k \varphi^W(\mathbf{r}; J^P = T_1^+, I=0)$$

$$V_C(r)^{(S,I)=(1,0)} \equiv V_0^{I=0}(r) + V_\sigma^{I=0}(r), \qquad V_X^{I=0} = V_X^0 - 3V_X^\tau.$$

$$\mathcal{P}\varphi^W_{\alpha\beta} \equiv P^{(A_1)}\varphi^W_{\alpha\beta}(\mathbf{r}; J^P = T_1^+, I=0) \qquad A_1^+(L = \text{"0"})$$

$$\mathcal{Q}\varphi^W_{\alpha\beta} \equiv P^{(E)}\varphi^W_{\alpha\beta}(\mathbf{r}; J^P = T_1^+, I=0) \simeq (1 - P^{(A_1)})\varphi^W_{\alpha\beta}(\mathbf{r}; J^P = T_1^+, I=0) \qquad \text{non } A_1^+(L = \text{"2"})$$

central
$$V_C(r)^{(1,0)} = E_k - \frac{1}{\Delta(\mathbf{r})}\left([\mathcal{Q}S_{12}\varphi^W]_{\alpha\beta}(\mathbf{r})H_0[\mathcal{P}\varphi^W]_{\alpha\beta}(\mathbf{r}) - [\mathcal{P}S_{12}\varphi^W]_{\alpha\beta}(\mathbf{r})H_0[\mathcal{Q}\varphi^W]_{\alpha\beta}(\mathbf{r})\right)$$

tensor
$$V_T(r) = \frac{1}{\Delta(\mathbf{r})}\left([\mathcal{Q}\varphi^W]_{\alpha\beta}(\mathbf{r})H_0[\mathcal{P}\varphi^W]_{\alpha\beta}(\mathbf{r}) - [\mathcal{P}\varphi^W]_{\alpha\beta}(\mathbf{r})H_0[\mathcal{Q}\varphi^W]_{\alpha\beta}(\mathbf{r})\right)$$

$$\Delta(\mathbf{r}) \equiv [\mathcal{Q}S_{12}\varphi^W]_{\alpha\beta}(\mathbf{r})[\mathcal{P}\varphi^W]_{\alpha\beta}(\mathbf{r}) - [\mathcal{P}S_{12}\varphi^W]_{\alpha\beta}(\mathbf{r})[\mathcal{Q}\varphi^W]_{\alpha\beta}(\mathbf{r}).$$

effective central potential

$$V_C^{\text{eff}}(r)^{(1,0)} = \frac{[E_k - H_0]\,\mathcal{P}\varphi^W_{\alpha\beta}(\mathbf{r})}{\mathcal{P}\varphi^W_{\alpha\beta}(\mathbf{r})} = V_C(r)^{(1,0)} + O(V_T^2)$$

3-4. A comparison with the finite volume method

- the local potential gives correct phase shift at $k = \sqrt{W^2/4 - m_N^2}$
- but it gives approximated phase shift at other k
- the finite size correction yo the potential is expected to be small
- the quark mass dependence of the potential is much smaller that that of the phase shift
- at the leading order, contaminations from L=4,6,... do not cause problems for the potential.

4. Results for nuclear potential from lattice QCD

4-1. Quenched QCD results for (effective) central potentials

- plaquette gauge action at β=5.7(quenched) on 32^4 lattice
  - a=0.137 fm from ρ meson mass ⇒ physical size: $(4.4\ \text{fm})^4$
- Wilson quark action
  - 3 quark masses

$$m_\pi = 370\ \text{MeV}(2000\ \text{conf}),\ m_\pi = 527\ \text{MeV}\ (2000\ \text{conf})$$

$$m_\pi = 732\ \text{MeV}(1000\ \text{conf})$$

Blue Gene/L @ KEK(stop operating in this January)

10 racks, 57.3 TFlops peak

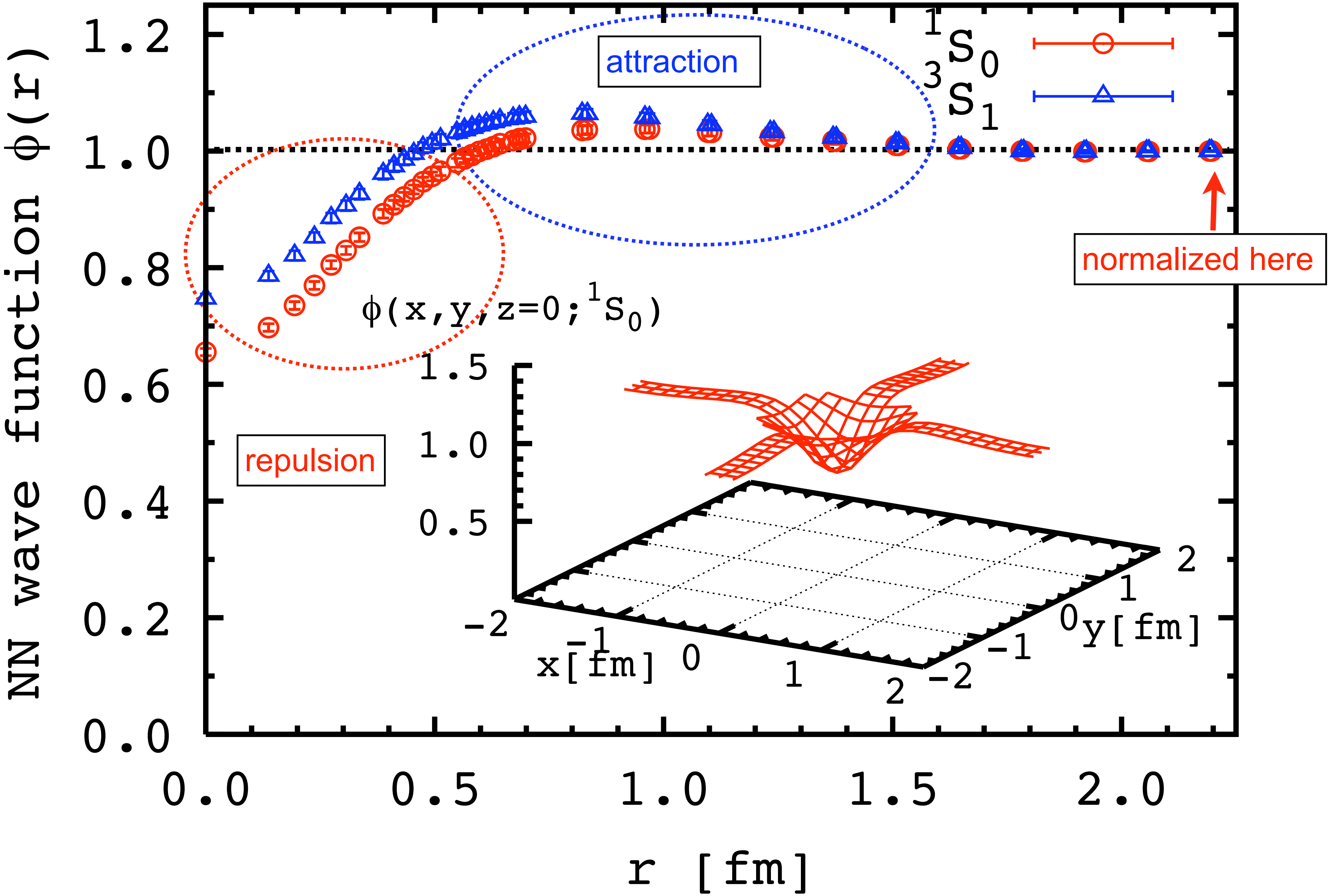
34-48 % of peak performance

about 4000 hours of
512 Node(half-rack, 2.87TFlops)

NN wave function

$m_\pi \simeq 0.53$ GeV

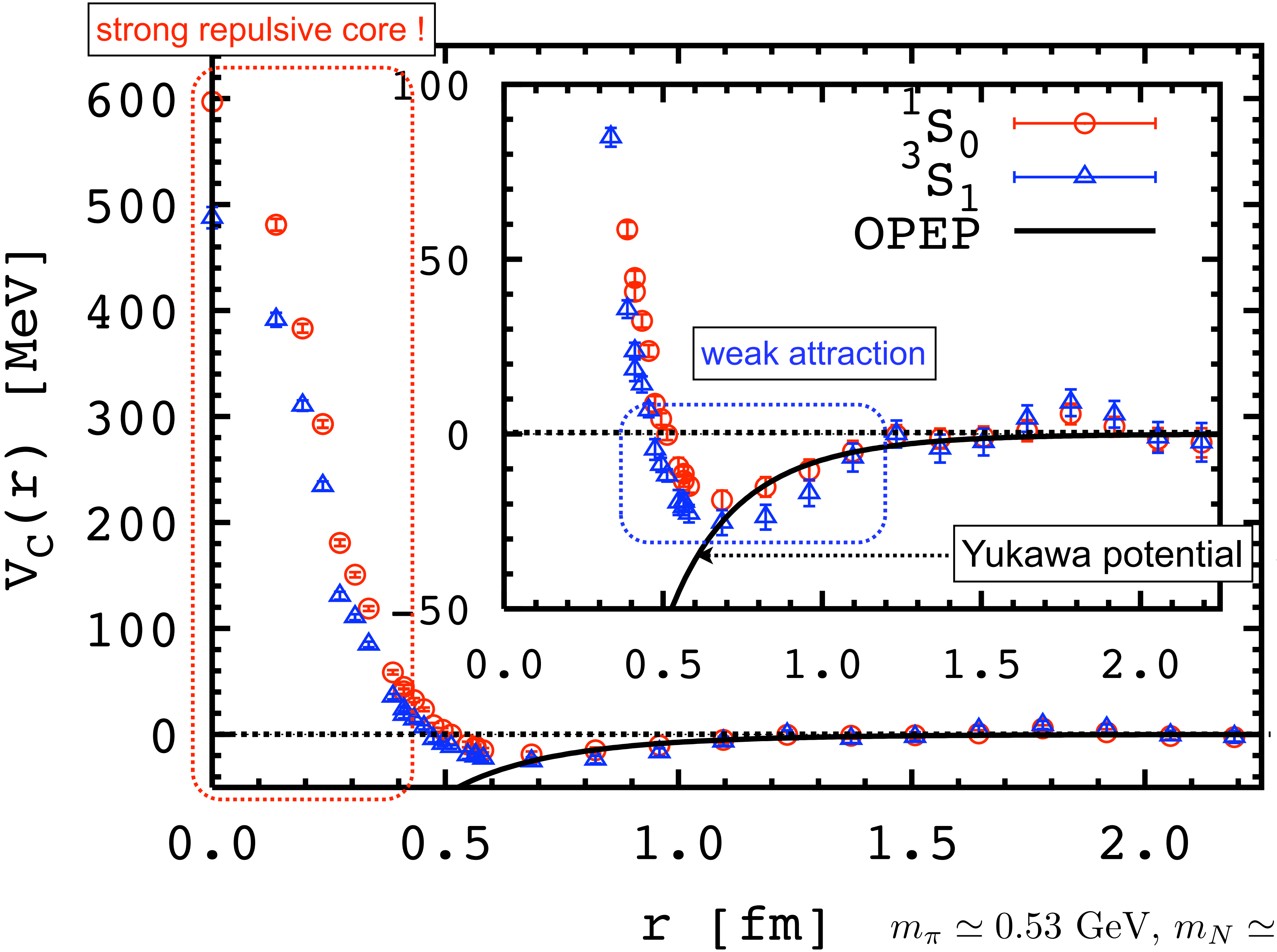
$t - t_s = 6$



NN (effective) central potentials

$m_\pi \simeq 0.53$ GeV

$t - t_s = 6$



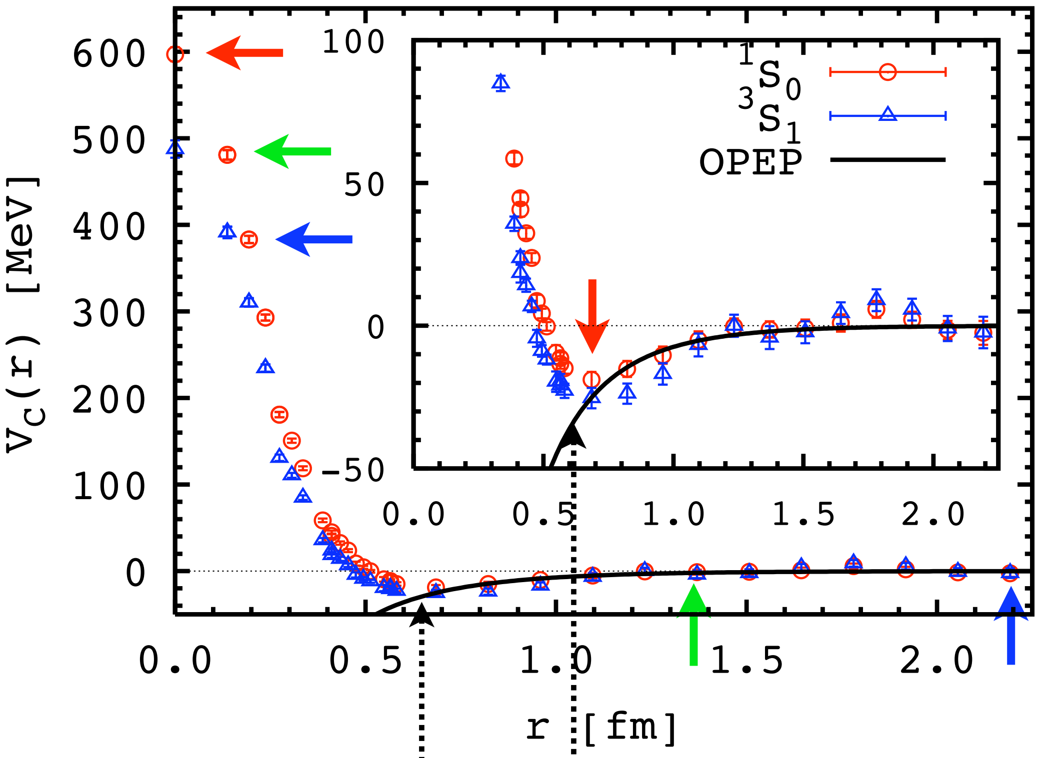
$\frac{g^2_{\pi N}}{4\pi} \simeq 14.0$

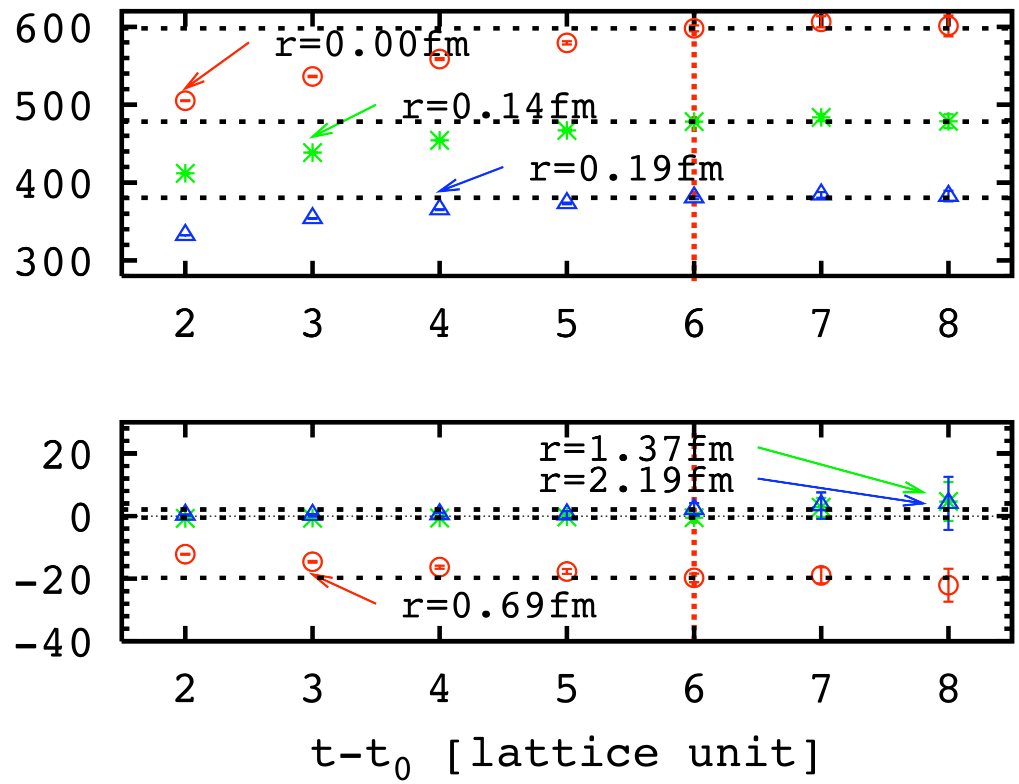
$m_\pi \simeq 0.53$ GeV, $m_N \simeq 1.34$ GeV

Ground state saturation

$^1S_0$

$m_\pi \simeq 0.53$ GeV





Yukawa potential

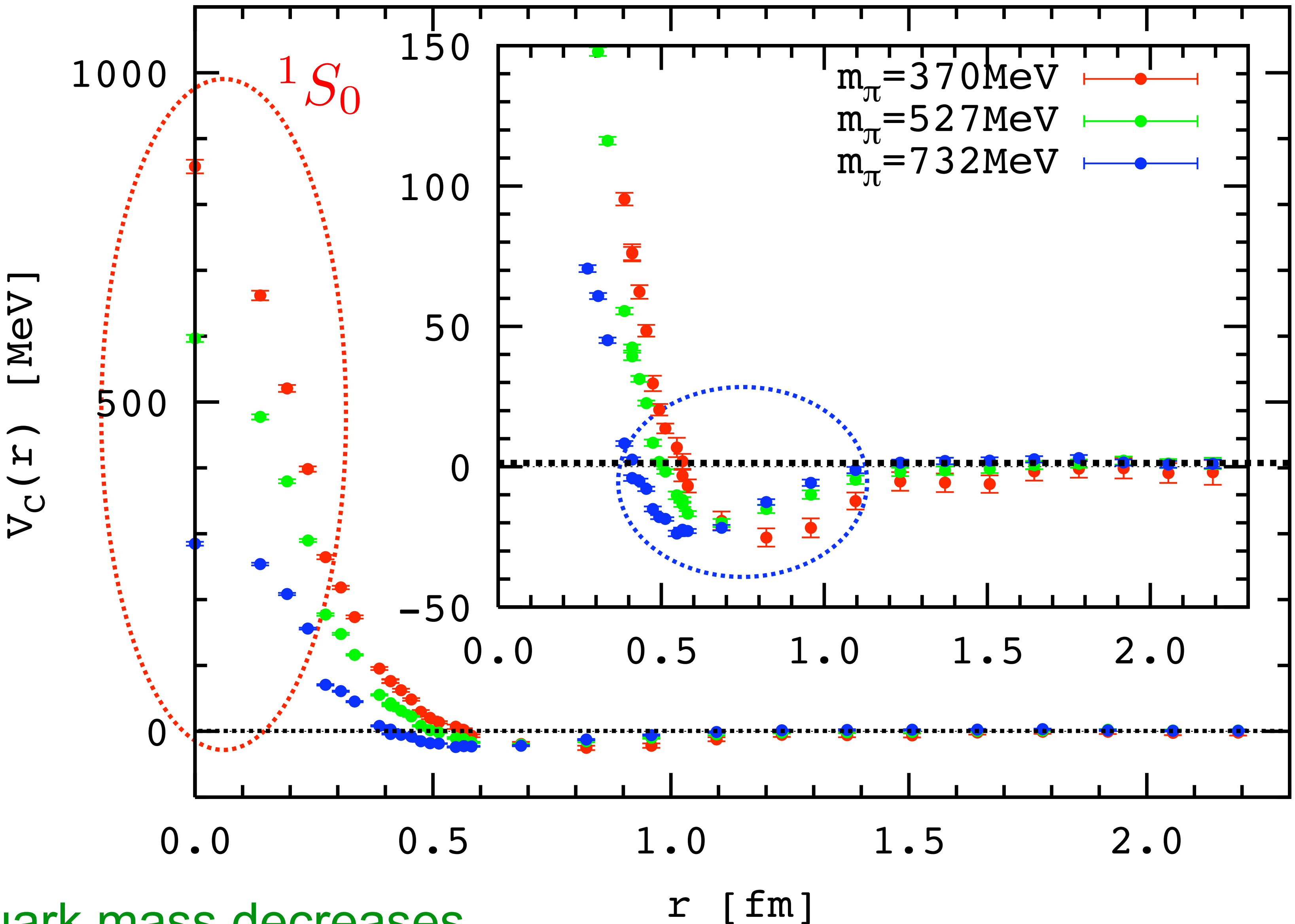
t=11 seems OK

$$V_C^\pi(r) = \frac{g_{\pi N}^2}{4\pi}\frac{(\tau_1\cdot\tau_2)(\sigma_1\cdot\sigma_2)}{3}\left(\frac{m_\pi}{2m_N}\right)^2\frac{e^{-m_\pi r}}{r}$$

