Monsoon Hadrons 2026

Testing the Standard Model with bottom-hadron semileptonic decays

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From the invitation email:

"We would be very grateful if you could deliver an online introductory lecture on heavy-hadron (semi)leptonic decays, with emphasis on baryons, phenomenology, experimental status, and prospects from lattice QCD. The audience will largely consist of participants already familiar with lattice methodologies, and therefore you may wish to focus primarily on the key phenomenological and lattice aspects of decay studies, without needing to discuss basic lattice concepts in detail."

Flavor in the Standard Model; the CKM matrix

The Standard Model of particle physics has the fascinating property that each of the five different fermion representations of the gauge group $SU(3) \times SU(2) \times U(1)$ comes in three copies – the *generations*:

$$
\begin{array}{rcl}
 & & \quad\;\; 1 \qquad\quad\;\; 2 \qquad\quad\;\; 3 \\
Q'_L & = & \left(\begin{pmatrix} u'_L \\ d'_L \end{pmatrix}, \begin{pmatrix} c'_L \\ s'_L \end{pmatrix}, \begin{pmatrix} t'_L \\ b'_L \end{pmatrix} \right), \\
U'_R & = & (\quad u'_R, \quad c'_R, \quad t'_R \quad), \\
D'_R & = & (\quad d'_R, \quad s'_R, \quad b'_R \quad), \\
L'_L & = & \left(\begin{pmatrix} {\nu_e}'_L \\ e'_L \end{pmatrix}, \begin{pmatrix} {\nu_\mu}'_L \\ \mu'_L \end{pmatrix}, \begin{pmatrix} {\nu_\tau}'_L \\ \tau'_L \end{pmatrix} \right), \\
E'_R & = & (\quad e'_R, \quad \mu'_R, \quad \tau'_R \quad).
\end{array}
$$

The species labels u, c, t, d, s, b, e, μ, τ are called *flavors*. The $'$ indicates the use of the gauge basis.

In the absence of flavor-violating interactions, we would have a $U(3)^5$ global flavor symmetry.

In the Standard Model, the only origin of flavor symmetry violation (and CP violation) is the Yukawa interaction of the fermions with the Higgs field ϕ:

$$\mathcal{L}_{\text{Yukawa}} = -\overline{Q'_{Li}} Y^U_{ij} U'_{Rj} \tilde{\phi} - \overline{Q'_{Li}} Y^D_{ij} D'_{Rj} \phi - \overline{L'_{Li}} Y^E_{ij} E'_{Rj} \phi + \text{h.c.}$$

When ϕ acquires its vacuum expectation value $\langle \phi \rangle = (0, v/\sqrt{2})$, these couplings produce the fermion mass terms. In the quark sector, the unitary field transformations that diagonalize the mass matrices,

$$\begin{aligned} U'_L &= V^U_L U_L, \quad & U'_R &= V^U_R U_R, \\ D'_L &= V^D_L D_L, \quad & D'_R &= V^D_R D_R, \end{aligned}$$

do not cancel in the charged current coupling to the W field,

$$\mathcal{L}_{\text{c.c.}} = -\frac{g}{\sqrt{2}} \overline{U'_{Li}} \gamma^\mu D'_{Li} \, W^+_\mu + \text{h.c.} = -\frac{g}{\sqrt{2}} \overline{U_{Li}} \underbrace{(V^{U\dagger}_L V^D_L)_{ij}}_{=V_{ij}} \gamma^\mu D_{Lj} \, W^+_\mu + \text{h.c.},$$

giving rise to the unitary Cabibbo-Kobayashi-Maskawa quark mixing matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

After eliminating unobservable phase factors, V can be written in terms of four parameters.

Some of the fundamental questions in flavor physics are:

- What is the origin of the three generations?
- What is the origin of the hierarchies in the fermion masses and mixing matrices?
- Are there other sources of flavor-violating interactions and CP violation beyond the Standard Model?

In most of the more fundamental theories that have been proposed to address the deficiencies of the Standard Model, the answer to the third question is “yes”. The precision study of flavor-changing processes is therefore a powerful tool for discovering new physics.

A first step is determining the parameters of the CKM matrix as precisely as possible, assuming the validity of the Standard Model.

A possible parametrization of the CKM matrix is in terms of θ_{12}, θ_{23}, θ_{13}, and δ as

$$
\begin{aligned}
V &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}
\end{aligned}
$$

where $s_{ij} = \sin(\theta_{ij})$, $c_{ij} = \cos(\theta_{ij})$.

It is known from experiments that V is close to the identity matrix, with

$$\begin{aligned}
|V_{ud}| \approx |V_{cs}| \approx |V_{tb}| &\approx 1, \\
|V_{us}| \approx |V_{cd}| &\approx 0.2, \\
|V_{cb}| \approx |V_{ts}| &\approx 0.04, \\
2|V_{ub}| \approx |V_{td}| &\approx 0.008.
\end{aligned}$$

This suggests parametrizing the elements using an expansion in powers of a small parameter $\lambda \approx 0.2$ [L. Wolfenstein, PRL 1983].

The observed orders

$$|V_{ud}| \sim |V_{cs}| \sim \lambda^0,$$
$$|V_{us}| \sim |V_{cd}| \sim \lambda^1,$$
$$|V_{cb}| \sim |V_{ts}| \sim \lambda^2,$$
$$|V_{ub}| \sim |V_{td}| \sim \lambda^3$$

and unitarity then lead to the form

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4),$$

where the new parameters λ, A, ρ, and η are called the Wolfenstein parameters.

Determination of $\lambda = \dfrac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}$

Assuming the CKM unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + \underbrace{|V_{ub}|^2}_{\approx 0} = 1$, a determination of $|V_{ud}|$ alone already gives us $|V_{us}|$ as well, or vice versa.

The most precise direct result for $|V_{ud}|$ comes from the study of superallowed $0^+ \to 0^+$ nuclear β decays, which are pure vector transitions and therefore fairly insensitive to nuclear/nucleon structure [E. Blucher, G. D'Ambrosio, and W. Marciano, 2026 Review of Particle Physics, Sec. 67]:

$$|V_{ud}| = 0.97367(11)_{\text{exp.}}(13)_{\text{RC}}(27)_{\text{NS}}.$$

We also have the following experimental results:

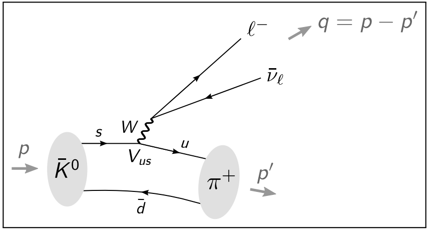
$$\frac{\Gamma(K^\pm \to \mu^\pm \nu[\gamma])}{\Gamma(\pi^\pm \to \mu^\pm \nu[\gamma])} = 1.3367(32),$$

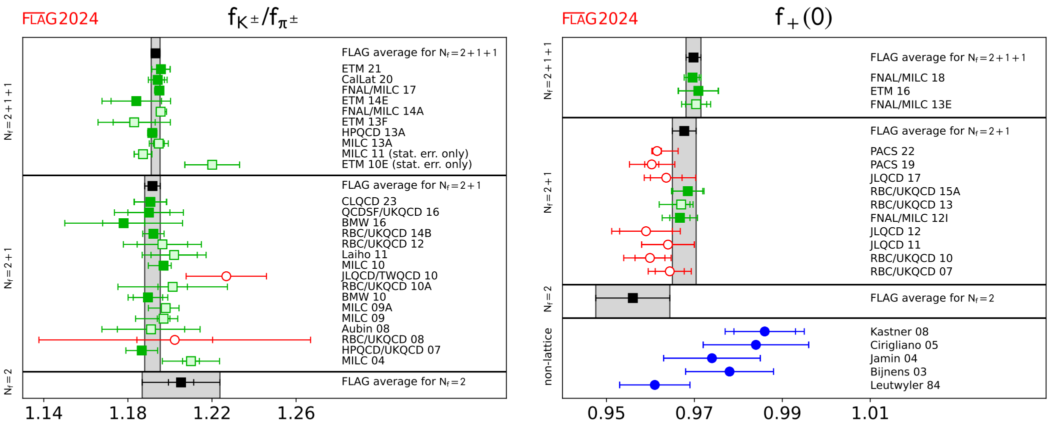
$$\frac{d\Gamma}{dq^2}(K \to \pi \ell \nu[\gamma]) \underset{\text{non-lattice theory}}{\Rightarrow} f_+(K \to \pi, q^2 = 0)|V_{us}| = 0.21656(35).$$

[E. Blucher, G. D'Ambrosio, and W. Marciano, 2026 Review of Particle Physics, Sec. 67]

To get $|V_{us}/V_{ud}|$ and $|V_{us}|$, we need lattice-QCD calculations of the ratio of decay constants $f_{K^\pm}/f_{\pi^\pm}$ and of the form factor $f_+(K \to \pi, q^2 = 0)$:

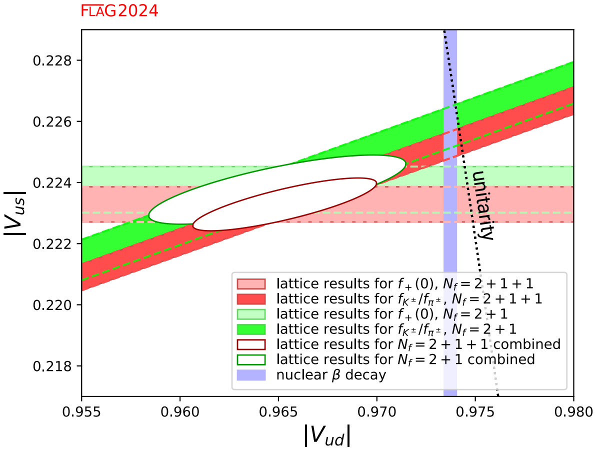
$$\begin{aligned}
\langle 0|\, \bar{u}\gamma^\mu\gamma_5 d\, |\pi^-(p)\rangle &= ip^\mu f_{\pi^-}, \\
\langle 0|\, \bar{u}\gamma^\mu\gamma_5 s\, |K^-(p)\rangle &= ip^\mu f_{K^-}, \\
\langle \pi^+(p')|\, \bar{u}\gamma^\mu s\, |\bar{K}^0(p)\rangle &= \left[(p+p')^\mu - \frac{m_K^2 - m_\pi^2}{q^2} q^\mu\right] f_+(K \to \pi, q^2) \\
&\quad + \frac{m_K^2 - m_\pi^2}{q^2} q^\mu f_0(K \to \pi, q^2).
\end{aligned}$$





[FLAG Review: 2411.04268/PRD 2026]

Here are the results for the CKM matrix elements compared to those from nuclear beta decays:



The results from $\dfrac{\Gamma(K^\pm \to \mu^\pm \nu[\gamma])}{\Gamma(\pi^\pm \to \mu^\pm \nu[\gamma])}$ shown here also use QED corrections calculated on the lattice [M. Di Carlo *et al.*, 1904.08731/PRD 2019]

There are some interesting tensions, but a fit that imposes unitarity yields

$$\lambda = 0.2251(6).$$

[http://utfit.org/UTfit/ResultsSummer2025SM]

Let's move to the next Wolfenstein parameter, A. Its definition is

$$A\lambda^2 = \frac{\lambda}{|V_{us}|}|V_{cb}|.$$

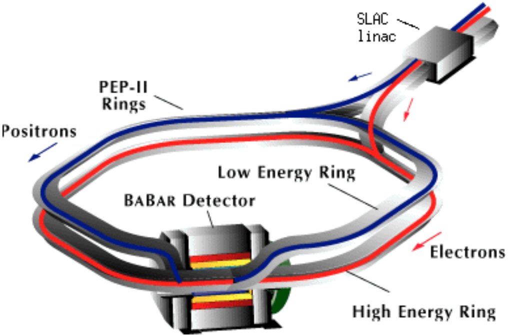
To get A, we therefore need to determine $|V_{cb}|$.

