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Finite Temperature QCD on the Lattice

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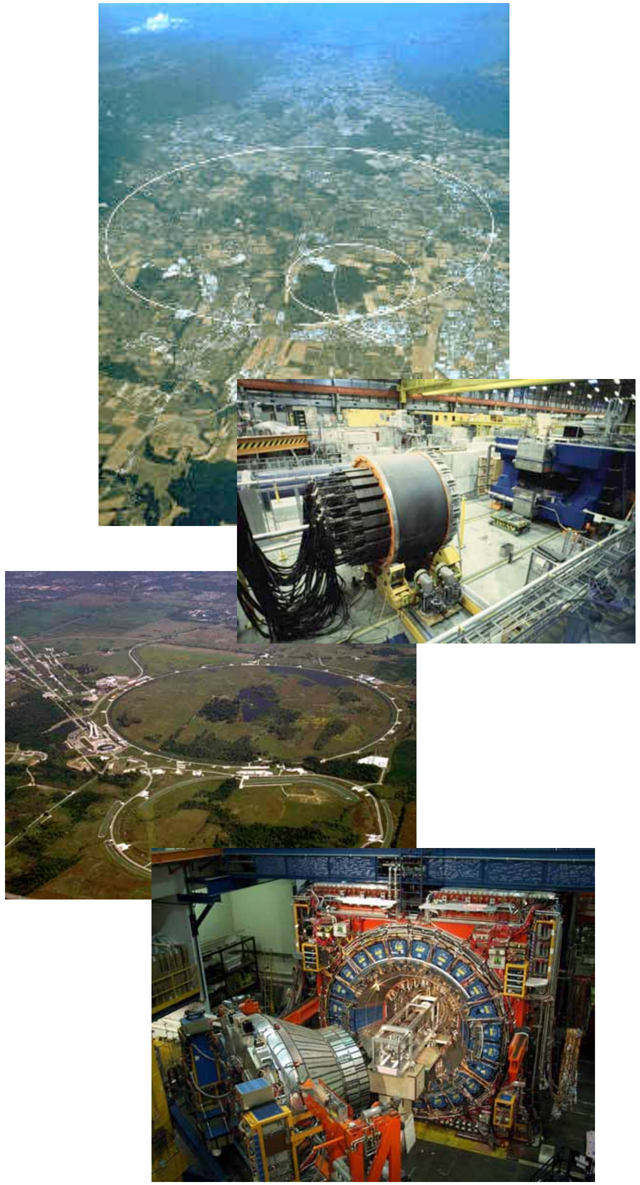
plan (hopefully)

- Lecture 1 (Mar.21)
  - motivations
  - formulation
  - phase transition and critical scaling
- Lecture 2 (Mar.22 ?)
  - QCD transition I: deconfinement transition
  - QCD transition II: chiral transition
- Lecture 3 (Mar.23 ??)
  - thermodynamic quantities

0. motivations

Why $T > 0$?



What do we know about particles?

We know from experiments that

strong interactions (QCD)

=> confinement,
spontaneous chiral breaking, etc.

electroweak interactions (WS)

=> <Higgs> ≠ 0,
$M_W \approx 80$GeV, $\theta_W \approx 30°$, etc.

They are the properties in the vacuum,
i.e. at $T \approx 0$ and $\mu \approx 0$.

T and μ : additional parameters to be explored.

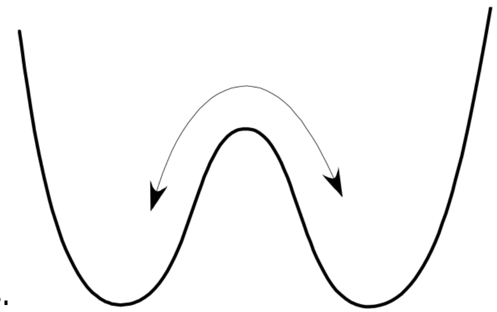
Something drastic may happen at high T and/or μ.

What may happen at T>0?

At very high T, we may expect

- restoration of spont. broken symmetries

  Thermal fluctuations will average over all sectors
  when the thermal energy > potential barrier between the sectors.