$\frac{g_{\pi N}^2}{4\pi} \simeq 14.0$

$m_\pi \simeq 0.53$ GeV, $m_N \simeq 1.34$ GeV

Quark mass dependence



as quark mass decreases

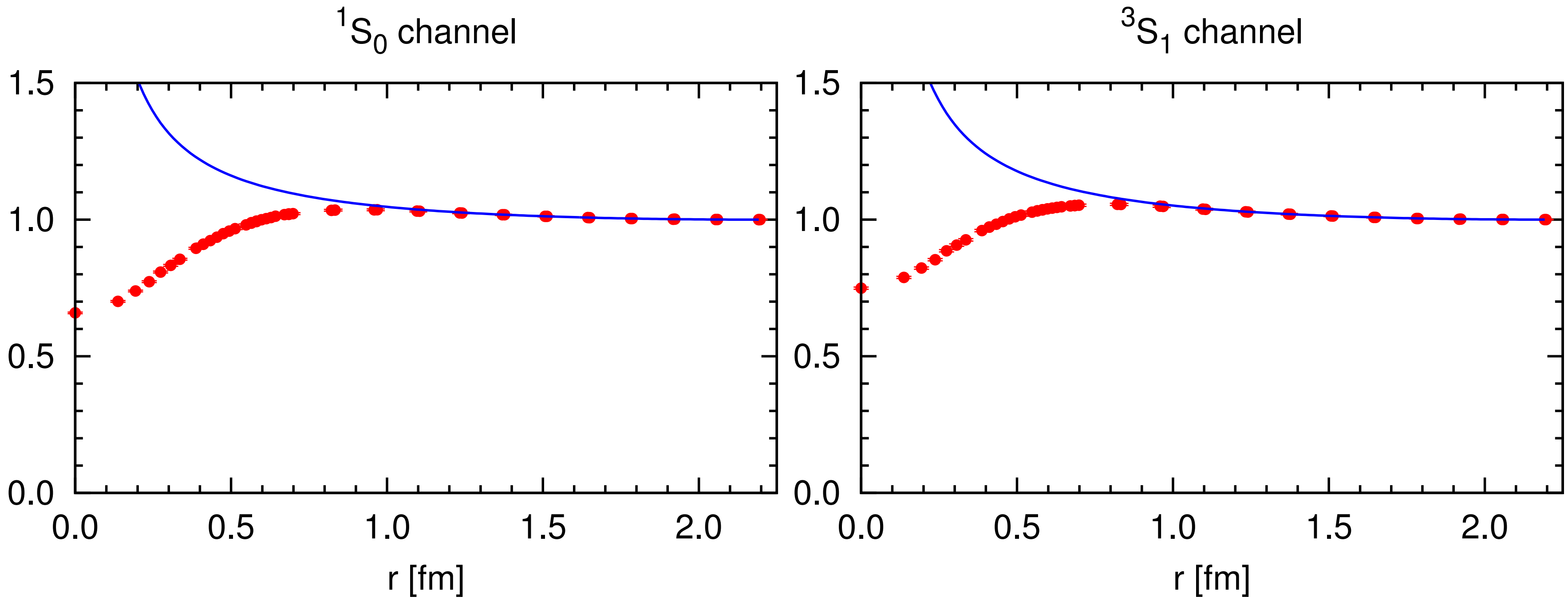
stronger repulsive core at short distance

a little stronger attraction at intermediate distance

Fit of wave function

$$G(\mathbf{r}; k^2) = \frac{1}{L^3} \sum_{\mathbf{n} \in \mathbf{Z}^3} \frac{e^{i(2\pi/L)\mathbf{n}\cdot\mathbf{r}}}{(2\pi/L)^2 \mathbf{n}^2 - \boxed{k^2}}$$

fit parameter



Lueshcer's formula

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; q^2) = \frac{1}{a_0} + O(k^2),$$

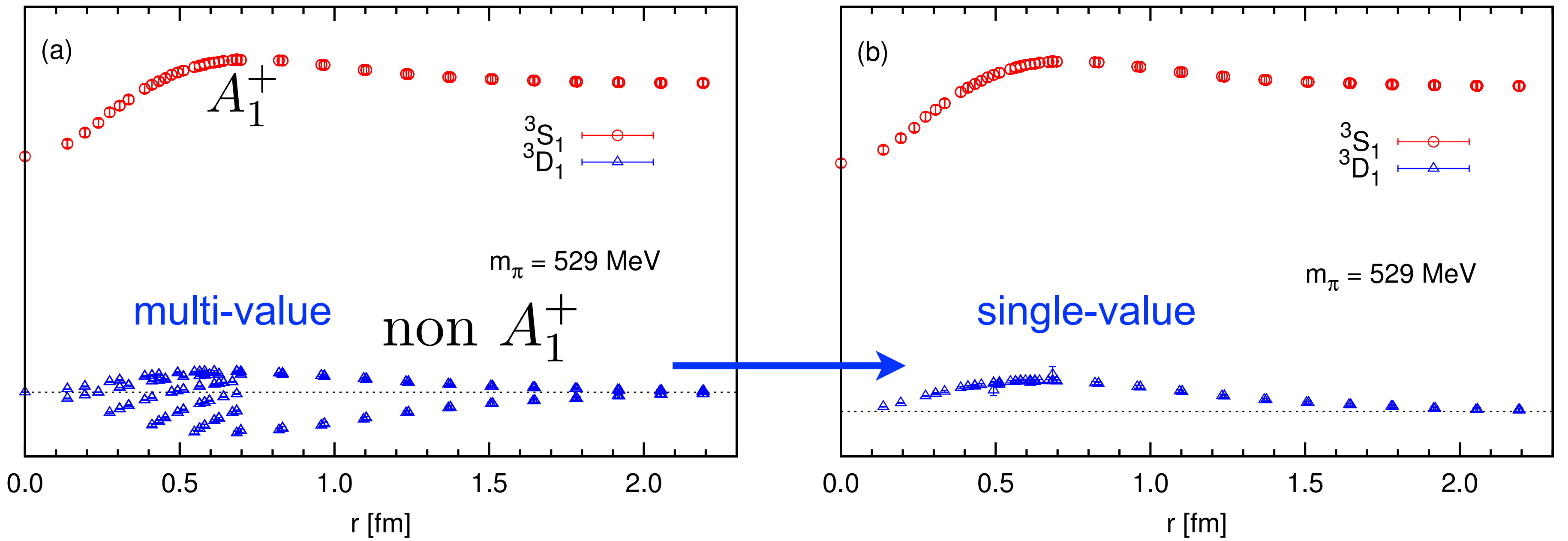
net attractions

	E[MeV]		a_0[fm]	
m_π[MeV]	spin-singlet	spin-triplet	$^1S_0$	$^3S_1$
731.1(4)	-0.40(8)	-0.48(10)	0.12(3)	0.14(3)
529.0(4)	-0.51(9)	-0.56(11)	0.13(3)	0.14(3)
379.7(9)	-0.68(26)	-0.97(37)	0.15(7)	0.23(10)

4-2. Tensor potential

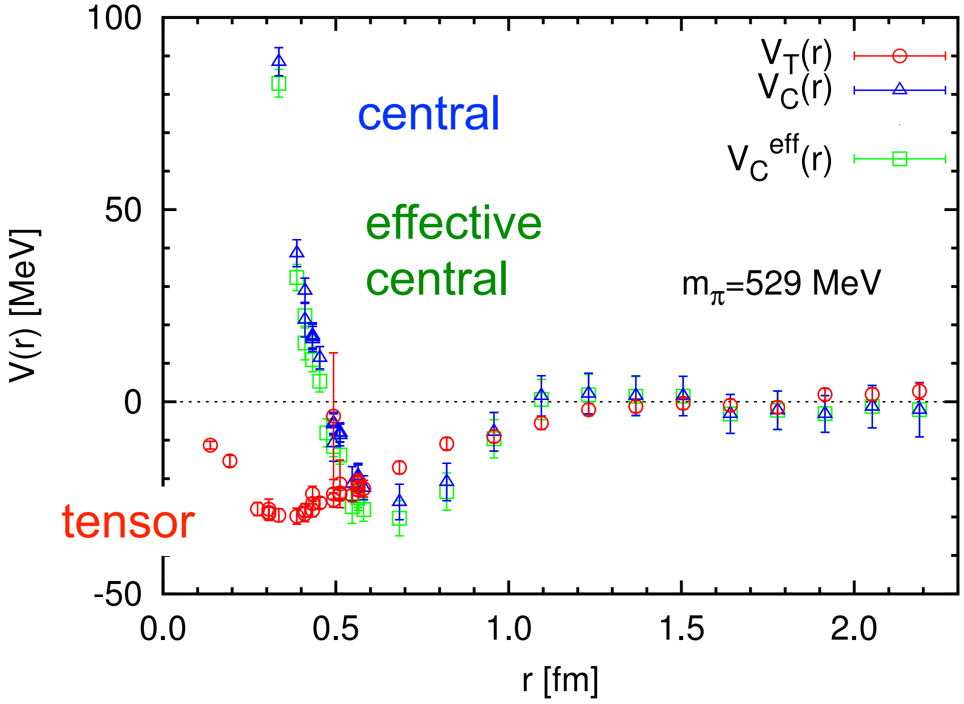
Quenched QCD

Wave functions

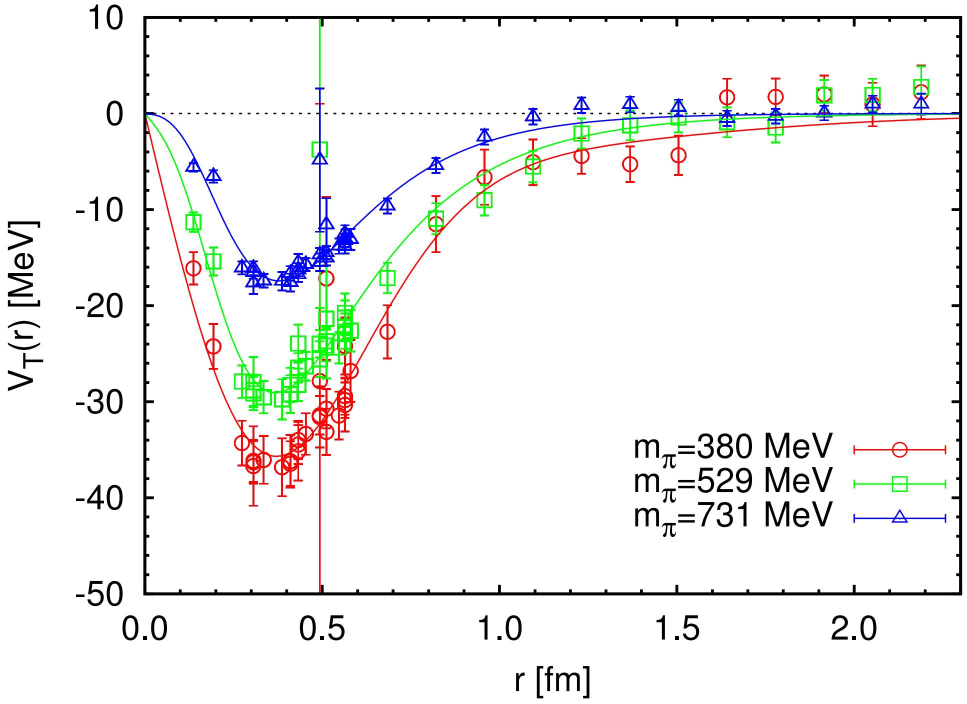


divided by $Y_{20}(\theta,\phi)$

Potentials



Quark mass dependence



- no repulsive core in the tensor potential.
- the central potential is roughly equal to the effective central potential.
  - the tensor potential is still small.
- the tensor potential increases as the pion mass decreases.
  - manifestation of one-pion-exchange ?
  - the fit below works well.

$$V_T(r) \;=\; b_1(1-e^{-b_2 r^2})^2\left(1+\frac{3}{m_\rho r}+\frac{3}{(m_\rho r)^2}\right)\frac{e^{-m_\rho r}}{r}+b_3(1-e^{-b_4 r^2})^2\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r}$$

4-3. Convergence of the velocity expansion

Non-local, E-independent ⟷ Local, E-dependent

$$\left(E+\frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y}) \qquad V_E(\mathbf{x})\varphi_E(\mathbf{x}) = \left(E+\frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x})$$

Numerical check in quenched QCD

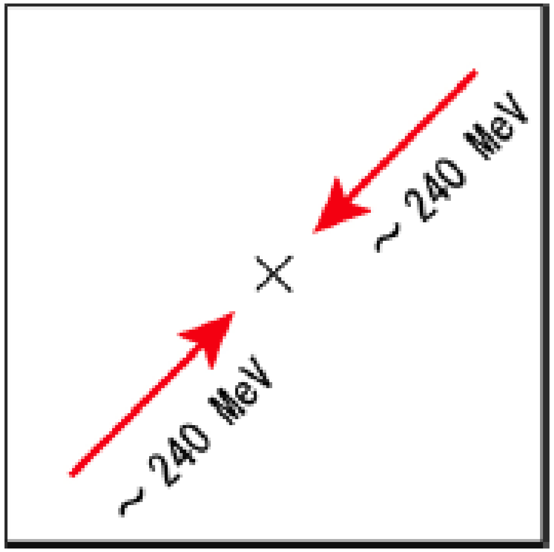
$m_\pi \simeq 0.53$ GeV

a=0.137fm

K. Murano, N. Ishii, S. Aoki, T. Hatsuda

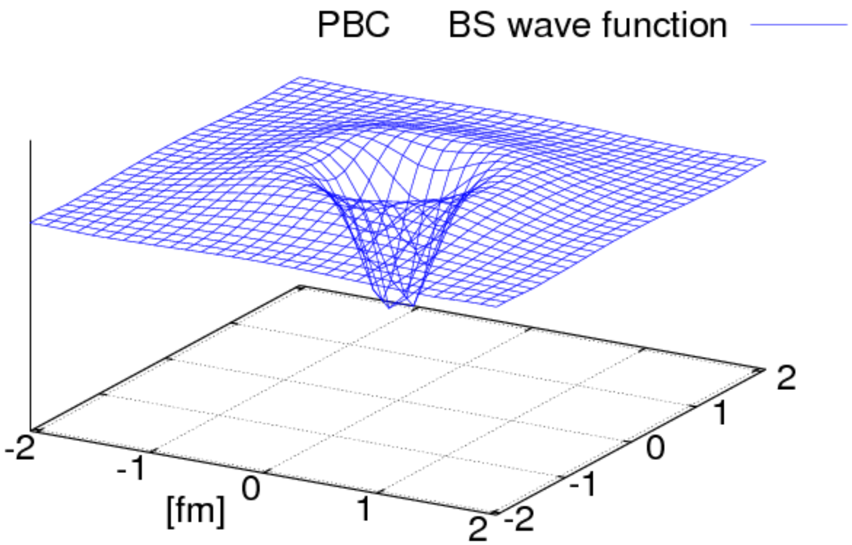
arXiv:1103.0619[hep-lat]

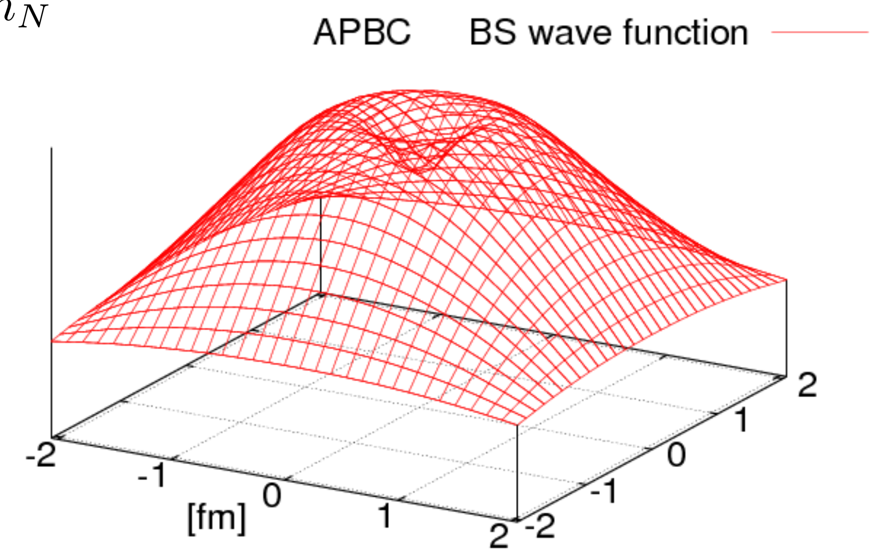
Anti-Periodic B.C.

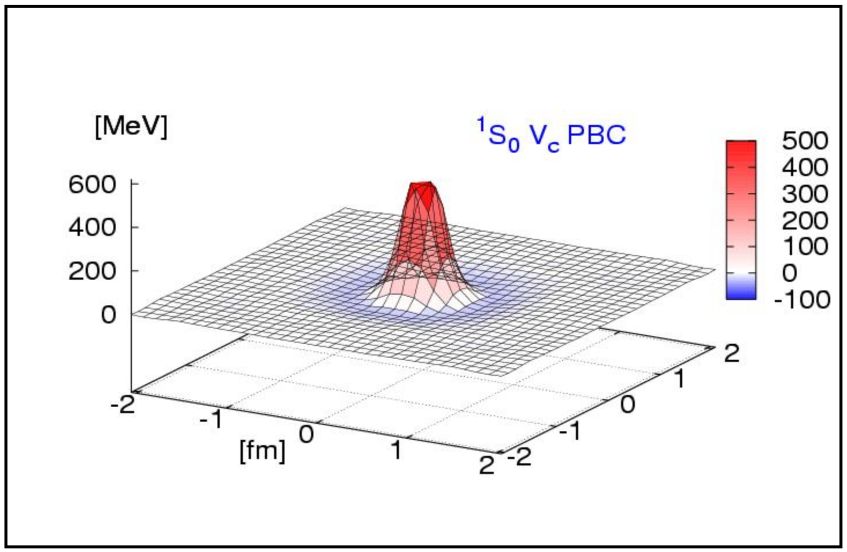


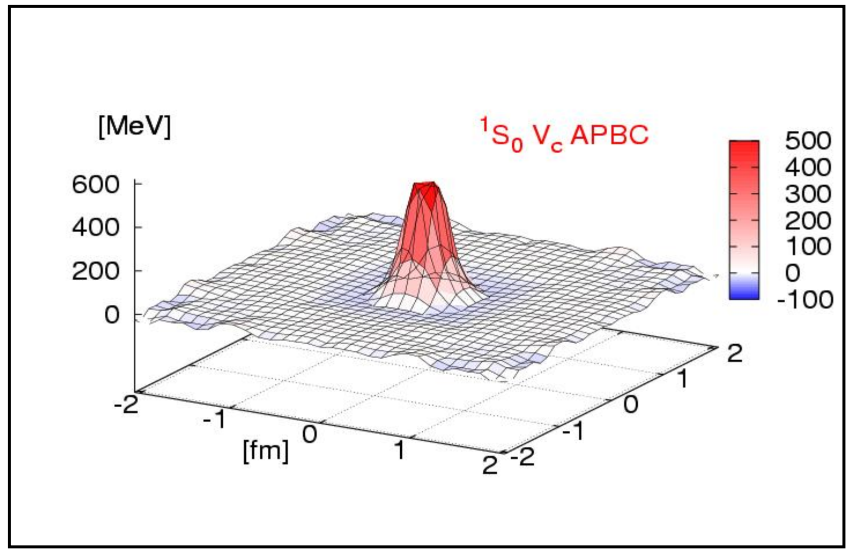
● PBC (E∼0 MeV) ● APBC (E∼46 MeV)

$$E = \frac{k^2}{m_N}$$



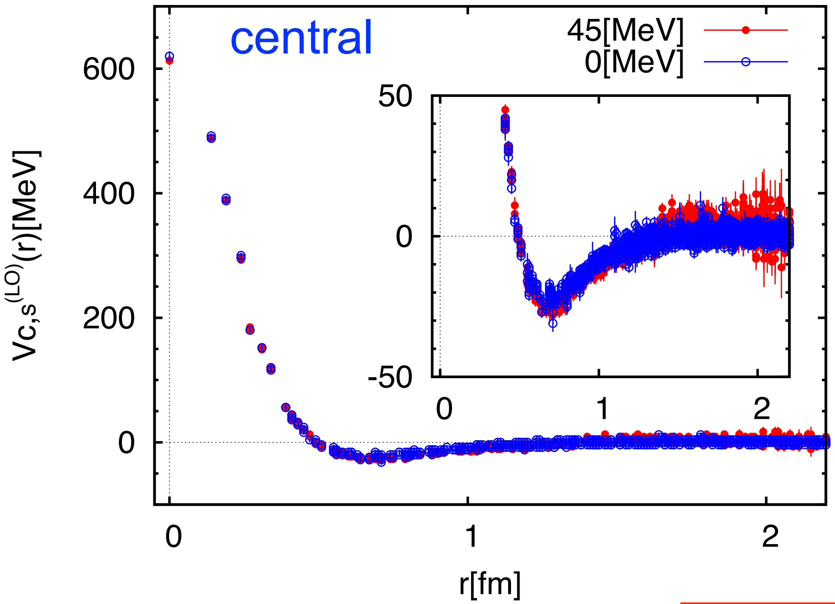




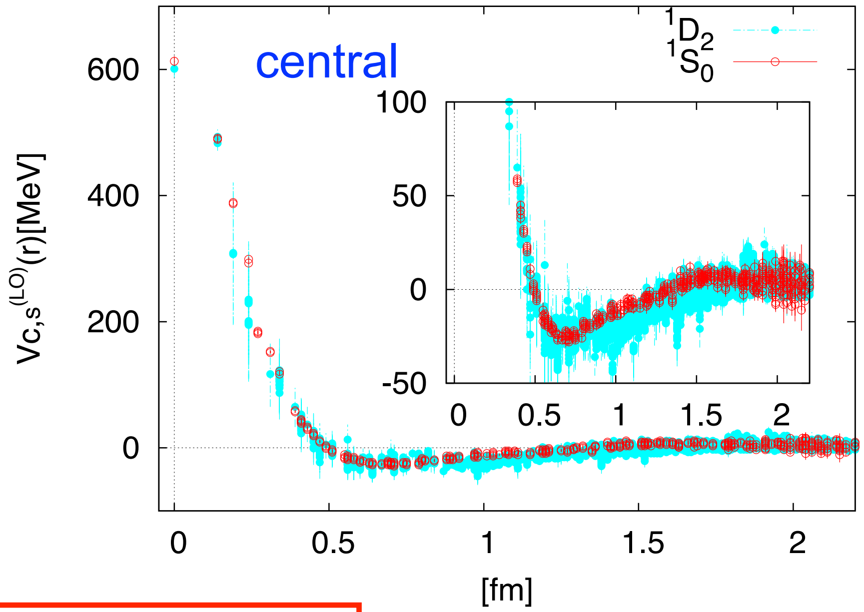


Spin-singlet

E-dep.



