Phenomenology of exotic hadrons: Positive parity open-charm mesons

Feng-Kun Guo

Institute of Theoretical Physics, Chinese Academy of Sciences

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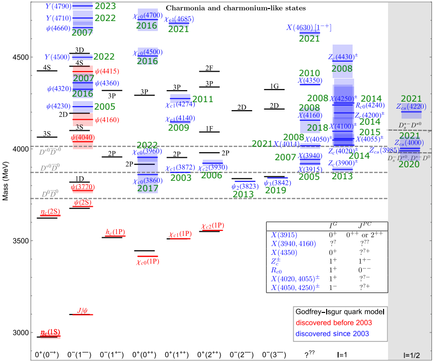
Exotic hadrons: beyond the quark model

- Conventional quark model (1964):
  mesons $\simeq q\bar{q}$, baryons $\simeq qqq$
- QCD does not forbid more:
  - ☞ multiquark: tetraquark $q\bar{q}q\bar{q}$, pentaquark $qqqq\bar{q}$, ...
  - ☞ hadronic molecule: hadrons bound by residual strong forces
  - ☞ hybrid $q\bar{q}g$, glueball gg
- Since 2003, dozens of XYZ, P_c, T_{cc} candidates, mostly near hadron thresholds

B factories, BESIII, LHCb, ...

- Their nature (compact vs. molecular) is the central question

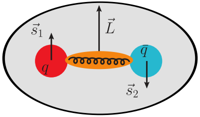


Charmonium(-like) spectrum: conventional & exotic candidates

- J^{PC} of regular $q\bar{q}$ mesons

  L: orbital angular momentum; $S = (0, 1)$: total $q\bar{q}$ spin

  $P = (-1)^{L+1}$, $C = (-1)^{L+S}$ (flavor-neutral mesons)



  ☞ $S = 0$: $J = L$, so $P = (-1)^{J+1}$, $C = (-1)^J \Rightarrow J^{PC} =$ even^{-+}, odd^{+-}

  ☞ $S = 1$: $P = C = (-1)^{L+1} \Rightarrow J^{PC} = 1^{--}, \{0, 1, 2\}^{++}, \{1, 2, 3\}^{--}, \ldots$

- Exotic J^{PC}, impossible for $q\bar{q}$:

$$J^{PC} = 0^{--},\ \text{even}^{+-},\ \text{odd}^{-+}$$

- Established 1^{-+} examples: $\pi_1(1600)$, $\eta_1(1855) \to \eta\eta'$ — hybrid? molecule?

  BESIII, PRL129(2022)192002

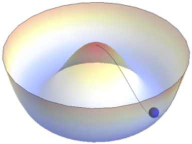
QCD symmetries

$$\mathcal{L}_{\rm QCD} = \sum_{f=u,d,s,c,b,t} \bar{q}_f \left(i \not{D} - m_f\right) q_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,\,a}$$

- Exact: Lorentz invariance, SU$(3)_c$ gauge, C, P, T
- Approximate, from the quark-mass hierarchy $m_{u,d,s} \ll \Lambda_{\rm QCD} \ll m_{c,b,t}$:

☞ light u, d, s: spontaneously broken chiral symmetry

SU$(N_f)_L \times$ SU$(N_f)_R \xrightarrow{\text{SSB}}$ SU$(N_f)_V$



$\Rightarrow \pi, K, \eta$ as (pseudo-)Goldstone bosons

☞ heavy c, b act as static color sources:

heavy quark spin symmetry (HQSS)

heavy quark flavor symmetry (HQFS)

heavy antiquark–diquark symmetry (HADS)

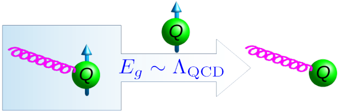
$\Rightarrow$ approximate spin & flavor multiplets, charm $\leftrightarrow$ bottom relations

Heavy quark spin symmetry (HQSS)

- Heavy quark (c, b) in a hadron: typical momentum transfer $\Lambda_{\text{QCD}} \ll m_Q$

  chromomagnetic interaction $\propto \dfrac{\boldsymbol{\sigma} \cdot \boldsymbol{B}}{m_Q} \to 0$

  $\Rightarrow$ the heavy-quark spin decouples



- Total angular momentum $\boldsymbol{J} = \boldsymbol{s}_Q + \boldsymbol{s}_\ell$

  $\boldsymbol{s}_Q$: heavy-quark spin; $\boldsymbol{s}_\ell$: spin of the light degrees of freedom

s_ℓ and s_Q are separately conserved in the heavy-quark limit

- Spin multiplets (degenerate up to $1/m_Q$ corrections):

  $\{D, D^*\}$, $\{B, B^*\}$ with $s_\ell^P = \frac{1}{2}^-$; quarkonia $\{\eta_c, J/\psi\}$, $\{\eta_b, \Upsilon\}$, ...

Heavy quark flavor symmetry (HQFS) & HADS

- Heavy quark flavor symmetry (HQFS) — for one heavy quark:

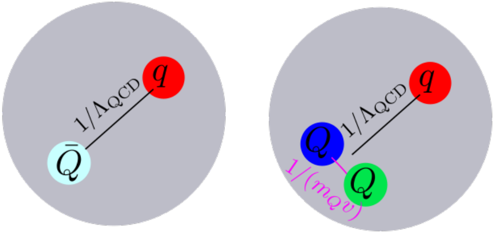
  velocity unchanged as $m_Q \to \infty$: $\Delta v = \dfrac{\Delta p}{m_Q} = \dfrac{\Lambda_{\text{QCD}}}{m_Q} \to 0$

  $\Rightarrow$ heavy quark $=$ static color-triplet source, m_Q irrelevant

  relates the charm and bottom sectors (used below to predict B^*_{s0}, B_{s1})

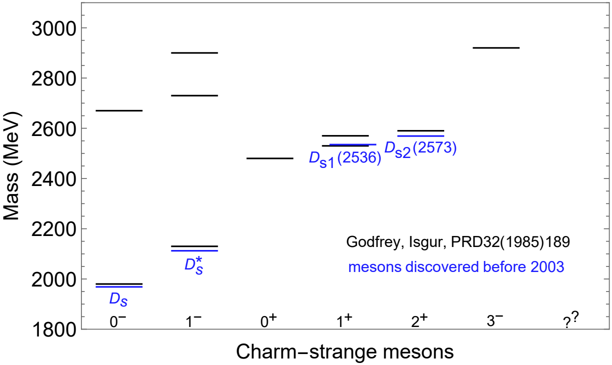
- Heavy antiquark–diquark symmetry (HADS) — Savage, Wise (1990)

  for $m_Q v \gg \Lambda_{\text{QCD}}$, a QQ diquark acts as a point-like color-$\bar{3}$ source, like a heavy antiquark $\Rightarrow$ relates doubly-heavy baryons to (anti-)heavy mesons



$Q\bar{q}$ heavy meson $\leftrightarrow$ QQq doubly-heavy baryon

Puzzles of charm mesons: Charm-strange



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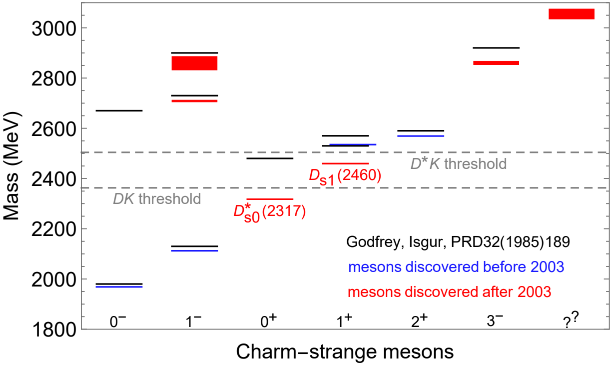


- $D^*_{s0}(2317)$: BaBar (2003)
  $J^P = 0^+$, $\Gamma < 3.8$ MeV
- $D_{s1}(2460)$: CLEO (2003)
  $J^P = 1^+$, $\Gamma < 3.5$ MeV
- no isospin partner observed, tiny widths
  $\Rightarrow I = 0$

- Puzzle 1: Mass problem: Why are $D^*_{s0}(2317)$ and $D_{s1}(2460)$ so light?
- Puzzle 2: Naturalness problem:

$$\text{Why } \underbrace{M_{D_{s1}(2460)} - M_{D^*_{s0}(2317)}}_{(141.8\pm0.8)\text{ MeV}} \simeq \underbrace{M_{D^{*\pm}} - M_{D^\pm}}_{(140.67\pm0.08)\text{ MeV}} ?$$

Puzzles of charm mesons: Charm-nonstrange (1)

Obervations of charm-nonstrange excited mesons in 2003

$B^- \to D^{(*)+}\pi^-\pi^-$ Belle, PRD69(2004)112002 [hep-ex/0307021]

- $D_0^*(2300)$: $J^P = 0^+$

  $\Gamma = 229 \pm 16$ MeV, M (MeV):

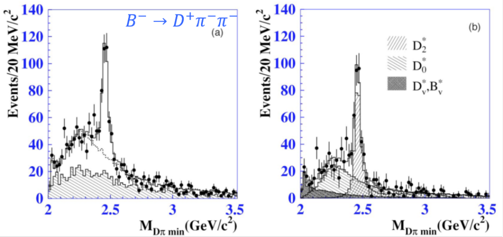
2343 ± 10	PDG2026	
2297 ± 22	BaBar	B decays
2308 ± 36	Belle	B decays
2401 ± 41	FOCUS	γA
2360 ± 34	LHCb	B, charged

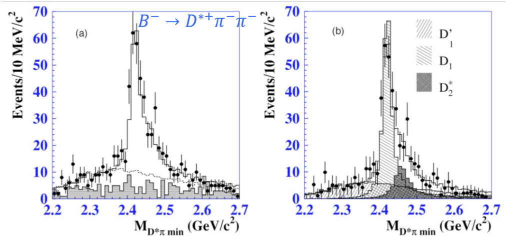
The $D_0^*(2300)$ was known as $D_0^*(2400)$ up to PDG2018.

- $D_1(2430)$: $J^P = 1^+$

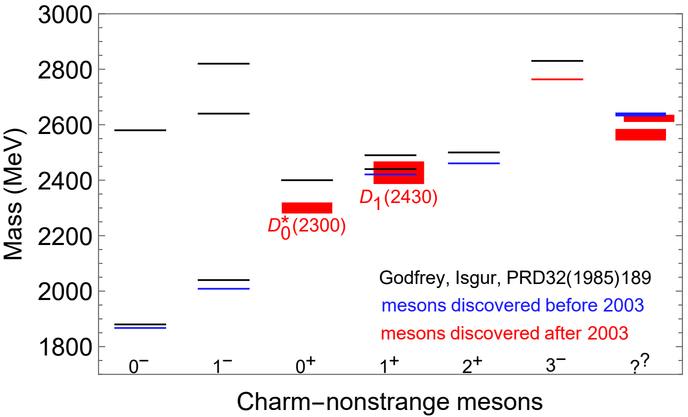
  $\Gamma = 314 \pm 29$ MeV PDG2026

  $M = (2412 \pm 9)$ MeV





Puzzles of charm mesons: Charm-nonstrange (2)



- Puzzle 3: Hierarchy problem:

  Why $M_{D_0^*(2300)} \gtrsim M_{D_{s0}^*(2317)}$ and $M_{D_1(2430)} \sim M_{D_{s1}(2460)}$?