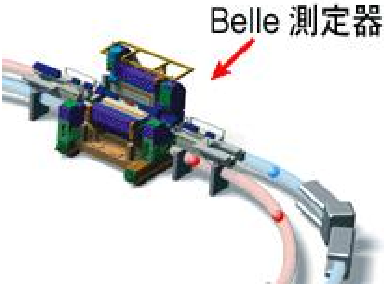
The most important processes currently used to determine $|V_{cb}|$ are

- Exclusive $\bar{B} \to D\ell\bar{\nu}$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments)
- Exclusive $\bar{B} \to D^*\ell\bar{\nu}$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments)
- Exclusive $\bar{B}_s \to D_s\mu\bar{\nu}$ (LCHb, using ratio to $\bar{B} \to D\mu\bar{\nu}$, taken from the above experiments)
- Exclusive $\bar{B}_s \to D_s^*\mu\bar{\nu}$ (LCHb, using ratio to $\bar{B} \to D^*\mu\bar{\nu}$, taken from the above experiments)
- Inclusive $\bar{B} \to X_c\ell\bar{\nu}$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments)

Absolute B decay branching fractions can be measured most precisely at dedicated e^+e^- B-factories, which operate through the process

$$e^+e^- \to \Upsilon(4S) \to B\,\bar{B}$$





1999-2008

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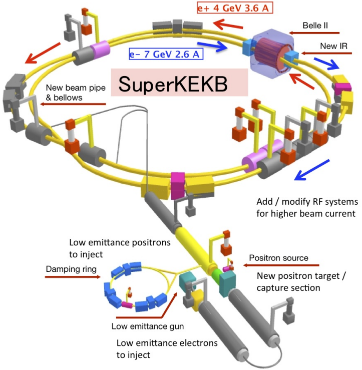
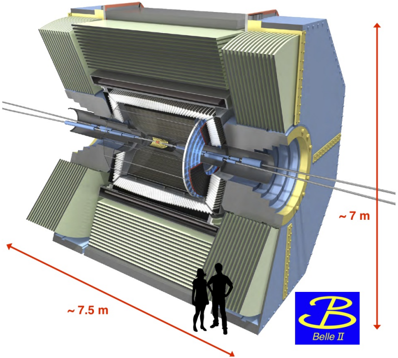
Absolute B decay branching fractions can be measured most precisely at dedicated e^+e^- B-factories, which operate through the process

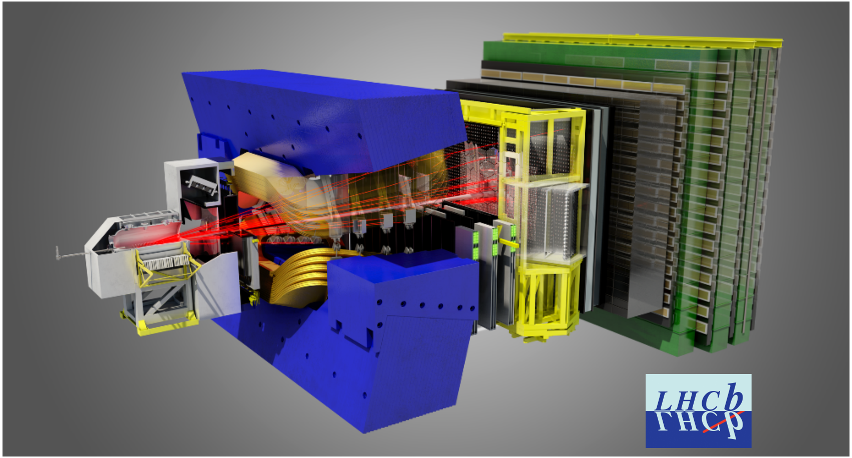
$$e^+e^- \to \Upsilon(4S) \to B\,\bar{B}$$



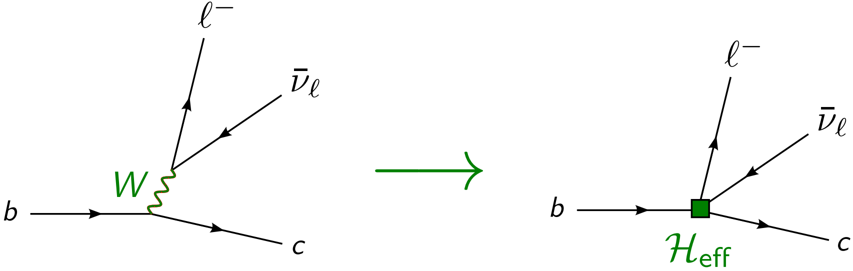


2019 $\sim$ 2035

The Large Hadron Collider produces a factor $\sim 10^3$ more b-hadrons, including b-baryons, but the backgrounds are higher, the initial longitudinal momentum of the b-hadron is unknown, and the production rates are not known with high precision.



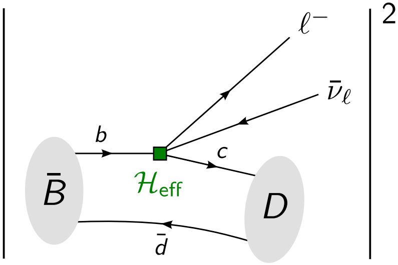
Low-energy effective Hamiltonian (density) for $b \to c\ell^-\bar{\nu}$



$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} \underbrace{\bar{c}\gamma^\mu(1-\gamma_5)b}_{\equiv J^\mu} \bar{\ell}\gamma_\mu(1-\gamma_5)\nu$$

Exclusive $\bar{B} \to D\ell\bar{\nu}$ decay rate

decay rate $\propto$



$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} \propto |V_{cb}|^2 \left| (\ldots)_\mu \underbrace{\langle D | J^\mu | \bar{B} \rangle}_{\text{lattice QCD}} \right|^2$$

Exclusive $\bar{B} \to D\ell\bar{\nu}$ decay rate

In detail,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2|V_{cb}|^2}{24\pi^3 m_B^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2 |\mathbf{p}_D|\left[\left(1+\frac{m_\ell^2}{2q^2}\right)m_B^2\mathbf{p}_D^2|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_B^2-m_D^2)^2|f_0(q^2)|^2\right]$$

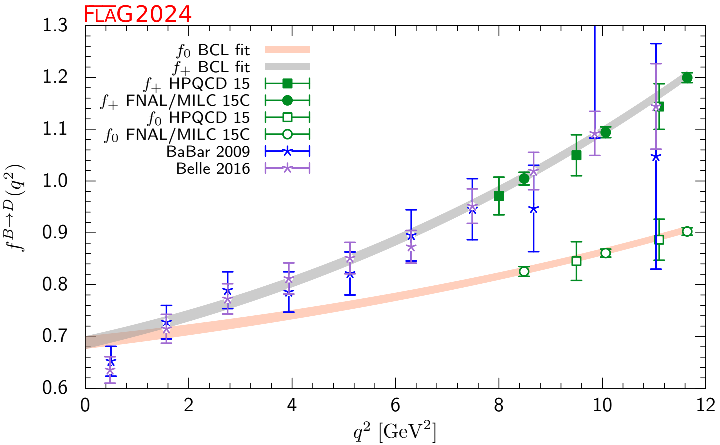
$$\begin{aligned}
\langle D(p')|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle &= \left[(p+p')^\mu - \frac{m_B^2-m_D^2}{q^2}q^\mu\right] f_+(q^2) + \frac{m_B^2-m_D^2}{q^2}q^\mu f_0(q^2),\\
\langle D(p')|\bar{c}\gamma^\mu\gamma_5 b|\bar{B}(p)\rangle &= 0,
\end{aligned}$$

$$q = p - p'.$$

For electrons and muons, the contribution from $f_0(q^2)$ is negligible, and the experimental data for $\mathrm{d}\Gamma/\mathrm{d}q^2$ can be recast into data for $|V_{cb}||f_+(q^2)|$

$|V_{cb}|$ from simultaneous fit to $\bar{B} \to D\ell\bar{\nu}$ experiment + lattice QCD

HPQCD 15: NRQCD b quarks, HISQ c quarks, 2+1 flavor AsqTad ensembles, $a \approx 0.12, 0.09$ fm
FNAL/MILC 15C: Fermilab b, c quarks, 2+1 flavor AsqTad ensembles, $a \approx 0.12, 0.09, 0.06, 0.045$ fm



$\to |V_{cb}| = 40.0(1.0) \times 10^{-3}$

$\bar{B} \to D^*$ form factors

Here, both the vector and axial-vector currents contribute:

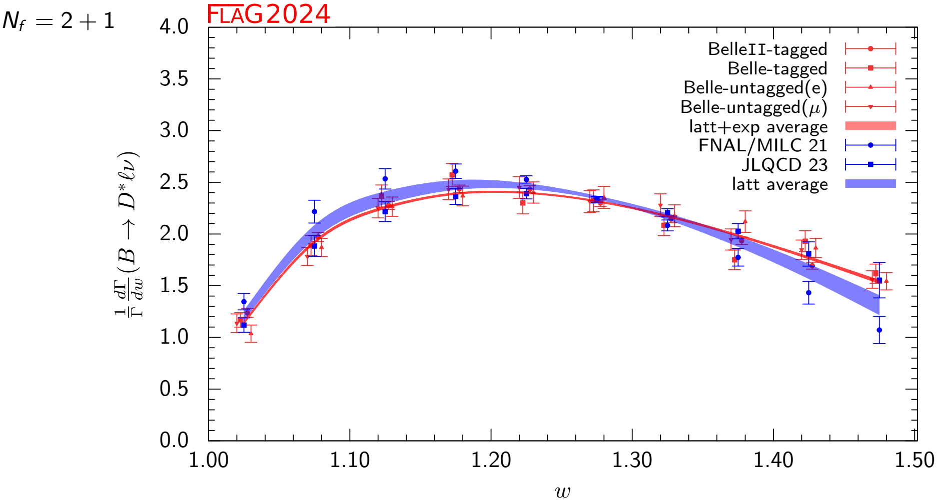
$$\langle D^* | \bar{c}\gamma^\mu b | \bar{B}\rangle \to 1 \text{ form factor},$$
$$\langle D^* | \bar{c}\gamma^\mu \gamma_5 b | \bar{B}\rangle \to 3 \text{ form factors}$$

Published lattice calculations:

	Fermilab/MILC 2105.14019/EPJC 2022	HPQCD 2304.03137/PRD 2023	JLQCD 2306.05657
$u, d, s, (c)$-quark action	AsqTad (2+1)	HISQ (2+1+1)	domain wall (2+1)
b-quark action	Fermilab clover	HISQ	domain wall
B-meson mass	$m_{\text{kin}} \approx m_{\text{phys}}$	$m \lesssim 0.93\, m_{\text{phys}}$	$m \lesssim 0.74\, m_{\text{phys}}$
m_π (MeV)	180 - 560*	135 - 329*	230 - 500
a (fm)	0.045 - 0.15	0.044 - 0.090	0.044 - 0.080
#(source-sink separations)	2 (T, $T+1$)	3	4

