XYZ Exotics: their nature and properties from NRQCD

Monsoon Hadrons 2026

Tata Institute of Fundamental Research, Mumbai

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Phys. Rev. D 110, 094040 (2024) (Editors Suggestion), Phys. Rev. Lett 135, 121902 (2025),
Phys. Rev. D 112, 114037 (2025), arXiv 2602.14916 (accepted in JHEP), arXiv 2604.12603

TECHNISCHE UNIVERSITÄT MÜNCHEN

भौतिक अनुसंधान प्रयोगशाला PHYSICAL RESEARCH LABORATORY

erc Advanced Grant EFT-XYZ

Outline

- The XYZ Experimental Landscape
- BOEFT Formalism
- Quarkonium, Tetraquark, Pentaquark
- Hadro-production

XYZ Experimental Landscape

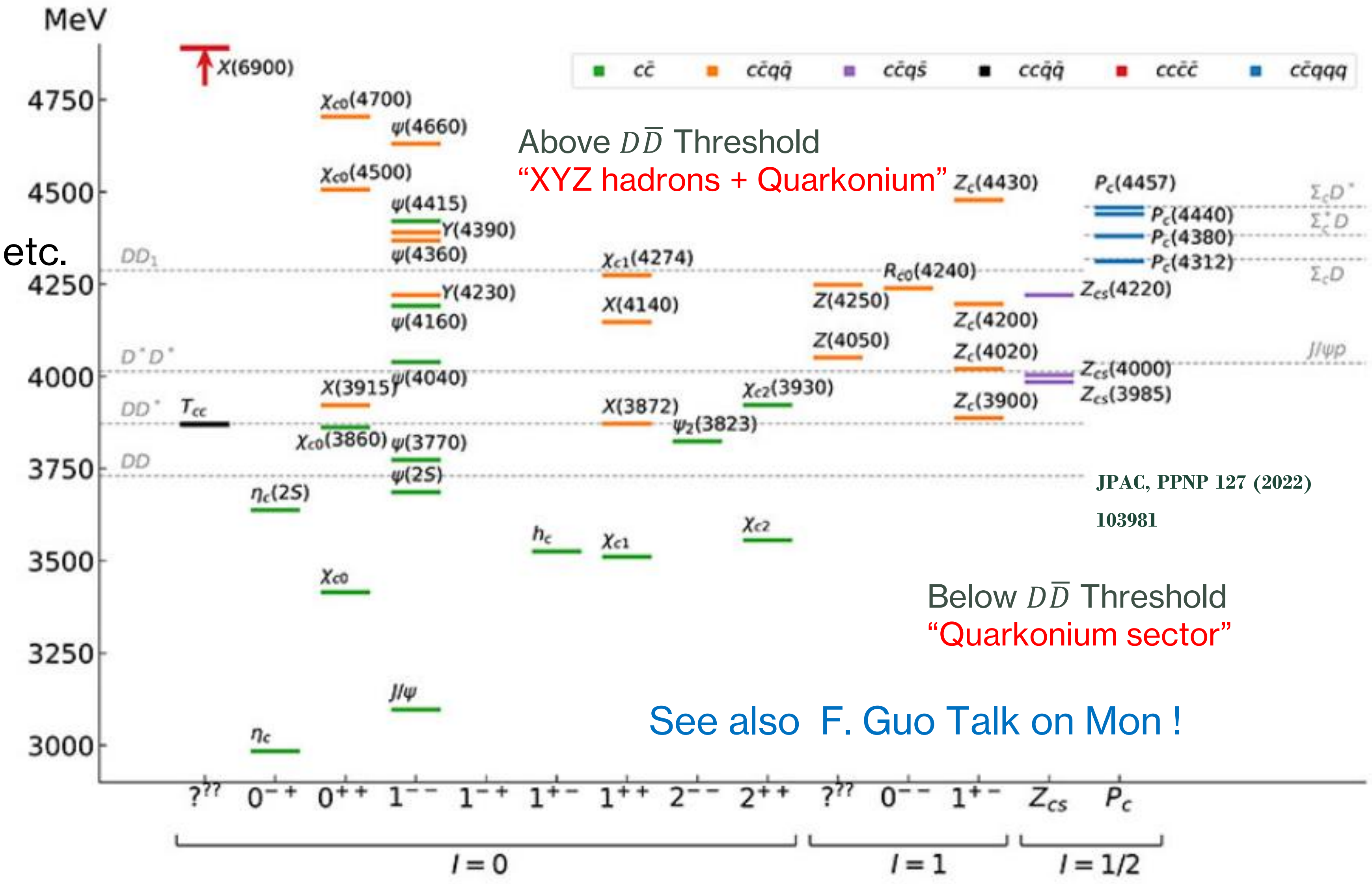
- ❑ **XYZ** Exotics: States with at-least 2-heavy quarks.
- ❑ Exotic $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc.
- ❑ Charge: minimal 4-quarks:

$Z_c(4430)^{\pm}$ $Z_b(10650)^{\pm}$, $T_{cc}(3875)^{+}$

For review see Brambilla et al. Phys. Reports. 873 (2020)

- ❑ Recent count including both **tetraquarks and pentaquarks**:
  - 54 in $c\bar{c}$ sector
  - 5 in $b\bar{b}$ sector
  - 4 with all c and $\bar{c}$
  - 1 with two charm quarks.

Lebed, arXiv 2308.00781 (2023)



Above $D\bar{D}$ Threshold
"XYZ hadrons + Quarkonium"

Below $D\bar{D}$ Threshold
"Quarkonium sector"

JPAC, PPNP 127 (2022) 103981

See also F. Guo Talk on Mon !

- ❑ CMS reported observation of new state $c\bar{c}c\bar{c}(7100)$ in Jan 2026

CMS 2602.02252

Nature of $\chi_{c1}(3872)$ state

Many experiments contribute to it:

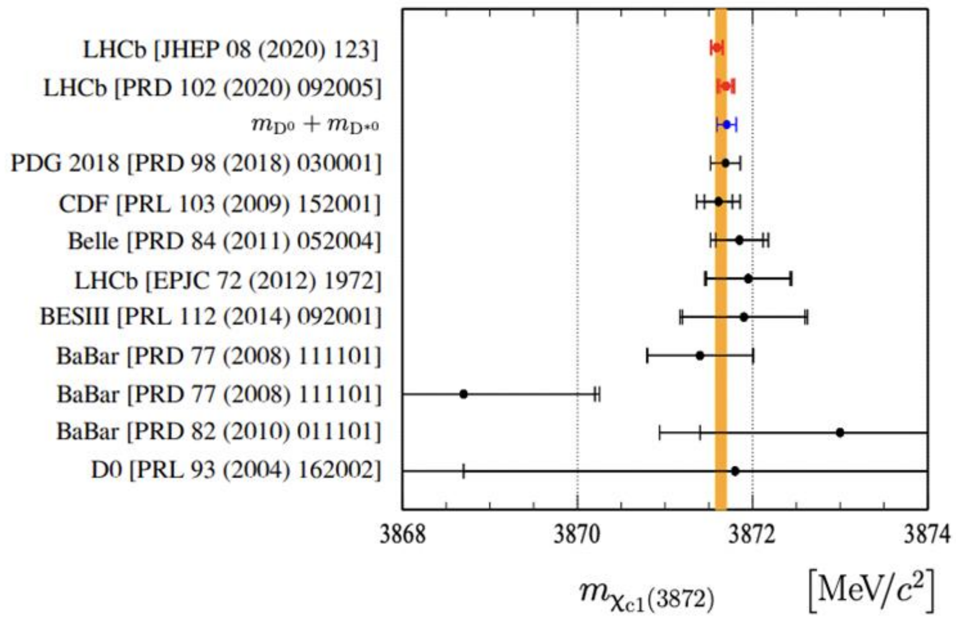
- Spin assignment: $J^{PC} = 1^{++}$
- Mass is consistent with **m(D⁰) + m(D*⁰)**
- Width is narrow compared to conventional $c\bar{c}$ states above threshold

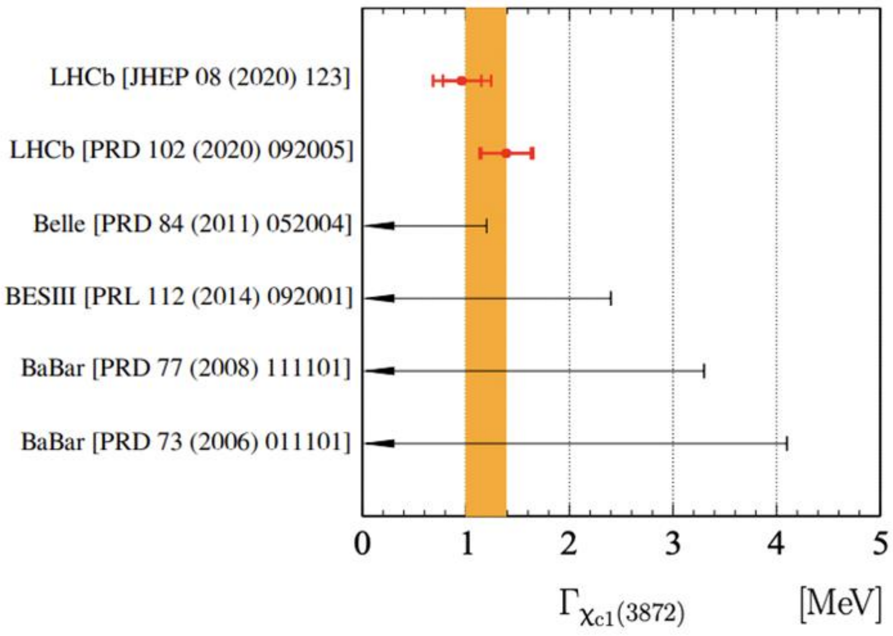
First exotic state discovered in 2003 by Belle.

Phys. Rev. Lett. 91, 262001 (2003)

Multiple debates in literature on whether conventional $\chi_{c1}(2^3P_1)$, molecular state, tetraquark, hybrid, or mixed?

JHEP 08 (2020) 123





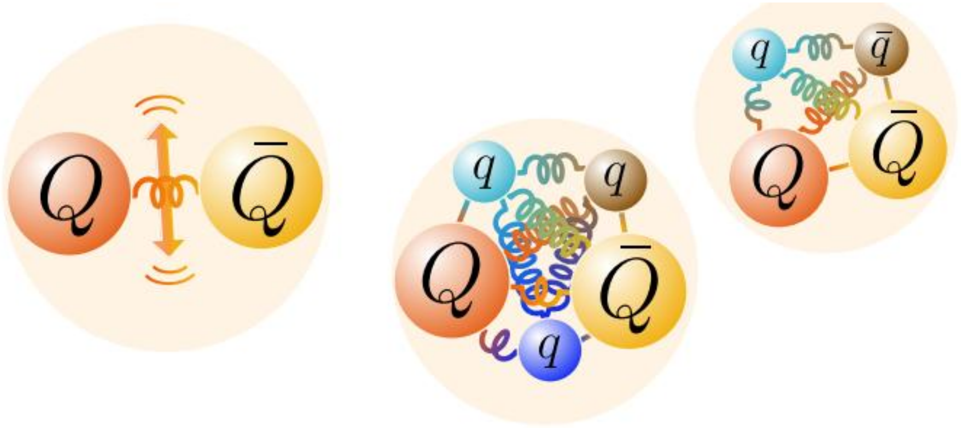
PRL. 110 (2013) 222001,
PRD 92 (2015) 011102(R)

Multiquark Hadron

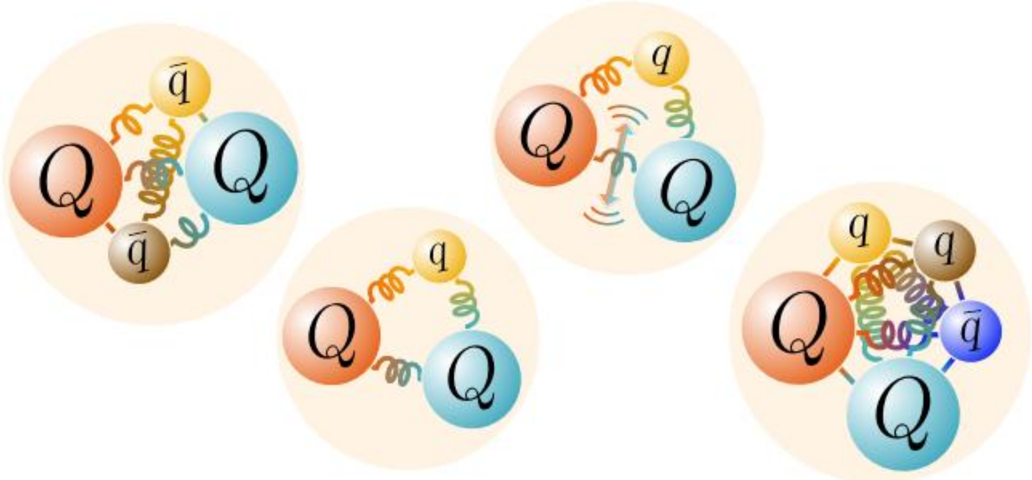
Hadrons with "two" heavy quarks

$$Q = b,\ c \quad , \quad q = u,\ d,\ s$$

- $Q\bar{Q}+$ light quarks and gluons
  - Heavy Quarkonium: $Q\bar{Q}$
  - Hybrids: $Q\bar{Q}g$
  - Tetraquarks: $Q\bar{Q}q\bar{q}$
  - Pentaquarks: $Q\bar{Q}qqq$
  - ...



- $QQ+$ light quarks and gluons
  - Double Heavy Baryons: QQq
  - Tetraquarks: $QQ\bar{q}\bar{q}$
  - Pentaquarks: $QQqq\bar{q}$
  - Hybrids: $QQqg$
  - ...



Born-Oppenheimer EFT

BOEFT: Exotic Hadron

- Exotic hadron $(Q\bar{Q}X, QQX, \ldots\ldots)$, X: any combination of light quark and gluons (LDF) for color singlet.

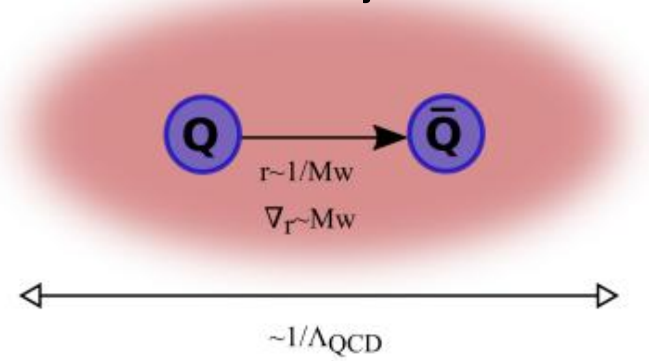
- Heavy quarks   Q = b,c → $m_Q \gg \Lambda_{QCD}$

Heavy quark move slowly $v \ll 1$, compared to light quarks or gluons (LDF)

- Multiple scales in Exotics:
  - ❖ Mass of heavy quark: m
  - ❖ Energy scale for LDF: Λ_{QCD}
  - ❖ Relative momentum between heavy quarks: $mv \sim 1/r$
  - ❖ Heavy Quark kinetic energy scale: mv^2

$$m \gg mv \gtrsim \Lambda_{QCD} \gg mv^2$$

Extended objects:

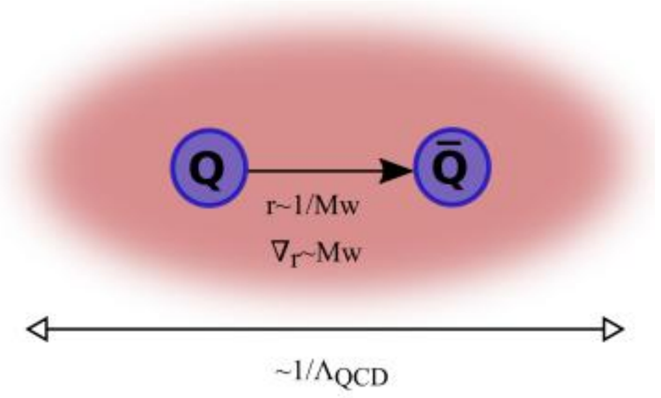


Heavy quark: slow-degrees of freedom ***X***: fast-degrees of freedom

BOEFT: Exotic Hadron

- Hierarchy of scales:

$$m \gg mv \gtrsim \Lambda_{\rm QCD} \gg mv^2$$



- Time scales:

$$\tau_{Q\bar{Q}} \sim 1/m_Q v^2 \gg 1/\Lambda_{QCD} \sim \tau_X$$

Adiabatic expansion

Born-Oppenheimer (BO) Approximation

M.Born, R. Oppenheimer, Annalen der Physik 389 (1927)

Heavy quarks **static** with respect to light quarks or gluons

- **Bound-state dynamics** at energy scale mv^2 ! Integrate out all energy scales above mv^2.

$$\rm QCD \to NRQCD \to pNRQCD/BOEFT$$

R. Oncala, J. Soto, Phys. Rev. D96 (2017)

Berwein, Brambilla, Castellà, Vairo Phys. Rev. D. 92 (2015)

Brambilla, Krein, Castellà, Vairo Phys. Rev. D. 97, (2018)

Castellà, Soto Phys. Rev. D. 102 (2020)

Berwein, Brambilla, AM, Vairo, arXiv 2408.04719

BOEFT: Quantum #'s

- **BO-quantum number (r $\neq$ 0)**: heavy quarks static, Cylindrical symmetry group $D_{\infty h}$

Labelling LDF static energies:

- ✓ Absolute value of component of **LDF angular momentum K**

$$|\boldsymbol{r} \cdot \boldsymbol{K}| \equiv \boldsymbol{\Lambda} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots \ldots .. (\mathbf{or}\ \boldsymbol{\Sigma}, \boldsymbol{\Pi}, \boldsymbol{\Delta}, \boldsymbol{\Phi}, \ldots ..)$$

- ✓ Product of charge conjugation and parity ($\boldsymbol{CP}$):

$$\boldsymbol{\eta} = +\mathbf{1}\ (\mathbf{g}), -\mathbf{1}\ (\boldsymbol{u})$$

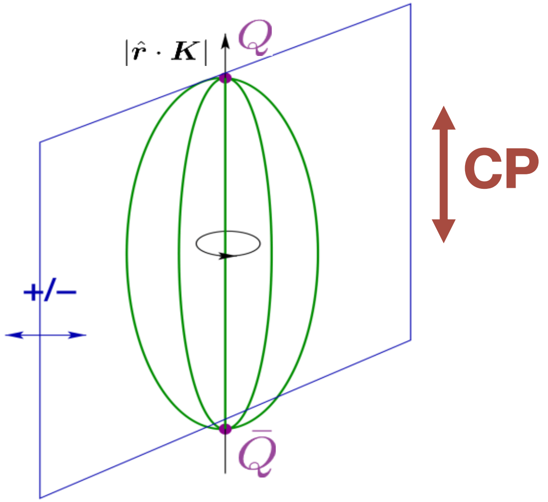
- ✓ $\boldsymbol{\sigma}$: Eigenvalue of reflection about a plane containing static sources.

$$\sigma = P\,(-1)^{K_{\text{light}}} = \pm 1$$

Born, Oppenheimer, Annalen der Physik 389 (1927)

Landau, Lifshitz & Pitaevskii, QM book

$$\Lambda_\eta^\sigma$$



Examples:

K^{PC}	Λ_η^σ
0^{++}	Σ_g^+
0^{+-}	Σ_u^+
1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
2^{--}	$\{\Sigma_g^-, \Pi_g, \Delta_g\}$

- **Spherical symmetry** restored in **r → 0** limit: Labelled by LDF quantum #'s:

$$\kappa = \{K^{PC}, f\}$$

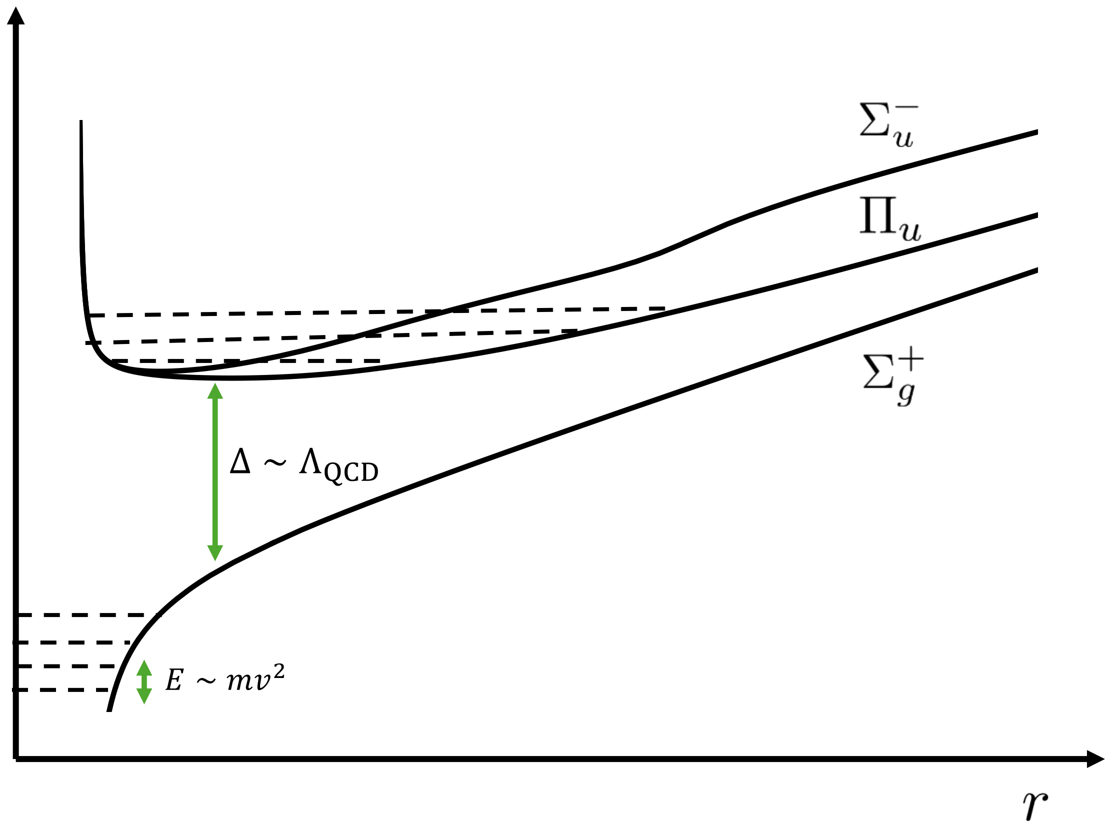
BOEFT

- BOEFT Lagrangian :

$$L_{\mathrm{BOEFT}} = L_{Q\bar{Q}} + L_{Q\bar{Q}g} + L_{Q\bar{Q}q\bar{q}} + L_{Q\bar{Q}qqq} + L_{\mathrm{mixing}} + \cdots$$

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040   Castellà , Soto Phys. Rev. D. 102, 014012 (2020)

Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)



$\Sigma_g^+, \Sigma_u^-, \Pi_u$: represent different potentials between heavy quark-antiquark

- Gap $\Lambda_{\mathrm{QCD}} \gg mv^2$ between low-lying states: quarkonium, hybrid, tetraquark etc.