Remarks on $D_0^*(2300)$ and $D_1(2430)$ resonance parameters (1)

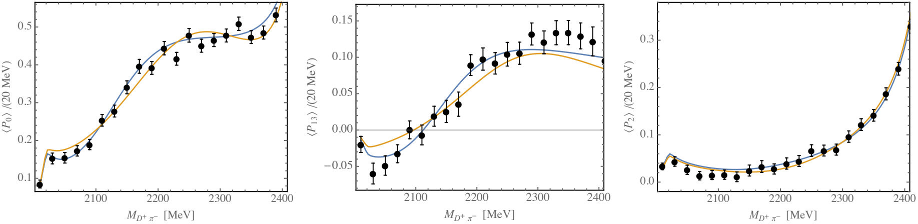
$D_{s0}^*(2300)$: $M = 2343 \pm 10$ MeV, $\Gamma = 229 \pm 16$ MeV PDG2026

- Extracted from the $D^{(*)}\pi$ invariant mass distribution using a single Breit-Wigner
- Issues:
  coupled channels ($D\eta$: 2.41 GeV; $D_s\bar{K}$: 2.46 GeV);
  chiral symmetry (derivative coupling $\propto E_\pi$ for S-wave)
- A simple BW tends to overshoot the mass when the chiral constraint is neglected:

$$\left|\frac{1}{s - m_0^2 + i m_0 \Gamma}\right| \text{ peaks at } \sqrt{s} = m_0;$$

$$\left|\frac{E_\pi}{s - m_0^2 + i m_0 \Gamma}\right| \text{ peaks at } \sqrt{s} = m_0 + \frac{E_{D,0}\Gamma^2}{4 m_0 E_{\pi,0} - \Gamma^2}$$

Fit with the usual BW and with the BW multiplied with an E_π factor:



Resonance parameters from the fits:

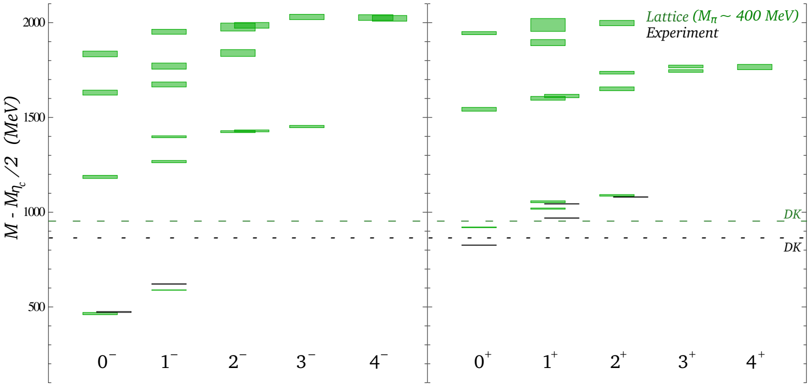
usual BW (χ^2/dof $= 1.8$): $M = 2319 \pm 9$ MeV, $\Gamma = 420 \pm 25$ MeV

BW multiplied with E_π factor (χ^2/dof $= 1.1$): $M = 2206 \pm 4$ MeV, $\Gamma = 341 \pm 17$ MeV

Coupled-channel effects still need to be incorporated

Lattice studies of the charmed scalar mesons: strange (1)

- Early studies using only $c\bar{s}$-type interpolators typically give mass larger than that for $D_{s0}^*(2317)$ G. Bali (2003); UKQCD (2003); ...



Lowest 0^+ below lattice DK threshold G. Moir et al. [HadSpec], JHEP05(2013)021

- $c\bar{s} + DK$ interpolators: ∼right mass D. Mohler et al., PRL111(2013)222001

$M_{D_{s0}^*} - \frac{1}{4}\left(M_{D_s} + 3M_{D_s^*}\right)$:

Mohler et al.	Expt.
(266 ± 16) MeV	(241.5 ± 0.8) MeV

- Calculation with $M_\pi = 150$ MeV G. Bali et al. [RQCD Col.], PRD96(2017)074501

	Energy [MeV]	Expt [MeV]
m_{0-}	1976.9(2)	1966.0(4)
m_{1-}	2094.9(7)	2111.3(6)
m_{0+}	2348(4)(+6)	2317.7(0.6)(2.0)
m_{1+}	2451(4)(+1)	2459.5(0.6)(2.0)

- Calculation at $M_\pi = 239, 391$ MeV G.K.C. Cheung et al. [HadSpec], JHEP02(2021)100
  - ☞ $I = 0\ DK$ bound state below threshold by 25(3) / 57(3) MeV ($M_\pi = 239/391$ MeV)
  - ☞ $I = 0\ D\bar{K}$: virtual state; $I = 1\ DK$: repulsive

Lattice studies of the charmed scalar mesons: nonstrange (1)

- $(S, I) = (0, \frac{1}{2})$: $c\bar{q} + D\pi$ interpolators:

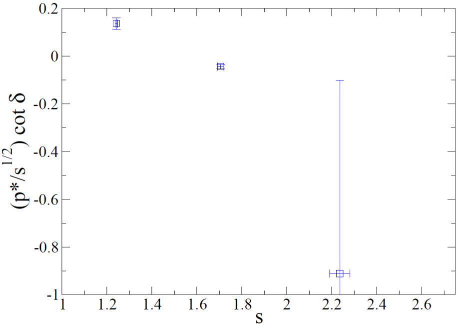
  D. Mohler et al., PRD87(2013)034501

  $M_\pi \approx 266$ MeV,

  $M_D \approx 1558$ MeV,

  $M_{D^*} \approx 1690$ MeV

  Lüscher's formula $\Rightarrow D\pi$ phase shifts



$\Rightarrow$ BW parameters of $D_0^*(2400)$ [the name of $D_0^*(2300)$ up to PDG18] consistent with PDG values

	D. Mohler et al.	PDG2024
$M_{D_0^*} - \frac{1}{4}(M_D + 3M_{D^*})$	(351 ± 21) MeV	(347 ± 29) MeV
$M_{D_1} - \frac{1}{4}(M_D + 3M_{D^*})$	(372 ± 10) MeV	(441 ± 9) MeV

Pole reported later in the Snowmass white paper: $m_{D_0^*} \simeq 2.12(3)$ GeV

J. Bulava et al., arXiv:2203.03230

Lattice studies of the charmed scalar mesons: nonstrange (2)

- $(S, I) = (0, \frac{1}{2})$: first coupled-channel lattice calculation including interpolating fields for $c\bar{q} + D\pi + D\eta + D_s\bar{K}$: G. Moir et al. [HadSpec], JHEP1610(2016)011
- $M_\pi = 391$ MeV, $M_D = 1885$ MeV: $D\pi$ threshold (2276.4 ± 0.9) MeV
- for coupled channels:

  parameterizing the T-matrix with the K-matrix formalism

$$T_{ij}^{-1}(s) = K_{ij}^{-1}(s) + I_{ij}(s)$$

$I_{ij}(s)$: 2-point loop function evaluated with a subtracted dispersion integral

$K_{ij}(s)$: different forms of the K-matrix were used, summarized as

$$K_{ij}(s) = \left(g_i^{(0)} + g_i^{(1)} s\right) \left(g_j^{(0)} + g_j^{(1)} s\right) \frac{1}{m^2 - s} + \gamma_{ij}^{(0)} + \gamma_{ij}^{(1)} s$$

- $\Rightarrow$ a pole below threshold (2275.9 ± 0.9) MeV. relation to $D_0^*(2300)$?
- Recent calculation by CLQCD at 4 pion masses: H. Yan et al., PRD111(2025)014503

  resonance for $M_\pi \simeq 133$ and 208 MeV; virtual state for $M_\pi \simeq 305$ and 317 MeV

Solutions to the first two puzzles in hadronic molecular model

- Not quark model $c\bar{s}$ mesons:
  $D_{s0}^*(2317)[\simeq DK(I=0)]$, $D_{s1}(2460)[\simeq D^*K(I=0)]$
  Barnes, Close, Lipkin (2003); van Beveren, Rupp (2003); Kolomeitsev, Lutz (2004); FKG et al. (2006); FKG, Hanhart, Meißner (2009); ...
- HQSS $\Rightarrow$ similar binding energies $M_D + M_K - M_{D_{s0}^*} \simeq 45$ MeV
  $$M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \simeq M_{D^*} - M_D \text{ is natural}$$
- HQFS $\Rightarrow$ predicting the 0^+ and 1^+ bottom-partner masses

$$M_{B_{s0}^*} \simeq M_B + M_K - 45\,\text{MeV} \simeq 5.730\text{ GeV}$$
$$M_{B_{s1}} \simeq M_{B^*} + M_K - 45\,\text{MeV} \simeq 5.776\text{ GeV}$$

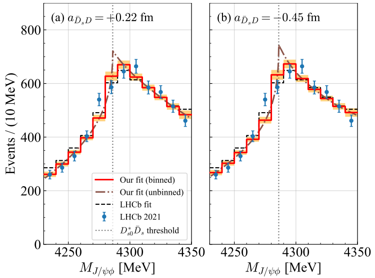
Lattice QCD results: C. Lang et al., PLB750(2015)17

$$M_{B_{s0}^*}^{\text{lat.}} = (5.711 \pm 0.013 \pm 0.019)\text{ GeV}$$
$$M_{B_{s1}}^{\text{lat.}} = (5.750 \pm 0.017 \pm 0.019)\text{ GeV}$$

Measuring the B_{s0}^* mass via a threshold cusp

H.-L. Fu, X. Zhang, FKG, C. Hanhart, U.-G. Meißner, M.-J. Yan, arXiv:2606.16976

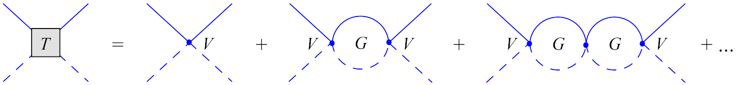
- Problem: B_{s0}^*, B_{s1} decay with photon(s) $(B_s^{(*)}\gamma, B_s\pi^0) \Rightarrow$ hard to detect at the LHC
- Idea: attractive S-wave coupling to a companion of precisely known mass $\Rightarrow$ a cusp at the pair threshold (or a peak just below if it binds) $\Rightarrow$ subtract the companion mass $\Rightarrow$ target mass, model-independently
- Proof of concept: LHCb $J/\psi\phi$: $X(4274) \simeq D_{s0}^*\bar{D}_s$ cusp $\Rightarrow m_{D_{s0}^*} = (2322 \pm 6)$ MeV [PDG $2317.8(5)$], favoring $J^{PC} = 0^{-+}$
- Prediction: a $\Upsilon\phi$ cusp at the $B_{s0}^*\bar{B}_s$ threshold $\simeq 11.09$ GeV



LHCb $J/\psi\phi$ data & our fit

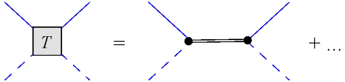
Interactions between charm and light mesons

- S-wave interactions between charm mesons (D, D_s) and light pseudoscalar mesons (π, K, η)