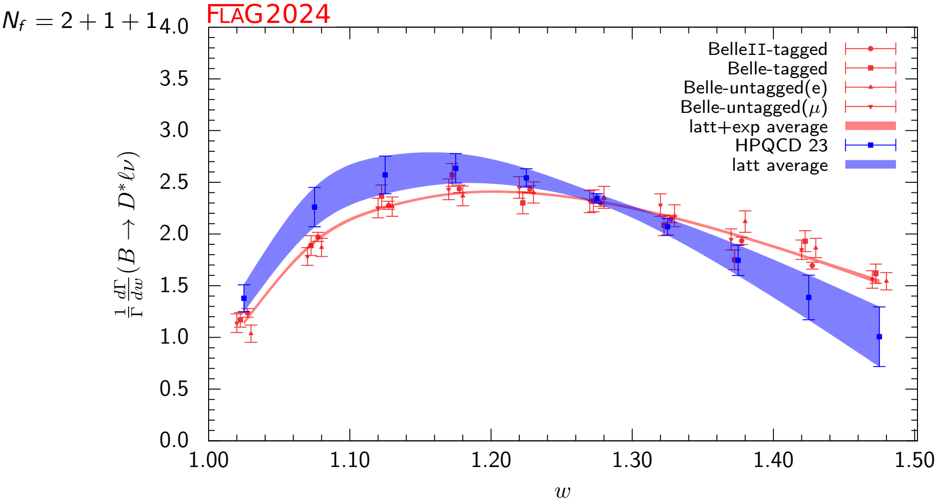
* These are the masses of the lightest pion (taste γ_5)

$|V_{cb}|$ from simultaneous fit to $\bar{B} \to D^* \ell \bar{\nu}$ experiment + lattice QCD



$w = v \cdot v'$. The fit includes additional angular observables (not shown). $\to |V_{cb}| = 39.23(65) \times 10^{-3}$

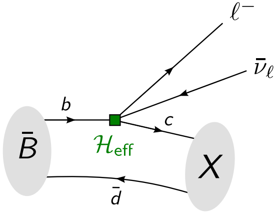
$|V_{cb}|$ from simultaneous fit to $\bar{B} \to D^* \ell \bar{\nu}$ experiment + lattice QCD



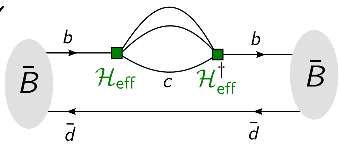
$w = v \cdot v'$. The fit includes additional angular observables (not shown). $\to |V_{cb}| = 39.44(89) \times 10^{-3}$

$|V_{cb}|$ from inclusive $\bar{B} \to X_c \ell \bar{\nu}$

decay rate $\propto \sum_X \Big|$



$\Big|^2 \propto \text{Im}\Big($



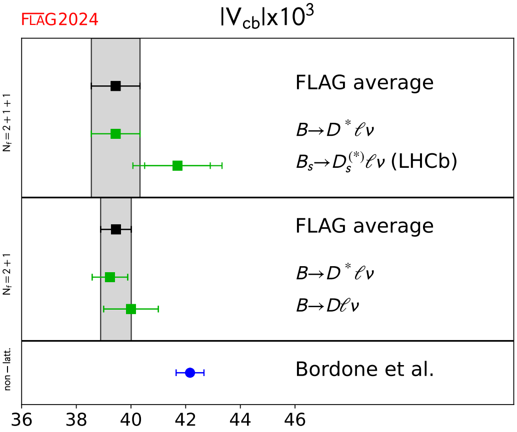
$\Big)$

$$\frac{d\Gamma}{dq^2 dE_\ell} \propto |V_{cb}|^2 (\ldots)_{\mu\nu} \ \text{Im} \underbrace{\left(-i \int d^4x \, e^{-iq\cdot x} \langle B| \, \mathbf{T} \, J^{\mu\dagger}(x) \, J^{\nu}(0) \, |B\rangle \right)}_{\text{OPE, HQET}}$$

For example, M. Bordone *et al.*: 2107.00604/PLB 2021 $\to |V_{cb}| = 42.16(51) \times 10^{-3}$

The inclusive decay rate can also be calculated on the lattice! [For example, A. Barone *et al.*, 2604.26381]

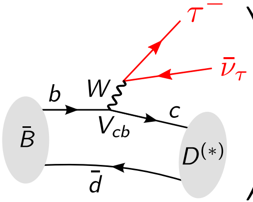
$|V_{cb}|$ summary

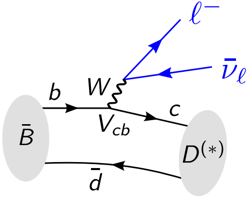


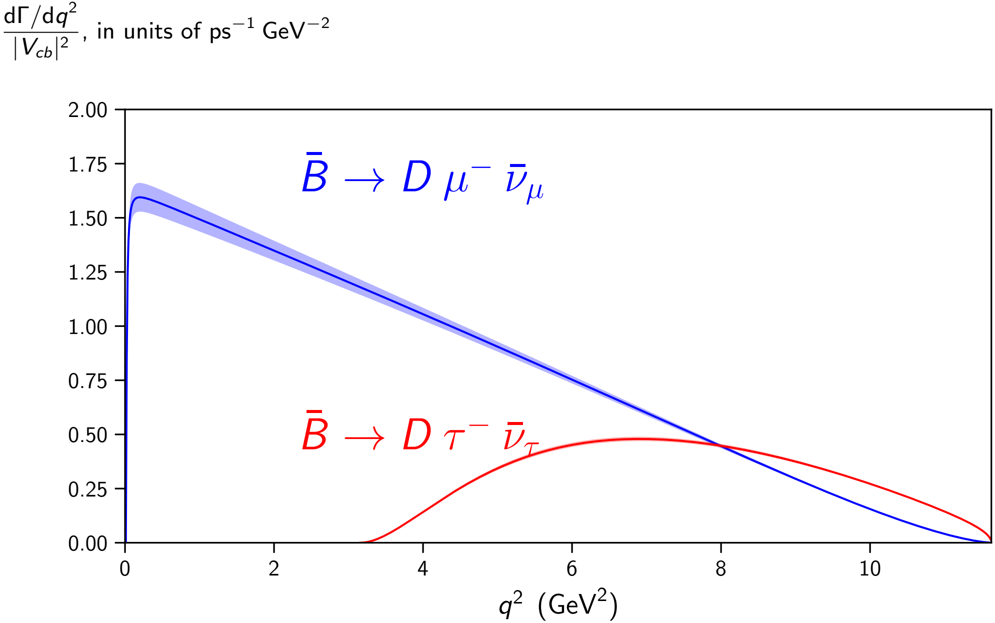
A significant discrepancy between exclusive and inclusive determinations that is long-standing and remains unexplained.

We will update the fits with **Belle II** data for the 2027 FLAG review.

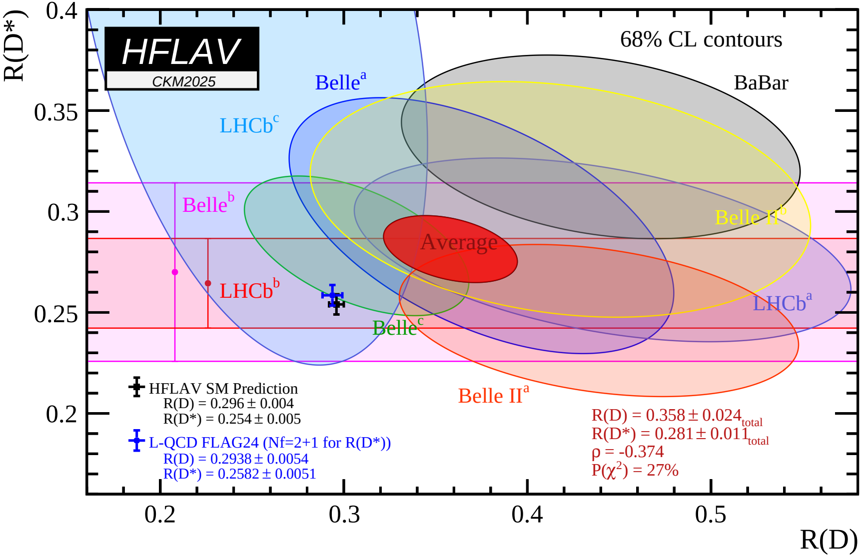
$R(D^{(*)})_{\mathrm{SM}} =$

$\Gamma\Big($  $\Big)$

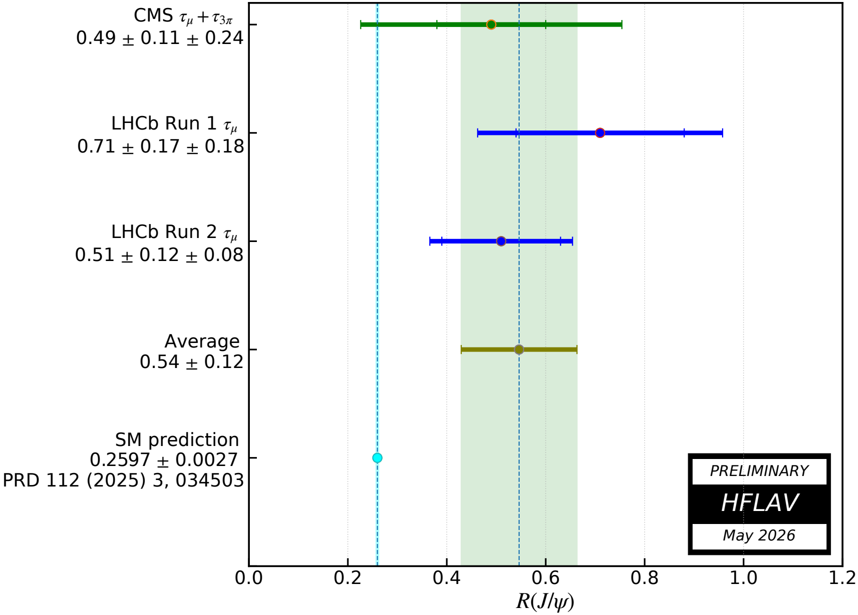
$\Gamma\Big($  $\Big)$



$\frac{d\Gamma/dq^2}{|V_{cb}|^2}$, in units of ps^{-1} GeV^{-2}
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
2
4
6
8
10
q^2 (GeV^2)
$\bar{B} \to D\,\mu^-\,\bar{\nu}_\mu$
$\bar{B} \to D\,\tau^-\,\bar{\nu}_\tau$



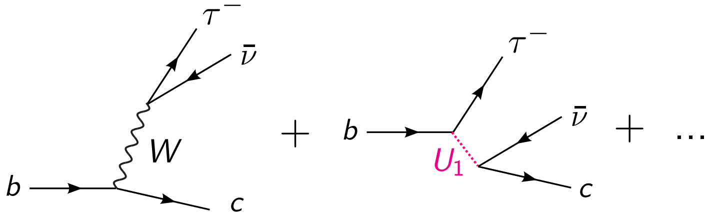
HFLAV
CKM2025
68% CL contours
R(D*)
R(D)
0.4
0.35
0.3
0.25
0.2
0.2
0.3
0.4
0.5
BaBar
Belle[a]
LHCb[c]
Belle[b]
LHCb[b]
Belle[c]
Average
Belle II[b]
LHCb[a]
Belle II[a]
HFLAV SM Prediction
R(D) = 0.296 ± 0.004
R(D*) = 0.254 ± 0.005
L-QCD FLAG24 (Nf=2+1 for R(D*))
R(D) = 0.2938 ± 0.0054
R(D*) = 0.2582 ± 0.0051
R(D) = 0.358 ± 0.024total
R(D*) = 0.281 ± 0.011total
ρ = -0.374
P(χ2) = 27%