  - WS: restoration of the electroweak symmetry

    T_c ~ O(1) TeV

  - QCD: resoration of the chiral symmetry

    T_c ~ O(100) MeV

- breakdown of confinement

  confinement <= strong attraction in the IR limit
  asymptotic freedom: $\alpha_s(T) \longrightarrow 0$ as $T \longrightarrow \infty$.

    T_c ~ O(100) MeV

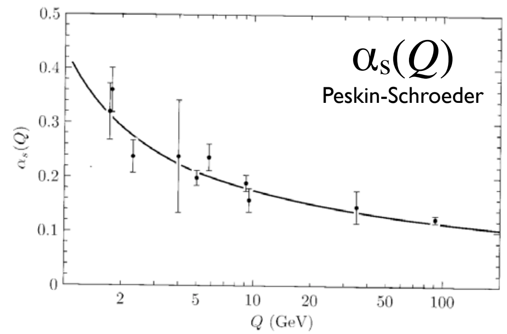


Figure 17.23. Measurements of α_s, plotted against the momentum scale Q at which the measurement was made. This figure was constructed by evolving the values of $\alpha_s(m_Z)$ listed in Table 17.1 back to the values of Q indicated in the table. The value for e^+e^- event shapes has been split into two points corresponding to experiments at the TRISTAN and LEP accelerators. These values are compared to the theoretical expectation from the renormalization group evolution with the initial condition $\alpha_s(m_Z) = 0.117$.

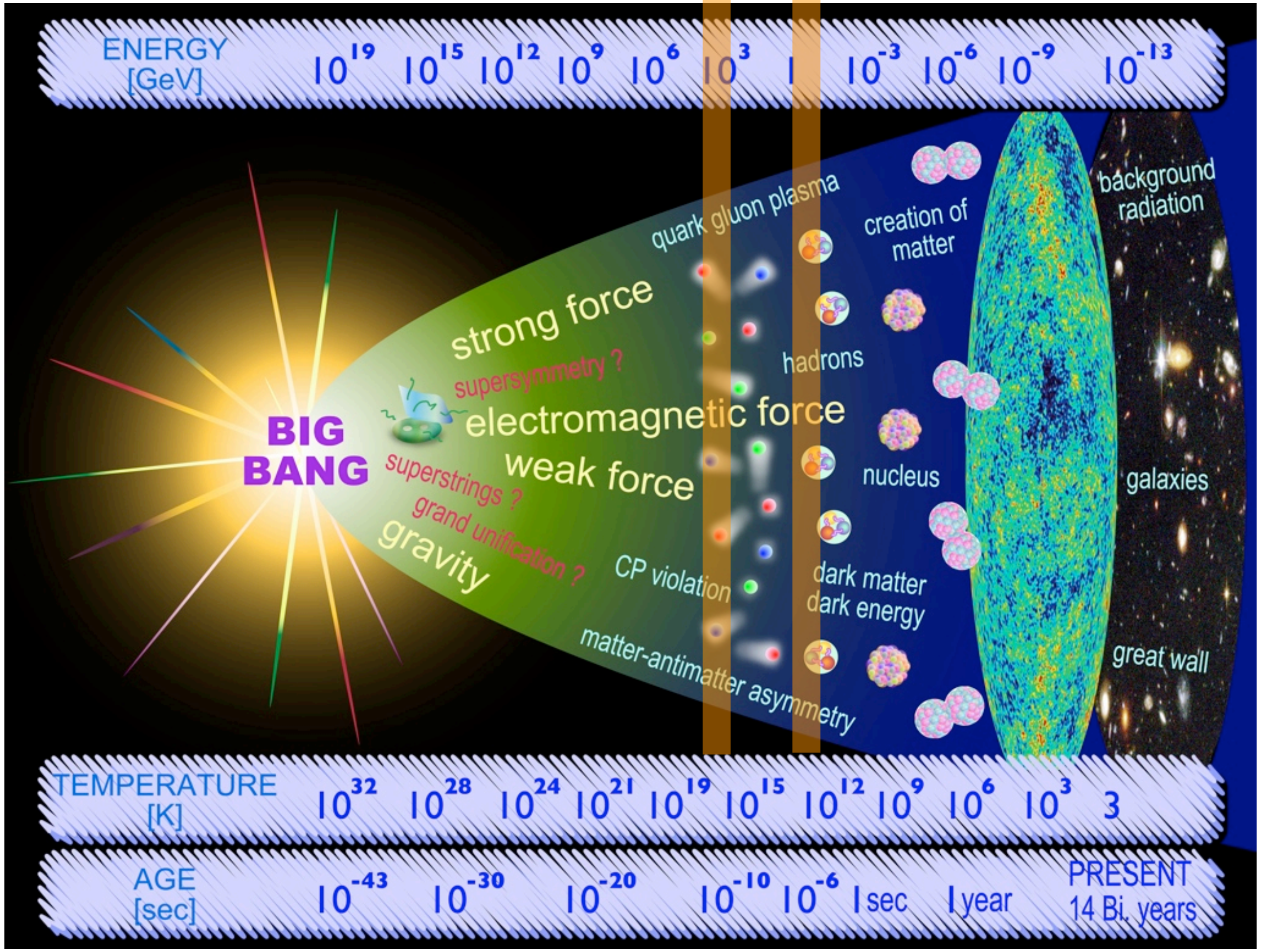
→ There will be phase transitions
with drastic phenomenological consequences!