L-dep. at E=46 MeV



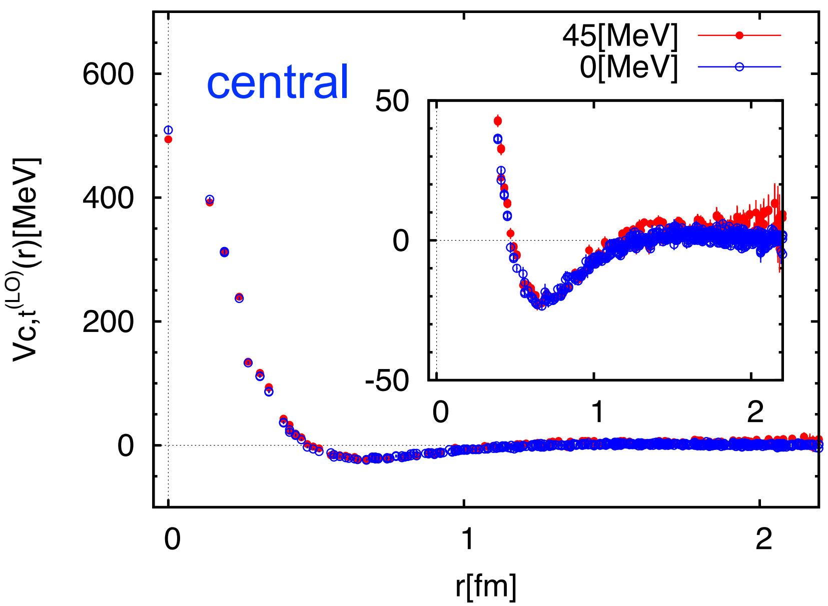
$T_2(L=2)$

$A_1(L=0)$

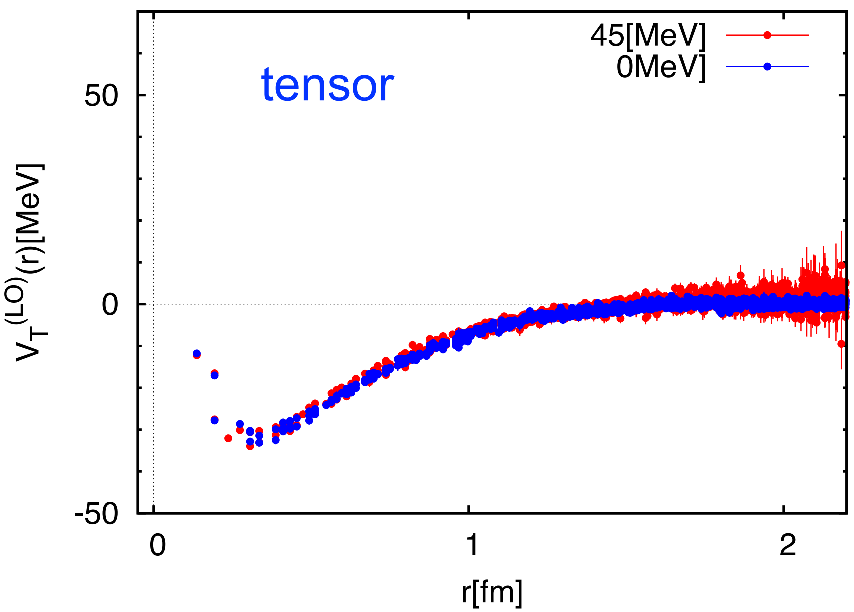
Non-locality turns out to be small in this setup.

Spin-triplet

E-dep.



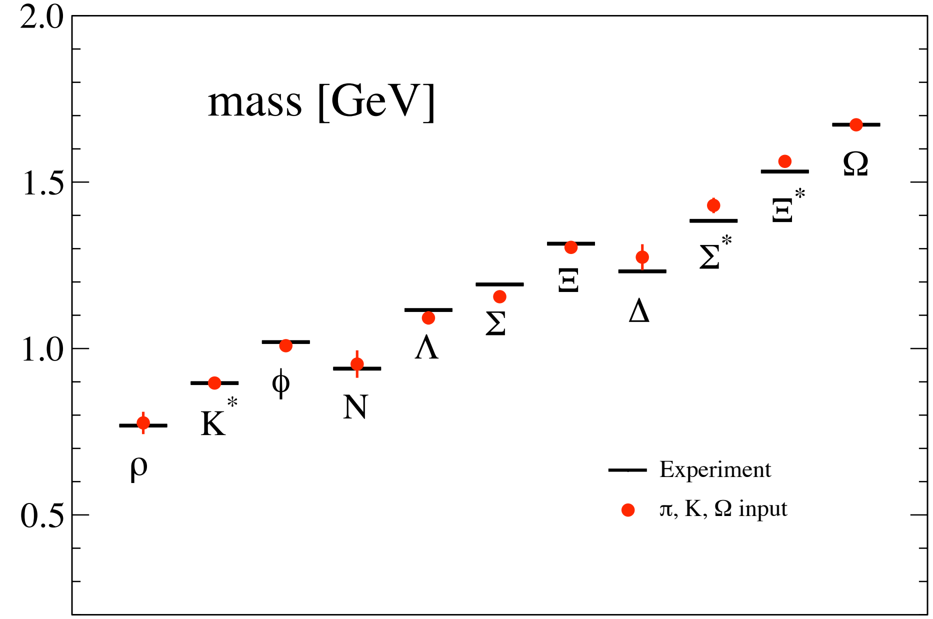
E-dep.



4-4. Full QCD results

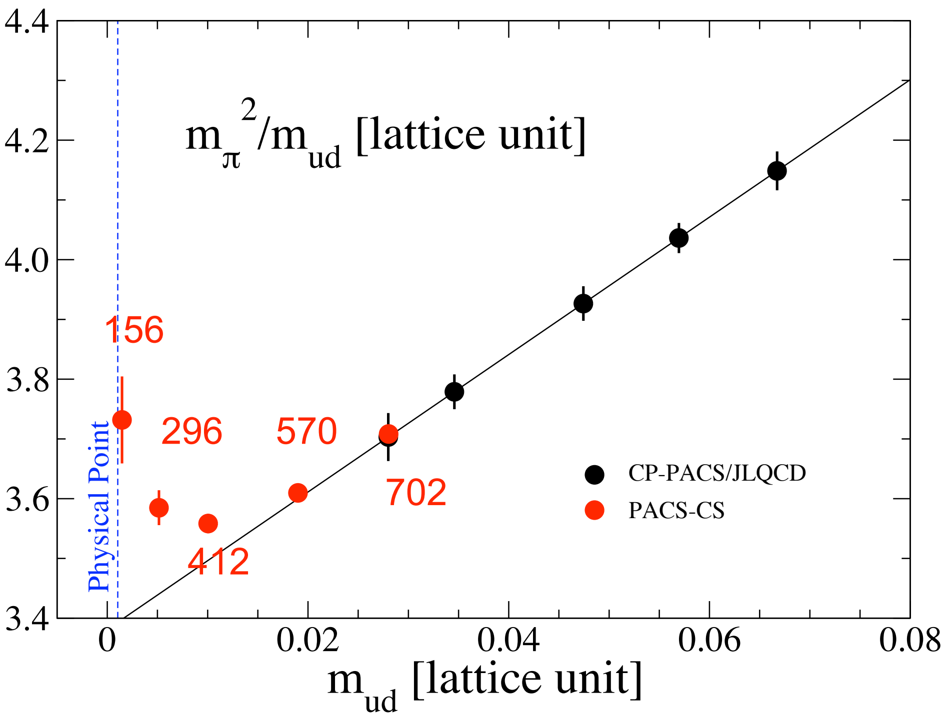
PACS-CS gauge configurations(2+1 flavors)

Phys. Rev. D79(2009)034503



$$a = 0.09 \text{ fm}$$
$$L = 2.9 \text{ fm}$$
$$m_\pi^{\text{min.}} = 156 \text{ MeV} \qquad m_\pi L = 2.3$$

We are almost on the “physical point”.



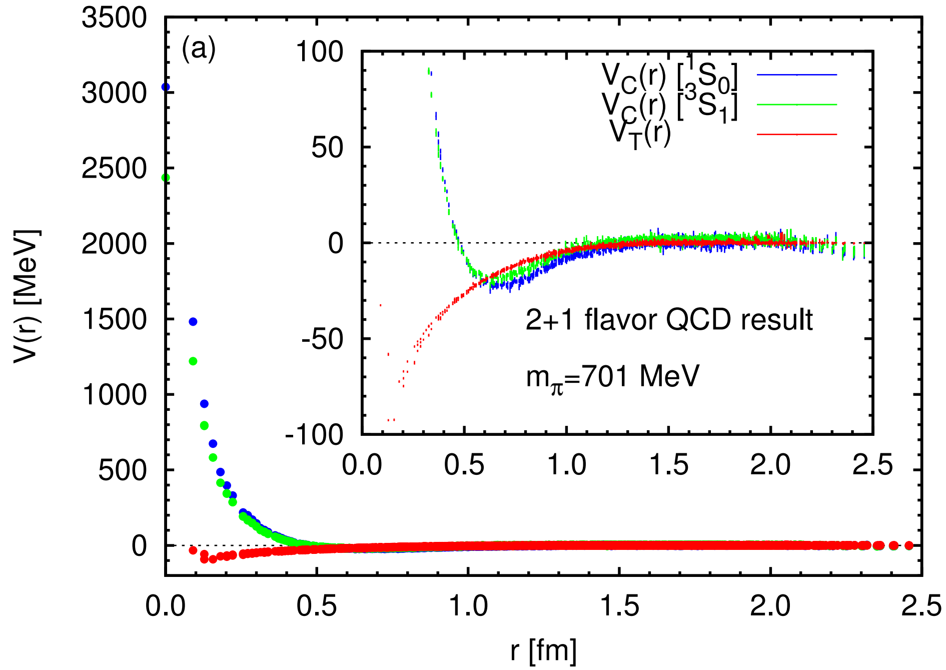
Calculations with L=5.8 fm and are on-going. $m_\pi \simeq 140 \text{ MeV}$

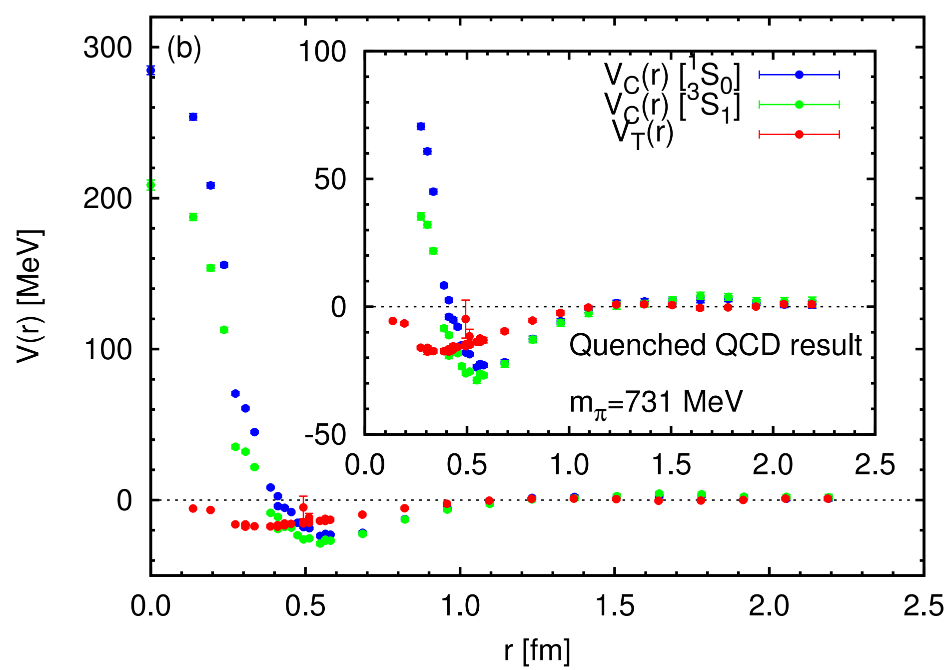
$$m_\pi L > 4$$

“Real QCD”

full QCD

quenched QCD



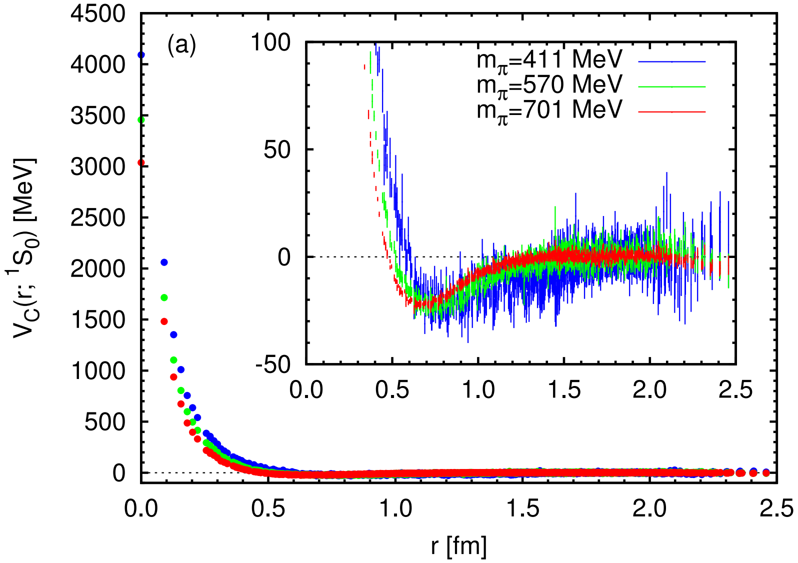


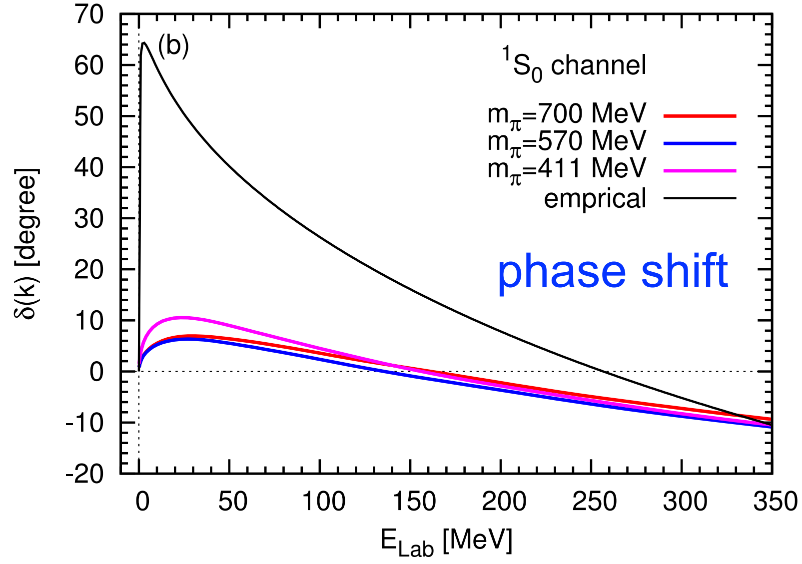
$a \simeq 0.091$ fm $\quad L \simeq 2.9$ fm $\quad a \simeq 0.137$ fm $\quad L \simeq 4.4$ fm

- both repulsive core at short distance and the tensor potential are enhanced in full QCD.
- the attraction at medium distance is shifted to outer region, while the magnitude remains almost unchanged.
- these differences may be caused by dynamical quark effects.
  - a more controlled comparison is needed.

Spin-singlet

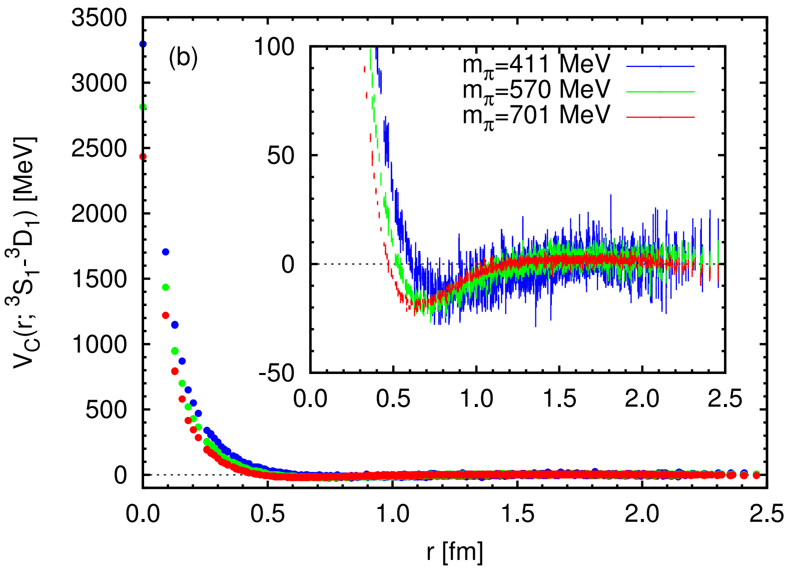
central

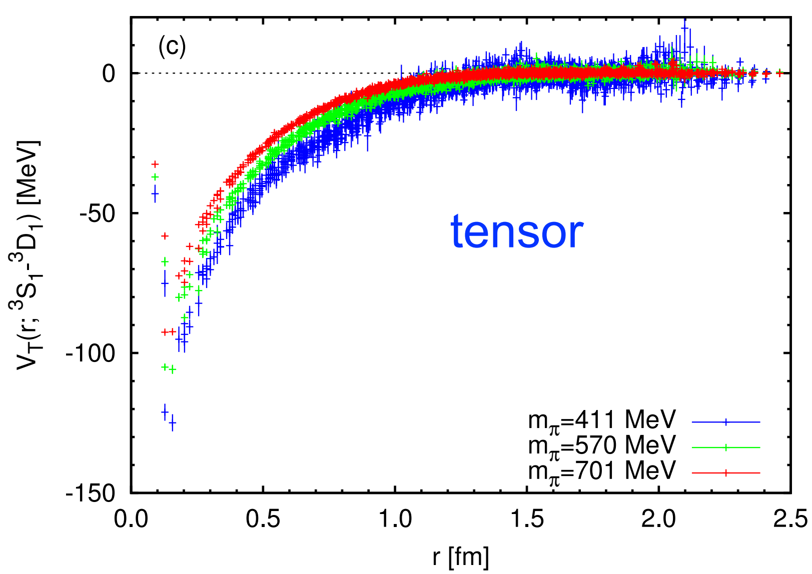




Spin-triplet

central

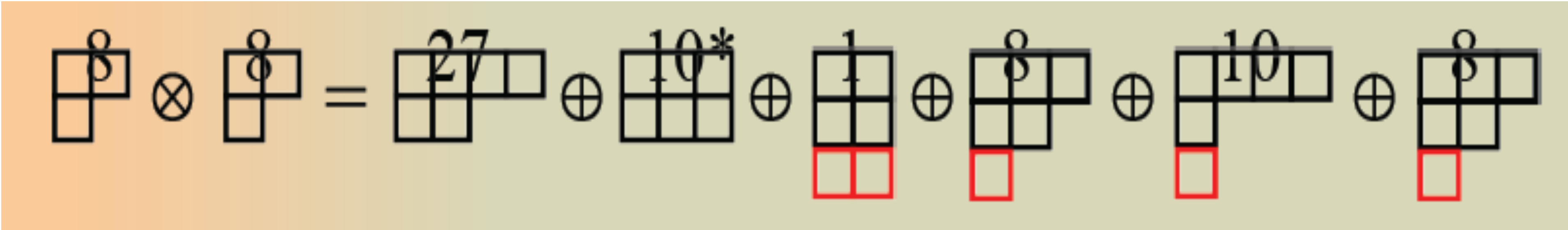


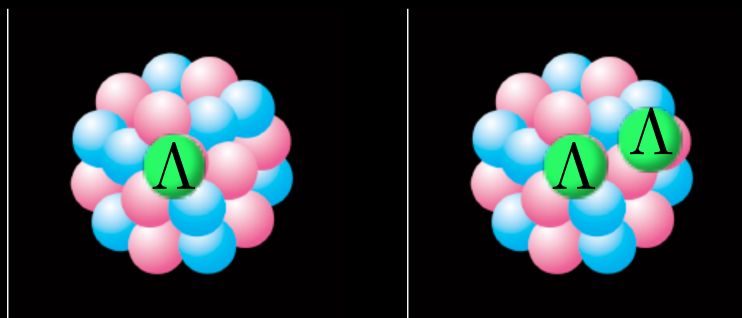


- both repulsive core and attractive pocket in the central increase as the pion mass decrease.
- the tensor potential also increases as the pion mass decreases.
- the phase shift is qualitatively similar in shape to but is much smaller in magnitude than experimental one.
  - simulations at physical point are needed.