CMS $\tau_\mu + \tau_{3\pi}$
0.49 ± 0.11 ± 0.24
LHCb Run 1 τ_μ
0.71 ± 0.17 ± 0.18
LHCb Run 2 τ_μ
0.51 ± 0.12 ± 0.08
Average
0.54 ± 0.12
SM prediction
0.2597 ± 0.0027
PRD 112 (2025) 3, 034503
PRELIMINARY
HFLAV
May 2026
0.0
0.2
0.4
0.6
0.8
1.0
1.2
$R(J/\psi)$

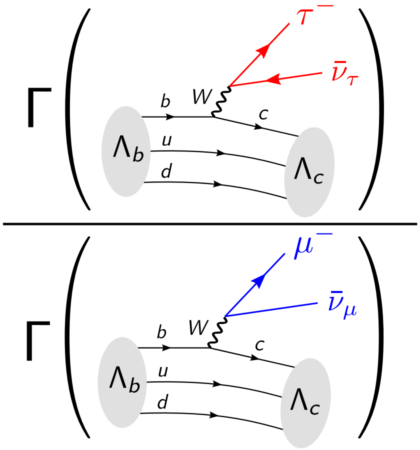
Violation of lepton flavor universality, if confirmed with higher significance by future experimental data, would be a clear sign of physics beyond the Standard Model. It could be caused by the quantum interference of very heavy elementary particles that have not yet been observed directly, for example a leptoquark as as shown below:

$$\text{Amplitude}(b \to c\tau^-\bar{\nu}) =$$



We can learn more by looking at the same quark transition inside baryons!

$$R(\Lambda_c)_{\text{SM}} =$$

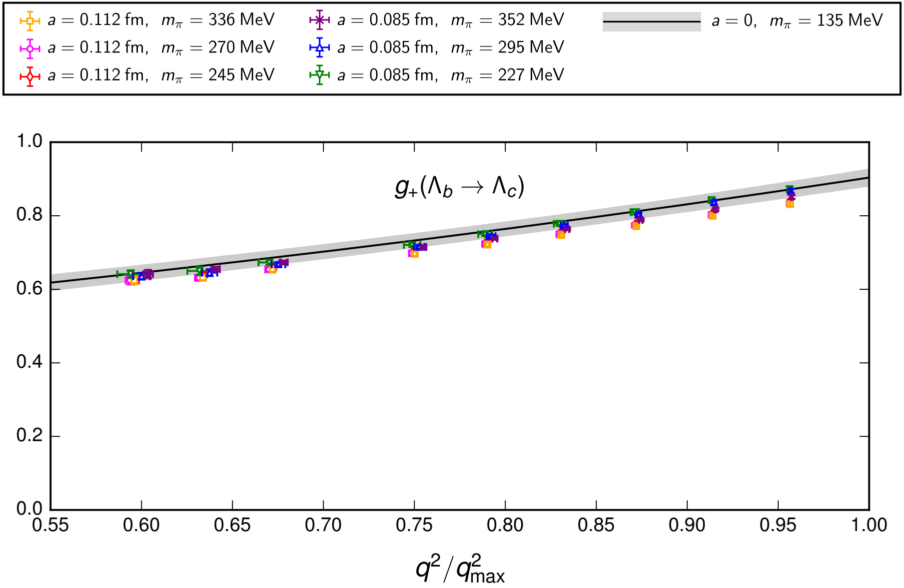


Vector and axial vector form factors:

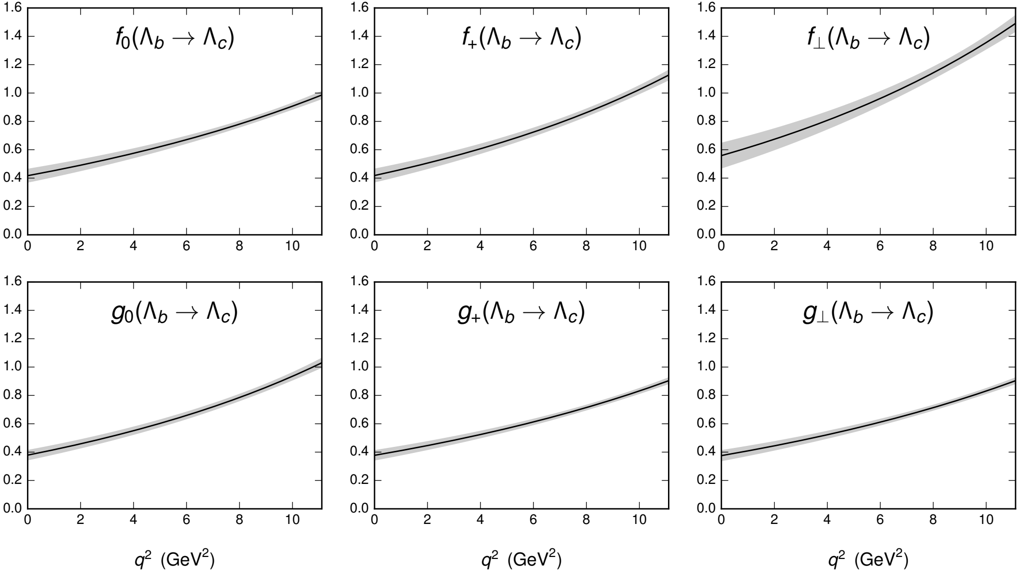
$$\begin{aligned}
\langle \Lambda_c | \bar{c}\gamma^\mu b | \Lambda_b \rangle &= \bar{u}_{\Lambda_c} \Bigg[(m_{\Lambda_b} - m_{\Lambda_c}) \frac{q^\mu}{q^2} f_0(q^2) \\
&\quad + \frac{m_{\Lambda_b} + m_{\Lambda_c}}{s_+} \left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_{\Lambda_c}^2) \frac{q^\mu}{q^2} \right) f_+(q^2) \\
&\quad + \left(\gamma^\mu - \frac{2m_{\Lambda_c}}{s_+} p^\mu - \frac{2m_{\Lambda_b}}{s_+} p'^\mu \right) f_\perp(q^2) \Bigg] u_{\Lambda_b},
\end{aligned}$$

$$\begin{aligned}
\langle \Lambda_c | \bar{c}\gamma^\mu \gamma_5 b | \Lambda_b \rangle &= -\bar{u}_{\Lambda_c} \gamma_5 \Bigg[(m_{\Lambda_b} + m_{\Lambda_c}) \frac{q^\mu}{q^2} g_0(q^2) \\
&\quad + \frac{m_{\Lambda_b} - m_{\Lambda_c}}{s_-} \left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_{\Lambda_c}^2) \frac{q^\mu}{q^2} \right) g_+(q^2) \\
&\quad + \left(\gamma^\mu + \frac{2m_{\Lambda_c}}{s_-} p^\mu - \frac{2m_{\Lambda_b}}{s_-} p'^\mu \right) g_\perp(q^2) \Bigg] u_{\Lambda_b},
\end{aligned}$$

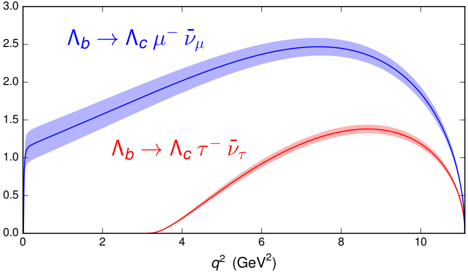
where $s_\pm = (m_{\Lambda_b} \pm m_{\Lambda_c})^2 - q^2$



[W. Detmold, C. Lehner, S. Meinel, 1503.01421/PRD 2015]

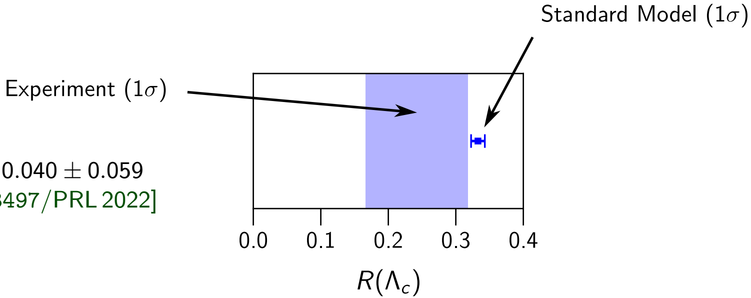


[W. Detmold, C. Lehner, S. Meinel, 1503.01421/PRD 2015]



$$R(\Lambda_c)_{\rm SM} = 0.3328 \pm 0.0074_{\rm stat} \pm 0.0070_{\rm syst}$$

[W. Detmold, C. Lehner, S. Meinel, 1503.01421/PRD 2015]

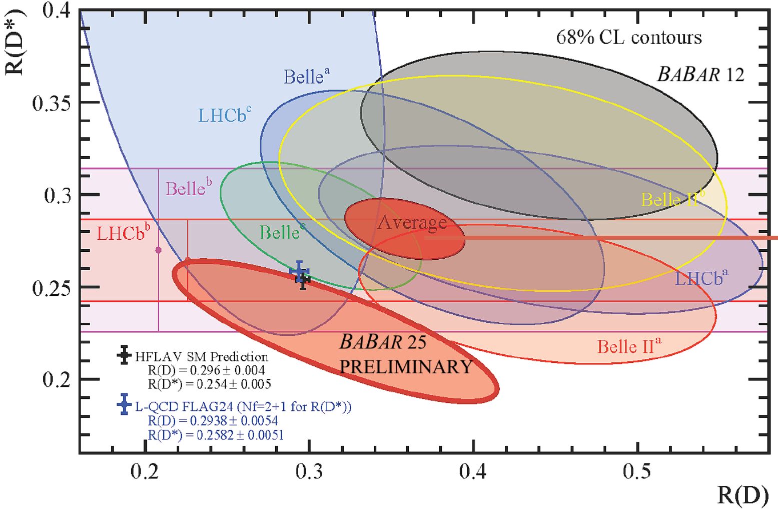


$$R(\Lambda_c)_{\rm expt.} = 0.242 \pm 0.026 \pm 0.040 \pm 0.059$$

[LHCb Collaboration, 2201.03497/PRL 2022]

No excess seen here, which disfavors new-physics explanations of the excesses seen in mesonic decays.

Preliminary new BaBar result first shown at "Violation of Fundamental Symmetries with *B* Mesons" workshop at Fermilab:

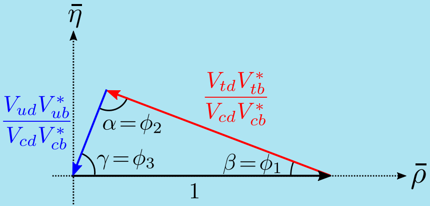


This average does not include the *BABAR* 25 preliminary result

The remaining two Wolfenstein parameters are ρ and η, or, to ensure exact unitarity, $\bar{\rho}$ and $\bar{\eta}$:

$$V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]} A\lambda^3(\bar{\rho} + i\bar{\eta}).$$

Also note that $\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$, and the orthogonality of the first and third columns of the CKM matrix, $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, can be represented as a triangle in the complex plane with apex $\bar{\rho} + i\bar{\eta}$:



The magnitude $|V_{ub}| = A\lambda^3\sqrt{\rho^2 + \eta^2}$ can be determined from b-hadron semileptonic decays.