- L_{mixing}: Mixing due to similar masses and same quantum-numbers.

  - Mixing at static potential level (avoided level crossing)
    Ex. Tetraquark-quarkonium mixing

  - Mixing between states suppressed by $O(1/m_Q)$
    Ex. Hybrid-quarkonium mixing

R. Oncala, J. Soto, Phys. Rev. D96 014004 (2017)

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

- BOEFT Lagrangian:

$$L_{\mathrm{BOEFT}} = \int d^3\boldsymbol{R} \int d^3\boldsymbol{r} \sum_{\kappa\lambda\lambda'} \mathrm{Tr}\Big\{ \Psi^{\dagger}_{\kappa\lambda}(\mathbf{r},\, \mathbf{R},\, t) \Big[i\partial_t\, \delta_{\lambda\lambda'} - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}(\theta,\phi)\, \frac{\boldsymbol{\nabla}^2_r}{m_Q} P^{i}_{\kappa\lambda'}(\theta,\phi) \Big] \Psi_{\kappa\lambda'}(\mathbf{r},\, \mathbf{R},\, t) \Big\}$$

LDF-quantum #: $\kappa = \{K^{PC}, f\}$ BO-quantum #: Λ^{σ}_{η} $\lambda = \pm\Lambda$

Projection vectors for $D_{\infty h}$: $P^{i}_{K\lambda}(\theta,\varphi) = D^{\lambda *}_{Ki}(0,\theta,\varphi)$

- **Good** quantum numbers:
  - ➢ BO-orbital momentum: $\boldsymbol{L} = \boldsymbol{L}_Q + \boldsymbol{K}$
  - ➢ Heavy quark Spin: $\boldsymbol{S}_Q$ (HQSS limit)
  - ➢ Total angular momentum: $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}_Q$

$\boldsymbol{K}$: LDF angular-momentum or spin

$\boldsymbol{L_Q}$: orbital-angular momentum of QQ or $Q\bar{Q}$ pair.

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

- **BO potentials: Potential between Q & $\bar{Q}$** due to LDF (light quarks, gluons).

Born-Oppenheimer (BO) potential:

$$V_{\kappa\lambda\lambda'}(r) = \boxed{E^{(0)}_{\kappa,|\lambda|}(r)}\delta_{\lambda\lambda'} + \boxed{\frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q}} + \dots ,$$

Brambilla, Lai, Segovia, Castellà, Phys. Rev. D. 101, (2020)

Static Energy

Spin-dependent potentials

- Static potentials with same LDF-quantum # κ degenerate in $r \to 0$ limit.
- Radial Coupled-Channel Schrödinger equation:

$$\sum_{\lambda}\left[-\frac{1}{m_Q r^2}\partial_r\, r^2\partial_r + \frac{1}{m_Q r^2}\boxed{M_{\lambda'\lambda}} + E^{(0)}_{\kappa,|\lambda|}(r)\delta_{\lambda\lambda'}\right]\psi^{(N)}_{\kappa\lambda}(r) = \mathcal{E}_N\, \psi^{(N)}_{\kappa\lambda'}(r)$$

1) Mixing term: Degenerate static potentials at short-distance corresponding to same LDF-quantum # κ

2) Mixing term: Degenerate static potentials at long or intermediate-distances corresponding to different LDF-quantum # κ **(Avoided Crossing)**

- BO potentials are inputs into Schroedinger equations.

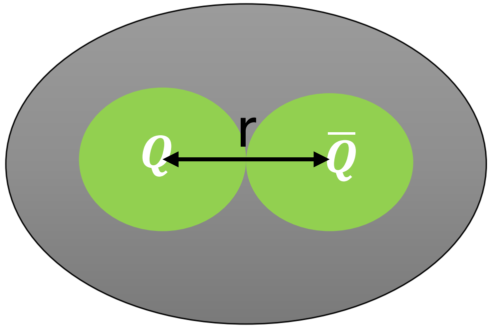
BOEFT: Potentials

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

LDF-quantum #: $\kappa = \{K^{PC}, f\}$

BO-quantum #: Λ_η^σ

$$V_s(r) = -\frac{4\alpha_s}{3r},\ V_o(r) = \frac{\alpha_s}{6r}$$
$$V_T(r) = -\frac{2\alpha_s}{3r},\ V_\Sigma(r) = \frac{\alpha_s}{3r}$$



$Q\bar{Q}$: $$E^{(0)}_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 + \cdots$$

$Q\bar{Q}X$: $$E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_o(r) + \Lambda_{H_\kappa} + b_{\Lambda_\eta^\sigma} r^2 + \cdots$$

QQX: $$E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_l(r) + \Lambda_{H_\kappa,l} + b_{\kappa\lambda,l}\, r^2 + \cdots \quad (l = T, \Sigma)$$

Short-distance ($r \to 0$)

$$\Lambda_{H_\kappa} = \lim_{T\to\infty} \frac{i}{T} \langle \text{vac}| H^a_\kappa(T/2,\ \boldsymbol{R})\, \phi^{ab}\, (T/2, -T/2)\, H^{a\dagger}_\kappa(-T/2,\ \boldsymbol{R})|\text{vac}\rangle$$

- Adjoint hadron mass for $Q\bar{Q}X$ states
- Triplet meson or baryon / Sextet meson or baryon mass for QQX states

Emergence of **diquark**: BOEFT operator for triplet meson coincides with good diquark in context of $T_{cc}(3875)^+$

Long-distance behavior of potential constrained by the BO-symmetry Λ_η^σ conservation.

BOEFT: Potentials

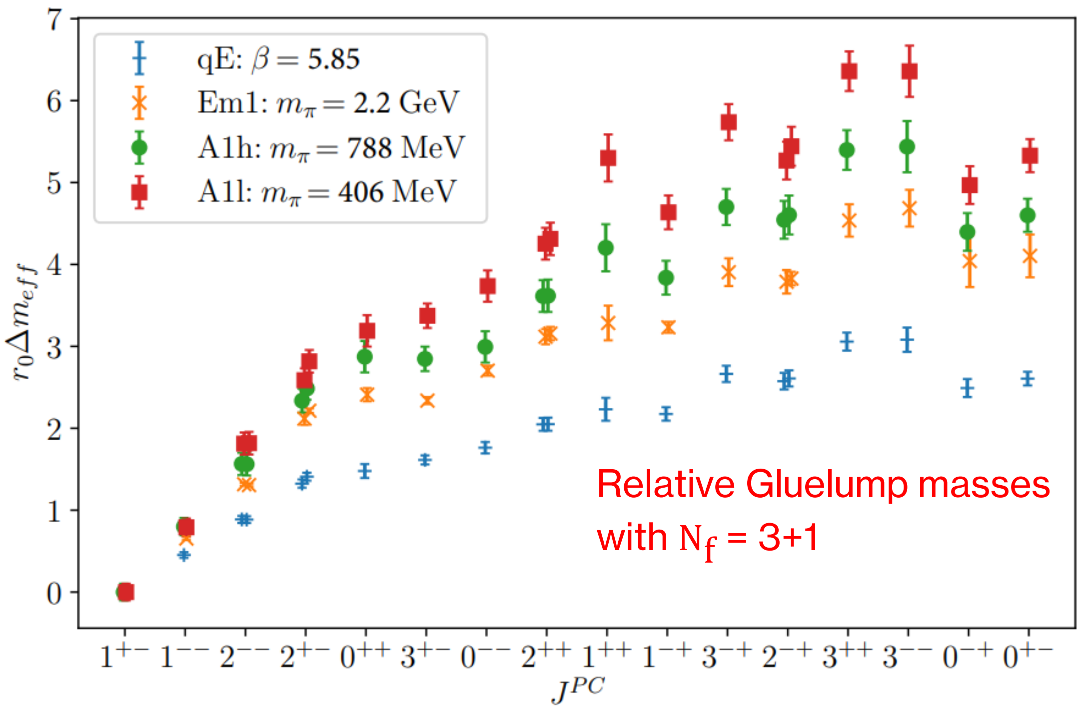
Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

$$\Lambda_{H_\kappa} = \lim_{T\to\infty} \frac{i}{T} \langle \text{vac}| H^a_\kappa(T/2,\, \boldsymbol{R})\, \phi^{ab}\,(T/2, -T/2)\, H^{a\dagger}_\kappa(-T/2,\, \boldsymbol{R})|\text{vac}\rangle$$

Foster, Michael (UKQCD) Phys. Rev. D 59 (1999)

Campbell, Jorysz, Michael Phys. Lett. B 167 (1986)

- Gluelump / adjoint meson or baryon mass for **$Q\bar{Q}X$** states
- Triplet meson or baryon / Sextet meson or baryon mass for **QQX** states



Höllwieser, Knechtli, Korzec, Peardon, Struckmeier, Andres Urea-Nino *PoS* LATTICE2024 (2025) 102

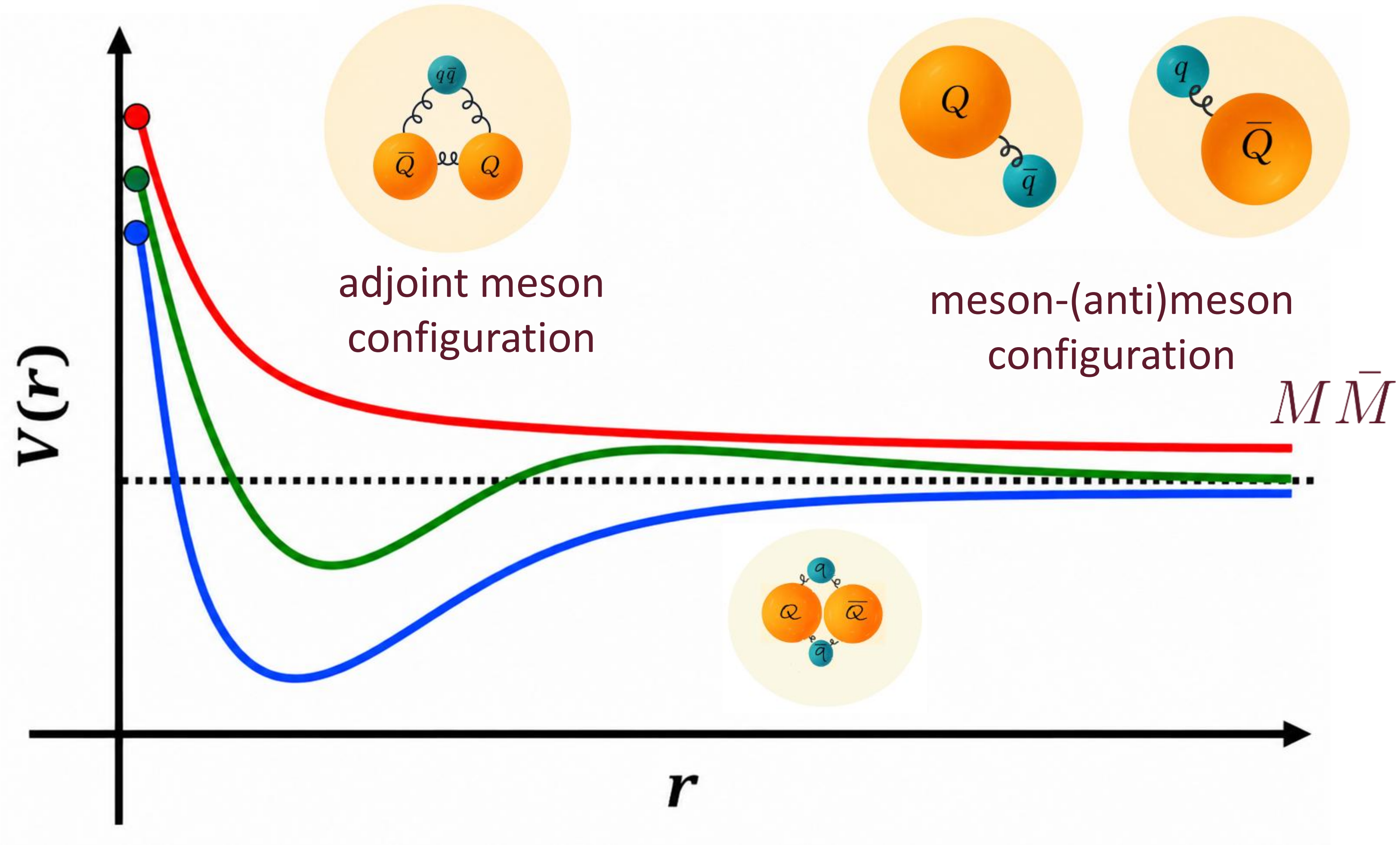
- Adjoint meson $(1^{--}$ & $0^{-+})$: quenched with valence quarks

Foster, Michael (UKQCD) Phys. Rev. D 59 (1999)

$$m_A\,(1^{--}) - m_G\,(1^{+-}) = -10(103)\ \text{MeV}$$
$$m_A\,(0^{-+}) - m_G\,(1^{+-}) = 34(161)\ \text{MeV}$$

- In context of **QQX states**, emergence of **diquark**: BOEFT operator for triplet meson coincides with good diquark but directly gauge-invariant !

Tetraquark potentials

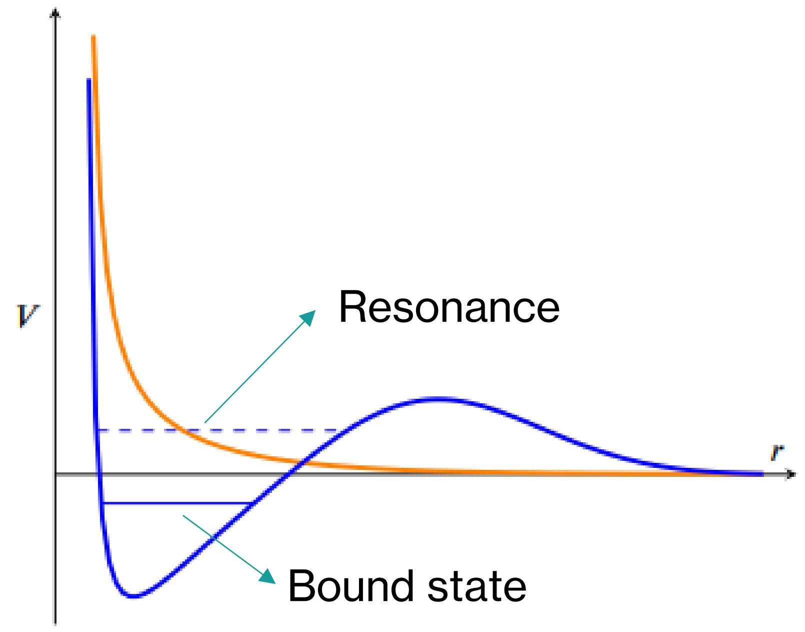


- Blue: may support bound states
- Green: may support both bound and resonance states
- Red: supports neither bound nor resonance states

No apriori assumption about the internal structure in BOEFT (compact, molecule, …)

These potentials predict a finite number of tetraquark bound states or resonances in the vicinity of meson-antimeson thresholds!

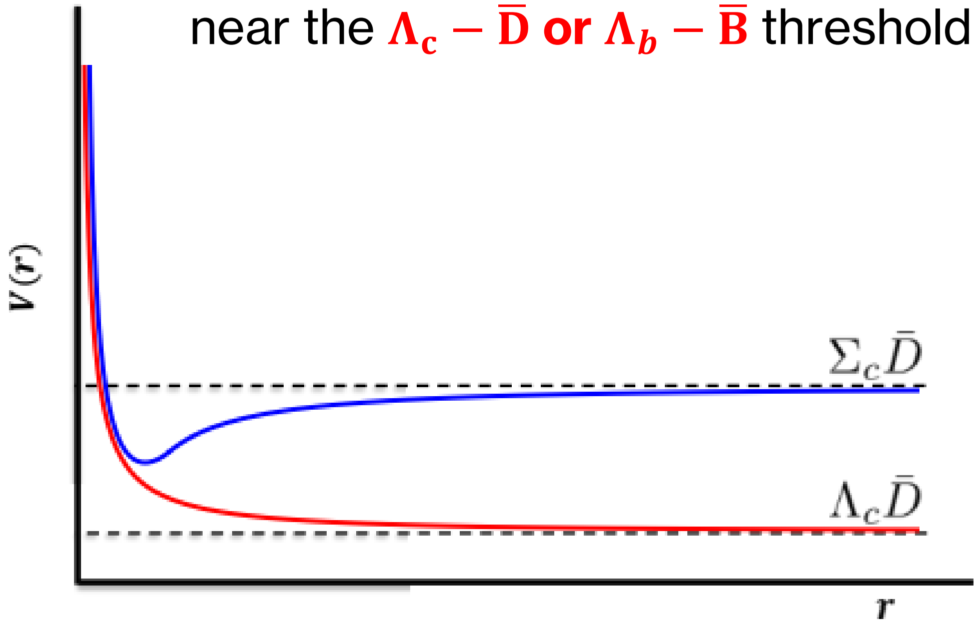
BO potentials



Braaten, Bruschini

Phys. Lett. B 863 (2025) 139386

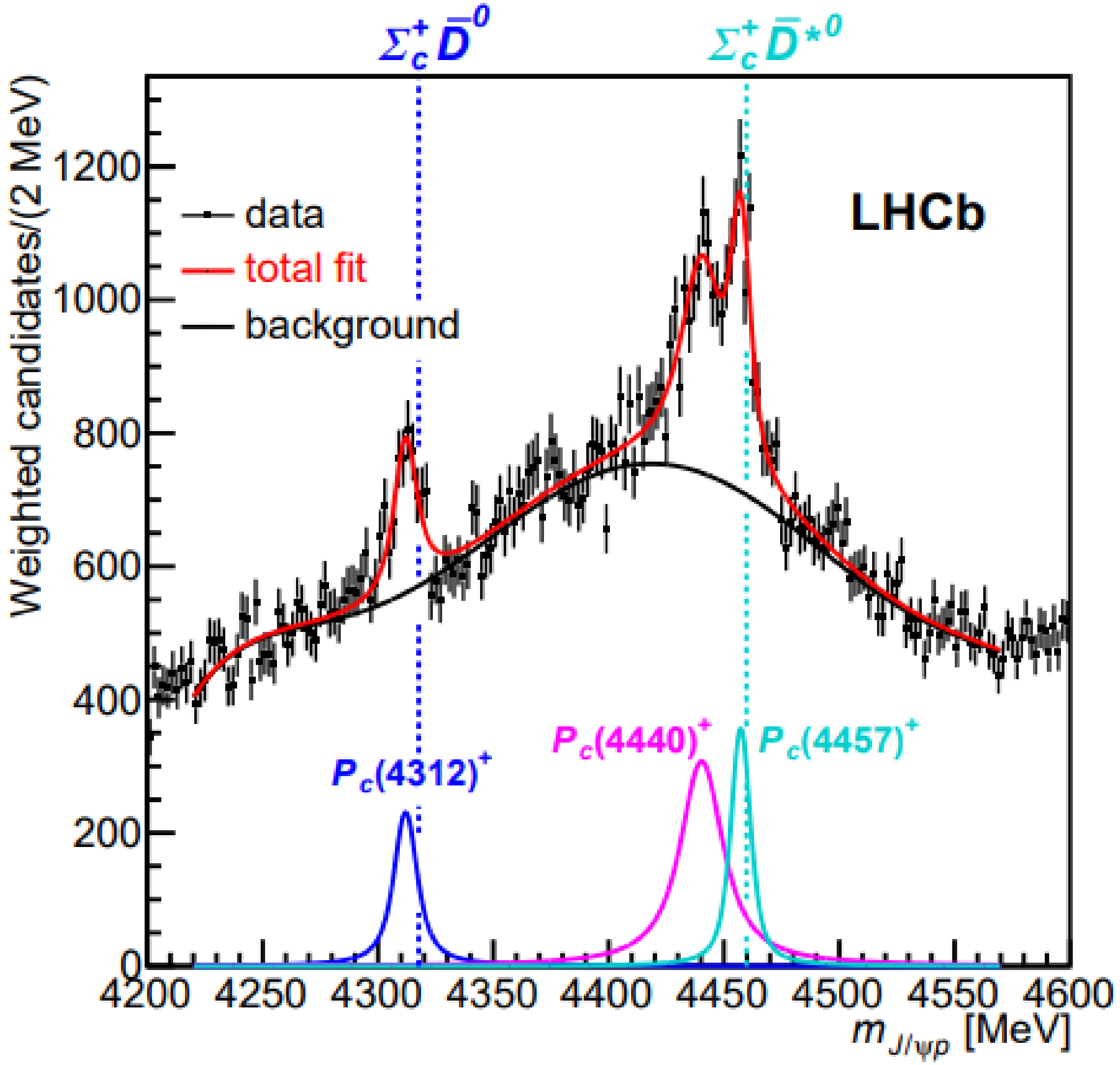
No pentaquark states observed near the $\mathbf{\Lambda_c - \bar{D}}$ **or** $\mathbf{\Lambda_b - \bar{B}}$ threshold



Intermediate region dictates if there are bound states or resonance !.