5. Hyperon Interactions

Octet Baryon interactions



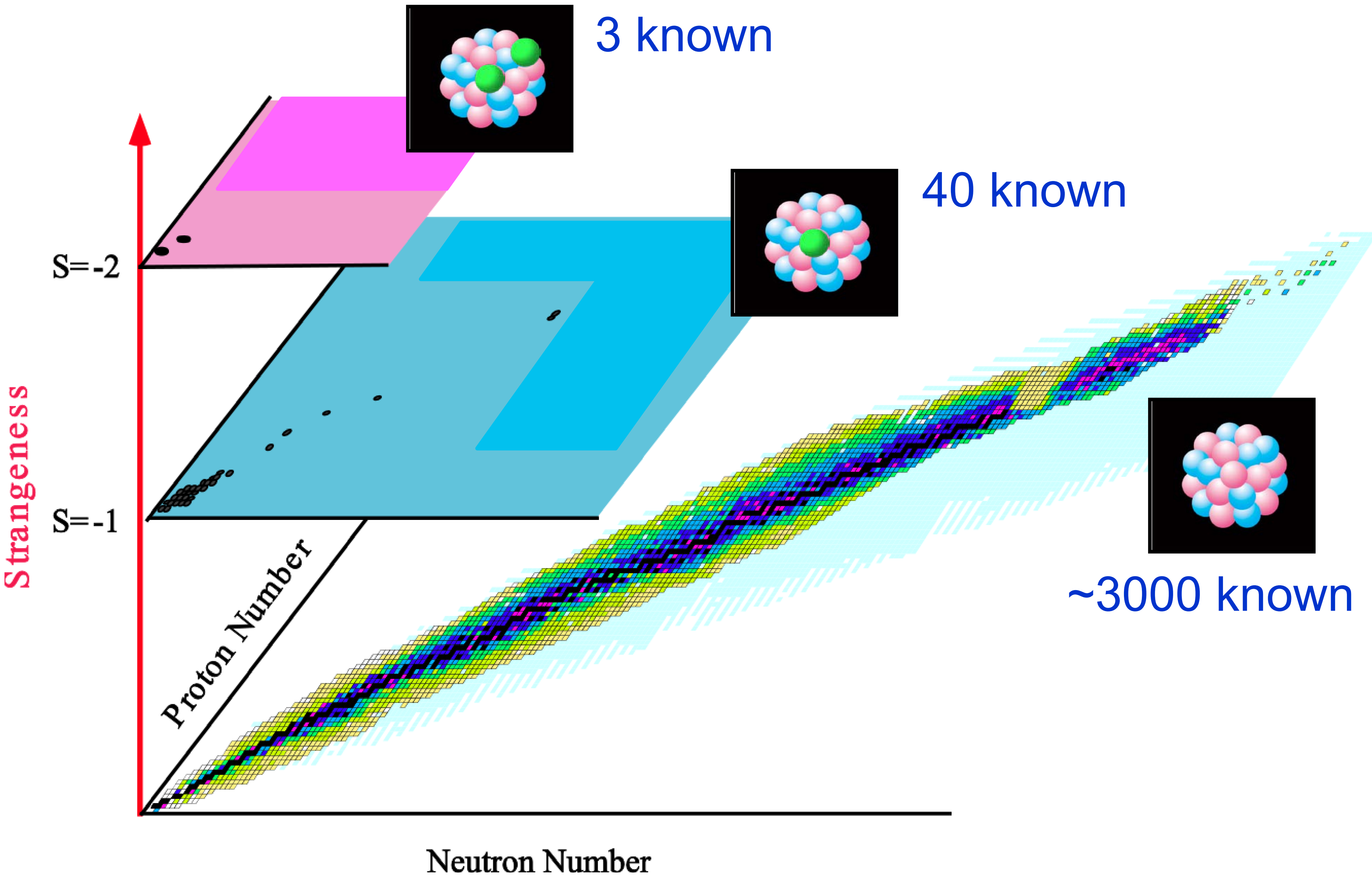


- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

also in GSI

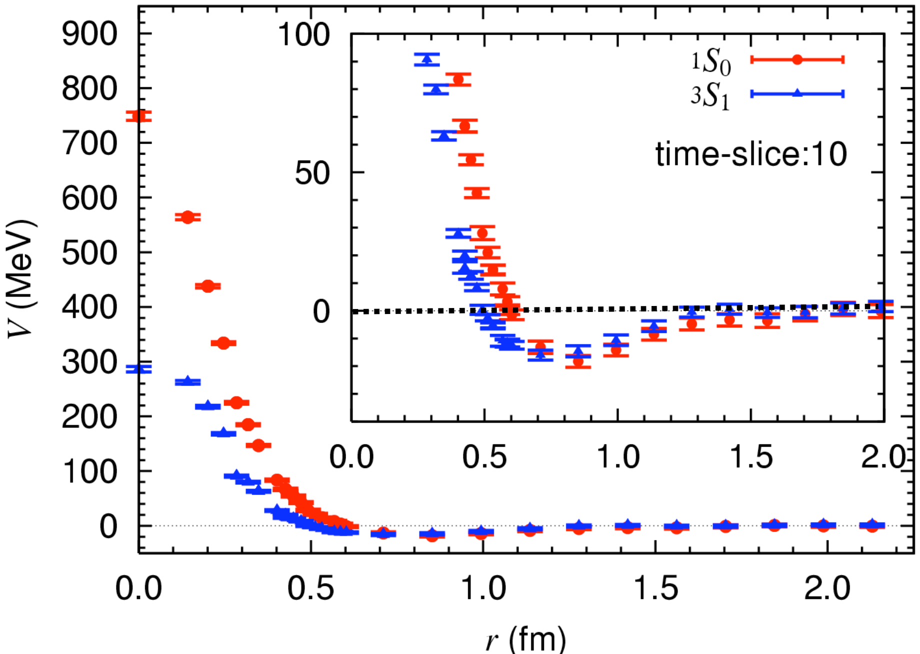
- prediction from lattice QCD
- difference between NN and YN ?

3D Nuclear chart



5-1. Quenched result for NΞ potential

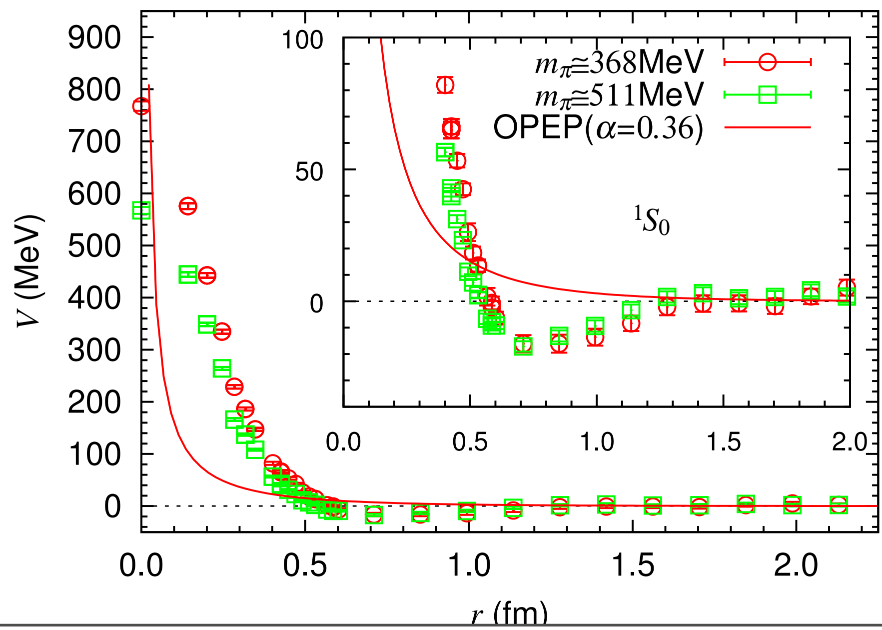
Nemura, Ishii, Aoki, Hatsuda,
Phys.Lett.B673 (2009)136



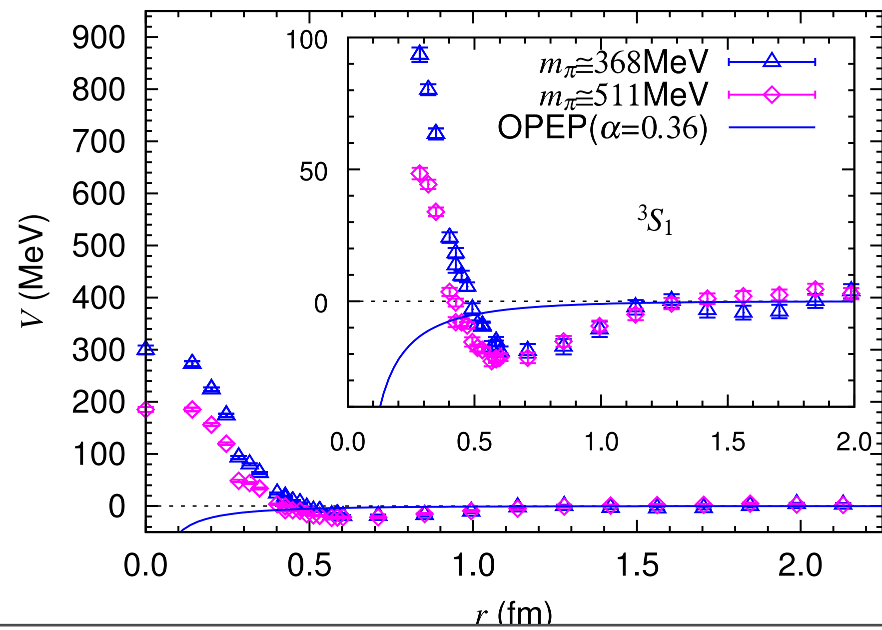
- clear spin dependence
- quark mass dependence is weaker than NN.
- No clear signature of one-pion exchange
  - state-independent attraction ?

$$V_C^\pi = -(1-2\alpha)\frac{g_{\pi NN}^2}{4\pi}\frac{(\vec{\tau}_N\cdot\vec{\tau}_\Xi)(\vec{\sigma}_N\cdot\vec{\sigma}_\Xi)}{3}\left(\frac{m_\pi}{2m_N}\right)^2\frac{e^{-m_\pi r}}{r}$$

Spin-singlet

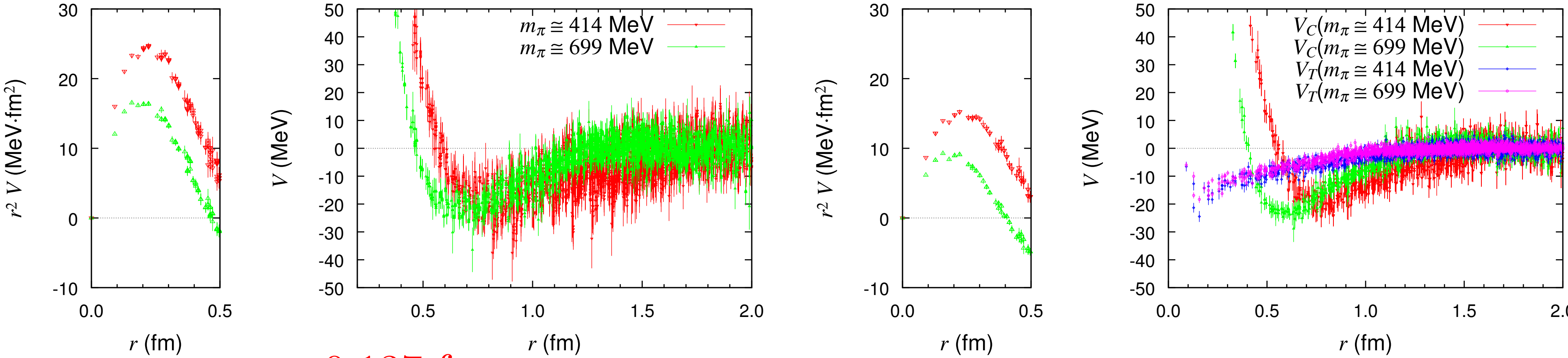


Spin-triplet

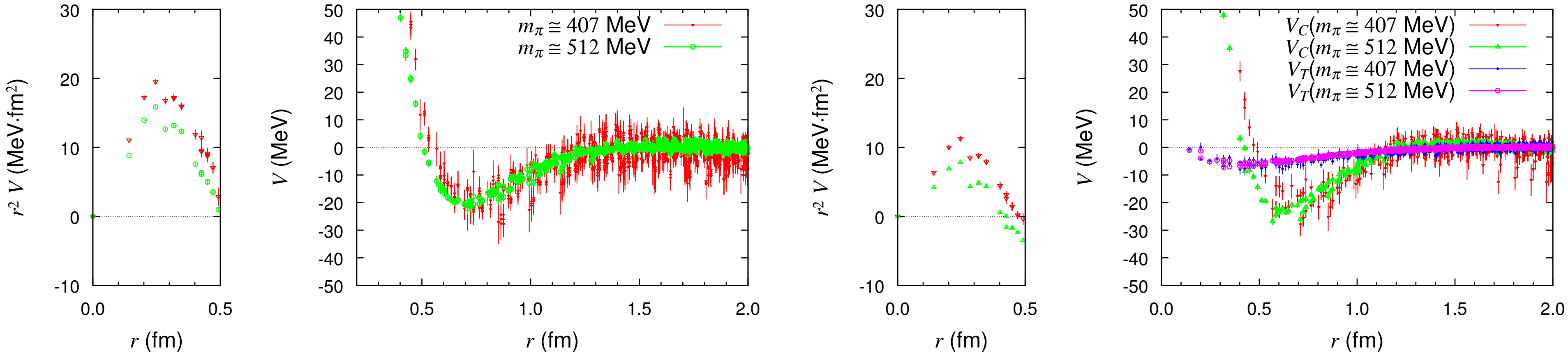


5-2. Full and quenched QCD result for NΛ potential

full QCD $a \simeq 0.091$ fm Spin-singlet $L \simeq 2.9$ fm Spin-triplet



quenched QCD $a \simeq 0.137$ fm $L \simeq 4.4$ fm

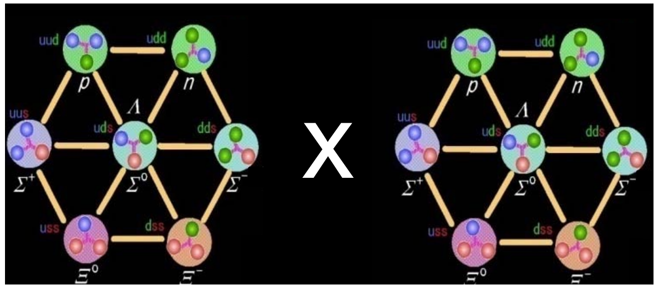


- attractive well moves to outer region but depth remains the same as pion mass decreases.
- tensor force: weaker than NN. quark mass dependence is also weak.
- repulsive core and the attractive well increase as pion mass decreases. Net attractions.
- full and quenched QCD are more or less similar.

5-3. Flavor SU(3) limit

$m_u = m_d = m_s$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric Anti-symmetric

6 independent potential in flavor-basis

$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \longleftarrow {}^1S_0$$

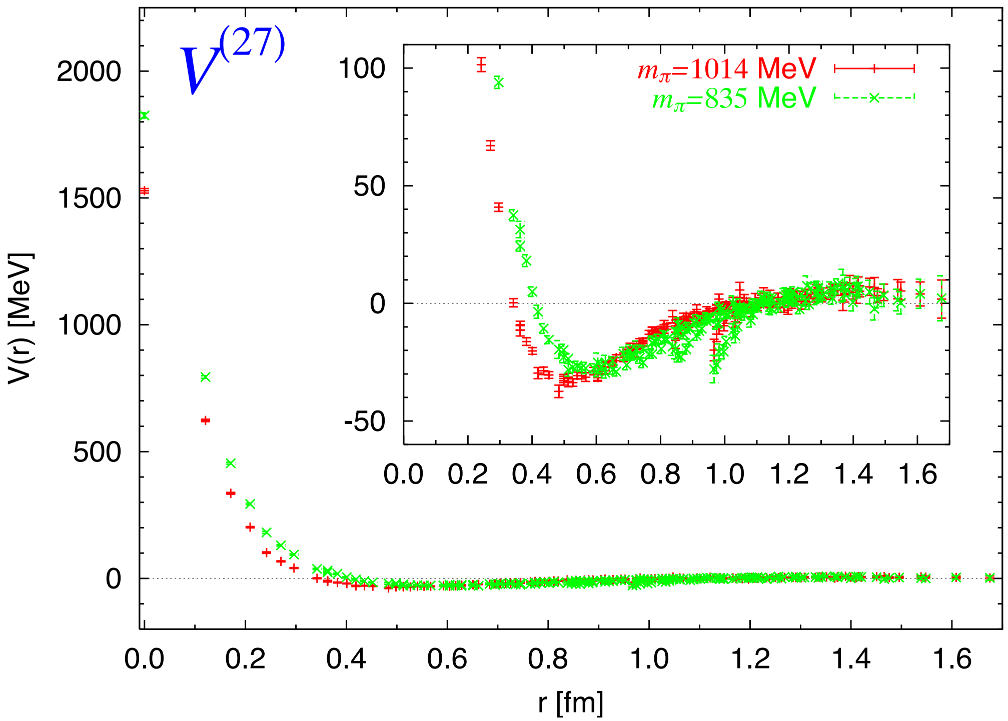
$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \longleftarrow {}^3S_1$$

Potentials in full QCD

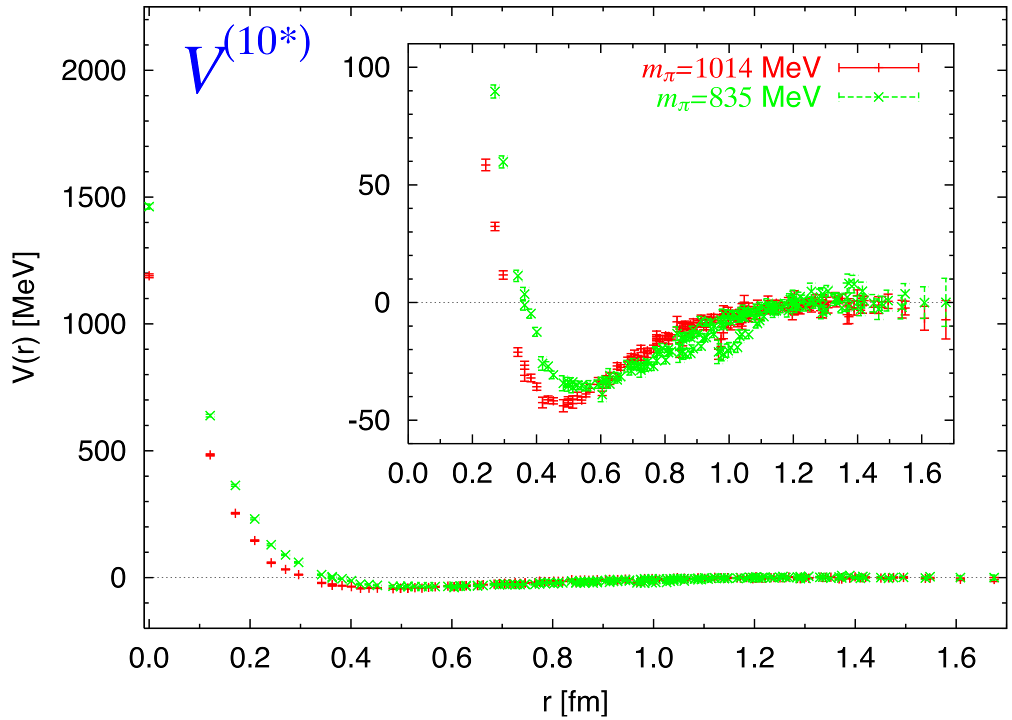
Inoue *et al.* (HAL QCD Coll.), PTP124(2010)591