- not far from the thresholds $\Rightarrow$ chiral EFT for matter field
- D^*_{s0}/D^*_0 should appear as poles in scattering amplitudes:

$$D^*_{s0} : DK, D_s\eta; \qquad D^*_0 : D\pi, D\eta, D_s\bar{K}$$



$\Rightarrow$ needs a nonperturbative treatment: ChPT + unitarization

Truong (1988); Oller, Oset (1997); Oller, Oset, Peláez (1998); Nieves, Ruiz Arriola (1999); Oller, Meißner (2001); . . .

$$T^{-1}(s) = V^{-1}(s) - G(s)$$

$V(s)$: from SU(3) chiral Lagrangian, 6 LECs up to NLO

$G(s)$: 2-point scalar loop functions, regularized with a subtraction constant $a(\mu)$

Chiral Lagrangian (1)

- The leading order Lagrangian:

$$\mathcal{L}_{\phi P}^{(1)} = D_\mu P D^\mu P^\dagger - m^2 P P^\dagger$$

with $P = (D^0, D^+, D_s^+)$ denoting the D mesons, and the covariant derivative being

$$\begin{aligned} D_\mu P &= \partial_\mu P + P\Gamma_\mu^\dagger, \quad D_\mu P^\dagger = (\partial_\mu + \Gamma_\mu)P^\dagger, \\ \Gamma_\mu &= \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right), \end{aligned}$$

where $u_\mu = i\left[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger\right], \quad u = e^{i\lambda_a\phi_a/(2F_0)}$

Burdman, Donoghue (1992); Wise (1992); Yan et al. (1992)

- this gives the Weinberg–Tomozawa term for $P\phi$ scattering: $\propto E_\phi + \mathcal{O}(1/M_D)$ (S-wave)

Chiral Lagrangian (2)

- At the next-to-leading order $\mathcal{O}\left(p^2\right)$: FKG, Hanhart, Krewald, Meißner, PLB**666**(2008)251

$$\mathcal{L}_{\phi P}^{(2)} = P\left[-h_0\langle\chi_+\rangle - h_1\chi_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu\right]P^\dagger$$
$$+D_\mu P\left[h_4\langle u_\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\}\right]D_\nu P^\dagger \; ,$$

$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u, \quad \chi = 2B_0\,\mathrm{diag}(m_u, m_d, m_s)$

- Low-energy constants:

$M_{D_s} - M_D \Rightarrow h_1 = 0.42$

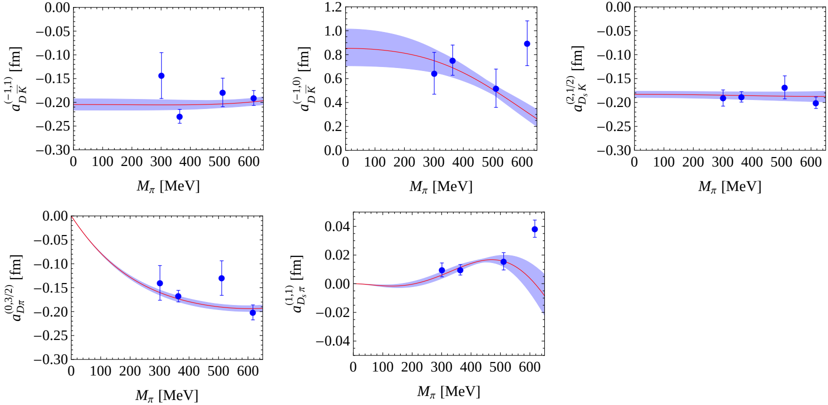
h_0: can be fixed from lattice results of quark mass dependence of charmed meson masses

$h_{2,3,4,5}$: to be fixed from lattice results on scattering lengths

Hierarchy from large N_c counting: $h_{1,3,5} = \mathcal{O}\left(N_c^0\right)$, $h_{0,2,4} = \mathcal{O}\left(N_c^{-1}\right)$

L. Liu, Orginos, FKG, Hanhart, Meißner, PRD86(2013)014508

- Fit to lattice data on scattering lengths in 5 simpler channels:
  $D\bar{K}(I=1, I=0)$, D_sK, $D\pi(I=3/2)$, $D_s\pi$: no disconnected contribution
  5 parameters: h_2, h_3, h_4, h_5 and $a(\mu)$



- N_c counting fulfilled: $\underbrace{h_2 \simeq 0.2,\ h_4\, M_D^2 \simeq -0.3,}_{\mathcal{O}(N_c^{-1})}$ $\underbrace{h_3 \simeq 2.1,\ h_5\, M_D^2 \simeq -1.9}_{\mathcal{O}(N_c^{0})}$

Predictions for 0^+ & 1^+ heavy mesons   M.-L. Du et al., PRD98(2018)094018

- Heavy-strange

meson	J^P	prediction (MeV)	PDG2024 (MeV)	lattice (MeV)
D_{s0}^*	0^+	2315^{+18}_{-28}	2317.8 ± 0.5	2348^{+7}_{-4}[1]

- Heavy-nonstrange, two $I = 1/2$ states $(M, \Gamma/2)$:

	Lower (MeV)	Higher (MeV)	PDG2024 (MeV)
D_0^*	$(2105^{+6}_{-8}, 102^{+10}_{-11})$	$(2451^{+36}_{-26}, 134^{+7}_{-8})$	$(2343 \pm 10, 115 \pm 8)$
D_1	$(2247^{+5}_{-6}, 107^{+11}_{-10})$	$(2555^{+47}_{-30}, 203^{+8}_{-9})$	$(2412 \pm 9, 157 \pm 15)$
B_0^*	$(5535^{+9}_{-11}, 113^{+15}_{-17})$	$(5852^{+16}_{-19}, 36 \pm 5)$	—
B_1	$(5584^{+9}_{-11}, 119^{+14}_{-17})$	$(5912^{+15}_{-18}, 42^{+5}_{-4})$	—

[1] Bali, Collins, Cox, Schäfer, PRD96(2017)074501

[2] Lang, Mohler, Prelovsek, Woloshyn, PLB750(2015)17

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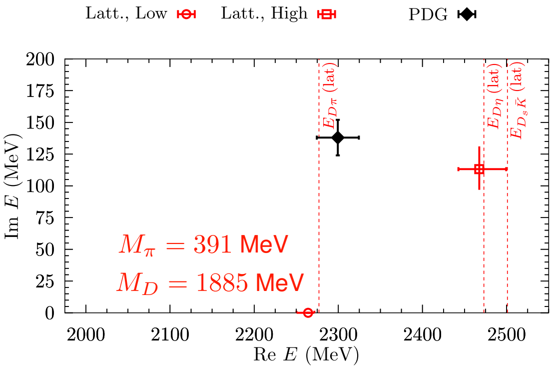
D_1	$(2247^{+5}_{-6}, 107^{+11}_{-10})$	$(2555^{+47}_{-30}, 203^{+8}_{-9})$	$(2412 \pm 9, 157 \pm 15)$
B_0^*	$(5535^{+9}_{-11}, 113^{+15}_{-17})$	$(5852^{+16}_{-19}, 36 \pm 5)$	–
B_1	$(5584^{+9}_{-11}, 119^{+14}_{-17})$	$(5912^{+15}_{-18}, 42^{+5}_{-4})$	–

[1] Bali, Collins, Cox, Schäfer, PRD96(2017)074501

[2] Lang, Mohler, Prelovsek, Woloshyn, PLB750(2015)17

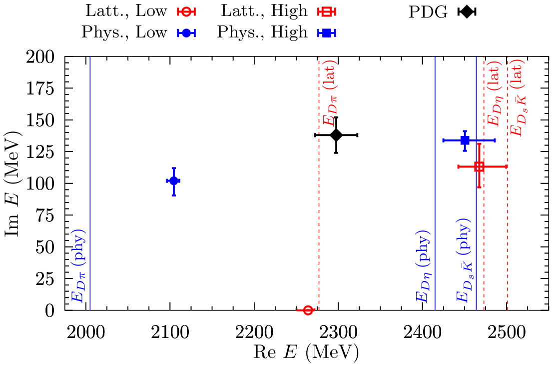
Pion mass dependence

Masses	M (MeV)	$\Gamma/2$ (MeV)	RS	$\lvert g_{D\pi}\rvert$	$\lvert g_{D\eta}\rvert$	$\lvert g_{D_s\bar{K}}\rvert$
lattice	$2264^{+\ 8}_{-14}$	0	$(+++)$	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	2468^{+32}_{-25}	113^{+18}_{-16}	$(--+)$	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$



Pion mass dependence

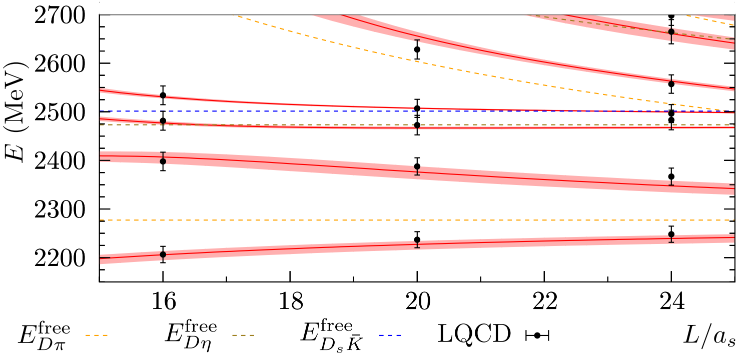
Masses	M (MeV)	$\Gamma/2$ (MeV)	RS	$\lvert g_{D\pi}\rvert$	$\lvert g_{D\eta}\rvert$	$\lvert g_{D_s\bar{K}}\rvert$
lattice	2264^{+8}_{-14}	0	$(+\,+\,+)$	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	2468^{+32}_{-25}	113^{+18}_{-16}	$(-\,-\,+)$	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$
physical	2105^{+6}_{-8}	102^{+10}_{-11}	$(-\,+\,+)$	$9.4^{+0.2}_{-0.2}$	$1.8^{+0.7}_{-0.7}$	$4.4^{+0.5}_{-0.5}$
	2451^{+36}_{-26}	134^{+7}_{-8}	$(-\,-\,+)$	$5.0^{+0.7}_{-0.4}$	$6.3^{+0.8}_{-0.5}$	$12.8^{+0.8}_{-0.6}$



Postdictions versus lattice results: charm-nonstrange

- Postdicted $I = 1/2\ D\pi, D\eta, D_s\bar{K}$ finite volume energy levels in the c.m. frame versus lattice QCD results by [G. Moir *et al.* [HadSpec], JHEP10(2016)011]