The most important processes currently used to determine $|V_{ub}|$ are

- Exclusive $\bar{B} \to \pi\ell^-\bar{\nu}$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments)
- Exclusive $B^+ \to \tau^+\nu$ (BaBar, Belle, Belle II, and older experiments)
- Inclusive $\bar{B} \to X_u\ell^-\bar{\nu}$ ($\ell = e, \mu$; BaBar, Belle, Belle II, and older experiments)

The ratio $|V_{ub}|/|V_{cb}|$ is also determined from

- Exclusive $B_s \to K\mu\nu/B_s \to D_s\mu\nu$ (LCHb)
- Exclusive $\Lambda_b \to p\mu\bar{\nu}/\Lambda_b \to \Lambda_c\mu\bar{\nu}$ (LCHb)

Why only $B \to \tau\nu$, not $B \to \mu\nu$ or $B \to e\nu$?

Answer:

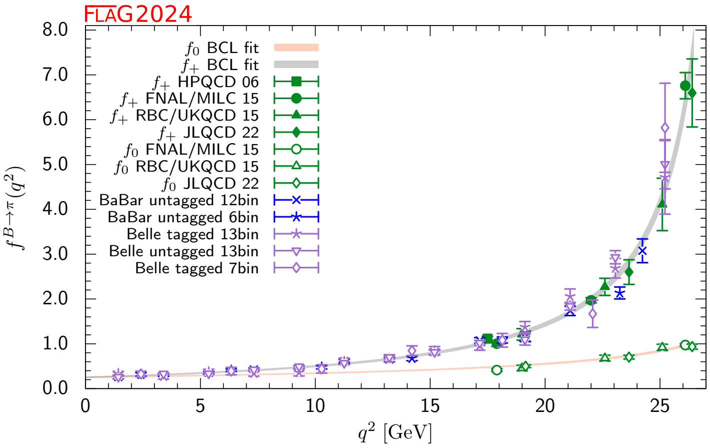
$$\Gamma(B \to \ell\nu) = \frac{m_B}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 \propto m_\ell^2$$

(helicity suppression).

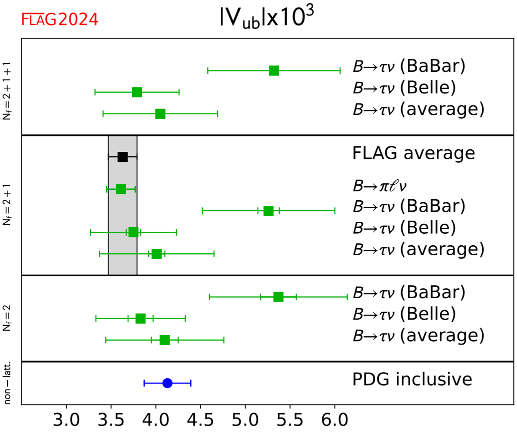
The decay constant f_B is known from lattice QCD to sub-percent precision, and the uncertainty of $|V_{ub}|$ from $B \to \tau\nu$ is dominated by experiment.

$|V_{ub}|$ from simultaneous lattice + experiment fit to $\bar{B} \to \pi \ell \bar{\nu}$

HPQCD 06	2+1 Asqtad, NRQCD b (not included in fit)
FNAL/MILC 15	2+1 Asqtad, Fermilab b
RBC/UKQCD 15	2+1 DWF, RHQ b
JLQCD 22	2+1 DWF, DWF b



$|V_{ub}|$ summary

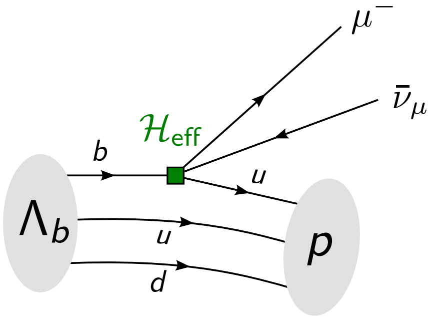


There used to be a more significant exclusive-inclusive discrepancy here, too, but it has weakened.

We will update the results with **Belle II** data for the 2027 FLAG review.

$|V_{ub}|/|V_{cb}|$ from LHCb

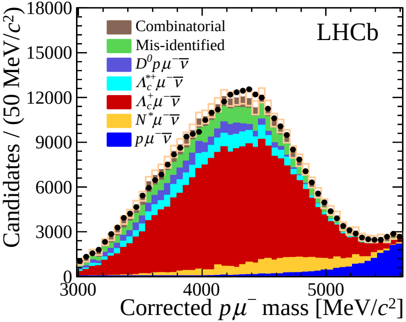
At LHCb, the $p\mu\bar{\nu}$ final state is easier to identify than $\pi\mu\bar{\nu}$



$|V_{ub}|/|V_{cb}|$ from LHCb

In 2015, LHCb published the measurement

$$\frac{\int_{15\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{d\Gamma(\Lambda_b \to p\,\mu^-\bar{\nu}_\mu)}{dq^2} dq^2}{\int_{7\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{d\Gamma(\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu)}{dq^2} dq^2} = (1.00 \pm 0.04 \pm 0.08) \times 10^{-2}$$



[LHCb Collaboration, 1504.01568/Nat. Phys. 2015]

$|V_{ub}|/|V_{cb}|$ from LHCb

While LHCb was working on this measurement, we were working on a lattice calculation of the relevant form factors. Using these form factors, we predicted

$$\frac{|V_{cb}|^2}{|V_{ub}|^2} \frac{\int_{15\ \mathrm{GeV}^2}^{q^2_{\max}} \frac{\mathrm{d}\Gamma(\Lambda_b \to p\,\mu^-\bar{\nu}_\mu)}{\mathrm{d}q^2}\mathrm{d}q^2}{\int_{7\ \mathrm{GeV}^2}^{q^2_{\max}} \frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c\,\mu^-\bar{\nu}_\mu)}{\mathrm{d}q^2}\mathrm{d}q^2} \quad = \quad 1.471 \pm 0.095_{\,\mathrm{stat.}} \pm 0.109_{\,\mathrm{syst.}}$$

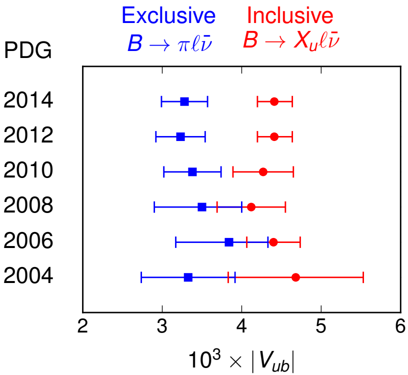
[W. Detmold, C. Lehner, S. Meinel, 1503.01421/PRD 2015]

Combined with the LHCb measurement, this gave

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004_{\,\mathrm{expt}} \pm 0.004_{\,\mathrm{lat}},$$

the first determination of this quantity at the Large Hadron Collider, and the first from baryons.

At that time, there was a fairly significant exclusive-inclusive tension,



and a possible new-physics explanation was a nonzero right-handed coupling,

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub}^L \left[(1+\epsilon_R)\bar{u}\gamma^\mu b - (1-\epsilon_R)\,\bar{u}\gamma^\mu\gamma_5 b\right]\,\bar{\ell}\gamma_\mu(1-\gamma_5)\nu$$

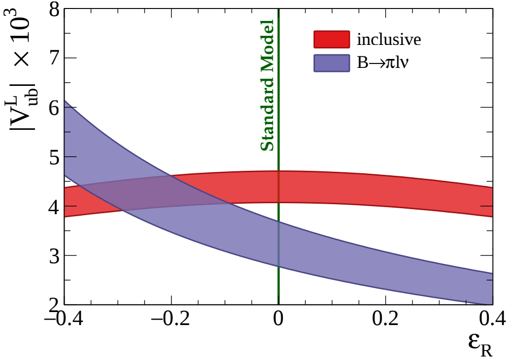
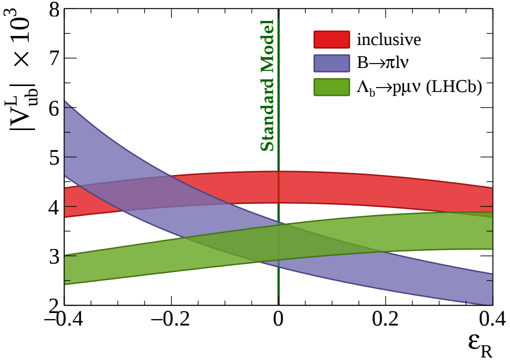


Figure (modified) from arXiv:1504.01568

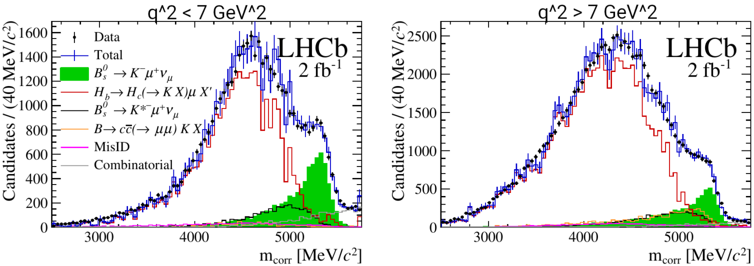
The baryon decay is sensitive to both V and A currents, and the measurement provides a complementary constraint that disfavors the right-handed coupling explanation.



(using $|V_{cb}|_{\text{excl.}}$ from 2014 PDG)

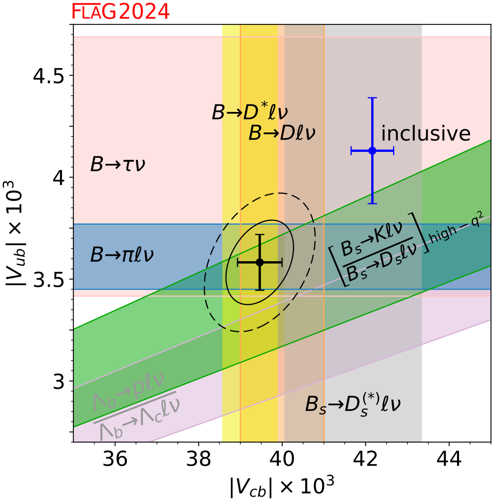
$|V_{ub}|/|V_{cb}|$ from LHCb

In 2020, LHCb published another determination of $|V_{ub}|/|V_{cb}|$ using $B_s \to K\mu\nu/B_s \to D_s\mu\nu$.