Where are they important?

- early Universe

electroweak transition

QCD transition



<= The last particle-level phase transition

V

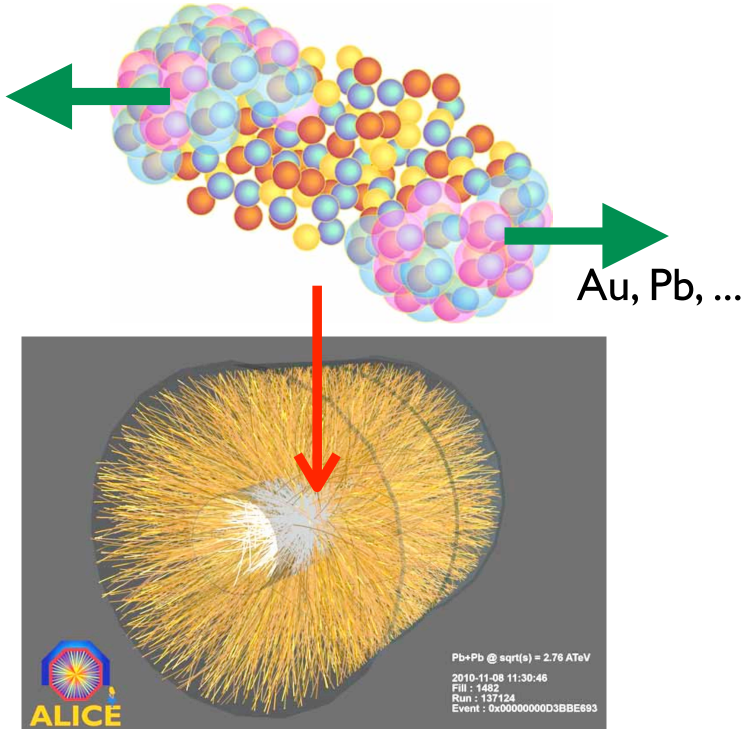
V

large implications to

- evolution of universe
- genesis of mass
- cosmic scales (galaxies, great walls, ...)
-

Where are they important?

- high-energy heavy-ion collision experiments @ RHIC, LHC, ...



* many expected/unexpected results being produced
* more than thousands of particles involved
* theoretical inputs indispensable

Deconfinement transition of QCD: essentially non-perturbative.

Even in the high T region, conventional pert. theory breaks down at high orders due to IR div's.

=> **Lattice**

1. formulation

finite temperature QCD

Partition function

$$Z = \mathrm{Tr} e^{-H/T} = \sum_n \langle n | e^{-H/T} | n \rangle, \quad k_B = 1$$

The calculation of r.h.s. is equivalent to the conventional calculation of the transition amplitude

$$\langle n | e^{-iHt} | m \rangle, \quad \hbar = 1$$

in quantum mechanics with $n = m$ and $t = 1/(iT)$.