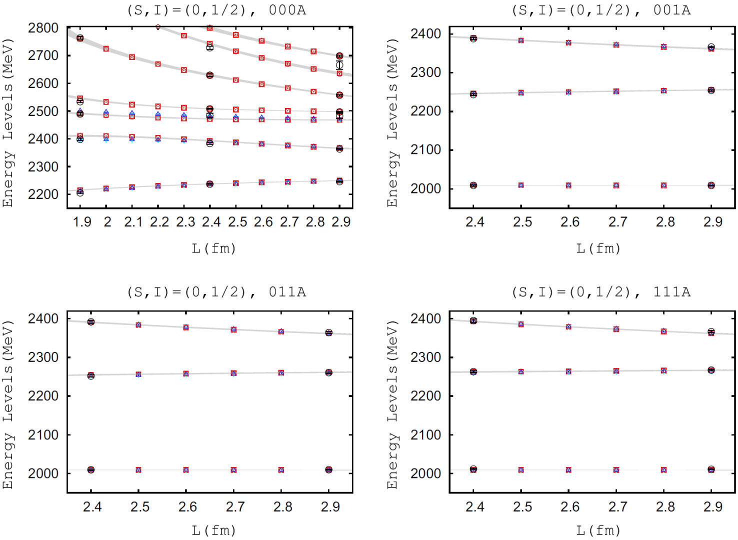
NOT a fit ! M. Albaladejo, P. Fernandez-Soler, FKG, J. Nieves, PLB767(2017)465



consequence of SU(3) + chiral

A more recent fit to lattice data including moving frame ones

Z.-H. Guo et al., EPJC79(2019)13



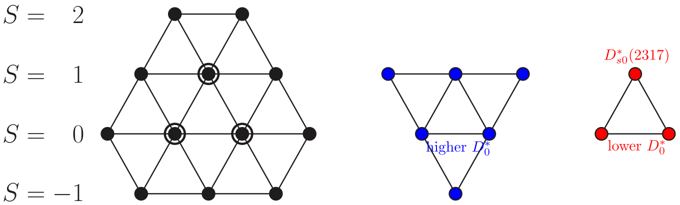
Determined parameters (from Fit IIB) are similar

Lattice data: G. Moir et al., JHEP10(2016)011

SU(3) analysis (1)

- SU(3) irreps: $\overline{\mathbf{3}} \otimes \mathbf{8} = \overline{\mathbf{15}} \oplus \mathbf{6} \oplus \overline{\mathbf{3}}$

M. Albaladejo, P. Fernandez-Soler, FKG, J. Nieves, PLB767(2017)465



- WT term: $\overline{\mathbf{15}}$: repulsive; $\mathbf{6}$: attractive; $\overline{3}$: most attractive

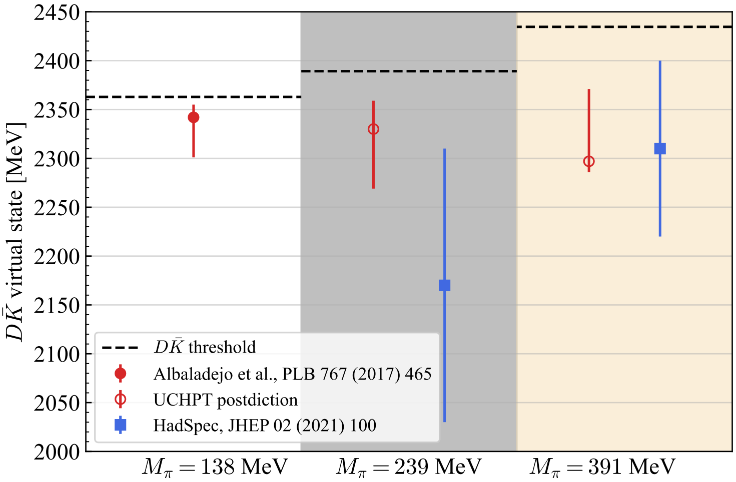
  $(S, I) = (1, 1)$: deep in the complex plane on wrong Riemann sheets

  $(S, I) = (-1, 0)$: virtual state at 2342^{+13}_{-41} MeV at the physical mass

SU(3) analysis (2)

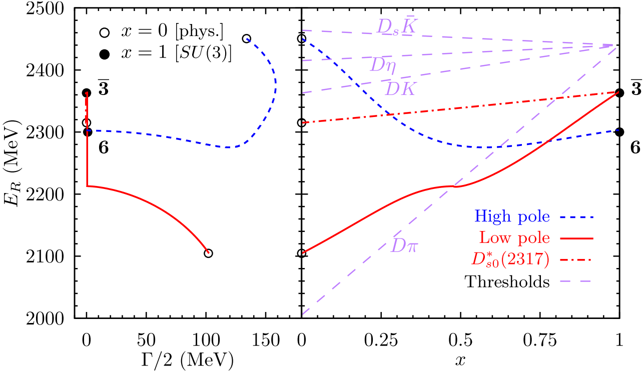
- Lattice QCD also found a virtual state for the sextet state $(S, I) = (-1, 0)\ D\bar{K}$

  G.K.C. Cheung et al. [HadSpec], JHEP 02 (2021) 100



FKG, PoS LATTICE2022, 232

- Evolution of the two poles from the physical to the SU(3) symmetric case



Here, $m_i = m^{\text{phy.}} + x(m_i - m_i^{\text{phy.}})$, taking $m_\phi = 490$ MeV, $M_H = 1.95$ GeV

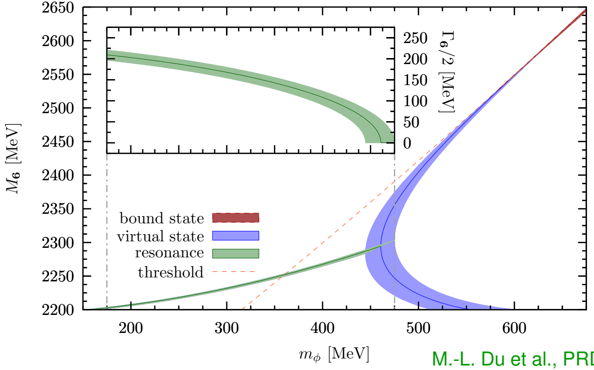
The sextet is clearly exotic beyond the $c\bar{q}$ configuration.

- Solution to puzzle 3: the SU(3) nonstrange partner of $D^*_{s0}(2317)$ is the lower D^*_0 state with a mass of about 2.1 GeV

Searching for the higher nonstrange state: lattice

- Tuning interaction strength by varying quark masses:

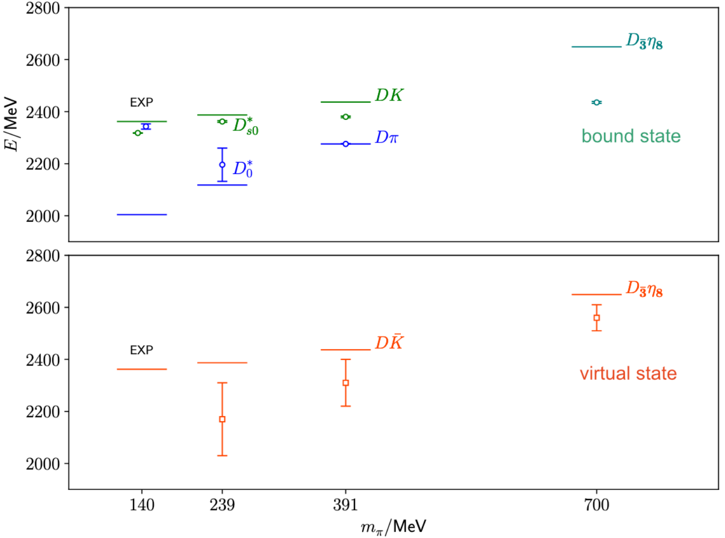
  Expectation: WT term $\propto E_\pi$, increasing M_π leads to stronger interaction increasing S-wave interaction strength $\Rightarrow$ resonance $\rightarrow$ below-th. resonance $\rightarrow$ virtual state $\rightarrow$ bound state, then easier for lattice to get a signal
- SU(3) symmetric, then the sextet decouples from the triplet; prediction (qualitative for large m_q), to check with large m_q on lattice:



M.-L. Du et al., PRD98(2018)094018

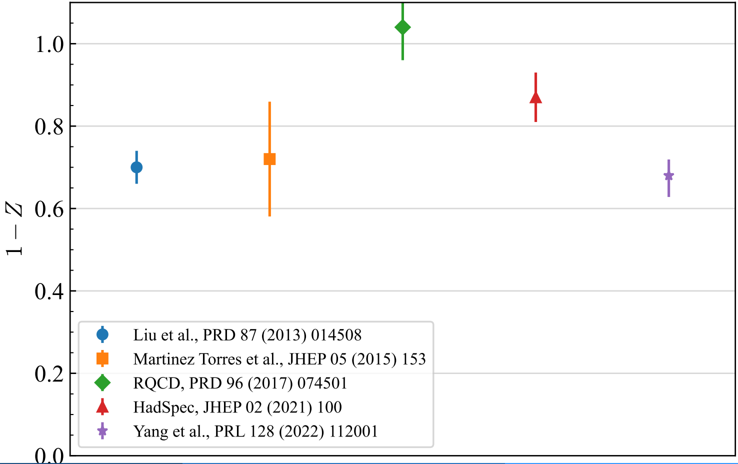
Last column: SU(3) sym. point with $M_\pi \simeq 700$ MeV Yeo et al. [HadSpec], JHEP07(2024)012

$\bar{\mathbf{3}}$: bound; $\mathbf{6}$: virtual



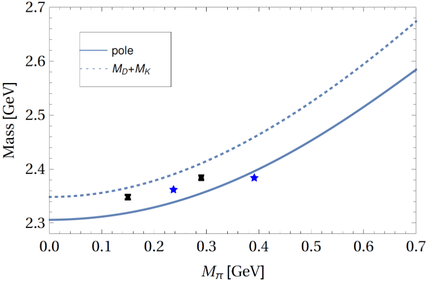
DK component from lattice QCD

- Postdicted mass for $D_{s0}^*(2317)$: 2315^{+18}_{-28} MeV
- Compositeness (DK component) for $D_{s0}^*(2317)$ from (in)direct lattice calculations: DK as the main component, it is in this sense we recognize it as a hadronic molecule



DK component from lattice QCD (2)

- Lattice results in G. Bali et al., PRD96(2017)074501



M_π [MeV]	150	290
$M_{D^*_{s0}(2317)}$ [MeV]	2348 ± 4	2384 ± 3
M_{D_s} [MeV]	1977 ± 1	1980 ± 1

strong M_π dependence!

curves: prediction in Du et al., EPJC77(2017)728

- Lattice results in HadSpec, JHEP 02 (2021) 100

M_π [MeV]	239	391
$M_{D^*_{s0}(2317)}$ [MeV]	2362 ± 3	2380 ± 3

Decay width of $D^*_{s0}(2317)$ (1)

- Partial widths:

	$\Gamma(D^*_{s0}(2317) \to D_s\pi^0)$[keV]	$\Gamma(D^*_{s0}(2317) \to D^*_s\gamma)$[keV]
had. mol.	133 ± 22 [1]	9.4 [2]
chiral doublet [3]	21.5	1.74

[1] L. Liu, Orginos, FKG, Hanhart, Meißner, PRD87(2013)014508

[2] Cleven, Grießhammer, FKG, Hanhart, Meißner, EPJA50(2014)149

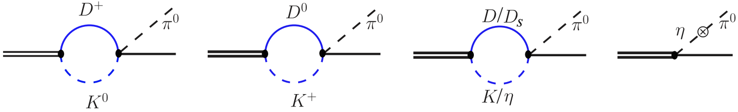
[3] Bardeen, Eichten, Hill, PRD68(2003)054024

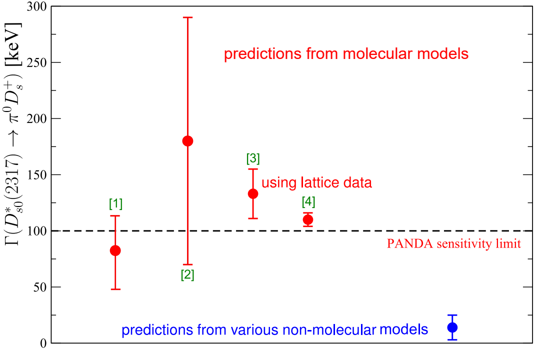
$\mathcal{B}(D^*_{s0}(2317) \to D_s\pi^0) \simeq 90\%$ in both models