a=0.12 fm, L=2 fm BG/L@KEK

Spin-singlet

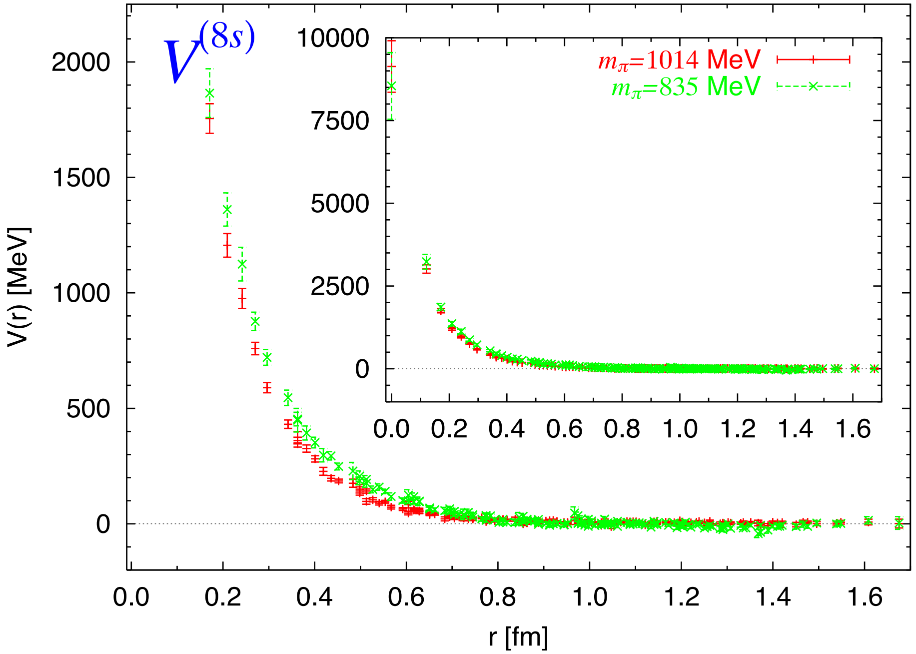


Spin-triplet

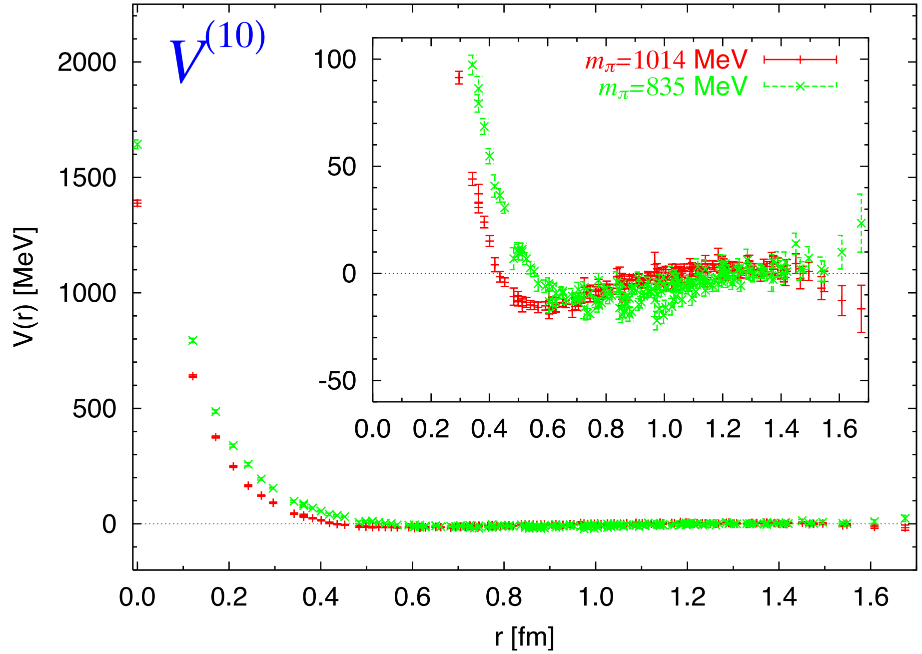


27, 10*: channels NN belongs to
same behaviors as NN potentials

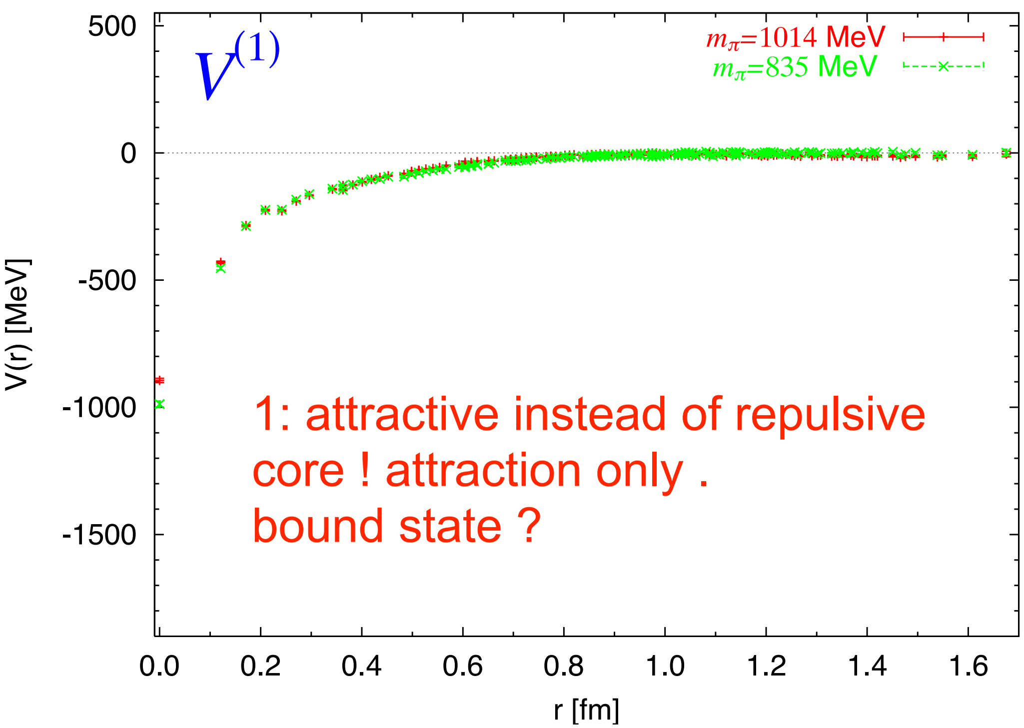
flavor multiplet	baryon pair (isospin)
27	{NN}(I=1), {NΣ}(I=3/2), {ΣΣ}(I=2), {ΣΞ}(I=3/2), {ΞΞ}(I=1)
$\mathbf{8}_s$	none
1	none
$\mathbf{10}^*$	[NN](I=0), [ΣΞ](I=3/2)
10	[NΣ](I=3/2), [ΞΞ](I=0)
$\mathbf{8}_a$	[NΞ](I=0)

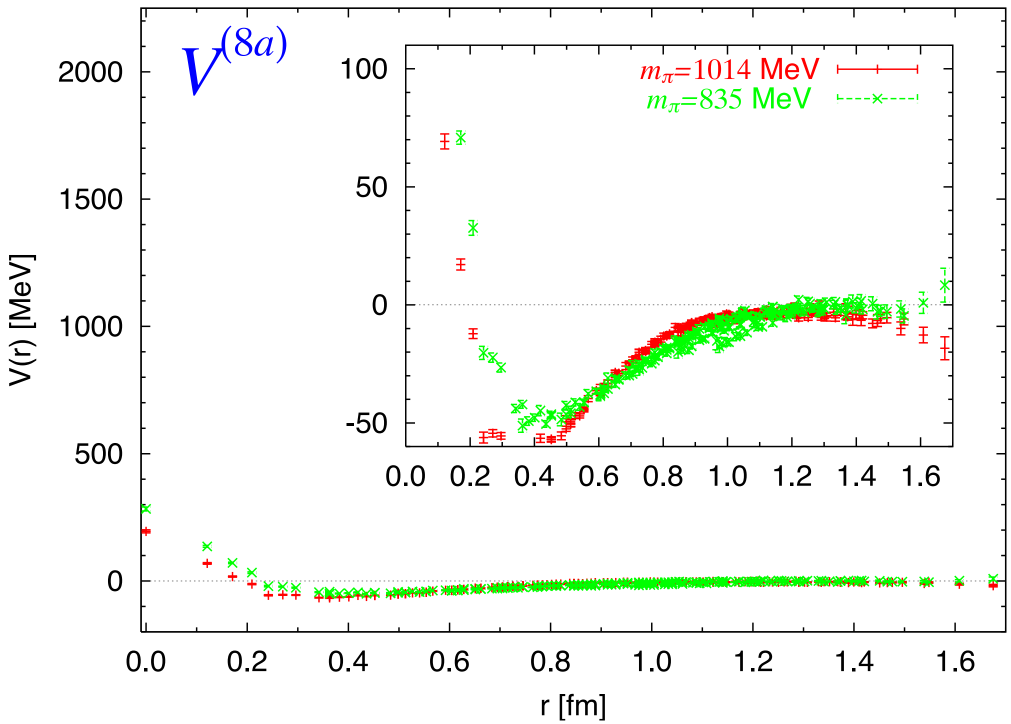


8s: strong repulsive core. repulsion only.



10: strong repulsive core. weak attraction.





8a: weak repulsive core. strong attraction.

Potential matrix in “baryon basis” (S=-2, I=0, spin-singlet)

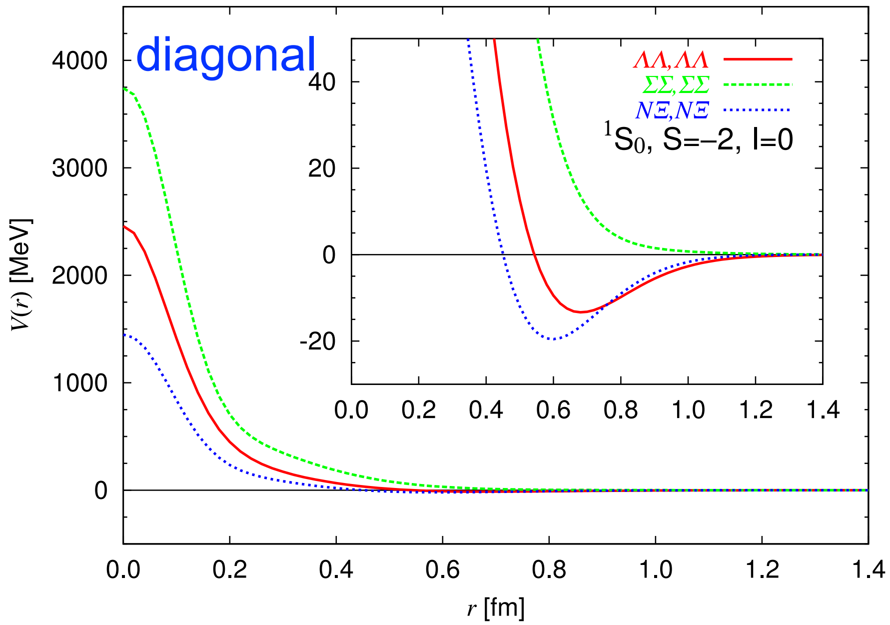
baryon flavor

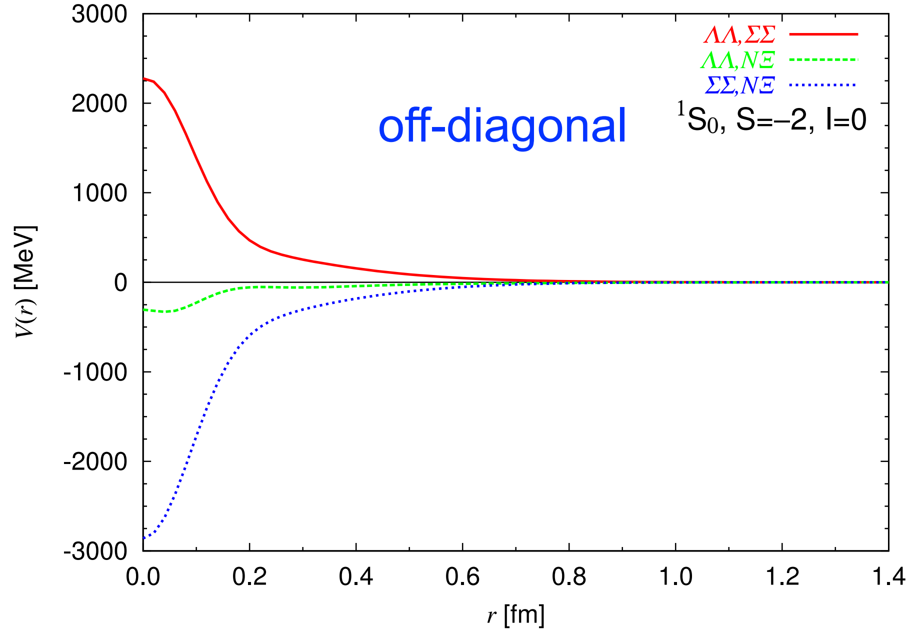
$$V_{ij}(r) = \sum_X U_{iX} V^{(X)}(r) U^\dagger_{Xj}$$

fit

$$V(r) = b_1 e^{-b_2 r^2} + b_3 (1 - e^{-b_4 r^2}) \left(\frac{e^{-b_5 r}}{r} \right)^2$$

$$\begin{pmatrix} |\Lambda\Lambda\rangle \\ |\Sigma\Sigma\rangle \\ |N\Xi\rangle \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{27}{40}} & -\sqrt{\frac{8}{40}} & -\sqrt{\frac{5}{40}} \\ -\sqrt{\frac{1}{40}} & -\sqrt{\frac{24}{40}} & \sqrt{\frac{15}{40}} \\ \sqrt{\frac{12}{40}} & \sqrt{\frac{8}{40}} & \sqrt{\frac{20}{40}} \end{pmatrix} \begin{pmatrix} |\mathbf{27}\rangle \\ |\mathbf{8}_s\rangle \\ |\mathbf{1}\rangle \end{pmatrix}$$





- attraction appeared in flavor-singlet channel can not be easily seen due to the strong repulsion in 8s channel. ΣΣ is the most repulsive due to the large coupling to 8s.
- NΞ has the strong attraction due to the large coupling to the singlet.
- off-diagonal parts are comparable to diagonal parts in magnitude.
  - full matrix is needed.
- “physics” can be easily seen in the “flavor” basis.

6. Origin of repulsive core

6-1. Operator Product Expansion(OPE) and repulsive core

NBS wave function $$\varphi_{AB}^E(\boldsymbol{r}) = \langle 0|T\{O_A(\boldsymbol{r}/2,0)O_B(-\boldsymbol{r}/2,0)\}|E\rangle$$

OPE $$O_A(\boldsymbol{r}/2,0)O_B(-\boldsymbol{r}/2,0) \simeq \sum_C D_{AB}^C(\boldsymbol{r})O_C(\boldsymbol{0},0).$$

short distance behavior $$D_{AB}^C(\boldsymbol{r}) \simeq r^{\alpha_C}(-\log r)^{\beta_C} f_C(\theta,\phi)$$

NBS wave function at short distance

$$\varphi_{AB}^E(\boldsymbol{r}) \simeq \sum_C r^{\alpha_C}(-\log r)^{\beta_C} f_C(\theta,\phi) D_C(E) \qquad D_C(E) = \langle 0|O_C(\boldsymbol{0},0)|E\rangle$$

ex: Ising field theory in 2-dim.

OPE $$\sigma(x,0)\sigma(0,0) \simeq G(r)\mathbf{1} + c\, r^{3/4} O_1(0) + \cdots, \qquad r = |x|$$

NBS wave function $$\varphi(r,E) \simeq r^{3/4}D(E) + O(r^{7/4}), \qquad D(E) = c\langle 0|O_1(0)|E\rangle$$

potential at short distance $$V(r) = \frac{\varphi''(r,E) + k^2\varphi(r,E)}{m\varphi(r,E)} \simeq -\frac{3}{16}\frac{1}{mr^2}$$

universal attraction
E-independent

QCD case

$\alpha_C = 0$ consider L=0 case

Take $\beta_C > \beta_{C'}$ for $\forall C' \neq C$ largest

$(1)\ \beta_C > \neq 0$ universal at short distance

$$V(r) \simeq -\frac{\beta_C}{mr^2(-\log r)}$$

attractive for $\beta_C > 0$

repulsive for $\beta_C < 0$

$(2)\ \beta_C = 0$ $\beta_{C'}$ the second largest

$$V(r) \simeq \frac{D_{C'}(E)}{D_C(E)} \frac{-\beta_{C'}}{mr^2}(-\log r)^{\beta_{C'}-1}$$

E-dependent non-universal

universal

Note that the potential does not diverge even at r=0 due to the lattice artifact in lattice QCD.

6-2. Renormalization Group and perturbation theory

1-loop calculation becomes exact at short distance in QCD due to the asymptotic freedom.

OPE

$$O_A(y/2)O_B(-y/2) = \sum_C D_{AB}^C(r,g,m,\mu)O_C(0)$$

RG analysis

fixed

$$\lim_{r\to 0} D_{AB}^C(r,g,m,\mu) = (-2\beta^{(1)}g^2\log r)^{\gamma_{AB}^{C,(1)}/(2\beta^{(1)})} D_{AB}^C(R,0,0,\mu).$$

$$r = e^{-t}R \to 0$$

beta function

$$\beta^{(1)} = \frac{1}{16\pi^2}\left(11 - \frac{2N_f}{3}\right)$$

anomalous dimensions

$$\gamma_{AB}^{C,(1)} = \gamma_C^{(1)} - \gamma_A^{(1)} - \gamma_B^{(1)} \equiv \frac{1}{48\pi^2}\gamma$$

$$\beta_C = \frac{\gamma_{AB}^{C,(1)}}{2\beta^{(1)}}$$

6-3. Results

Spin-singlet　　Spin-triplet

2-flavors

(2) $\beta_C = 0$

$$\beta_{C'}^{S=0} = -\frac{6}{33-2N_f}, \quad \beta_{C'}^{S=1} = -\frac{2}{33-2N_f}$$

$$V_C^S(r) \simeq \frac{D_{C'}(E)}{D_C(E)} \frac{-\beta_{C'}^S(-\log r)^{\beta_{C'}^S - 1}}{m_N r^2}, \qquad r \to 0.$$

$$V_T(r) \simeq 0$$

no divergence in tensor

non-relativisitic quark model

$$\frac{D_{C'}(E)}{D_C(E)}(S=0) \simeq \frac{D_{C'}(E)}{D_C(E)}(S=1) \simeq 2.$$

repulsive core !

3-flavors

$$\beta_C = \frac{\gamma^{(X)}}{33-2N_f}.$$

attraction in MC

X	$\mathbf{27}$	$\mathbf{8}_s$	$\mathbf{1}$	$\overline{\mathbf{10}}$	$\mathbf{10}$	$\mathbf{8}_a$
$\gamma^{(X)}$	0	6	12	0	0	4
Non-relativistic op.	yes	no	yes	yes	yes	yes

strong repulsion in MC but no non-rel. op. ($\mathbf{8}_s$)

weak repulsion in MC ($\mathbf{8}_a$)

universal attractive core in 1, 8s,8a

$$V_C^{(X)}(r) \simeq -\frac{\gamma^{(X)}}{(33-2N_f)} \frac{1}{m_B r^2(-\log r)}$$

7. Extension of the potential method

7-1. Inelastic scatterings

inelastic due to the change of particle species

$$A + B \to C + D$$

$$m_A + m_B < m_C + m_D < W \qquad W = E_k^A + E_k^B = E_q^C + E_q^D \qquad E_k^X = \sqrt{m_X^2 + \boldsymbol{k}^2}$$

QCD eigenstate

$$\begin{aligned} |W\rangle &= c_{AB}|AB, W\rangle + c_{CD}|CD, W\rangle + \cdots \\ |AB, W\rangle &= |A, \boldsymbol{k}\rangle_{\rm in} \otimes |B, -\boldsymbol{k}\rangle_{\rm in}, \quad |CD, W\rangle = |C, \boldsymbol{q}\rangle_{\rm in} \otimes |D, -\boldsymbol{q}\rangle_{\rm in} \end{aligned}$$

NBS wave functions

$$\begin{aligned} \varphi_{AB}(\boldsymbol{r}, \boldsymbol{k})e^{-Wt} &= \langle 0|T\{\varphi_A(\boldsymbol{x}+\boldsymbol{r}, t)\varphi_B(\boldsymbol{x}, t)\}|W\rangle \\ \varphi_{CD}(\boldsymbol{r}, \boldsymbol{q})e^{-Wt} &= \langle 0|T\{\varphi_C(\boldsymbol{x}+\boldsymbol{r}, t)\varphi_D(\boldsymbol{x}, t)\}|W\rangle \end{aligned}$$

$$\varphi_{XY}(\boldsymbol{r}, \boldsymbol{k}) = 4\pi \sum_{l,m} i^l \varphi_{XY}^\ell(r, k) Y_{lm}(\Omega_{\boldsymbol{r}})\overline{Y_{lm}(\Omega_{\boldsymbol{k}})}$$

partial wave decomposition

$$\begin{aligned} \begin{pmatrix} \varphi_{AB}^\ell(r,k) \\ \varphi_{CD}^\ell(r,q) \end{pmatrix} &\simeq \begin{pmatrix} j_l(kr) & 0 \\ 0 & j_l(qr) \end{pmatrix} \begin{pmatrix} c_{AB} \\ c_{CD} \end{pmatrix} + \begin{pmatrix} n_l(kr) + i j_l(kr) & 0 \\ 0 & n_l(qr) + i j_l(qr) \end{pmatrix} \\ &\times \ O(W) \begin{pmatrix} e^{i\delta_l^1(W)} \sin\delta_l^1(W) & 0 \\ 0 & e^{i\delta_l^2(W)} \sin\delta_l^2(W) \end{pmatrix} O^{-1}(W) \begin{pmatrix} c_{AB} \\ c_{CD} \end{pmatrix}, \end{aligned}$$

$$(\nabla^2 + \boldsymbol{k}^2)\varphi_{AB}(\boldsymbol{r}, \boldsymbol{k}) = 0, \quad (\nabla^2 + \boldsymbol{q}^2)\varphi_{CD}(\boldsymbol{r}, \boldsymbol{q}) = 0$$