Pentaquarks

Pentaquark



LHCb, Phys. Rev. Lett. 122, (2019), 222001

Observed states

- **4** states with isospin $\boldsymbol{I = 1/2}$:

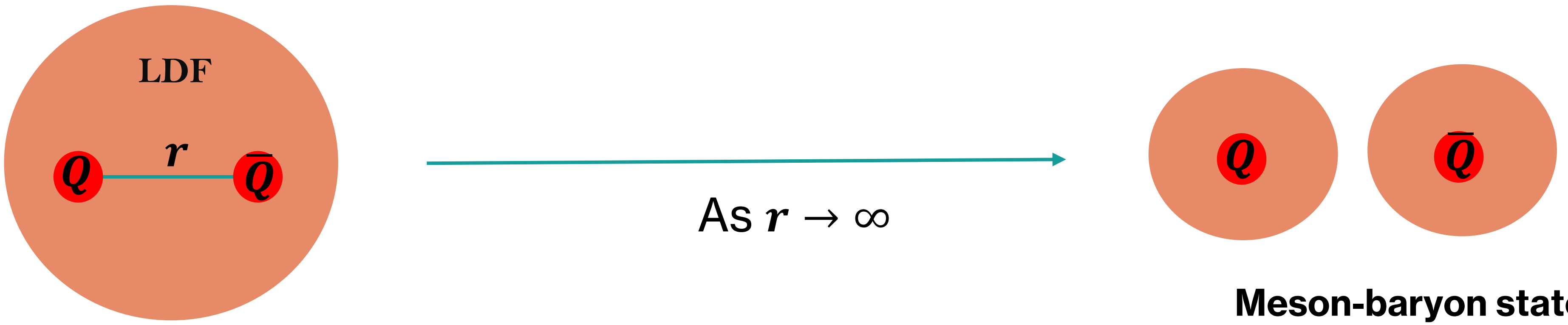
$P_{c\bar{c}}(4312)^+$, $P_{c\bar{c}}(4380)^+$, $P_{c\bar{c}}(4440)^+$, $P_{c\bar{c}}(4457)^+$

J^P quantum numbers not established

All pentaquark states seen by LHCb experiment

PDG 2025

BO potentials: Pentaquarks



Meson-baryon state

Consider $Q\bar{Q}qqq$ system:

BO-quantum # Λ_η^σ as $r \to 0$:

$Q\bar{Q}$ color state	Light spin k^P	BO quantum # $D_{\infty h}$
Octet **8**	$(1/2)^+$	$(1/2)_g$
	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$

There are **two** $k^P = (1/2)^+$ states

BO-quantum # Λ_η^σ for meson-baryon as $r \to \infty$

$k^P_{qq} \otimes k^P_q$	k^P	BO quantum # $D_{\infty h}$	
$0^+ \otimes (1/2)^+$	$(1/2)^+$	$(1/2)_g$	$\Lambda_c - \bar{D}$ threshold
$1^+ \otimes (1/2)^+$	$(1/2)^+$	$(1/2)_g$	$\Sigma_c - \bar{D}$ threshold
	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$	

BO-quantum # Λ_η^σ conservation: Potentials at small r should smoothly connect to meson-baryon threshold at large r !

Brambilla, AM, Vairo, arXiv 2508.13050

Brambilla, AM, Vairo, Phys. Rev. D 112 (2025), 114037

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Pentaquark

Lowest pentaquark multiplets:

$Q\bar{Q}$ color state	Light spin k^P	BO quantum # $D_{\infty h}$	l	J^P $\{S_Q = 0, S_Q = 1\}$
Octet	$(1/2)^+$	$(1/2)_g$	$1/2$	$\{1/2^-, (1/2, 3/2)^-\}$
8	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$	$3/2$	$\{3/2^-, (1/2, 3/2, 5/2)^-\}$

No lattice QCD results on adjoint baryon mass: $\Lambda_{(1/2)^+}$ and $\Lambda_{(3/2)^+}$

Treat $\Lambda_{(1/2)^+}$ and $\Lambda_{(3/2)^+}$ as **free parameter** to fix on $P_{c\bar{c}}$ spectrum.

Coupled-channel Equations $k^P = (1/2)^+$:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\, \partial_r + \frac{(l-1/2)(l+1/2)}{m_Q r^2} + V_{(1/2)_g}\right]\psi^{(N)}_{(1/2)^+} = \mathcal{E}_{1/2}\,\psi^{(N)}_{(1/2)^+}$$

$l = 1/2$

Coupled-channel Equations $k^P = (3/2)^+$:

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l-1)-\frac{9}{4} & -\sqrt{3l(l+1)-\frac{9}{4}} \\ -\sqrt{3l(l+1)-\frac{9}{4}} & l(l+1)-\frac{3}{4}\end{pmatrix} + \begin{pmatrix} V_{(1/2)'_g} & 0 \\ 0 & V_{(3/2)_g}\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{1/2} \\ \psi^{(N)}_{3/2}\end{pmatrix} = \mathcal{E}_{3/2}\begin{pmatrix}\psi^{(N)}_{1/2} \\ \psi^{(N)}_{3/2}\end{pmatrix}$$

$l = 3/2$

Brambilla, AM, Vairo, Phys. Rev. D 112 (2025), 114037

Pentaquark

Spin-corrections through spin-splittings in meson – baryon threshold :

$$V_{SS} = \underbrace{\frac{2\Delta_1^Q}{3} \boldsymbol{S}_1 \cdot \boldsymbol{K}_1}_{\text{Splitting in } \Sigma_c \text{ or } \Sigma_b \text{ baryon}} + \underbrace{\Delta_2^Q \boldsymbol{S}_2 \cdot \boldsymbol{K}_2}_{\text{Splitting in } D \text{ or } B \text{ meson}}$$

Splitting in Σ_c or Σ_b baryon

Splitting in D or B meson

$Q\bar{Q}$ color state	Light spin k^P	BO quantum # $D_{\infty h}$	l	J^P $\{S_Q=0, S_Q=1\}$
Octet	$(1/2)^+$	$(1/2)_g$	1/2	$\{1/2^-, (1/2, 3/2)^-\}$
8	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$	3/2	$\{3/2^-, (1/2, 3/2, 5/2)^-\}$

First-order perturbation theory in Vss:

$$E^{s=0}_{\frac{1}{2}\frac{1}{2}} = \mathcal{E}_{1/2}, \qquad E^{s=1}_{\frac{1}{2}\frac{1}{2}} = \mathcal{E}_{1/2} - \frac{4\Delta_1^Q}{9} + \frac{\Delta_2^Q}{6},$$

$$E^{s=1}_{\frac{1}{2}\frac{3}{2}} = \mathcal{E}_{1/2} + \frac{2\Delta_1^Q}{9} - \frac{\Delta_2^Q}{12},$$

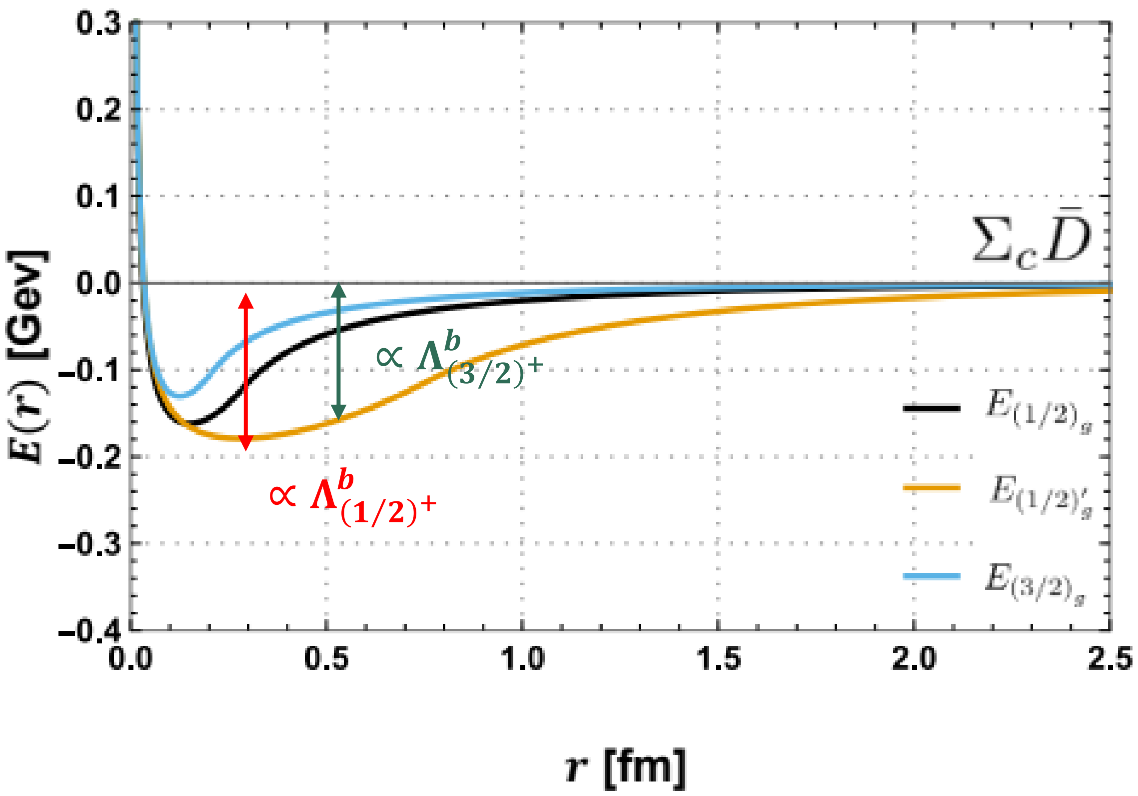
$$E^{s=0}_{\frac{3}{2}\frac{3}{2}} = \mathcal{E}_{3/2}, \qquad E^{s=1}_{\frac{3}{2}\frac{1}{2}} = \mathcal{E}_{3/2} - \frac{5\Delta_1^Q}{9} - \frac{5\Delta_2^Q}{12},$$

$$E^{s=1}_{\frac{3}{2}\frac{3}{2}} = \mathcal{E}_{3/2} - \frac{2\Delta_1^Q}{9} - \frac{\Delta_2^Q}{6}, \qquad E^{s=1}_{\frac{3}{2}\frac{5}{2}} = \mathcal{E}_{3/2} + \frac{\Delta_1^Q}{3} + \frac{\Delta_2^Q}{4}$$

If $\varepsilon_{1/2}$ and $\varepsilon_{3/2}$ nearly degenerate, use degenerate-perturbation theory for each J^P.

Brambilla, AM, Vairo, Phys. Rev. D 112 (2025), 114037

Pentaquark



$$\Lambda_\eta \in \left[(1/2)_g, \{(1/2)_g^{+\prime}, (3/2)_g\}\right]$$

$$E_{(\Lambda)_\eta}(r) = \begin{cases} V_o^{\mathrm{RS}}(r, \nu_f) + \Lambda_\kappa + A_{\Lambda_\eta} r^2 & r < R_{\Lambda_\eta} \\ F_{\Lambda_\eta} e^{-r/d}/r & r > R_{\Lambda_\eta} \end{cases}$$

One-pion exchange potential at large distance between meson and baryon

Can also consider two-pion exchange potentials.

$\Lambda^b_{(1/2)^+}$ and $\Lambda^b_{(3/2)^+}$ only changes within 100 MeV.

Adjoint baryon $(qqq)_8$ energy:

$\Lambda^b_{(1/2)^+} \approx \mathbf{1125}$ MeV, $\Lambda^b_{(3/2)^+} \approx \mathbf{1152}$ MeV (RS-scheme)

Brambilla, AM, Vairo, Phys. Rev. D 112 (2025), 114037

Pentaquark

Spin-corrections through spin-splittings in meson – baryon threshold :

3×3 submatrix for $J^P = (1/2)^-$ is given by

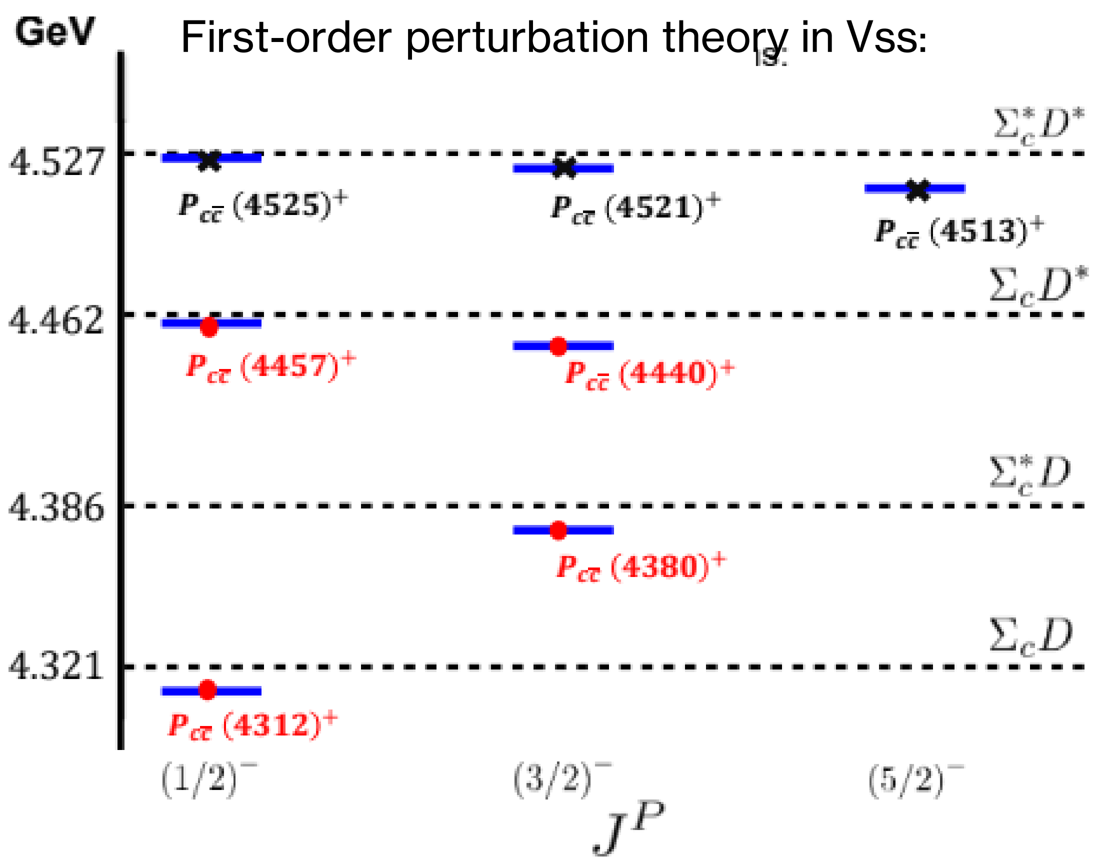
$$M_{(1/2)^-} = \begin{pmatrix} E^{s=0}_{\frac{1}{2}\frac{1}{2}} & -\frac{2\Delta_1^Q}{3\sqrt{3}} - \frac{\Delta_2^Q}{4\sqrt{3}} & -\frac{2\Delta_1^Q}{3\sqrt{6}} - \frac{\Delta_2^Q}{\sqrt{6}} \\ -\frac{2\Delta_1^Q}{3\sqrt{3}} - \frac{\Delta_2^Q}{4\sqrt{3}} & E^{s=1}_{\frac{1}{2}\frac{1}{2}} & \frac{\sqrt{2}\Delta_1^Q}{9} - \frac{\Delta_2^Q}{3\sqrt{2}} \\ -\frac{2\Delta_1^Q}{3\sqrt{6}} - \frac{\Delta_2^Q}{\sqrt{6}} & \frac{\sqrt{2}\Delta_1^Q}{9} - \frac{\Delta_2^Q}{3\sqrt{2}} & E^{s=1}_{\frac{3}{2}\frac{1}{2}} \end{pmatrix},$$

Similarly, for $J^P = (3/2)^-$

Fitting to 2 lowest pentaquark states $P_{c\bar{c}}(4312)^+$, $P_{c\bar{c}}(4380)^+$:

$$\mathcal{E}_{1/2} = -0.5 \text{ MeV and } \mathcal{E}_{3/2} = -14 \text{ MeV}$$

First-order perturbation theory in Vss:



- ✓ Compact pentaquark model: 10 states (some with parity = +)

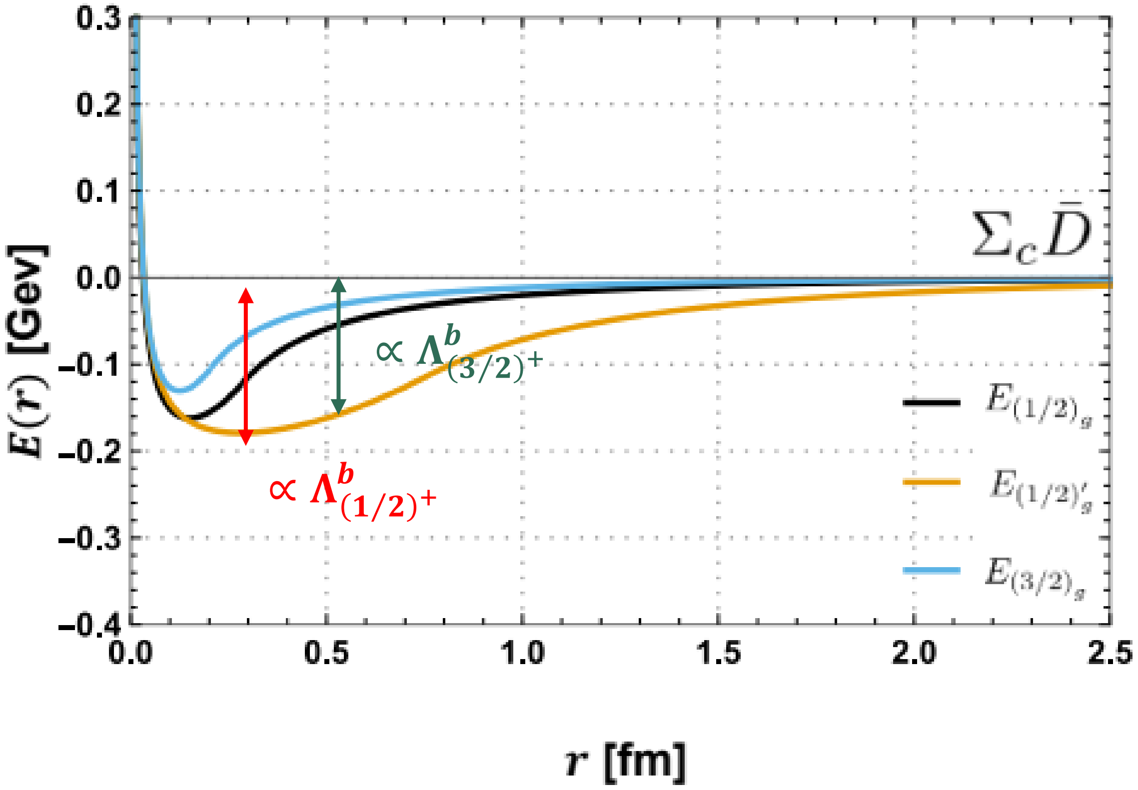
Maiani, Polosa, Riquer Phys. Lett. B, 749 (2015) Ali, Parkhomenko, Phys. Lett. B, 793 (2019)

- ✓ Molecular model: 7 states

Du, Baru, Guo, Hanhart, Meissner, Oller, Wang, JHEP 08, 157, Phys. Rev. Lett. 124, 072001 (2020)

Brambilla, AM, Vairo, Phys. Rev. D 112 (2025), 114037

Pentaquark: $P_{b\bar{b}}$



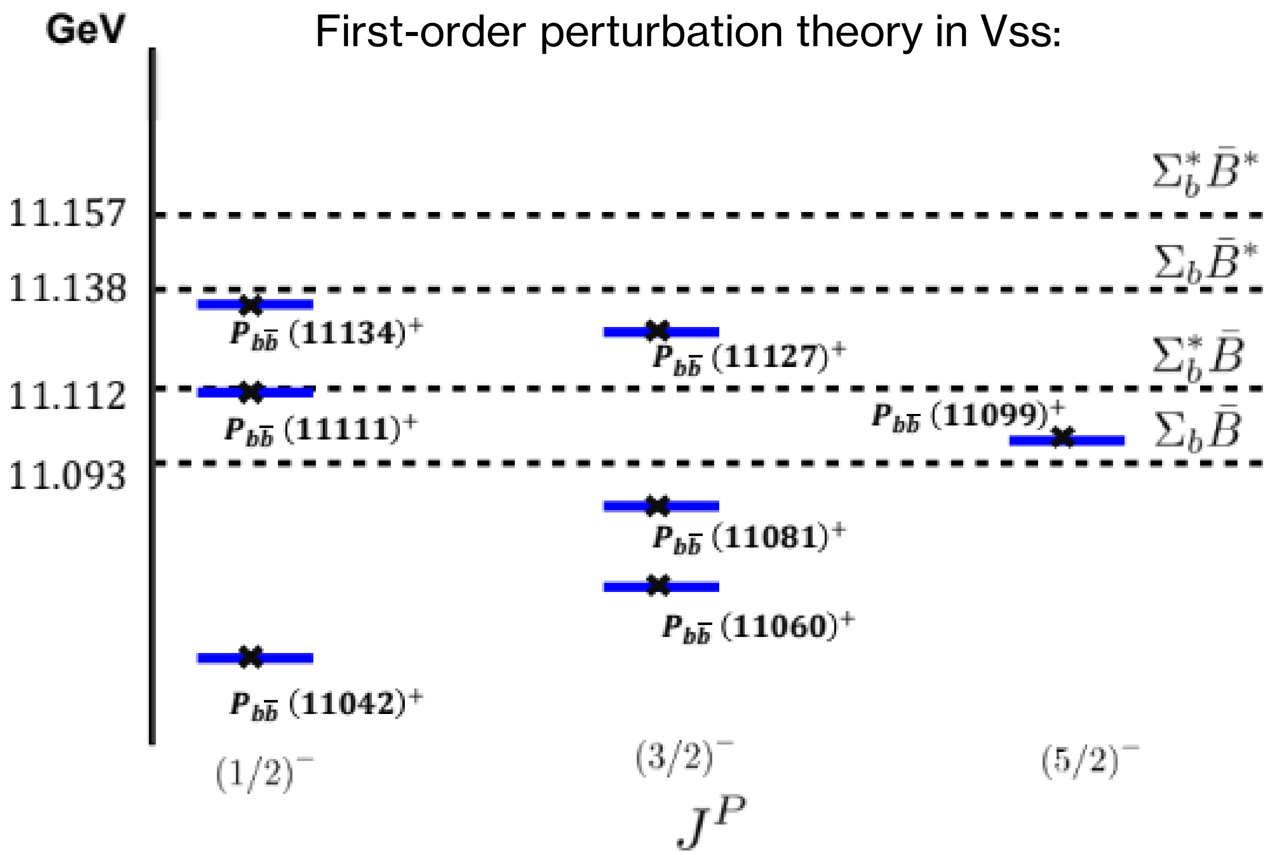
Adjoint baryon $(qqq)_8$ energy:

$\Lambda^b_{(1/2)^+} \approx$ **1125** MeV, $\Lambda^b_{(3/2)^+} \approx$ **1152** MeV (RS-scheme)

Spin-splitting in meson - baryon :

$$V_{SS} = \frac{2\Delta_1^Q}{3} \boldsymbol{S}_1 \cdot \boldsymbol{K}_1 + \Delta_2^Q \boldsymbol{S}_2 \cdot \boldsymbol{K}_2$$

First-order perturbation theory in Vss:



Brambilla, AM, Vairo, Phys. Rev. D 112 (2025), 114037

Pentaquark: Decays

- Semi-inclusive decays (Multipole Expansion): M1 transitions

$$P_{c\bar{c}} \to Q_n + Y$$

Q_n: low-lying quarkonium **Y**: light hadrons

- ✓ ΔE: Large energy difference ⇒ $\Delta E \equiv E_{P_{c\bar{c}}} - E_n \gtrsim 1$ **GeV**.
- ✓ Hierarchy of scales: $\Delta E \gg \Lambda_{QCD} \gg mv^2$ ($\Delta E \gg \Lambda_{QCD}$: Perturbative computation)
- ✓ Decays to J/ψ through **spin-flipping M1 transitions.**

Reported in PDG 2024:

$P_{c\bar{c}}$	Γ (total width)
$P_{c\bar{c}}(4312)$	10 ± 5 MeV
$P_{c\bar{c}}(4380)$	210 ± 90 MeV
$P_{c\bar{c}}(4440)$	21^{+10}_{-11} MeV
$P_{c\bar{c}}(4457)$	$6.4^{+6}_{-2.8}$ MeV

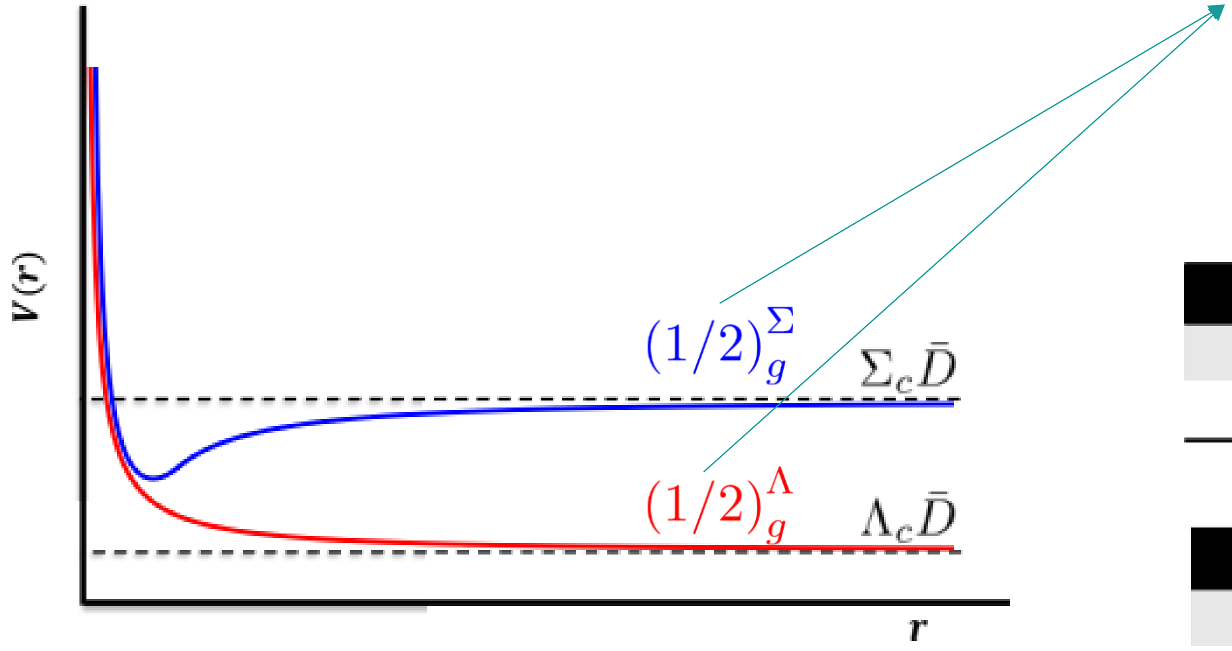
Semi-inclusive decays:

$P_{c\bar{c}}$	J^P	$\Gamma\,(P_{c\bar{c}} \to J/\psi + \cdots)$
$P_{c\bar{c}}(4312)$	$(1/2)^-$	$2^{+1\,+1}_{-1\,-1}$ MeV
$P_{c\bar{c}}(4457)$	$(1/2)^-$	$1^{+0.5\,+0.4}_{-0.3\,-0.3}$ MeV
$P_{c\bar{c}}(4380)$	$(3/2)^-$	$14^{+6\,+5}_{-3\,-4}$ MeV
$P_{c\bar{c}}(4440)$	$(3/2)^-$	$20^{+9\,+8}_{-5\,-6}$ MeV

- ➢ $P_{c\bar{c}}\left[J^P = (5/2)^-\right]$ only decays to η_c !!!

Pentaquark: Decays

- **Ratio of decays** to $\Lambda_c - \bar{D}$ and $\Lambda_c - \overline{D^*}$: $\frac{\Gamma(P_{c\bar{c}} \to \Lambda_c - \bar{D})}{\Gamma(P_{c\bar{c}} \to \Lambda_c - \overline{D^*})}$



- Decay happens through **transition amplitude $g_{(1/2)}^{\Sigma-\Lambda}$**
- $g_{(1/2)}^{\Sigma-\Lambda}$ currently unknown can be **determined by lattice QCD**

$J^P = (1/2)^-$

	$P_{c\bar{c}}(4312)^+$	$P_{c\bar{c}}(4440)^+$	$P_{c\bar{c}}(4507)^+$
$\Lambda_c \bar{D}$	0	3.52	5.48
$\Lambda_c \bar{D}^*$	3.59	3.24	2.17

$J^P = (3/2)^-$

	$P_{c\bar{c}}(4380)^+$	$P_{c\bar{c}}(4457)^+$	$P_{c\bar{c}}(4515)^+$
$\Lambda_c \bar{D}$	0	0	0
$\Lambda_c \bar{D}^*$	3.74	1.17	4.09

$J^P = (5/2)^-$

	$P_{c\bar{c}}(4526)^+$
$\Lambda_c \bar{D}$	0
$\Lambda_c \bar{D}^*$	0

- $P_{c\bar{c}}(4380)$, narrower state, compared to experimental broad width
- $P_{c\bar{c}}\left[J^P = (5/2)^-\right]$ decays to $\Lambda_c - \bar{D}$ and $\Lambda_c - \overline{D^*}$ in d-wave !!!

Pentaquark

Alasiri, Braaten, Bruschini, Phys. Lett. B 873 (2026) 140162

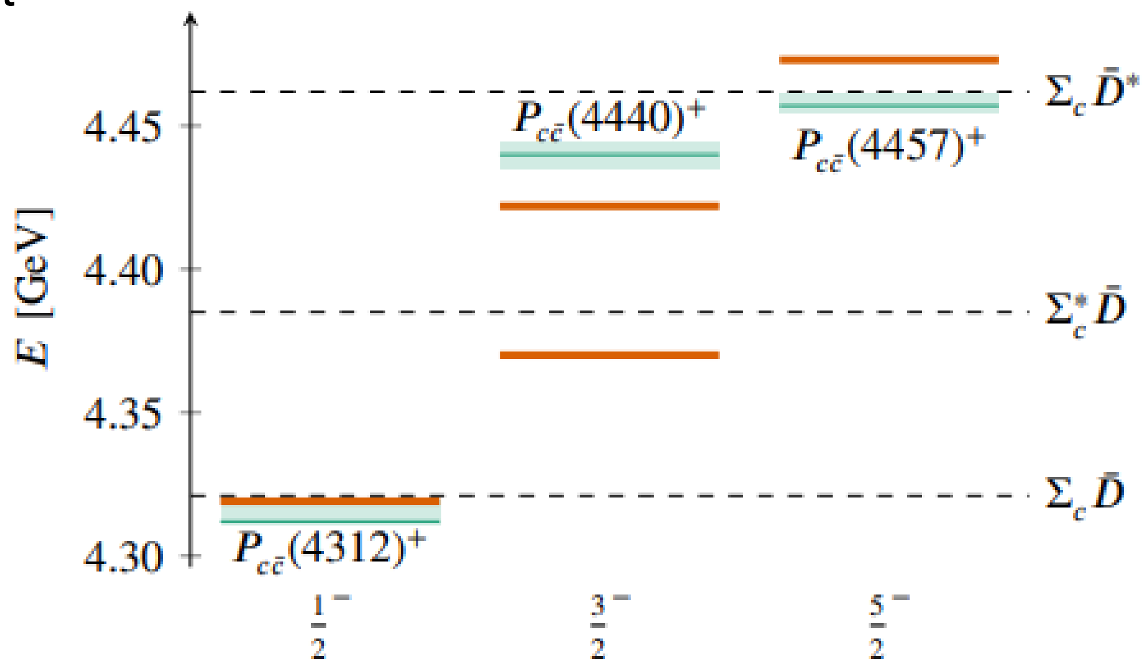
Spin-splitting in meson - baryon :

$$V_{SS} = \frac{2\Delta_1^Q}{3} \boldsymbol{S}_1 \cdot \boldsymbol{K}_1 + \Delta_2^Q \boldsymbol{S}_2 \cdot \boldsymbol{K}_2$$

First-order perturbation theory in Vss:

Assuming no bound state in adjoint baryon $\Lambda^b_{(1/2)^+}$ that connects to $\Sigma_c - \bar{D}$ threshold.

Only states in adjoint baryon 3/2



Adjoint baryon $(qqq)_8$ energy:

$\Lambda^b_{(3/2)^+} \approx \mathbf{1070}$ MeV (RS-scheme)

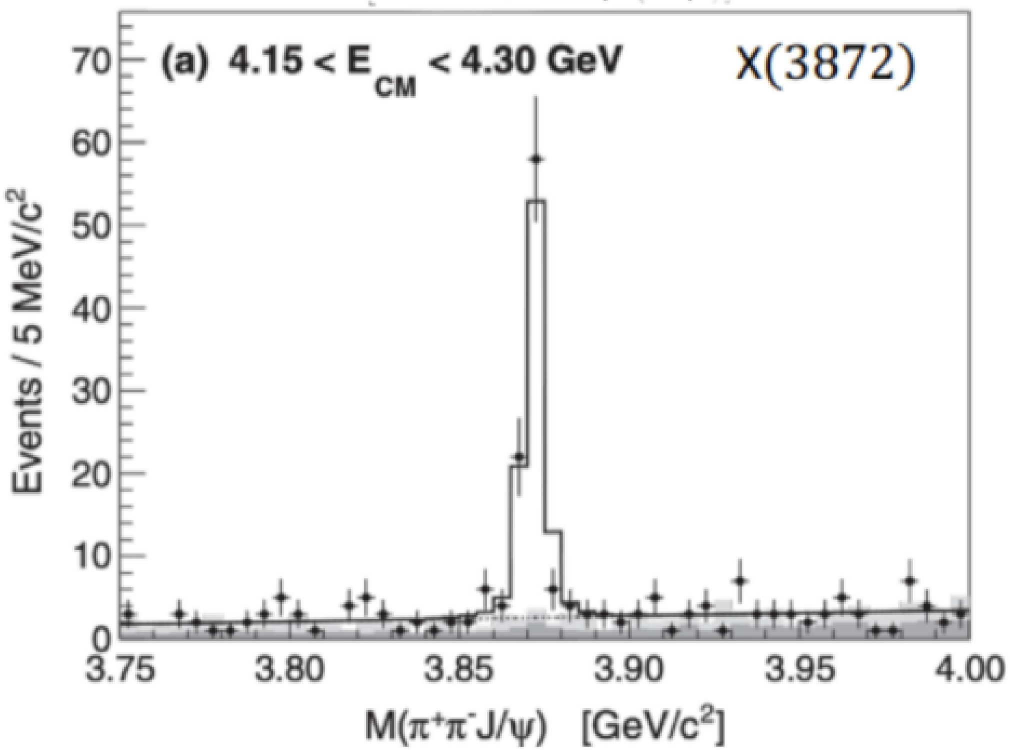
$\Lambda^b_{(1/2)^+}$: Much higher value

$\chi_{c1}(3872)$ &
String breaking corrections

$\chi_{c1}(3872)$

$e^+e^- \to \gamma X(3872)$; $X(3872) \to \pi^+\pi^- J/\psi$

BesIII coll. PRL 122 (2019) 202001

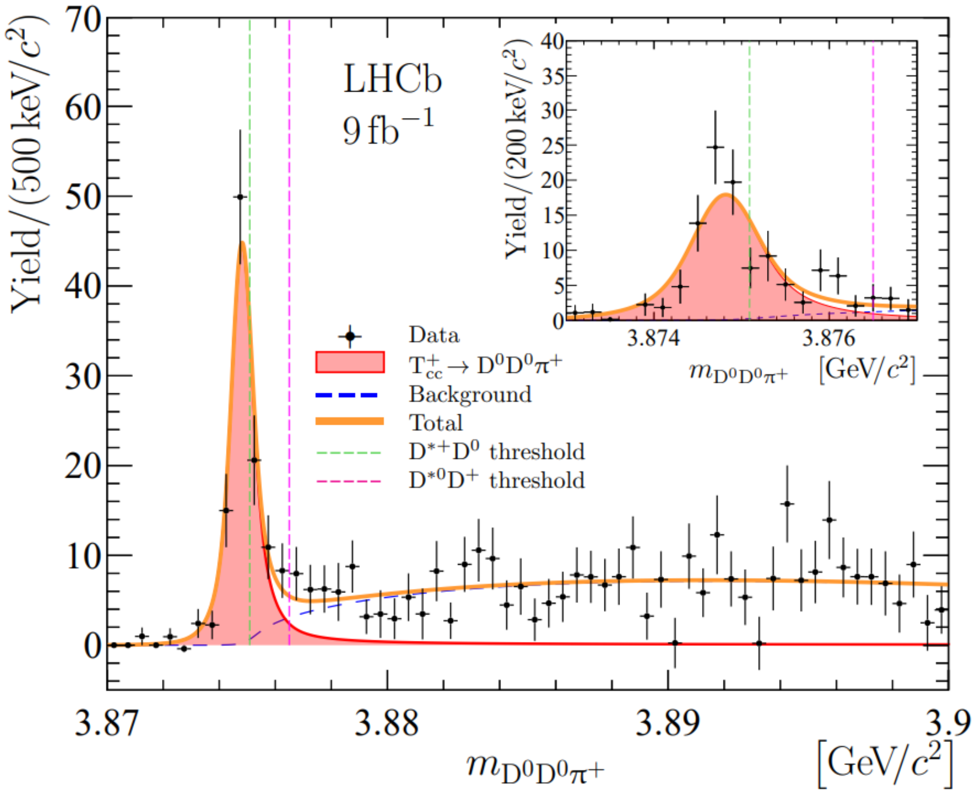


$$m_{\chi_{c1}(3872)} - (m_{D^{*0}} + m_{\bar{D}^0}) = -0.07 \pm 0.12 \text{ MeV}.$$

LHCb, JHEP 08 (2020) 123

- **Quantum numbers**: **$J^{PC}=1^{++}$ (Isospin=0)**

$T_{cc}^+(3875)$

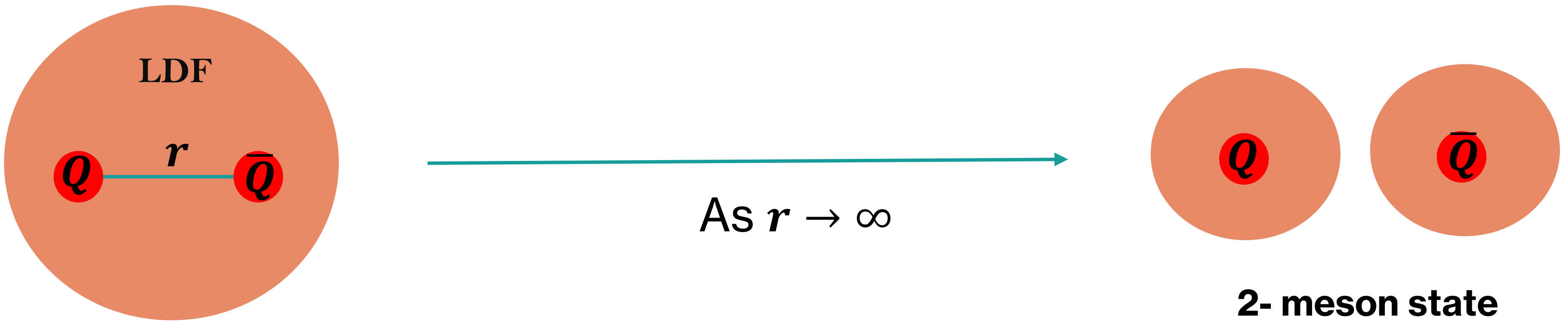


LHCb (Nature Phys. 18 (2022) 7, 751; Nature Comm. 13 (2022) 3351)

$$m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) = -0.27 \pm 0.06 \text{ MeV}.$$

- Consistent with **isoscalar** with **$J^P=1^+$**
- **Longest lived** Exotic particle: $\Gamma \sim 50$ keV

BO potentials: Tetraquarks



Consider $Q\bar{Q}q\bar{q}$ system:

BO-quantum # Λ_η^σ as $r \to 0$:

$Q\bar{Q}$ (color)	Light Spin K^{PC}	Λ_η^σ $(D_{\infty h})$
Octet	0^{-+}	Σ_u^-
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

BO-quantum # Λ_η^σ for meson-antimeson as $r \to \infty$

$K_{\bar{q}}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

s-wave+s-wave
Ex. $\mathbf{D\bar{D}}$ threshold

BO-quantum # Λ_η^σ conservation: potentials at small r should smoothly connect to meson pair threshold at large r !

Berwein, Brambilla, AM, Vairo,
Phys. Rev. D. 110, (2024), 094040

Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

BO potential

Bulava, Hoerz, Knechtli, Koch, Moir, Morningstar, Peardon, Phys. Lett. B. 793 (2019)

Bulava, Knechtli, Koch, Morningstar, Peardon, Phys. Lett. B. 854 (2024)

String breaking: quarkonium & meson-antimeson

$$H(r) = \begin{pmatrix} \hat{V}(r) & \sqrt{2}g_l & g_s \\ \sqrt{2}g_l & \hat{E}_1 & 0 \\ g_s & 0 & \hat{E}_2 \end{pmatrix}, \hat{V}(r) = \hat{V}_0 + \sigma r + \gamma/r$$

Eigenenergies: Adiabatic energy levels

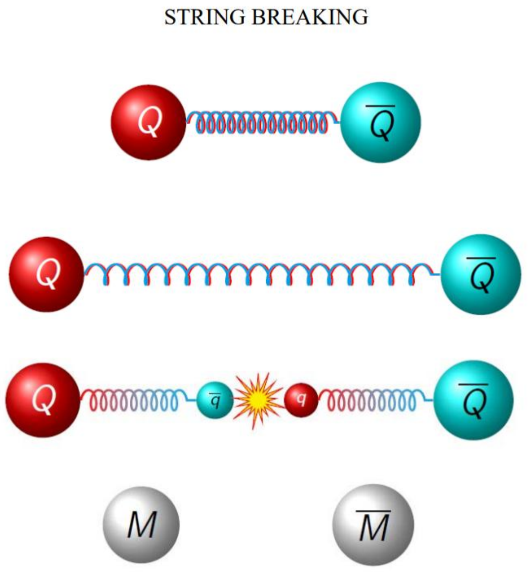
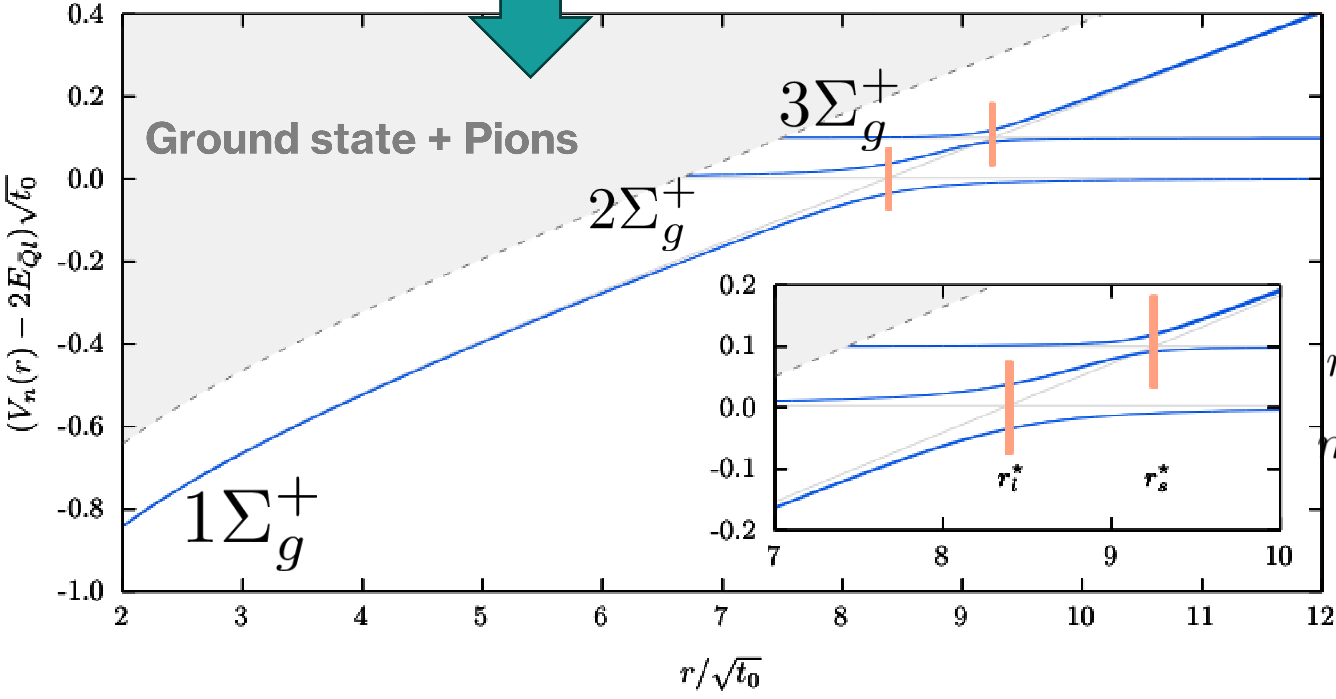


Figure from R. Bruschini talk



String breaking radius ≈ 1.22 fm

$m_\pi \approx 200 - 340$ MeV

$m_K \approx 440 - 480$ MeV

Avoided crossing between **degenerate** static potentials with **same** BO-quantum # Σ_g^+