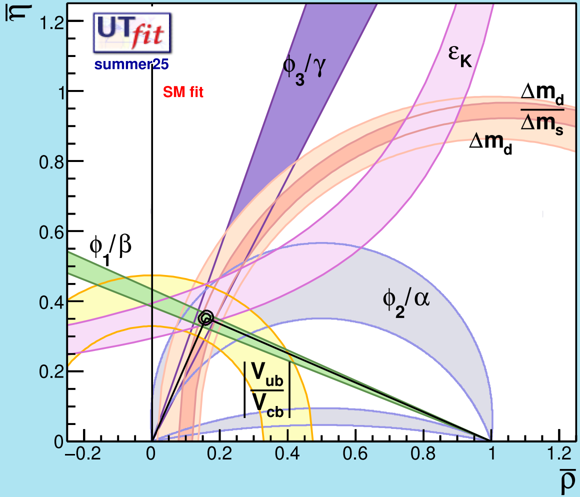


[2012.05143/PRL 2021]

FLAG combination of $|V_{ub}|$ and $|V_{cb}|$



Other constraints on the Wolfenstein parameters $\bar{\rho}$, $\bar{\eta}$



- α from *CP* violation in e.g. $B^0(\overline{B^0}) \to \pi\pi, \pi\rho, \rho\rho$
- β from *CP* violation in e.g. $B^0(\overline{B^0}) \to J/\psi K_S$
- γ from *CP* violation in e.g. $B^- \to D^0(\overline{D^0})(\to f)K^-$
- Δm_d, $\dfrac{\Delta m_d}{\Delta m_s}$: $B^0/\overline{B^0}$, $B_s^0/\overline{B_s^0}$ mixing mass differences – uses hadronic matrix elements from lattice QCD
- ϵ_K: indirect *CP* violation in the neutral kaon system – uses hadronic matrix elements from lattice QCD
- ϵ'_K (not shown): direct *CP* violation in the neutral kaon system – uses hadronic matrix elements from lattice QCD

NB: much of the uncertainty in ϵ_K and Δm_d comes from $|V_{cb}|$.

The 2025 Standard-Model global fit of Wolfenstein parameters by UTfit gives

$$\begin{aligned}\lambda &= 0.2251(6),\\ A &= 0.826(9),\\ \bar{\rho} &= 0.160(9),\\ \bar{\eta} &= 0.350(11),\end{aligned}$$

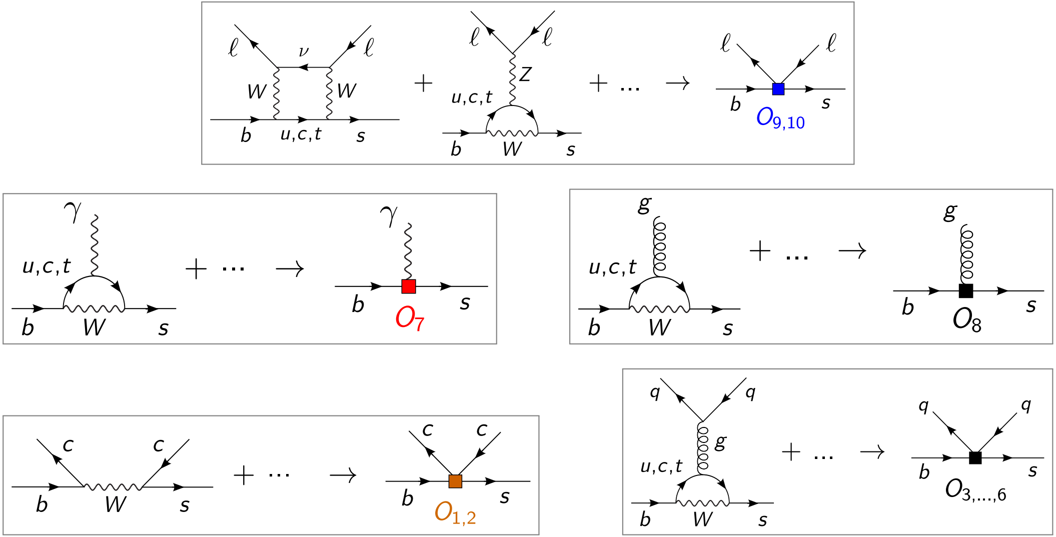
which corresponds to

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97433(14) & 0.22507(62) & 0.003736(78)\, e^{-i(65.5(1.1))^o} \\ -0.22492(62)\, e^{+i(0.03537(74))^o} & 0.97347(14)\, e^{-i(0.001888(38))^o} & 0.04206(40) \\ 0.008590(90)\, e^{-i(22.66(53))^o} & -0.04117(36)\, e^{+i(1.064(25))^o} & 0.999115(15) \end{pmatrix}.$$

[UTfit Collaboration, http://www.utfit.org/UTfit/ResultsSummer2025SM]

Rare $b \to s\ell^+\ell^-$ decays

Weak effective Hamiltonian for $b \to s\ell^+\ell^-$ decays



[B. Grinstein, M. J. Savage, M. B. Wise, Nucl. Phys. B 319, 271 (1989)]

Weak effective Hamiltonian for $b \to s\ell^+\ell^-$ decays

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i$$

with

$$\begin{aligned}
O_1 &= \bar{c}^b \gamma^\mu b_L^a \;\; \bar{s}^a \gamma_\mu c_L^b, \\
O_2 &= \bar{c}^a \gamma^\mu b_L^a \;\; \bar{s}^b \gamma_\mu c_L^b, \\
O_7 &= \frac{e\, m_b}{16\pi^2} \;\bar{s}\sigma^{\mu\nu} b_R \;\; F_{\mu\nu}^{(\text{e.m.})}, \\
O_9 &= \frac{e^2}{16\pi^2} \;\bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \ell, \\
O_{10} &= \frac{e^2}{16\pi^2} \;\bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \gamma_5 \ell, \\
&\ldots
\end{aligned}$$

In the Standard Model, $\overline{\text{MS}}$ scheme, at $\mu = 4.2$ GeV,

C_1	C_2	C_7	C_9	C_{10}	...
−0.288	1.010	−0.336	4.275	−4.160	...

[Computed using EOS, `https://eos.github.io/`]

Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays

For a generic decay $H_b \to H_s \ell^+\ell^-$:

Contributions from O_7, O_9, O_{10}: $\langle H_s(p')| \bar{s}\Gamma b |H_b(p)\rangle$ $\to$ local form factors, can be calculated using lattice QCD.

Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays

Contributions from $O_{1,\dots,6}$, O_8 (most significant: O_1, O_2):

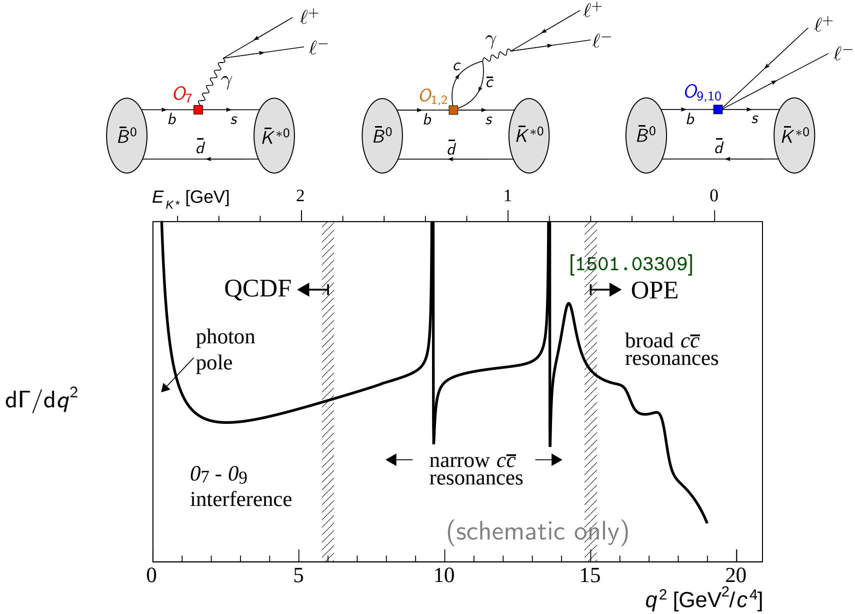
$$\int \mathrm{d}^4x\;\; e^{iq\cdot x}\, \langle H_s(p')|\, \mathrm{T}\left\{O_i(0)\; J^{\mu}_{\mathrm{e.m.}}(x)\right\} |H_b(p)\rangle$$

$\to$ nonlocal form factors. Very challenging for lattice QCD.

Typically treated approximately using continuum methods: local OPE at high q^2 and QCDF/light-cone OPE at low q^2:

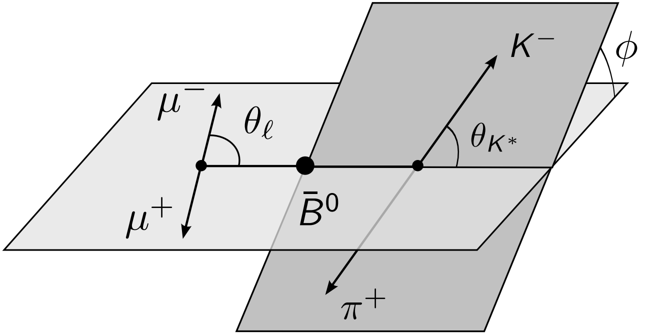
$$i \sum_{i=1,\dots,6,8} C_i \int d^4x\, e^{iq\cdot x}\, \mathrm{T}\left\{O_i(0)\; J^{\mu}_{\mathrm{e.m.}}(x)\right\}$$

$$= \frac{1}{16\pi^2}\left[\left(q_\mu q_\rho - q^2 g_{\mu\rho}\right) \Delta C_9(q^2)\, \bar{s}\gamma^\rho b_L + 2im_b\, q^\rho\, \Delta C_7(q^2)\, \bar{s}\sigma_{\mu\rho} b_R\right] + \text{higher powers}$$



$$q^2 = (p_{\ell^+} + p_{\ell^-})^2 = (p_B - p_{K^*})^2$$

$\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\mu^+\mu^-$ angular distribution



$\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\mu^+\mu^-$ angular distribution

In the narrow-width approximation,

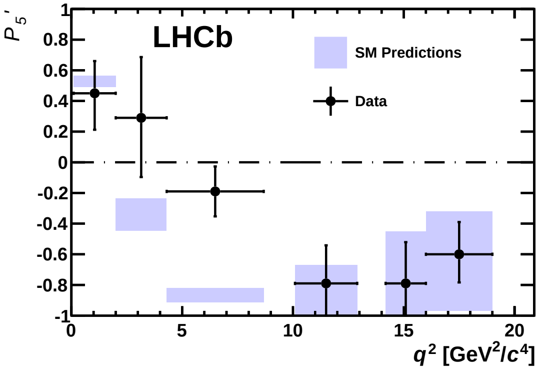
$$
\begin{aligned}
\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi} = \frac{9}{32\pi}\Big[& I_1^s \sin^2\theta_{K^*} + I_1^c \cos^2\theta_{K^*} \\
& + (I_2^s \sin^2\theta_{K^*} + I_2^c \cos^2\theta_{K^*}) \cos 2\theta_\ell \\
& + I_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi \\
& + I_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi \\
& + I_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi \\
& + (I_6^s \sin^2\theta_{K^*} + I_6^c \cos^2\theta_{K^*}) \cos\theta_\ell \\
& + I_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi \\
& + I_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi \\
& + I_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi \Big],
\end{aligned}
$$

where the functions $I_i^{(a)}$ depend only on q^2

[F. Krüger *et al.*, hep-ph/9907386/PRD 2000]

2013: The " P_5' anomaly" in $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\mu^+\mu^-$