Euclidian field theory

Wick rotation:

$$x_0 = t \longrightarrow -ix_4 \qquad \gamma_0 \longrightarrow \gamma_4^E \qquad \{\gamma_\mu^E, \gamma_\nu^E\} = 2\delta_{\mu\nu}$$

$$d^4x \longrightarrow -id^4x_E \qquad \gamma_i \longrightarrow -i\gamma_i^E$$

$$A_\mu A^\mu \longrightarrow -A_\mu A_\mu \qquad i\gamma_\mu(\partial^\mu - igA^\mu) \longrightarrow -\gamma_\mu^E(\partial_\mu - igA_\mu)$$

Path-integral repr. of the partition function

Field theory in an Euclidian space-time with a finite extent in the "temporal" direction.

$$Z = \mathrm{Tr} e^{-H/T} = \int_{b.c.} \mathcal{D}\phi \, e^{-S} \qquad S = \int_0^{1/T} d^4x_E \, \mathcal{L}_E(\phi(x_E))$$

Matsubara formalism

$$Z = \mathrm{Tr} e^{-H/T} = \int_{b.c.} \mathcal{D}\phi \, e^{-S}$$

Tr calculation realized by appropriate **boundary conditions** in the Euclidian time direction.

$$A_\mu(x_4 = 0, \vec{x}) \;=\; A_\mu(x_4 = 1/T, \vec{x})$$

For quarks, due to a property of Grassmann integration: $\mathrm{Tr} M = \int \langle \xi | M | -\xi \rangle \, d\xi$

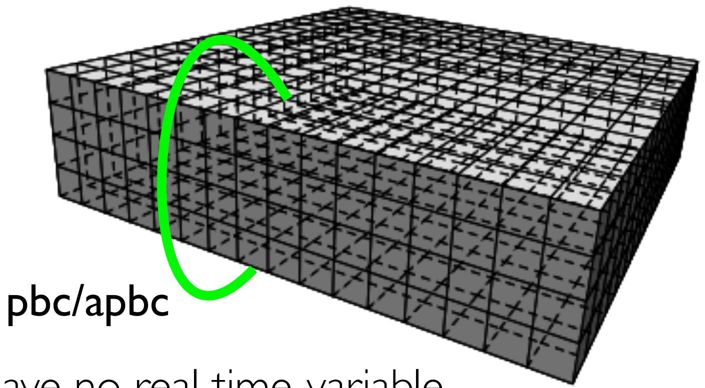
anti-periodic for quarks.

$$q(x_4 = 0, \vec{x}) \;=\; -q(x_4 = 1/T, \vec{x})$$

<= perturbation theory, approximate treatments such as MF, high T effective theories, etc.

To study non-perturbative properties,
the most powerful approach is

Lattice discretization + simulation

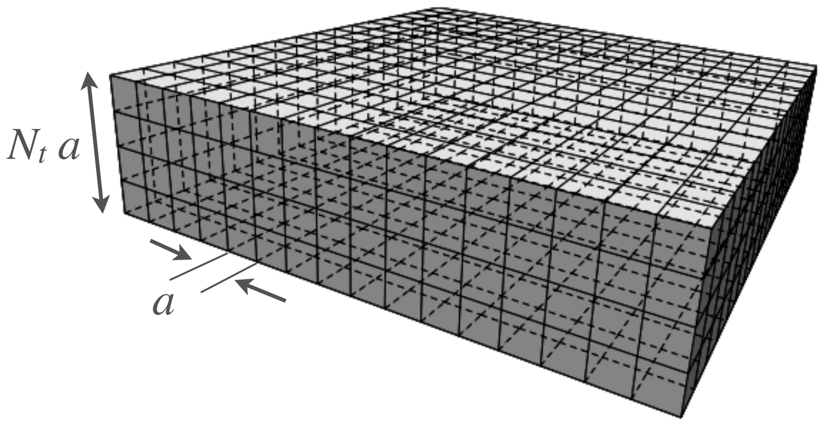


Notes

- Systems in thermal equilibrium are considered. We thus have no real time-variable. The Euclidian time is mere mathematical.
- The spatial volume must be sufficiently large to mimic the thermodynamic limit.