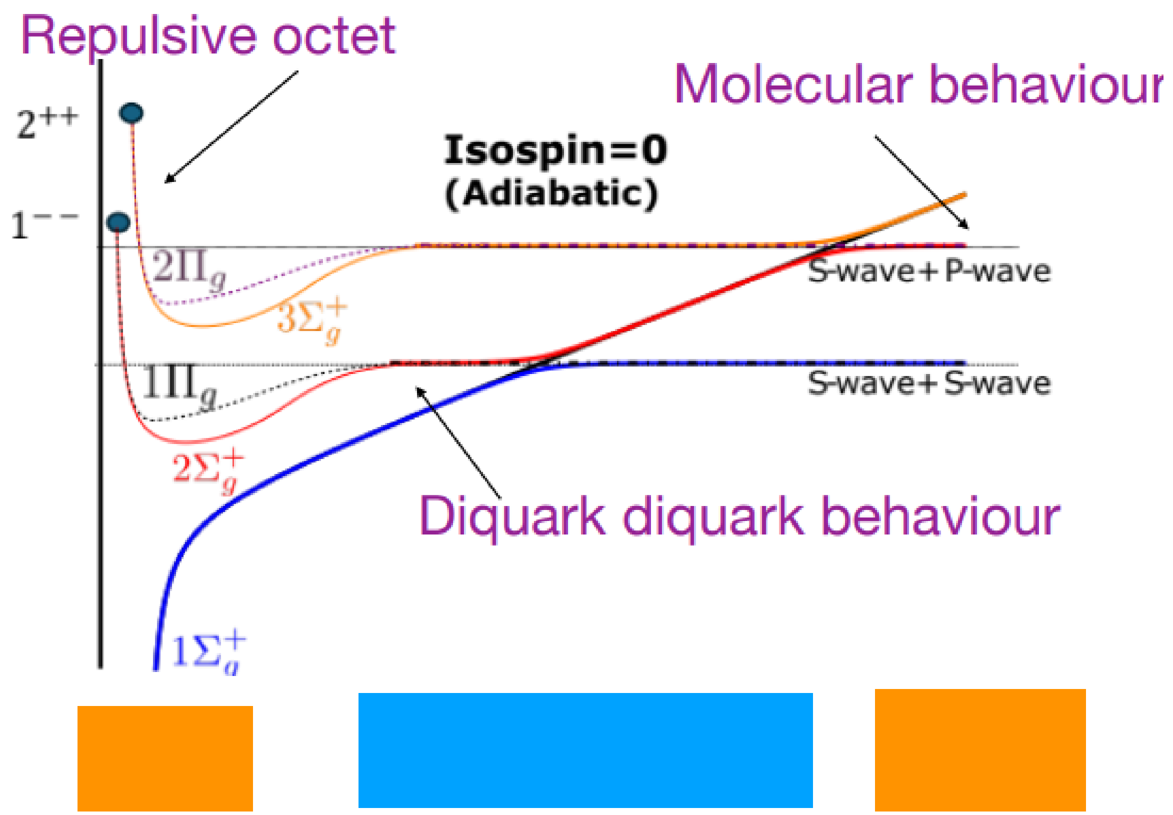
BO potentials

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

The BOEFT

Behavior of tetraquark static energy:

- ❑ Adjoint meson behavior at small r $(r \to 0)$
- ❑ Heavy meson pair threshold at large r $(r \to \infty)$



Fixed by symmetry And perturbation theory

to be calculated on the lattice

Fixed by symmetry

BOEFT Could subdue molecular and compact tetraquark pictures

BOEFT: $Q\bar{Q}q\bar{q}$ multiplets

Berwein, Brambilla, AM, Vairo,

Phys. Rev. D. 110, (2024), 094040

Isospin-0

$q\bar{q}$ spin k^{PC}	BO quantum # Λ_η^σ	l	J^{PC} $\{S_Q = 0, S_Q = 1\}$
1^{--}	$\Sigma_g^{+\prime}, \Pi_g$	1	$\{1^{+-}, (0,1,2)^{++}\}$
	$\Sigma_g^{+\prime}$	0	$\{0^{-+}, 1^{--}\}$
	Π_g	1	$\{1^{-+}, (0,1,2)^{--}\}$
	$\Sigma_g^{+\prime}, \Pi_g$	2	$\{2^{-+}, (1,2,3)^{--}\}$

Lowest multiplet with $\chi_{c1}(3872)$ **J^{PC}=1^{++}**

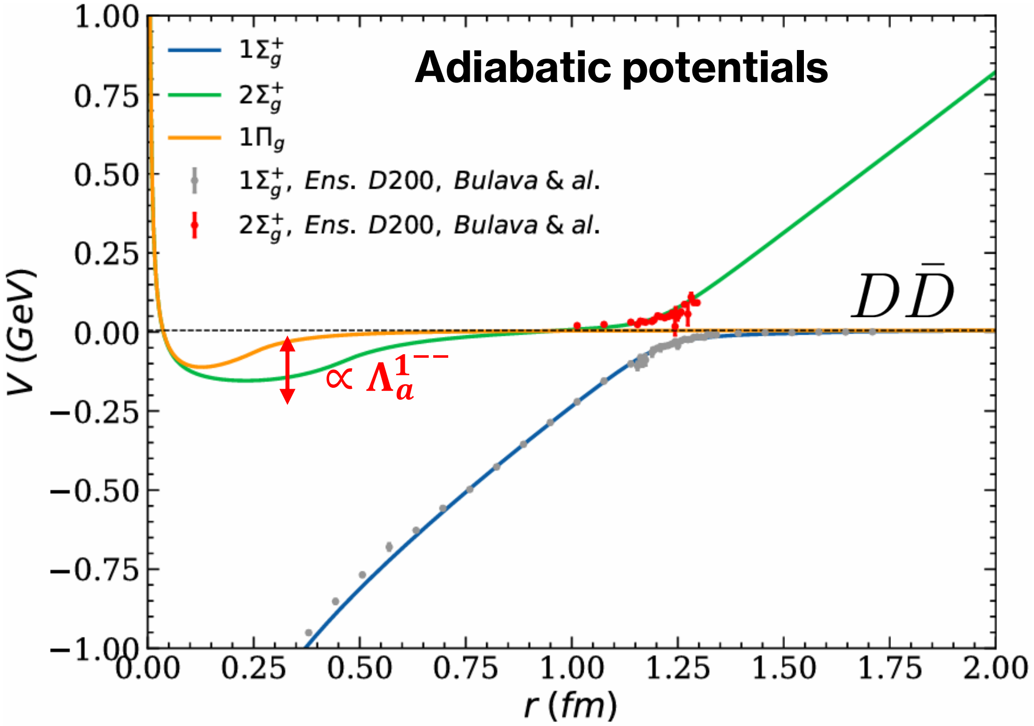
1^{--} Adjoint meson mass unknown.
Fix it to reproduce $\chi_{c1}(3872)$.

Coupled-channel Equations:

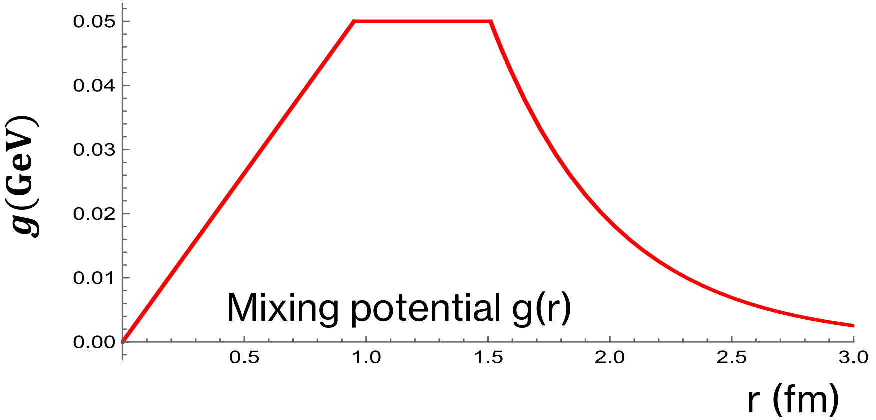
$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix} \psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi \end{pmatrix} = \mathcal{E}\begin{pmatrix} \psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi \end{pmatrix}$$

Brambilla, AM, Scirpa, Vairo

Phys. Rev. Lett. 135 (2025), 131902



Bulava et al Phys. Lett. B. 854 (2024)



$\chi_{c1}(3872)$

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Brambilla, AM, Scirpa, Vairo Phys. Rev. Lett. 135 (2025), 131902

Coupled-channel Equations:

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix}$$

$l = 1$

Results:

1) 2P Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8\ \%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\ \%$, $|\psi_\Pi|^2 \approx 54\ \%$
3) Radius ~ 15 fm.
4) Deeper state in bottom sector: 15 MeV below spin-isospin averaged $B\bar{B}$ threshold.

Adjoint $(q\bar{q})_8$: $\Lambda_a^{1^{--}} \approx -228$ MeV
$\Lambda_a^{1^{--}} \approx 915$ MeV (pole mass scheme)

$\chi_{c1}\,(3872)$

Brambilla, AM, Scirpa, Vairo
Phys. Rev. Lett. 135 (2025), 131902

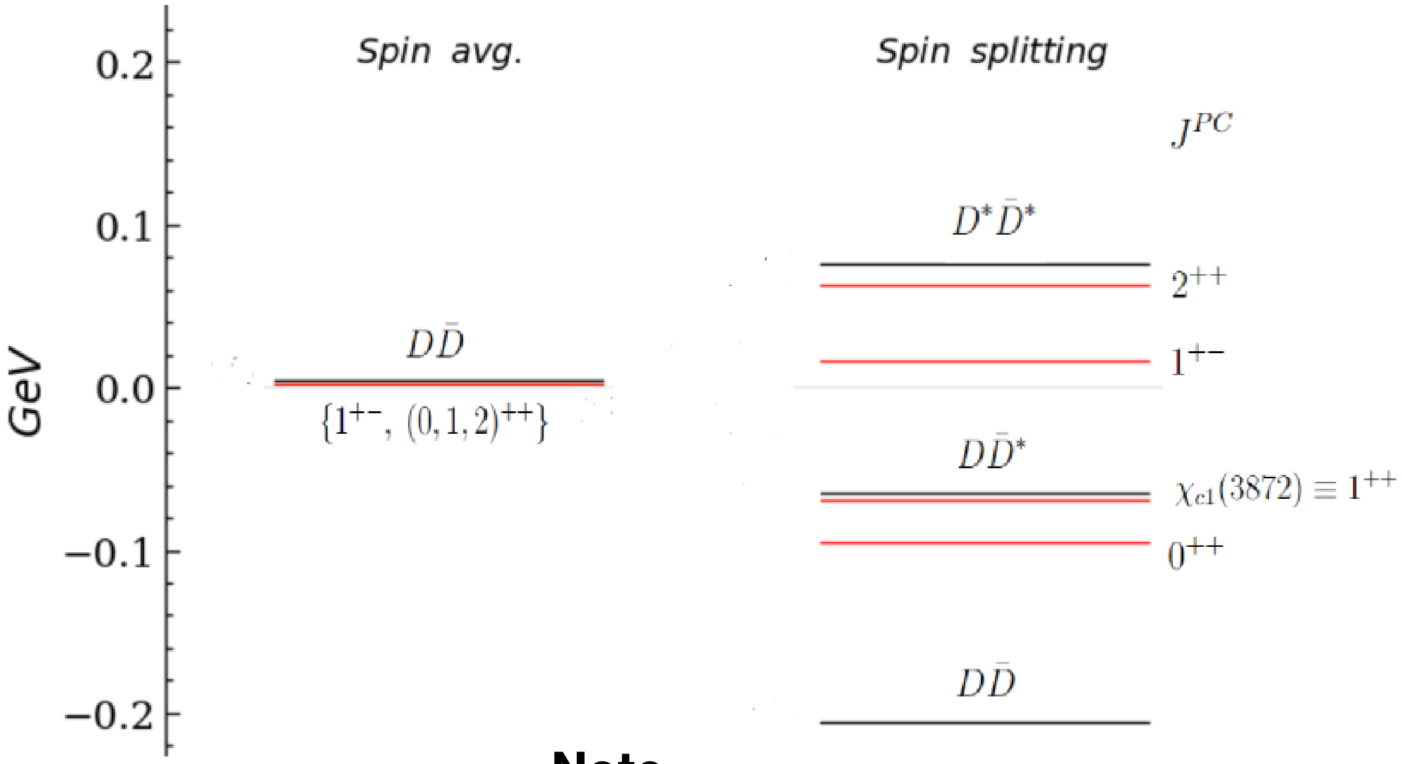
Spin-splitting in meson pairs:

$$V_{SS} = \delta_Q\,(\boldsymbol{S}_1.\boldsymbol{K}_1 + \boldsymbol{S}_2.\boldsymbol{K}_2)$$

First-order perturbation theory in Vss:

Adjoint $(q\bar{q})_8$ energy: $\Lambda_a^{1^{--}} \approx \mathbf{915}$ MeV:
No bound states in higher multiplets

Multiplet T_1^1: $\{1^{+-}, (0,1,2)^{++}\}$



1^{++} state: Identified with $\chi_{c1}(3872)$

1^{+-} state: Mass around 3.957 (11) GeV.

2^{++} state: Mass around 4.004 (14) GeV.

0^{++} state: Mass around 3.846 (11) GeV.

Note:

$\boldsymbol{\chi_{c1}(3872)}$ and $\boldsymbol{\chi_{c1}(2P)}$ both with **J^{PC}=1^{++}** are two distinct states.

$\boldsymbol{\chi_{c1}(2P) = \chi_{c1}(4010)}$ seen recently by LHCb

LHCb, Phys. Rev. Lett. 133, 131902 (2024)

$\chi_{c1}(3872)$

Brambilla, AM, Scirpa, Vairo

Phys. Rev. Lett. 135 (2025), 131902

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8\,\%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\,\%$, $|\psi_\Pi|^2 \approx 54\,\%$
3) Radius > 15 fm.

We **naturally** get 8 % quarkonium component in $\chi_{c1}(3872)$ due to **avoided level crossing**

Radiative decays:

$$\mathcal{R}_{\gamma\psi} = \frac{\Gamma_{\chi_{c1}(3872)\to\gamma\psi(2s)}}{\Gamma_{\chi_{c1}(3872)\to\gamma J\psi}},$$

Our estimate: $\boldsymbol{R_{\gamma\psi} = 2.99 \pm 2.36}$ (assuming only through $\boldsymbol{\chi_{c1}(2P)}$ **component**)

LHCb: $\boldsymbol{R_{\gamma\psi} = 1.67 \pm 0.25}$ Aaij et al arXiv: 2406.17006

Compositeness:

Z=0 purely molecular state, Z=1 purely compact state

Weinberg, Phys. Rev. 137, B672 (1965)

BES III: $\boldsymbol{Z = 0.18^{+0.20}_{-0.23}}$

Ablikim et al. Phys. Rev. Lett 132, 151903 (2024)

EMPPR: 0.052 < Z < 0.14

Esposito, Maiani, Pilloni, Polosa, Riquer

Phys. Rev. D 105, L031503 (2022)

Our estimate

$Z = 0.08$

assuming quarkonium as compact component

Lattice QCD:

$c\bar{c}$ operator along with $D\bar{D}^*$ relevant for $\chi_{c1}(3872)$ signal

Padmanath, Lang, Prelovsek Phys. Rev. D 92, 034501 (2015)

Prelovsek and Leskovec Phys. Rev. Lett 111, 192001 (2013)

X_b

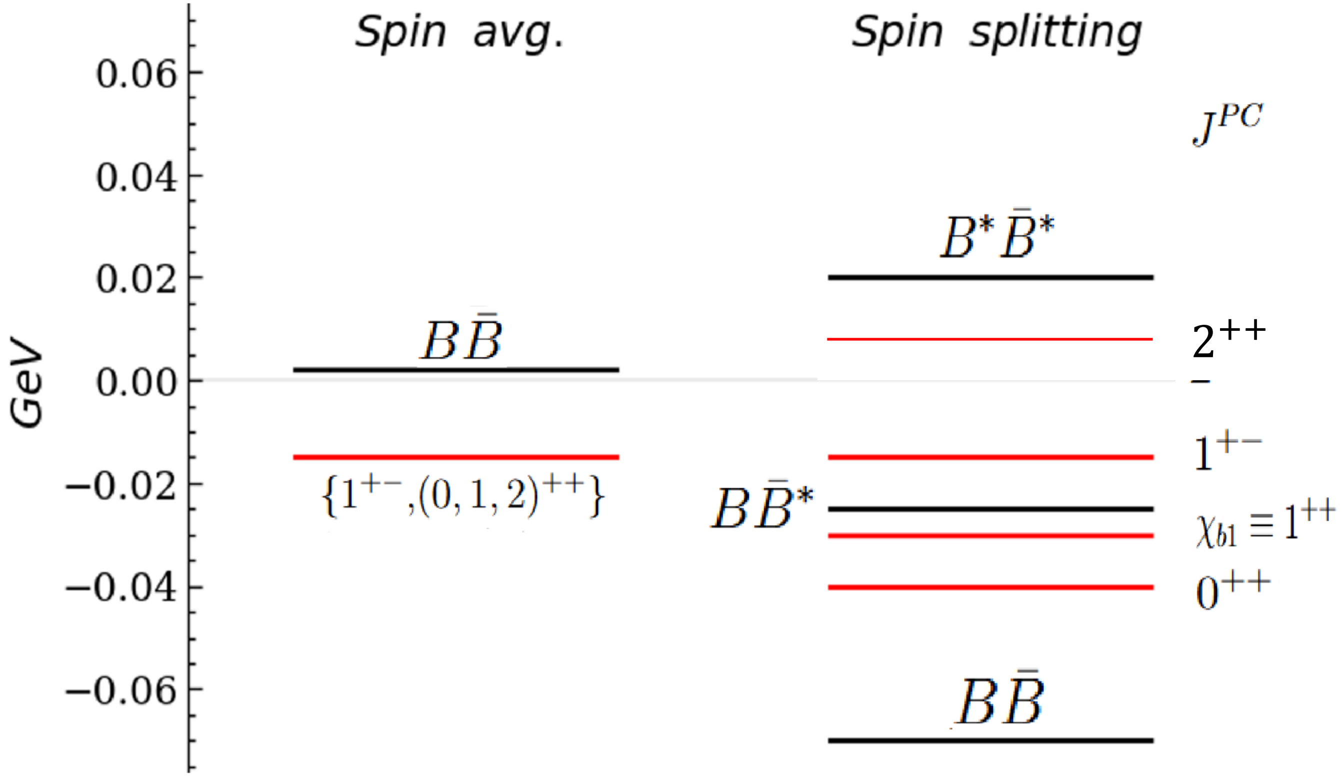
Brambilla, AM, Scirpa, Vairo

Phys. Rev. Lett. 135 (2025), 131902

Results with
adjoint meson energy $\approx$ **915** MeV

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 1.5\ \%$
2) Tetraquark percentage:
$|\psi_{\Sigma'}|^2 \approx 45.4\ \%,\ |\psi_\Pi|^2 \approx\ 53.1\ \%$

First-order perturbation theory in Vss:



Multiplet T_1^1: $\{1^{+-}, (0, 1, 2)^{++}\}$

1^{++}: identified with X_b: Mass around 10.595 GeV

1^{+-} state: Mass around 10.612 GeV.

2^{++} state: Mass around 10.635 GeV.

0^{++} state: Mass around 10.576 GeV.

String-breaking corrections

Brambilla, Mohapatra, TS, Vairo arxiv: 2604.12603

Charmonium spectrum below the $D\bar{D}$ threshold

$nl\ (c\bar{c})$	$M^{th.}(M^{exp.})$ (MeV)	% Σ_g^+	% $\Sigma_g^{+\prime}$	% Π_g	$\Delta E_{gap}^{D\bar{D}}$ (MeV)	$\Delta E_{nl}^{str.br.}$ (MeV)
$1S$	3127.6 (3068.7)	99.9	0.1		−818.4	−0.4
$2S$	3706.3 (3674.0)	99.0	1.0		−239.7	−3.8
$1P$	3536.9 (3525.3)	99.7	0.2	0.1	−409.1	−1.6
$\chi_{c1}(3872)$	3945.9	8.6	37.4	54.0	−0.1	}
$2P$	3989.0	...	...	...	...	...
$1D$	3819.0	98.6	1.2	0.2	−127.0	−4.1

String-breaking corrections

$$\Delta E_{nl}^{str.br.} = \mathcal{E}_{nl} - E_{nl}^{V_{\Sigma_g^+ - \Sigma_g{}'} \to 0}$$

Mass difference with and without considering the quarkonium-threshold mixing

$\chi_{c1}(3872)$ and $\chi_{c1}(2P)$ are two different states in our calculation

Brambilla, Bruschini, Mohapatra, Peng, TS, in preparation

- String-breaking corrections considerably smaller than phenomenological models predictions (3P0, CCCM) $\mathcal{O}(10 - 100)MeV$
- Conventional quarkonium states far below threshold have marginal tetraquark components (few %)

M.R.Pennington et al. ,Phys.Rev.D76, 077502 (2007), T.Barnes et al. Phys.Rev.C77,055206 (2008), E. Eichten et al., Phys. Rev. D 17, 3090 (1978), Phys. Rev. D 21, 203 (1980),

String-breaking corrections

Brambilla, Mohapatra, TS, Vairo arxiv: 2604.12603

Bottomonium spectrum below the $B\bar{B}$ threshold

$nl\ (b\bar{b})$	$M^{th.} (M^{exp.})$ (MeV)	% Σ_g^+	% $\Sigma_g^{+\prime}$	% Π_g	$E_{gap}^{B\bar{B}}$ (MeV)	$\Delta E_{nl}^{str.br.}$ (MeV)
$1S$	9444.9 (9445.0)	100	-		-1182.1	-0.1
$2S$	9987.7 (10017.3)	99.9	0.01		-639.2	-1.0
$3S$	10327.7	99.1	0.9		-299.3	-3.5
$4S$	10593.6	78.8	21.2		-33.4	-16.0
$1P$	9882.9 (9899.7)	99.9	0.1	-	-744.1	-0.5
$2P$	10235.0 (10260.2)	99.5	0.5	-	-320.0	-2.3
$3P$	10516.2	95.8	4.1	0.1	-110.8	-7.6
X_b	10613.4	1.5	44.9	53.6	-13.5	
$1D$	10124.4	99.8	0.2	-	-502.6	-1.2
$2D$	10419.0	98.7	1.3	-	-208.0	-4.2