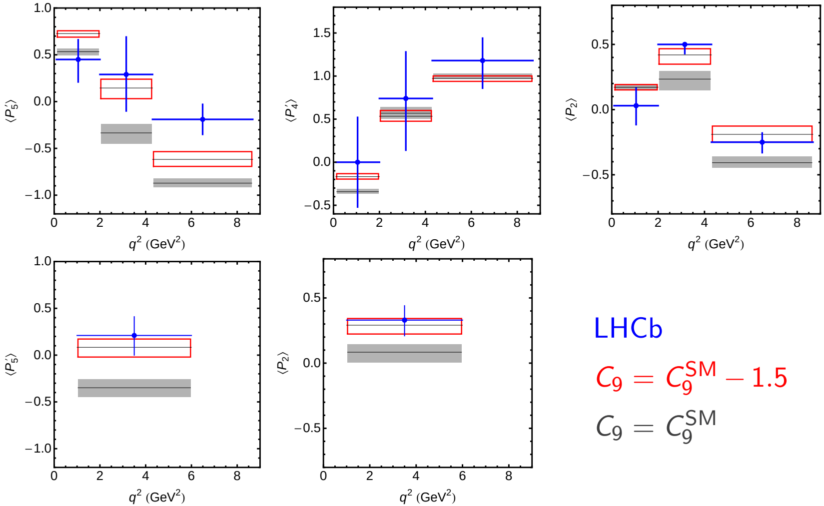
$$P_5' = \frac{I_5}{2\sqrt{-I_2^s \, I_2^c}}$$



[LHCb Collaboration, 1308.1707/PRL 2013]

- SM predictions: S. Descotes-Genon, T. Hurth, J. Matias, J. Virto, 1303.5794/JHEP 2013
- Hadronic matrix elements calculated using QCD factorization and light-cone sum-rules (good at low q^2, i.e., high K^* momentum)

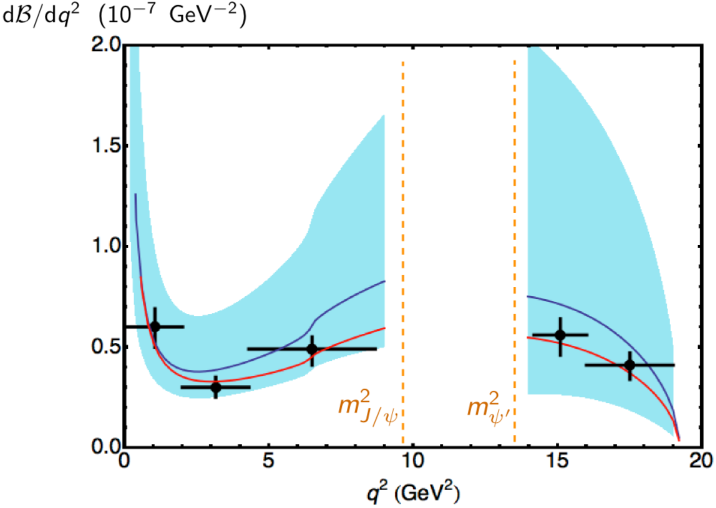
2013: A negative shift in the Wilson coefficient C_9?



[S. Descotes-Genon, J. Matias, J. Virto, 1307.5683/PRD 2013]

(NB: normalization of observables defined differently)

2013: $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\mu^+\mu^-$ differential branching fraction calculated using light-cone sum rules



$C_9 = C_9^{SM}$

$C_9 = C_9^{SM} - 1.5$

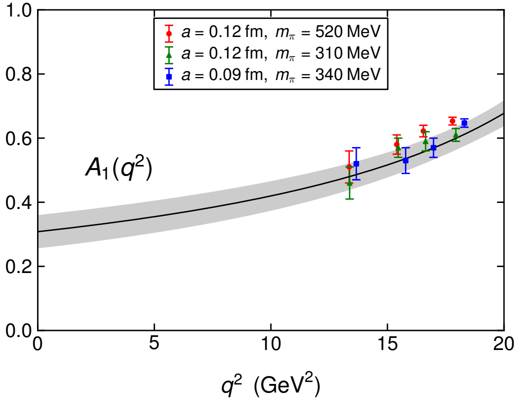
Form factor uncertainty from light cone sum rules

[S. Descotes-Genon, J. Matias, J. Virto, 1311.3876]

2013: $B \to K^*$ and $B_s \to \phi$ form factors from lattice QCD

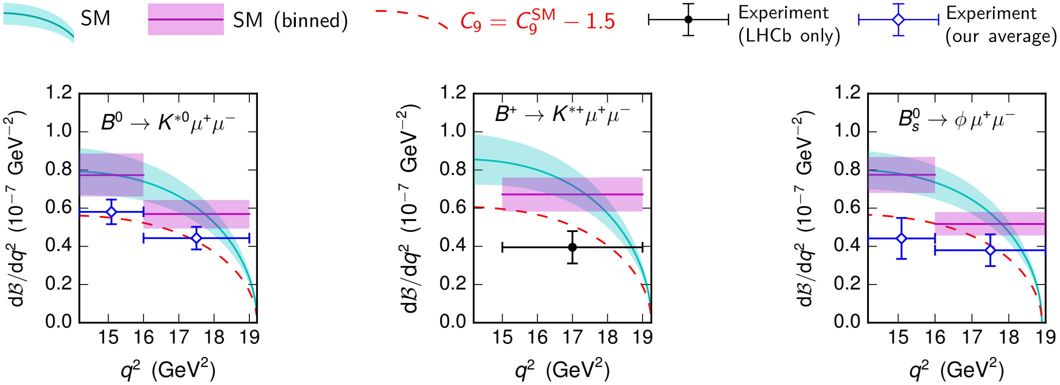
First complete lattice-QCD calculation (using single-hadron approach for K^* and ϕ), NRQCD b, MILC AsqTad 2+1 flavor
[R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, 1310.3722/PRD 2014]

Example (one of seven form factors):



2013: $B \to K^*\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ differential BF's calculated using lattice QCD

Contributions from $O_{1...6;8}$ treated with OPE ($\sim$ 10% effect)
[B. Grinstein and D. Pirjol, hep-ph/0404250/PRD 2004]



[R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, 1310.3887/PRL 2014]

2014: Ratio of $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$ branching fractions

$$R_K \equiv \frac{\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} \frac{\mathrm{d}\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathrm{d}q^2}\mathrm{d}q^2}{\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} \frac{\mathrm{d}\mathcal{B}(B^+ \to K^+e^+e^-)}{\mathrm{d}q^2}\mathrm{d}q^2},$$

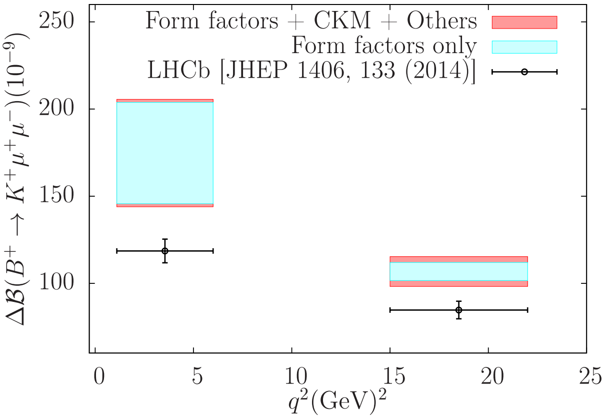
Standard Model:

$$R_K = 1 + \mathcal{O}(10^{-3})$$

2014 LHCb measurement [1406.6482/PRL 2014]:

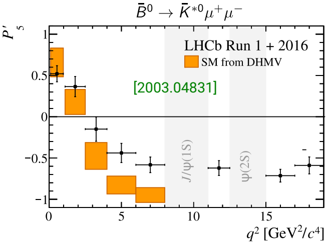
$$R_K = 0.745^{+0.090}_{-0.074}(\mathrm{stat}) \pm 0.036(\mathrm{syst})$$

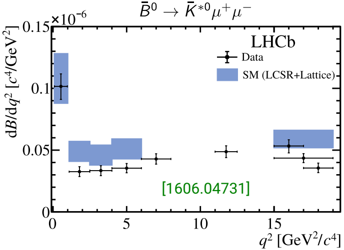
2015: $B^+ \to K^+\mu^+\mu^-$ differential BF calculated using form factors from lattice QCD

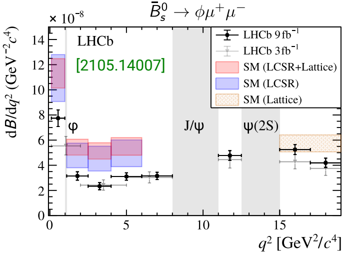


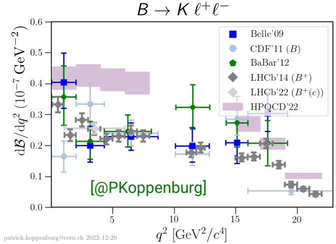
[D. Du *et al.* (Fermilab Lattice and MILC Collaborations), 1510.02349/PRD 2016; see also earlier work: C. Bouchard *et al.*, (HPQCD Collaboration), 1306.2384/PRD 2013]

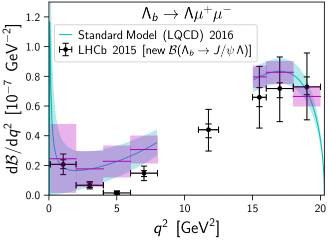
Latest results for P_5' and for differential branching fractions

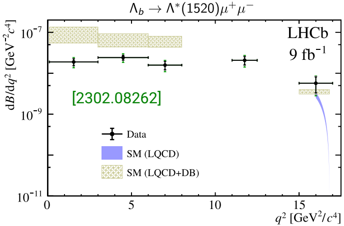








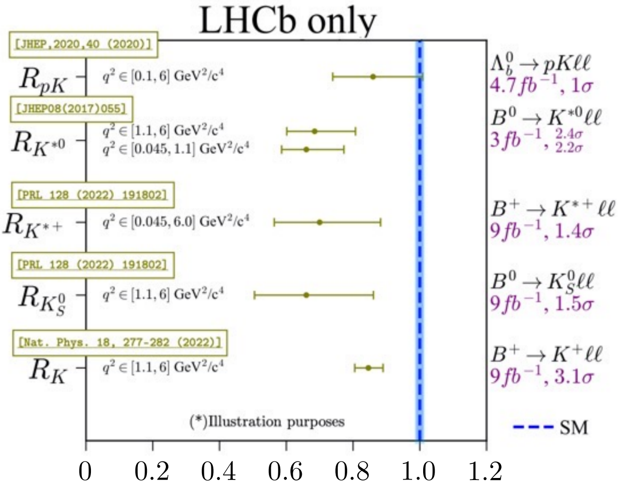




SM predictions using lattice-QCD form factors from:

R. Horgan, Z. Liu, S. Meinel, M. Wingate, 1310.3722/PRD 2014; W. Detmold, S. Meinel, 1602.01399 /PRD 2016; S. Meinel, G. Rendon, 2107.13140/PRD 2022; W. Parrott, C. Bouchard, C. Davies, 2207.12468 /PRD 2023

LFUV ratios like $R_K \equiv \frac{\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \frac{\mathrm{d}\mathcal{B}(B^+\to K^+\mu^+\mu^-)}{\mathrm{d}q^2}\mathrm{d}q^2}{\int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} \frac{\mathrm{d}\mathcal{B}(B^+\to K^+e^+e^-)}{\mathrm{d}q^2}\mathrm{d}q^2}$ as of November 2022



[P. de Simone, Talk at ALPS 2023]

At that time, a good description of all $b \to s\ell^+\ell^-$ observables ($\ell = e, \mu$) was possible using