T>0 QCD on the lattice

$$Z = \int [dU][dq][d\bar{q}]\, e^{-S}$$



$$T = \frac{1}{N_t a}$$

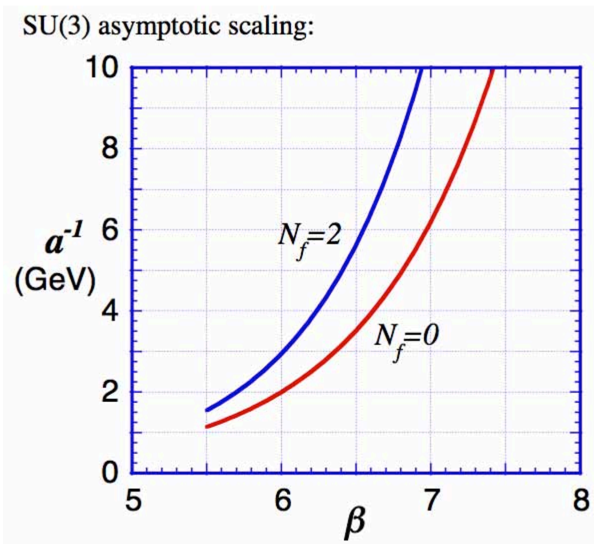
N_t : lattice size in the euclidian time direction
a : lattice spacing

For a given T,
continuum limit: $N_t \longrightarrow \infty$

To vary T,

conventionally, a is varied at a fixed N_t where a is controlled by $\beta = 1/g^2$ ("fixed Nt approach").

small β	large β
small a^{-1}	large a^{-1}
low T	high T



T>0 QCD on the lattice

$$T = \frac{1}{N_t a}$$

Alternatively,

we may vary N_t, with a fixed ("**fixed scale approach**").

large N_t → small N_t

low T → high T

Both methods have pros and cons, and are complementary with each other.

roughly speaking:

	fixed Nt	fixed scale
low T	☹	☺
high T	☺	☹

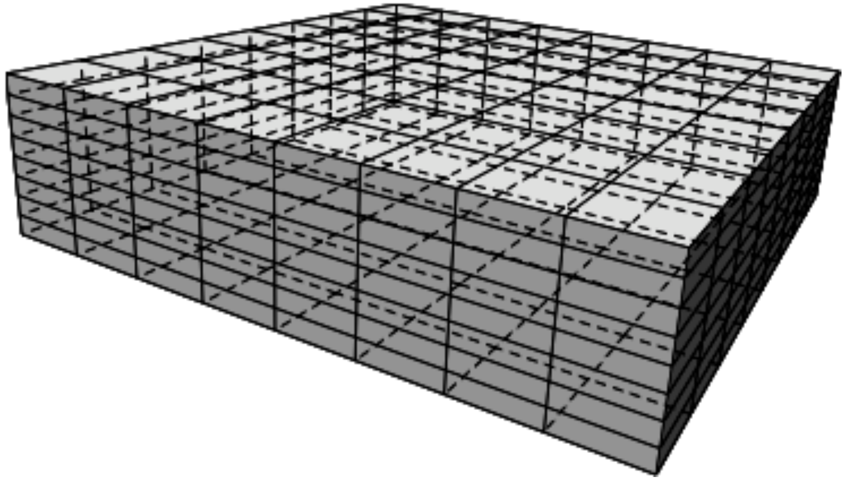
We may choose/combine them according to the objective of the project (see discussions later).

T>0 QCD on the lattice

Anisotropic lattices

$$a_s \neq a_t$$

$$1/T = N_t a_t, \quad V = (N_s a_s)^3$$



Different motivations

- More points in the euclidian-time direction

  => maximum-entropy method, variational method, thermal masses, ...
- Reduce lattice artifacts due to quite small *Nt* => continuum extrapolation
- More tuning parameters for improvement => heavy quarks on the lattice
- Finer *T*-resolution in the fixed-scale approach
- etc.