String-breaking corrections

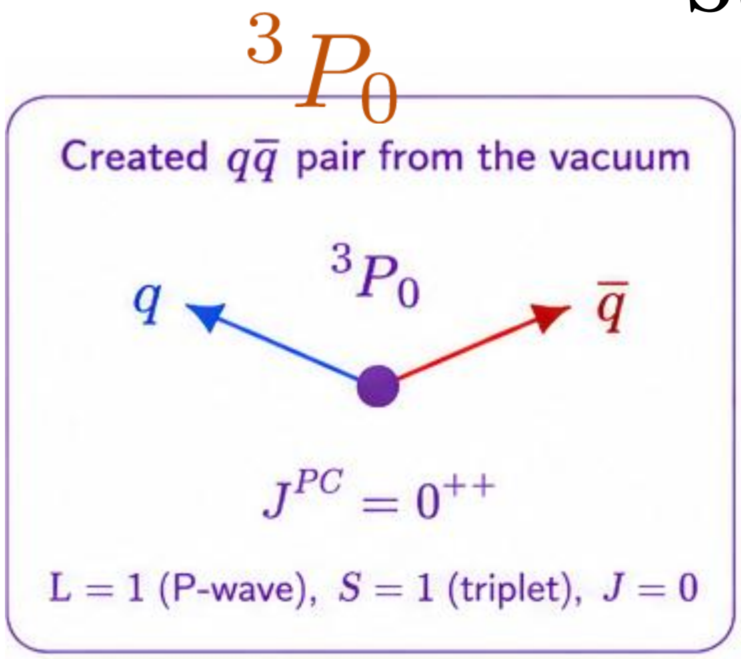
$$\Delta E_{nl}^{str.br.} = \mathcal{E}_{nl} - E_{nl}^{V_{\Sigma_g^+ - \Sigma_g{}'} \to 0}$$

Mass difference with and without considering the quarkonium-threshold mixing

- Quarkonium 4S state acquire a significant tetraquark component $\sim 20\%$ and receive sizable string-breaking corrections $\sim O(20)\ MeV$

String-breaking corrections

Brambilla, Mohapatra, TS, Vairo arxiv: 2604.12603



BOEFT mixing potential expressed i.t.o. 3P0 parameters

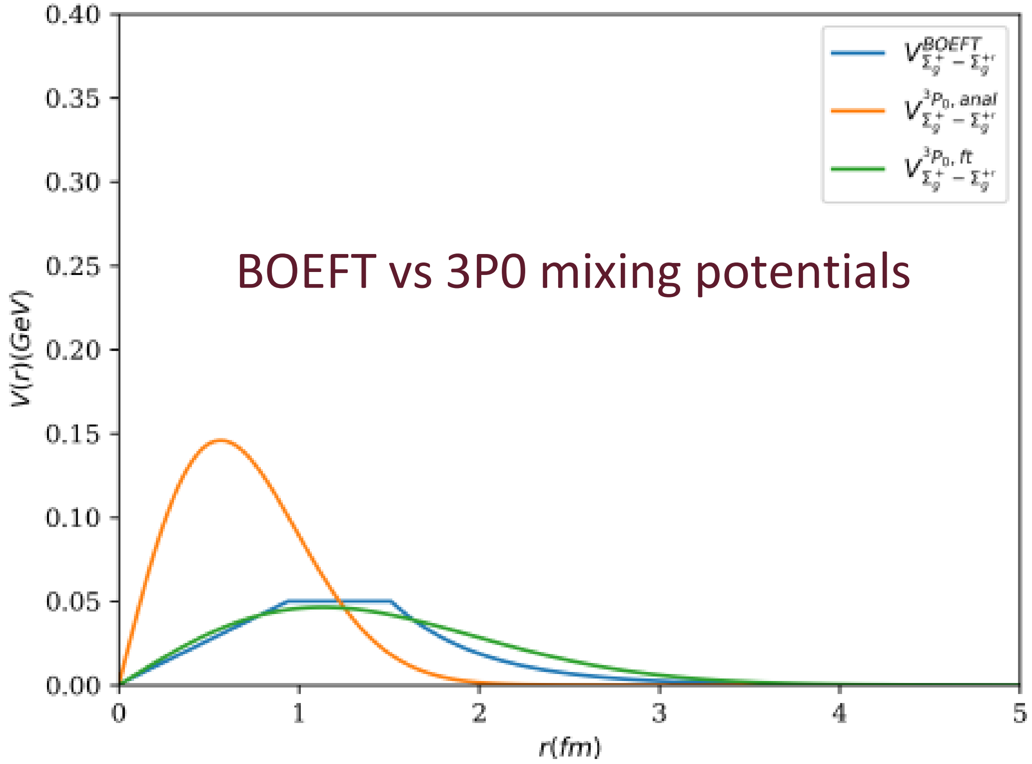
$$V^{^3P_0,\,anal.}_{\Sigma_g^+ - \Sigma_g^{+\prime}} = \frac{\sqrt{\pi}}{8\sqrt{2}L^2}\gamma r e^{-r^2/4L^2}$$

assuming heavy-light mesons gaussian wavefunctions

Free parameters

- L: heavy-light meson size
- γ : quark-pair creation constant

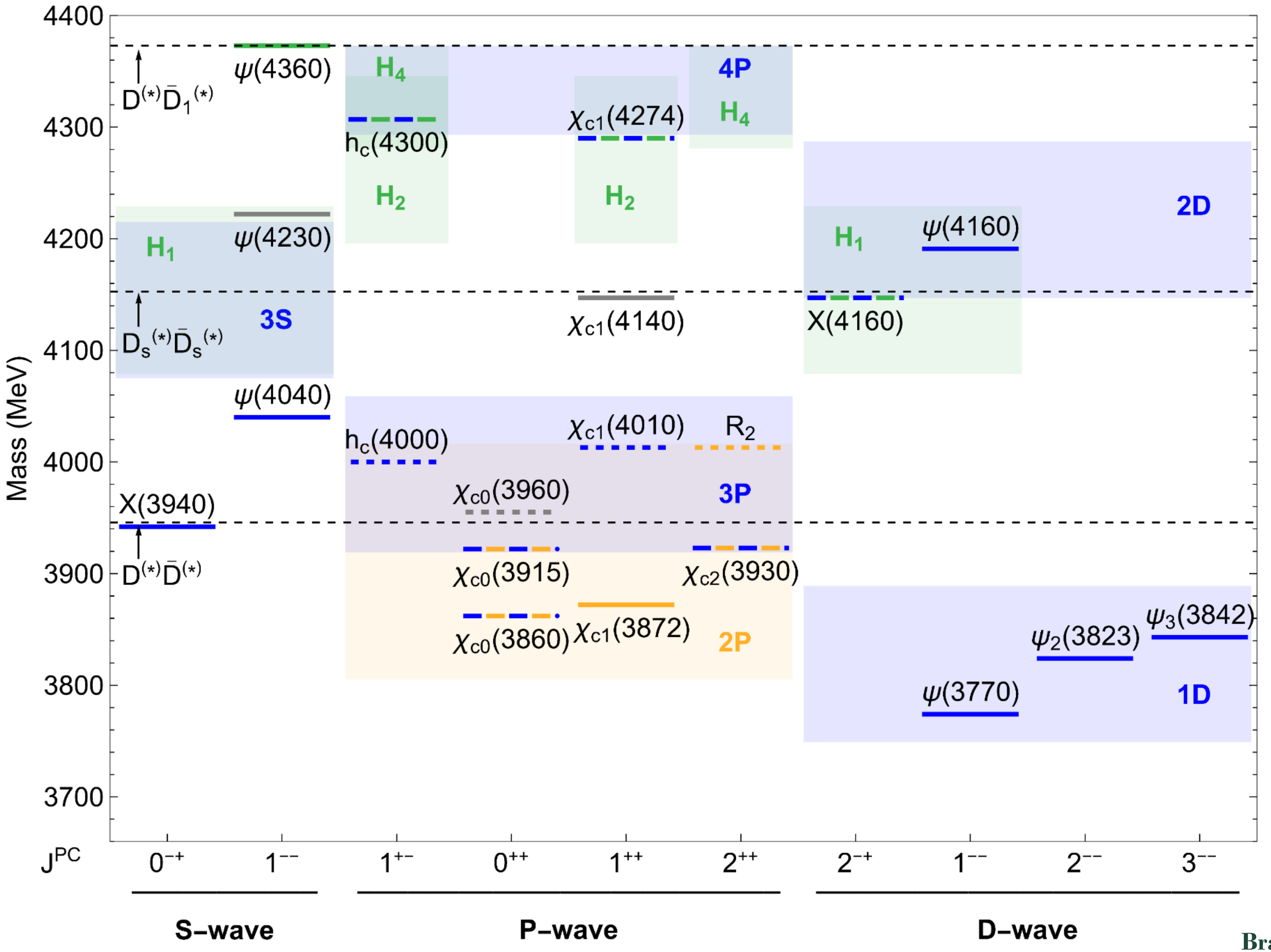
fitted on the spectrum!



3P0 parameters from Ono, Phys. Rev. D 23, 1118 (1981)

BOEFT provides **unambiguous** definitions of all potentials computable in LQCD.

BOEFT results vs Experimental Data: Charm

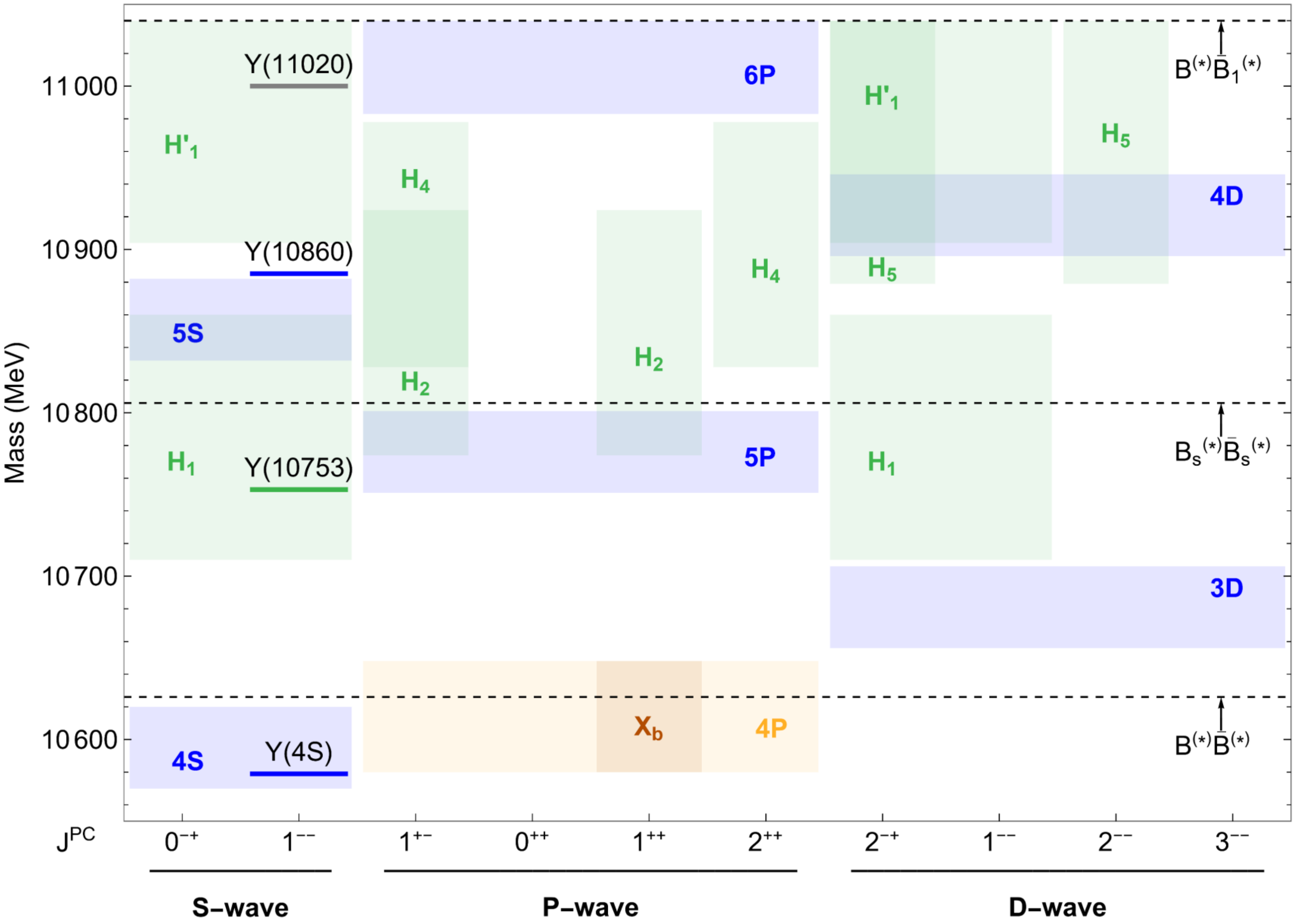


- Spin-splittings not accounted in the calculation
- Strange meson-antimeson threshold not accounted
- Result obtained with adjoint meson mass fixed to $\chi_{c1}(3872)$

$\Lambda_a^{1^{--}} \approx \mathbf{915}$ MeV (pole mass scheme)

Brambilla, Bruschini, Mohapatra, Peng, TS, in preparation

BOEFT results vs Experimental Data: Bottom



- Spin-splittings not accounted in the calculation
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- Result obtained with adjoint meson mass fixed to $\chi_{c1}(3872)$

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Brambilla, Bruschini, Mohapatra, Peng, TS, in preparation

$T_{cc}^{+}\ (3875)$

BOEFT: $QQ\bar{q}\bar{q}$ multiplets

Berwein, Brambilla, AM, Vairo,
Phys. Rev. D. 110, (2024), 094040

light antiquarks

{qq'}, 1^+ [qq'], 0^+

"Bad diquark" **"Good diquark"**

Defines the Born-Oppenheimer static potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

Bad diquark – Good diquark $\approx$ 200 MeV

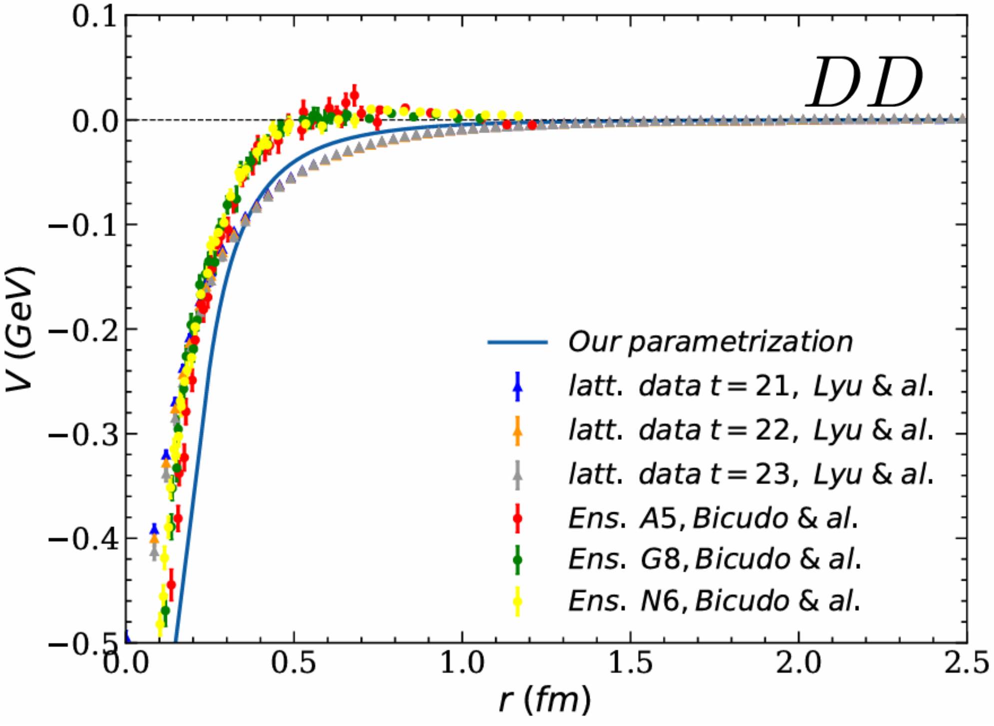
QQ color state	Light spin K^{PC}	Static energies	Isospin I	l	J^{PC} $S_Q = 0$	J^{PC} $S_Q = 1$
anti-triplet $\bar{3}$	0^+	$\{\Sigma_g^+\}$	0	0	—	1^+
				1	1^-	—
	1^+	$\{\Sigma_g^-, \Pi_g\}$	1	0	0^-	—
				1	1^-	$(0, 1, 2)^+$

J^P for T_{cc}^+

$T_{cc}^{+}(3875)$

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Brambilla, AM, Scirpa, Vairo Phys. Rev. Lett. 135 (2025), 131902



Good diquark : $\Lambda_t^{0^+} \approx 664$ MeV (pole mass scheme)

Lyu, Aoki, Doi, Hatsuda, Ikeda, Meng, Phys. Rev. Lett. 131, 161901 (2023)

Bicudo, Marinkovic, Mueller, Wagner, arXiv 2409.10786

Schrödinger Equation:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{m_Q r^2} + V_{\Sigma_g^+}\right]\psi_{\Sigma_g^+} = \mathcal{E}_N\, \psi_{\Sigma_g^+}\,.$$

$l = 0$

Results:

1) T_{CC} state : 320 keV below DD threshold
2) Radius ~ 8 fm or larger.
3) Deeper bound state in bb sector: T_{bb} 116 MeV below BB threshold.
4) Deeper bound state in bc sector: T_{bc} 25 MeV below DB threshold.

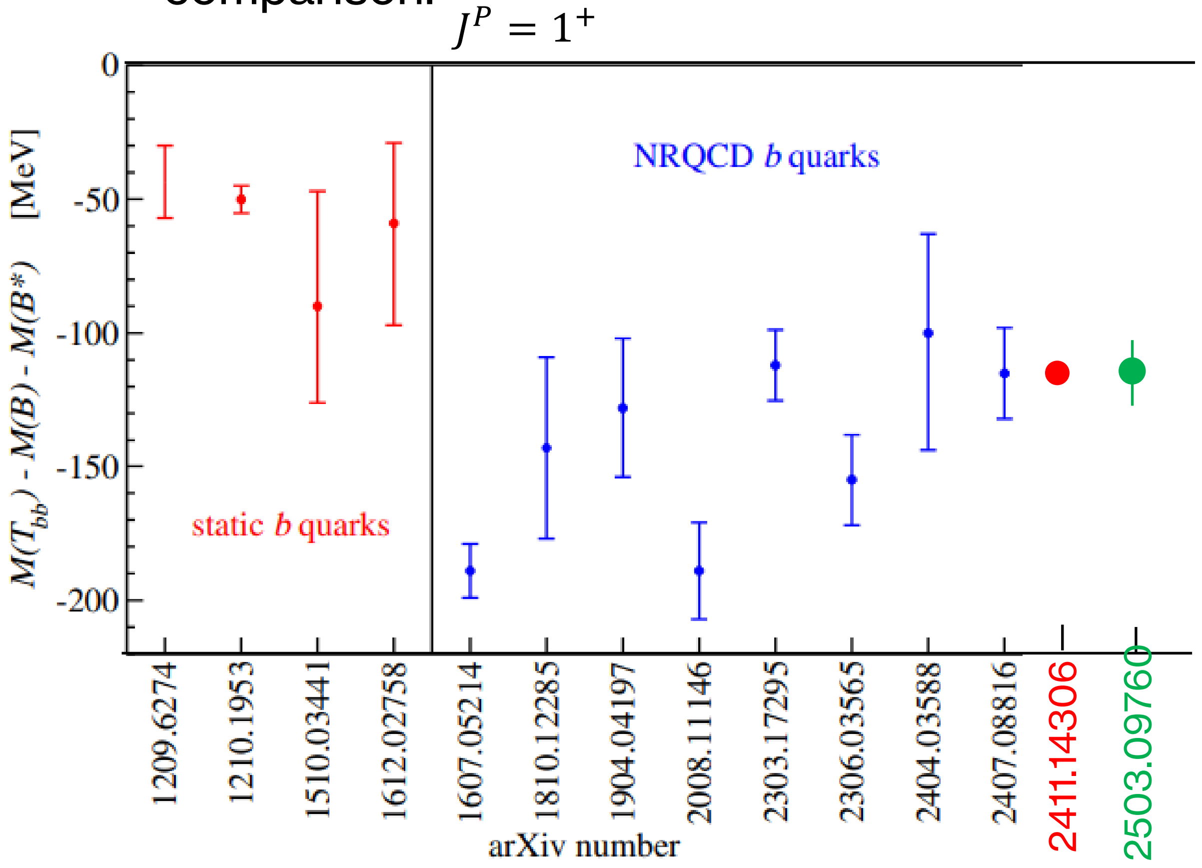
Good diquark energy $\Lambda_t^{0^+}$ can be confirmed by lattice QCD !!

T_{bb} & T_{bc}

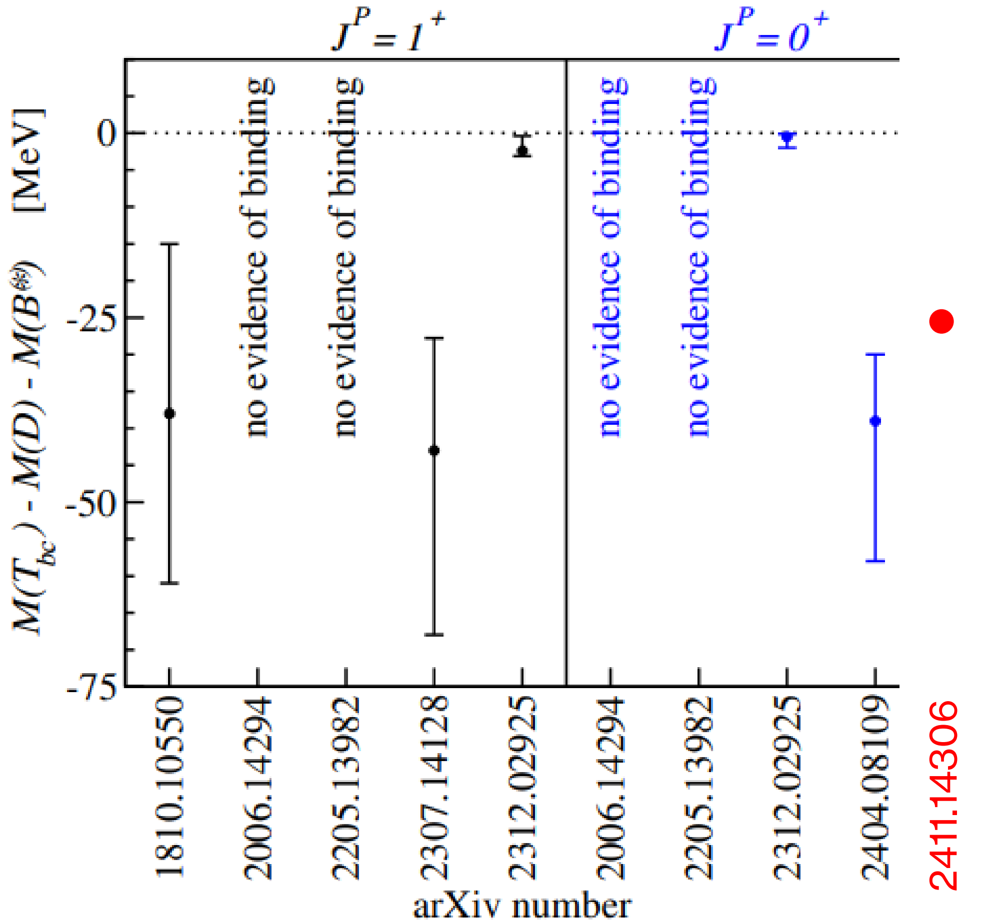
Brambilla, AM, Scirpa, Vairo

Phys. Rev. Lett. 135 (2025), 131902

T_{bb} binding energy comparison:



T_{bc} binding energy comparison:



EFT and heavy-quark-diquark symmetry prediction for T_{bb}: 133 ± 25 MeV

Our result 25 MeV for both $J^P = \{0^+, 1^+\}$

Braaten, He, AM, Phys. Rev. D. 103, 016001 (2021)

Figures from Randy Lewis talk: LepageFest 2024

Hadro-production

Inclusive Hadroproduction

- NRQCD factorization for hadro-production cross-sections:

Brambilla, Butenschoen, Hibler, AM, Vairo, Wang
arXiv 2602.14916

$$\sigma_{\mathcal{Q}+X} = \sum_N \underbrace{\sigma_{Q\bar{Q}(N)}}_{} \underbrace{\langle\Omega|\, \mathcal{O}^{\mathcal{Q}}(N)\, |\Omega\rangle}_{}$$

N: denotes color state and spin

Bodwin, Braaten, Lepage,
Phys. Rev. D 51 (1995), 1125-1171

Short-distance Coefficients (SDC) (perturbative !!)