Finite volume

two QCD eigenstates $|W_i\rangle \qquad W_i = W + O(V^{-1}), i = 1,2$

NBS wave functions

$$
\begin{aligned}
\varphi_{AB}(\boldsymbol{r},\boldsymbol{k}_i)e^{-W_i t} &= \langle 0|T\{\varphi_A(\boldsymbol{x}+\boldsymbol{r},t)\varphi_B(\boldsymbol{x},t)\}|W_i\rangle \\
\varphi_{CD}(\boldsymbol{r},\boldsymbol{q}_i)e^{-W_i t} &= \langle 0|T\{\varphi_C(\boldsymbol{x}+\boldsymbol{r},t)\varphi_D(\boldsymbol{x},t)\}|W_i\rangle
\end{aligned}
$$

coupled channel Schroedinger eq.

$$
\begin{aligned}
\left[\frac{k_i^2}{2\mu_{AB}} - H_0\right]\varphi_{AB}(\boldsymbol{x},\boldsymbol{k}_i) &= \int d^3y\, U_{AB,AB}(\boldsymbol{x};\boldsymbol{y})\,\varphi_{AB}(\boldsymbol{y},\boldsymbol{k}_i) + \int d^3y\, U_{AB,CD}(\boldsymbol{x};\boldsymbol{y})\,\varphi_{CD}(\boldsymbol{y},\boldsymbol{q}_i) \\
\left[\frac{q_i^2}{2\mu_{CD}} - H_0\right]\varphi_{CD}(\boldsymbol{x},\boldsymbol{k}_i) &= \int d^3y\, U_{CD,AB}(\boldsymbol{x};\boldsymbol{y})\,\varphi_{AB}(\boldsymbol{y},\boldsymbol{k}_i) + \int d^3y\, U_{CD,CD}(\boldsymbol{x};\boldsymbol{y})\,\varphi_{CD}(\boldsymbol{y},\boldsymbol{q}_i)
\end{aligned}
$$

velocity expansion

$$
\begin{aligned}
K_{AB}(\boldsymbol{x},\boldsymbol{k}_i) \equiv \left[\frac{k_i^2}{2\mu_{AB}} - H_0\right]\varphi_{AB}(\boldsymbol{x},\boldsymbol{k}_i) &= V_{AB,AB}(\boldsymbol{x})\,\varphi_{AB}(\boldsymbol{x},\boldsymbol{k}_i) + V_{AB,CD}(\boldsymbol{x})\,\varphi_{CD}(\boldsymbol{x},\boldsymbol{q}_i) \\
K_{CD}(\boldsymbol{x},\boldsymbol{q}_i) \equiv \left[\frac{q_i^2}{2\mu_{CD}} - H_0\right]\varphi_{CD}(\boldsymbol{x},\boldsymbol{k}_i) &= V_{CD,AB}(\boldsymbol{x})\,\varphi_{AB}(\boldsymbol{x},\boldsymbol{k}_i) + V_{CD,CD}(\boldsymbol{x})\,\varphi_{CD}(\boldsymbol{x},\boldsymbol{q}_i)
\end{aligned}
$$

$$
\begin{pmatrix} V_{AB,AB}(\boldsymbol{x}) & V_{AB,CD}(\boldsymbol{x}) \\ V_{CD,AB}(\boldsymbol{x}) & V_{CD,CD}(\boldsymbol{x}) \end{pmatrix}
= \begin{pmatrix} K_{AB}(\boldsymbol{x},\boldsymbol{k}_1) & K_{AB}(\boldsymbol{x},\boldsymbol{k}_2) \\ K_{CD}(\boldsymbol{x},\boldsymbol{q}_1) & K_{CD}(\boldsymbol{x},\boldsymbol{q}_2) \end{pmatrix}
\times \begin{pmatrix} \varphi_{AB}(\boldsymbol{x},\boldsymbol{k}_1) & \varphi_{AB}(\boldsymbol{x},\boldsymbol{k}_2) \\ \varphi_{CD}(\boldsymbol{x},\boldsymbol{q}_1) & \varphi_{CD}(\boldsymbol{x},\boldsymbol{q}_2) \end{pmatrix}^{-1}
$$

inelastic due to particle production

$$A+B \to A+B \text{ and } A+B \to A+B+C$$

$$m_A + m_B + m_C < W < m_A + m_B + 2m_C$$

NBS wave functions

$$\begin{aligned}
\varphi^W_{AB}(\boldsymbol{x})e^{-Wt} &= \langle 0|T\{\varphi_A(\boldsymbol{r}+\boldsymbol{x},t)\varphi_B(\boldsymbol{r},t)\}|W\rangle \\
\varphi^W_{ABC}(\boldsymbol{x},\boldsymbol{y})e^{-Wt} &= \langle 0|T\{\varphi_A\left(\boldsymbol{r}+\boldsymbol{x}+\frac{\boldsymbol{y}\,\mu_{BC}}{m_C},t\right)\varphi_B(\boldsymbol{r}+\boldsymbol{y},t)\varphi_C(\boldsymbol{r},t)\}|W\rangle
\end{aligned}$$

couple channel eq. + velocity expansion

$$\begin{aligned}
K^W_{AB}(\boldsymbol{x}) &= V_{AB,AB}(\boldsymbol{x})\varphi^W_{AB}(\boldsymbol{x}) + \int d^3w\; V_{AB,ABC}(\boldsymbol{x},\boldsymbol{w})\varphi^W_{ABC}(\boldsymbol{x},\boldsymbol{w}) \\
K^W_{ABC}(\boldsymbol{x},\boldsymbol{y}) &= V_{ABC,AB}(\boldsymbol{x},\boldsymbol{y})\varphi^W_{AB}(\boldsymbol{x}) + V_{ABC,ABC}(\boldsymbol{x},\boldsymbol{y})\varphi^W_{ABC}(\boldsymbol{x},\boldsymbol{y}).
\end{aligned}$$

$$K^W_{AB}(\boldsymbol{x}) \equiv \left[\frac{\boldsymbol{k}^2}{2\mu_{AB}} - H_0^{AB}\right]\varphi^W_{AB}(\boldsymbol{x}) \qquad K^W_{ABC}(\boldsymbol{x},\boldsymbol{y}) \equiv \left[\frac{\boldsymbol{q}_x^2}{2\mu_{A,BC}} + \frac{\boldsymbol{q}_y^2}{2\mu_{BC}} - H_0^{A,BC} - H_0^{BC}\right]\varphi^W_{ABC}(\boldsymbol{x},\boldsymbol{y})$$

$$\begin{aligned}
\begin{pmatrix} V_{ABC,AB}(\boldsymbol{x},\boldsymbol{y}) & V_{ABC,ABC}(\boldsymbol{x},\boldsymbol{y}) \end{pmatrix} &= \begin{pmatrix} K^{W_1}_{ABC}(\boldsymbol{x},\boldsymbol{y}) & K^{W_2}_{ABC}(\boldsymbol{x},\boldsymbol{y}) \end{pmatrix} \\
&\times \begin{pmatrix} \Psi^{W_1}_{AB}(\boldsymbol{x}) & \Psi^{W_2}_{AB}(\boldsymbol{x}) \\ \Psi^{W_1}_{ABC}(\boldsymbol{x},\boldsymbol{y}) & \Psi^{W_2}_{ABC}(\boldsymbol{x},\boldsymbol{y}) \end{pmatrix}^{-1}
\end{aligned}$$

$$V_{AB,AB}(\boldsymbol{x}) = \frac{1}{\Psi^W_{AB}(\boldsymbol{x})}\left[K^W_{AB}(\boldsymbol{x}) - \int d^3w\; \underline{V_{ABC,AB}(\boldsymbol{x},\boldsymbol{w})}\Psi^W_{ABC}(\boldsymbol{x},\boldsymbol{w})\right]$$

hermiticity $V_{AB,ABC}(\boldsymbol{x},\boldsymbol{y}) = V_{ABC,AB}(\boldsymbol{x},\boldsymbol{y})$

7-2. S=-2, I=0 coupled channel

$$m_N = 939 \text{ MeV}, \; m_\Lambda = 1116 \text{ MeV}, \; m_\Sigma = 1193 \text{ MeV}, \; m_\Xi = 1318 \text{ MeV}$$

S=-2 System(I=0)

$$M_{\Lambda\Lambda} = 2232 \text{ MeV} < M_{N\Xi} = 2257 \text{ MeV} < M_{\Sigma\Sigma} = 2386 \text{ MeV}$$

local potentials

$$V_{AB,CD}(\boldsymbol{r}) = \sum_i K^i_{AB}(\boldsymbol{r}) \left[\varphi^{W_i}_{CD}(\boldsymbol{r}, \boldsymbol{k}^i_{CD}) \right]^{-1}$$

3 x 3(CD,i)

NBS wave functions

$$\varphi^{W_i}_{AB}(\boldsymbol{r}, \boldsymbol{k}^i_{AB}) e^{-W_i t} = \langle 0 | T\{\varphi_A(\boldsymbol{r}+\boldsymbol{x}, t)\varphi_B(\boldsymbol{r}, t)\} | W_i \rangle$$

$$AB = \Lambda\Lambda, \; N\Xi \text{ and } \Sigma\Sigma$$

$$K^i_{AB}(\boldsymbol{r}) = \frac{1}{2\mu_{AB}} \left(\nabla^2 + (\boldsymbol{k}^i_{AB})^2 \right) \varphi^{W_i}_{AB}(\boldsymbol{r}, \boldsymbol{k}^i_{AB})$$

$$W_i = \sqrt{(\boldsymbol{k}^i_{AB})^2 + m_A^2} + \sqrt{(\boldsymbol{k}^i_{AB})^2 + m_B^2}$$

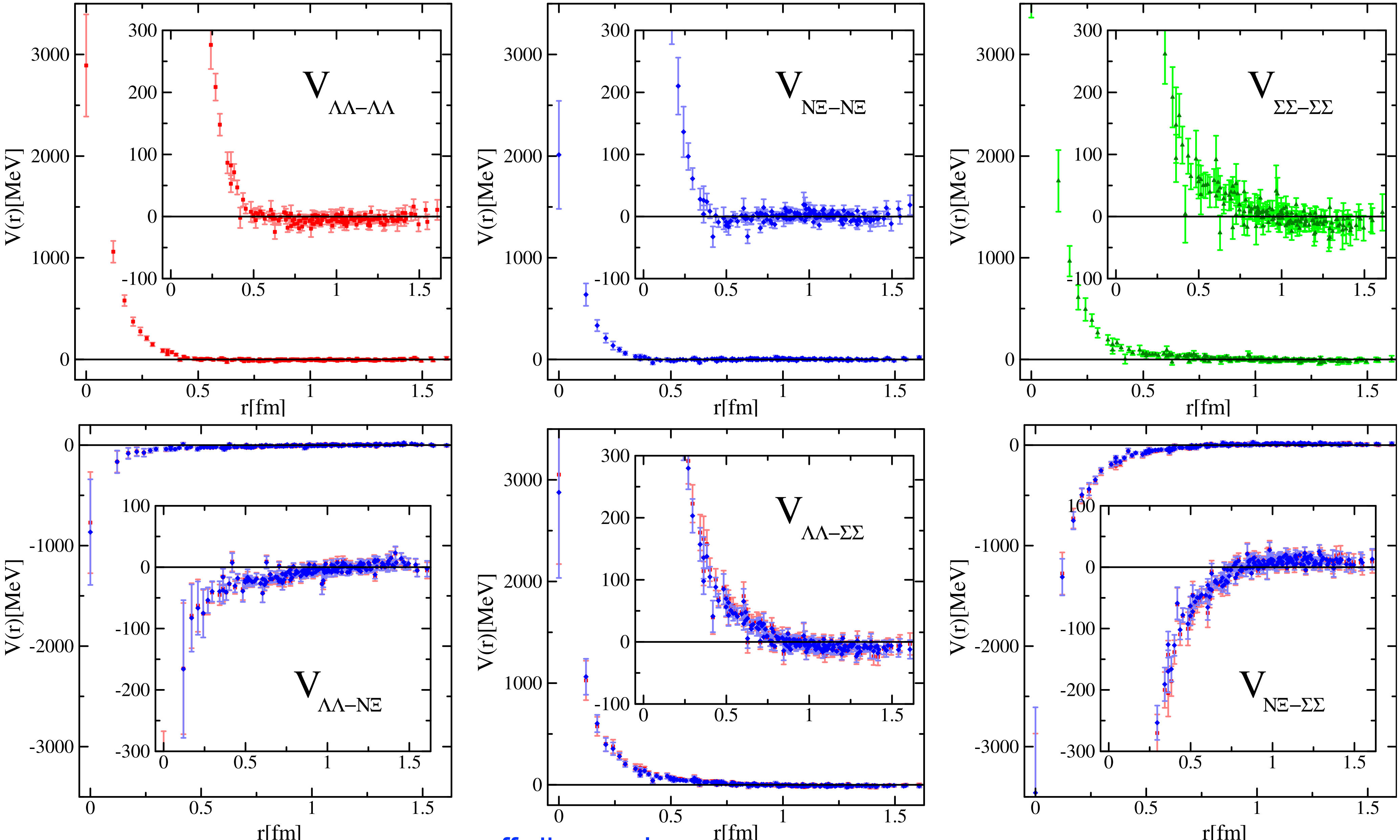
CP-PACS/JLQCD gauge configuration

$$a \simeq 0.12 \text{ fm} \qquad L \simeq 1.9 \text{ fm}$$

	N_{conf}	m_π	m_K	m_N	m_Λ	m_Σ	m_Ξ
Set 1	700	875(1)	916(1)	1806(3)	1835(3)	1841(3)	1867(2)
Set 2	800	749(1)	828(1)	1616(3)	1671(2)	1685(2)	1734(2)
Set 3	800	661(1)	768(1)	1482(3)	1557(3)	1576(3)	1640(3)

Set 1

diagonal

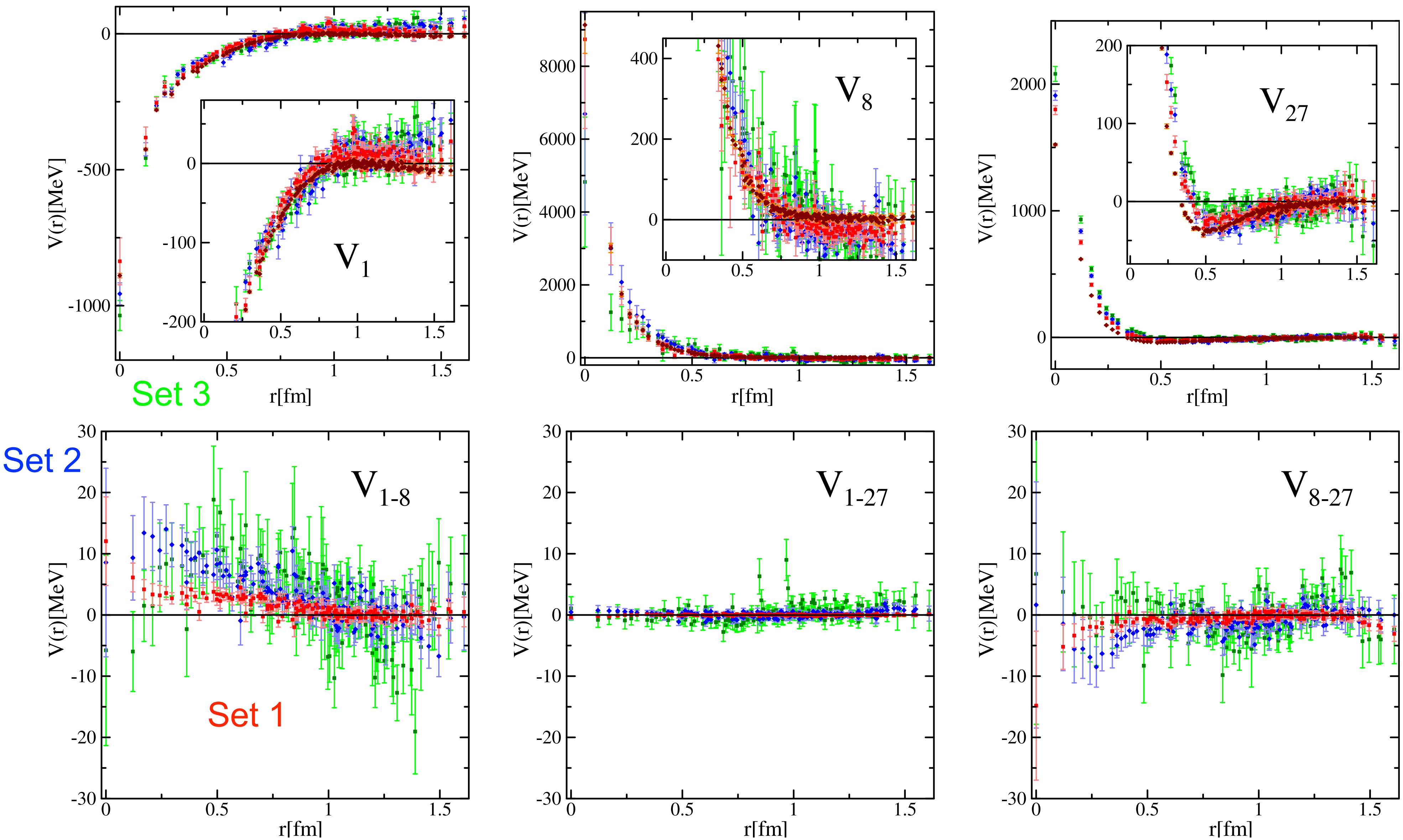


off-diagonal

$V_{AB} \simeq V_{BA}$: hermiticity

off-diagonal $\simeq$ diagonal

Flavor SU(3) basis



similar to SU(3) limit

off-diagonal $\ll$ diagonal

SU(3) breaking becomes manifest in V1-8 and increases as decrease pion mass

7-3. Time dependent method

Ground state saturation should be satisfied for all r.

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r}, t-t_0) = A_0 \varphi^{W_0}(\mathbf{r}) e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$ difficult !

Consider $$F(\boldsymbol{r}, t) = \sum_{W \leq W_{\rm th}} A_W \phi^W(\boldsymbol{r}) e^{-Wt} + O(e^{-W_{\rm th} t})$$ which satisfies

$$H_0 F(\boldsymbol{r}, t) \simeq \sum_W A_W \int d^3\boldsymbol{r}' [E_W \delta^{(3)}(\boldsymbol{r}-\boldsymbol{r}') - U(\boldsymbol{r}, \boldsymbol{r}')] \varphi^W(\boldsymbol{r}') e^{-Wt}$$

Using $W = 2\sqrt{k_W^2 + m_N^2} = 2m_N + k_W^2/m_N + O(k_W^4/m_N^3)$ (non-rel. expansion)

velocity expansion

$$\left[H_0 + \frac{d}{dt} + 2m_N\right] F(\boldsymbol{r}, t) = -\int d^3\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}') F(\boldsymbol{r}, t) \simeq -V^{\rm LO}(\boldsymbol{r}) F(\boldsymbol{r}, t)$$

$$\Rightarrow \quad V^{\rm LO}(\boldsymbol{r}) = -\frac{\left[H_0 + \frac{d}{dt}\right] R(\boldsymbol{r}, t)}{R(\boldsymbol{r}, t)}$$

$$R(\boldsymbol{r}, t) = F(\boldsymbol{r}, t)/e^{-(m_A+m_B)t}.$$

Relativistic formulation

$$m_A = m_B$$

$$V^{\mathrm{LO}}(\boldsymbol{r}) = \frac{\left[-H_0 + \frac{1}{4m_N}\frac{d^2}{dt^2} - m_N\right] F(\boldsymbol{r}, t)}{F(\boldsymbol{r}, t)}$$

Non-local potential

symmetric correlation function

$$F(\boldsymbol{x}, \boldsymbol{y}, t) = \int d^3\boldsymbol{x}_1 d^3\boldsymbol{y}_1 \langle 0|T\{N(\boldsymbol{x}_1 + \boldsymbol{x}, t)N(\boldsymbol{x}_1, t)\}T\{\overline{N}(\boldsymbol{y}_1 + \boldsymbol{y}, 0)\overline{N}(\boldsymbol{y}_1, 0)\}|0\rangle$$

$$\left[H_0 - \frac{1}{4m_N}\frac{d^2}{dt^2} + m_N\right] R(\boldsymbol{x}, \boldsymbol{y}, t) = -\int d^3\boldsymbol{z}\, U(\boldsymbol{x}, \boldsymbol{z}) F(\boldsymbol{z}, \boldsymbol{y}, t)$$

$$\Rightarrow \quad U(\boldsymbol{x}, \boldsymbol{y}) = \int d^3\boldsymbol{z} \left[-H_0 + \frac{1}{4m_N}\frac{d^2}{dt^2} - m_N\right] F(\boldsymbol{x}, \boldsymbol{z}, t) \cdot \tilde{F}^{-1}(\boldsymbol{z}, \boldsymbol{y}, t)$$

eigenvector of F

$$\tilde{F}^{-1}(\boldsymbol{x}, \boldsymbol{y}, t) = \sum_{\lambda_n \neq 0} \frac{1}{\lambda_n(t)} v_n(\boldsymbol{x}, t) v_n^\dagger(\boldsymbol{y}, t)$$