- BESIII measurement: BESIII, PRD97(2018)051103

$$\mathcal{B}(D^*_{s0}(2317) \to D_s\pi^0) = 1.00^{+0.00}_{-0.14} \pm 0.14$$

Decay width of $D_{s0}^*(2317)$ (2): smoking gun





measurement planned at $\overline{\text{P}}$ANDA

molecular: [1] Faessler et al., PRD76(2007)014005; [2] FKG et al.,PLB666(2008)251; [3] L. Liu et al.,PRD87(2013)014508; [4] X. Guo, Heo, Lutz,PRD98(2018)014510

non-molecular: e.g., Colangelo, De Fazio, PLB570(2003)180; Bardeen, Eichten, Hill, PRD68(2003)054024

Hadronic decays of $D_{s1}(2460)$

- Experimental measurement:

$$\frac{\Gamma\left(D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-\right)}{\Gamma\left(D_{s1}(2460)^+ \to D_s^{*+}\pi^0\right)} = \begin{cases} 0.14 \pm 0.04 \pm 0.02 & \text{Belle, PRL 92(2004)012002} \\ 0.09 \pm 0.02 & \text{PDG fit} \end{cases}$$

- Isospin breaking $D_{s1} \to D_s^*\pi^0$: (111 ± 15) keV   H.-L. Fu et al., EPJA58(2022)70
- How about $D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$?
  - CHPT result from   S. Fajfer, A. Prapotnik Brdnik, PRD92(2015)074047

$$\Gamma\left(D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-\right) = 0.25(4)(7)\left({}^{+2}_{-4}\right)\,\mathrm{keV}$$

    Both D_s and D_{s1} were treated statically, P-wave happens between π^+ and $\pi^- \Rightarrow I(\pi^+\pi^-) = 1$, isospin breaking
  - But isospin is conserved (!!) for P-wave between D_s and isoscalar $\pi^+\pi^-$

$D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ (1)

M.-N. Tang, Y.-H. Lin, FKG, C. Hanhart, U.-G. Meißner, Commun.Theor.Phys. 75 (2023) 055203

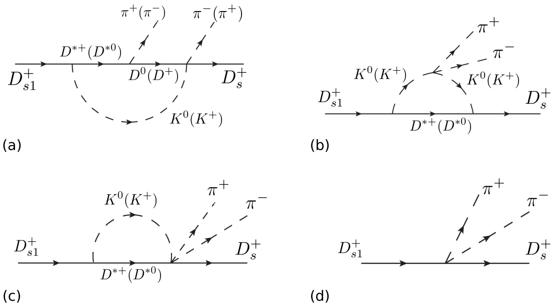
- Hadronic molecule v.s. other (compact) components

  Effective coupling contains crucial information S. Weinberg, PR137(1965)B672

$$g^2 \propto (1 - Z)\sqrt{2\mu E_B}$$

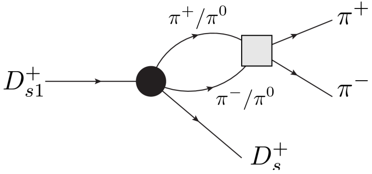
  is maximized for a pure molecular state

- Diagrams for $D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$: (a,b,c): leading for molecular state



$D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ (2)

- Phase space: $m_{\pi^+\pi^-} \in [2M_{\pi^\pm}, 0.49\text{ GeV}]$
- $\pi\pi$ final state interaction (FSI) is important in S-wave: $f_0(500)$ (σ) meson



Black filled circle: all the one-loop diagrams in the previous page, $\hat{\mathcal{A}}_L(s)$

- Framework: dispersion relation, use the $\pi\pi$ S-wave scattering phase shifts $\delta_0(s)$ as input;

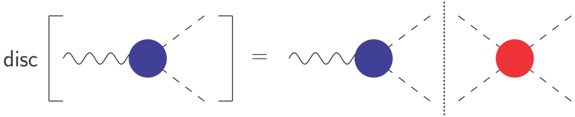
  $\delta_0(s)$ values are precisely known from solution of Roy-type equations;

  the $f_0(500)$ meson is included automatically

  R. Ananthanarayan et al., Phys.Rept.353(2001)207; R. García-Martin et al., PRD83(2011)074004; J. Peláez, Phys.Rept.658(2016)1

Two-body FSI and Omnès representation

- Two-body final state interaction (FSI) below inelastic threshold, unitarity $\Rightarrow$



$$\frac{1}{2i}\text{disc} f_L(s) = \text{Im} f_L(s) = f_L(s)\,\theta\,(s - s_{\text{th}})\,\sin\delta_L(s)\,\exp[-i\delta_L(s)]$$

☞ Watson's FSI theorem:

the phase of $f_L(s)$ is given by $\delta_L(s)$ (modulo $n\pi$) Watson (1953,1954)

☞ $\text{disc}\,[\log f_L(s)] = 2i\delta_L\,\theta\,(s - s_{\text{th}}) \Rightarrow$ dispersion relation for $\log f_L(s)$ gives

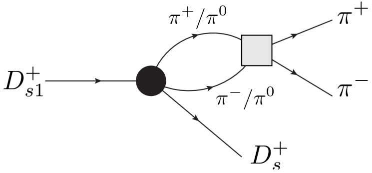
- Omnès representation Omnès (1958)

$$f_L(s) = P_L(s)\Omega_L(s), \qquad \Omega_L(s) = \exp\left[\frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} dz \frac{\delta_L(z)}{z(z-s)}\right]$$

$P_l(s)$: polynomial

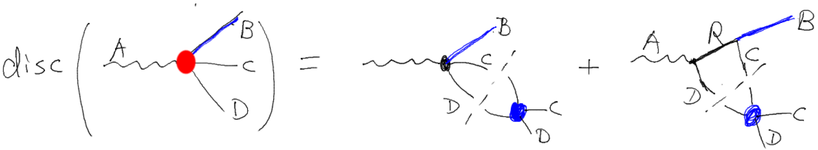
- Input: scattering phase shifts, $f_0(500)$ included automatically; can be applied to, e.g., $\pi\pi$ FSI in $D^+ \to \pi^+\pi^- e^+\nu_e$, $\tau \to \pi^-\pi^0\nu_\tau$, $\psi' \to J/\psi\pi^+\pi^-$, ...

- $\pi\pi$ FSI is more complicated for 3-body final state, possible nontrivial effects from crossed channels



Black filled circle: all the one-loop diagrams in the previous page, $\hat{\mathcal{A}}_L(s)$

- Unitarity $\Rightarrow$ (let R in the 3rd diagram denote all 1-loop diagrams)



$$\frac{1}{2i}\,\text{disc}\left[\mathcal{A}_L(s) + \underbrace{\hat{\mathcal{A}}_L(s)}_{\text{no right-hand cut}}\right] = \mathcal{A}_L(s)\rho(s)T_L^*(s) + \hat{\mathcal{A}}_L(s)\rho(s)\underbrace{T_L^*(s)}_{\pi\pi\text{ scattering}}$$

$D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ (4)

- $f_0(980)$ and $f_2(1270)$ are far away, thus, consider only S-wave single-channel $\pi\pi$ FSI
- Single subtraction is enough (a: subtraction constant)

$$\mathcal{A}_{\text{tot},0}(s) = \hat{\mathcal{A}}_0(z) + \Omega_0(s)\left[a + \frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} dz \frac{\hat{\mathcal{A}}_0(z)\sin\delta_L(z)}{z(z-s)|\Omega_0(z)|}\right]$$

- Subtraction constant matched to the $D_{s1}D_s^\dagger\pi^+\pi^-$ contact term
- Physical meaning:
  - ☞ a: tree-level diagram from contact term for $D_{s1} \to D_s\pi^+\pi^-$
  - ☞ $a\Omega_0(s)$: two-body $\pi\pi$ FSI connected to the contact term
  - ☞ $\hat{\mathcal{A}}_0(z)$: D^*DK, D^*KK and D^*K 1-loop diagrams
  - ☞ $\pi\pi$ FSI connected to the 1-loop diagrams is given by

$$\Omega_0(s)\frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} dz \frac{\hat{\mathcal{A}}_0(z)\sin\delta_L(z)}{z(z-s)|\Omega_0(z)|}$$

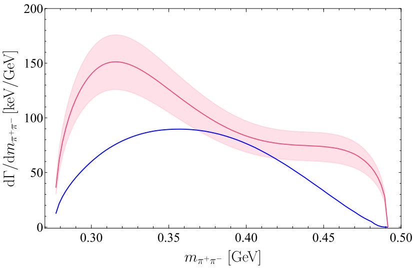
- If D_{s1} is dominantly compact: $|a| \gg |\hat{\mathcal{A}}_0(s)|$; otherwise, D^*K loops should be important

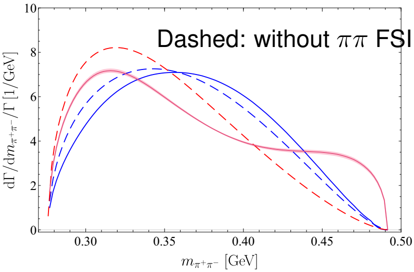
$D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ (5)

- Double bump structure in the $\pi\pi$ invariant mass distribution as a feature of the hadronic molecular picture

  ⋆ Red: molecular, assuming the $D_{s1}D_s\pi\pi$ contact term to vanish + loops with the same cutoff as fixed from scattering

  ⋆ Blue: compact, without D^*K loops

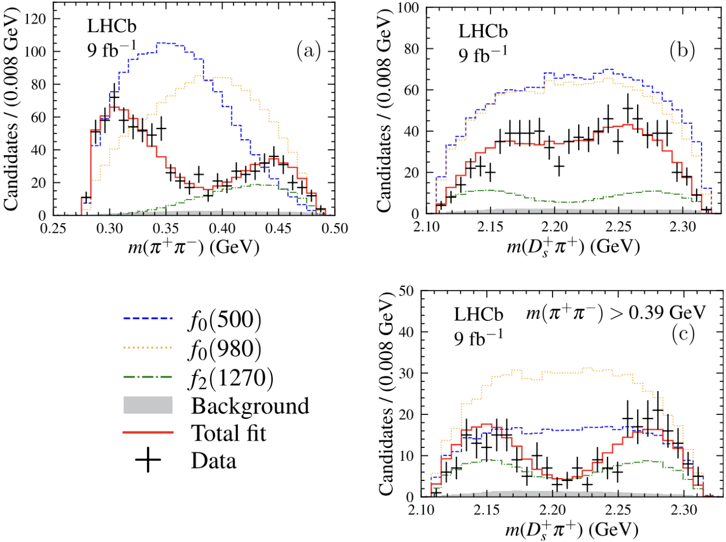




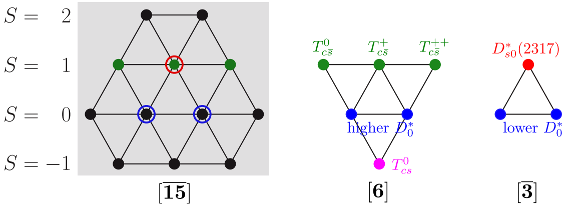
- agree with Belle measurement $0.14 \pm 0.04 \pm 0.02$:

$$\left.\frac{\Gamma\left(D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-\right)}{\Gamma\left(D_{s1}(2460)^+ \to D_s^{*+}\pi^0\right)}\right|_{\text{mol.}} = 0.19^{+0.07}_{-0.05}$$