Long-distance matrix elements (LDME's) (non-perturbative !!)

Dominant color-octet $(Q\bar{Q})_8$ matrix elements at leading power in v :

$$\sigma_{\chi_{c1}(3872)+X} = \sigma_{Q\bar{Q}(^3S_1^{[8]})} \langle\Omega|\, \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})\, |\Omega\rangle\,,$$

$$\langle\Omega|\, \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})\, |\Omega\rangle = \langle\Omega|\, \chi^\dagger \sigma^i T^A \psi \Phi_\ell^{\dagger AB}\, \boxed{\mathcal{P}_{\chi_{c1}(3872)}}\, \Phi_\ell^{BC} \psi^\dagger \sigma^i T^C \chi\, |\Omega\rangle$$

Projection vector that projects onto unpolarized $\chi_{c1}(3872)$ at rest + other LDF

- Fix LDME on B-hadron decays

$$\boldsymbol{Br}(B \to \chi_{c1}(3872) + X) = \boldsymbol{Br}(b \to c\bar{c}(^3S_1^{[8]}) + X)\, \langle\Omega|\mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})|\Omega\rangle$$

Beneke, Maltoni, Rothstein, Phys.Rev.D 59 (1999) 054003

Inclusive Hadroproduction

Brambilla, Butenschoen, Hibler, AM, Vairo, Wang

arXiv 2602.14916

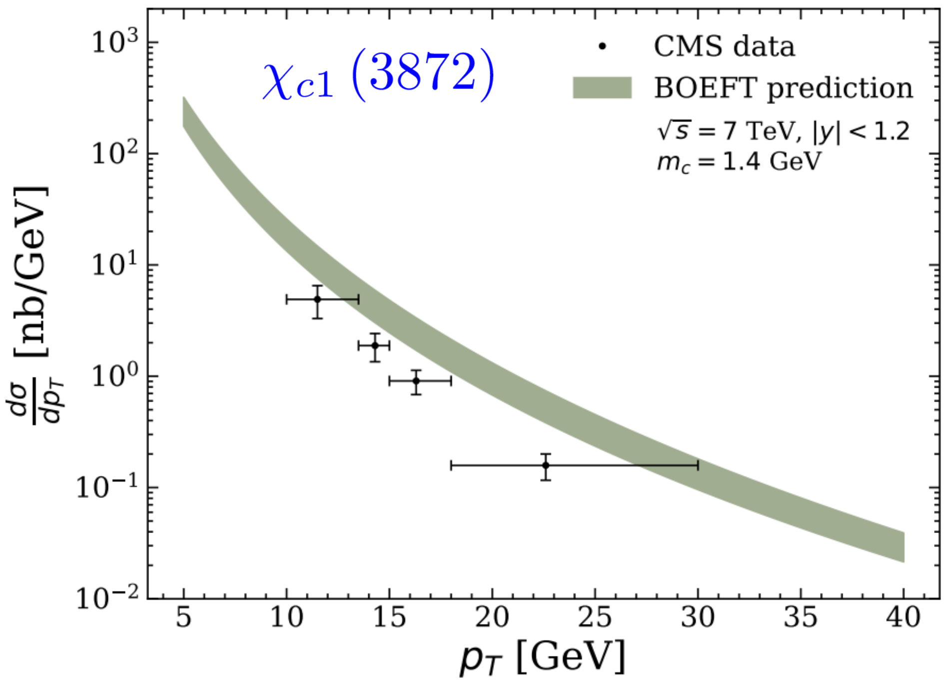
LDME $\chi_{c1}(3872)$

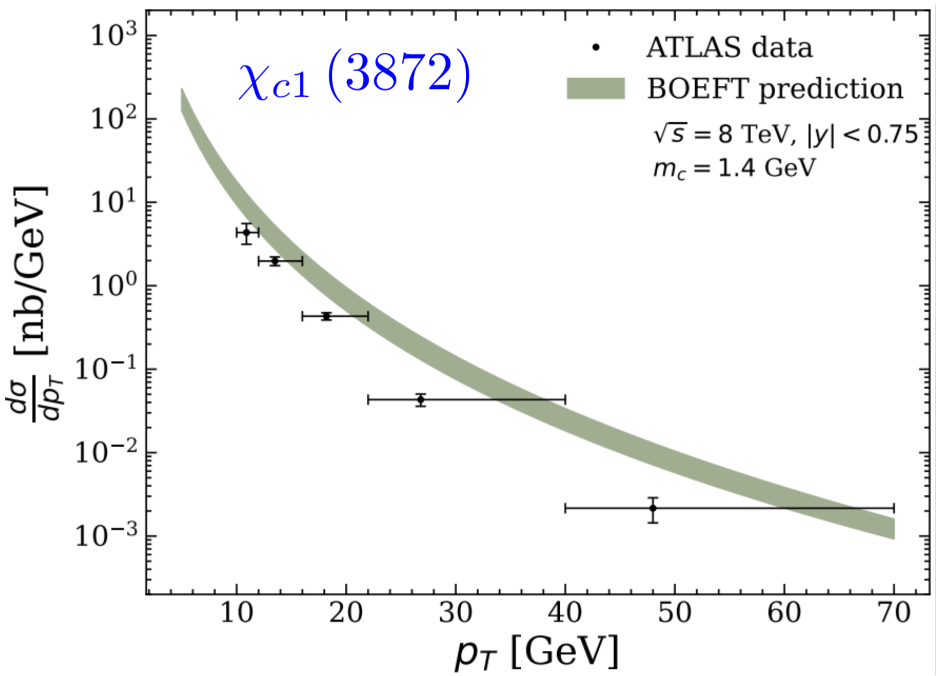
$$\langle \mathcal{O}^{\chi_{c1}(3872)}({}^3S_1^{[8]})\rangle = \frac{3}{2\pi}\,|\phi_S(0)|^2\,\mathcal{M}_S$$

- Wave-function computed from Schrodinger equation while $\mathcal{M}_S$ is universal, determined from B-decays.

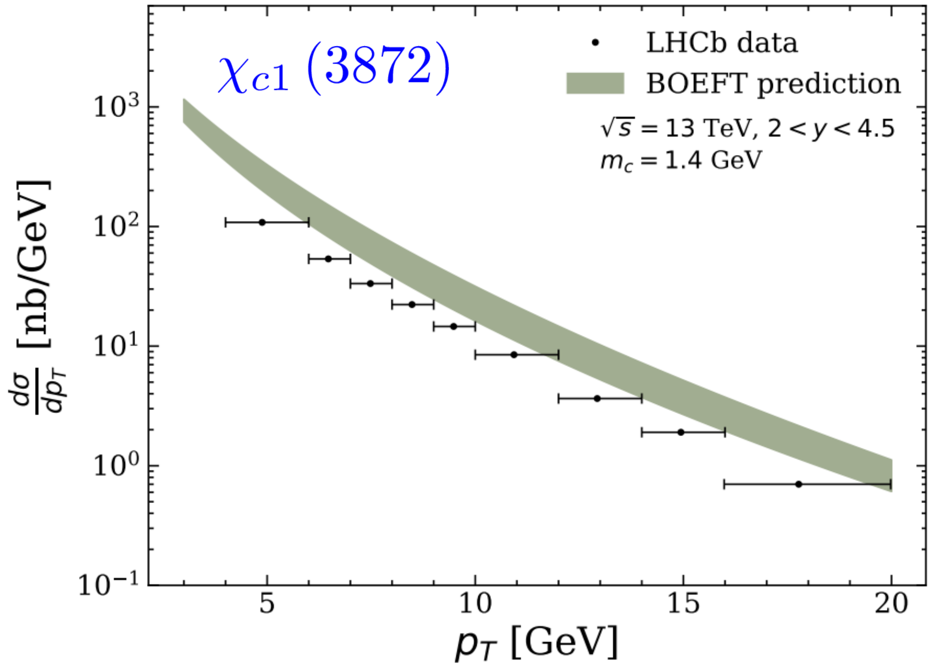
$$\langle \mathcal{O}^{\chi_{c1}(3872)}({}^3S_1^{[8]})\rangle \lesssim \frac{3}{2\pi} \times |\phi(0)|^2 \times \frac{4}{3}$$

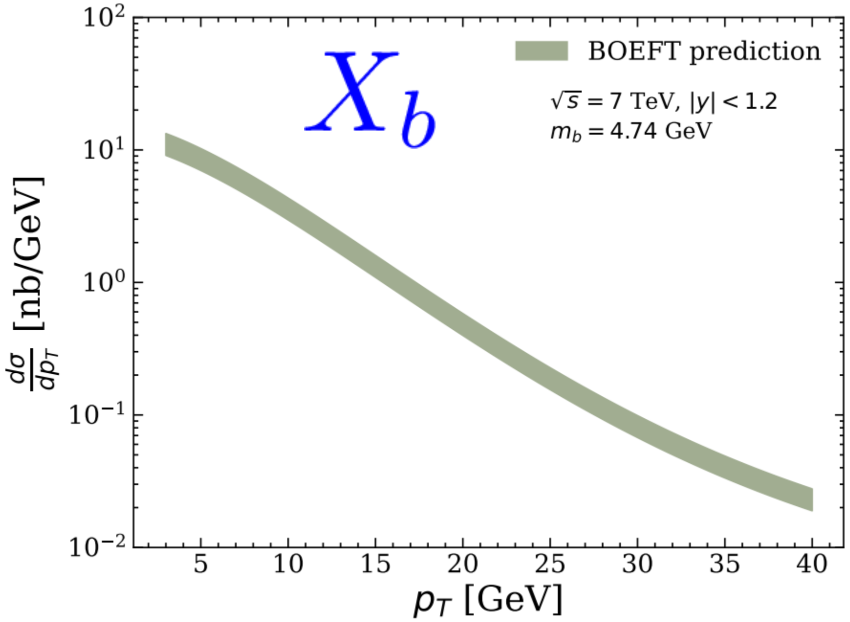
(Upper bound on LDME from positivity and completeness set by BOEFT!)





Error band from SDC and LDME uncertainties. No additional free parameters





Predictions in agreement with experimental data

NOTE:
LDME was not fixed on Hadro-production Data to get the results.

Summary

- Born-Oppenheimer EFT (BOEFT):
  - ❑ Theoretical framework based on scale separation, symmetries with some inputs from lattice QCD.
  - ❑ Quarkonium and Exotic XYZ mesons can be described in the same framework.
- Exotic dynamics governed by Schrödinger equations. Universal non-perturbative parameters same for both charm and bottom.
- Tetraquark / pentaquark results from BOEFT:
  - ❑ finite number of bound states & resonances in proximity of meson-antimeson thresholds (no a priori assumption on structure)
  - ❑ Adjoint or triplet hadrons mass: Key parameter for understanding exotics nature
  - ❑ BOEFT subdues “compact” or “molecular” debate as it spans both short & long distances.
- Leading order production cross-section for $\chi_{c1}(3872)$ at high pT slightly above the experimental results but in agreement within error bars.
- BOEFT predictions are already in contrast to existing model predictions .

Backup Slides

BO potentials : Quenched

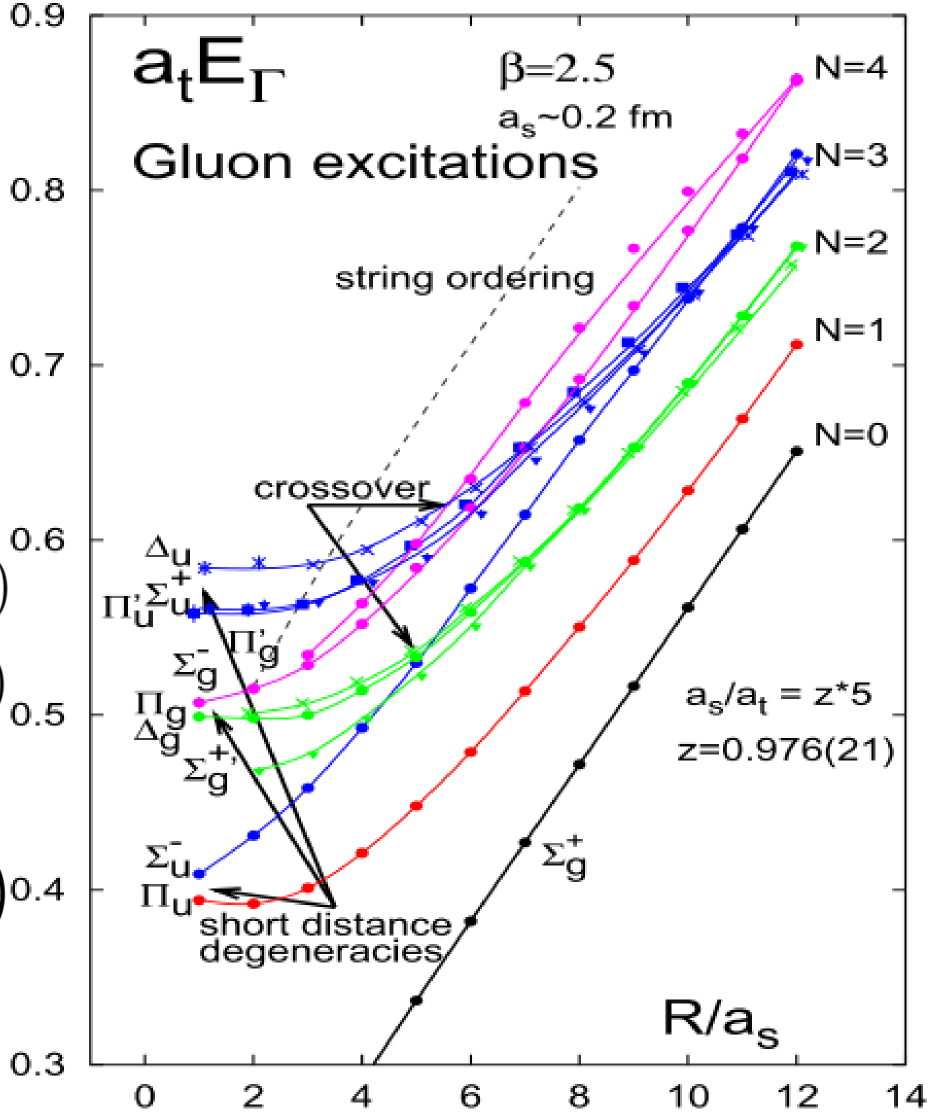
Λ_η^σ
corresponding to **gluelump** quantum # K^{PC}

$2^{+-}\ (\Sigma_u^+, \Pi_u', \Delta_u)$

$2^{--}\ (\Delta_g, \Sigma_g^-, \Pi_g')$

$1^{--}\ (\Sigma_g^{+\prime}, \Pi_g)$

$1^{+-}\ (\Pi_u, \Sigma_u^-)$



0 ← r → ∞

$N = 3\ (\Sigma_u^-, \Sigma_u^+, \Pi_u', \Delta_u, \cdots)$

$N = 2\ (\Sigma_g^{+\prime}, \Pi_g, \Delta_g)$

$N = 1\ (\Pi_u)$

$N = 0\ (\Sigma_g^+)$

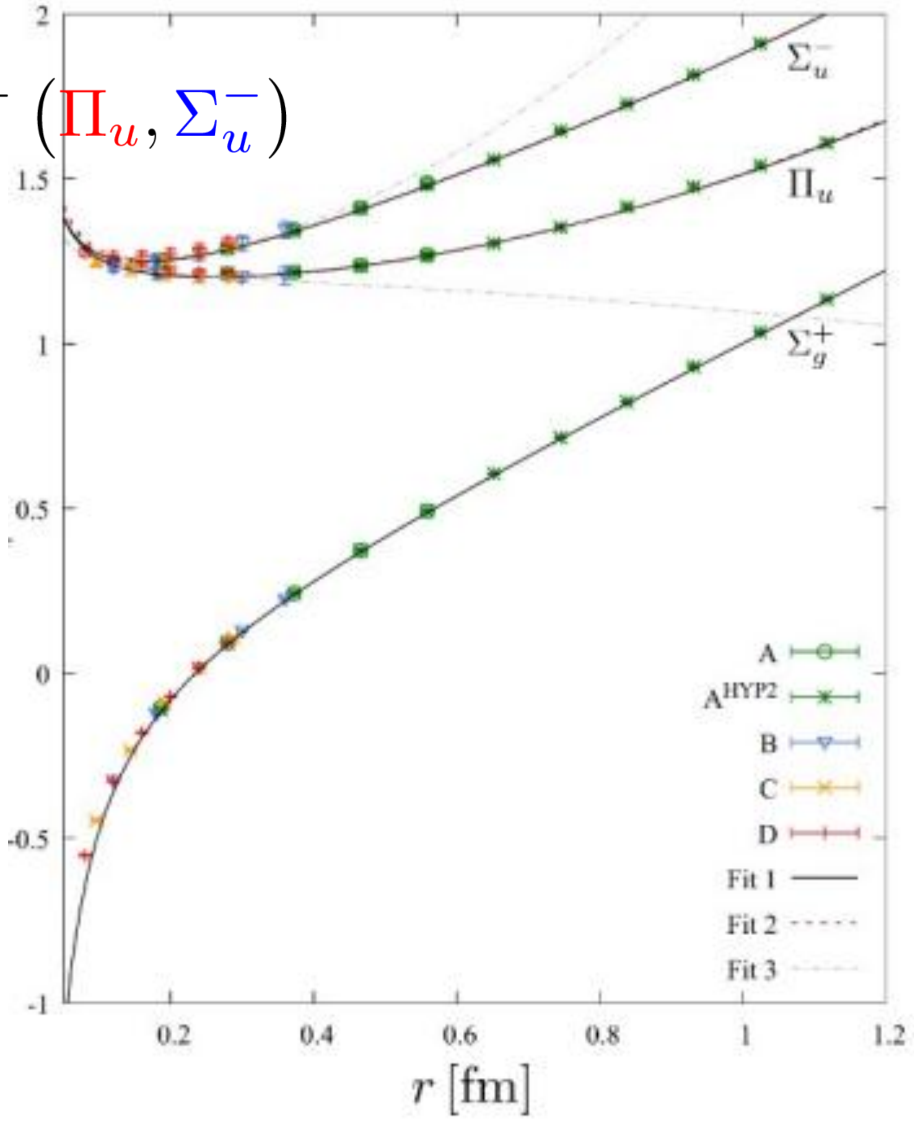
Observation:
BO-quantum # Λ_η^σ **conserved** at all values of r

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

BO potentials: Quenched

gluelump
quantum # K^{PC}

$1^{+-}\ (\Pi_u, \Sigma_u^-)$



Σ_g^+: Quarkonium Potential

(Σ_u^-, Π_u): Quarkonium Hybrid potentials

Observation:
BO-quantum # Λ_η^σ
conserved
at all values of r

Schlosser and Wagner
Phys. Rev. D. 105, (2022)

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Castellà , Soto Phys. Rev. D. 102, 014012 (2020)

NRQCD operator (gauge invariant) for exotic hadron $Q\bar{Q}X$ or QQX :

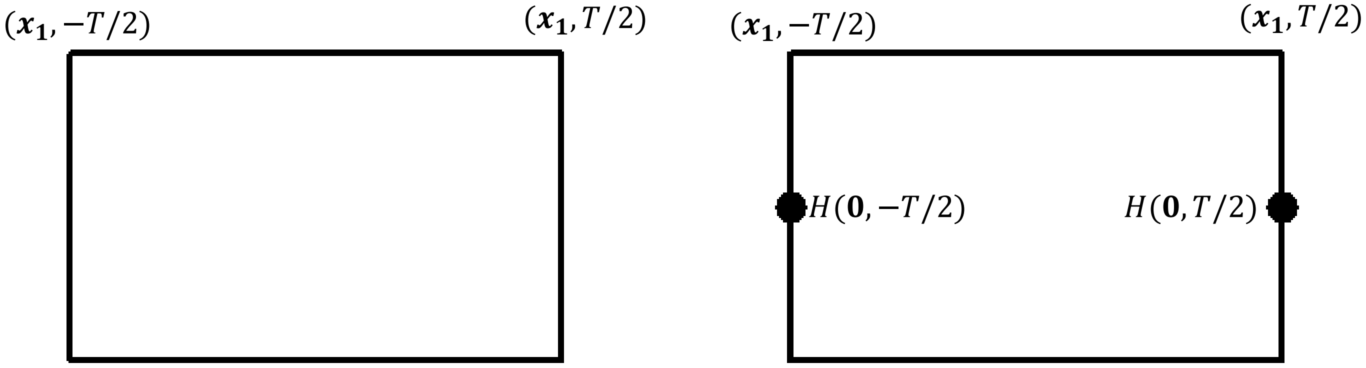
$$\mathcal{O}_{\kappa,\lambda}(t,\boldsymbol{r}) = \chi^\dagger(t,\boldsymbol{r}/2)\,\phi(t;\boldsymbol{r}/2,\boldsymbol{0})\,P^{\alpha\dagger}_{\kappa,\lambda}H^{\alpha}_{\kappa}(t,\boldsymbol{0})\,\phi(t;\boldsymbol{0},-\boldsymbol{r}/2)\,\psi(t,-\boldsymbol{r}/2)$$

H^{α}_{κ} : LDF (gluon or light-quarks) operator characterizing X based on quantum # κ (isospin, color etc..