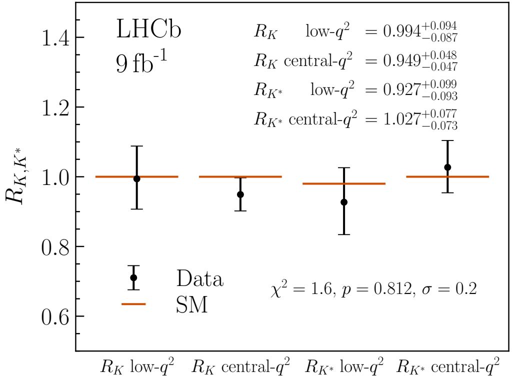
$$C_{9\mu} = C_{9\mu}^{\text{SM}} + C_{9\mu}^{\text{NP}}, \qquad C_{9e} = C_{9e}^{\text{SM}},$$

i.e., with new physics coupling to muons (and taus), but not to electrons, with $C_{9\mu}^{\text{NP}} \approx -1$. Global fits showed $\gtrsim 5\sigma$ significance
[For example, M. Algueró *et al.*, 2104.08921/EPJC 2022]

There were many viable new-physics models, including combined explanations of the $b \to s\ell^+\ell^-$ and $b \to c\tau^-\bar{\nu}$ anomalies
[For example, D. Buttazzo, A. Greljo, G. Isidori, D. Marzocca, 1706.07808/JHEP 2017]

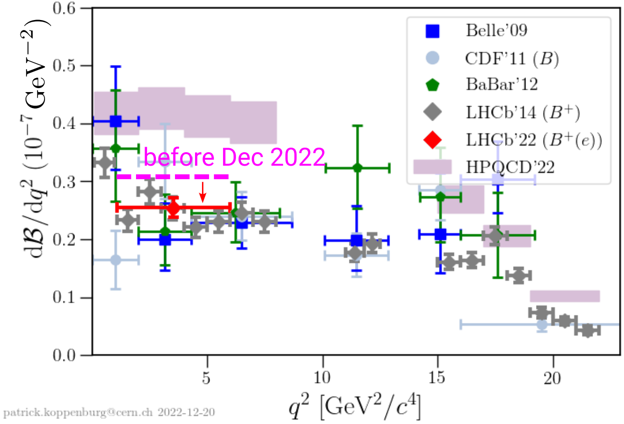
December 2022: a major change of the picture

It turned out that the LHCb results for the LFUV ratios had an error (hadrons misidentified as electrons). In December 2022, LHCb published a new analysis:

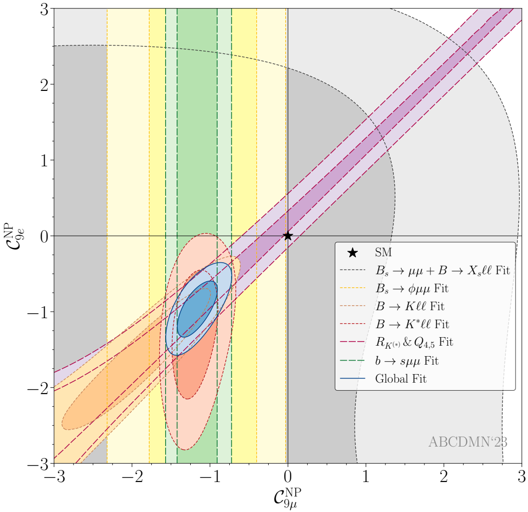


[LHCb Collaboration, 2212.09153/PRD 2023]

Note that only the $B \to K^{(*)}e^+e^-$ decay rate measurements have changed, and are now lower:



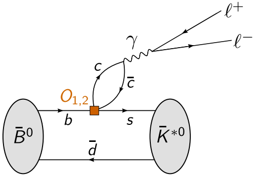
To get a good fit, one now must allow for new physics affecting **both electrons and muons**. For example, from M. Algueró *et al.*, 2304.07330/EPJC 2023:



This changes the direction for BSM model building, but there are still options. See, e.g., A. Greljo, J. Salko, A. Smolkovič, P. Stangl, 2212.10497/JHEP 2023

New physics or charm loops?

Recall, the contribution from the charm four-quark operators mimic a shift in C_9 (equal for electrons and muons):



$$i \sum_{i=1,\dots,6,8} C_i \int d^4x\, e^{iq\cdot x}\, \mathrm{T}\left\{O_i(0)\, J^{\mu}_{\text{e.m.}}(x)\right\}$$
$$= \frac{1}{16\pi^2}\left[\left(q_\mu q_\rho - q^2 g_{\mu\rho}\right) \Delta C_9(q^2)\, \bar{s}\gamma^\rho b_L + 2im_b\, q^\rho\, \Delta C_7(q^2)\, \bar{s}\sigma_{\mu\rho} b_R\right] + \text{higher powers}$$

This is already included in the Standard-Model calculations, but are the approximations used under control?

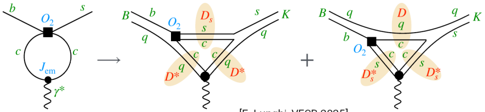
Recent progress on the charm loops

Not a complete list:

- Dispersive bounds with inputs from light-cone sum rules and $\mathcal{B}(B \to K^{(*)} J/\psi)$

  N. Gubernari, D. van Dyk, J. Virto, 2011.09813/JHEP 2021; N. Gubernari, M. Reboud, D. van Dyk, J. Virto 2206.03797/JHEP 2022

- HHχPT-inspired estimates of charm-meson rescattering

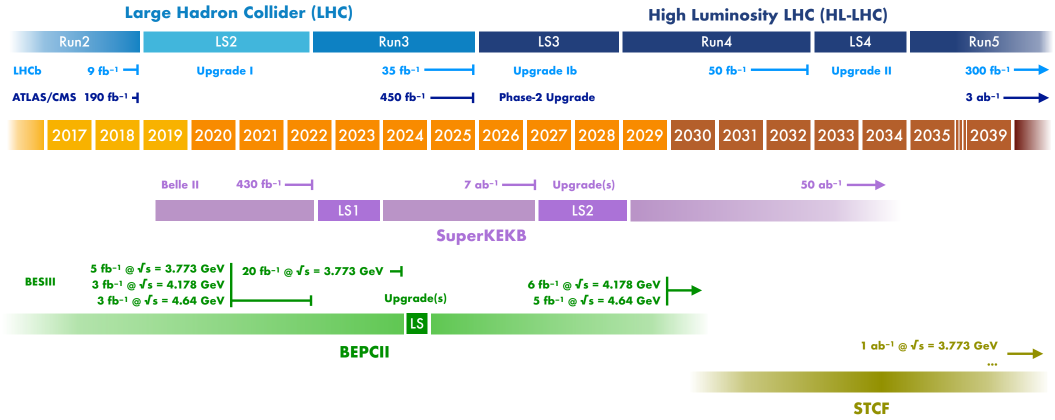


  G. Isidori, Z. Polonsky, A. Tinari, 2405.17551/PRD 2025; 2507.17824/EPJC 2025

- Framework for a lattice-QCD calculation using spectral reconstruction

  R. Frezotti *et al.*, 2508.03655/PRD 2026

The experimental prospects are exciting!



+ FCC!

Lots of work to do for the lattice-QCD community.

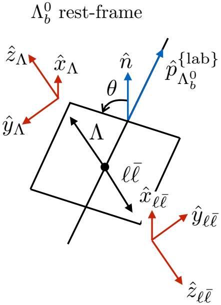
Extra slides

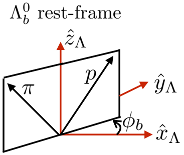
$\Lambda_b \to \Lambda(\to p^+\pi^-)\mu^+\mu^-$ decay angles

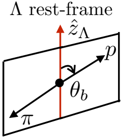
$\hat{n} = \hat{p}_{\Lambda_b} \times \hat{p}_{\text{beam}}$

$$\hat{z}_\Lambda = \hat{p}_\Lambda^{\{\Lambda_b^0\}}$$
$$\hat{y}_\Lambda = \hat{n} \times \hat{p}_\Lambda^{\Lambda_b^0}$$

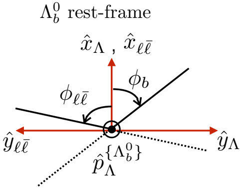
$$\hat{z}_{\ell\bar{\ell}} = \hat{p}_{\ell\bar{\ell}}^{\{\Lambda_b^0\}}$$
$$\hat{y}_{\ell\bar{\ell}} = \hat{n} \times \hat{p}_{\ell\bar{\ell}}^{\{\Lambda_b^0\}}$$







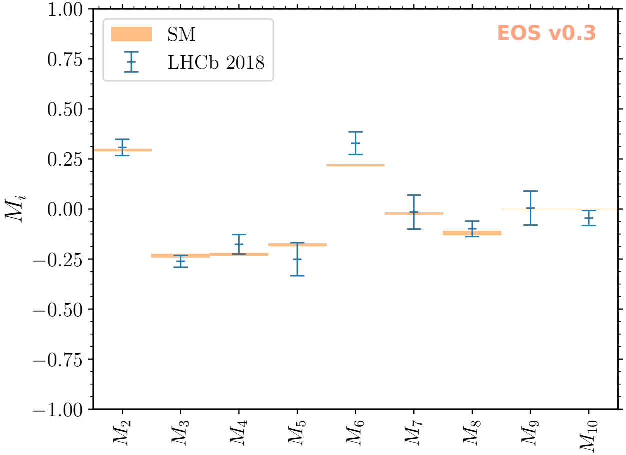
$$\hat{z}_\Lambda^{\{\Lambda\}} = -\hat{p}_{\ell\bar{\ell}}^{\{\Lambda\}}$$



[P. Böer, T. Feldmann, D. van Dyk, 1410.2115/JHEP 2015; T. Blake, M. Kreps, arXiv:1710.00746/JHEP 2017]

$\Lambda_b \to \Lambda(\to p^+\pi^-)\mu^+\mu^-$ angular observables

In the bin $15 < q^2 < 20\ \mathrm{GeV}^2$ [Experimental results: LHCb, arXiv:1808.00264/JHEP 2018]



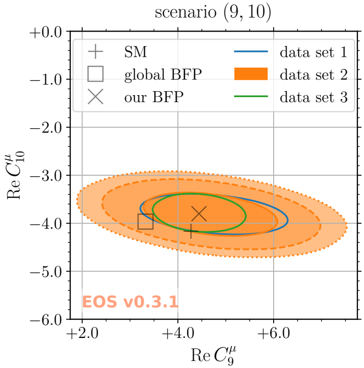
[T. Blake, S. Meinel, D. van Dyk, arXiv:1912.05811/PRD 2020]

Fitting Wilson coefficients to $\Lambda_b \to \Lambda(\to p^+\pi^-)\mu^+\mu^-$ data

A fit of C_9^μ and C_{10}^μ to

- The 2018 LHCb data for all $\Lambda_b \to \Lambda(\to p^+\pi^-)\mu^+\mu^-$ angular observables
- The world average of $\mathcal{B}(B_s \to \mu^+\mu^-)$ from LHCb, ATLAS, and CMS

gives the following ("data set 2"):



[T. Blake, S. Meinel, D. van Dyk, arXiv:1912.05811/PRD 2020]

The best-fit point is consistent with global fits to mesonic data, but also consistent with the SM, given current uncertainties.