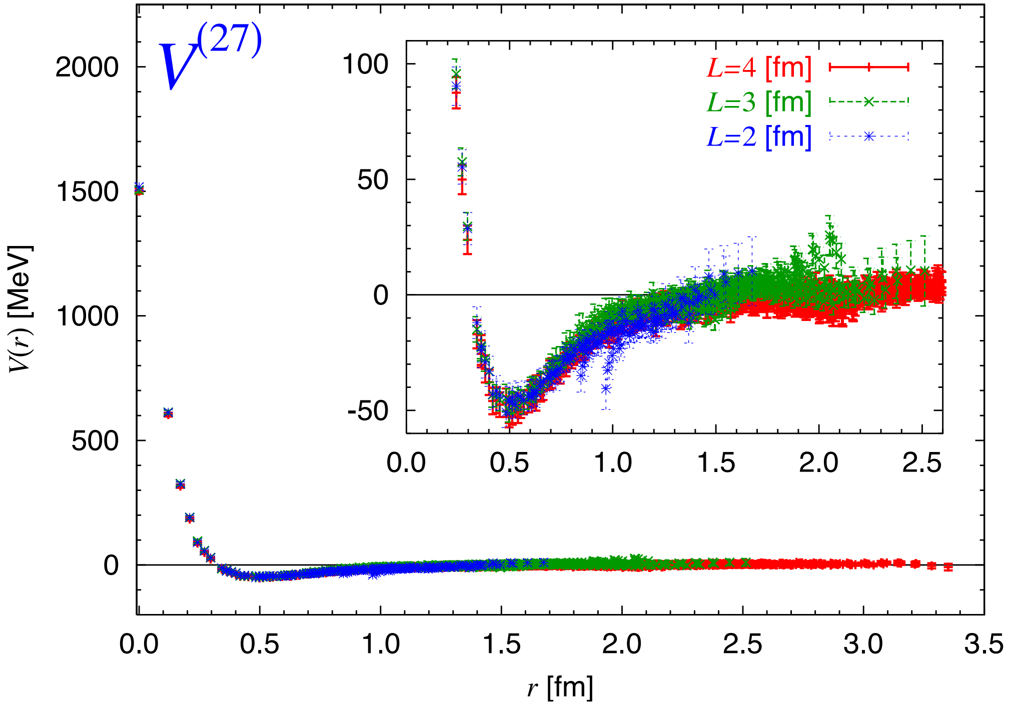
eigenvalue of F

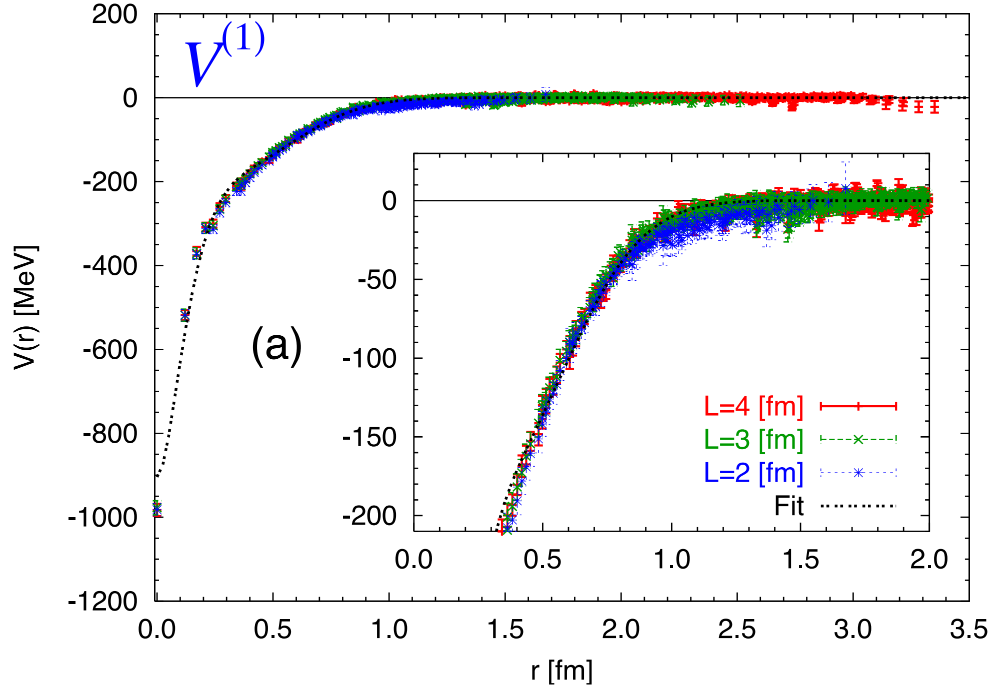
7-4. Bound H dibaryon in flavor SU(3) limit

Large volumes and time dependent method

$$V_C^{(X)}(r) = -\frac{\left[H_0 + \frac{d}{dt}\right] R(\boldsymbol{r}, t-t_0)}{R(\boldsymbol{r}, t-t_0)}$$

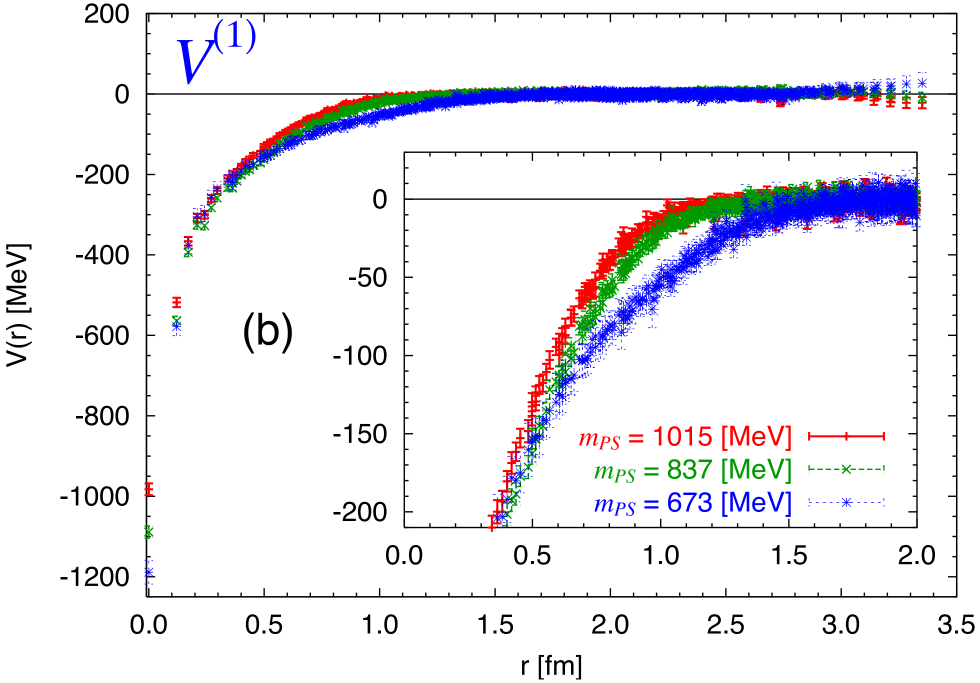
volume dependence





L=3 fm is enough for the potential.

pion mass dependence



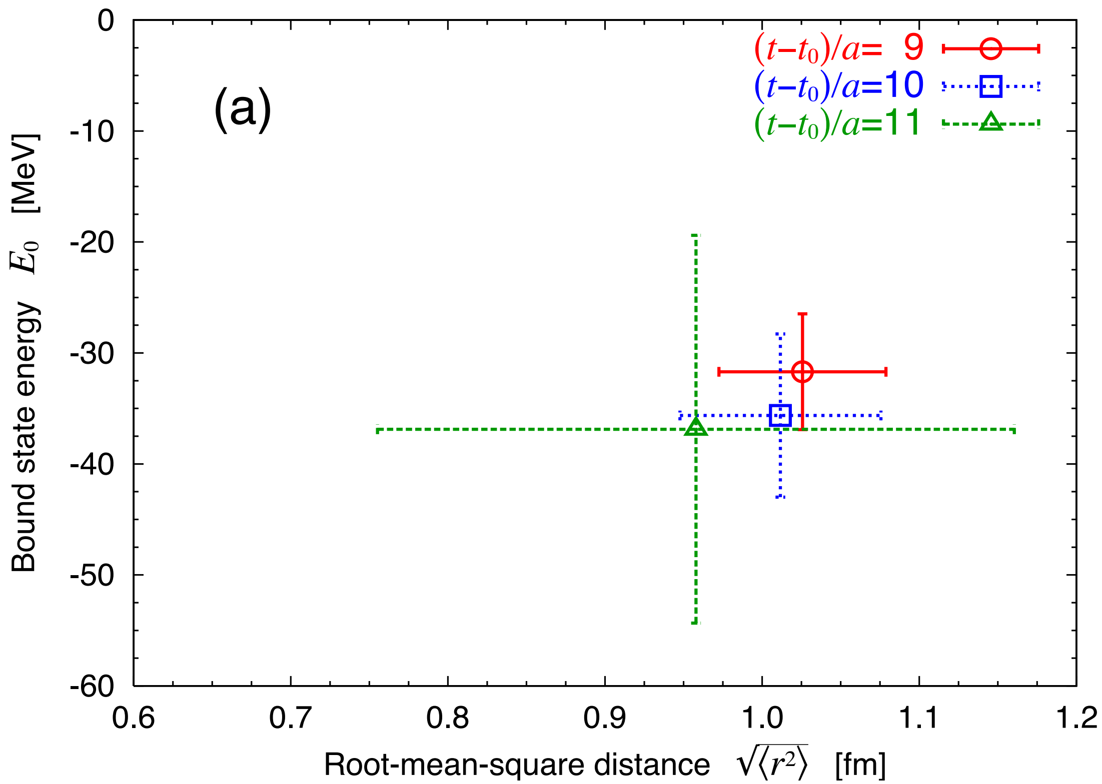
lighter the pion mass, stronger the attraction

fit the potential at L=4 fm by

$$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$$

solve Schroedinger equation with this potential in the infinite volume.

A bound state (H dibaryon) exists !

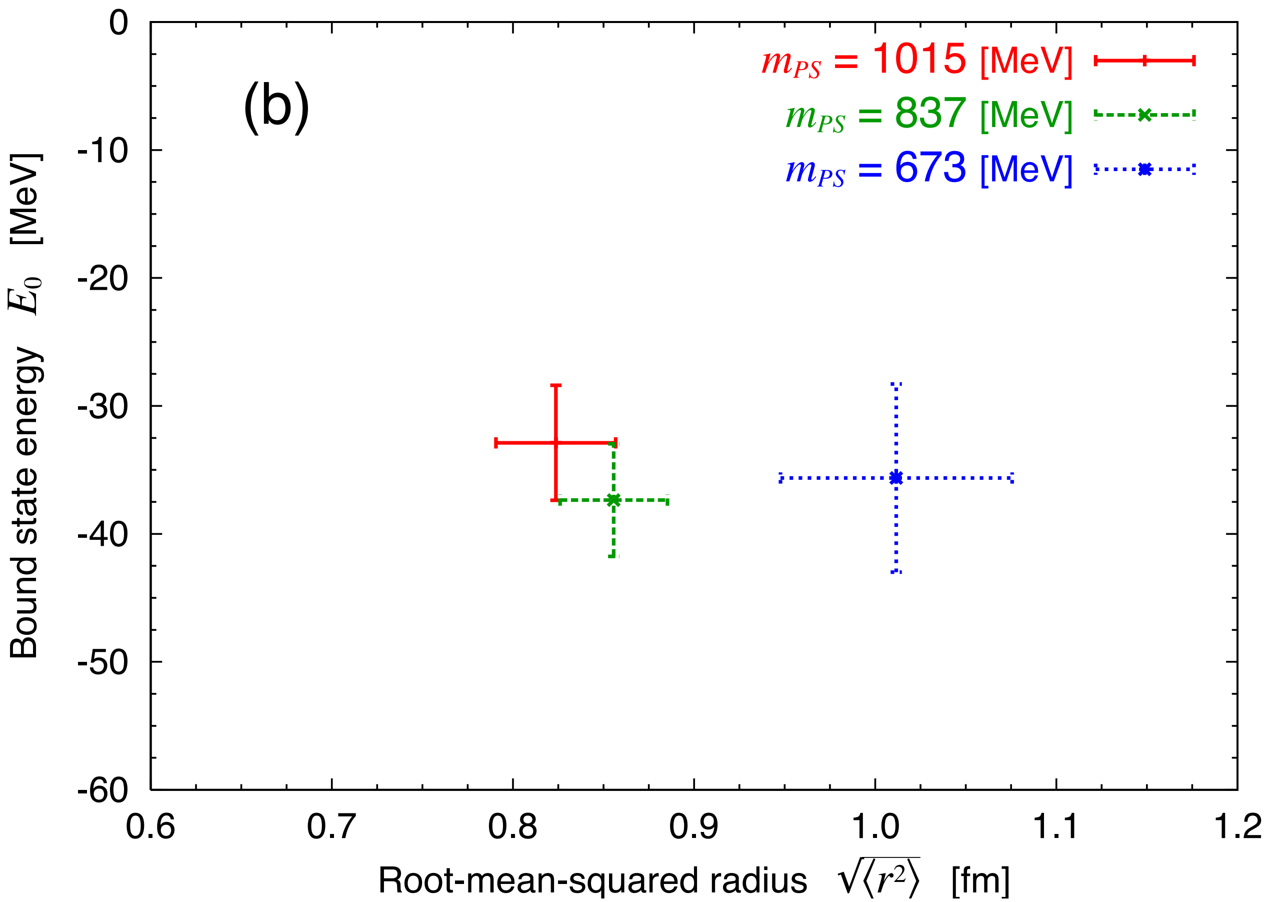


t-dependence

t=10a is enough.

t=9a,11a ⟶ symmetric errors

pion mass dependence

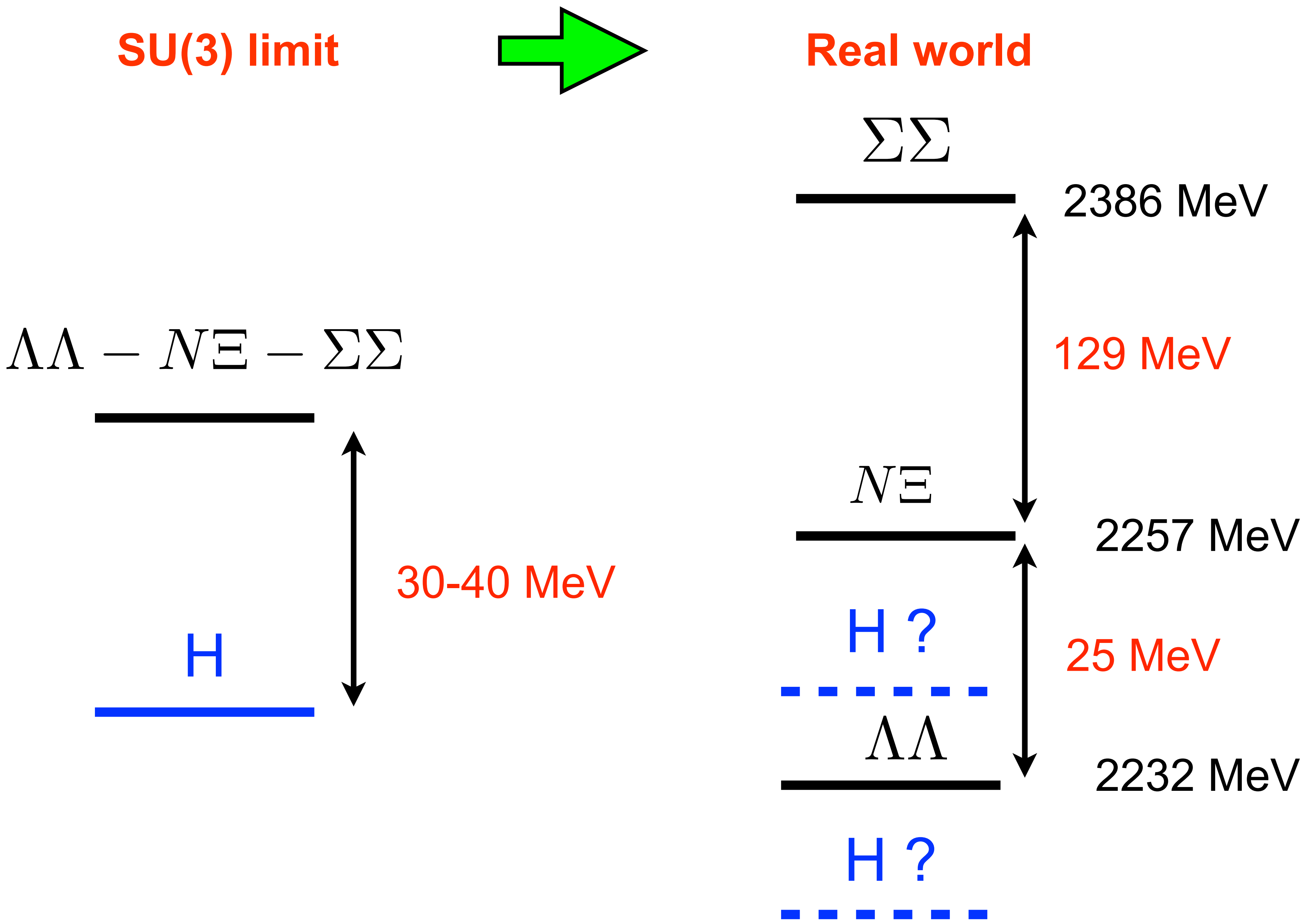


$$
\begin{aligned}
m_{ps} = 1015 \text{ MeV}: \quad \tilde{B}_H &= 32.9(4.5)(6.6) \text{ MeV} \\
\sqrt{\langle r^2 \rangle} &= 0.823(33)(40) \text{ fm} \\
m_{ps} = 837 \text{ MeV}: \quad \tilde{B}_H &= 37.4(4.4)(7.3) \text{ MeV} \\
\sqrt{\langle r^2 \rangle} &= 0.855(29)(61) \text{ fm} \\
m_{ps} = 673 \text{ MeV}: \quad \tilde{B}_H &= 35.6(7.4)(4.0) \text{ MeV} \\
\sqrt{\langle r^2 \rangle} &= 1.011(63)(68) \text{ fm}
\end{aligned}
$$

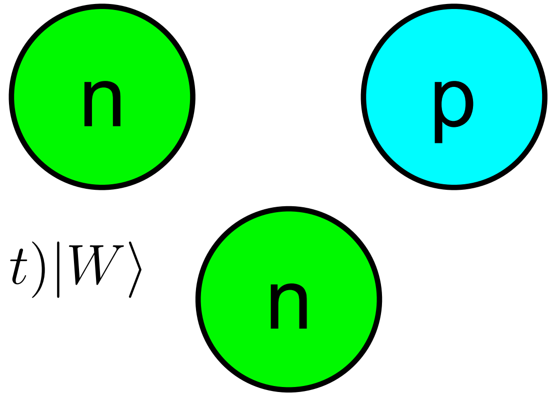
An H-dibaryon exists
in the flavor SU(3) limit !
Binding energy = 30-40 MeV,
weak quark mass dependence.

Real world ?

coupled channel analysis with SU(3) breaking is needed.



7-5. Three nucleon force (TNF)



NBS wave function

$$\varphi^W(\boldsymbol{r}_{12}, \boldsymbol{r}_{123}) e^{-Wt} = \langle 0 | N(\boldsymbol{x}_1, t) N(\boldsymbol{x}_2, t) N(\boldsymbol{x}_3, t) | W \rangle$$

$$\left[-\frac{1}{2\mu_{12}} \nabla^2_{r_{12}} - \frac{1}{2\mu_{123}} \nabla^2_{r_{123}} + \sum_{i<j} V_{2\mathrm{N},ij}(\boldsymbol{x}_i - \boldsymbol{x}_j) + V_{\mathrm{TNF}}(\boldsymbol{r}_{12}, \boldsymbol{r}_{123}) \right] \varphi^W(\boldsymbol{r}_{12}, \boldsymbol{r}_{123}) = E \varphi^W(\boldsymbol{r}_{12}, \boldsymbol{r}_{123})$$

TNF

numerically demanding L^3 more expensive than NN potential

Effective NN potential

$$\varphi^W(\boldsymbol{r}_{12}) = \sum_{\boldsymbol{x}_3} \varphi^W(\boldsymbol{r}_{12}, \boldsymbol{r}_{123}) = \sum_{\boldsymbol{r}_{123}} \varphi^W(\boldsymbol{r}_{12}, \boldsymbol{r}_{123}).$$

integration over the position

$$\left[-\frac{1}{2\mu_{12}} \nabla^2_{r_{12}} + V_{\mathrm{eff}}(\boldsymbol{r}_{12}) \right] \varphi^W(\boldsymbol{r}_{12}) = E \varphi^W(\boldsymbol{r}_{12})$$

effective NN

$V_{eff}(\vec{r}) - V_{2N}(\vec{r})$ “finite density effect” in 3N system

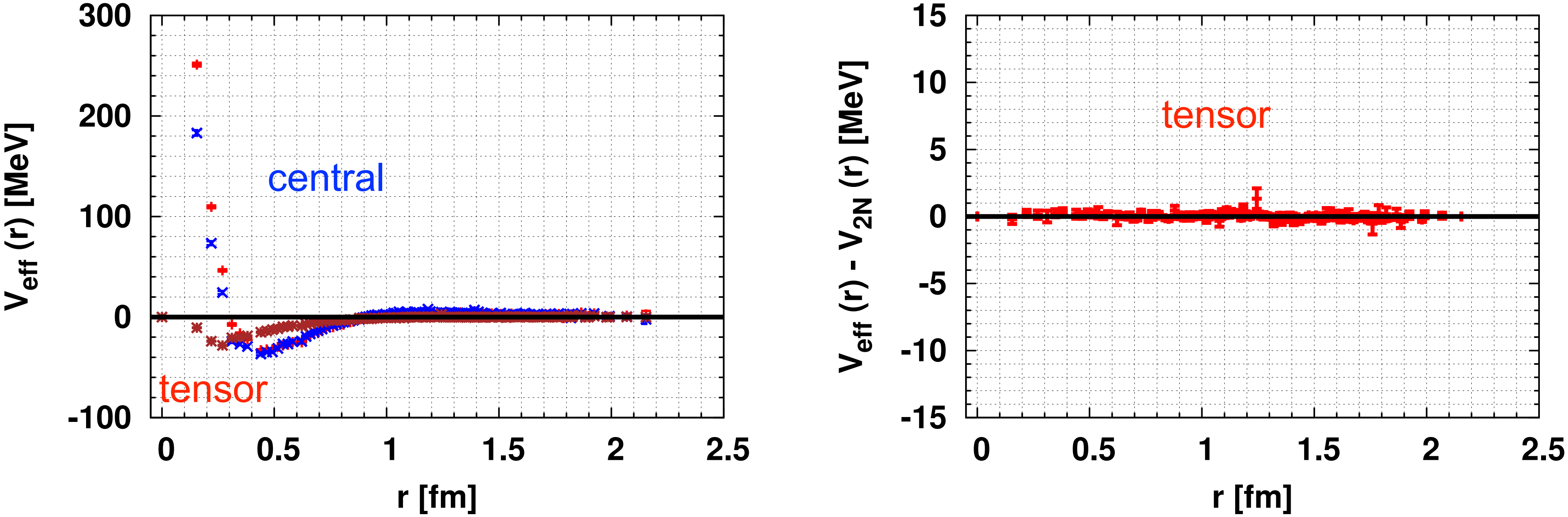
2-flavor QCD

$a \simeq 0.16$ fm, $L \simeq 2.6$ fm

Doi for HAL QCD

$m_\pi \simeq 1.13$ GeV, $m_N \simeq 2.15$ GeV

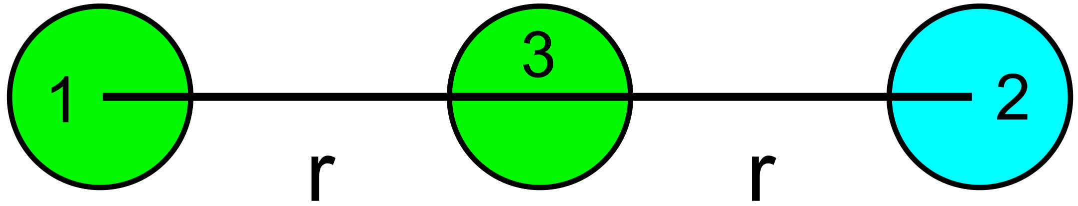
Triton$(I = 1/2,\ J^P = 1/2^+)$



- effective NN can be obtained in good precision.
- The difference is consistent with zero within MeV statistical error.
    - almost no “density effect”
    - heavy pion ?
    - TNF effect is suppressed by integration ?