$P^{\alpha}_{\kappa,\lambda}$: Projection vectors for projecting onto cyclindrical symmetry $D_{\infty h}$ representations.

$$E^{(0)}_{\kappa,|\lambda|}(r) = \lim_{T\to\infty}\frac{i}{T}\log\left[\langle\text{vac}|\mathcal{O}_{\kappa,\lambda}(T/2,\,\boldsymbol{r},\,\boldsymbol{R})\,\mathcal{O}^\dagger_{\kappa,\lambda}(-T/2,\,\boldsymbol{r},\,\boldsymbol{R})|\text{vac}\rangle\right]$$

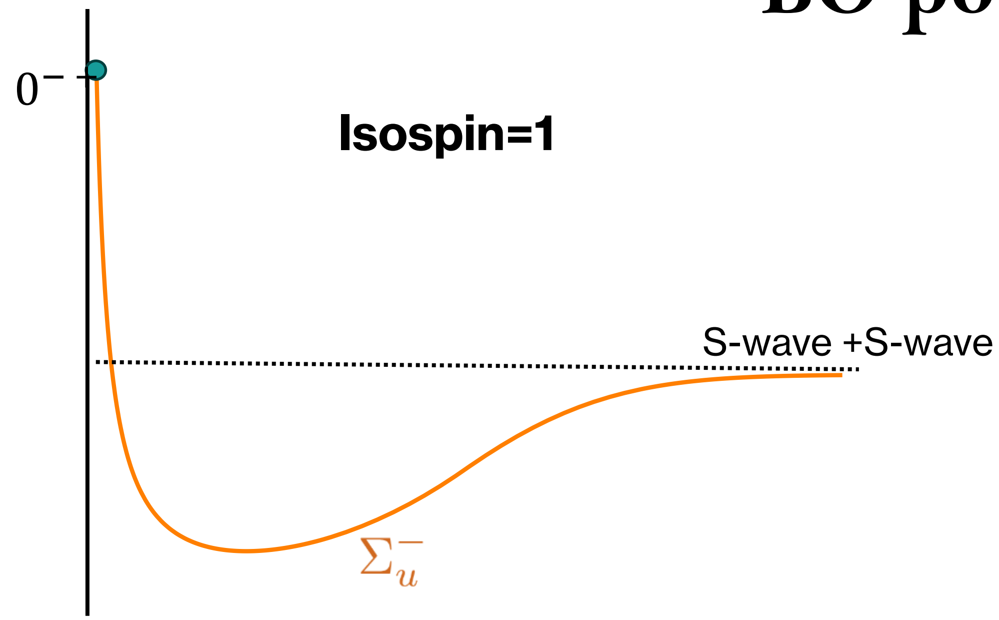
See Joan Soto and Marc Wagner talk on Mon !



(a) Quarkonium

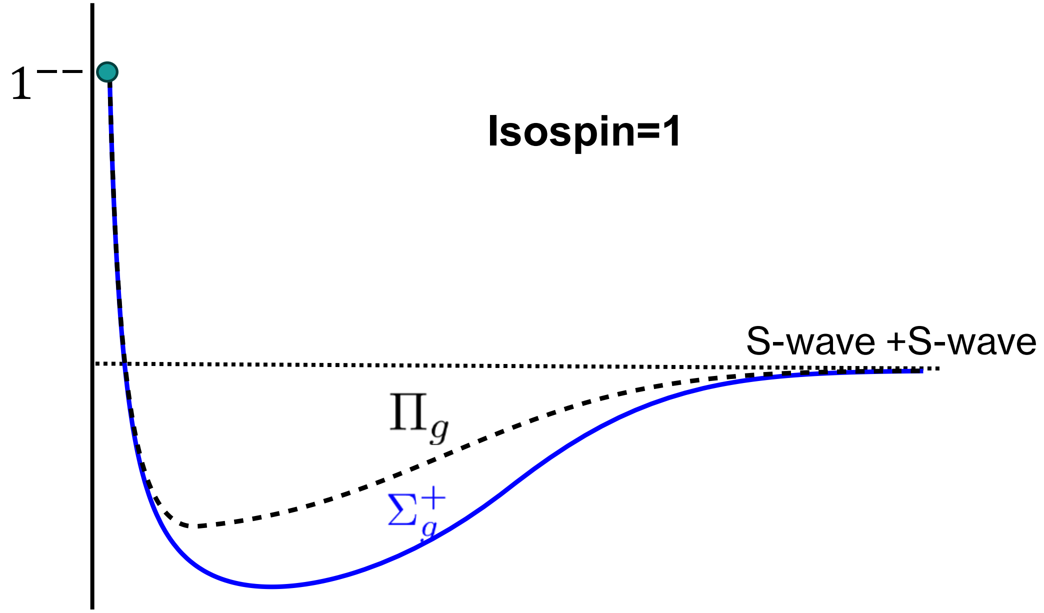
(b) Wilson loop for exotics

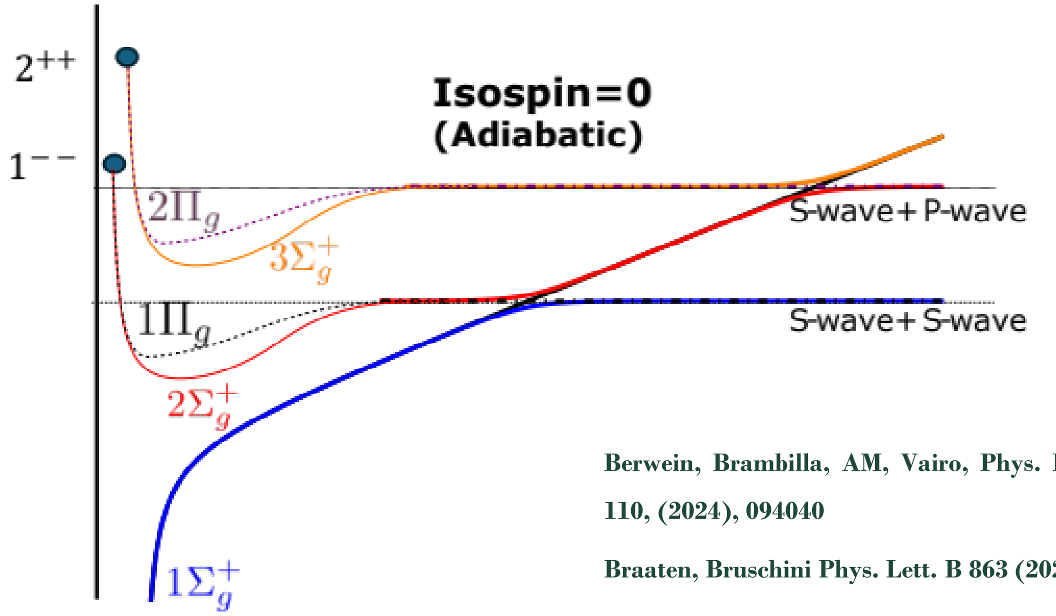
BO potentials: Tetraquarks



Key Takeaways: Tetraquark static potential behavior

- ❑ Repulsive behavior at small r due to adjoint color $(r \to 0)$
- ❑ Heavy meson pair threshold at large r $(r \to \infty)$
- ❑ Avoided crossing with quarkonium static energy (Isospin=0)





Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

Static potentials parametrizations for $\chi_{c1}(3872)$

Quarkonium\tetraquark potentials parametrizations

$$V_{\Sigma_g^+}(r) = V_0 + \frac{\gamma}{r} + \sigma r,$$

→ Cornell potential

$$\Lambda_\eta^\sigma = \{\Sigma_g^{+\prime}, \Pi_g\} \qquad V_{\Lambda_\eta^\sigma}(r) = \begin{cases} \kappa_8/r + \bar{\Lambda}_{1^{--}} + A_{\Lambda_\eta^\sigma}\, r^2 + B_{\Lambda_\eta^\sigma}\, r^4 & r < R_{\Lambda_\eta^\sigma} \\ F_{\Lambda_\eta^\sigma}\, e^{-r/d}/r + E_1 & r > R_{\Lambda_\eta^\sigma} \end{cases}$$

Short-distance parametrization

- k_8 : color factor from pert. theory
- $\bar{\Lambda}_{1^{--}} \simeq 1 GeV$ (pole mass scheme): adjoint meson mass. Unknown parameter fixed on $\chi_{c1}(3872)$
- $A_{\Lambda_\eta^\sigma}, B_{\Lambda_\eta^\sigma}$: computed in LQCD in pure gauge for static hybrid potentials

Alasiri, Braaten, Mohapatra Phys. Rev. D 110, 054029 (2024)

Long-distance parametrization

- one-pion exchange interaction

Brambilla, Mohapatra, TS, Vairo Phys. Rev. Lett. 135 (2025), 131902

Brambilla, Mohapatra, TS, Vairo arxiv: 2604.12603

→ Value consistent with the former LQCD calculation of **Foster, Michael (UKQCD) Phys. Rev. D 59, 094509 (1999)**

$V_{\Sigma_g^+-\Sigma_g^{+\prime}}$ mixing potential for $\chi_{c1}(3872)$

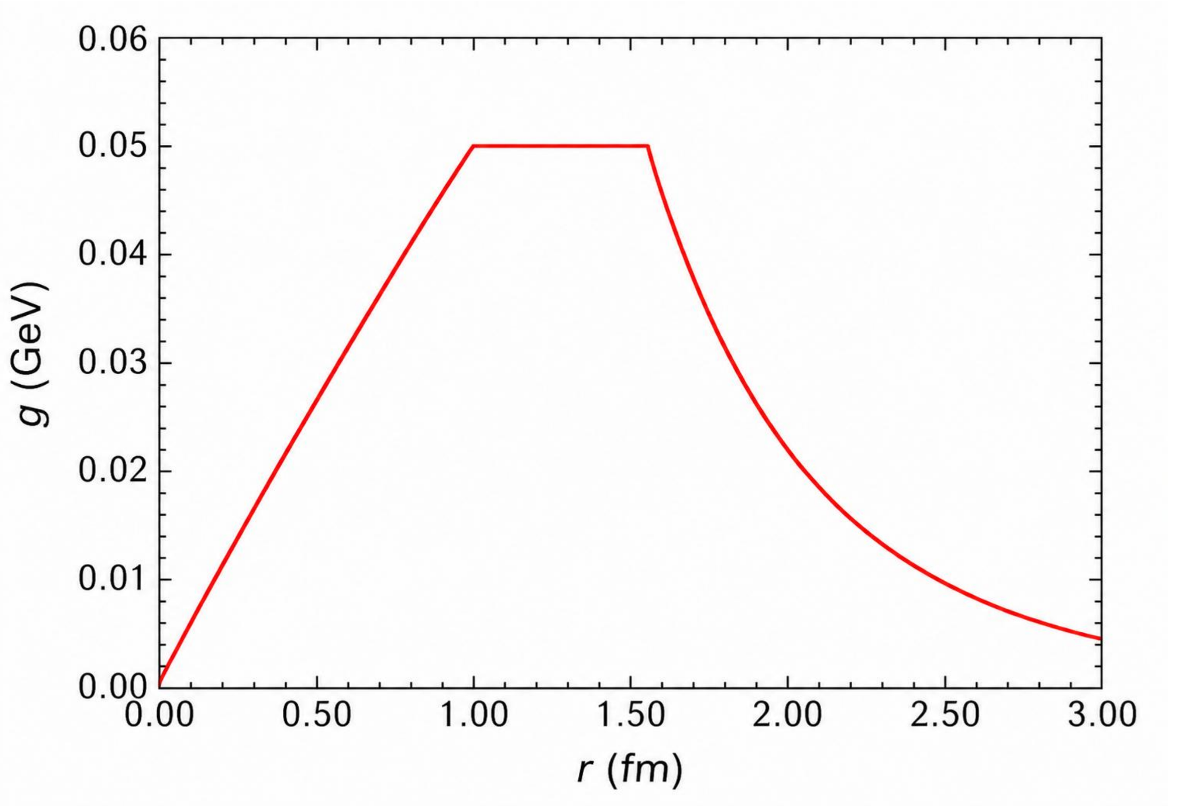
Parametrization

$$V_{\Sigma_g^+-\Sigma_g^{+\prime}}(r) = \begin{cases} g\, r/r_1 & r < r_1 \\ g & r_1 \le r \le r_2 \\ g\, e^{-(r-r_2)/r_0} & r > r_2 \end{cases}$$

$$r_1 = 0.95\, fm \quad r_2 = 1.51\, fm$$

- linear at small r due to color singlet-octet transitions Castellà Phys. Rev. D. 106, 094020 (2022)
- constant in the intermediate region from LQCD
  Bulava, Knechtli, Koch, Morningstar, Peardon, Phys. Lett. B. 854 (2024)
- tending to zero at large r due to the energy gap between the quarkonium $V_{\Sigma_g^+}$ and tetraquark potentials. $V_{\Sigma_g^{+\prime}}$

Plot



Brambilla, Mohapatra, TS, Vairo Phys. Rev. Lett. 135 (2025), 131902

Brambilla, Mohapatra, TS, Vairo arxiv: 2604.12603

Static potentials parametrizations for $T_{cc}^{+}(3875)$

Tetraquark potential parametrization

$$V_{\Sigma_g^+} = \begin{cases} \kappa_3/r + \Lambda_{0^+} + A_{\Sigma_g^+}\, r^2 & r < R_{\Sigma_g^+} \\ F_{\Sigma_g^+}\, e^{-r/d}/r^2 & r > R_{\Sigma_g^+} \end{cases}$$

Short-distance parametrization

- k_3 known from pert. theory, $A_{\Sigma_g^+}$ computed in LQCD in pure gauge
- $\Lambda_0^+ \simeq 0.7\, GeV$ (pole mass scheme): triplet meson mass, fixed on $T_{cc}^{+}(3875)$

Long-distance parametrization

- pion exchange dynamics

Brambilla, Mohapatra, TS, Vairo Phys. Rev. Lett. 135 (2025), 131902

Isospin =1
Adjoint meson spectrum

Z_b & Z_c

$Q\bar{Q}$ color state	$q\bar{q}$ spin k^{PC}	BO quantum # Λ_η^σ	l	J^{PC} $\{S=0, S=1\}$
Octet **8**	0^{-+}	Σ_u^-	0	$\{0^{++}, 1^{+-}\}$
			1	$\{1^{--}, (0,1,2)^{-+}\}$
	1^{--}	$\Sigma_g^{+\prime}, \Pi_g$	1	$\{1^{+-}, (0,1,2)^{++}\}$
		$\Sigma_g^{+\prime}$	0	$\{0^{-+}, 1^{--}\}$
		Π_g	1	$\{1^{-+}, (0,1,2)^{--}\}$

Isospin-1 channel: $Z_c\,(3900), Z_c\,(4200), Z_b(10610),\ Z_b(10610)$

Mixing between adjoint mesons $K^{PC} = 0^{-+}$ and $K^{PC} = 1^{--}$

Light-quark spin-symmetry !!

Berwein, Brambilla, AM, Vairo,
Phys. Rev. D. 110, (2024), 094040

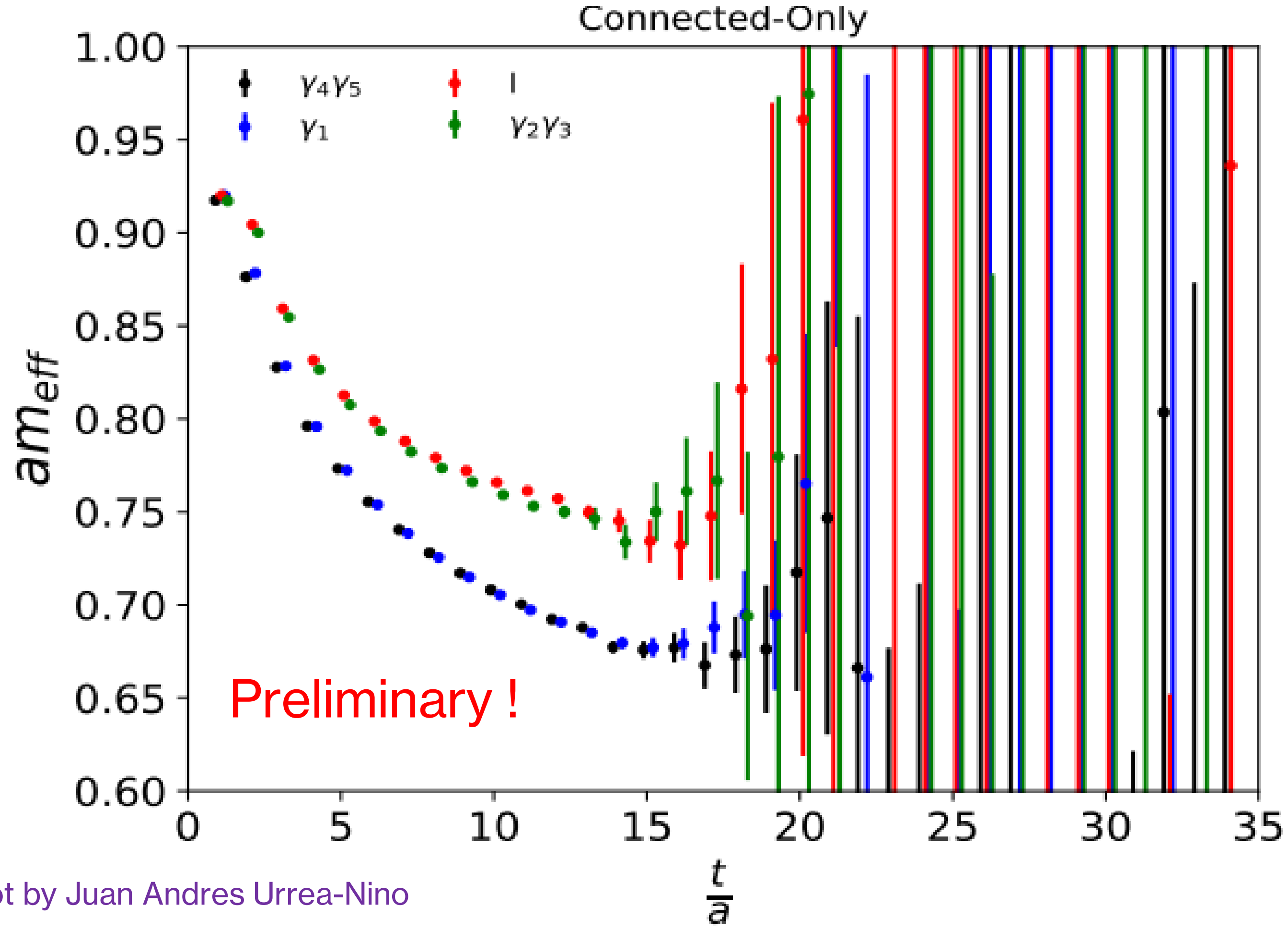
Voloshin, Phys. Rev. D. 93, 074011 (2016)

Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

Isospin=1 adjoint spectrum

Slide: Sipaz Sharma talk, Lattice 2025, Mumbai.

$$am^i_{eff}(t, t+1) = \log \frac{C_{ii}(t)}{C_{ii}(t+1)};\ C_{ij}(t) = \sum_{n=1}^{\infty} e^{-E_n t}\langle \hat{O}_i|n\rangle\langle n|\hat{O}_j\rangle$$



Plot by Juan Andres Urrea-Nino

$N_f = 3+1$ ensemble, $m_\pi = 406$ MeV, $a = 0.05359$ fm

$\gamma_1 : 1^{--}$ $\gamma_4\gamma_5 : 0^{-+}$

- ❑ 0^{-+} and 1^{--} adjoint mesons are degenerate as predicted by BOEFT + experimental input.

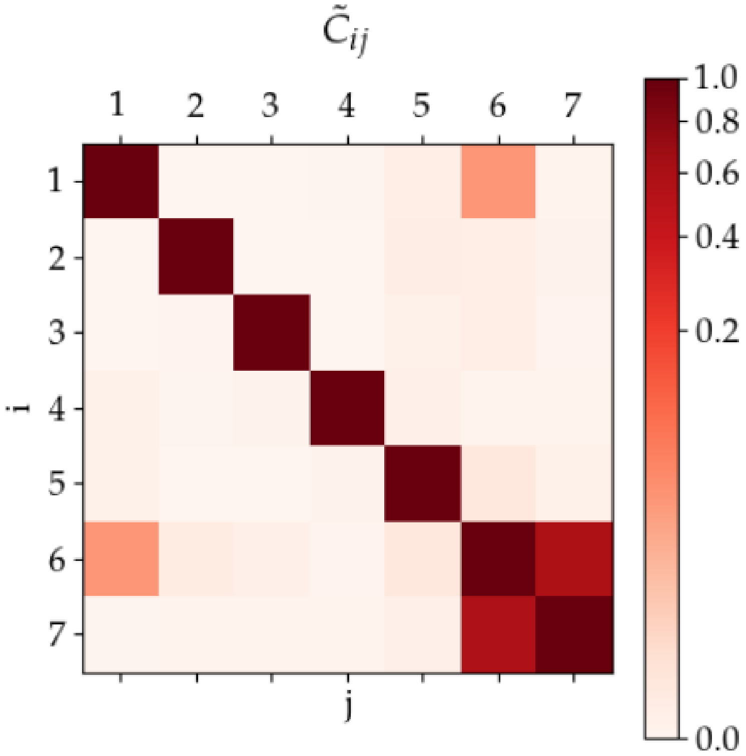
Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

$1 : 0^{++}$ $\gamma_2\gamma_3 : 1^{+-}$

- ❑ 0^{++} and 1^{+-} adjoint mesons are degenerate
- ❑ There are 6 adjoint mesons including 0^{++} and 1^{+-} associated with s-wave + p-wave meson-pair threshold.

Preliminary steps towards extracting Z_b static energies

- We constructed a 7×7 correlation matrix, with
  - Op1: $\Upsilon\pi(0)$
  - Op2,3,4: $\Upsilon\pi(1,2,3)$
  - Op5: $\Upsilon b_1(0)$
  - Op6: $B\bar{B}^*$
  - Op7: O_{BO} with 0^{-+} LDF configuration (Σ_u^-)
- $\tilde{C}_{ij}(r) = \dfrac{|\sum_{t=1}^{n_t} C_{ij}(r,t)|}{\sqrt{|\sum_{t=1}^{n_t} C_{ii}(r,t)||\sum_{t=1}^{n_t} C_{jj}(r,t)|}}$
- Op6 has a visible overlap with Op1 but Op7 does not have a visible overlap with Op1.



$r/a = 1$

Slide: Sipaz Sharma talk, Lattice 2025, Mumbai.