Double-bump structure observed by LHCb Collaboration LHCb, Sci.Bull.70(2025)3219

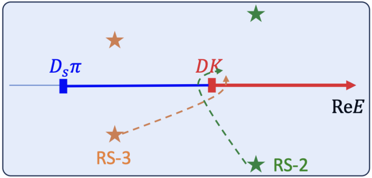


- More about the sextet — FKG, B.-S. Zou, Sci.Bull.70(2025)3425



- Isospin triplet $T_{c\bar{s}0}$?
  - ☞ Mixing between $[6]$ and $[\overline{15}]$: less attraction than $I = 0$ $D\bar{K}$ (T_{cs}, virtual state)
  - ☞ Pole deep in the complex plane: insignificant effect, cusp at DK threshold

(S, I)	$h'_5 = +1$			$h'_5 = -1$		
	Re	Im	RS	Re	Im	RS
$(0, \frac{1}{2})$	2107	± 123	II	2107	± 105	II
	2452	± 17	III	2519	± 69	III
$(0, \frac{1}{2})$ $(V_{ii} = 0)$	2466	± 24	III	2388	± 49	III
$(1, 0)$	2318	0	I	2318	0	I
$(1, 1)$ DK	2309	± 111	III	2283	± 196	III



old prediction without lattice input — FKG, C. Hanhart, U.-G. Meißner, EPJA40(2009)179

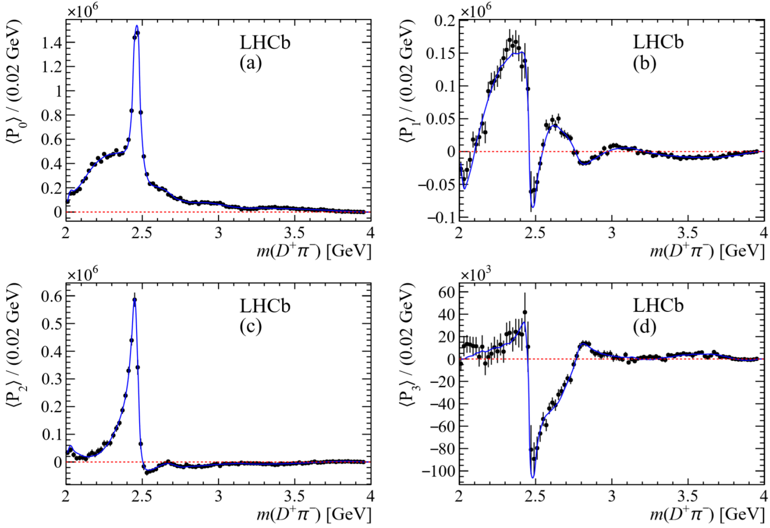
with lattice input: $2.22 \pm 0.17i$ GeV (RS-3); $2.45 \pm 0.27i$ GeV (RS-2)

LHCb data can be well described — H.-L. Fu et al., in preparation

$B^- \to D^+\pi^-\pi^-$ (1)

LHCb, PRD94(2016)072001

Angular moments: $$\langle P_L \rangle \propto \int_{-1}^{+1} d\cos\theta \, P_L(\cos\theta) \, \frac{d\Gamma}{dm_{D\pi}\, d\cos\theta}$$

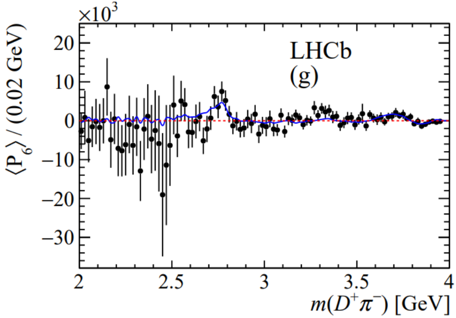


- Consider only S, P, D waves, up to around 2.5 GeV:

$$\mathcal{A}(B^- \to D^+\pi^-\pi^-) = \mathcal{A}_0(s) + \sqrt{3}\mathcal{A}_1(s)P_1(z) + \sqrt{5}\mathcal{A}_2(s)P_2(z);$$

higher partial waves negligible:

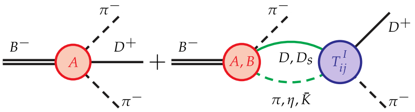
$$\langle P_6 \rangle \propto |\mathcal{A}_3|^2$$



- P-wave: D^*, $D^*(2680)$ [$M = 2681$ MeV, $\Gamma = 187$ MeV]; D-wave: $D_2(2460)$ parametrized (with the same masses and widths) as in the LHCb paper: Breit–Wigner with Bleit–Weisskopf barrier factors, one constant phase for each as free parameters

$B^- \to D^+\pi^-\pi^-$ (3)

M.-L. Du et al., PRD98(2018)094018



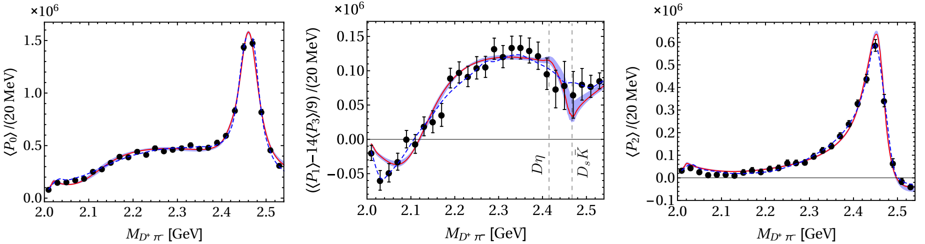
- SU(3) symmetry   Savage, Wise (1989)
- S-wave: FSI, two new parameters
- P, D-wave: BWs from the LHCb fit

Angular moments:

$$\langle P_0\rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2\,, \quad \langle P_2\rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2|\cos(\delta_2 - \delta_0)\,,$$

$$\langle P_{13}\rangle \equiv \langle P_1\rangle - \frac{14}{9}\langle P_3\rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1|\cos(\delta_1 - \delta_0)$$

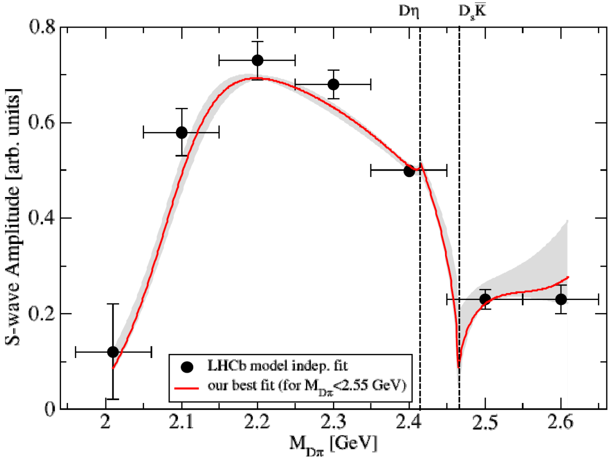
Data: LHCb, PRD94(2016)072001



- Fast variation in [2.4, 2.5] GeV in $\langle P_{13}\rangle$: cusps at $D\eta$ and $D_s\bar{K}$ thresholds

$B^- \to D^+\pi^-\pi^-$ (4)

$D\pi$ S-wave amplitude: comparison with the LHCb determination



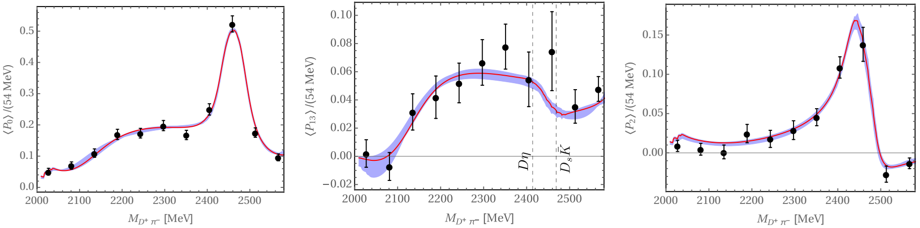
Coupled-channel threshold cusps, effects enhanced by the pole at $(2.45 - i\,0.13)$ GeV

More B decays

M.-L. Du, FKG, U.-G. Meißner, PRD99(2019)114002

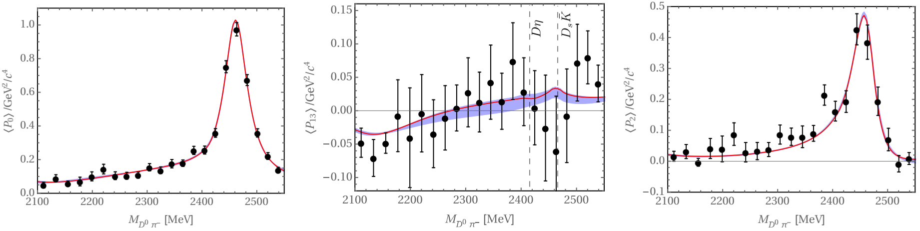
Fit to data of $B^- \to D^+\pi^-K^-$

Data: LHCb, PRD91(2015)092002



Fit to data of $B^0 \to \bar{D}^0\pi^-\pi^+$

Data: LHCb, PRD92(2015)032002



and also $B^0 \to \bar{D}^0\pi^-K^+$, $B^- \to D^+\pi^-K^-$, $B_s^0 \to \bar{D}^0K^-\pi^+$

Where is the lightest D_0^* (1)

M.-L. Du, FKG, Hanhart, Kubis, Meißner, PRL126(2021)192001

- The S-wave $D\pi$ phase in the decay can be extracted from the phase moments:

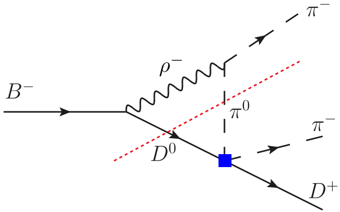
$$\cos(\delta_0 - \delta_1) = \sqrt{\frac{3}{10}} \frac{\langle P_{13} \rangle}{\sqrt{\langle P_2 \rangle} \sqrt{\langle P_0 \rangle - \frac{5}{2} \langle P_2 \rangle}}$$

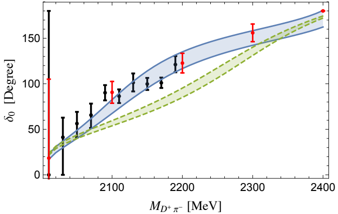
- Soft-pion theorems: W.L.Lin, C.C.Chiang, Lett. Nuovo Cim. 38(1983)508

$$\mathcal{A}(B^- \to D^+ \pi^- \pi^-)|_{p_{\pi^-} \to 0} = \frac{1}{F_\pi} \mathcal{A}(\underbrace{B^0 \to \bar{D}^0 \pi^0}_{\mathcal{B}=2.6\times 10^{-4}}) ,$$

$$\mathcal{A}(B^- \to D^0 \pi^0 \pi^-)|_{p_{\pi^0} \to 0} = -\frac{1}{F_\pi} \mathcal{A}(\underbrace{B^- \to D^0 \pi^-}_{\mathcal{B}=4.7\times 10^{-3}})$$