Linear setup

Triton$(I = 1/2,\ J^P = 1/2^+)$

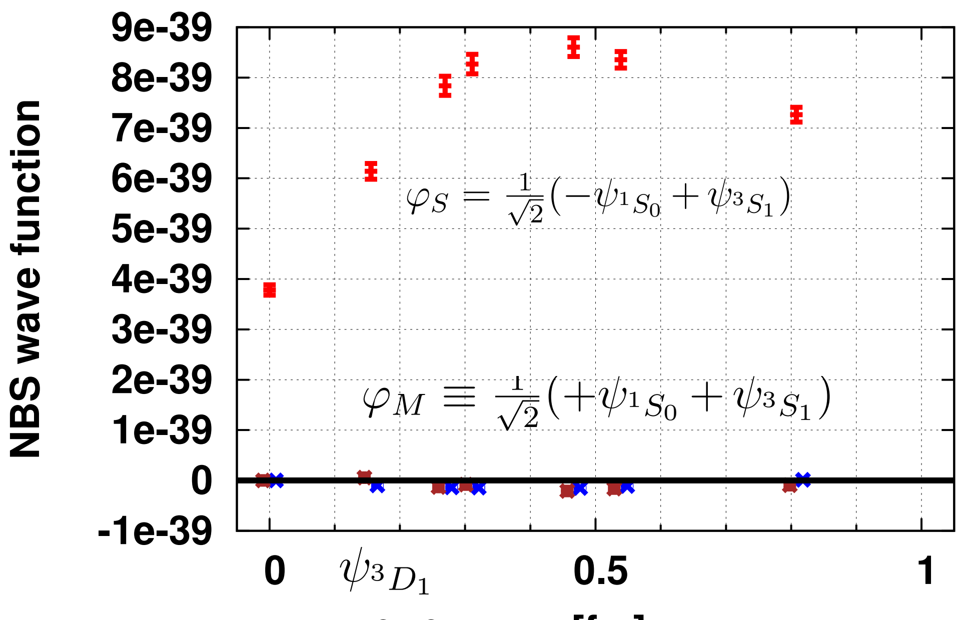


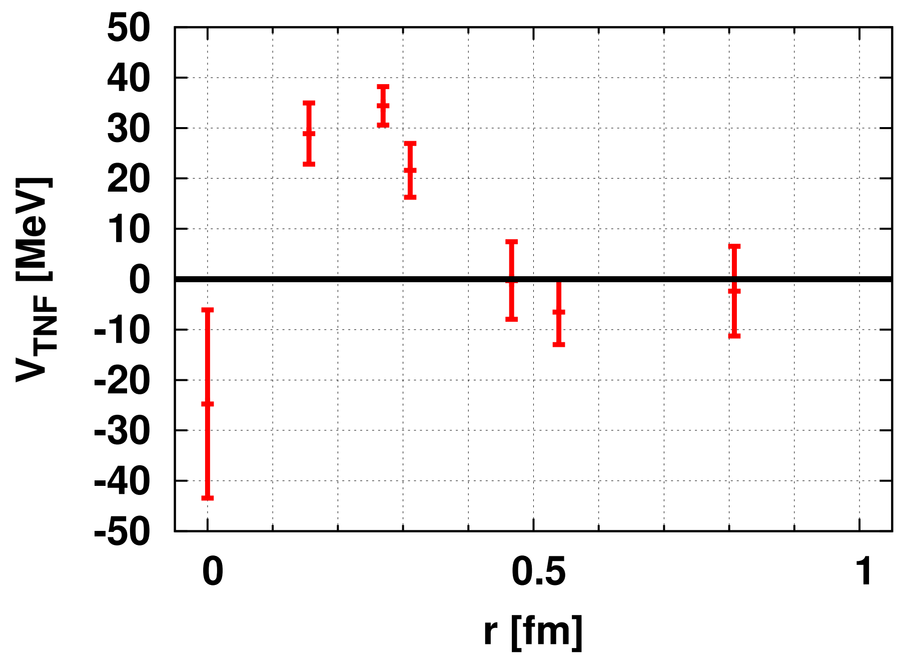
S-wave only

(1,2) pair $\quad {}^1S_0,\ {}^3S_1,\ {}^3D_1$

$$\varphi_S \equiv \frac{1}{\sqrt{6}}\left[-p_\uparrow n_\uparrow n_\downarrow + p_\uparrow n_\downarrow n_\uparrow - n_\uparrow n_\downarrow p_\uparrow + n_\downarrow n_\uparrow p_\uparrow + n_\uparrow p_\uparrow n_\downarrow - n_\downarrow p_\uparrow n_\uparrow\right]$$

symmetric wave function





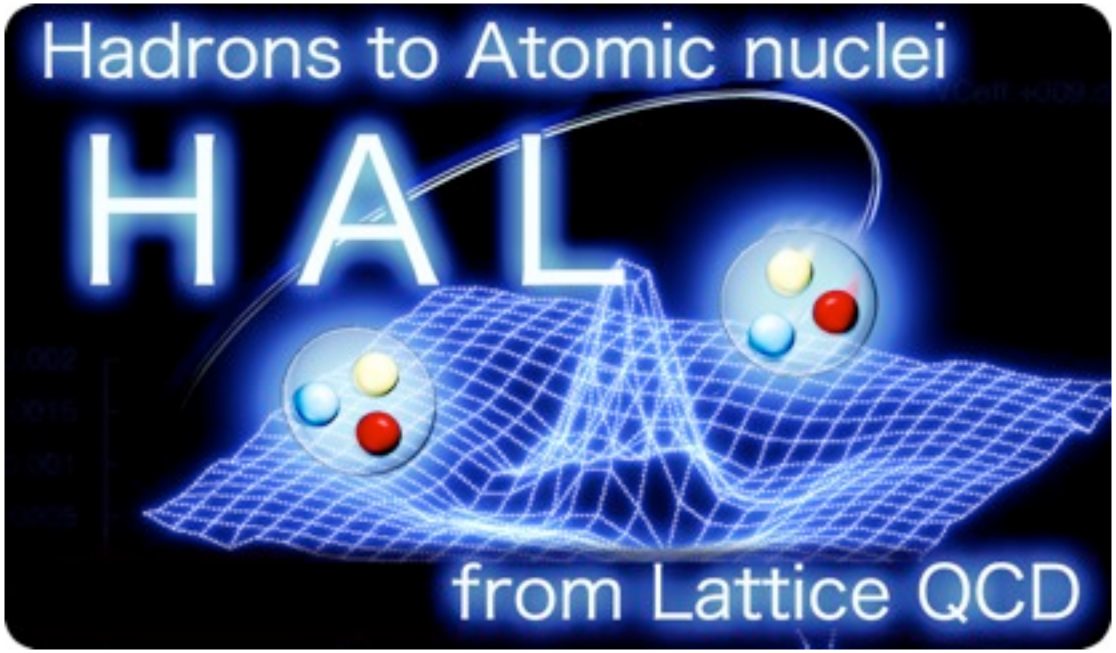
scalar/isoscalar TNF is observed at short distance.

further study is needed to confirm this result.

8. Conclusions

- the potential method is new but very useful to investigate baryon interactions in lattice QCD.
- the method can be easily also applied to meson-baryon and meson-meson interactions.
- three body force can be analyzed.
- various extensions of the method will be looked for.

HAL QCD Collaboration



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Back up slides

BS amplitude

$$A = (k^a, s^a) = (k_0, \vec{k}, s^a)$$

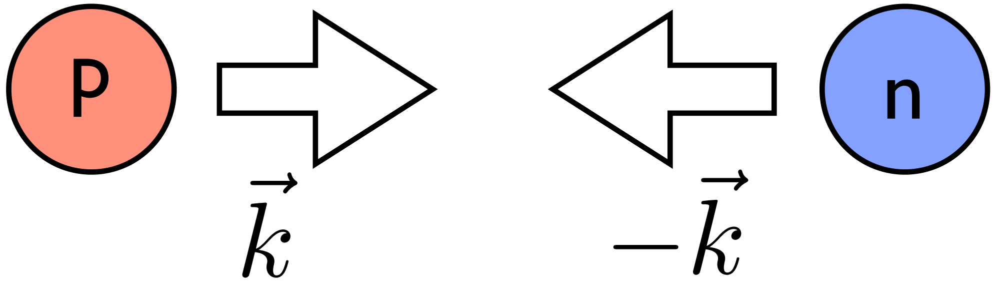
$$k_0 = \sqrt{\vec{k}^2 + m_N^2}$$ helicity

$$B = (k^b, s^b) = (k_0, -\vec{k}, s^b)$$

$$\varphi_{\alpha\beta}(x, y; AB) = \langle 0 | T\{N_\alpha^a(x) N_\beta^b(y)\} | N^a(A) N^b(B) \rangle_{\rm in}$$

$$N_\alpha^a = \varepsilon^{ABC}(u^A C\gamma_5 d^B) q_\alpha^{C,a} = \begin{pmatrix} p \\ n \end{pmatrix}$$

$$C = \gamma_2\gamma_4 \qquad q = \begin{pmatrix} u \\ d \end{pmatrix}$$



For simplicity, we assume $a \neq b$ ($I = 0$ or 1) and $x_0 > y_0$.

Complete set

$$\mathbf{1} = \sum_{s,c} \int \frac{d^3p}{(2\pi)^3 2p_0} |N^c(\vec{p}, s)\rangle \langle N^c(\vec{p}, s)| + \sum_{X \neq N^c} |X\rangle \frac{1}{2E_X} \langle X|$$

1 particle out-state

others

$$\varphi_{\alpha\beta}(x, y, AB) = \varphi_{\alpha\beta}^{\rm elastic}(x, y) + \varphi_{\alpha\beta}^{\rm inelastic}(x, y)$$

$$\varphi_{\alpha\beta}^{\rm elastic}(x, y) = \sum_{s,c} \int \frac{d^3p}{(2\pi)^3 2p_0} \langle 0 | N_\alpha^a(x) | N^c(\vec{p}, s)\rangle \langle N^c(\vec{p}, s) | N_\beta^b(y) | N^a N^b \rangle$$

$$\varphi_{\alpha\beta}^{\rm inelastic}(x, y) = \sum_{X \neq N^c} \langle 0 | N_\alpha^a(x) | X \rangle \frac{1}{2E_X} \langle X | N_\beta^b(y) | N^a N^b \rangle$$

Elastic contribution

$$\langle 0|N^a_\alpha(x)|N^c(\vec{p},s)\rangle = \delta^{ab}\sqrt{Z^a}u_\alpha(\vec{p},s)e^{-ipx}$$

spinors

$$\varphi^{\rm elastic}_{\alpha\beta}(x,y) = \sum_s \int \frac{d^3p}{(2\pi)^3 2p_0}\sqrt{Z^a}u_\alpha(\vec{p},s)e^{-ipx}\boxed{\langle N^a(\vec{p},s)|N^b_\beta(y)|N^aN^b\rangle}$$

reduction formula

$$N^a_\alpha(p) = \int d^4x\, e^{ipx}\frac{1}{\sqrt{Z^a}}N^a_\alpha(x)$$

$$\boxed{} = \langle 0|B^a_{\rm out}(\vec{p},s)N^b_\beta(y)|N^aN^b\rangle$$

$$= \boxed{-\langle 0|N^b_\beta(y)B^a_{\rm in}(\vec{p},s)|N^aN^b\rangle} - iu(\vec{p},s)\underbrace{\boxed{(\gamma\cdot p - m_N)}}_{D(p)}\boxed{\langle 0|T\{N^a(p)N^b_\beta(y)\}|N^aN^b\rangle}$$

$$\boxed{} = -(2\pi)^3 2k_0\delta_{ss^a}\delta^{(3)}(p-k)\sqrt{Z^b}u(-\vec{k},s^b)e^{-ik^by}$$

$$\boxed{} = -\sqrt{Z^b}\int\frac{d^4q}{(2\pi)^4}e^{-iqy}\boxed{\langle 0|T\{N^a(p)N^b_\beta(q)N^a(k^a)N^b(k^b)\}|0\rangle}D(k^a)u(\vec{k},s^a)D(k^b)u(-\vec{k},s^b)$$

$$\rightarrow i(2\pi)^4\delta(p+k-k^a-k^b)G^{abab}_{\alpha\beta\gamma\delta}$$

"Off-shell T matrix"

$$\hat{T}^{abab}_{s\beta s^a s^b}(p,q,k^a,k^b) \quad = \quad i[\bar{u}D]_\alpha(\vec{p},s)iD_{\beta\beta'}(q)G^{abab}_{\alpha\beta'\gamma\delta}(p,q,k^a,k^b)[Du]_\gamma(\vec{k},s^a)[Du]_\delta(-\vec{k},s^b)$$

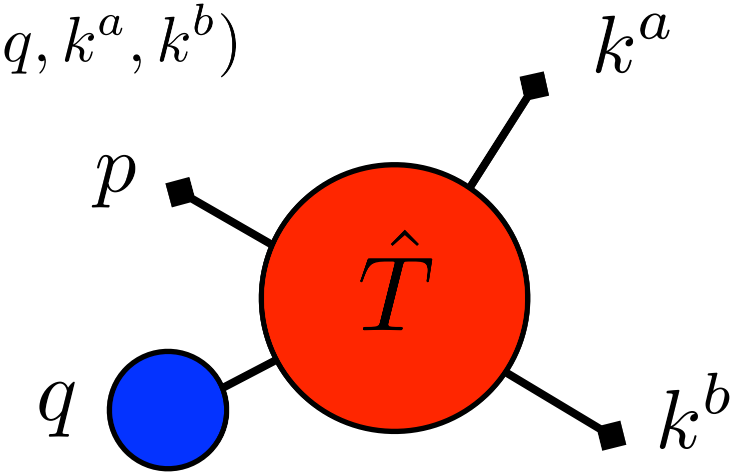
$$q = k^a + k^b - p = (2k_0 - p_0, -\vec{p})$$

$$\varphi^{\text{elastic}}_{\alpha\beta}(x,y) = -\sqrt{Z^a Z^b}\, u_\alpha(\vec{k}, s^a) u_\beta(-\vec{k}, s^b) e^{-ik_0(x+y)_0} e^{i\vec{k}\vec{r}}$$ incoming wave

$$+ \sqrt{Z^a Z^b} \sum_s \int \frac{d^3p}{(2\pi)^3 2p_0} u_\alpha(\vec{p}, s) e^{-ip_0x_0 - q_0y_0} e^{i\vec{p}\vec{r}}$$

$\vec{r} = \vec{x} - \vec{y}$

$$\times \left[\frac{1}{m - \gamma\cdot q - i\epsilon}\right]_{\beta\beta'} \hat{T}^{abab}_{s\beta' s^a s^b}(p, q, k^a, k^b)$$



Using $$\sum_s u(-\vec{p}, s)\bar{u}(-\vec{p}, s) = m + \gamma\cdot q$$

$$\frac{1}{m - \gamma\cdot q - i\epsilon} = (m + \gamma q)\left[i\frac{\pi}{4p}\delta(p-k) + P\frac{p_0 + k_0}{4k_0(p^2 - k^2)}\right]$$

on-shell T matrix $$T^{abab}_{ss^d s^a s^b}(\vec{p}, -\vec{p}, \vec{k}, -\vec{k}) = \bar{u}_\beta(-\vec{p}, s_d)\hat{T}^{abab}_{s\beta s^a s^b}(p, q, k^a, k^b)|_{q_0 = k_0}$$

Final result

$$\frac{\varphi^{\text{elastic}}_{\alpha\beta}(x,y)}{\sqrt{Z^a Z^b}} = -u_\alpha(\vec{k}, s^a) u_\beta(-\vec{k}, s^b) e^{-ik_0(x+y)_0} e^{i\vec{k}\vec{r}}$$

$$+ \frac{ik}{32\pi k_0}\int_{-1}^{1} d\cos\theta\, e^{-ik_0(x+y)_0} e^{ikr\cos\theta} u_\alpha(\vec{p}, s) u_\beta(-\vec{p}, s^d) T^{abab}_{ss^d s^a s^b}(\vec{p}, -\vec{p}, \vec{k}, -\vec{k})|_{p=k}$$

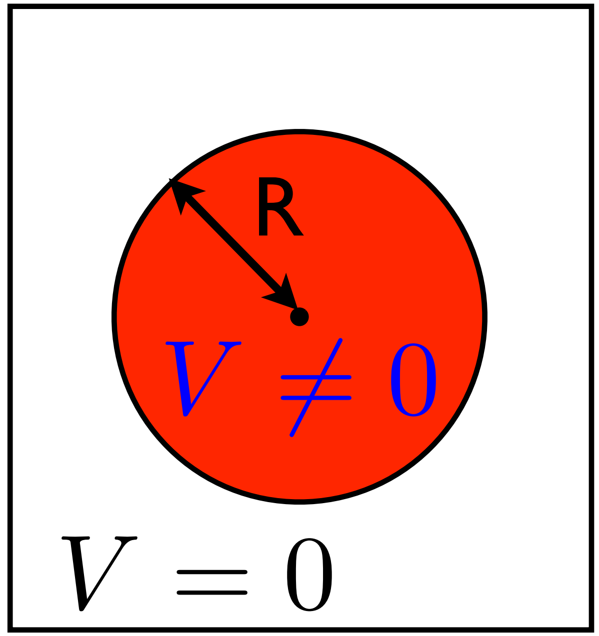
$$+ \sum_s \int \frac{d^3p}{(2\pi)^3} u_\alpha(\vec{p}, s) e^{-ip_0x_0 - q_0y_0} e^{i\vec{p}\vec{r}} (m + \gamma\cdot q)_{\beta\beta'} \frac{p_0 + k_0}{8p_0k_0} \frac{1}{p^2 - k^2} \hat{T}^{abab}_{s\beta' s^a s^b}(p, q, k^a, k^b)$$

Similarly "inelastic contributions" can be evaluated.

Take $x_0 \to y_0$ and consider the spin singlet state $({}^1S_0)$.

at $r = |\vec{r}| > R$ Interaction range R

$k = |\vec{k}|$



$$V^{{}^1S_0}(r,k) = -(\nabla^2 + k^2)\varphi^{{}^1S_0}(\vec{r},k) \to 0$$

$r > R$ $\quad E < E_{\text{inelastic}}$

$$\varphi^{{}^1S_0}(r,k) \sim j_0(kr) + \frac{k}{16\pi k_0} T_{{}^1S_0}(|p| = k, k)[n_0(kr) + i j_0(kr)]$$

$$\frac{k}{16\pi k_0} T_{{}^1S_0}(|p| = k, k) = e^{i\delta_0(k)} \sin\delta_0(k)$$

phase shift

$$\varphi^{{}^1S_0}(r,k) \sim \frac{e^{i\delta_0(k)}}{kr} \sin(kr + \delta_0(k))$$