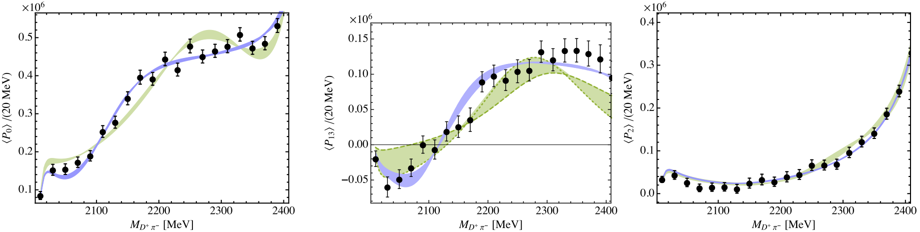
- Phase described well by UCHPT, but not by the BW for $D_0^*(2300)$





Where is the lightest D_0^* (2)

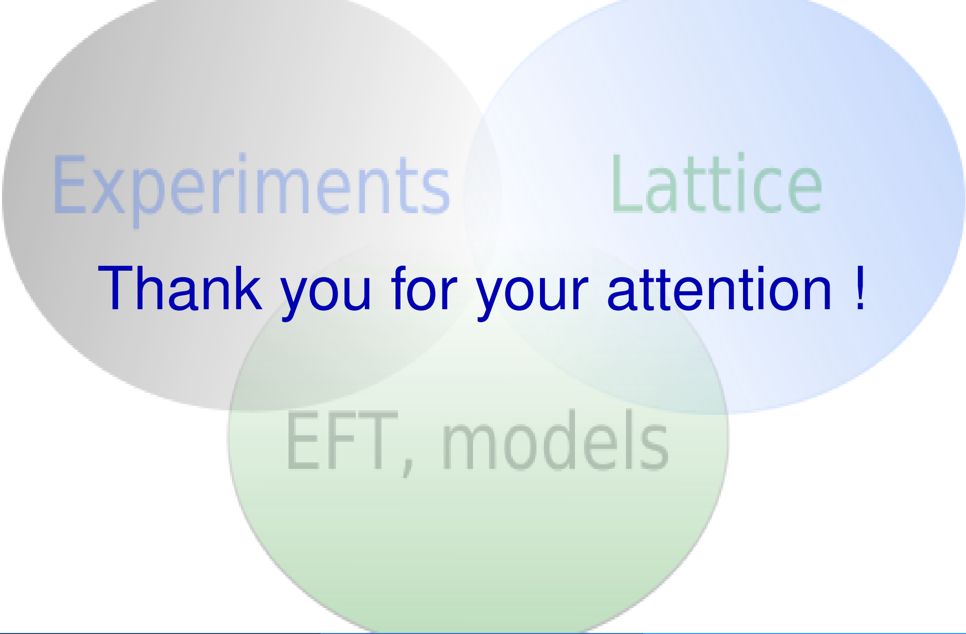
- Fits with the Khuri-Treiman equation taking into account three-body unitarity: using S-wave $D\pi$ scattering phase from UCHPT ($\chi^2/\text{d.o.f.} = 1.2$) and from BW ($\chi^2/\text{d.o.f.} = 2.0$)



- The LHCb data are well described with UCHPT amplitude with two D_0^* states; the lowest has a mass about 2.1 GeV
- Support from recent lattice results: L. Gayer et al. [HadSpec], JHEP 07 (2021) 123
  D_0^* with $M \approx 2.2$ GeV and $\Gamma \approx 0.4$ GeV obtained using $M_\pi \approx 239$ MeV

Summary

- Exp + lattice + EFT(-like): Unitarized chiral approach can be regarded as a parameterization of T-matrix, bridging the lattice results and experimental measurements to reach a PRECISION hadron spectroscopy
- Puzzles of positive-parity charmed mesons naturally understood in the hadronic moelcular picture; two D_0^*
- The picture is consistent with available experimental and lattice data
- π, K, η are pseudo-Goldstone bosons, interactions for other hadrons could be more strong
  $\Rightarrow$ importance of S-wave multi-hadron channels should be generally expected
  $\Rightarrow$ new paradigm shifted from quark model (old paradigm)
- $D^{(*)}\bar{K}$ form $I = 0$ virtual states; it's reasonable to expect $D^{(*)}\bar{K}^*$ would have bound states. $D^*\bar{K}^*$ molecule: $X(2900)$ found by LHCb?
  Then there must be partners:
  spin partner $D\bar{K}^*$:~ 2760 MeV
  bottom partners BK^* :~ 6175 MeV; B^*K^* :~ 6220 MeV
  Then there might be more deeply bound $D^{(*)}K^*/B^{(*)}\bar{K}^*$ isoscalar states.



Thank you for your attention !

Energy levels in a finite volume

- To compare with lattice data (energy levels), framework in a finite volume (FV)
- In a FV, momentum gets quantized: $\vec{q} = \frac{2\pi}{L}\vec{n},\, \vec{n} \in \mathbb{Z}^3$
- Loop integral $G(s)$ gets modified: $\int d^3\vec{q} \to \frac{1}{L^3}\sum_{\vec{q}}$, and one gets

M. Döring, U.-G. Meißner, E. Oset, A. Rusetsky, EPJA47(2011)139

$$\widetilde{G}(s,L) = G(s) + \lim_{\Lambda\to+\infty} \underbrace{\left[\frac{1}{L^3} \sum_{\vec{n}}^{|\vec{q}|<\Lambda} I(\vec{q}) - \int_0^\Lambda \frac{q^2 \mathrm{d}q}{2\pi^2}\, I(\vec{q}) \right]}_{\text{finite volume effect}}$$

$I(\vec{q})$: loop integrand

- FV energy levels obtained by as poles of $\widetilde{T}(s,L)$:

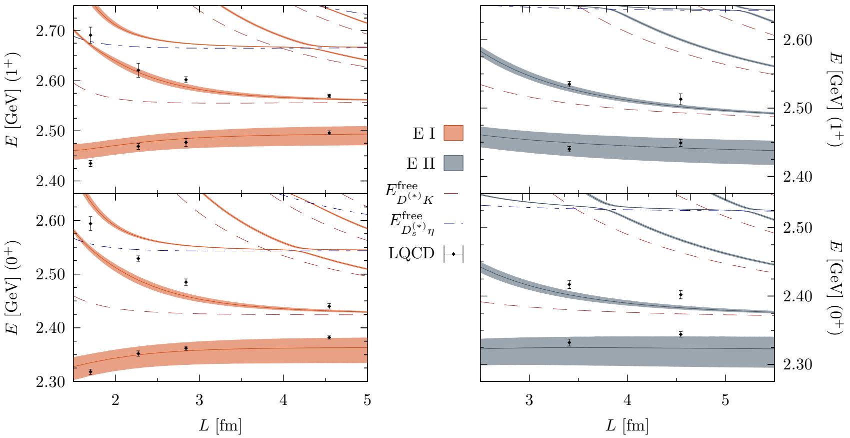
$$\widetilde{T}^{-1}(s,L) = V^{-1}(s) - \widetilde{G}(s,L)$$

Postdictions versus lattice results: charm-strange

- Postdicted finite volume energy levels for $(S, I) = (1, 0)\ D^{(*)}K$, $J^P = 1^+$ & 0^+ versus lattice QCD results by [G. Bali, S. Collins, A. Cox, A. Schäfer, PRD96(2017)074501]

M. Albaladejo, P. Fernandez-Soler, J. Nieves, P. G. Ortega, EPJC78(2018)722

E I: $M_\pi = 290$ MeV E II: $M_\pi = 150$ MeV



$D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ (more details)

- Using $\rho(s)T_L^*(s) = \sin\delta_L(s)e^{-i\delta_L(s)}$, we get

$$\frac{1}{2i}\,\mathrm{disc}\,\mathcal{A}_L(s) = \left[\mathcal{A}_L(s) + \hat{\mathcal{A}}_L(s)\right]\sin\delta_L(s)e^{-i\delta_L(s)}$$

- Rewrite the unitarity relation using $\mathrm{disc}\,\mathcal{A}_L(s) = \mathcal{A}_L(s+i\epsilon) - \mathcal{A}_L(s-i\epsilon)$:

$$\frac{1}{2i}\left[\mathcal{A}_L(s+i\epsilon)e^{-i\delta_L(s)} - \mathcal{A}_L(s-i\epsilon)e^{i\delta_L(s)}\right] = \hat{\mathcal{A}}_L(s)\sin\delta_L(s)$$

- Using properties of the Omnès function:

$$|\Omega_L(s)| = \Omega_L(s+i\epsilon)e^{-i\delta_L(s)} = \Omega_L(s-i\epsilon)e^{i\delta_L(s)},$$

$$\frac{1}{2i}\left[\frac{\mathcal{A}_L(s+i\epsilon)}{\Omega_L(s+i\epsilon)} - \frac{\mathcal{A}_L(s-i\epsilon)}{\Omega_L(s-i\epsilon)}\right] = \frac{\hat{\mathcal{A}}_L(s)\sin\delta_L(s)}{|\Omega_L(s)|}$$

- Writing down the dispersion relation for $\mathcal{A}_L(s)/\Omega_L(s)$, one gets the total decay amplitude: $\mathcal{A}_{\text{tot}}(s,\cos\theta) = \sum_L \left[\mathcal{A}_L(s) + \hat{\mathcal{A}}_L(s)\right]P_L(\cos\theta)$

$$\mathcal{A}_L(s) = \Omega_L(s)\left[P_{n-1}(s) + \frac{s^n}{\pi}\int_{s_{\text{th}}}^{\infty} dz\,\frac{\hat{\mathcal{A}}_L(z)\sin\delta_L(z)}{z^n(z-s)|\Omega_L(z)|}\right]$$

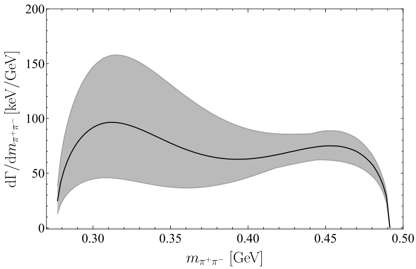
$D_{s1}(2460)^+ \to D_s^+\pi^+\pi^-$ (more results)

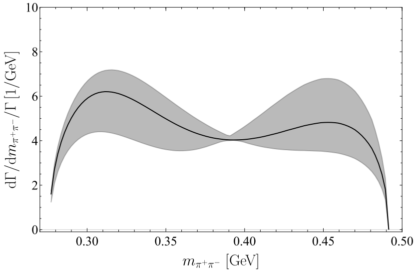
- With the contact term fixed from reproducing the Belle measurement (consistent with zero)

  Partial width

$$\Gamma\left(D_{s1}^+ \to D_s^+\pi^+\pi^-\right) = (16^{+7}_{-5})\ \mathrm{keV}$$

  Double-bump structure in $\pi^+\pi^-$ invariant mass distribution:

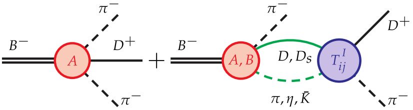




- Prediction for the bottom analogue:

$$\Gamma\left(B_{s1}^0 \to B_s^0\pi^+\pi^-\right) = (3 \pm 1)\ \mathrm{keV}$$

- $B^- \to D^+\pi^-\pi^-$ contains coupled-channel $D\pi$ FSI
- consider S, P, D waves: $\mathcal{A}(B^- \to D^+\pi^-\pi^-) = \mathcal{A}_0(s) + \mathcal{A}_1(s) + \mathcal{A}_2(s)$
  - ☞ P-wave: D^*, $D^*(2680)$; D-wave: $D_2(2460)$ as in the LHCb paper
  - ☞ S-wave: use the coupled-channel (1: $D\pi$; $2: D\eta$; $3: D_s\bar{K}$) amplitudes with all parameters fixed before



  - ☞ only 2 parameters in S-wave: C and a subtraction constant in $G_i(s)$

SU(3)+chiral ⇒
$$\mathcal{A}_0(s) \propto E_\pi\left[2 + G_{D\pi}(s)\left(\frac{5}{3}T^{1/2}_{11}(s) + \frac{1}{3}T^{3/2}(s)\right)\right] + \frac{1}{3}E_\eta G_{D\eta}(s)T^{1/2}_{21}(s) + \sqrt{\frac{2}{3}}E_{\bar{K}}G_{D_s\bar{K}}(s)T^{1/2}_{31}(s) + C\,E_\eta G_{D\eta}(s)T^{1/2}_{21},$$

Im $G_i(s) = -\rho_i(s) \Rightarrow$ Unitarity: Im$\mathcal{A}_{0,i}(s) = -\sum_j T^*_{ij}(s)\rho_j(s)\mathcal{A}_{0,j}(s)$

More similar states?

FKG, U.-G. Meißner, PRD84(2011)014013

- Chiral symmetry $\Rightarrow$ universal Weinberg–Tomozawa term
  applicable to any hadrons with a small width $\Gamma \ll$ inverse of force range
- nice candidates: $D_1(2420)$ & $D_2(2460)$, $\Gamma \sim 30$ MeV
  more speculative (using the same subtraction constant) predictions of
  $D_1(2420)K$ and $D_2(2460)K$ bound states

Constituents	$D_1(2420)K$	$D_2(2460)K$	$\bar{B}_1(5720)K$	$\bar{B}_2(5747)K$
J^P	1^-	2^-	1^-	2^-
Predictions	2870 ± 9	2910 ± 9	6151 ± 33	6169 ± 33
Decays	$D^{(*)}K, D_s^{(*)}\eta$	$D^*K, D_s^*\eta$	$\bar{B}^{(*)}K, \bar{B}_s^{(*)}\eta$	$\bar{B}^*K, \bar{B}_s^*\eta$

- $D_{s1}^*(2860)$ is probably the $D_1(2420)K$ bound state!

What is $D_s^*(2860)$?

- $D_{s1}^*(2860)$: puzzling decay pattern: $\Gamma(D^*K)/\Gamma(DK) = 1.10 \pm 0.24$

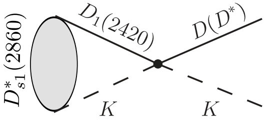
  Predictions from HQSS: P.Colangelo et al.,PRD77(2008)014012

$D_{sJ}(2860)$	$D_{sJ}(2860) \to DK$	$\frac{\Gamma(D_{sJ}\to D^*K)}{\Gamma(D_{sJ}\to DK)}$
$s_\ell^P = \frac{1}{2}^-, J^P = 1^-, n = 1$	p-wave	1.23
$s_\ell^P = \frac{1}{2}^+, J^P = 0^+, n = 1$	s-wave	0
$s_\ell^P = \frac{3}{2}^+, J^P = 2^+, n = 1$	d-wave	0.63
$s_\ell^P = \frac{3}{2}^-, J^P = 1^-, n = 0$	p-wave	0.06
$s_\ell^P = \frac{5}{2}^-, J^P = 3^-, n = 0$	f-wave	0.39

  but, better candidate for $(2S, 1^-)$: $D_{s1}^*(2700)$ $\Gamma(D^*K)/\Gamma(DK) = 0.91 \pm 0.18$

  $M(2P, 2^+) \sim 3.16$ GeV M. Di Pierro, E. Eichten, PRD64(2001)114004

- A natural explanation of the decay pattern:



:

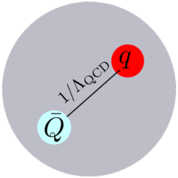
$$\frac{\Gamma(D_{s1}^*(2860) \to D^*K)}{DK} \simeq 2\frac{M_{D^*}}{M_D}\left|\frac{\vec{k}_{D^*}}{\vec{k}_D}\right|^3 = 1.23$$

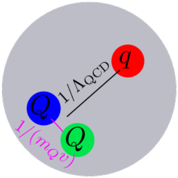
- Heavy anti-quark–diquark symmetry (HADS): Savage, Wise (1990)

  $m_Q v \gg \Lambda_{\text{QCD}}$,

  the diquark serves as a point-like color-$\bar{3}$ source, like a heavy anti-quark.

  doubly-heavy baryons $\Leftrightarrow$ anti-heavy mesons

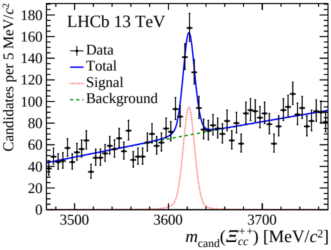


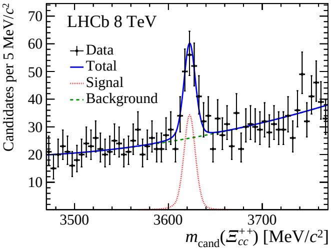


- HADS + CHPT with virtual photons: Brodsky, FKG, Hanhart, Meißner, PLB698(2011)251

  $$M_{D^+} - M_{D^0} \Rightarrow M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^{+}} = (1.5 \pm 2.7)\ \text{MeV}$$

- LHCb observation of Ξ_{cc}^{++}: $M = (3621.40 \pm 0.78)$ MeV LHCb, PRL119(2017)112001





Doubly-charmed baryons with $J^P = 1/2^-$

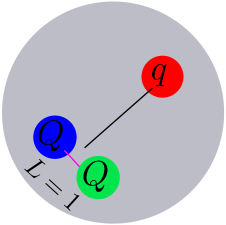
- P-wave QQ, excitation energy Mehen, PRD96(2017)094028

  $\sim \frac{1}{2}(M_{h_c} - M_{J/\psi}) = \mathcal{O}\,(200\text{ MeV})$

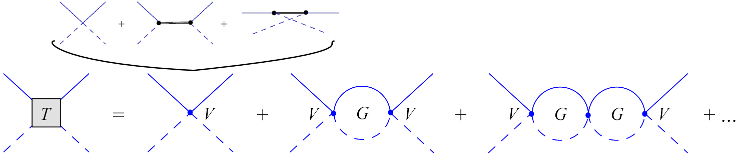
  rel. QM ($q[QQ]$, linear conf.): ~ 220 MeV

  Ebert et al., PRD66(2002)014008

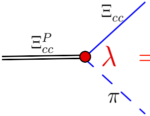
  [but it might be much larger ... (Eichten's talk)]



- if small $\Rightarrow$ $\Xi_{cc}^P, \Omega_{cc}^P$ as dynamical degrees of freedom



- S-wave QQ: spin $s_{QQ} = 1$, P-wave QQ: spin $s_{QQ} = 0$



$\lambda = \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_Q}\right)$

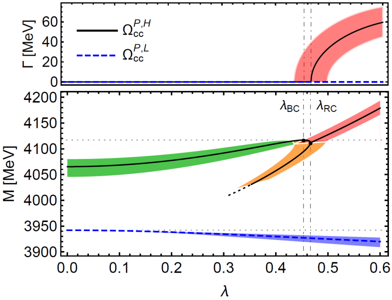
Doubly-charmed strange baryons with $J^P = 1/2^-$: Ω_{cc}^P

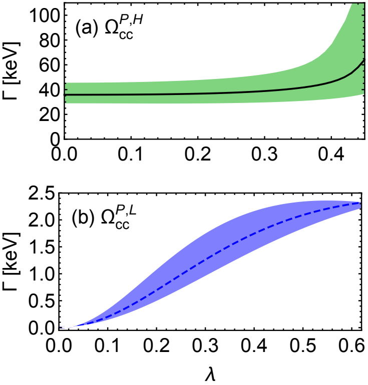
M.-J. Yan, X.-H. Liu et al., PRD98(2018)091502(R)

- Likely two states below the $\Xi_{cc}\bar{K}$ threshold

  Inputs: bare $\mathring{M}_{\Xi_{cc}^P} = 3838$ MeV from quark model D. Ebert et al., PRD96(2002)024008

$$M_{\Omega_{cc}} - M_{\Xi_{cc}} = M_{D_s} - M_D, \quad \mathring{M}_{\Omega_{cc}^P} - M_{\Omega_{cc}} = 217 \text{ MeV}$$



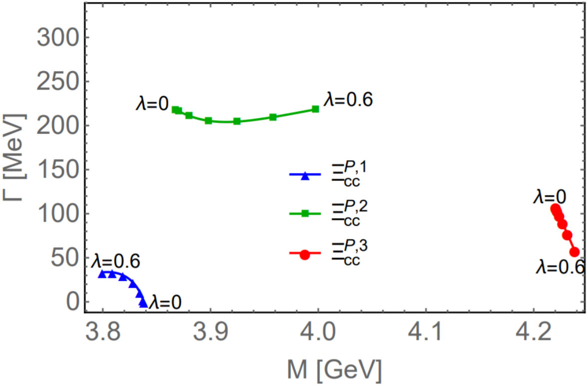


- Tiny widths due to isospin breaking: $\Omega_{cc}^P \to \Omega_{cc}\pi^0$

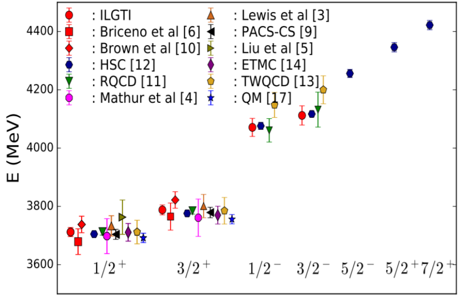
Doubly-charmed nonstrange baryons with $J^P = 1/2^-$: Ξ_{cc}^P

M.-J. Yan, X.-H. Liu et al., PRD98(2018)091502(R)

- Three $\frac{1}{2}^-$ Ξ_{cc}^P states below 4.2 GeV:

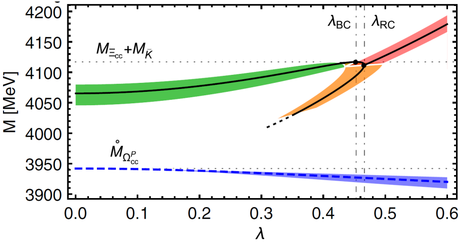


Lattice results



N. Mathur, M. Padmanath, PRD99(2019)031501

- $M = 4071 \pm 25 \pm 18$ MeV
- three-quark operators only
- $M_\pi \simeq 650$ MeV, no chiral extrapolation



Remarks:

- $\Xi_{cc}\bar{K}$ threshold $\simeq 4.12$ GeV
- importance of $\Xi_{cc}\bar{K}$ type operators
- M_π dependence could be sizeable